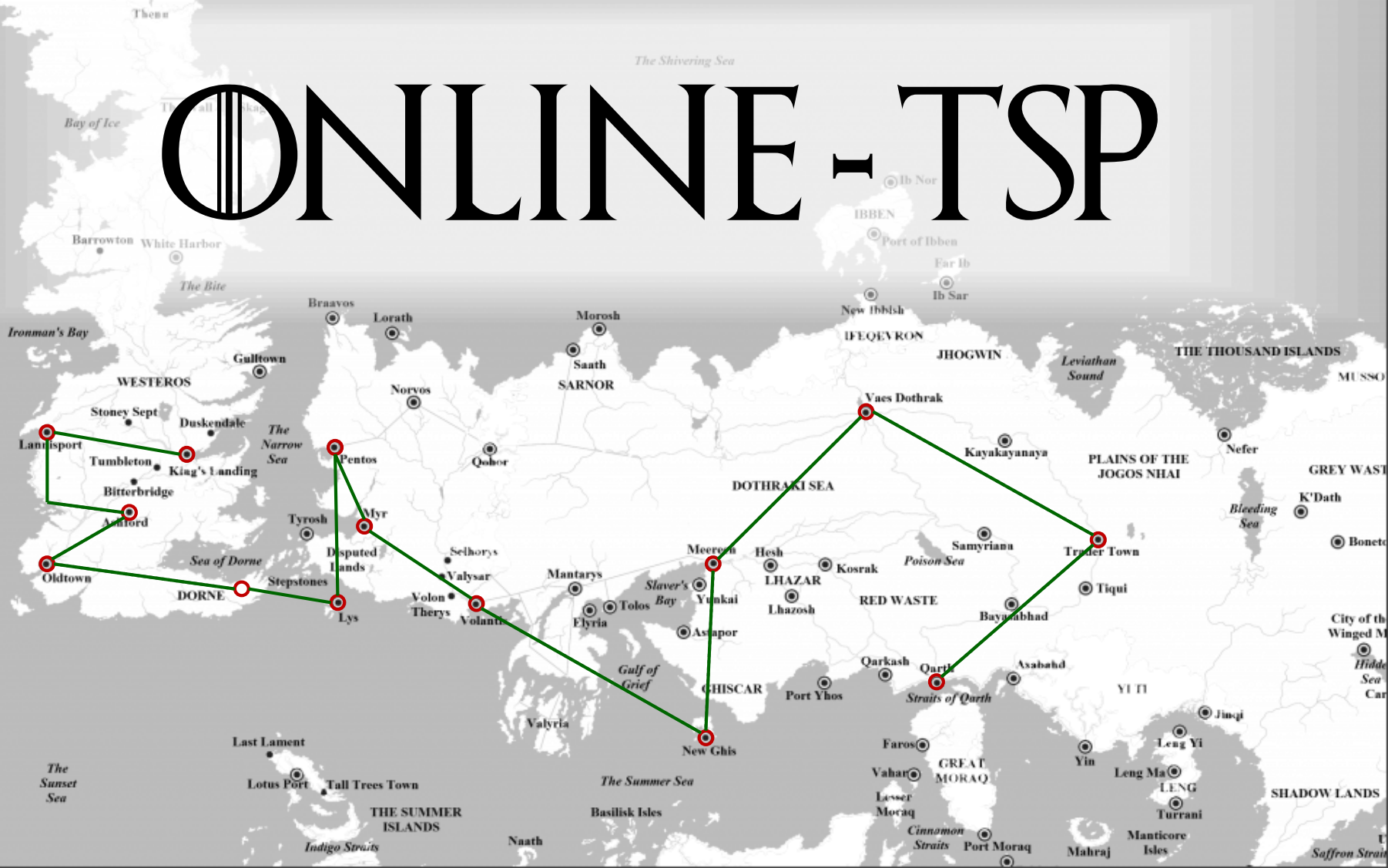


# ONLINE - TSP

# ONLINE - TSP



# Online-TSP

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# Online-TSP

---

(metric)

# Online-TSP

---

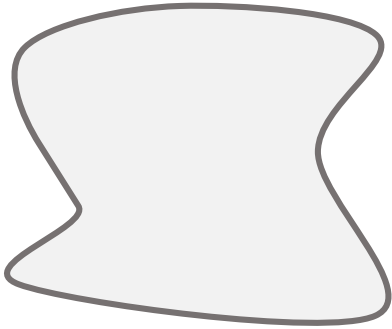
(metric)

INPUT:

# Online-TSP

(metric)

INPUT:

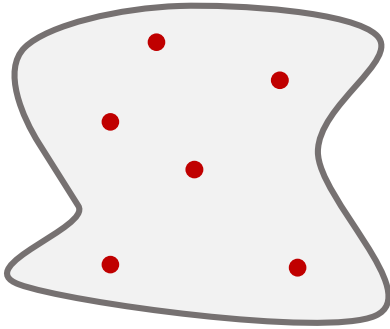


- metric space:  $M$   
(with metric  $d$ )

# Online-TSP

(metric)

INPUT:

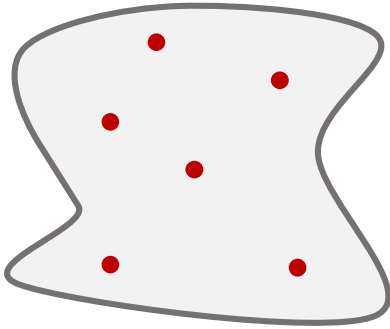


- metric space:  $M$   
(with metric  $d$ )
- places to visit:  $S$

# Online-TSP

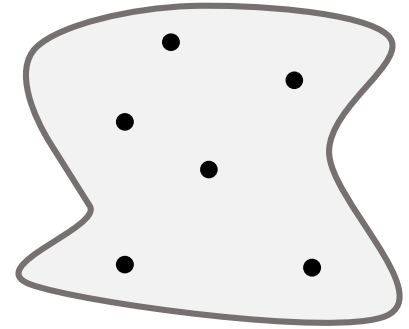
(metric)

INPUT:



- metric space:  $M$   
(with metric  $d$ )
- places to visit:  $\mathcal{S}$

OUTPUT:

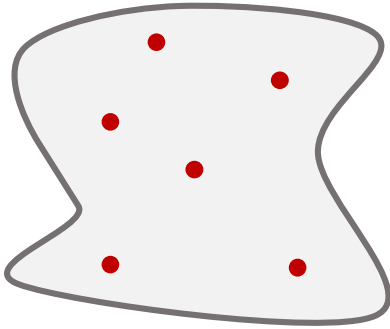




# Online-TSP

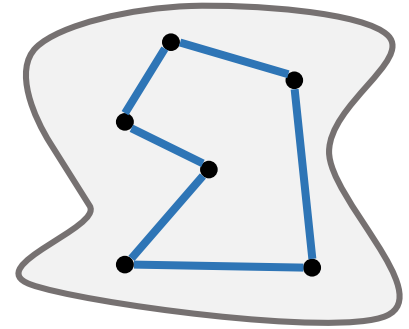
(metric)

INPUT:



- metric space:  $M$   
(with metric  $d$ )
- places to visit:  $\mathcal{S}$

OUTPUT:

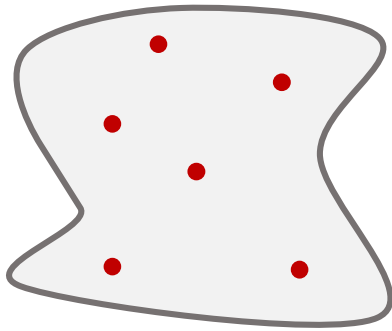


Shortest tour  $\mathcal{T}^*$   
through  $\mathcal{S}$

# Online-TSP

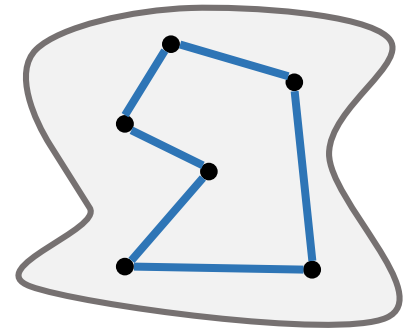
(metric)

INPUT:



NP-hard!

OUTPUT:



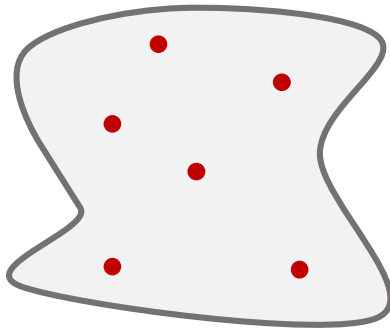
- metric space:  $M$   
(with metric  $d$ )
- places to visit:  $\mathcal{S}$

Shortest tour  $\mathcal{T}^*$   
through  $\mathcal{S}$

# Online-TSP

(metric)

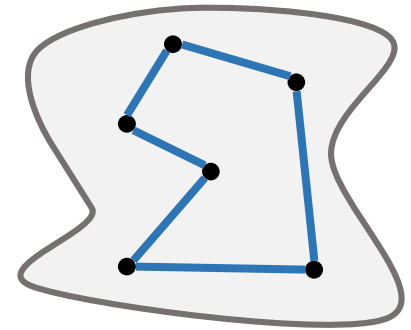
INPUT:



NP-hard!

ALG

OUTPUT:



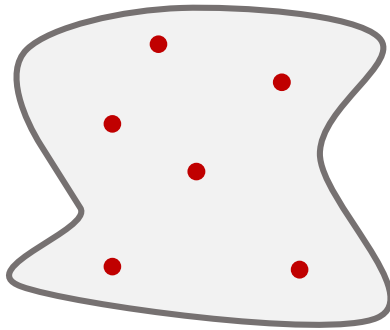
- metric space:  $M$   
(with metric  $d$ )
- places to visit:  $\mathcal{S}$

Shortest tour  $\mathcal{T}^*$   
through  $\mathcal{S}$

# Online-TSP

(metric)

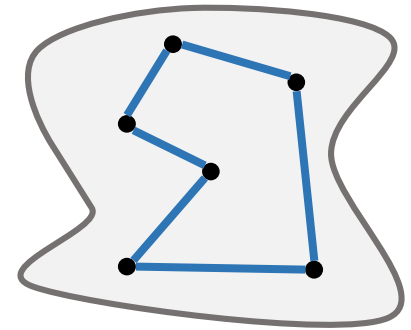
INPUT:



NP-hard!

ALG

OUTPUT:



- metric space:  $M$   
(with metric  $d$ )

- Superpolynomial Alg.

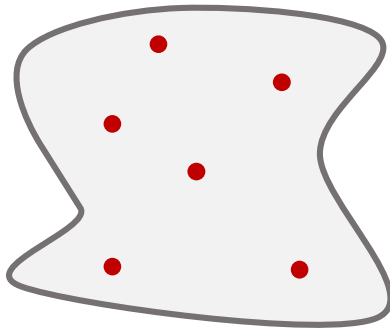
Shortest tour  $\mathcal{T}^*$   
through  $\mathcal{S}$

- places to visit:  $\mathcal{S}$

# Online-TSP

(metric)

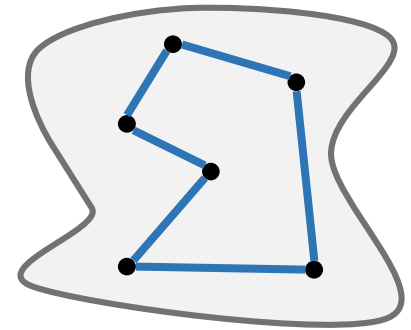
INPUT:



NP-hard!

ALG

OUTPUT:



- metric space:  $M$   
(with metric  $d$ )
- places to visit:  $\mathcal{S}$

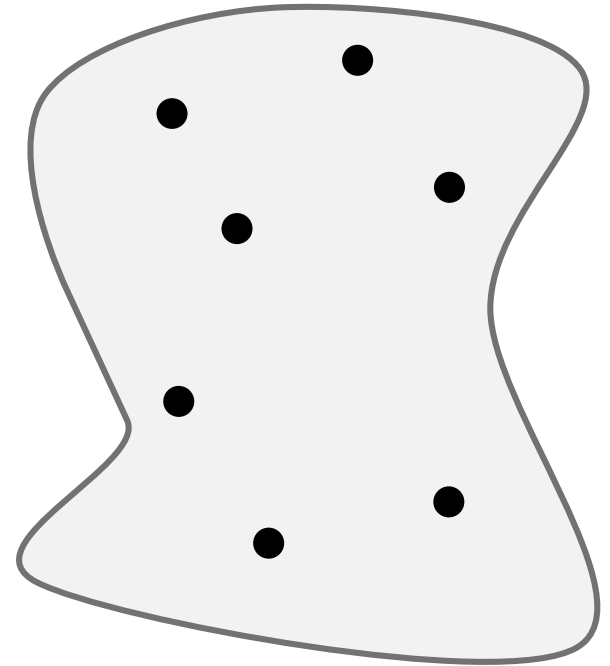
- Superpolynomial Alg.
- Approximation Alg.  
e.g. *Christofides*

Shortest tour  $\mathcal{T}^*$   
through  $\mathcal{S}$

# Online-TSP

(metric)

Christofides Algorithm:

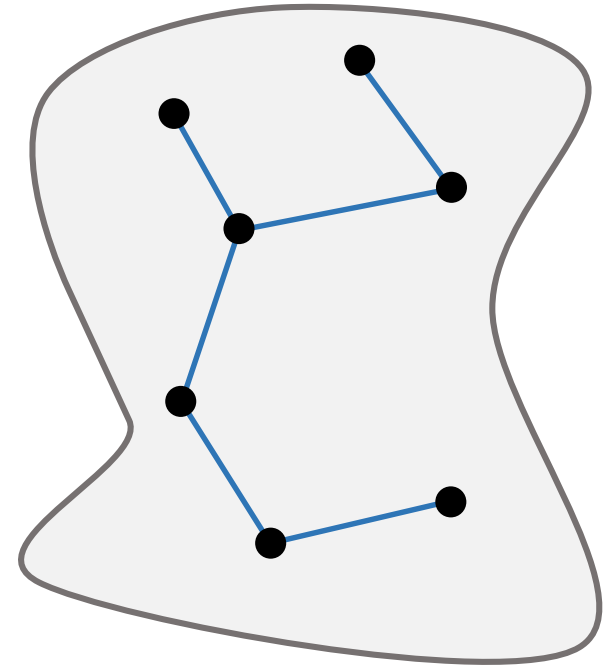


# Online-TSP

(metric)

## Christofides Algorithm:

(1) minimal spanning tree

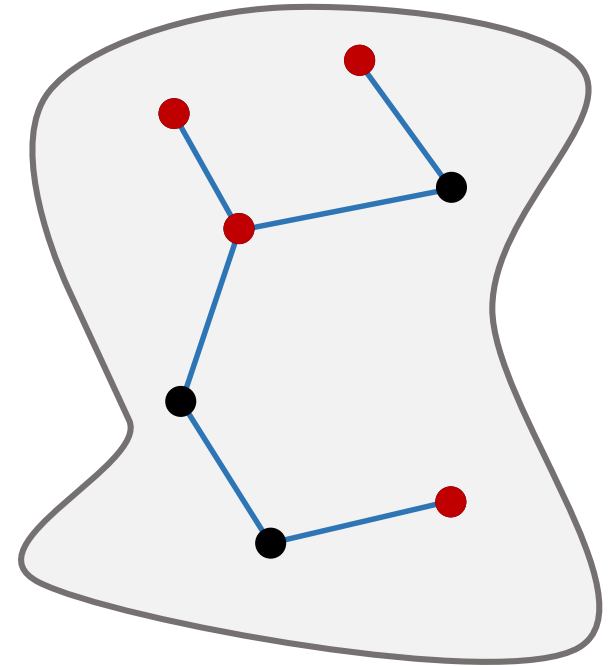


# Online-TSP

(metric)

## Christofides Algorithm:

(1) minimal spanning tree



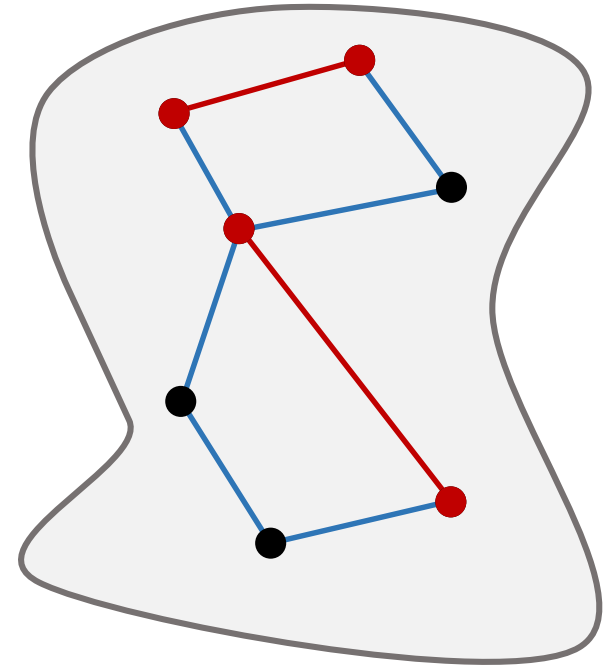


# Online-TSP

(metric)

## Christofides Algorithm:

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices

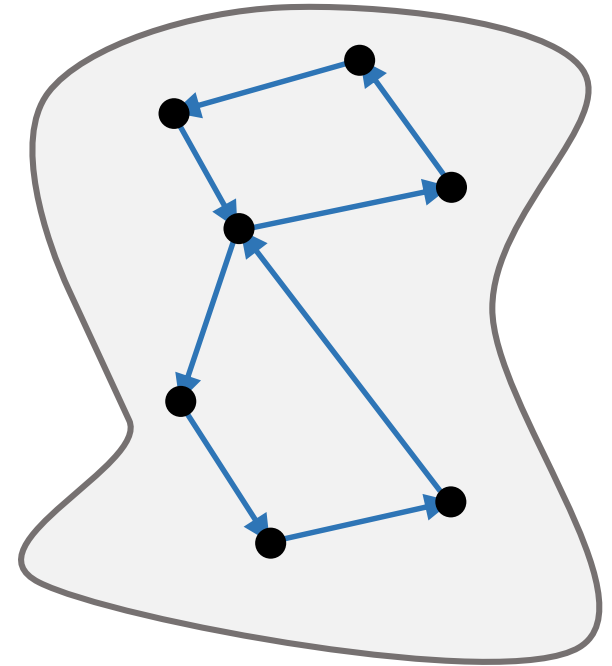


# Online-TSP

(metric)

## Christofides Algorithm:

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour

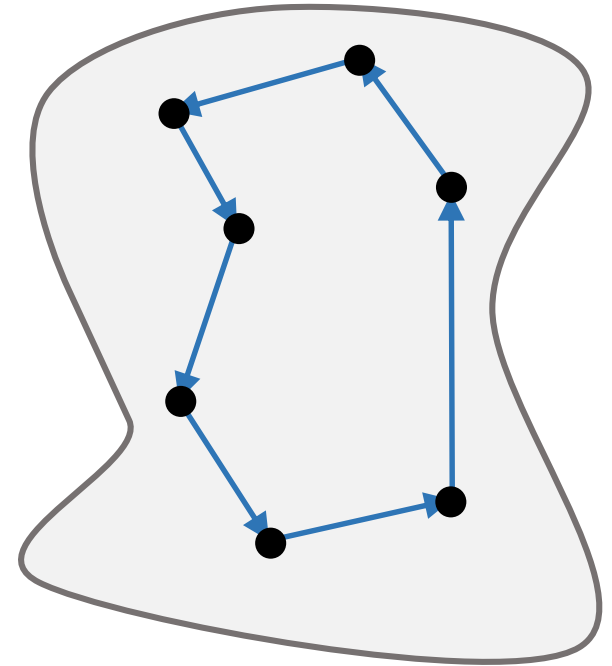


# Online-TSP

(metric)

## Christofides Algorithm:

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices



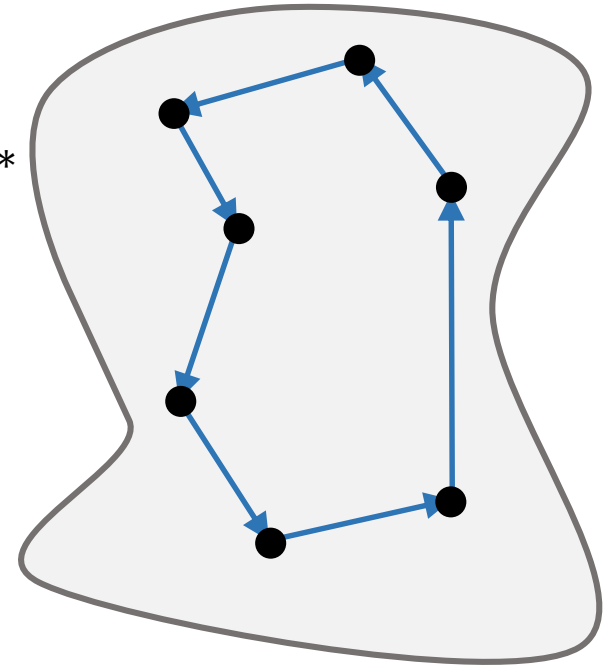
# Online-TSP

(metric)

## Christofides Algorithm:

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices

$$\leq 1.5 \cdot \mathcal{T}^*$$



# Online-TSP

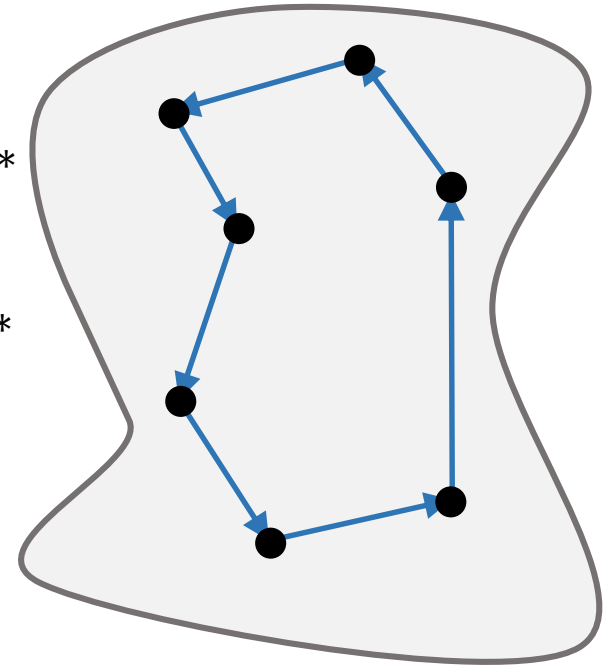
(metric)

## Christofides Algorithm:

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices

$$\leq 1 \cdot \mathcal{T}^*$$

$$\leq \frac{1}{2} \cdot \mathcal{T}^*$$



# Online-TSP

(metric)

## Christofides Algorithm:

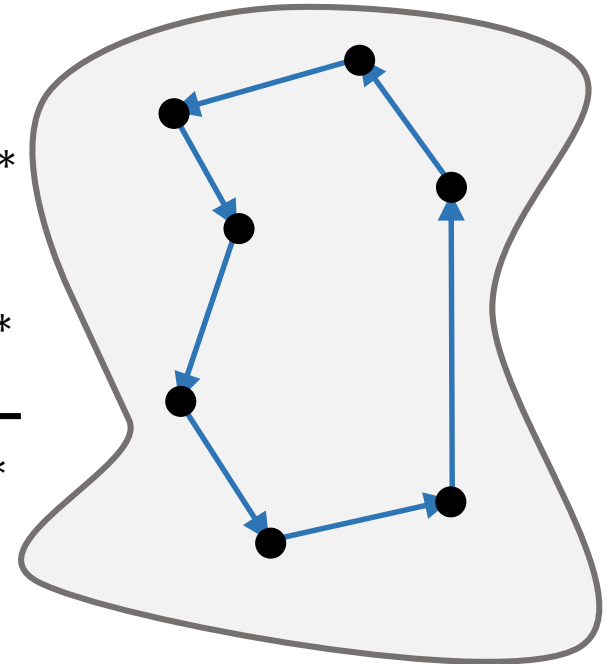
- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices

$$\leq 1 \cdot \mathcal{T}^*$$

$$\leq \frac{1}{2} \cdot \mathcal{T}^*$$

---

$$\leq \frac{3}{2} \cdot \mathcal{T}^*$$



# Online-TSP

(metric)

## Christofides Algorithm:

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices

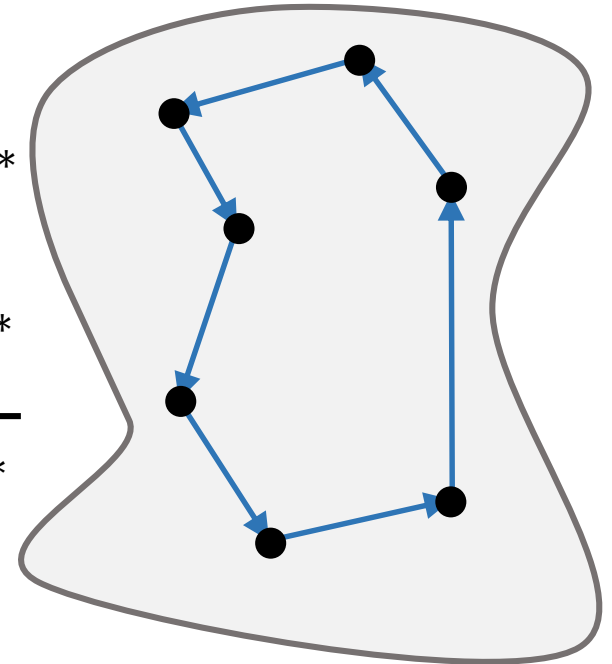
$$\leq 1 \cdot \mathcal{T}^*$$

$$\leq \frac{1}{2} \cdot \mathcal{T}^*$$

$$\leq \frac{3}{2} \cdot \mathcal{T}^*$$



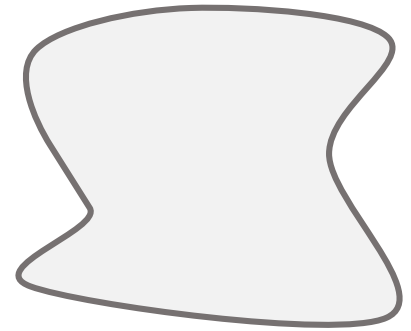
1,5-approximative solution



# Online-TSP

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INPUT:

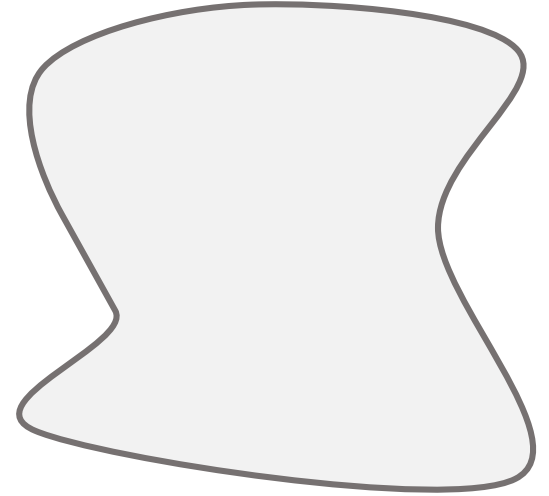




# Online-TSP

INPUT:

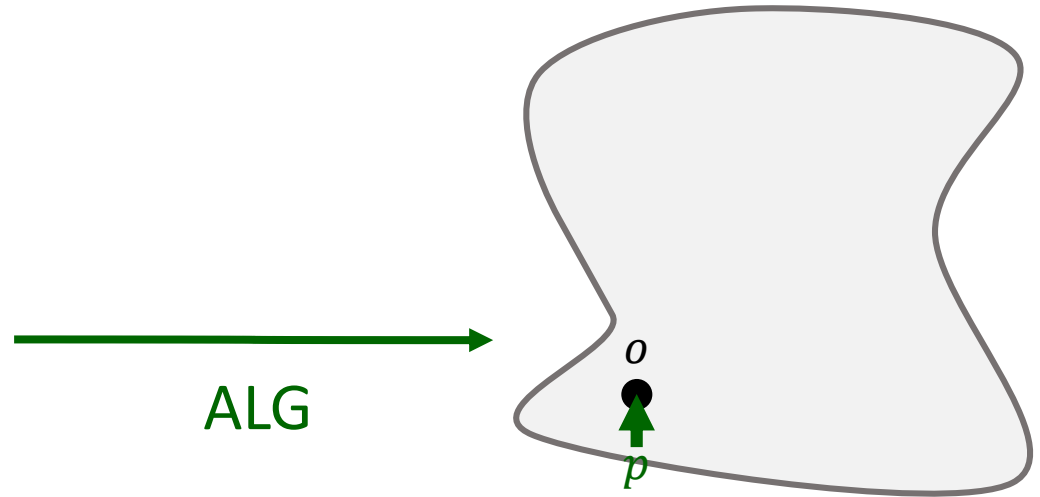
- metric space



# Online-TSP

INPUT:

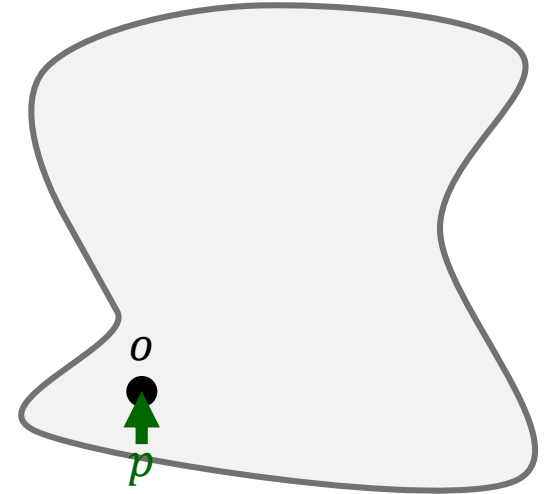
- metric space
- starting-point:  $o$



# Online-TSP

INPUT:

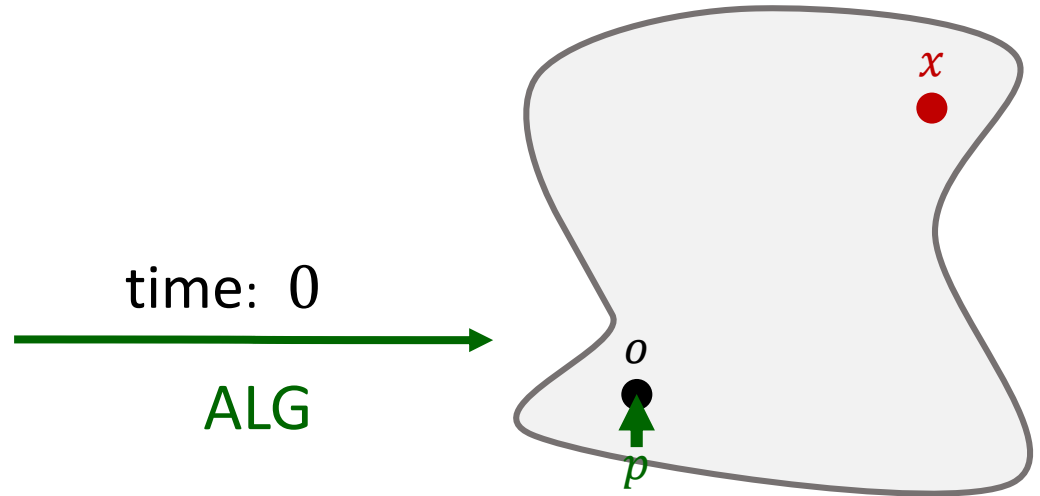
- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :



# Online-TSP

INPUT:

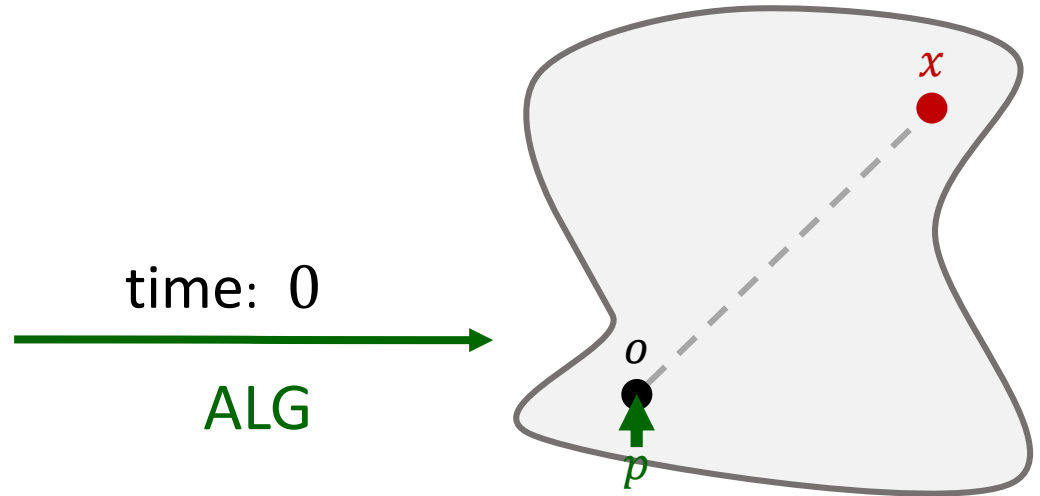
- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x)$



# Online-TSP

INPUT:

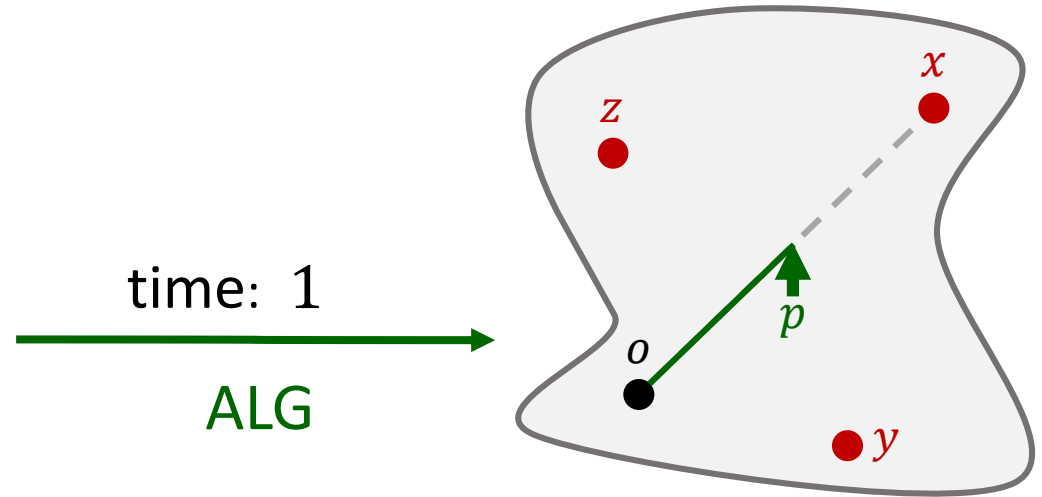
- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x)$



# Online-TSP

INPUT:

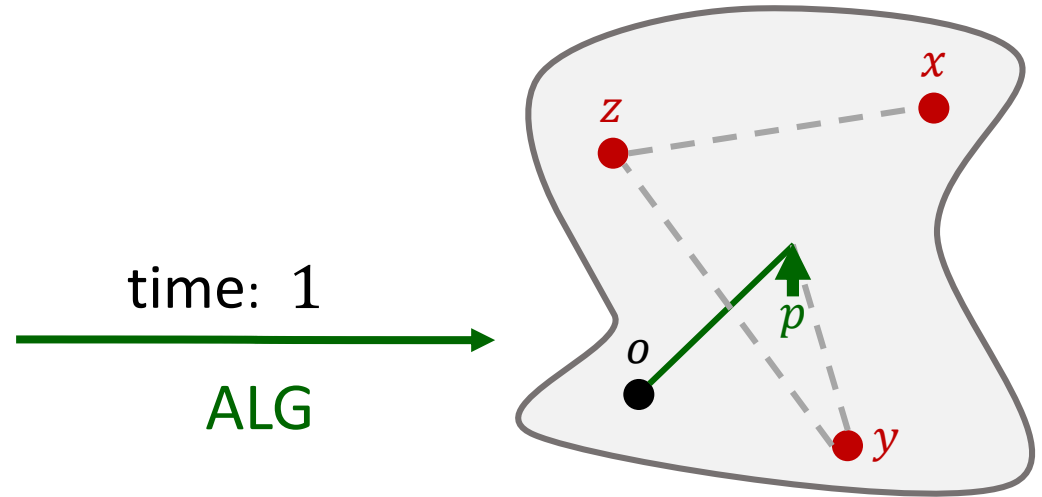
- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x), (1, y), (1, z)$



# Online-TSP

INPUT:

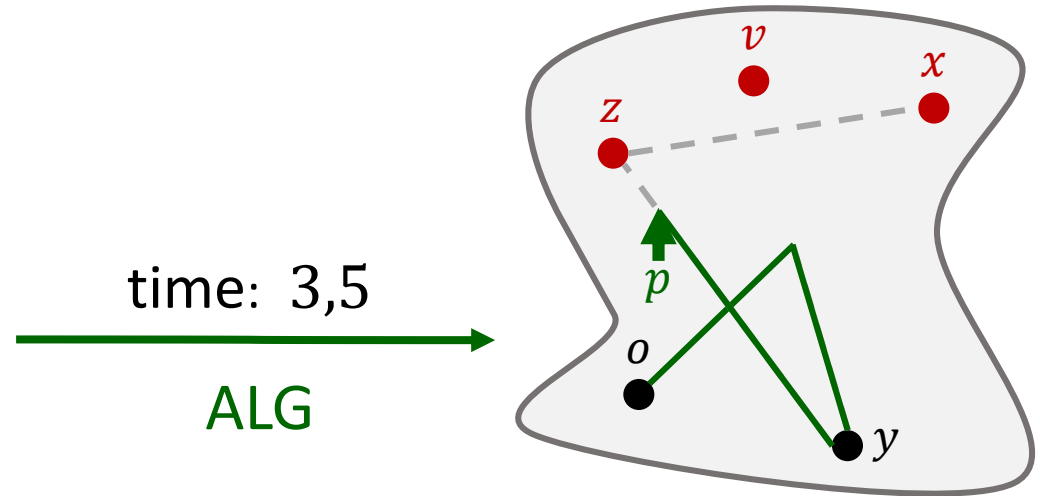
- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x), (1, y), (1, z)$



# Online-TSP

INPUT:

- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x), (1, y), (1, z), \dots$



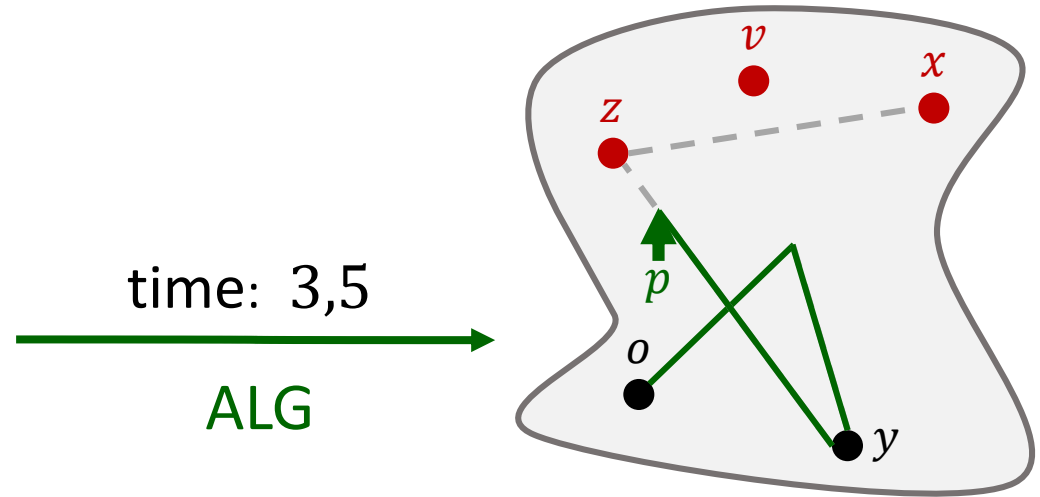


# Online-TSP

INPUT:

- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x), (1, y), (1, z), \dots$

N-OLTSP  
“nomadic”

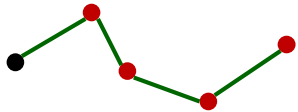


# Online-TSP

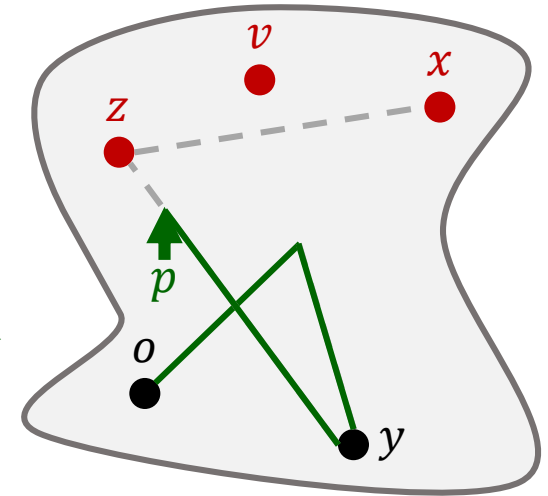
INPUT:

- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x), (1, y), (1, z), \dots$

N-OLTSP  
“nomadic”



time: 3,5  
ALG



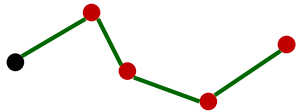
# Online-TSP

INPUT:

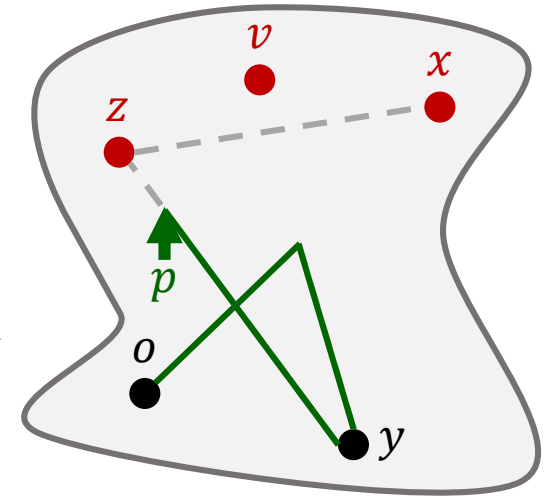
- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x), (1, y), (1, z), \dots$

N-OLTSP  
“nomadic”

H-OLTSP  
“homing”



time: 3,5  
ALG

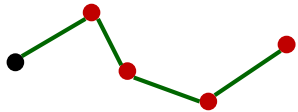


# Online-TSP

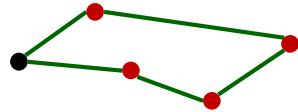
INPUT:

- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x), (1, y), (1, z), \dots$

N-OLTSP  
“nomadic”

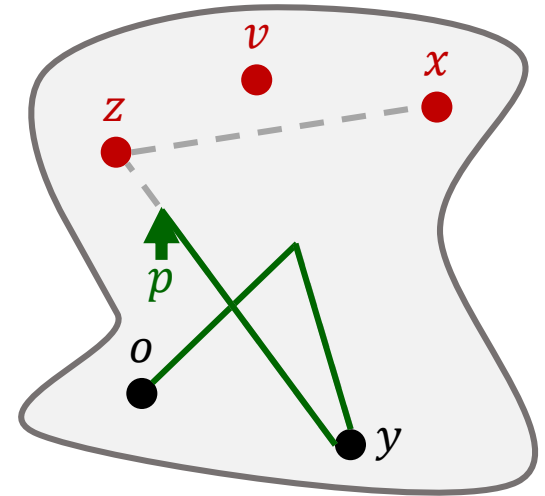


H-OLTSP  
“homing”



time: 3,5

ALG

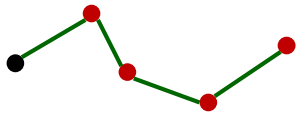


# Online-TSP

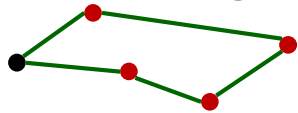
INPUT:

- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x), (1, y), (1, z), \dots$

N-OLTSP  
“nomadic”

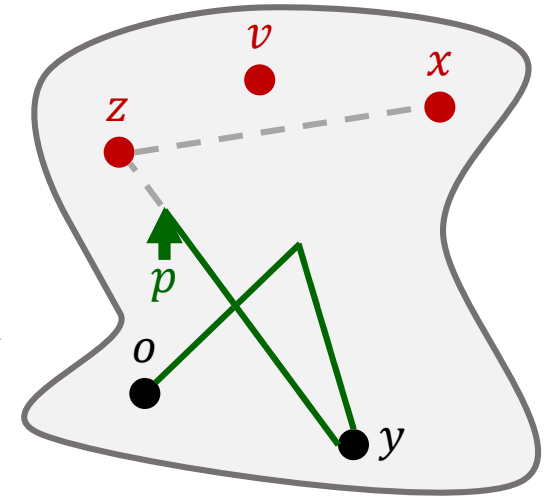


H-OLTSP  
“homing”



time: 3,5

ALG



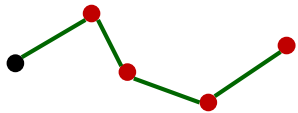
DEF: ALG is  $\rho$ -competitive

# Online-TSP

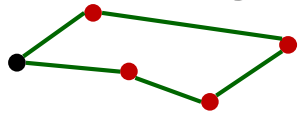
INPUT:

- metric space
- starting-point:  $o$
- request-sequence  $\sigma$ :  
 $(0, x), (1, y), (1, z), \dots$

N-OLTSP  
“nomadic”

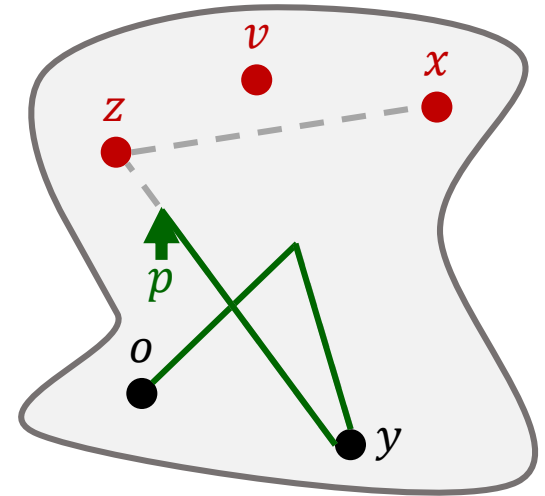


H-OLTSP  
“homing”



time: 3,5

ALG



DEF: ALG is  $\rho$ -competitive

$$\Leftrightarrow |\mathcal{T}^{\text{ALG}}| \leq \rho \cdot |\mathcal{T}^{\text{OPT}}|$$

for all request- sequences

# Goals

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# Goals

---

- I. Find online-algorithms



# Goals

---

- I. Find online-algorithms (superpolynomial)

# Goals

---

- I. Find online-algorithms (superpolynomial)
- II. Find lower bounds

# Goals

---

- I. Find online-algorithms (superpolynomial)
- II. Find lower bounds
- III. Find *polynomial* online-algorithms

# An algorithm for N-OLSTP

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

H-OLTSP 

# An algorithm for N-OLSTP

*Invariant:* always on shortest path between points in  $S$

$S :=$  places requested until  $t$

Online-TSP

I. Algorithms

II. Lower Bounds

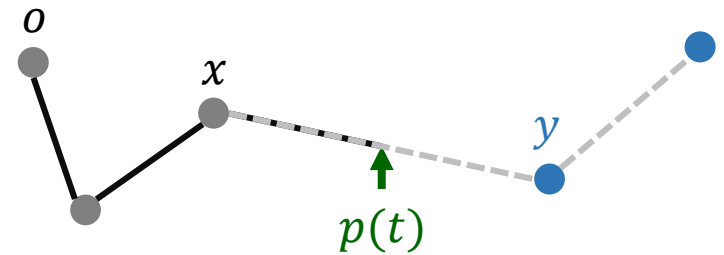
III. Polynomial Algorithms

N-OLTSP 

H-OLTSP 

# An algorithm for N-OLSTP

*Invariant:* always on shortest path between points in  $S$



$S :=$  places requested until  $t$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

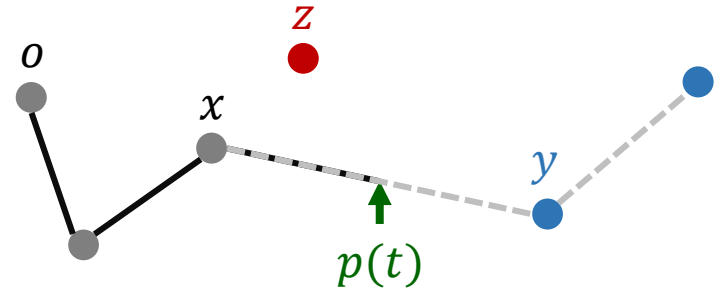
N-OLTSP

H-OLTSP

# An algorithm for N-OLSTP

Invariant: always on shortest path between points in  $S$

- (1) **New request  $(t, z)$**  at time  $t$   
and ALG between  $x$  and  $y$



$S :=$  places requested until  $t$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

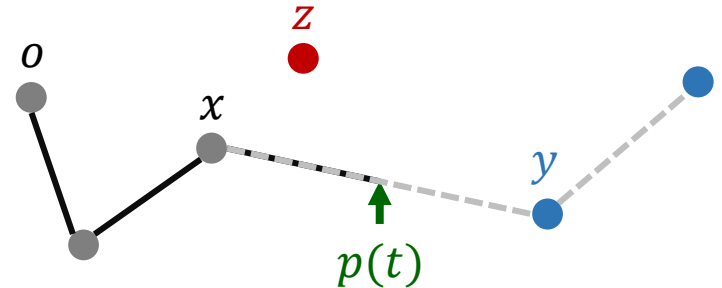
N-OLTSP 

H-OLTSP 

# An algorithm for N-OLSTP

Invariant: always on shortest path between points in  $S$

(1) Add  $z$  to  $U$



$S :=$  places requested until  $t$   
 $S \supseteq U :=$  places yet to visit at  $t$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

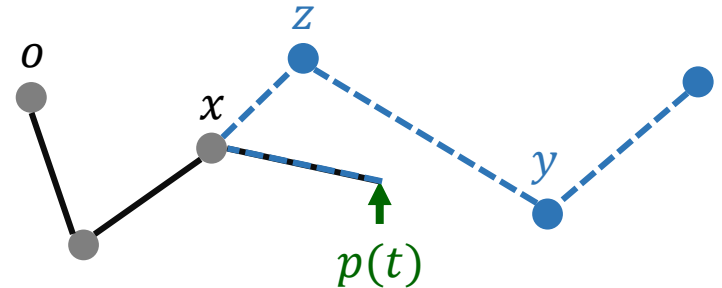
H-OLTSP 



# An algorithm for N-OLSTP

Invariant: always on shortest path between points in  $S$

- (1) Add  $z$  to  $U$
- (2) Follow shortest path through  $\mathcal{U}$  beginning with  $x$  or  $y$



$S :=$  places requested until  $t$   
 $S \supseteq U :=$  places yet to visit at  $t$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

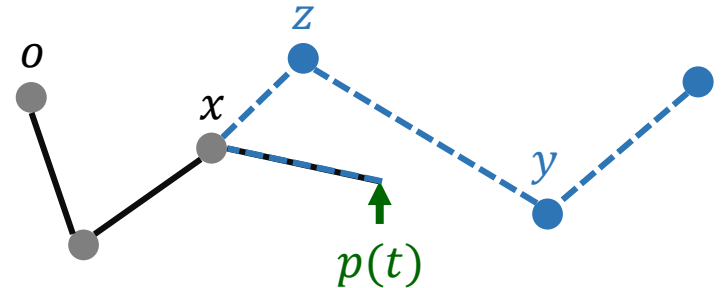
N-OLTSP 

H-OLTSP 

# An algorithm for N-OLSTP

Invariant: always on shortest path between points in  $S$

- (1) Add  $z$  to  $U$
- (2) Follow shortest path through  $\mathcal{U}$  beginning with  $x$  or  $y$



$S :=$  places requested until  $t$   
 $S \supseteq U :=$  places yet to visit at  $t$

## Greedly Travelling between Requests (GTR)

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

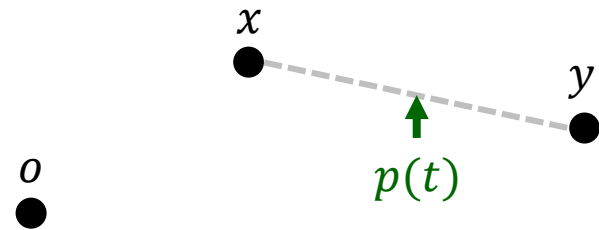
N-OLTSP

H-OLTSP

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) New request  $(t, z)$  at time  $t$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

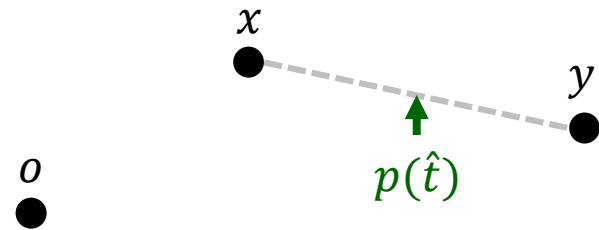
N-OLTSP 

H-OLTSP 

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

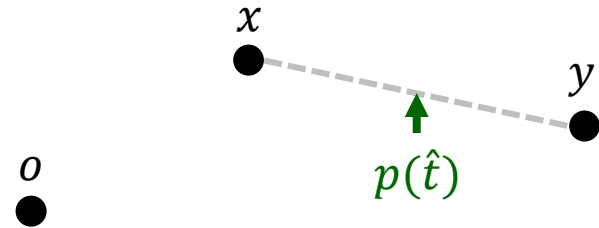
N-OLTSP 

H-OLTSP 

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



path found by GTR

$$\downarrow$$
$$|\mathcal{T}^{\text{GTR}}|$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

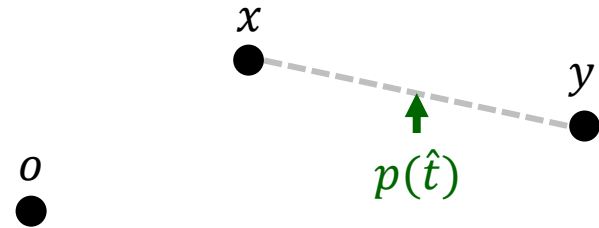
N-OLTSP 

H-OLTSP 

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



path found by GTR

$$|\mathcal{J}^{\text{GTR}}| \leq \hat{t}$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

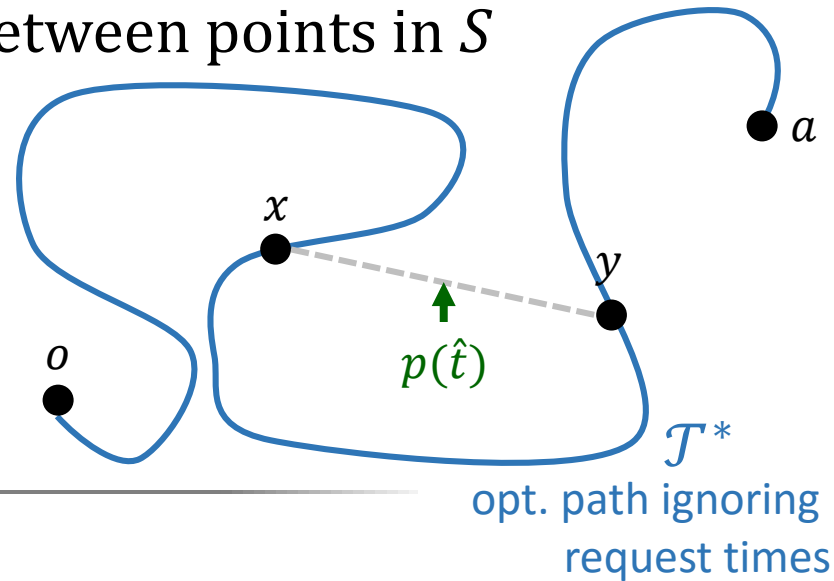
N-OLTSP 

H-OLTSP 

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



path found by GTR

$$|\mathcal{J}^{\text{GTR}}| \leq \hat{t}$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

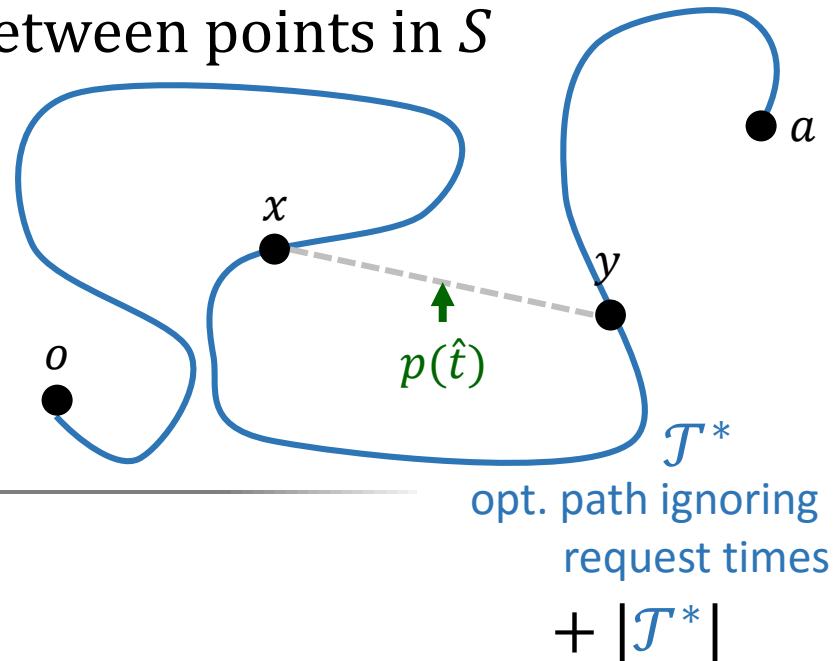
N-OLTSP

H-OLTSP

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



path found by GTR

$$|\mathcal{J}^{\text{GTR}}| \leq \hat{t}$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

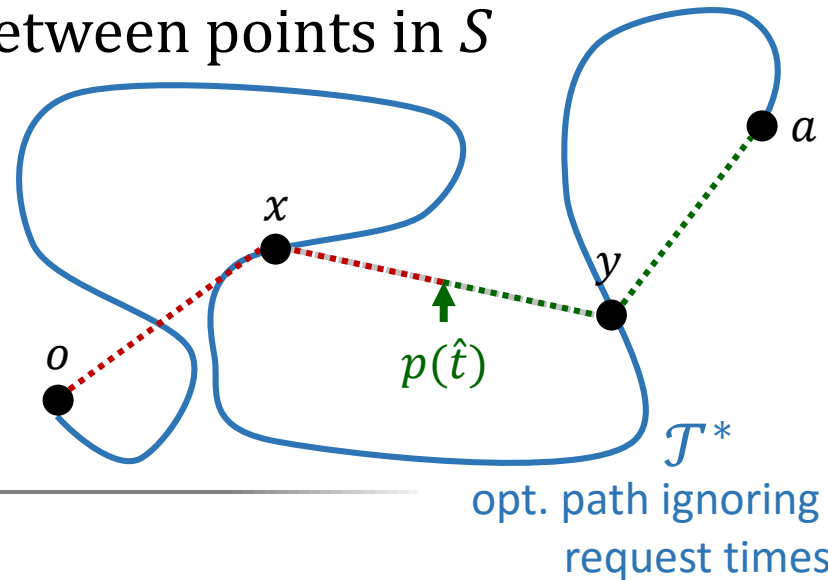
H-OLTSP



# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$  and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$  beginning with  $x$  or  $y$



path found by GTR

$$|\mathcal{T}^{\text{GTR}}| \leq \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, a)\} + |\mathcal{J}^*|$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

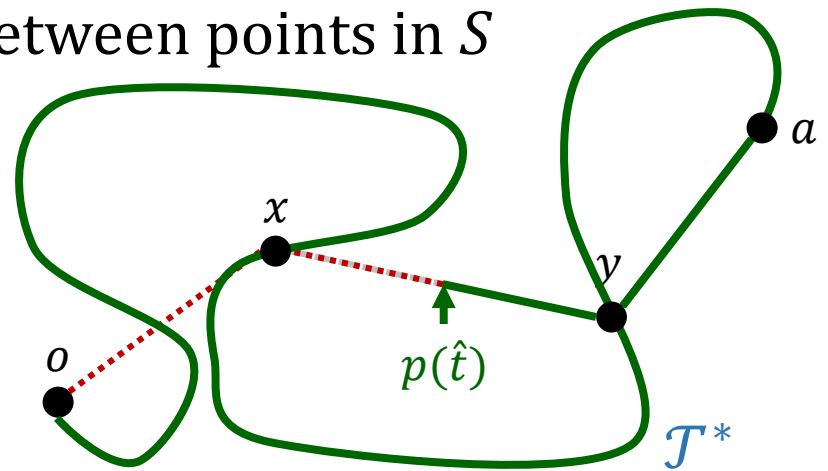
N-OLTSP 

H-OLTSP 

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



path found by GTR

$$\begin{aligned}
 |\mathcal{T}^{\text{GTR}}| &\leq \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, a)\} + |\mathcal{J}^*| \\
 &\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{J}^*| + |\mathcal{J}^*|
 \end{aligned}$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

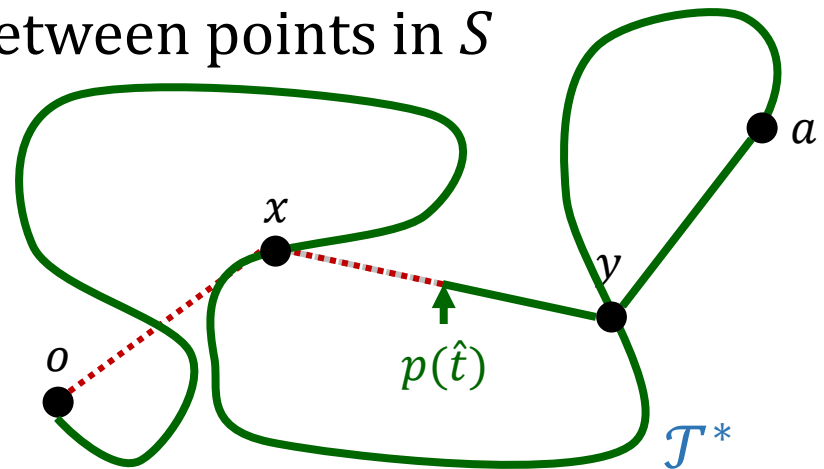
N-OLTSP

H-OLTSP

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$  and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$  beginning with  $x$  or  $y$



path found by GTR

$$|\mathcal{T}^{\text{GTR}}| \leq \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, a)\} + |\mathcal{J}^*|$$

$$\begin{aligned} &\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{J}^*| + |\mathcal{J}^*| \\ &\leq |\mathcal{T}^{\text{OPT}}| + \end{aligned}$$

path found by opt. offline-ALG

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

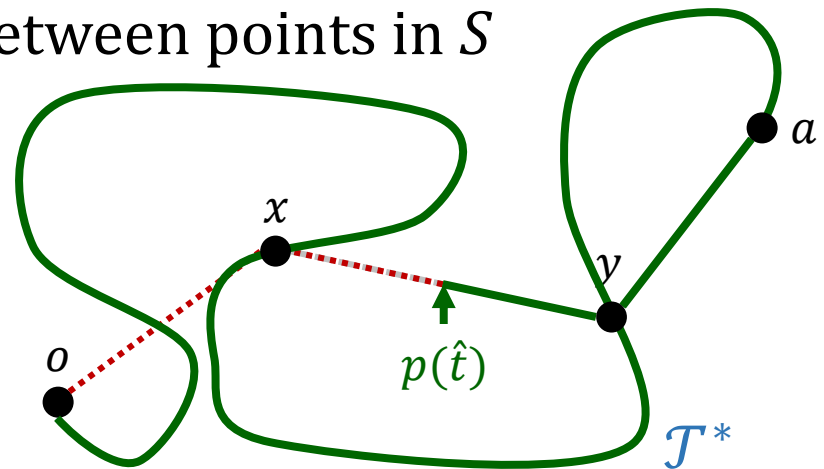
N-OLTSP

H-OLTSP

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



path found by GTR

$$|\mathcal{J}^{\text{GTR}}| \leq \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, a)\} + |\mathcal{J}^*|$$

$$\leq \hat{t} +$$

$$\frac{1}{2} \cdot |\mathcal{J}^*|$$

$$+ |\mathcal{J}^*|$$

path found by  
opt. offline-ALG

$$\leq |\mathcal{J}^{\text{OPT}}|$$

+

$$\frac{3}{2} \cdot |\mathcal{J}^{\text{OPT}}|$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

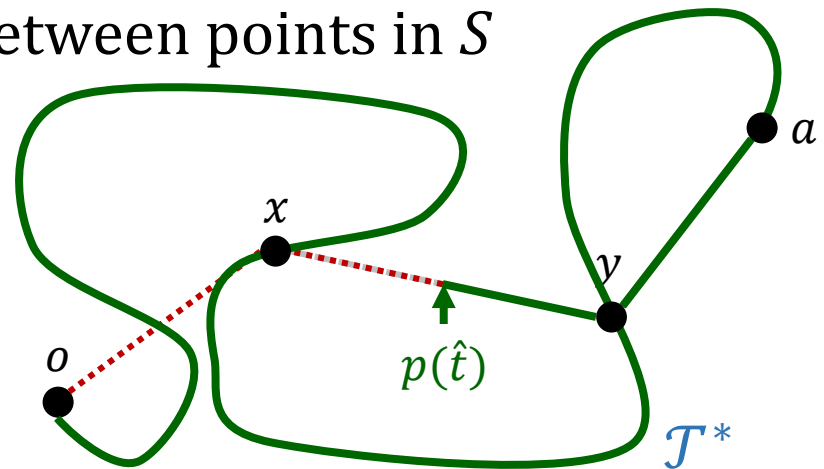
N-OLTSP

H-OLTSP

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$  and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$  beginning with  $x$  or  $y$



path found by GTR

$$|\mathcal{J}^{\text{GTR}}| \leq \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, a)\} + |\mathcal{J}^*|$$

$$\begin{aligned} &\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{J}^*| + |\mathcal{J}^*| \\ \text{path found by opt. offline-ALG} &\leq |\mathcal{J}^{\text{OPT}}| + \frac{3}{2} \cdot |\mathcal{J}^{\text{OPT}}| = \frac{5}{2} \cdot |\mathcal{J}^{\text{OPT}}| \end{aligned}$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

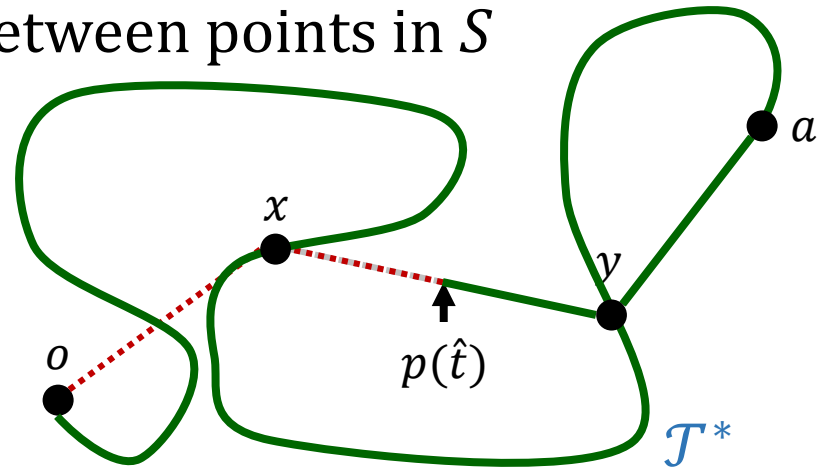
N-OLTSP 

H-OLTSP 

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



$$|\mathcal{J}^{\text{GTR}}| \leq |\mathcal{J}^{\text{OPT}}| + \frac{3}{2} \cdot |\mathcal{J}^{\text{OPT}}| = \frac{5}{2} \cdot |\mathcal{J}^{\text{OPT}}|$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

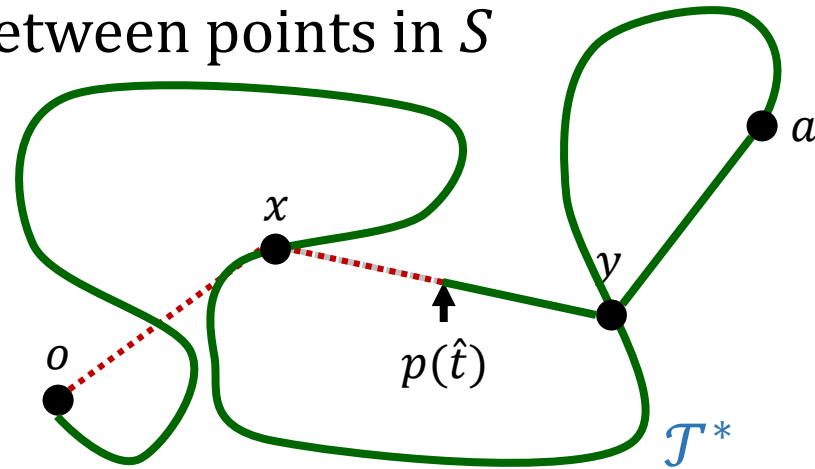
N-OLTSP 

H-OLTSP 

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$   
beginning with  $x$  or  $y$



**THEOREM:** GTR is 2,5-competitive for N-OLTSP

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

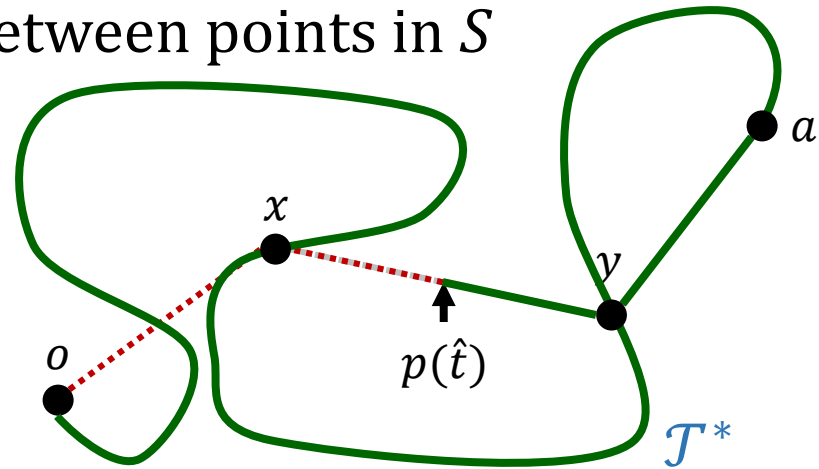
GTR (Greedy):  $\rho = 2,5$

H-OLTSP

# Competitiveness of GTR

*Invariant:* always on shortest path between points in  $S$

- (1) Last request  $(\hat{t}, z)$  at time  $\hat{t}$  and ALG between  $x$  and  $y$
- (3) Follow shortest path through  $U$  beginning with  $x$  or  $y$



**THEOREM:** GTR is 2,5-competitive for N-OLTSP

**REMARK:** GTR is also 2,5-competitive for H-OLTSP

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

H-OLTSP 

GTR (Greedy):  $\rho = 2,5$

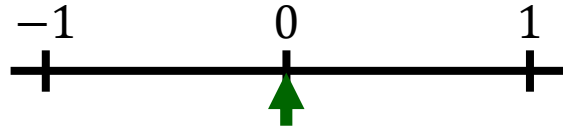


# Lower Bound for N-OLTSP

time, request

Online-ALG

0



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

**N-OLTSP** 

GTR (Greedy):  $\rho = 2,5$

**H-OLTSP** 

GTR (Greedy):  $\rho = 2,5$

# Lower Bound for N-OLTSP

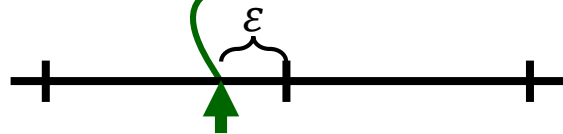
time, request

Online-ALG

0



1



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

H-OLTSP 

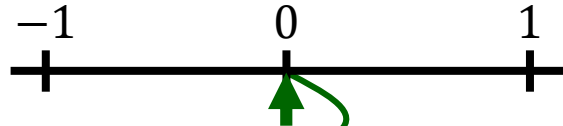
GTR (Greedy):  $\rho = 2,5$

# Lower Bound for N-OLTSP

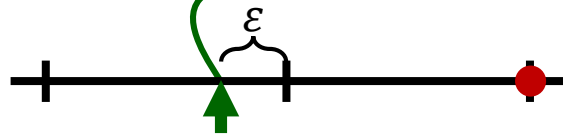
time, request

Online-ALG

0



1, 1



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

H-OLTSP 

GTR (Greedy):  $\rho = 2,5$

# Lower Bound for N-OLTSP

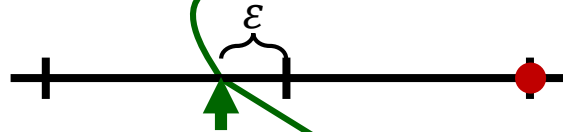
time, request

Online-ALG

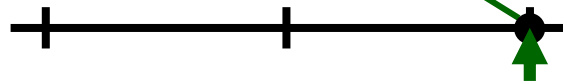
0



1, 1



$2 + \varepsilon$



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

H-OLTSP 

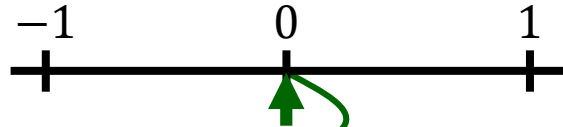
GTR (Greedy):  $\rho = 2,5$

# Lower Bound for N-OLTSP

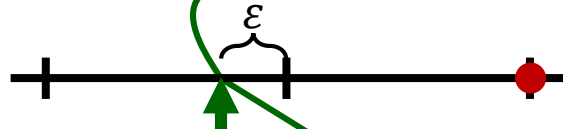
time, request

Online-ALG

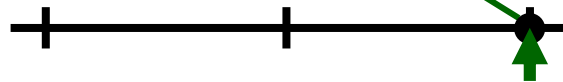
0



1, 1



$2 + \varepsilon$



$$\Rightarrow |\mathcal{T}^{\text{ALG}}| = 2 + \varepsilon$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

H-OLTSP 

GTR (Greedy):  $\rho = 2,5$

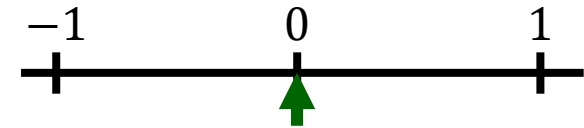
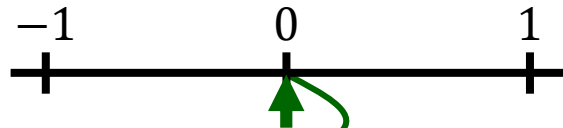
# Lower Bound for N-OLTSP

time, request

Online-ALG

Opt. offline-ALG

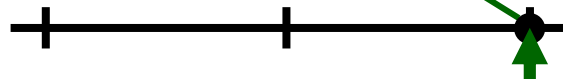
0



1, 1



$2 + \varepsilon$



$$\Rightarrow |\mathcal{T}^{\text{ALG}}| = 2 + \varepsilon$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

H-OLTSP 

GTR (Greedy):  $\rho = 2,5$

# Lower Bound for N-OLTSP

time, request

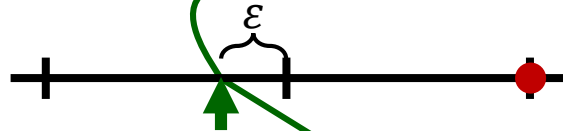
Online-ALG

Opt. offline-ALG

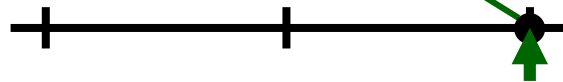
0



1, 1



$2 + \varepsilon$



$$\Rightarrow |\mathcal{T}^{\text{ALG}}| = 2 + \varepsilon$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

H-OLTSP 

GTR (Greedy):  $\rho = 2,5$

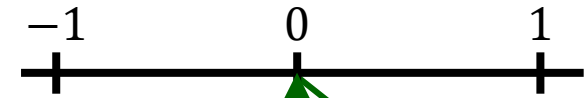
# Lower Bound for N-OLTSP

time, request

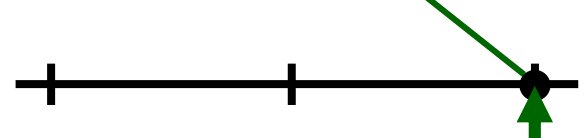
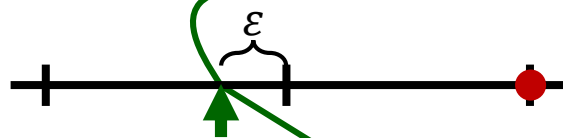
Online-ALG

Opt. offline-ALG

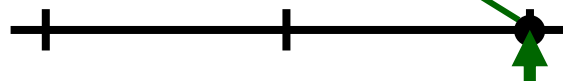
0



1, 1



$2 + \varepsilon$



$$\Rightarrow |\mathcal{T}^{\text{ALG}}| = 2 + \varepsilon$$

$$\Rightarrow |\mathcal{T}^{\text{OPT}}| = 1$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

H-OLTSP 

GTR (Greedy):  $\rho = 2,5$



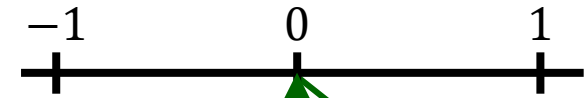
# Lower Bound for N-OLTSP

time, request

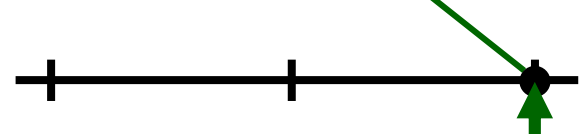
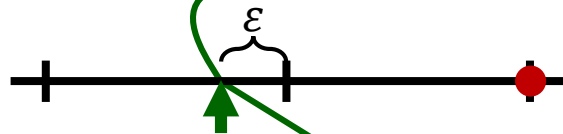
Online-ALG

Opt. offline-ALG

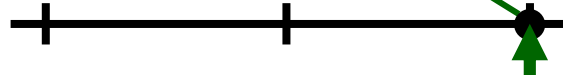
0



1, 1



$2 + \varepsilon$



$$\Rightarrow |\mathcal{T}^{\text{ALG}}| = 2 + \varepsilon$$

$$\Rightarrow |\mathcal{T}^{\text{OPT}}| = 1$$

**THEOREM:** Any  $\rho$ -competitive ALG for N-OLTSP has  $\rho \geq 2$ .

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

H-OLTSP 

GTR (Greedy):  $\rho = 2,5$

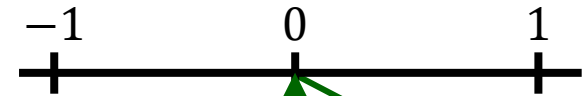
# Lower Bound for H-OLTSP

time, request

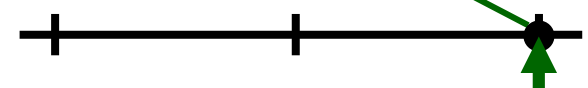
Online-ALG

Opt. offline-ALG

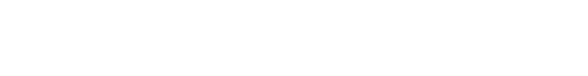
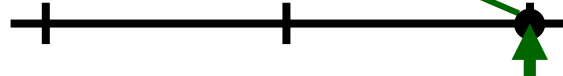
0



1, 1



$2 + \varepsilon$



$$\Rightarrow |\mathcal{T}^{\text{ALG}}| = 2 + \varepsilon$$

$$\Rightarrow |\mathcal{T}^{\text{OPT}}| = 1$$

**THEOREM:** Any  $\rho$ -competitive ALG for N-OLTSP has  $\rho \geq 2$ .

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

H-OLTSP

GTR (Greedy):  $\rho = 2,5$

# Lower Bound for H-OLTSP

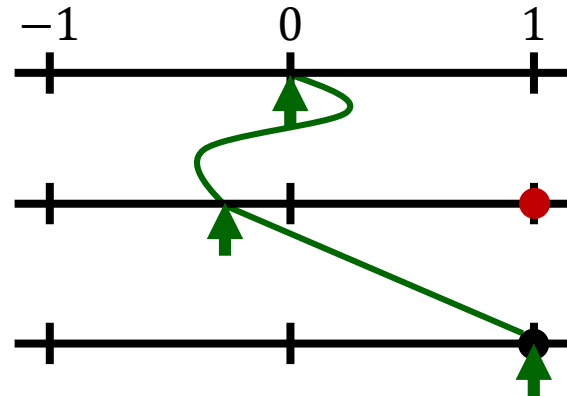
time, request

0

1, 1

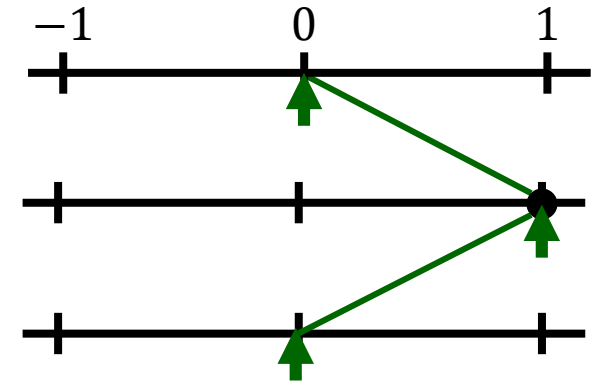
$2 + \varepsilon$

Online-ALG



$$\Rightarrow |\mathcal{T}^{\text{ALG}}| = 2 + \varepsilon$$

Opt. offline-ALG



$$\Rightarrow |\mathcal{T}^{\text{OPT}}| = 2$$

**THEOREM:** Any  $\rho$ -competitive ALG for N-OLTSP has  $\rho \geq 2$ .

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

H-OLTSP

GTR (Greedy):  $\rho = 2,5$

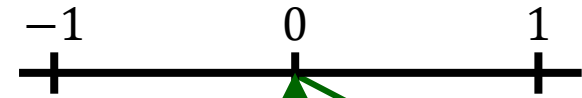
# Lower Bound for H-OLTSP

time, request

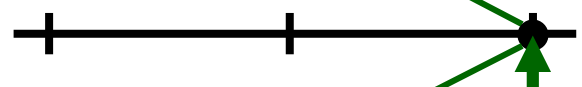
Online-ALG

Opt. offline-ALG

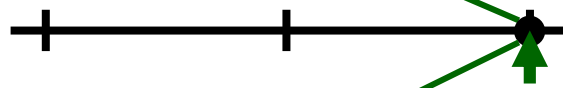
0



1, 1



$2 + \varepsilon$



$3 + \varepsilon$



$$\Rightarrow |\mathcal{T}^{\text{ALG}}| = 3 + \varepsilon$$

$$\Rightarrow |\mathcal{T}^{\text{OPT}}| = 2$$

**THEOREM:** Any  $\rho$ -competitive ALG for N-OLTSP has  $\rho \geq 2$ .

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

H-OLTSP

GTR (Greedy):  $\rho = 2,5$

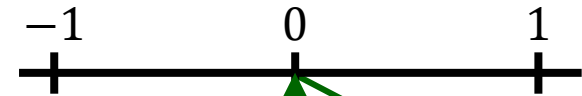
# Lower Bound for H-OLTSP

time, request

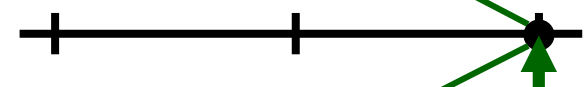
Online-ALG

Opt. offline-ALG

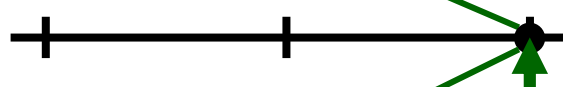
0



1, 1



$2 + \varepsilon$



$3 + \varepsilon$



$$\Rightarrow |\mathcal{T}^{\text{ALG}}| = 3 + \varepsilon$$

$$\Rightarrow |\mathcal{T}^{\text{OPT}}| = 2$$

**THEOREM:** Any  $\rho$ -competitive ALG for H-OLTSP has  $\rho \geq 1,5$ .

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

**N-OLTSP**

GTR (Greedy):  $\rho = 2,5$

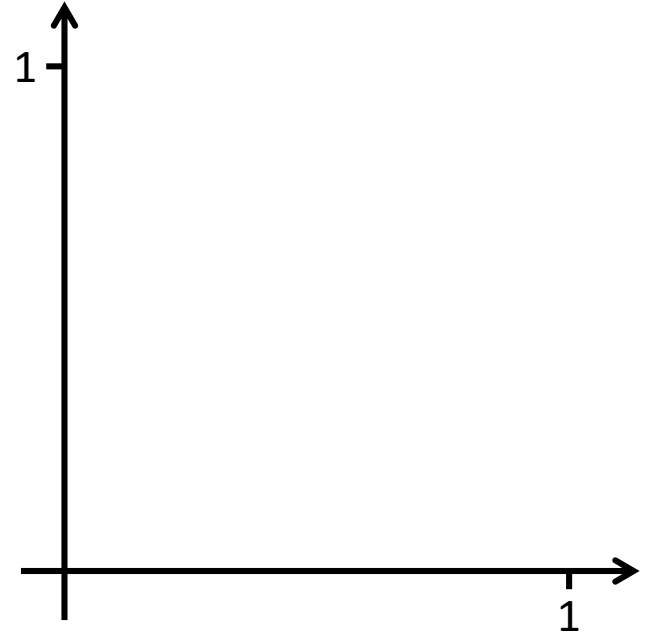
$$\rho \geq 2$$

**H-OLTSP**

GTR (Greedy):  $\rho = 2,5$

$$\rho \geq 1,5$$

# Lower Bound for H-OLTSP



Online-TSP

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**N-OLTSP** 

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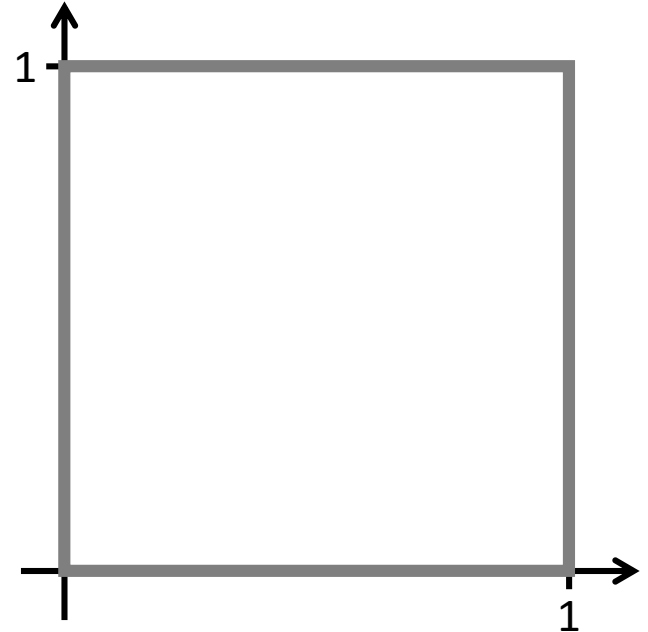
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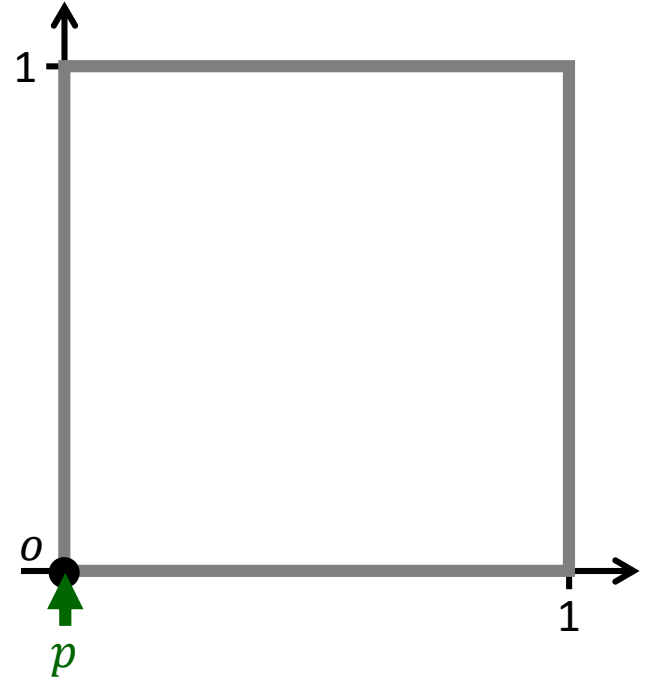
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# Lower Bound for H-OLTSP



Online-TSP

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II. Lower Bounds

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$\rho \geq 2$

**H-OLTSP**

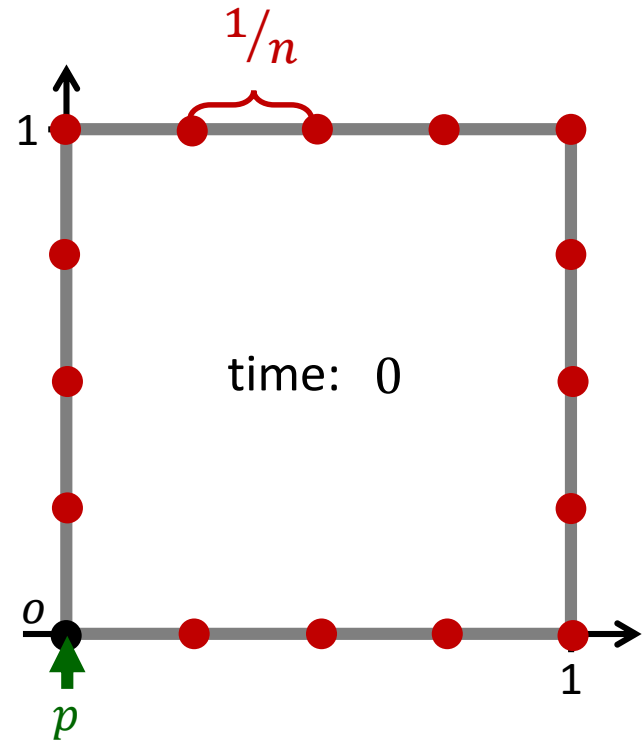
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# Lower Bound for H-OLTSP

- requests at time 0



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II. Lower Bounds

III. Polynomial Algorithms

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$\rho \geq 2$

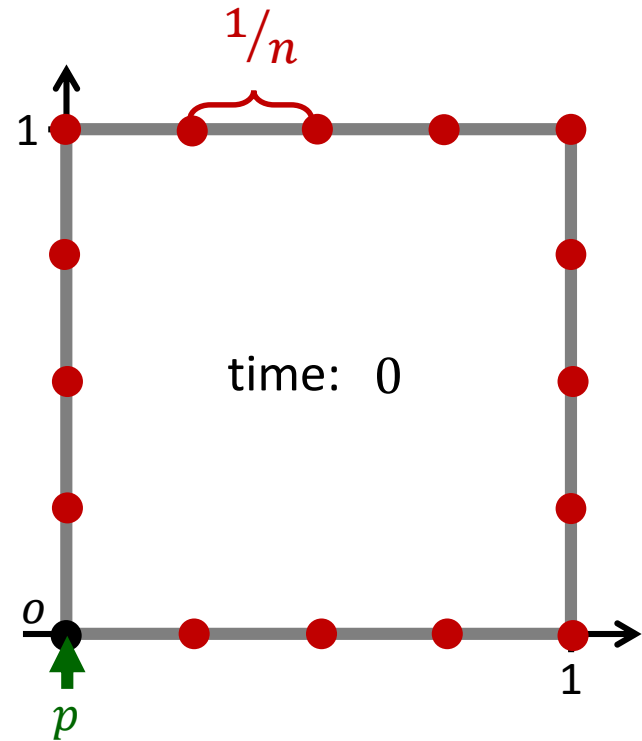
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# Lower Bound for H-OLTSP

- requests at time 0
- wait until time  $t = 2$



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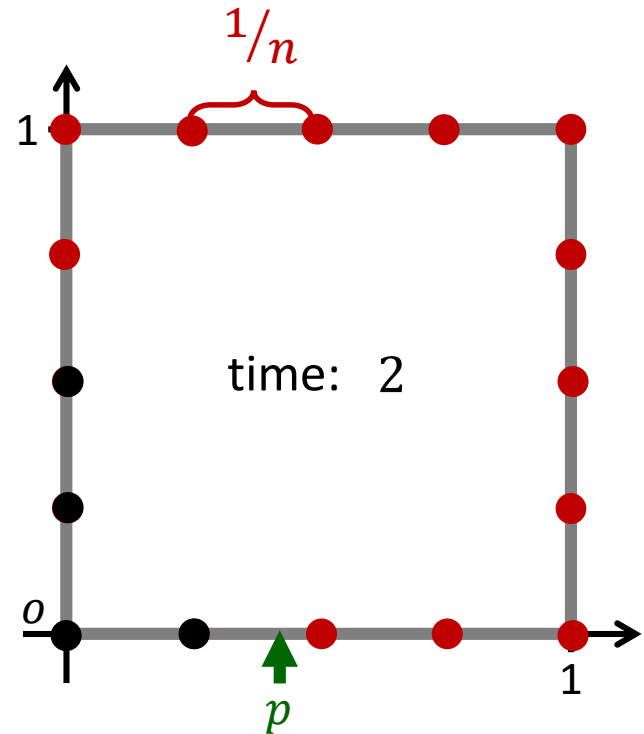
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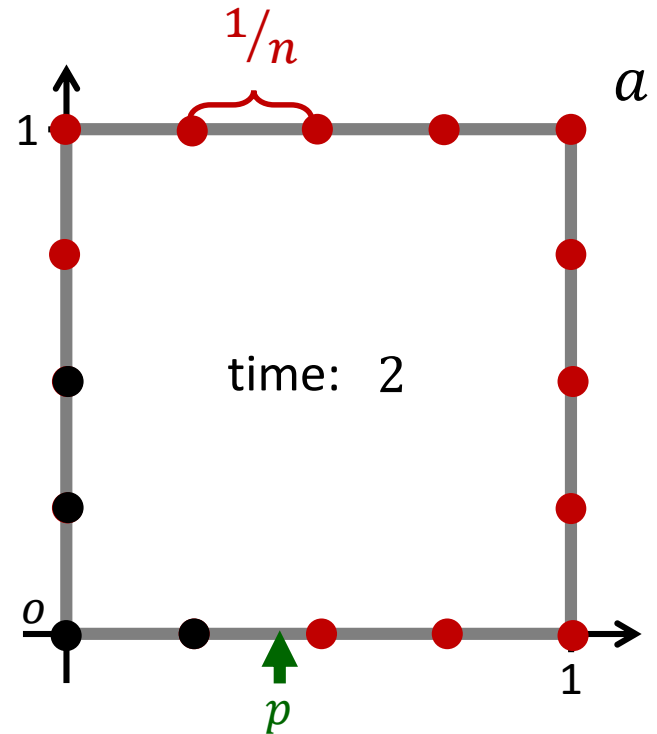
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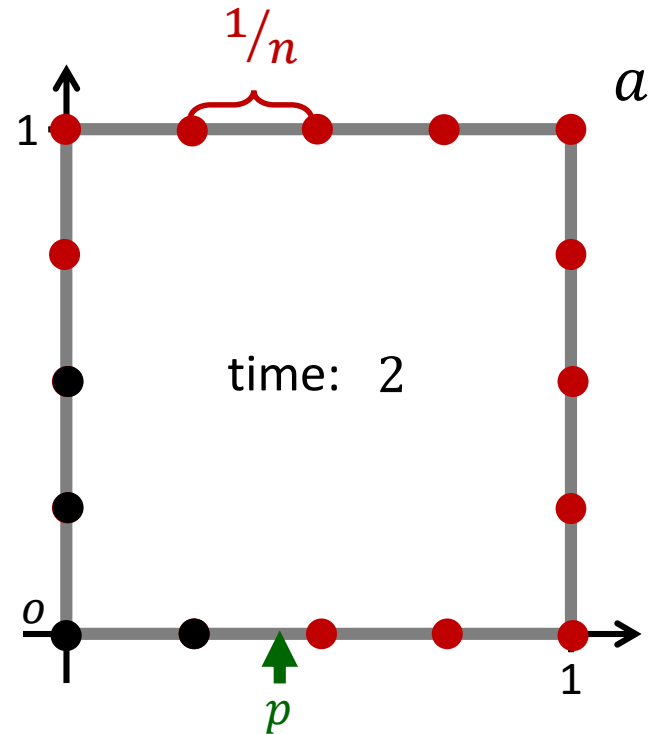
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# Lower Bound for H-OLTSP

- requests at time 0
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- wait until  $d(p, a) = t - 2$



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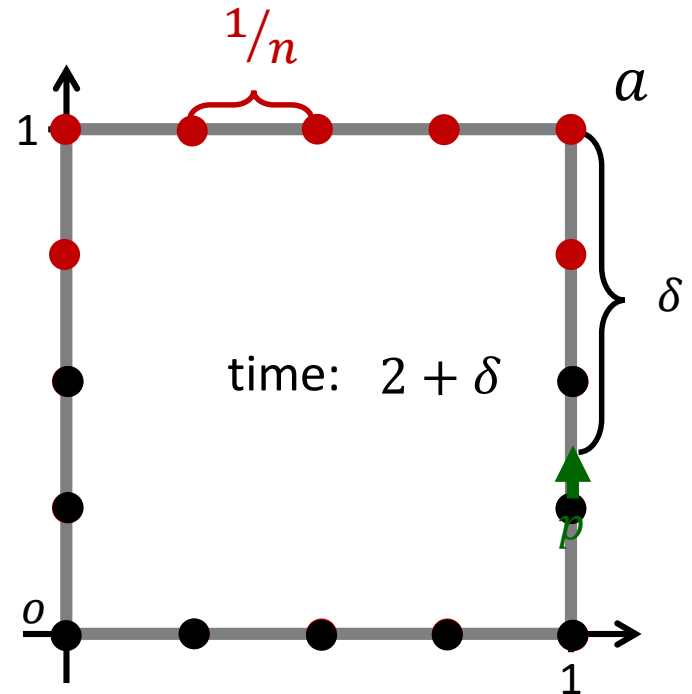
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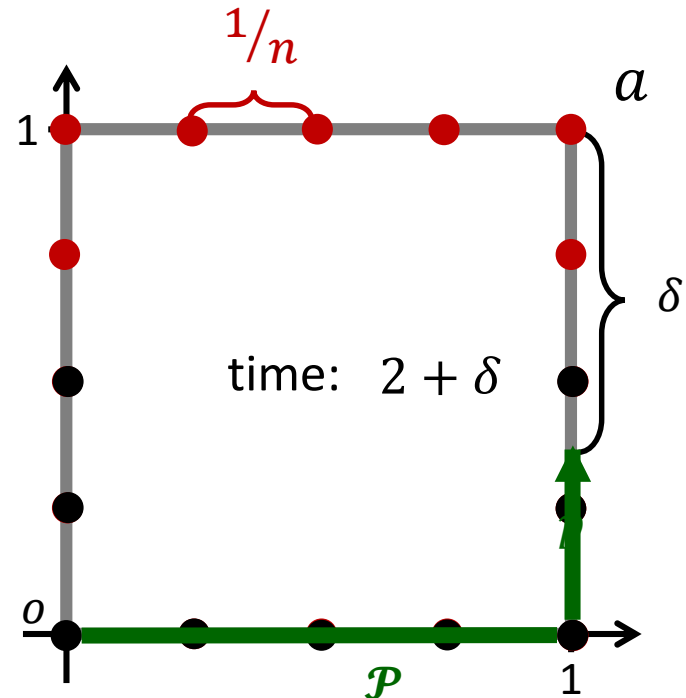
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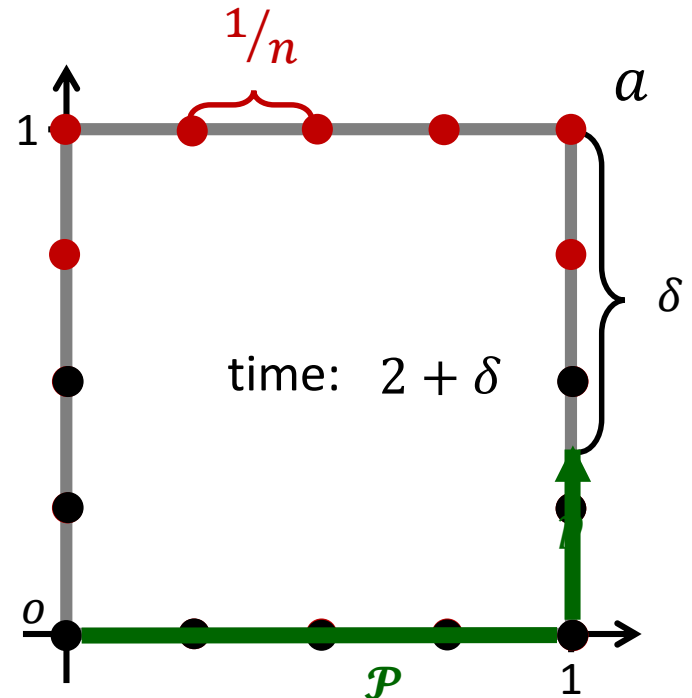
H-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 1,5$

# Lower Bound for H-OLTSP

- requests at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$



$$|\mathcal{P}| =$$

Online-TSP

I. Algorithms

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GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

H-OLTSP

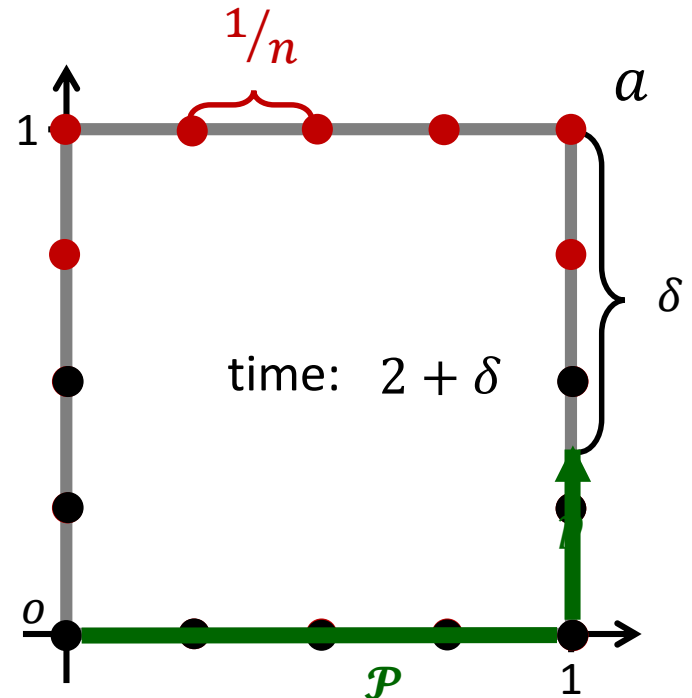
GTR (Greedy):  $\rho = 2,5$

$\rho \geq 1,5$



# Lower Bound for H-OLTSP

- requests at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$



$$|\mathcal{P}| = 2 - \delta$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

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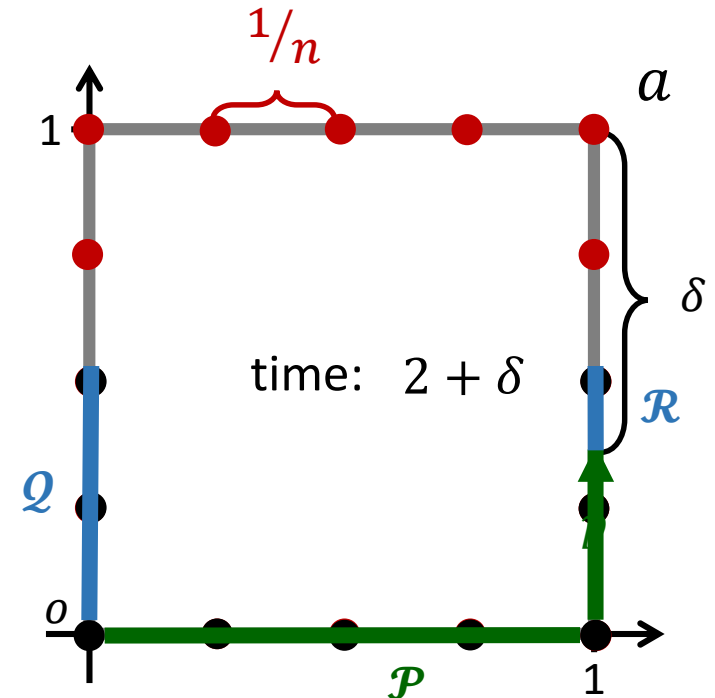
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# Lower Bound for H-OLTSP

- requests at time 0
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II. Lower Bounds

III. Polynomial Algorithms

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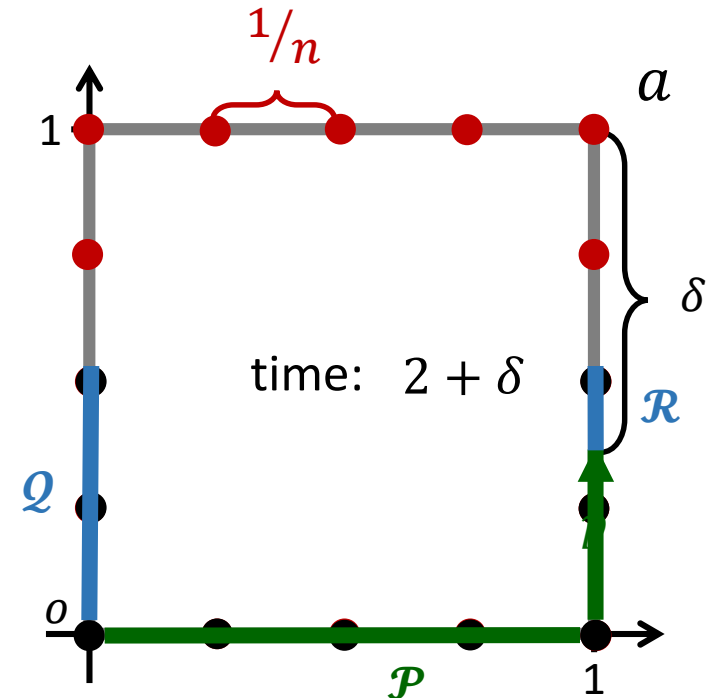
H-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 1,5$

# Lower Bound for H-OLTSP

- requests at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$



$$|\mathcal{P}| = 2 - \delta$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

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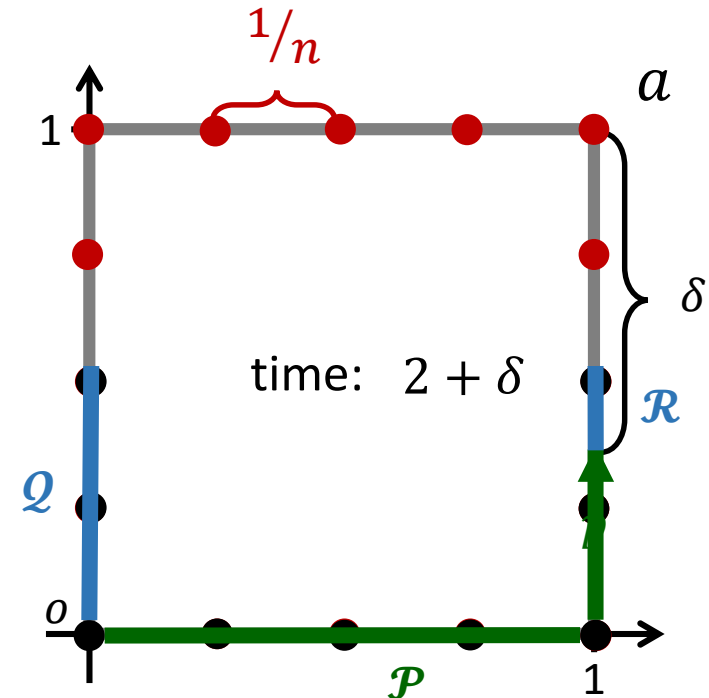
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$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq 2 + \delta$$

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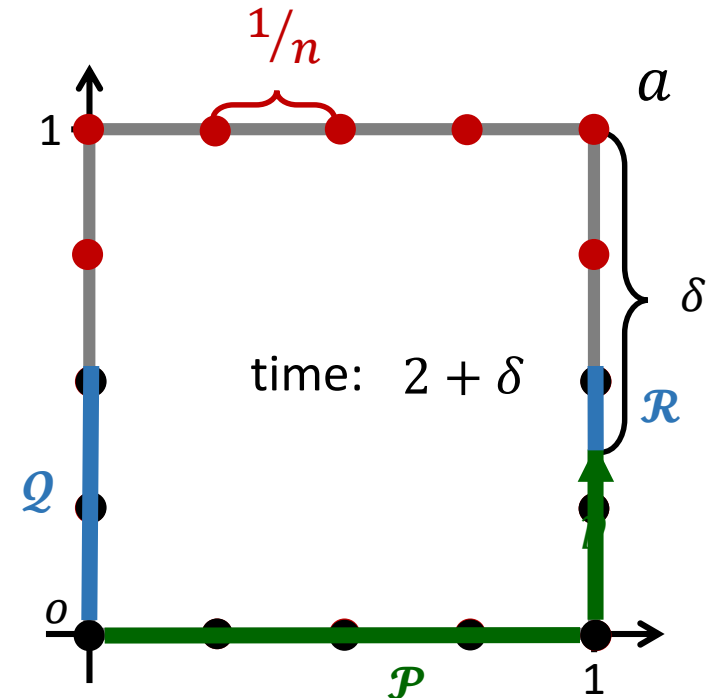
H-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 1,5$

# Lower Bound for H-OLTSP

- requests at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$



$$\left. \begin{array}{l} |\mathcal{P}| = 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq 2 + \delta \end{array} \right\} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}|$$

Online-TSP

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II. Lower Bounds

III. Polynomial Algorithms

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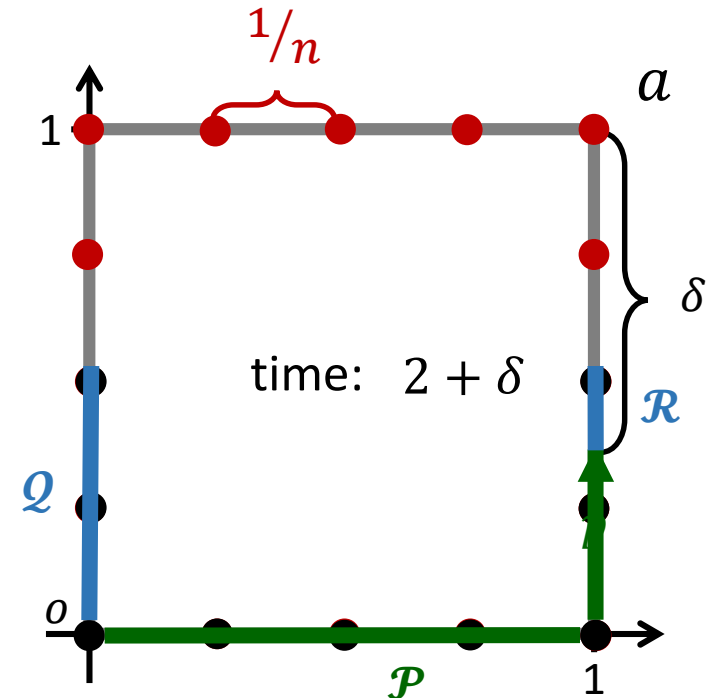
H-OLTSP

GTR (Greedy):  $\rho = 2,5$

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# Lower Bound for H-OLTSP

- requests at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$



$$\left. \begin{array}{l} |\mathcal{P}| = 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq 2 + \delta \end{array} \right\} \quad |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2$$

Online-TSP

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II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

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$\rho \geq 2$

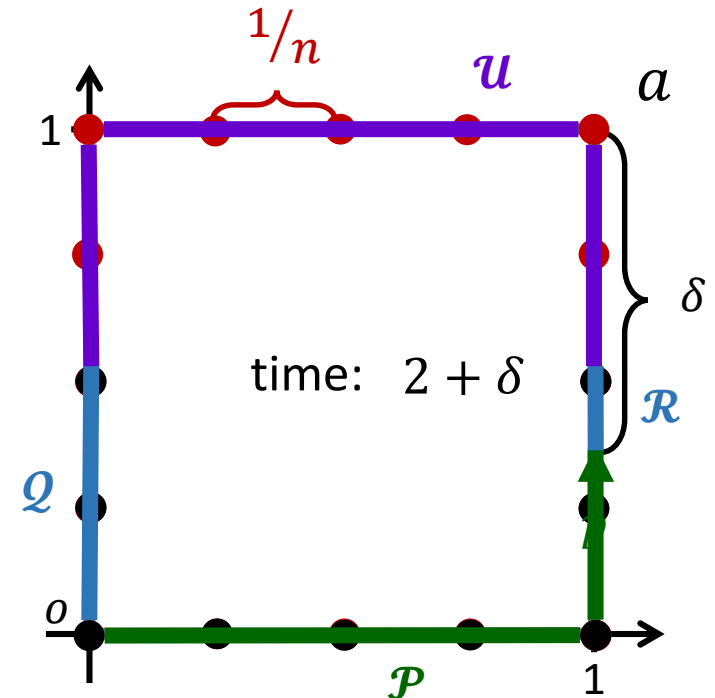
H-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 1,5$

# Lower Bound for H-OLTSP

- **requests** at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$



$$\left. \begin{array}{l} |\mathcal{P}| = 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq 2 + \delta \end{array} \right\} \begin{array}{l} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2 \\ \Rightarrow |\mathcal{U}| \end{array}$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

**N-OLTSP**

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

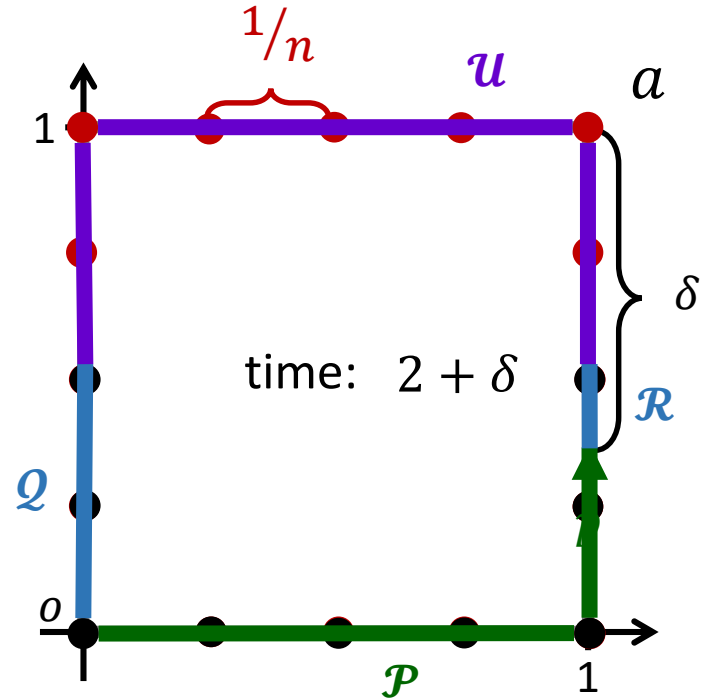
**H-OLTSP**

GTR (Greedy):  $\rho = 2,5$



$\rho \geq 1,5$

# Lower Bound for H-OLTSP

- requests at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$



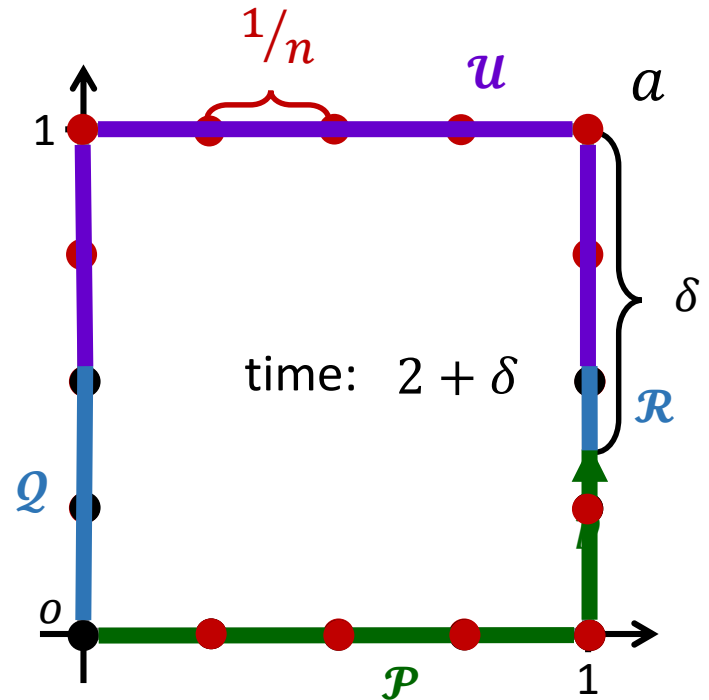
$$\left. \begin{array}{l} |\mathcal{P}| = 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq 2 + \delta \end{array} \right\} \begin{array}{l} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2 \\ \Rightarrow |\mathcal{U}| \geq 2 \end{array}$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
<b>H-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 1,5$	





# Lower Bound for H-OLTSP

- requests at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$
- new requests on  $\mathcal{P}$

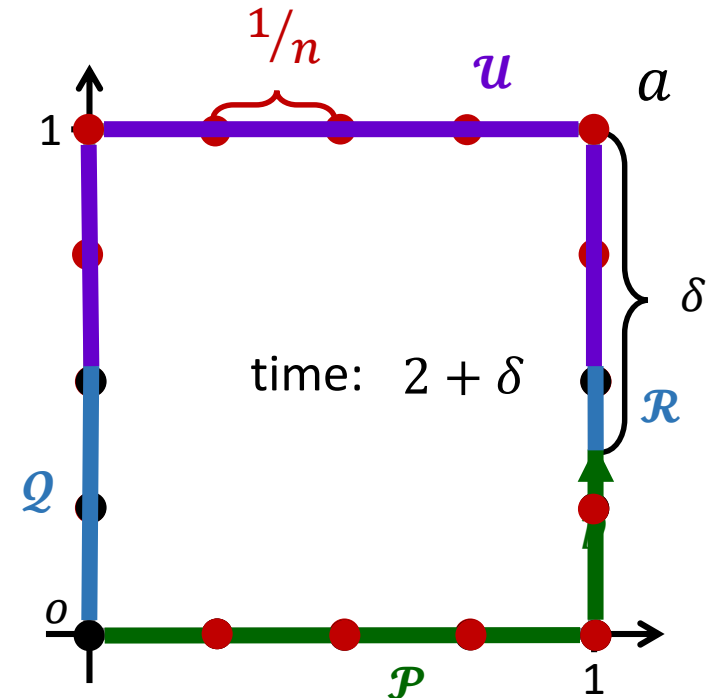


$$\left. \begin{array}{l} |\mathcal{P}| = 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq 2 + \delta \end{array} \right\} \begin{array}{l} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2 \\ \Rightarrow |\mathcal{U}| \geq 2 \end{array}$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
<b>H-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 1,5$	

# Lower Bound for H-OLTSP

- **requests** at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$
- **new requests** on  $\mathcal{P}$
- OPT finishes at  $t = 4$



$$\left. \begin{array}{l} |\mathcal{P}| = 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq 2 + \delta \end{array} \right\} \begin{array}{l} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2 \\ \Rightarrow |\mathcal{U}| \geq 2 \end{array}$$

Online-TSP

I. Algorithms

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GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

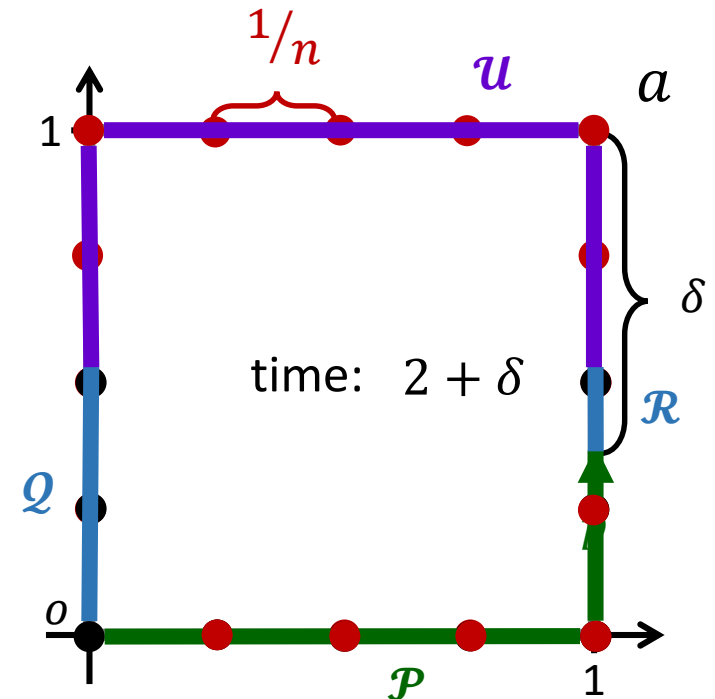
H-OLTSP

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# Lower Bound for H-OLTSP

- **requests** at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$
- **new requests** on  $\mathcal{P}$
- OPT finishes at  $t = 4$
- ALG finishes at  $t \geq$



$$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2$$

$$\Rightarrow |\mathcal{U}| \geq 2$$

Online-TSP

I. Algorithms

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N-OLTSP

GTR (Greedy):  $\rho = 2,5$

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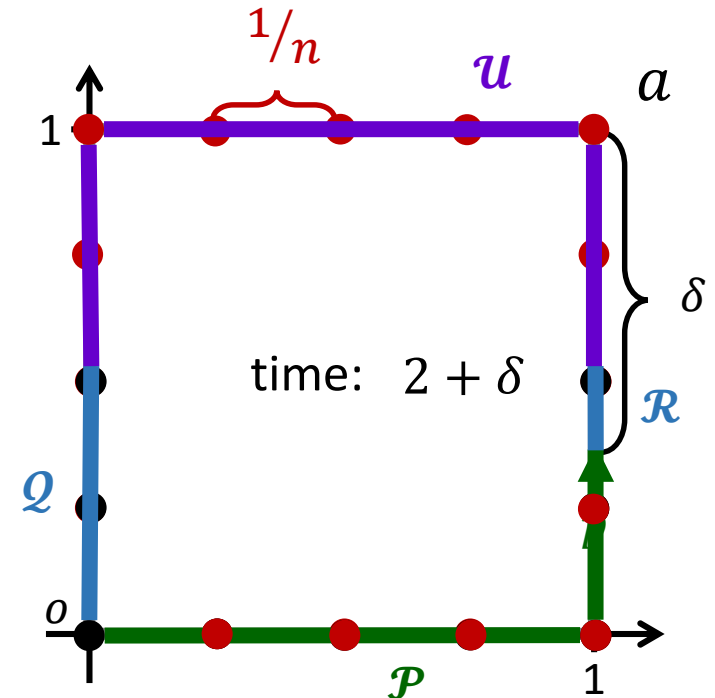
H-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 1,5$

# Lower Bound for H-OLTSP

- **requests** at time 0
- wait until time  $t = 2$
- wait until  $d(p, a) = t - 2$
- **new requests** on  $\mathcal{P}$
- OPT finishes at  $t = 4$
- ALG finishes at  $t \geq$



$$|Q| + |P| + |R| \leq 2$$

$$\Rightarrow |U| \geq 2$$

Online-TSP

I. Algorithms

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H-OLTSP

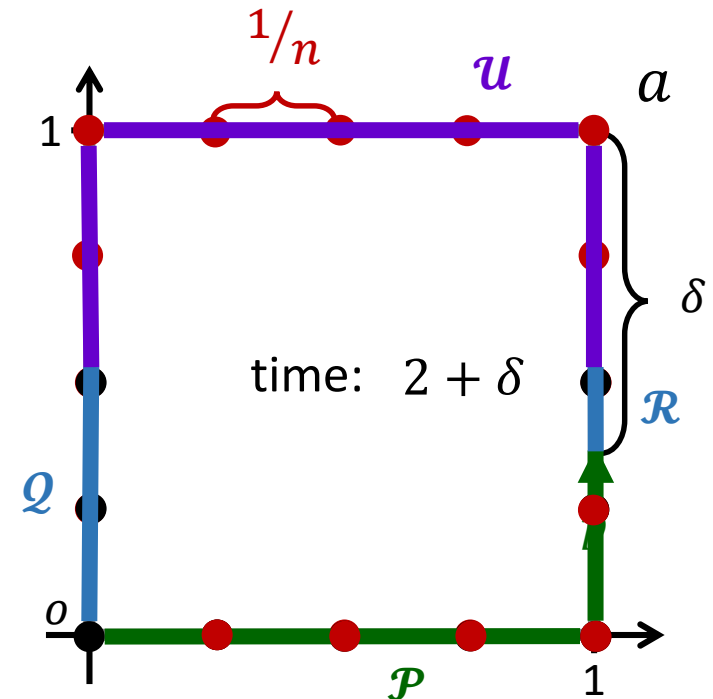
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# Lower Bound for H-OLTSP

- **requests** at time 0
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- **new requests** on  $\mathcal{P}$
- OPT finishes at  $t = 4$
- ALG finishes at  $t \geq$

$$\begin{cases} \geq 2 + \delta + 2 \cdot \left(2 - \frac{1}{n}\right) + 2 - \delta \\ \geq 2 + \delta + 4 - \frac{1}{n} + (2 - \delta) - \frac{1}{n} \end{cases}$$



$$\begin{aligned} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| &\leq 2 \\ \Rightarrow |\mathcal{U}| &\geq 2 \end{aligned}$$

Online-TSP

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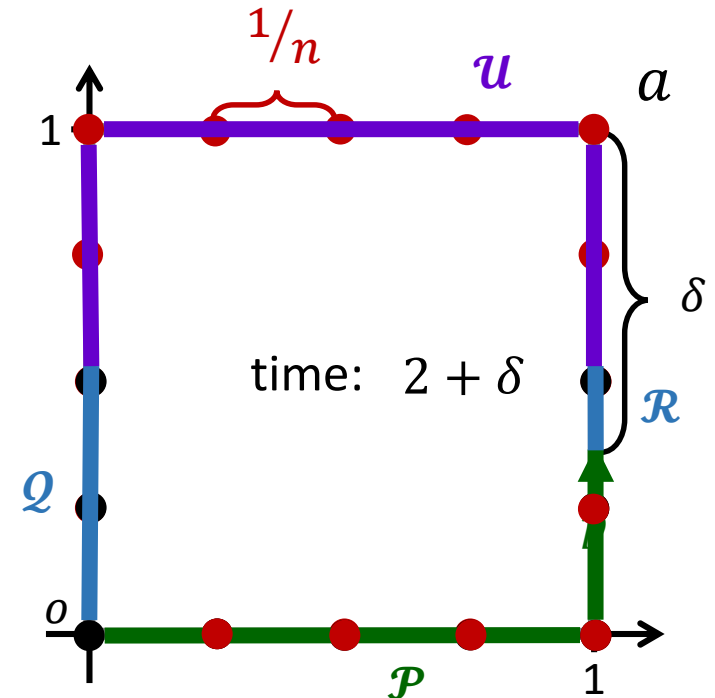
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- **requests** at time 0
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- OPT finishes at  $t = 4$
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$$\begin{cases} \geq 2 + \delta + 2 \cdot \left(2 - \frac{1}{n}\right) + 2 - \delta \\ \geq 2 + \delta + 4 - \frac{1}{n} + (2 - \delta) - \frac{1}{n} \end{cases}$$



$$|Q| + |P| + |R| \leq 2$$

$$\Rightarrow |U| \geq 2$$

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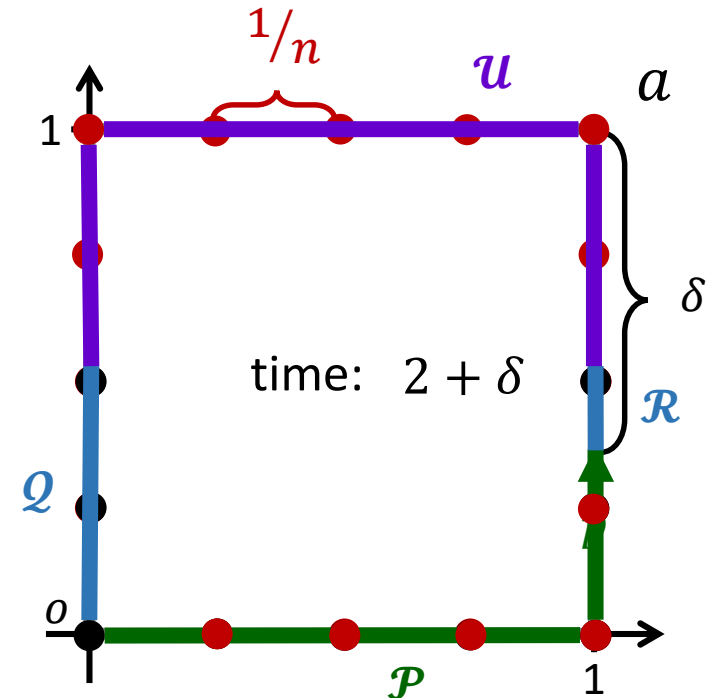
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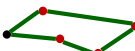
$\rho \geq 2$

H-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

# A better algorithm for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
<b>H-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	



# A better algorithm for H-OLTSP



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

**N-OLTSP** 

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

**H-OLTSP** 

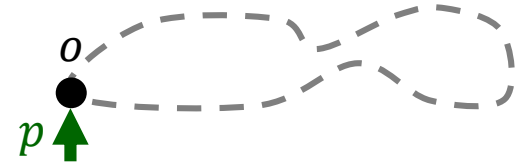
GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

# A better algorithm for H-OLTSP

$U :=$  places yet to visit

(1) At  $o$ : start optimal tour through  $U$



Online-TSP

I. Algorithms

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GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

**H-OLTSP**

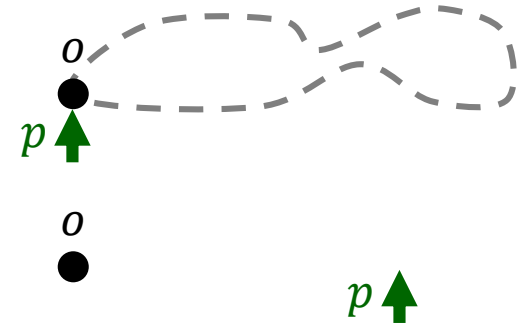
GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

# A better algorithm for H-OLTSP

$U :=$  places yet to visit

- (1) At  $o$ : start optimal tour through  $U$
- (2) For new request  $(t, x)$ :



Online-TSP

I. Algorithms

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III. Polynomial Algorithms

**N-OLTSP**

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$\rho \geq 2$

**H-OLTSP**

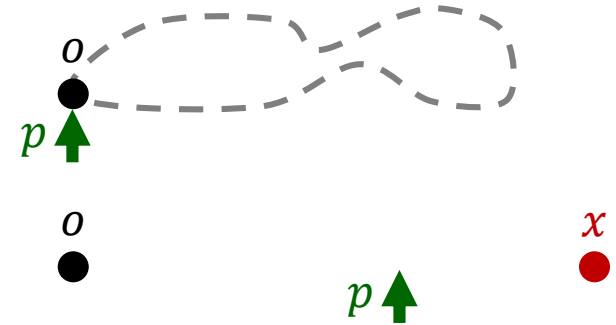
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$\rho \geq 2$

**H-OLTSP**

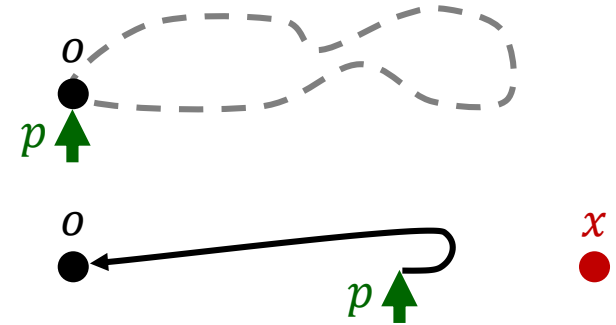
GTR (Greedy):  $\rho = 2,5$


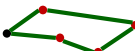
$\rho \geq 2$

# A better algorithm for H-OLTSP

$U :=$  places yet to visit

- (1) At  $o$ : start optimal tour through  $U$
- (2) For new request  $(t, x)$ :
  - a) If  $d(x, o) > d(p, o)$ : go back to  $o$

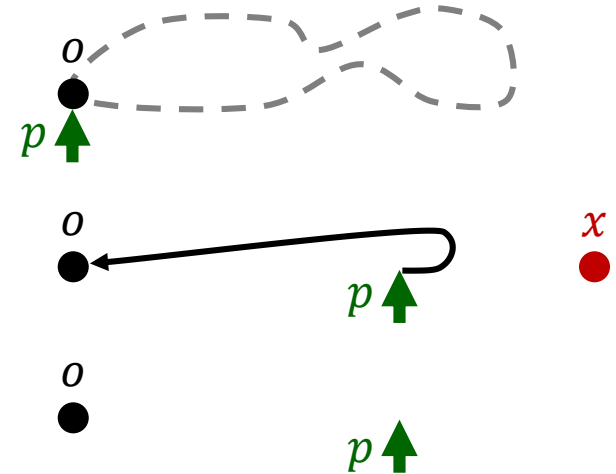


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
<b>H-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	

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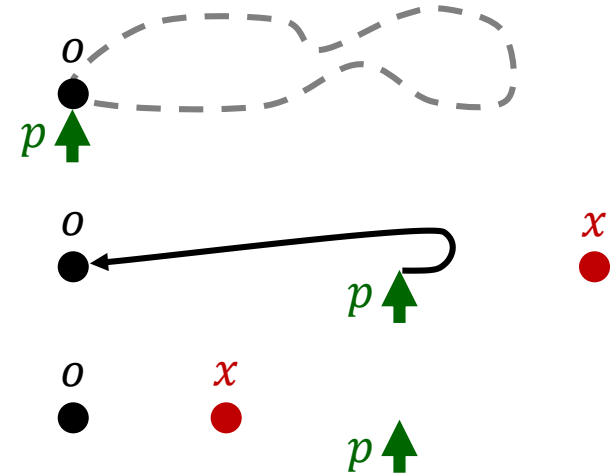


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

**N-OLTSP**

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

**H-OLTSP**

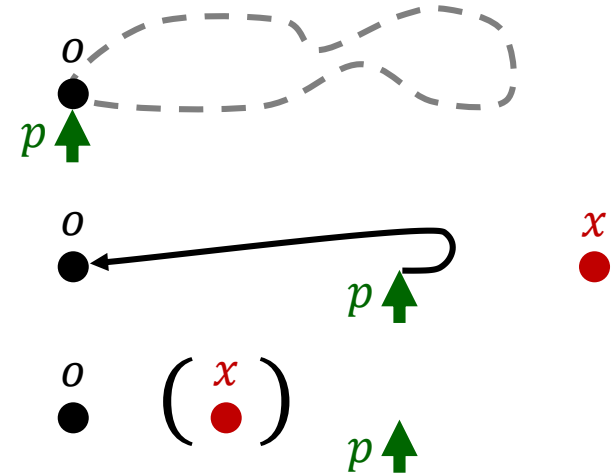
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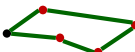
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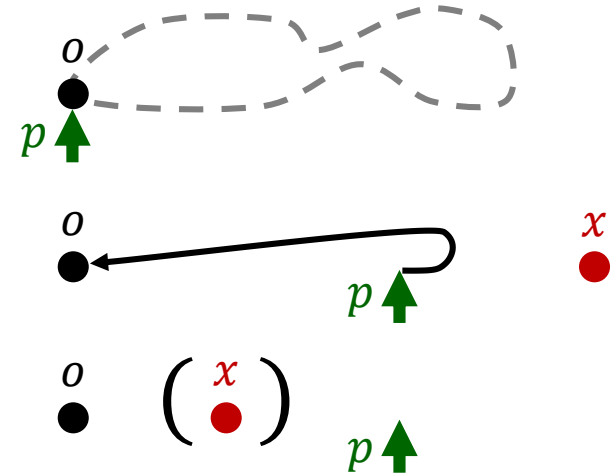
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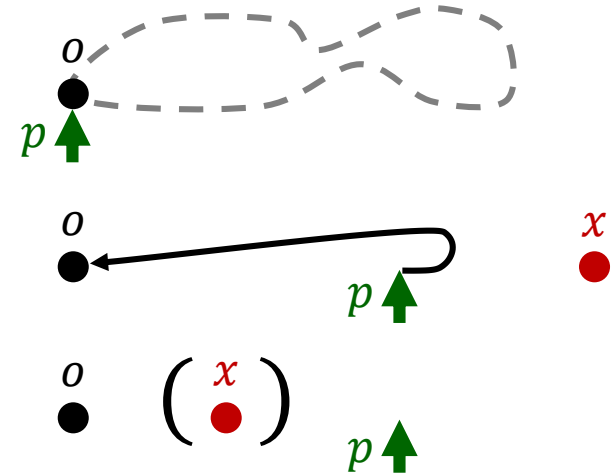
## Plan At Home (PAH)

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b>	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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## Plan At Home (PAH)

**GOAL:** PAH is 2-competitive for H-OLTSP

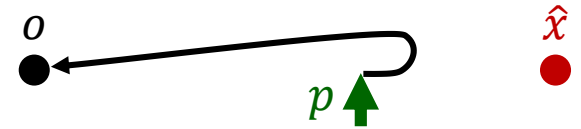
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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# Competitiveness of PAH


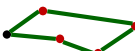
$U :=$  places yet to visit,  $(\hat{t}, \hat{x})$  last request

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**GOAL:** PAH is 2-competitive for H-OLTSP

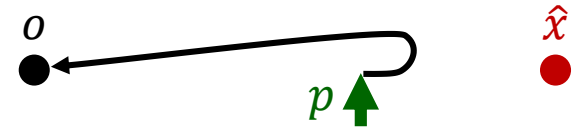
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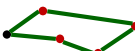
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$|\mathcal{T}^{\text{PAH}}|$

**GOAL:** PAH is 2-competitive for H-OLTSP

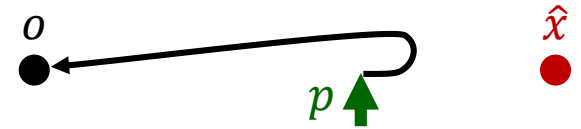
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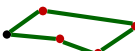
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$$|\mathcal{T}^{\text{PAH}}| = \hat{t} +$$

**GOAL:** PAH is 2-competitive for H-OLTSP

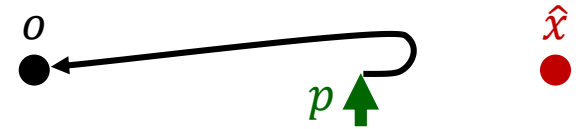
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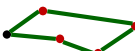
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$$|\mathcal{T}^{\text{PAH}}| = \hat{t} + d(p, o)$$

**GOAL:** PAH is 2-competitive for H-OLTSP

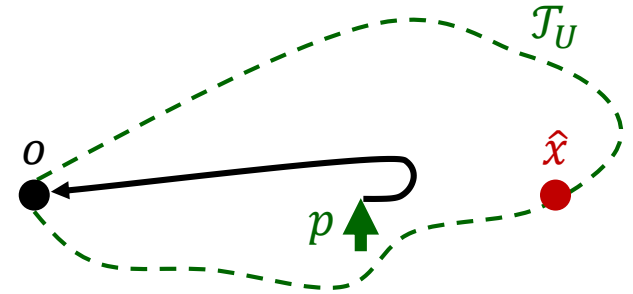
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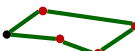
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$$|\mathcal{J}^{\text{PAH}}| = \hat{t} + d(p, o) + |\mathcal{J}_U|$$

**GOAL:** PAH is 2-competitive for H-OLTSP

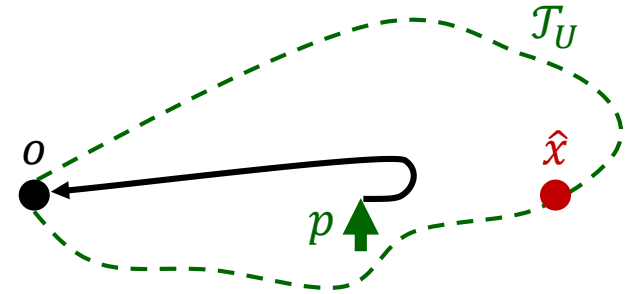
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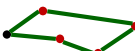
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$$\leq |\mathcal{J}^{\text{OPT}}|$$

**GOAL:** PAH is 2-competitive for H-OLTSP

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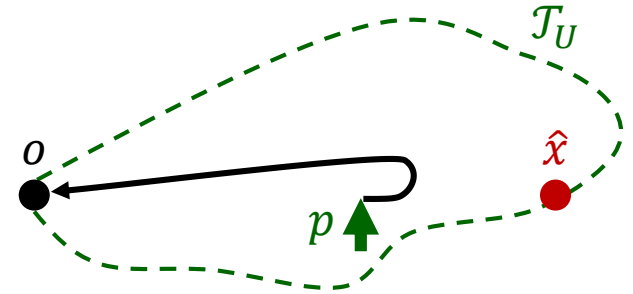


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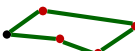
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$$|\mathcal{J}^{\text{PAH}}| \leq \hat{t} + d(\hat{x}, o) + |\mathcal{J}_U|$$

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**GOAL:** PAH is 2-competitive for H-OLTSP

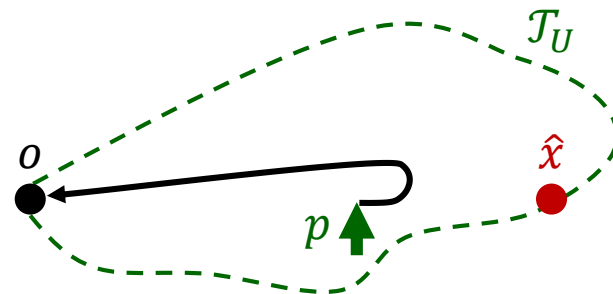
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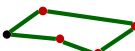
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$$|\mathcal{T}^{\text{PAH}}| \leq \underbrace{\hat{t} + d(\hat{x}, o)} + |\mathcal{J}_U|$$

$$\leq |\mathcal{T}^{\text{OPT}}|$$

**GOAL:** PAH is 2-competitive for H-OLTSP

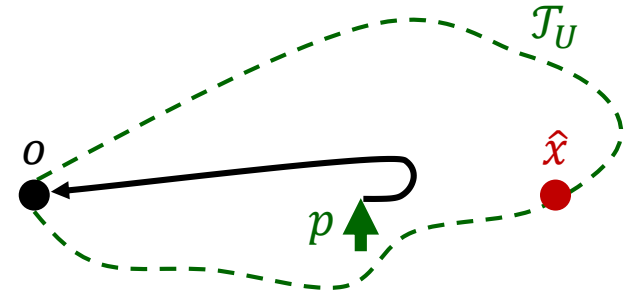
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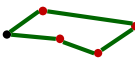
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$$\begin{aligned}
 |\mathcal{J}^{\text{PAH}}| &\leq \underbrace{\hat{t} + d(\hat{x}, o)} + |\mathcal{J}_U| \\
 &\leq |\mathcal{J}^{\text{OPT}}| + |\mathcal{J}^{\text{OPT}}|
 \end{aligned}$$

**GOAL:** PAH is 2-competitive for H-OLTSP

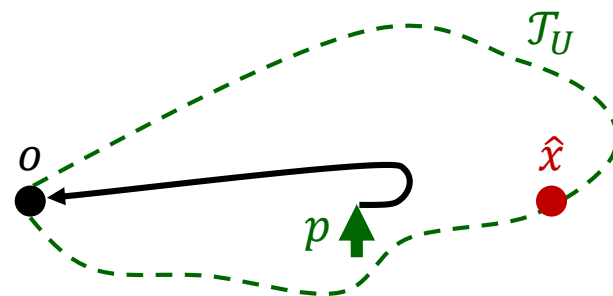
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$$\begin{aligned}
 |\mathcal{J}^{\text{PAH}}| &\leq \underbrace{\hat{t} + d(\hat{x}, o)} + |\mathcal{J}_U| \\
 &\leq |\mathcal{J}^{\text{OPT}}| + |\mathcal{J}^{\text{OPT}}| = 2 \cdot |\mathcal{J}^{\text{OPT}}|
 \end{aligned}$$

**GOAL:** PAH is 2-competitive for H-OLTSP

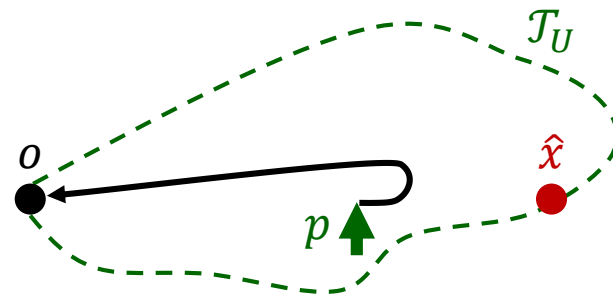
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$$\begin{aligned}
 |\mathcal{T}^{\text{PAH}}| &\leq \underbrace{\hat{t} + d(\hat{x}, o)} + |\mathcal{J}_U| \\
 &\leq |\mathcal{T}^{\text{OPT}}| + |\mathcal{T}^{\text{OPT}}| = 2 \cdot |\mathcal{T}^{\text{OPT}}| \quad \checkmark
 \end{aligned}$$

GOAL:

PAH is 2-competitive for H-OLTSP

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

H-OLTSP

GTR (Greedy):  $\rho = 2,5$

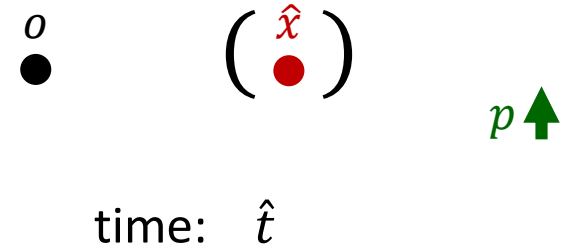
$\rho \geq 2$

# Competitiveness of PAH


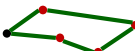
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**GOAL:** PAH is 2-competitive for H-OLTSP

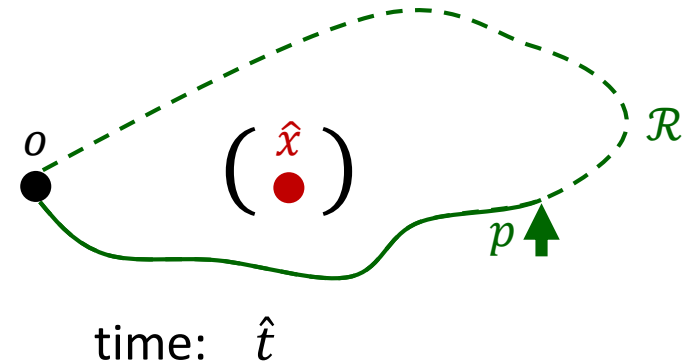
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**GOAL:** PAH is 2-competitive for H-OLTSP

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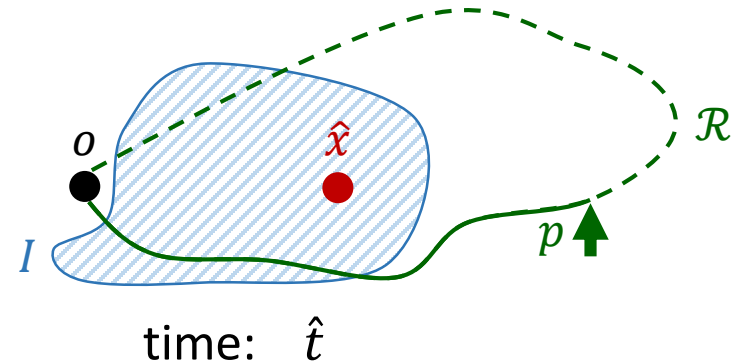
# Competitiveness of PAH

$U :=$  places yet to visit,  $(\hat{t}, \hat{x})$  last request

$I :=$  ignored requests

(2) For new request  $(\hat{t}, \hat{x})$ :

b)  $d(\hat{x}, o) > d(p, o)$ : ignore  $\hat{x}$  ...



**GOAL:** PAH is 2-competitive for H-OLTSP

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

**N-OLTSP**

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$\rho \geq 2$

**H-OLTSP**

GTR (Greedy):  $\rho = 2,5$

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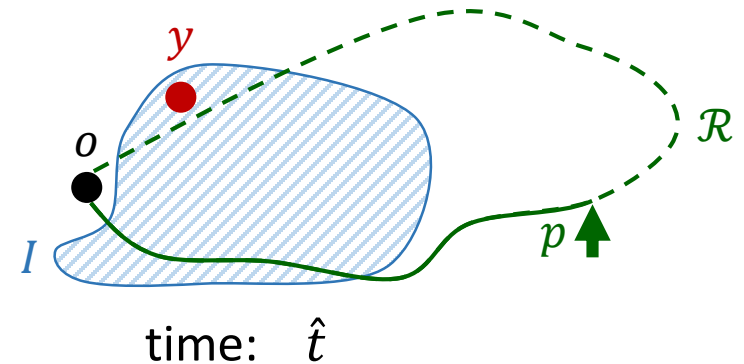
# Competitiveness of PAH

$U$  := places yet to visit,  $(\hat{t}, \hat{x})$  last request, **y** **fst** place in  $I$  visited by OPT

$I$  := ignored requests

(2) For new request  $(\hat{t}, \hat{x})$ :

b)  $d(\hat{x}, o) > d(p, o)$ : ignore  $\hat{x}$  ...



**GOAL:** PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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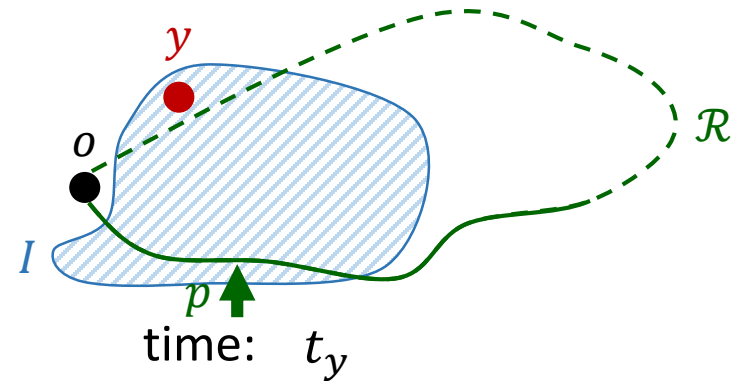
# Competitiveness of PAH

$U :=$  places yet to visit,  $(\hat{t}, \hat{x})$  last request,  $y$  1st place in  $I$  visited by OPT

$I :=$  ignored requests

(2) For new request  $(\hat{t}, \hat{x})$ :

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GOAL:

PAH is 2-competitive for H-OLTSP

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

GTR (Greedy):  $\rho = 2,5$

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H-OLTSP

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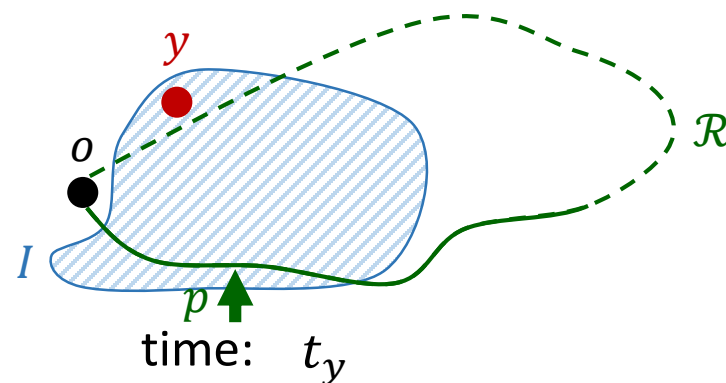
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$U :=$  places yet to visit,  $(\hat{t}, \hat{x})$  last request,  $y$  1st place in  $I$  visited by OPT

$I :=$  ignored requests


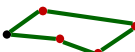
(2) For new request  $(t_y, y)$ :

b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...



GOAL:

PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	

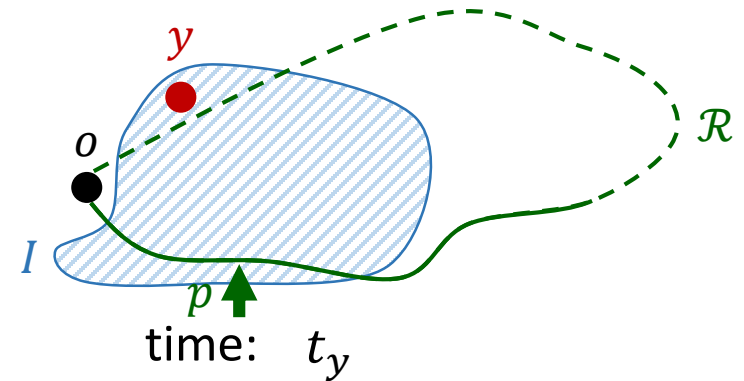
# Competitiveness of PAH

$U :=$  places yet to visit,  $(\hat{t}, \hat{x})$  last request,  $y$  1st place in  $I$  visited by OPT

$I :=$  ignored requests

(2) For new request  $(t_y, y)$ :


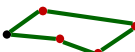
b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...



$|\mathcal{T}^{\text{PAH}}|$

GOAL:

PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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# Competitiveness of PAH

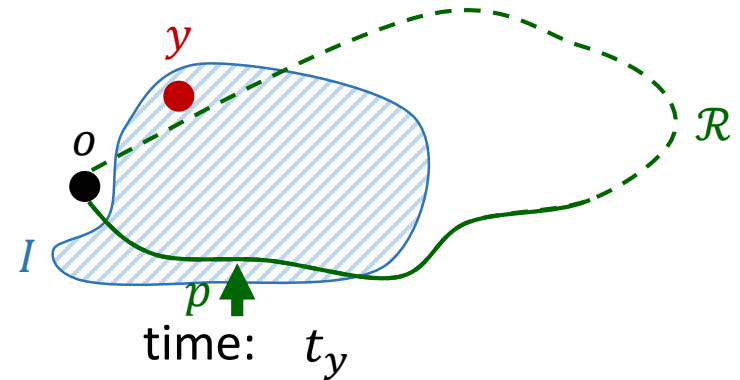
$U$  := places yet to visit,  $(\hat{t}, \hat{x})$  last request,  $y$  1st place in  $I$  visited by OPT

$I$  := ignored requests

(2) For new request  $(t_y, y)$ :

b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...

$$|\mathcal{T}^{\text{PAH}}| \leq t_y$$



GOAL:

PAH is 2-competitive for H-OLTSP

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

GTR (Greedy):  $\rho = 2,5$

$$\rho \geq 2$$

H-OLTSP

GTR (Greedy):  $\rho = 2,5$

$$\rho \geq 2$$

# Competitiveness of PAH

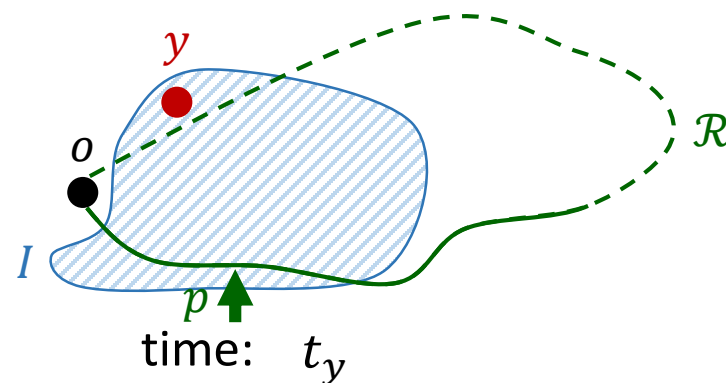
$U :=$  places yet to visit,  $(\hat{t}, \hat{x})$  last request,  $y$  1st place in  $I$  visited by OPT

$I :=$  ignored requests


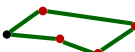
(2) For new request  $(t_y, y)$ :

b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...

$$|\mathcal{T}^{\text{PAH}}| \leq t_y + |\mathcal{R}|$$



**GOAL:** PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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# Competitiveness of PAH

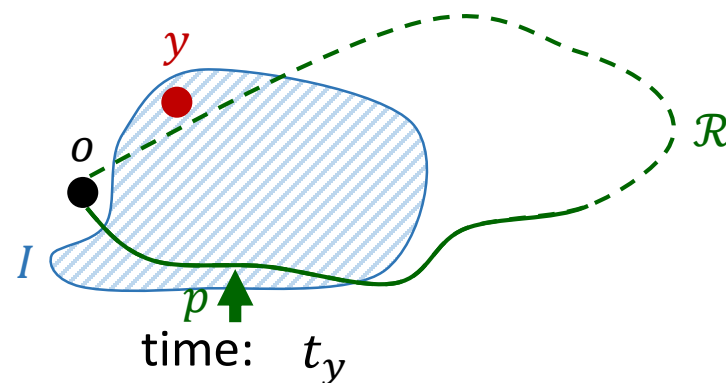
$U :=$  places yet to visit,  $(\hat{t}, \hat{x})$  last request,  $y$  1st place in  $I$  visited by OPT

$I :=$  ignored requests

(2) For new request  $(t_y, y)$ :

b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...

$$|\mathcal{T}^{\text{PAH}}| \leq t_y + |\mathcal{R}| - d(o, y)$$



GOAL:

PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	

# Competitiveness of PAH

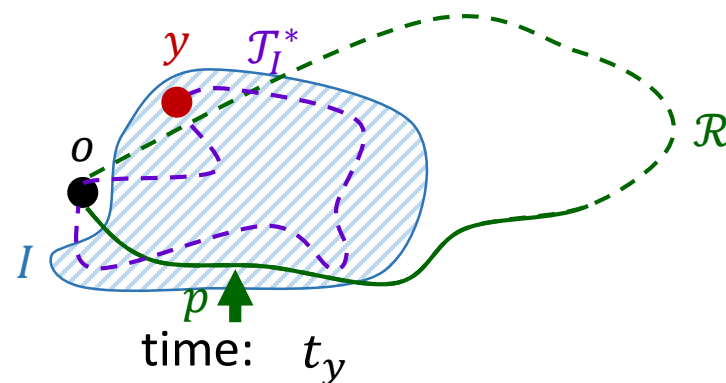
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GOAL: PAH is 2-competitive for H-OLTSP

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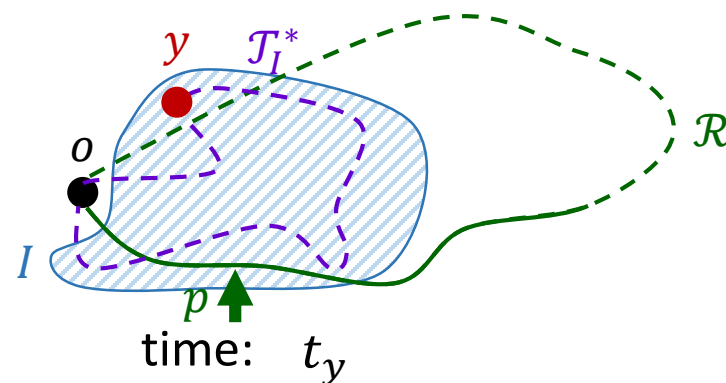
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(2) For new request  $(t_y, y)$ :

b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...



$$|\mathcal{T}^{\text{PAH}}| \leq t_y + |\mathcal{R}| - d(o, y) + |\mathcal{J}_I^*|$$

**GOAL:** PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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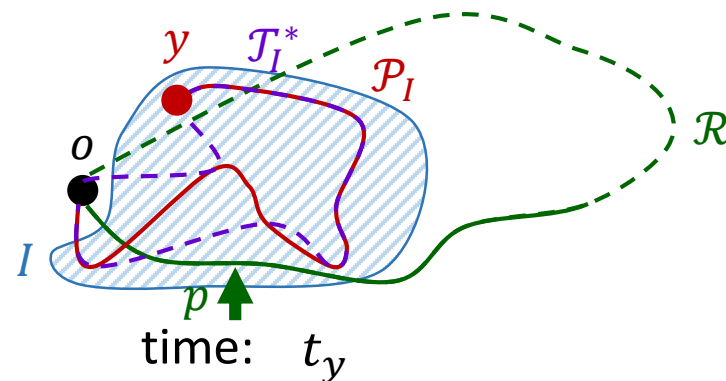
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$$|\mathcal{T}^{\text{PAH}}| \leq t_y + |\mathcal{R}| - d(o, y) + |\mathcal{J}_I^*|$$

GOAL:

PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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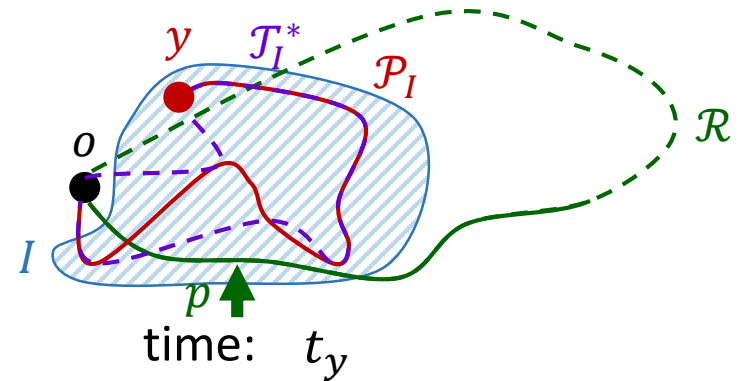
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$$|\mathcal{T}^{\text{PAH}}| \leq t_y + |R| - \underbrace{d(o, y) + |J_I^*|}$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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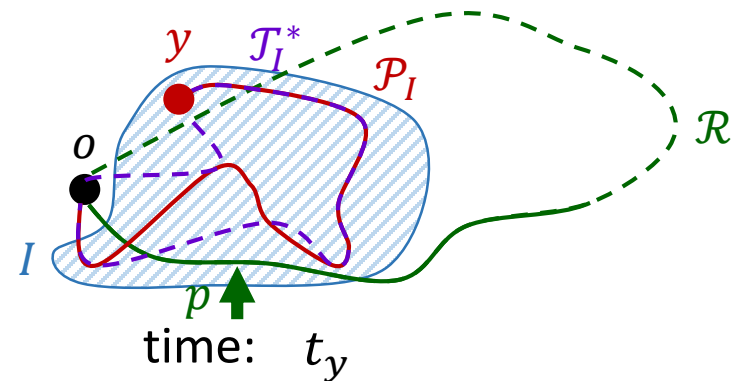
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$$|\mathcal{T}^{\text{PAH}}| \leq t_y + |\mathcal{R}| \underbrace{- d(o, y) + |\mathcal{J}_I^*|}_{|\mathcal{P}_I|}$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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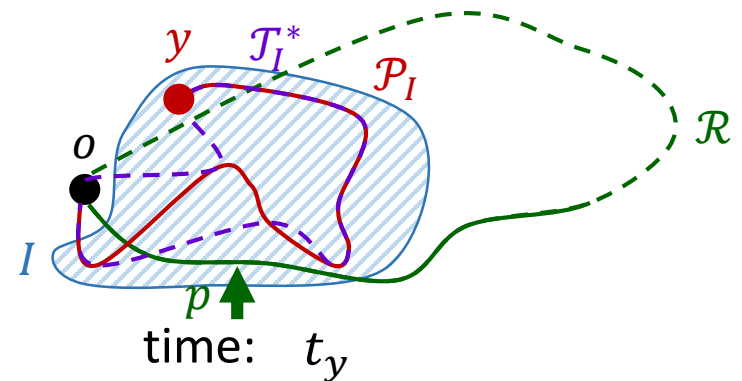
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$$|\mathcal{T}^{\text{PAH}}| \leq t_y + |\mathcal{R}| \underbrace{- d(o, y) + |\mathcal{J}_I^*|}_{\leq t_y + |\mathcal{R}| + |\mathcal{P}_I|}$$

**GOAL:** PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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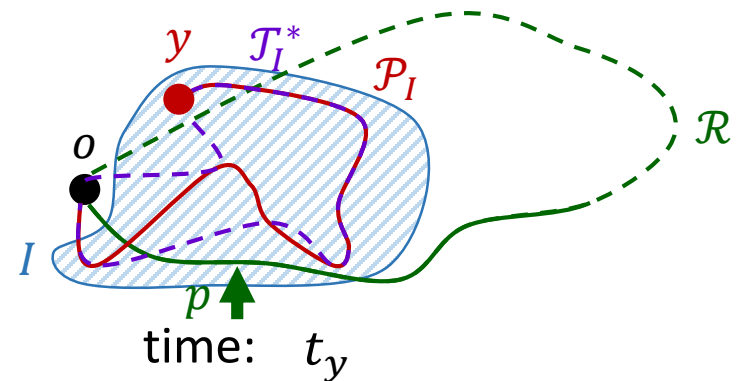
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(2) For new request  $(t_y, y)$ :

b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...



$$\begin{aligned}
 |\mathcal{T}^{\text{PAH}}| &\leq t_y + |\mathcal{R}| + \underbrace{-d(o, y) + |\mathcal{J}_I^*|}_{|\mathcal{P}_I|} \\
 &\leq t_y + |\mathcal{R}| + |\mathcal{P}_I| = t_y + |\mathcal{P}_I| + |\mathcal{R}|
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**GOAL:** PAH is 2-competitive for H-OLTSP

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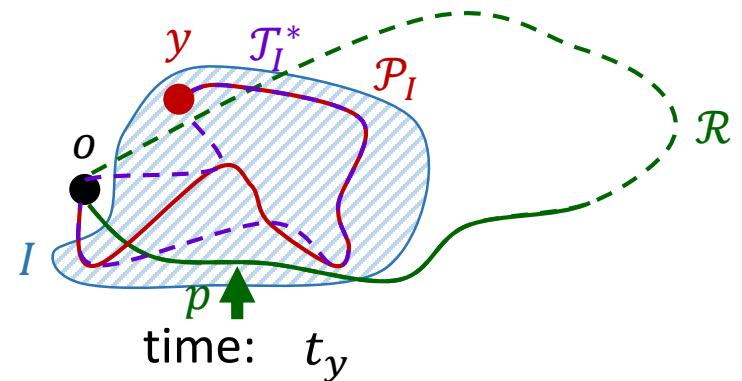
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 |\mathcal{T}^{\text{PAH}}| &\leq t_y + |\mathcal{R}| - \underbrace{d(o, y) + |\mathcal{J}_I^*|}_{|\mathcal{P}_I|} \\
 &\leq t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|}_{|\mathcal{R}|} + |\mathcal{R}|
 \end{aligned}$$

**GOAL:** PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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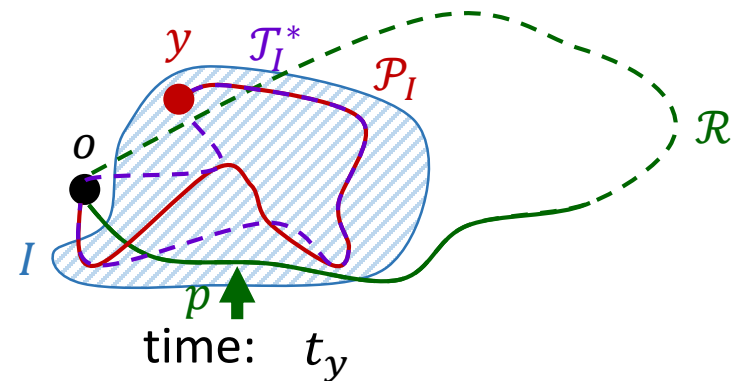
# Competitiveness of PAH

$U :=$  places yet to visit,  $(\hat{t}, \hat{x})$  last request,  $y$  1st place in  $I$  visited by OPT

$I :=$  ignored requests

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b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...



$$\begin{aligned}
 |\mathcal{T}^{\text{PAH}}| &\leq t_y + |\mathcal{R}| + \underbrace{-d(o, y) + |\mathcal{J}_I^*|}_{\leq |\mathcal{P}_I|} \\
 &\leq t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|}_{\leq |\mathcal{T}^{\text{OPT}}|} + |\mathcal{R}| \\
 &\leq |\mathcal{T}^{\text{OPT}}|
 \end{aligned}$$

**GOAL:** PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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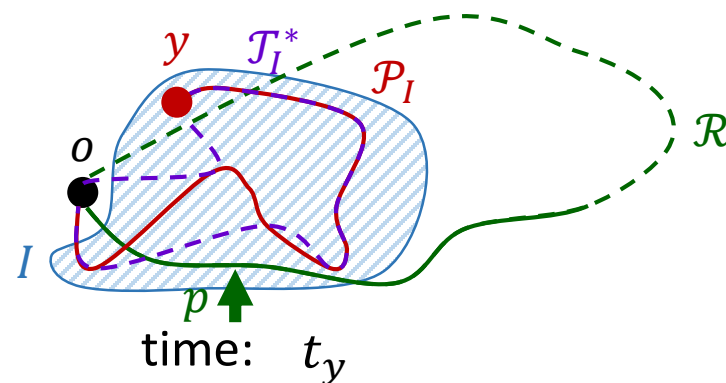
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$$\begin{aligned}
 |\mathcal{T}^{\text{PAH}}| &\leq t_y + |\mathcal{R}| - \underbrace{d(o, y) + |\mathcal{J}_I^*|}_{\leq |\mathcal{P}_I|} \\
 &\leq t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|}_{\leq |\mathcal{T}^{\text{OPT}}|} + |\mathcal{R}| \\
 &\leq |\mathcal{T}^{\text{OPT}}| + |\mathcal{T}^{\text{OPT}}|
 \end{aligned}$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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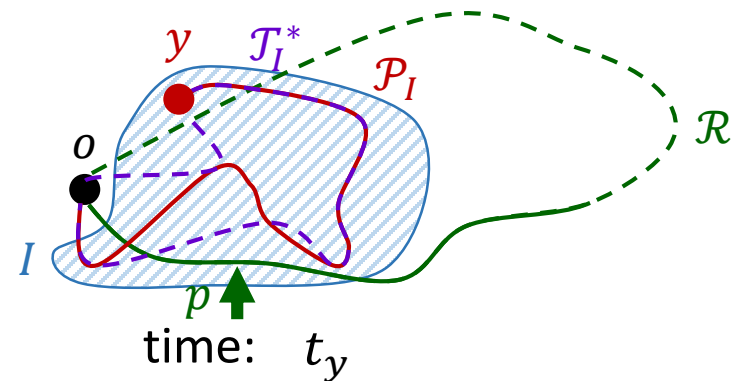
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$U :=$  places yet to visit,  $(\hat{t}, \hat{x})$  last request,  $y$  1st place in  $I$  visited by OPT

$I :=$  ignored requests

(2) For new request  $(t_y, y)$ :

b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...



$$\begin{aligned}
 |\mathcal{T}^{\text{PAH}}| &\leq t_y + |\mathcal{R}| - d(o, y) + |\mathcal{J}_I^*| \\
 &\leq t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|}_{|\mathcal{T}^{\text{OPT}}|} + |\mathcal{R}| \\
 &\leq |\mathcal{T}^{\text{OPT}}| + |\mathcal{T}^{\text{OPT}}| = 2 \cdot |\mathcal{T}^{\text{OPT}}|
 \end{aligned}$$

**GOAL:** PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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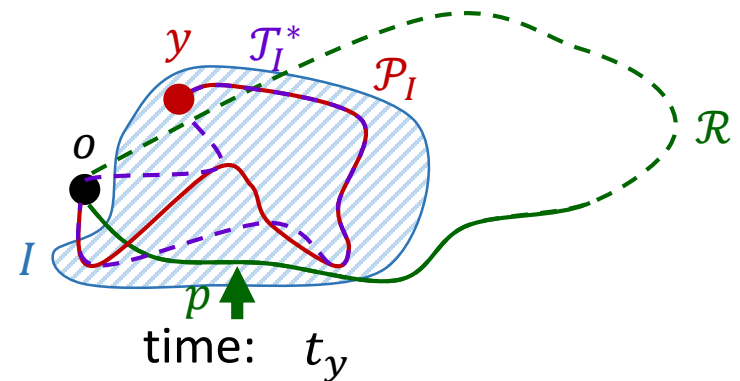
# Competitiveness of PAH

$U$  := places yet to visit,  $(\hat{t}, \hat{x})$  last request,  $y$  1st place in  $I$  visited by OPT

$I$  := ignored requests

(2) For new request  $(t_y, y)$ :

b)  $d(y, o) > d(p, o)$ : ignore  $y$  ...



$$\begin{aligned}
 |\mathcal{T}^{\text{PAH}}| &\leq t_y + |\mathcal{R}| - d(o, y) + |\mathcal{J}_I^*| \\
 &\leq t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|}_{|\mathcal{T}^{\text{OPT}}|} + |\mathcal{R}| \\
 &\leq |\mathcal{T}^{\text{OPT}}| + |\mathcal{T}^{\text{OPT}}| = 2 \cdot |\mathcal{T}^{\text{OPT}}|
 \end{aligned}$$

GOAL: PAH is 2-competitive for H-OLTSP



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

H-OLTSP

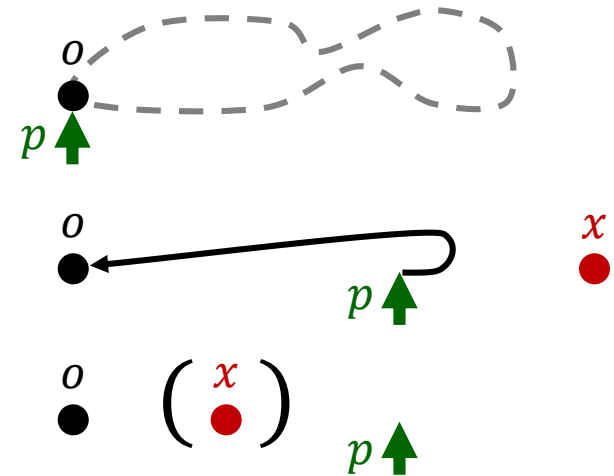
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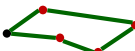
# Competitiveness of PAH

$U$  := places yet to visit

- (1) At  $o$ : start optimal tour through  $U$
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  - a) If  $d(x, o) > d(p, o)$ : go back to  $o$
  - b) Else: ignore  $x$  until back at  $o$



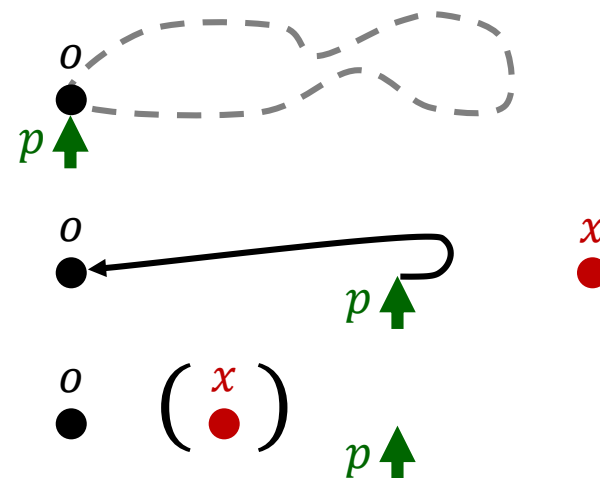
**GOAL:** PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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# Competitiveness of PAH

$U$  := places yet to visit

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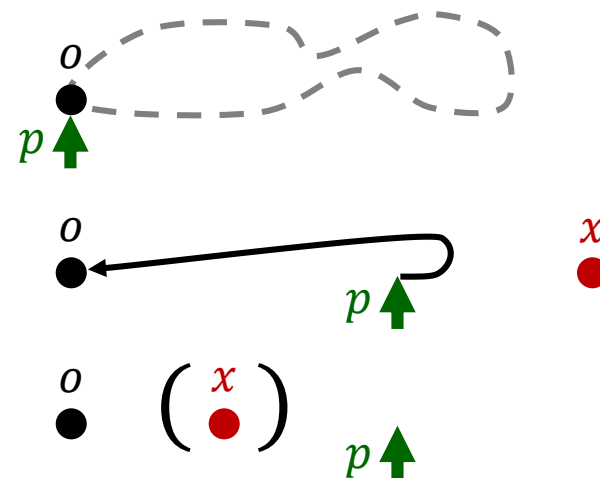
**THEOREM:** PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

# Competitiveness of PAH


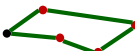
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- (1) At  $o$ : start optimal tour through  $U$
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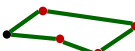


**THEOREM:** PAH is 2-competitive for H-OLTSP

**REMARK:** PAH is optimal online algorithm for H-OLTSP


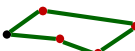
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
<b>H-OLTSP</b> 	PAH: $\rho = 2$	$\rho \geq 2$	

# Polynomial Algorithm for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
<b>H-OLTSP</b> 	PAH: $\rho = 2$	$\rho \geq 2$	

# Polynomial Algorithm for H-OLTSP

*Invariant:* always on shortest path between points in  $S$

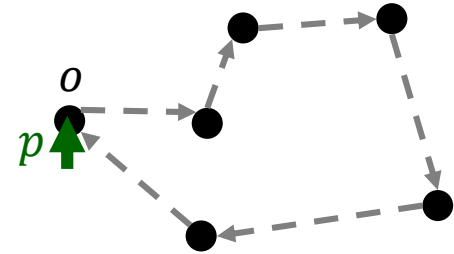
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
<b>H-OLTSP</b> 	PAH: $\rho = 2$	$\rho \geq 2$	



# Polynomial Algorithm for H-OLTSP

Invariant: always on shortest path between points in  $S$

- (1) At  $o$ : Find tour through  $U \cup \{o\}$   
with Christofides-Heuristic



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

H-OLTSP 

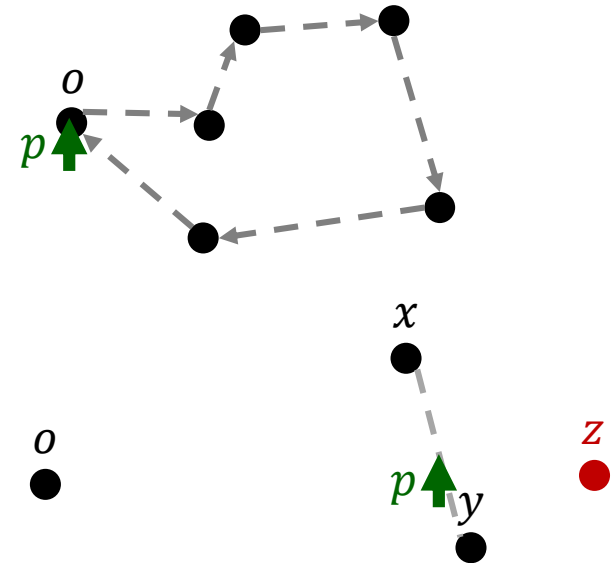
PAH:  $\rho = 2$


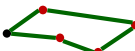
$\rho \geq 2$

# Polynomial Algorithm for H-OLTSP

Invariant: always on shortest path between points in  $S$

- (1) At  $o$ : Find tour through  $U \cup \{o\}$  with Christofides-Heuristic
- (2) For new request  $(t, z)$  at time  $t$  and ALG between  $x$  and  $y$ :

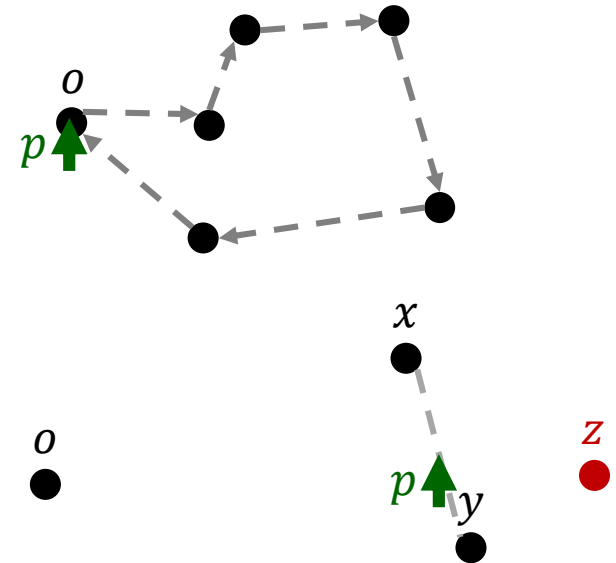



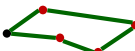
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	

# Polynomial Algorithm for H-OLTSP

Invariant: always on shortest path between points in  $S$

- (1) At  $o$ : Find tour through  $U \cup \{o\}$  with Christofides-Heuristic
- (2) For new request  $(t, z)$  at time  $t$  and ALG between  $x$  and  $y$ :
  - Add  $z$  to  $U$

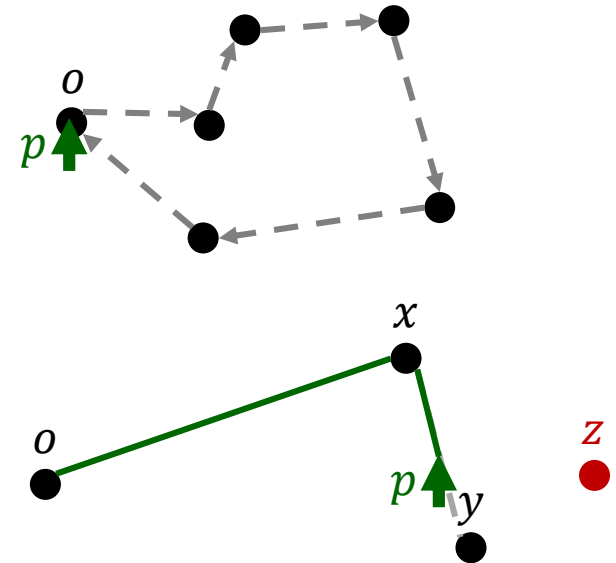



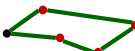
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	

# Polynomial Algorithm for H-OLTSP

Invariant: always on shortest path between points in  $S$

- (1) At  $o$ : Find tour through  $U \cup \{o\}$  with Christofides-Heuristic
- (2) For new request  $(t, z)$  at time  $t$  and ALG between  $x$  and  $y$ :
  - Add  $z$  to  $U$
  - go back to  $o$  via  $x$  or  $y$  (take shortest path)

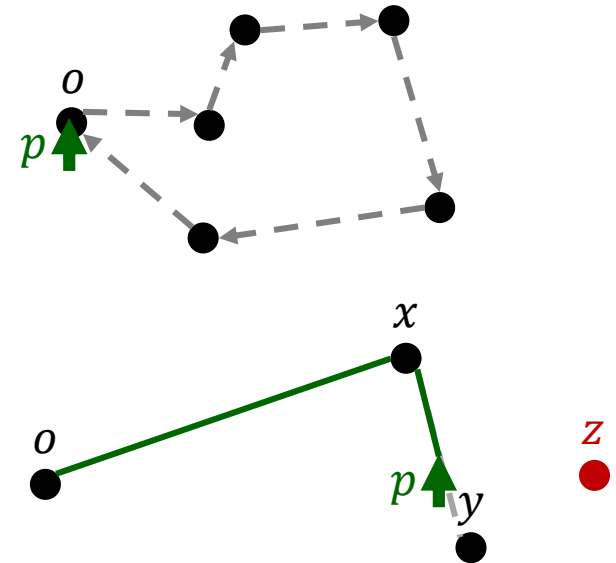


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	


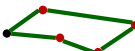
# Polynomial Algorithm for H-OLTSP

Invariant: always on shortest path between points in  $S$

- (1) At  $o$ : Find tour through  $U \cup \{o\}$  with Christofides-Heuristic
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  - Add  $z$  to  $U$
  - go back to  $o$  via  $x$  or  $y$  (take shortest path)



**THEOREM:** CHR is a polynomial (and correct).

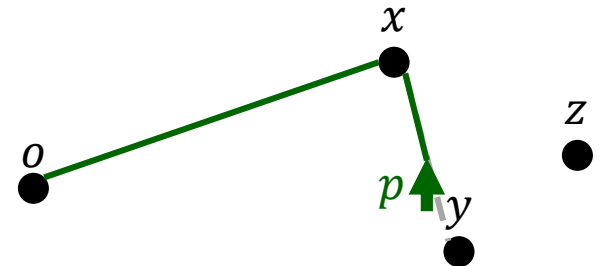
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N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	

# Competitiveness of CHR


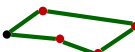
*Invariant:* always on shortest path between points in  $S$

(2) For last request  $(\hat{t}, z)$  at time  $\hat{t}$   
and ALG between  $x$  and  $y$ :

- go back to  $o$  via  $x$  or  $y$   
(take shortest path)



**GOAL:** CHR is  $\frac{1}{2}$ -competitive.

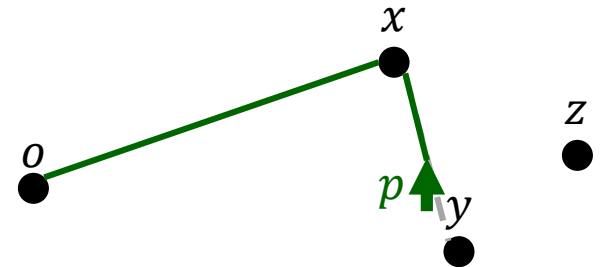
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
<b>H-OLTSP</b> 	PAH: $\rho = 2$	$\rho \geq 2$	

# Competitiveness of CHR

*Invariant:* always on shortest path between points in  $S$


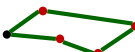
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- go back to  $o$  via  $x$  or  $y$   
(take shortest path)



$$|\mathcal{T}^{\text{CHR}}| = \hat{t} +$$

**GOAL:** CHR is  $\_$ -competitive.

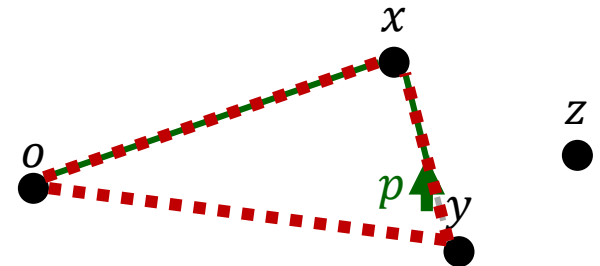
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<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
<b>H-OLTSP</b> 	PAH: $\rho = 2$	$\rho \geq 2$	

# Competitiveness of CHR

*Invariant:* always on shortest path between points in  $S$


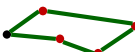
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- go back to  $o$  via  $x$  or  $y$   
(take shortest path)



$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\}$$

**GOAL:** CHR is  $\_$ -competitive.

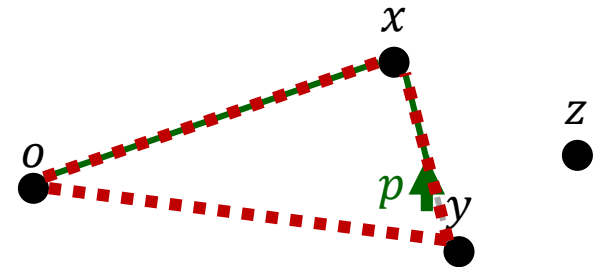
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# Competitiveness of CHR


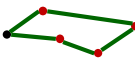
*Invariant:* always on shortest path between points in  $S$

- (1) At  $o$ : Find tour through  $U \cup \{o\}$   
with Christofides-Heuristic



$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\}$$

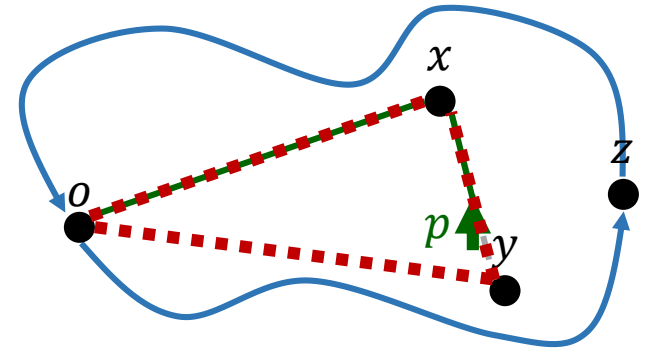
**GOAL:** CHR is  $\frac{3}{2}$ -competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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# Competitiveness of CHR


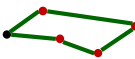
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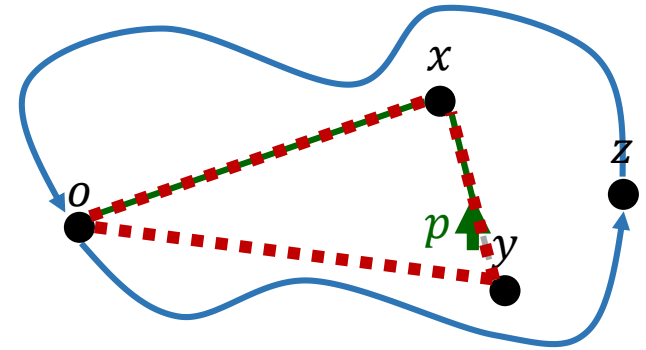
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# Competitiveness of CHR

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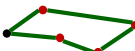
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$$\leq |\mathcal{T}^{\text{OPT}}|$$

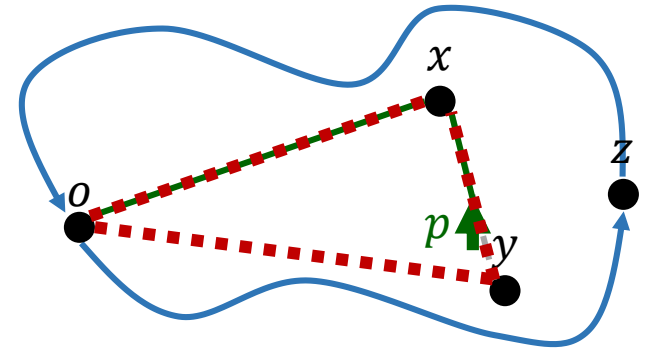
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# Competitiveness of CHR


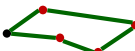
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- (1) At  $o$ : Find tour through  $U \cup \{o\}$   
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$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \underbrace{\min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\}}_{\text{CHR}(U)} + \text{CHR}(U) \leq |\mathcal{T}^{\text{OPT}}|$$

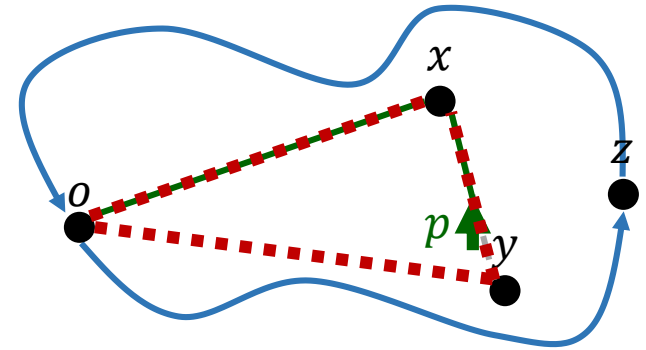
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# Competitiveness of CHR

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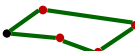
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$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \underbrace{\min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\}}_{\leq |\mathcal{T}^{\text{OPT}}|} + \text{CHR}(U)$$

$$\leq |\mathcal{T}^{\text{OPT}}| + \frac{1}{2} \cdot |\mathcal{T}^{\text{OPT}}|$$

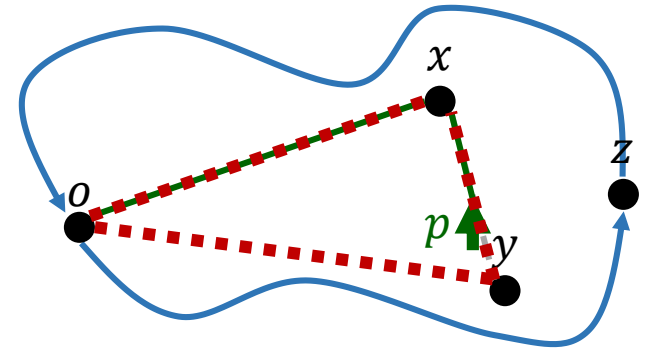
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# Competitiveness of CHR

*Invariant:* always on shortest path between points in  $S$


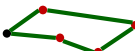
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$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \underbrace{\min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\}}_{\leq |\mathcal{T}^{\text{OPT}}|} + \text{CHR}(U)$$

$$\leq |\mathcal{T}^{\text{OPT}}| + \frac{1}{2} \cdot |\mathcal{T}^{\text{OPT}}| + \frac{3}{2} \cdot |\mathcal{T}^{\text{OPT}}|$$

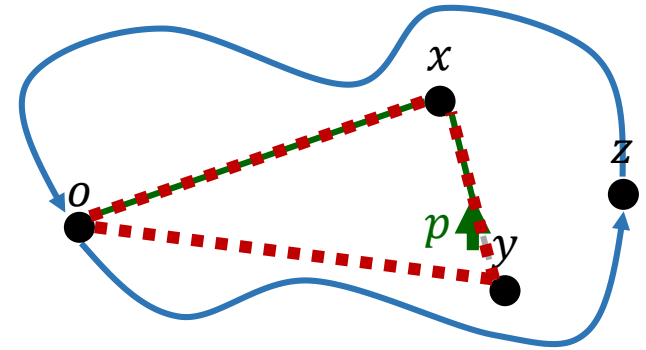
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# Competitiveness of CHR

*Invariant:* always on shortest path between points in  $S$


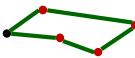
- (1) At  $o$ : Find tour through  $U \cup \{o\}$   
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$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$

$$\leq |\mathcal{T}^{\text{OPT}}| + \frac{1}{2} \cdot |\mathcal{T}^{\text{OPT}}| + \frac{3}{2} \cdot |\mathcal{T}^{\text{OPT}}| =$$

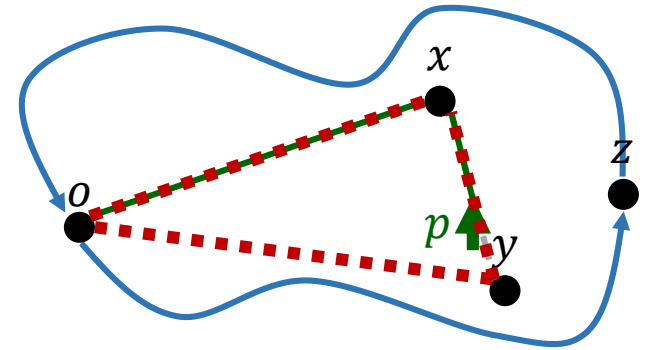
**GOAL:** CHR is  $\frac{3}{2}$ -competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	

# Competitiveness of CHR

*Invariant:* always on shortest path between points in  $S$


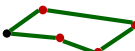
- (1) At  $o$ : Find tour through  $U \cup \{o\}$   
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$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$

$$\leq |\mathcal{T}^{\text{OPT}}| + \frac{1}{2} \cdot |\mathcal{T}^{\text{OPT}}| + \frac{3}{2} \cdot |\mathcal{T}^{\text{OPT}}| = 3 \cdot |\mathcal{T}^{\text{OPT}}|$$

**GOAL:** CHR is  $\frac{3}{2}$ -competitive.

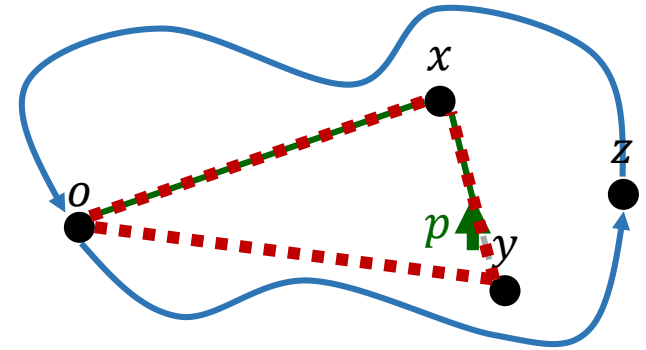
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
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# Competitiveness of CHR

*Invariant:* always on shortest path between points in  $S$


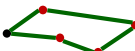
- (1) At  $o$ : Find tour through  $U \cup \{o\}$   
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$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$

$$\leq |\mathcal{T}^{\text{OPT}}| + \frac{1}{2} \cdot |\mathcal{T}^{\text{OPT}}| + \frac{3}{2} \cdot |\mathcal{T}^{\text{OPT}}| = 3 \cdot |\mathcal{T}^{\text{OPT}}|$$

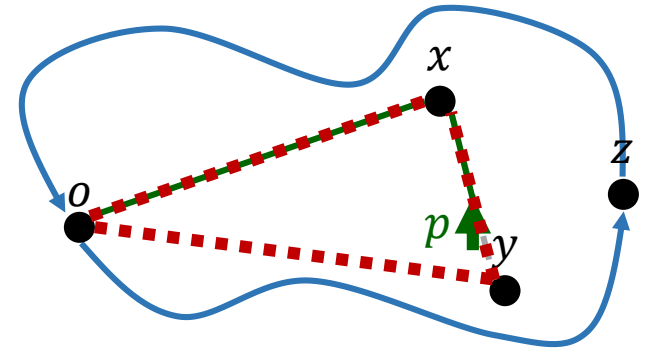
**THEOREM:** CHR is 3-competitive for H-OLTSP.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

# Competitiveness of CHR

*Invariant:* always on shortest path between points in  $S$

- (1) At  $o$ : Find tour through  $U \cup \{o\}$   
with Christofides-Heuristic



$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$


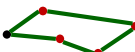
$$\leq |\mathcal{T}^{\text{OPT}}| + \frac{1}{2} \cdot |\mathcal{T}^{\text{OPT}}| + \frac{3}{2} \cdot |\mathcal{T}^{\text{OPT}}| = 3 \cdot |\mathcal{T}^{\text{OPT}}|$$

**REMARK:** There is a 3-competitive algorithm for N-OLTSP.


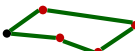
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b>	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	MST: $\rho = 3$
<b>H-OLTSP</b>	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

# Credits & References

- Based on **Algorithms for the On-Line Travelling Salesman** by G. Ausiello, E. Feuerstein, S. Leonardi, L. Stougie, and M. Talamo in *Algorithmica* (2001) 29: 560–581, DOI: 10.1007/s004530010071 (<http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.8.5620>)
- Titlepage:
  - Map: <http://awoiaf.westeros.org/index.php/File:WorldofIceandFire.png>
  - Font by Charlie Samways: <http://www.fonts4free.net/game-of-thrones-font.html>

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

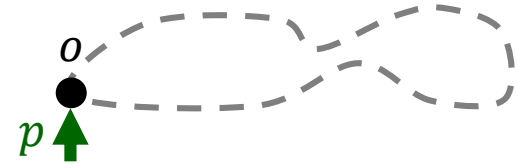
# N-OLTSP on the Real Line

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
<b>N-OLTSP</b> 	GTR (Greedy): $\rho = 2,5$	$\rho \geq 2$	MST: $\rho = 3$
<b>H-OLTSP</b> 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

# Competitiveness of PAH

$U$  := places yet to visit,  $I$  := ignored requests,  $(\hat{t}, \hat{x})$  last request

(1) At  $o$ : start optimal tour through  $U$



$$|\mathcal{T}^{\text{PAH}}| \leq \hat{t} + |\mathcal{T}^*| \leq |\mathcal{T}^{\text{OPT}}| + |\mathcal{T}^{\text{OPT}}| = 2 \cdot |\mathcal{T}^{\text{OPT}}| \quad |\mathcal{T}^*|$$

GOAL:

PAH is 2-competitive for H-OLTSP

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP 

GTR (Greedy):  $\rho = 2,5$

$\rho \geq 2$

H-OLTSP 

PAH:  $\rho = 2$

$\rho \geq 2$