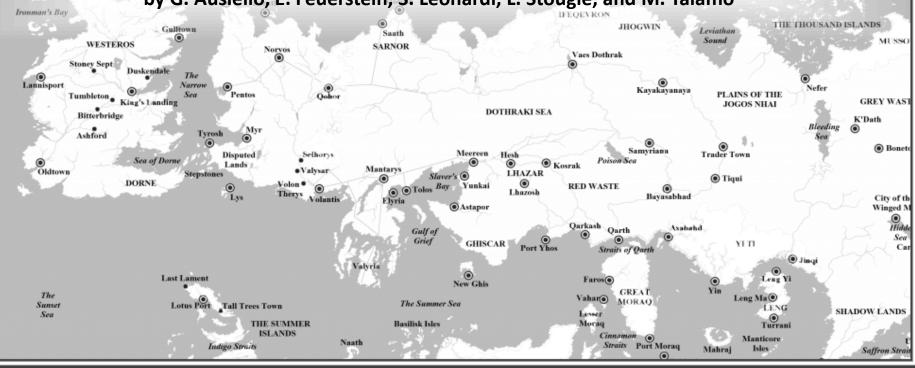
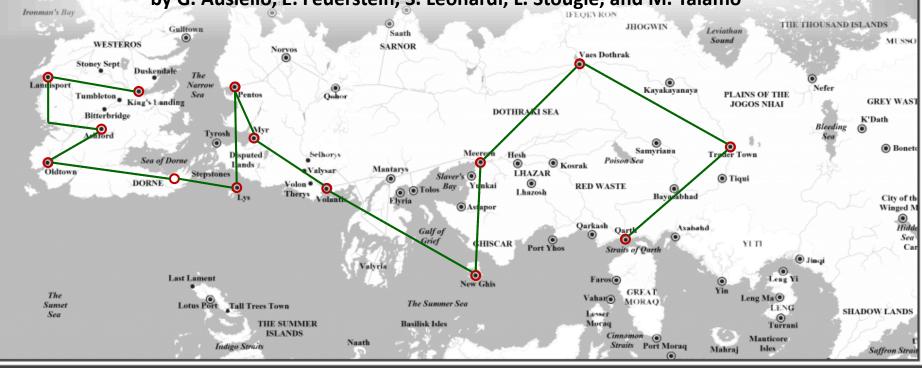


Presenting results from the paper "Algorithms for the On-Line Travelling Salesman" by G. Ausiello, E. Feuerstein, S. Leonardi, L. Stougie, and M. Talamo





Presenting results from the paper "Algorithms for the On-Line Travelling Salesman" by G. Ausiello, E. Feuerstein, S. Leonardi, L. Stougie, and M. Talamo

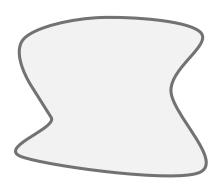


Online-TSP (metric)

Online-TSP (metric)

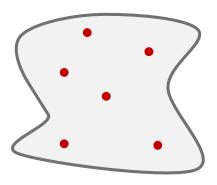
(metric)

INPUT:



metric space: M
 (with metric d)

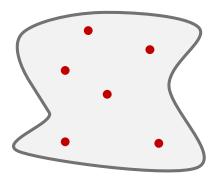
(metric)



- metric space: M
 (with metric d)
- places to visit: S

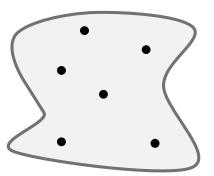
(metric)

INPUT:



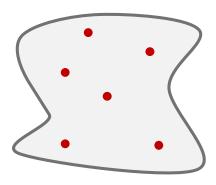
- metric space: M
 (with metric d)
- places to visit: S

OUTPUT:



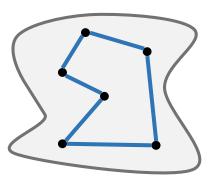
(metric)

INPUT:



- metric space: M
 (with metric d)
- places to visit: S

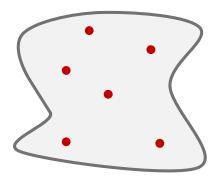
OUTPUT:



Shortest tour \mathcal{T}^* through \mathcal{S}

(metric)

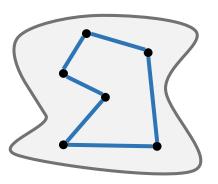
INPUT:



- metric space: M
 (with metric d)
- places to visit: S

NP-hard!

OUTPUT:



Shortest tour \mathcal{T}^* through \mathcal{S}

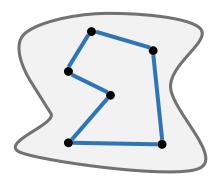
(metric)

INPUT:

NP-hard!

ALG

OUTPUT:



Shortest tour \mathcal{T}^* through \mathcal{S}

- metric space: M
 (with metric d)
- places to visit: S

(metric)

INPUT:

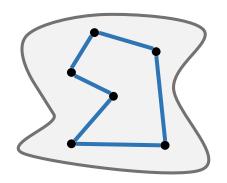
NP-hard!

ALG

Superpolynomial Alg.

- metric space: M
 (with metric d)
- places to visit: S

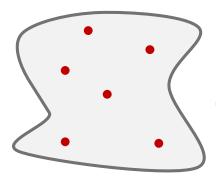
OUTPUT:



Shortest tour \mathcal{T}^* through \mathcal{S}

(metric)

INPUT:

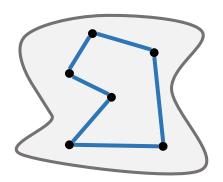


NP-hard!

ALG

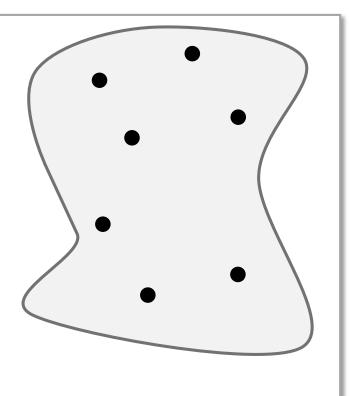
- metric space: M
 (with metric d)
- places to visit: S
- Superpolynomial Alg.
- Approximation Alg. e.g. Christofides

OUTPUT:



Shortest tour \mathcal{T}^* through \mathcal{S}

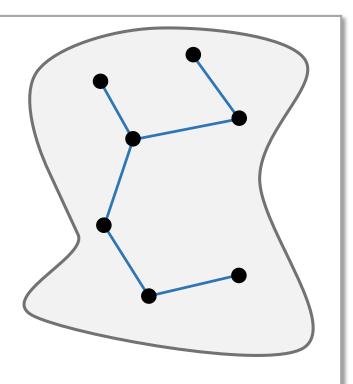
(metric)



(metric)

Christofides Algorithm:

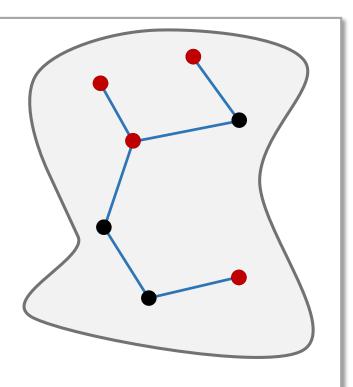
(1) minimal spanning tree



(metric)

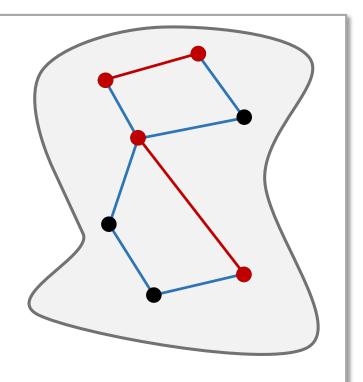
Christofides Algorithm:

(1) minimal spanning tree



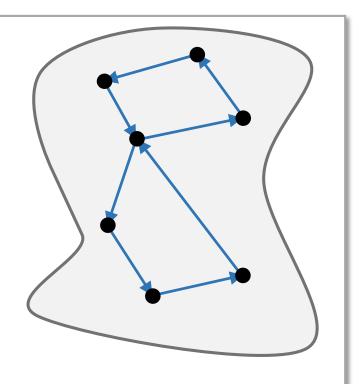
(metric)

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices



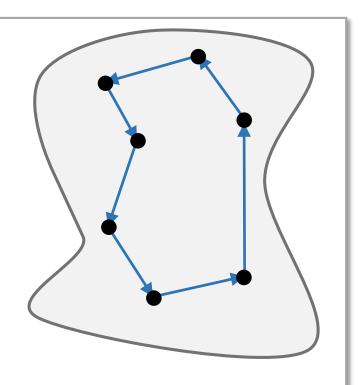
(metric)

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour



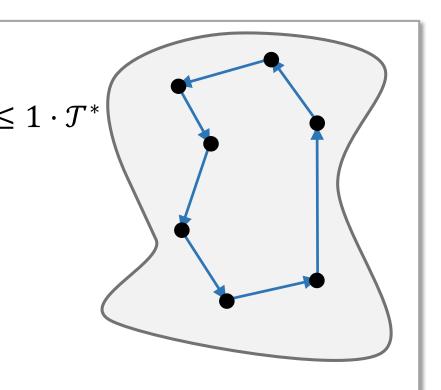
(metric)

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices



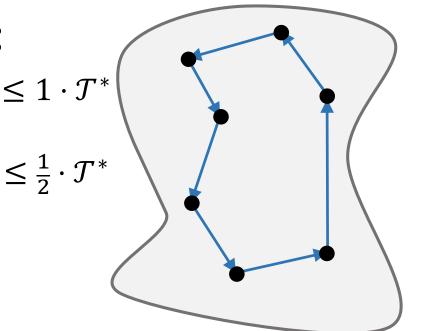
(metric)

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices



(metric)

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices



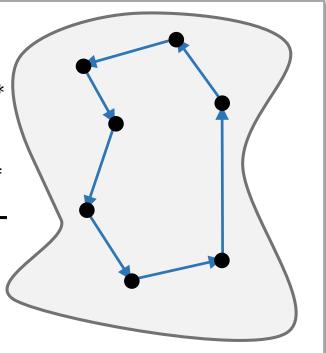
(metric)

- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices



$$\leq \frac{1}{2} \cdot \mathcal{T}^*$$

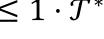
$$\leq \frac{3}{2} \cdot \mathcal{T}^*$$



(metric)

Christofides Algorithm:

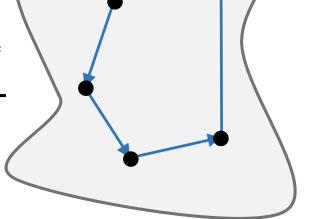
- (1) minimal spanning tree
- (2) minimum weighted perfect matching of odd vertices
- (3) Euler tour
- (4) Skip double visited vertices



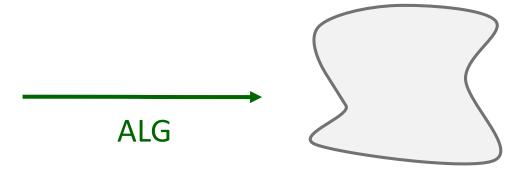
$$\leq \frac{1}{2} \cdot \mathcal{T}^*$$

$$\leq \frac{3}{2} \cdot \mathcal{T}^*$$



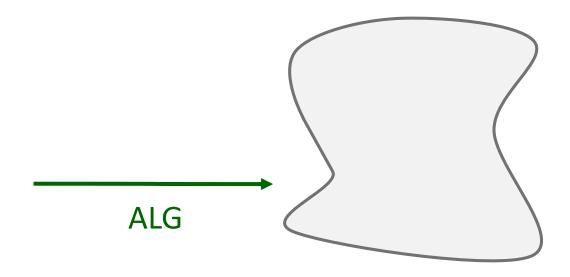


1,5-approximative solution

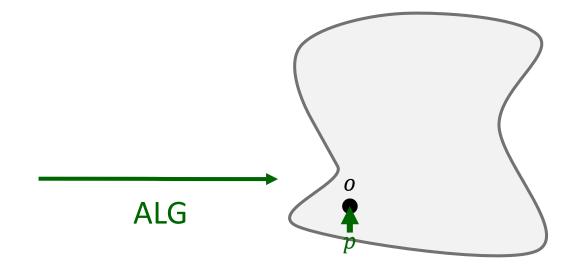


INPUT:

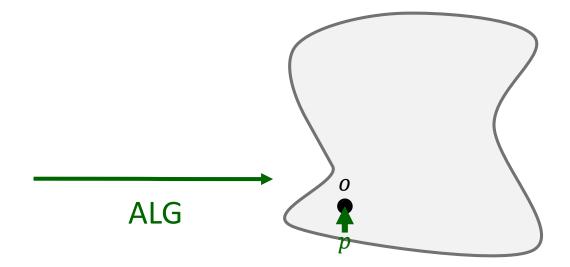
• metric space



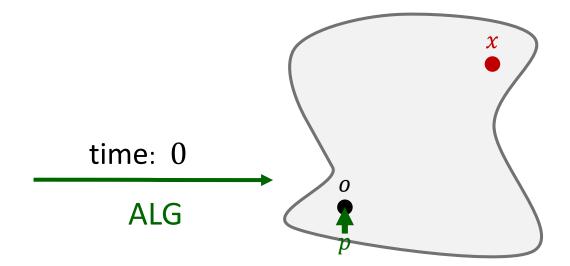
- metric space
- starting-point: *o*



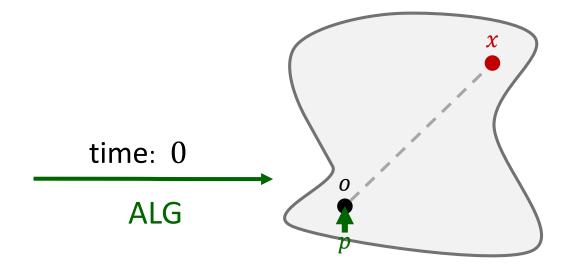
- metric space
- starting-point: o
- request-sequence σ :



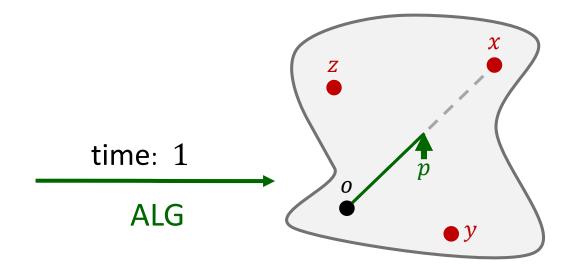
- metric space
- starting-point: o
- request-sequence σ : (0, x)



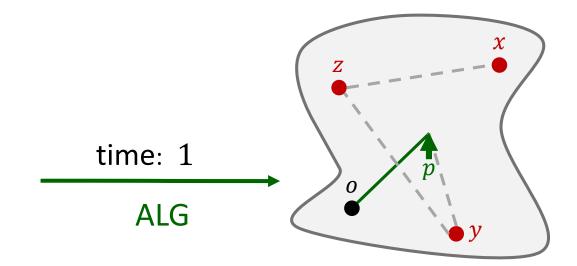
- metric space
- starting-point: o
- request-sequence σ : (0, x)



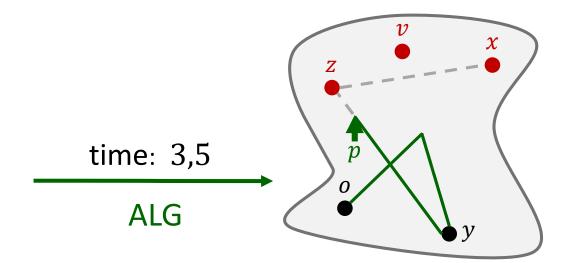
- metric space
- starting-point: o
- request-sequence σ : (0, x), (1, y), (1, z)



- metric space
- starting-point: o
- request-sequence σ : (0, x), (1, y), (1, z)



- metric space
- starting-point: o
- request-sequence σ : (0, x), (1, y), (1, z), ...



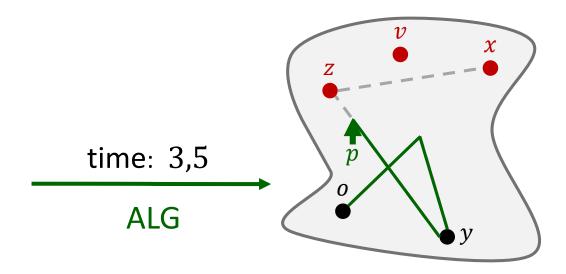
INPUT:

- metric space
- starting-point: o
- request-sequence σ : (0, x), (1, y), (1, z), ...



N-OLTSP

"nomadic"



INPUT:

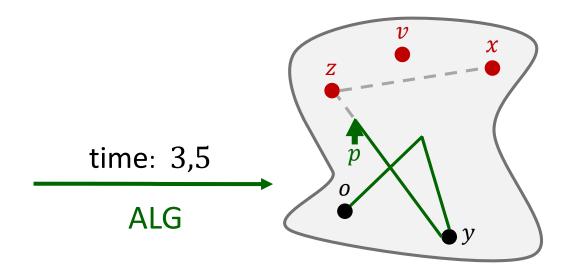
- metric space
- starting-point: o
- request-sequence σ : (0, x), (1, y), (1, z), ...



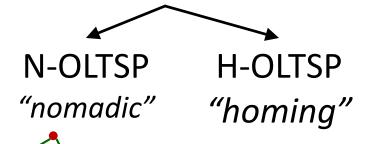
N-OLTSP

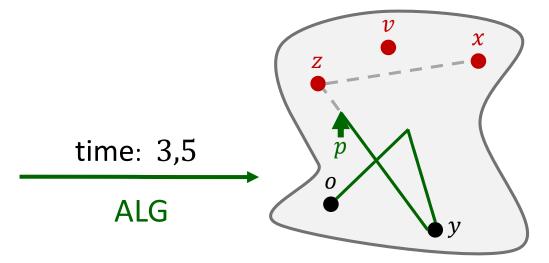
"nomadic"



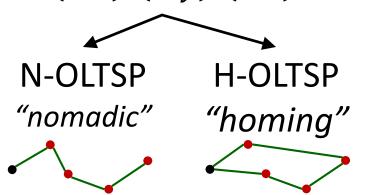


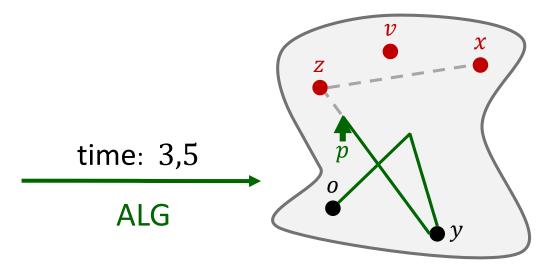
- metric space
- starting-point: o
- request-sequence σ : (0, x), (1, y), (1, z), ...





- metric space
- starting-point: o
- request-sequence σ : (0, x), (1, y), (1, z), ...

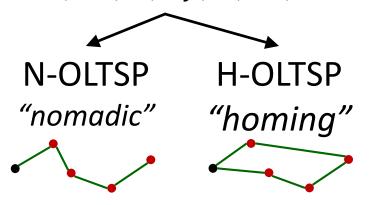


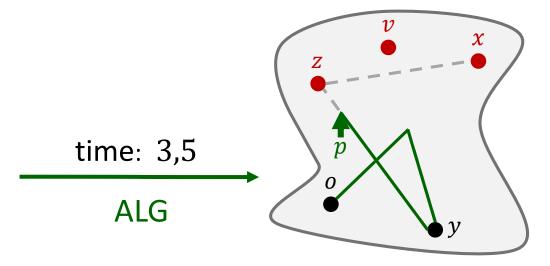


Online-TSP

INPUT:

- metric space
- starting-point: o
- request-sequence σ : (0, x), (1, y), (1, z), ...



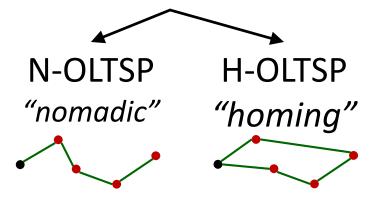


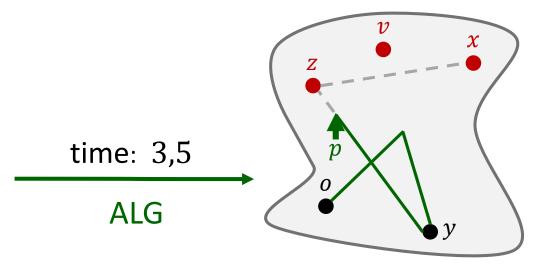
DEF: ALG is ρ -competitive

Online-TSP

INPUT:

- metric space
- starting-point: o
- request-sequence σ : (0, x), (1, y), (1, z), ...





DEF: ALG is ρ -competitive

$$\Leftrightarrow |\mathcal{T}^{ALG}| \leq \rho \cdot |\mathcal{T}^{OPT}|$$

for all request- sequences

I. Find online-algorithms

I. Find online-algorithms (superpolynomial)

I. Find online-algorithms (superpolynomial)

II. Find lower bounds

Find online-algorithms (superpolynomial)

II. Find lower bounds

III. Find *polynomial* online-algorithms

Online-TSP

I. Algorithms

II. Lower Bounds

Invariant: always on shortest path between points in *S*

S := places requested until t

Online-TSP

I. Algorithms

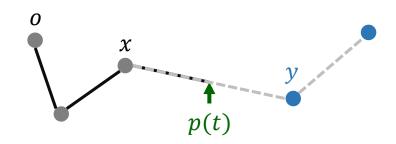
II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP <



Invariant: always on shortest path between points in *S*



S :=places requested until t

Online-TSP

I. Algorithms

II. Lower Bounds

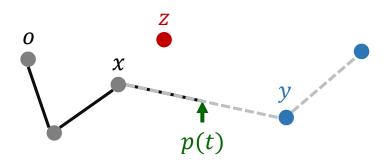
III. Polynomial Algorithms

N-OLTSP <



Invariant: always on shortest path between points in S

(1) New request (t, z) at time t and ALG between x and y



S :=places requested until t

Online-TSP

I. Algorithms

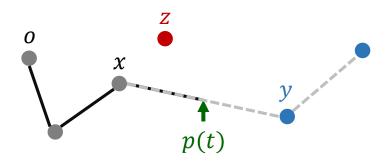
II. Lower Bounds





Invariant: always on shortest path between points in S

(1) Add z to U



S := places requested until t $S \supseteq U :=$ places yet to visit at t

Online-TSP

I. Algorithms

II. Lower Bounds

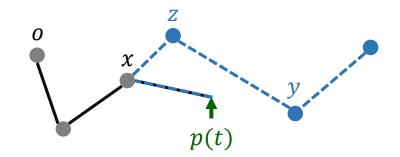
III. Polynomial Algorithms

N-OLTSP /



Invariant: always on shortest path between points in S

- (1) Add z to U
- (2) Follow shortest path through ${\cal U}$ beginning with x or y



S := places requested until t $S \supseteq U :=$ places yet to visit at t

Online-TSP

I. Algorithms

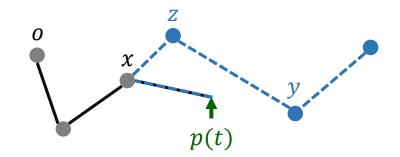
II. Lower Bounds





Invariant: always on shortest path between points in S

- (1) Add z to U
- (2) Follow shortest path through ${\cal U}$ beginning with x or y



S := places requested until t $S \supseteq U :=$ places yet to visit at t

Greedily Travelling between Requests (GTR)

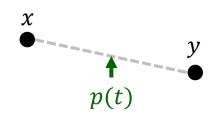
Online-TSP

I. Algorithms

II. Lower Bounds

Invariant: always on shortest path between points in *S*

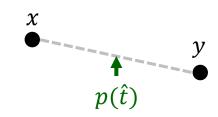
- (1) New request (t, z) at time t and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



Online-TSP

Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*

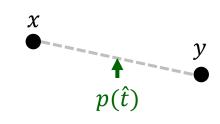


•

Online-TSP

Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



path found by GTR $|\mathcal{T}^{\mathsf{GTR}}|$

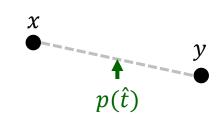
N-OLTSP /

Online-TSP



Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



path found by GTR
$$\left|\mathcal{T}^{\overset{ullet}{\mathsf{GTR}}}
ight| \leq$$

Online-TSP

I. Algorithms

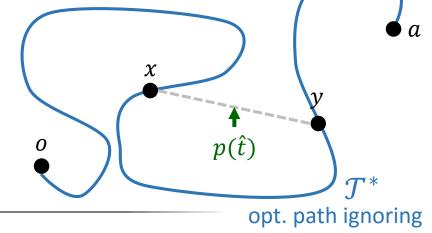
II. Lower Bounds





Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



path found by GTR
$$\left|\mathcal{T}^{\overset{\bullet}{\mathsf{GTR}}}\right| \leq \hat{t}$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

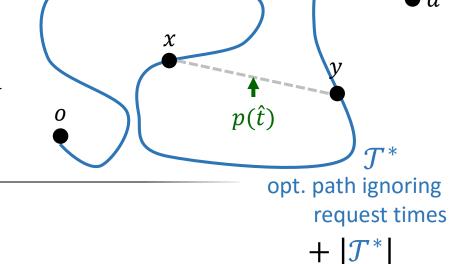
request times





Invariant: always on shortest path between points in S

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



path found by GTR
$$\left|\mathcal{T}^{\mathsf{GTR}}\right| \leq \hat{t}$$

Online-TSP

I. Algorithms

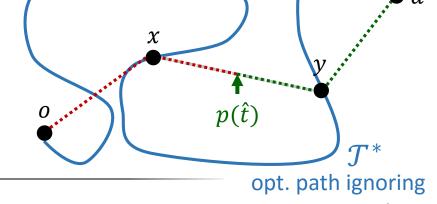
II. Lower Bounds





Invariant: always on shortest path between points in S

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



path found by GTR opt. path ignoring request times
$$\left|\mathcal{T}^{\text{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

Online-TSP

I. Algorithms

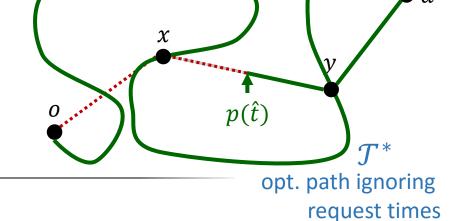
II. Lower Bounds





Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



path found by GTR
$$|\mathcal{T}^{\mathsf{GTR}}| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

Online-TSP

I. Algorithms

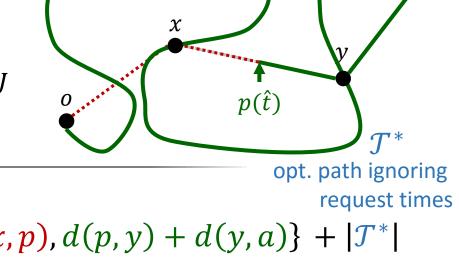
II. Lower Bounds





Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



path found by GTR
$$|\mathcal{T}^{GTR}| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$
 path found by
$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$
 popt. offline-ALG
$$\leq |\mathcal{T}^{OPT}| +$$

Online-TSP

I. Algorithms

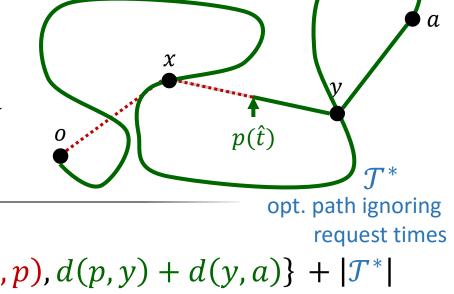
II. Lower Bounds





Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



path found by GTR
$$|\mathcal{T}^{GTR}| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$
 path found by opt. offline-ALG
$$\leq |\mathcal{T}^{OPT}| + \frac{3}{2} \cdot |\mathcal{T}^{OPT}|$$

Online-TSP

I. Algorithms

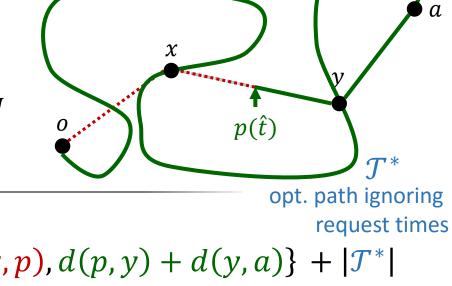
II. Lower Bounds





Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



path found by GTR
$$|\mathcal{T}^{\mathsf{GTR}}| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$
 path found by opt. offline-ALG
$$\leq |\mathcal{T}^{\mathsf{OPT}}| + \frac{3}{2} \cdot |\mathcal{T}^{\mathsf{OPT}}| = \frac{5}{2} \cdot |\mathcal{T}^{\mathsf{OPT}}|$$

Online-TSP

I. Algorithms

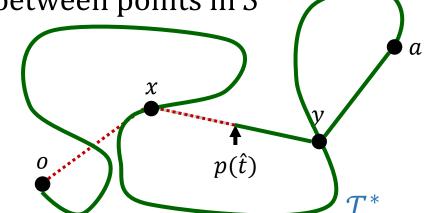
II. Lower Bounds





Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

$$\frac{3}{2} \cdot |\mathcal{T}^{OPT}|$$

$$= \frac{5}{2} \cdot \left| \mathcal{T}^{OPT} \right|$$

Online-TSP

I. Algorithms

II. Lower Bounds

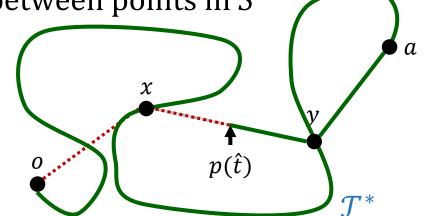
III. Polynomial Algorithms

N-OLTSP /



Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



THEOREM: GTR is 2,5-competitive for N-OLTSP

Online-TSP

I. Algorithms

II. Lower Bounds

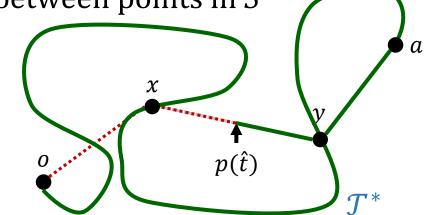
III. Polynomial Algorithms





Invariant: always on shortest path between points in *S*

- (1) Last request (\hat{t}, z) at time \hat{t} and ALG between x and y
- (3) Follow shortest path through *U* beginning with *x* or *y*



ms

THEOREM: GTR is 2,5-competitive for N-OLTSP

REMARK: GTR is also 2,5-competitive for H-OLTSP

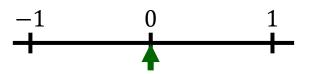
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithr
N-OLTSP /	GTR (Greedy): $ ho=2$,5		

H-OLTSP GTR (Greedy):
$$\rho=2.5$$

time, request

Online-ALG

0



N-OLTSP /

Online-TSP

GTR (Greedy): $\rho = 2.5$

I. Algorithms

H-OLTSP

GTR (Greedy): $\rho=$ 2,5

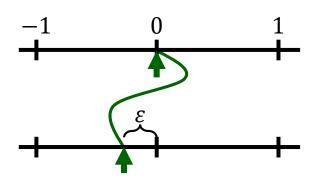
II. Lower Bounds

time, request

Online-ALG

0

1



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP /

GTR (Greedy): $\rho = 2.5$

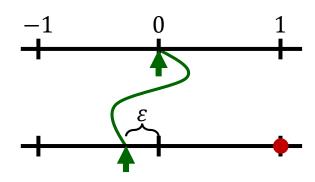
H-OLTSP \checkmark

time, request

Online-ALG

0

1



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP /

GTR (Greedy): $\rho = 2.5$

H-OLTSP \checkmark

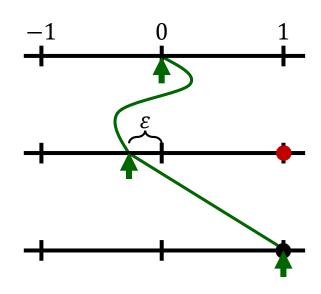
time, request

Online-ALG

0

1. 1

 $2 + \varepsilon$



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP /

GTR (Greedy): $\rho = 2.5$

H-OLTSP $\overline{}$

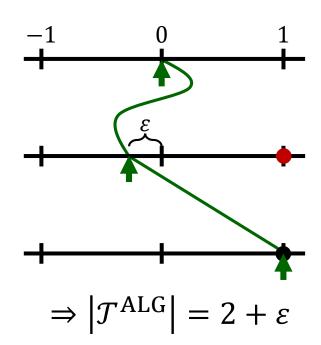
time, request

Online-ALG

0

1, 1

 $2 + \varepsilon$



Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP /

GTR (Greedy): $\rho = 2.5$

H-OLTSP GT

time, request

Online-ALG

Opt. offline-ALG

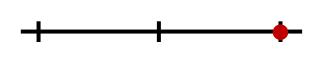
0

-1 0 1

-1 0 1

1,

1



 $2 + \varepsilon$

$$\Rightarrow |\mathcal{T}^{ALG}| = 2 + \varepsilon$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP /

GTR (Greedy): $\rho = 2.5$

H-OLTSP

time, request

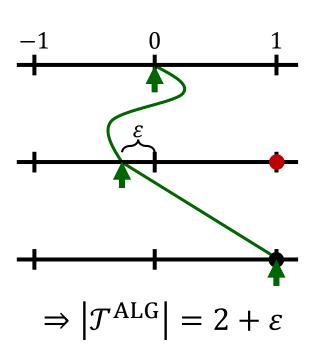
Online-ALG

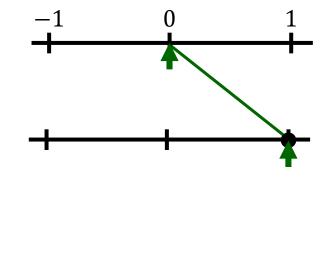
Opt. offline-ALG

0

l. 1

 $2 + \varepsilon$





Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP /

GTR (Greedy): $\rho = 2.5$

H-OLTSP 🗢

time, request

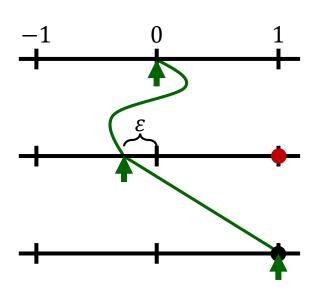
Online-ALG

Opt. offline-ALG

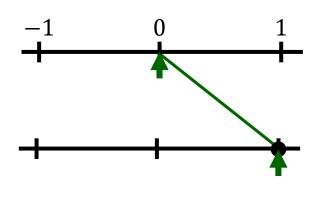
0

. 1

 $2 + \varepsilon$



 $\Rightarrow |\mathcal{T}^{ALG}| = 2 + \varepsilon$



$$\Rightarrow |\mathcal{T}^{OPT}| = 1$$

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

N-OLTSP /

GTR (Greedy): $\rho = 2.5$

H-OLTSP $\overline{\hspace{1cm}}$

time, request

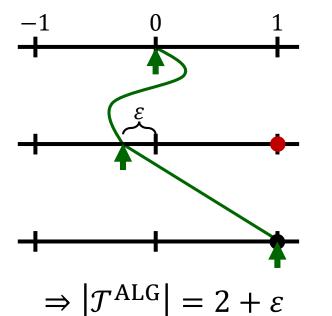
Online-ALG

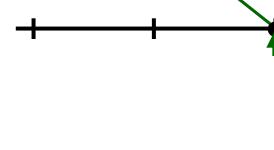
Opt. offline-ALG

0

. 1

 $2 + \varepsilon$





 $\Rightarrow |\mathcal{T}^{OPT}| = 1$

Any ρ -competitive ALG for N-OLTSP has $\rho \geq 2$.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OITSP /	GTR (Greedy): $a = 2.5$		

time, request

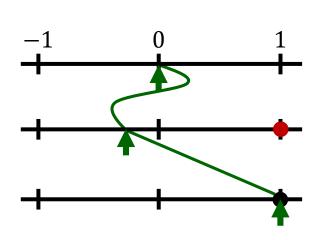
Online-ALG

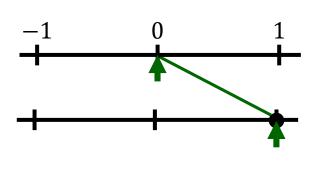
Opt. offline-ALG

0

1, 1

 $2 + \varepsilon$





$$\Rightarrow |\mathcal{T}^{ALG}| = 2 + \varepsilon$$

$$\Rightarrow \left| \mathcal{T}^{\text{OPT}} \right| = 1$$

THEOREM:

Any ρ -competitive ALG for N-OLTSP has $\rho \geq 2$.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP \leftarrow	GTR (Greedy): $\rho = 2.5$		

time, request

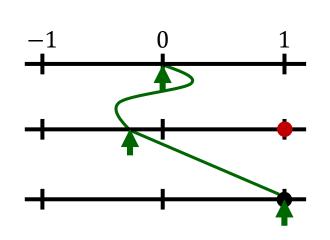
Online-ALG

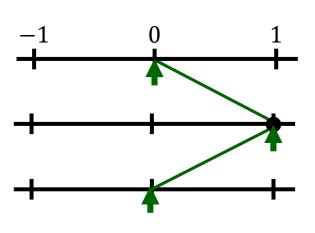
Opt. offline-ALG

0

1, 1

 $2 + \varepsilon$





$$\Rightarrow |\mathcal{T}^{ALG}| = 2 + \varepsilon$$

$$\Rightarrow \left| \mathcal{T}^{\text{OPT}} \right| = 2$$

THEOREM:

Any ρ -competitive ALG for N-OLTSP has $\rho \geq 2$.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	

GTR (Greedy): $\rho=$ 2,5

time, request

Online-ALG

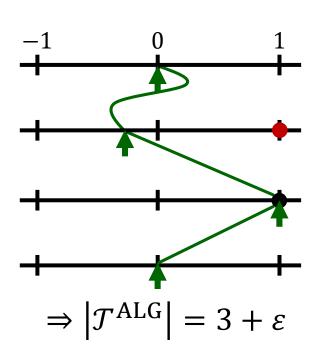
Opt. offline-ALG

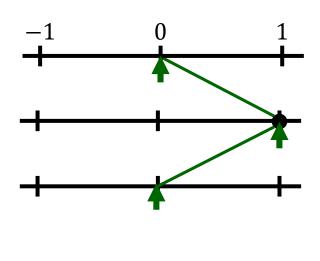
0

[, 1

 $2 + \varepsilon$

 $3 + \varepsilon$





$$\Rightarrow \left| \mathcal{T}^{\text{OPT}} \right| = 2$$

THEOREM:

Any ρ -competitive ALG for N-OLTSP has $\rho \geq 2$.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

GTR (Greedy): $\rho=$ 2,5

time, request

Online-ALG

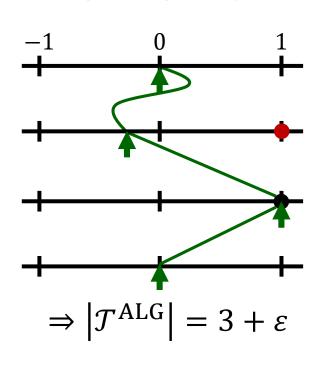
Opt. offline-ALG

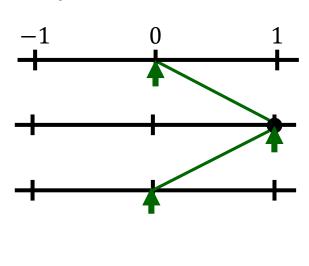
0

[, 1

 $2 + \varepsilon$

 $3 + \varepsilon$



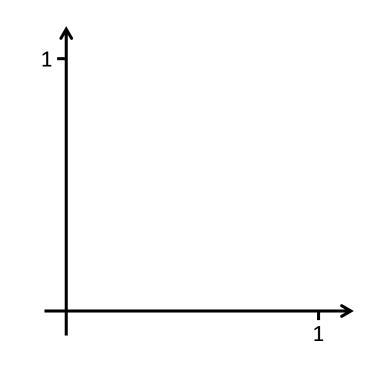


$$\Rightarrow \left| \mathcal{T}^{\text{OPT}} \right| = 2$$

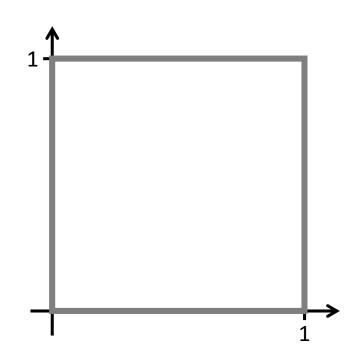
THEOREM:

Any ρ -competitive ALG for H-OLTSP has $\rho \geq 1.5$.

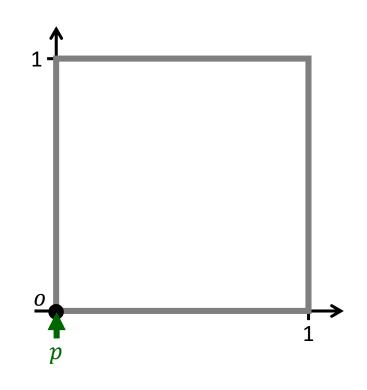
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP	GTR (Greedy): $ ho=2,5$	$\rho \geq 1,5$	



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $\rho = 2.5$	$\rho \geq 1.5$	

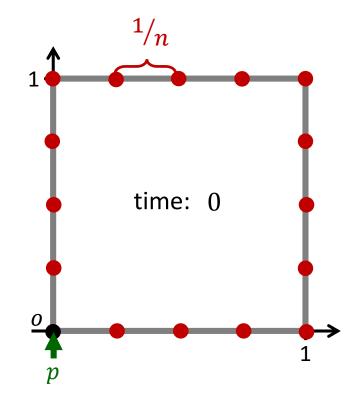


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $\rho=2.5$	$ ho \geq 1,5$	



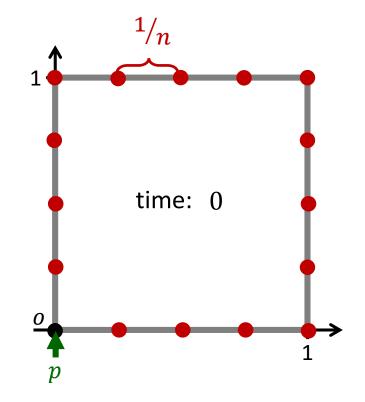
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho = 2.5$	$\rho \geq 1.5$	

requests at time 0



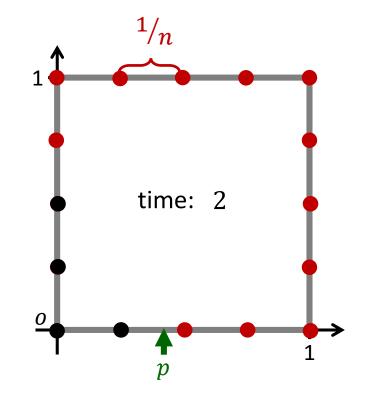
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $\rho=2.5$	$\rho \geq 1,5$	

- requests at time 0
- wait until time t=2



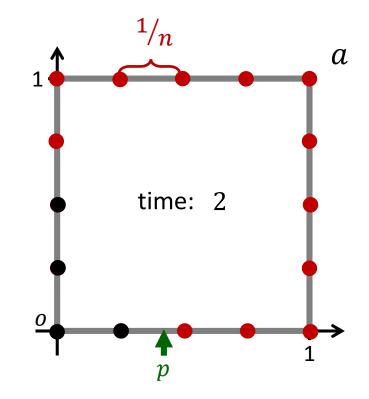
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP \angle	GTR (Greedy): $\rho = 2.5$	$\rho > 1.5$	

- requests at time 0
- wait until time t = 2



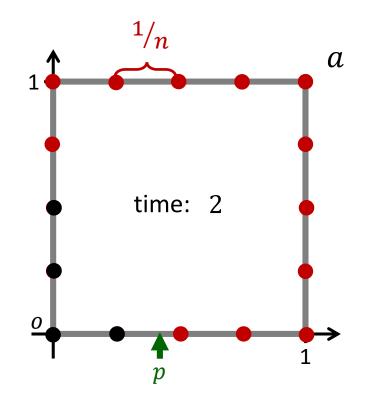
Online TCD	I. Alma vitlava a	II. Lavran Davrada	III Dalamanaial Alaamithaa
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho = 2.5$	$\rho \geq 1.5$	

- requests at time 0
- wait until time t = 2



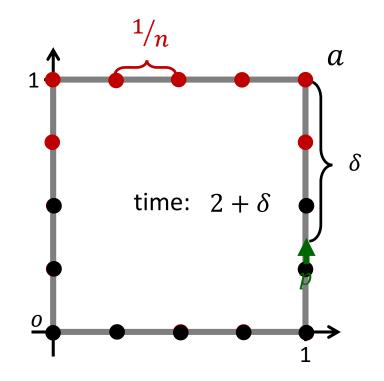
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho = 2.5$	$\rho \geq 1.5$	

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2



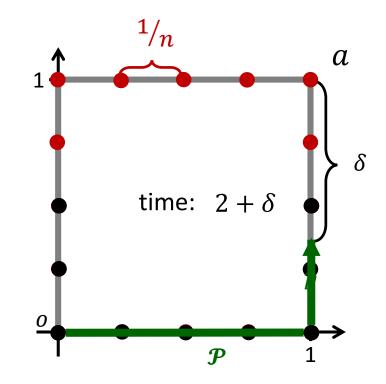
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho = 2.5$	$\rho \geq 1.5$	

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2



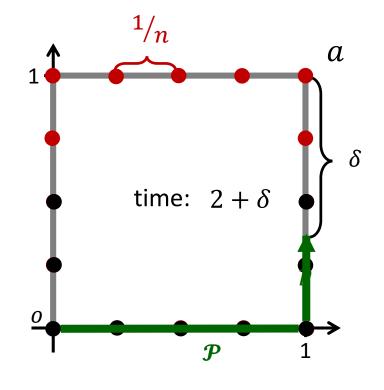
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP \leftarrow	GTR (Greedy): $\rho = 2.5$	$\rho \geq 1.5$	

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP $ line $	GTR (Greedy): $\rho = 2.5$	$\rho > 1.5$	

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2

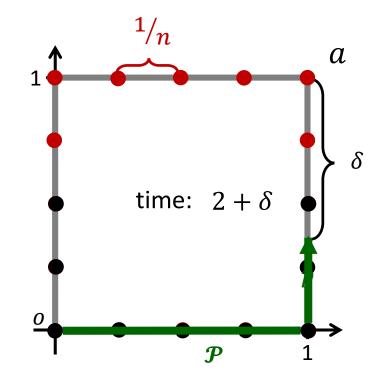


lgorithms

$$|\mathcal{P}|$$
 =

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Al
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $\rho=2.5$	$ ho \geq 1,5$	

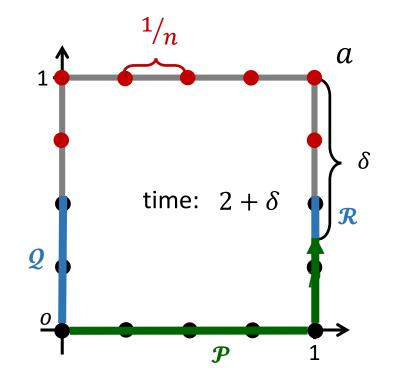
- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2



$$|\mathcal{P}| = 2 - \delta$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithm
N-OLTSP /	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	
H-OLTSP	GTR (Greedy): $a = 2.5$	o > 1.5	

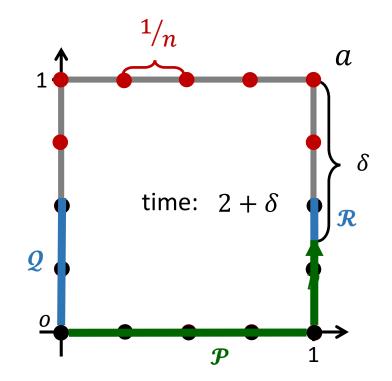
- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2



$$|\mathcal{P}| = 2 - \delta$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP \angle	GTR (Greedy): $\rho = 2.5$	$\rho > 1.5$	

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2

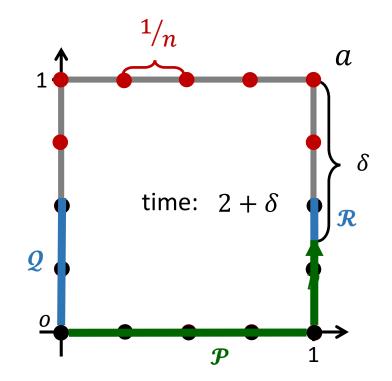


$$|\mathcal{P}| = 2 - \delta$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \le$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $\rho=2.5$	$\rho \geq 1.5$	

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2

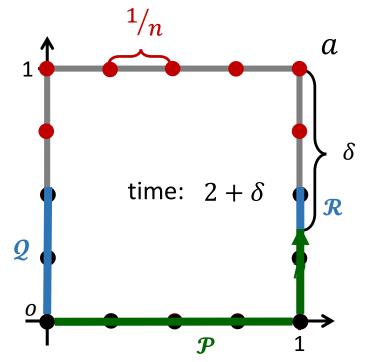


$$|\mathcal{P}| = 2 - \delta$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \le 2 + \delta$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho = 2.5$	$\rho \geq 1.5$	

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2



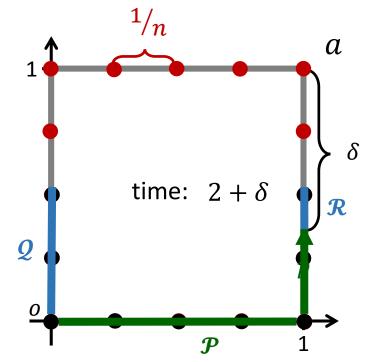
$$|\mathcal{P}| = 2 - \delta$$

$$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}|$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \le 2 + \delta$$

Online-TSP	I. Algorithms	II. Lower Bounds
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$
H-OLTSP $\overline{\hspace{1cm}}$	GTR (Greedy): $ ho=2.5$	$\rho \geq 1,5$

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2



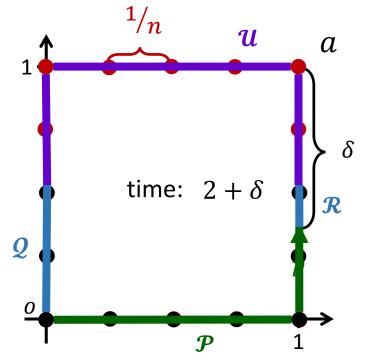
$$|\mathcal{P}| = 2 - \delta$$

$$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \le 2$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \le 2 + \delta$$

Online-TSP	I. Algorithms	II. Lower Bounds
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$
H-OLTSP $\overline{}$	GTR (Greedy): $\rho=2.5$	$\rho \geq 1,5$

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2

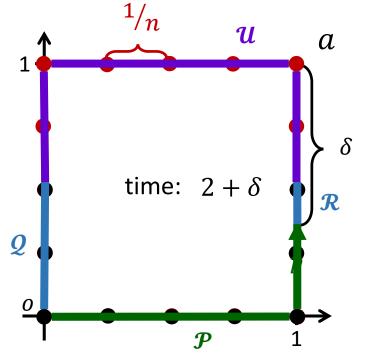


$$|\mathcal{P}| = 2 - \delta \} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \le 2$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \le 2 + \delta \} \Rightarrow |\mathcal{U}|$$

Online-TSP	I. Algorithms	II. Lower Bounds
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$
H-OLTSP $\overline{\hspace{1cm}}$	GTR (Greedy): $ ho=2.5$	$\rho \geq 1,5$

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2

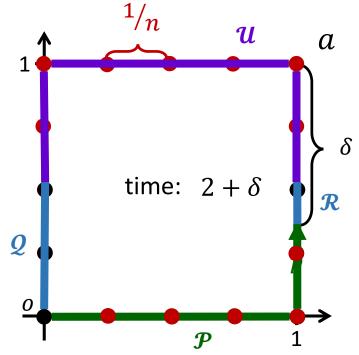


$$|\mathcal{P}| = 2 - \delta \} |Q| + |\mathcal{P}| + |\mathcal{R}| \le 2$$

$$|\mathcal{P}| + 2|Q| + 2|\mathcal{R}| \le 2 + \delta \} \Rightarrow |\mathcal{U}| \ge 2$$

Online-TSP	I. Algorithms	II. Lower Bounds
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2.5$	$\rho \geq 1,5$

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2
- new requests on ${\mathcal P}$

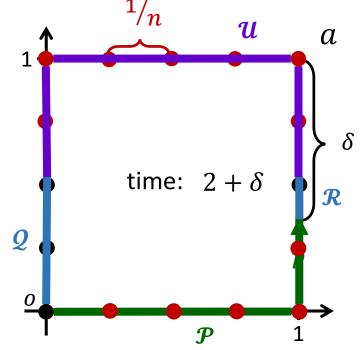


$$|\mathcal{P}| = 2 - \delta \} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \le 2$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \le 2 + \delta \} \Rightarrow |\mathcal{U}| \ge 2$$

Online-TSP I. Algorithms		II. Lower Bounds
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$
H-OLTSP $\overline{\hspace{1cm}}$	GTR (Greedy): $ ho=2.5$	$\rho \geq 1,5$

- requests at time 0
- wait until time t = 2
- wait until d(p,a) = t-2
- new requests on ${\mathcal P}$
- OPT finishes at t=4

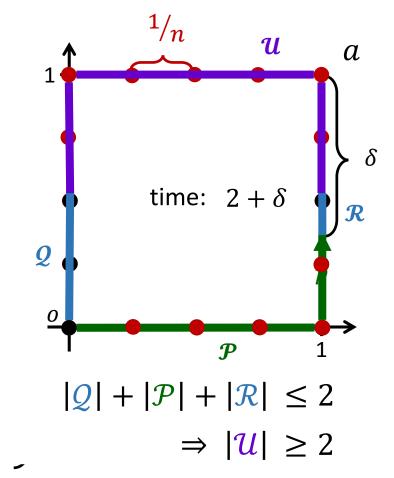


$$|\mathcal{P}| = 2 - \delta \} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \le 2$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \le 2 + \delta \} \Rightarrow |\mathcal{U}| \ge 2$$

Online-TSP	I. Algorithms	II. Lower Bounds
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$
H-OLTSP $\overline{}$	GTR (Greedy): $\rho=2,5$	$\rho \geq 1,5$

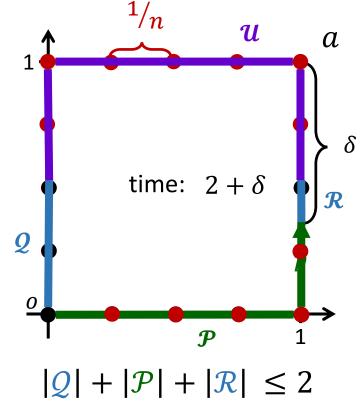
- requests at time 0
- wait until time t = 2
- wait until d(p,a) = t-2
- new requests on ${\mathcal P}$
- OPT finishes at t = 4ALG finishes at $t \ge$



Online-TSP	I. Algorithms	II. Lower Bounds
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 1,5$

- requests at time 0
- wait until time t=2
- wait until d(p,a) = t-2
- new requests on ${\mathcal P}$
- OPT finishes at t=4ALG finishes at $t \ge$





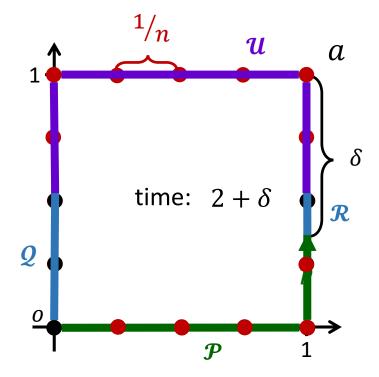
$$|Q| + |P| + |R| \le 2$$

$$\Rightarrow |U| \ge 2$$

Online-TSP	I. Algorithms	II. Lower Bounds
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$
H-OLTSP $\overline{}$	GTR (Greedy): $\rho=2.5$	$ ho \geq 1,5$

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2
- new requests on ${\mathcal P}$
- OPT finishes at t = 4ALG finishes at $t \ge$

$$\begin{cases} \ge 2 + \delta + 2 \cdot \left(2 - \frac{1}{n}\right) + 2 - \delta \\ \ge 2 + \delta + 4 - \frac{1}{n} + (2 - \delta) - \frac{1}{n} \end{cases}$$



$$|Q| + |P| + |R| \le 2$$

$$\Rightarrow |U| \ge 2$$

III. Polynomial Algorithms

N-OLTSP	1
M-OLISP	

Online-TSP

GTR (Greedy): $\rho = 2.5$

I. Algorithms

 $\rho \geq 2$

II. Lower Bounds

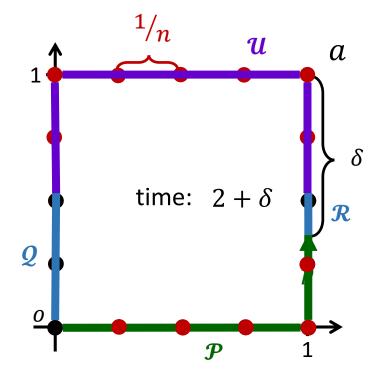
H-OLTSP

GTR (Greedy): $\rho = 2.5$

 $\rho \geq 1.5$

- requests at time 0
- wait until time t=2
- wait until d(p,a) = t-2
- new requests on ${\mathcal P}$
- OPT finishes at t=4ALG finishes at $t \ge 8 - \frac{2}{n}$

$$\begin{cases} \ge 2 + \delta + 2 \cdot \left(2 - \frac{1}{n}\right) + 2 - \delta \\ \ge 2 + \delta + 4 - \frac{1}{n} + (2 - \delta) - \frac{1}{n} \end{cases}$$



$$|Q| + |P| + |R| \le 2$$

$$\Rightarrow |U| \ge 2$$

N-OLTSP	•	

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

Online-TSP

GTR (Greedy): $\rho = 2.5$

 $\rho \geq 2$

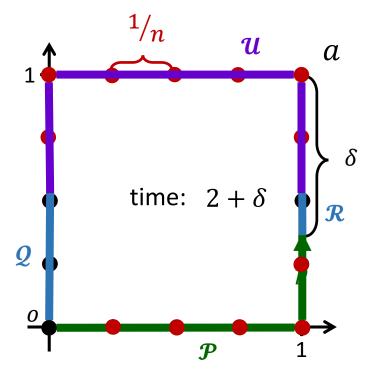
H-OLTSP

GTR (Greedy): $\rho = 2.5$

 $\rho \geq 1.5$

- requests at time 0
- wait until time t = 2
- wait until d(p, a) = t 2
- new requests on ${\mathcal P}$
- OPT finishes at t=4ALG finishes at $t \ge 8 - \frac{2}{n}$

$$\begin{cases} \ge 2 + \delta + 2 \cdot \left(2 - \frac{1}{n}\right) + 2 - \delta \\ \ge 2 + \delta + 4 - \frac{1}{n} + (2 - \delta) - \frac{1}{n} \end{cases}$$



$$|Q| + |P| + |R| \le 2$$

$$\Rightarrow |U| \ge 2$$

N-OLTSP	1
II OLIGI	

Online-TSP

GTR (Greedy): $\rho = 2.5$

I. Algorithms

 $\rho \geq 2$

II. Lower Bounds

H-OLTSP $\overline{}$

GTR (Greedy): $\rho = 2.5$

 $\rho \geq 2$

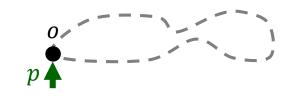
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho = 2.5$	$\rho \geq 2$	



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho = 2.5$	$\rho \geq 2$	

U :=places yet to visit

(1) At o: start optimal tour through U

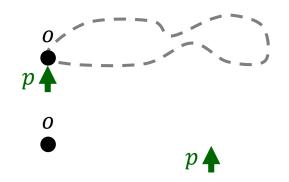


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2.5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $\rho=2.5$	$\rho \geq 2$	

U :=places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):

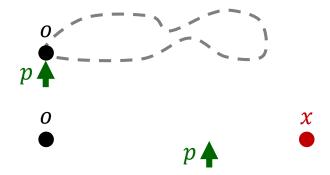


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho=2.5$	$\rho \geq 2$	

U :=places yet to visit

(1) At *o*: start optimal tour through *U*

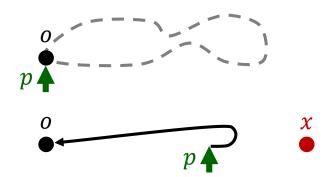
(2) For new request (t, x):



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP \angle	GTR (Greedy): $\rho = 2.5$	$\rho > 2$	

U :=places yet to visit

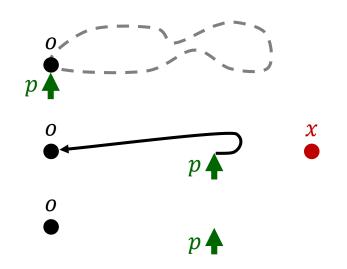
- (1) At *o*: start optimal tour through *U*
- (2) For new request (t, x):
 - a) If d(x, o) > d(p, o): go back to o



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $\rho = 2.5$	$\rho \geq 2$	

U :=places yet to visit

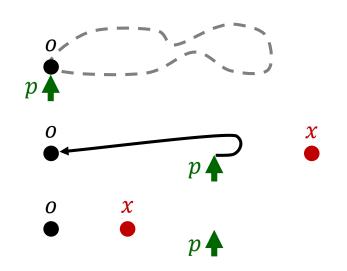
- (1) At o: start optimal tour through U
- (2) For new request (t, x):
 - a) If d(x, o) > d(p, o): go back to o



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $\rho = 2.5$	$\rho \geq 2$	

U :=places yet to visit

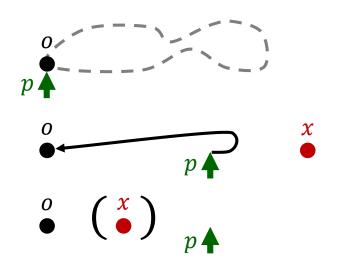
- (1) At *o*: start optimal tour through *U*
- (2) For new request (t, x):
 - a) If d(x, o) > d(p, o): go back to o



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OITSP /	GTR (Greedy): $a = 2.5$	o > 2	

U :=places yet to visit

- (1) At *o*: start optimal tour through *U*
- (2) For new request (t, x):
 - a) If d(x, o) > d(p, o): go back to o
 - b) Else: ignore x until back at o

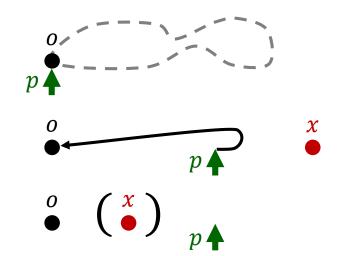


hms

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorith
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP $ left = left$	GTR (Greedy): $\rho = 2.5$	$\rho > 2$	

U :=places yet to visit

- (1) At *o*: start optimal tour through *U*
- (2) For new request (t, x):
 - a) If d(x, o) > d(p, o): go back to o
 - b) Else: ignore x until back at o

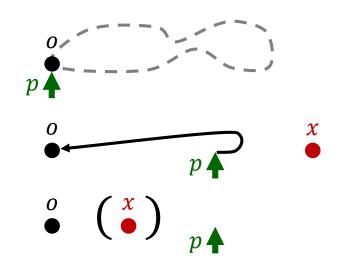


Plan At Home (PAH)

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

U :=places yet to visit

- (1) At *o*: start optimal tour through *U*
- (2) For new request (t, x):
 - a) If d(x, o) > d(p, o): go back to o
 - b) Else: ignore x until back at o

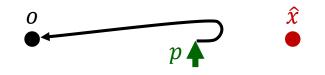


Plan At Home (PAH)

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o

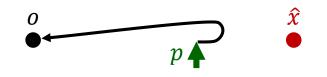


GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP \leftarrow	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



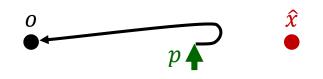
$$|\mathcal{T}^{\mathsf{PAH}}|$$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



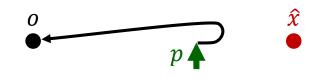
$$|\mathcal{T}^{PAH}| = \hat{t} +$$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



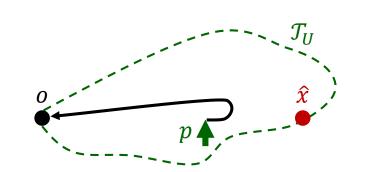
$$\left|\mathcal{T}^{\text{PAH}}\right| = \hat{t} + d(p, o)$$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



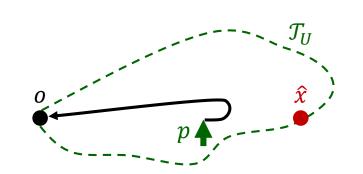
$$|\mathcal{T}^{\text{PAH}}| = \hat{t} + d(p, o) + |\mathcal{T}_U|$$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \hat{t} + d(p, o) + \left|\mathcal{T}_{U}\right|$$

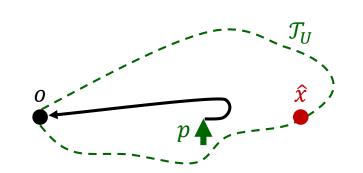
$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



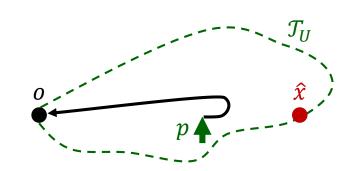
$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq \hat{t} + d(\hat{x}, o) + \left|\mathcal{T}_{U}\right|$$
 $\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



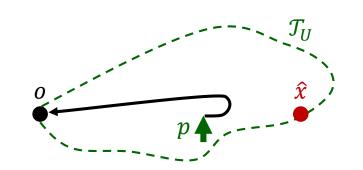
$$\left|\mathcal{T}^{\text{PAH}}\right| \leq \underbrace{\hat{t} + d(\hat{x}, o)}_{\leq} + \left|\mathcal{T}_{U}\right|$$
 $\leq \left|\mathcal{T}^{\text{OPT}}\right|$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



$$\left|\mathcal{T}^{\text{PAH}}\right| \leq \underbrace{\hat{t} + d(\hat{x}, o)}_{\text{ }} + \left|\mathcal{T}_{U}\right|$$

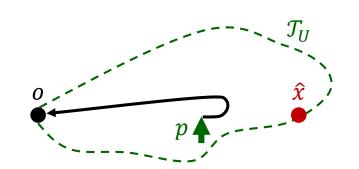
$$\leq \left|\mathcal{T}^{\text{OPT}}\right| + \left|\mathcal{T}^{\text{OPT}}\right|$$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



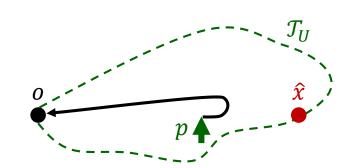
$$\begin{aligned} \left| \mathcal{T}^{\text{PAH}} \right| &\leq \underbrace{\hat{t} + d(\hat{x}, o)}_{} + \left| \mathcal{T}_{U} \right| \\ &\leq \left| \mathcal{T}^{\text{OPT}} \right| + \left| \mathcal{T}^{\text{OPT}} \right| = 2 \cdot \left| \mathcal{T}^{\text{OPT}} \right| \end{aligned}$$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - a) If $d(\hat{x}, o) > d(p, o)$: go back to o



$$\begin{aligned} \left| \mathcal{T}^{\text{PAH}} \right| &\leq \underbrace{\hat{t} + d(\hat{x}, o)}_{} + \left| \mathcal{T}_{U} \right| \\ &\leq \left| \mathcal{T}^{\text{OPT}} \right| + \left| \mathcal{T}^{\text{OPT}} \right| = 2 \cdot \left| \mathcal{T}^{\text{OPT}} \right| \end{aligned}$$

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - b) $d(\hat{x}, o) > d(p, o)$: ignore \hat{x} ...

 $\stackrel{o}{\bullet} \qquad \left(\begin{smallmatrix} \hat{\chi} \\ \bullet \end{smallmatrix} \right)$

 $p \blacktriangle$

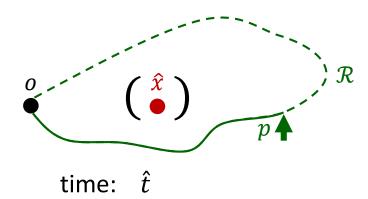
time: \hat{t}

GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

- (2) For new request (\hat{t}, \hat{x}) :
 - b) $d(\hat{x}, o) > d(p, o)$: ignore \hat{x} ...

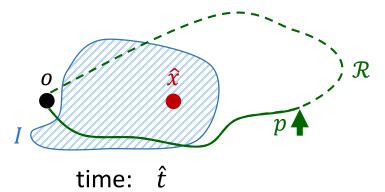


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request

I := ignored requests

- (2) For new request (\hat{t}, \hat{x}) :
 - b) $d(\hat{x}, o) > d(p, o)$: ignore \hat{x} ...



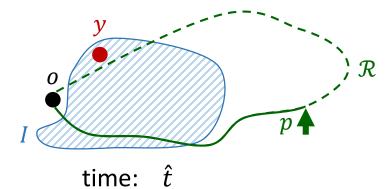
GOAL: PAH is 2-coi

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

 $I \coloneqq \text{ignored requests}$

- (2) For new request (\hat{t}, \hat{x}) :
 - b) $d(\hat{x}, o) > d(p, o)$: ignore \hat{x} ...

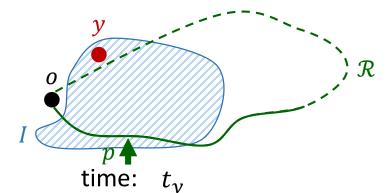


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP \leftarrow	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

 $I \coloneqq \text{ignored requests}$

- (2) For new request (\hat{t}, \hat{x}) :
 - b) $d(\hat{x}, o) > d(p, o)$: ignore \hat{x} ...



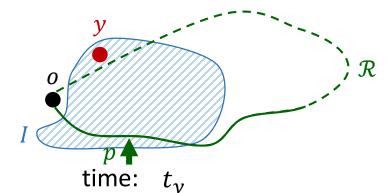
GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_y, y) :
 - b) d(y,o) > d(p,o): ignore y ...



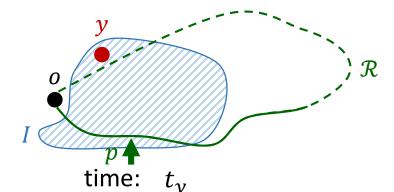
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_y, y) :
 - b) d(y,o) > d(p,o): ignore y ...

 $|\mathcal{T}^{\mathsf{PAH}}|$



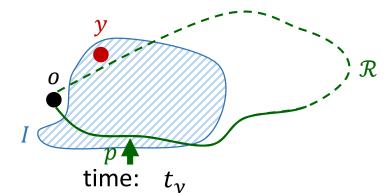
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_y, y) :
 - b) d(y,o) > d(p,o): ignore y ...

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_{v}$$



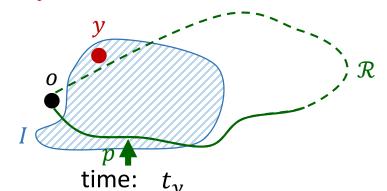
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...

$$\left|\mathcal{T}^{\text{PAH}}\right| \leq t_y + |\mathcal{R}|$$



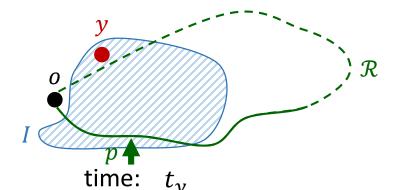
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2$,5	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=$ 2,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...

$$|\mathcal{T}^{\text{PAH}}| \le t_y + |\mathcal{R}| - d(o, y)$$



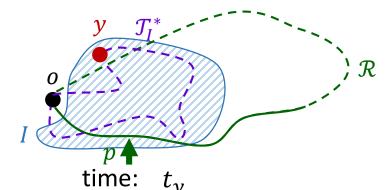
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...

$$|\mathcal{T}^{\text{PAH}}| \le t_y + |\mathcal{R}| - d(o, y)$$



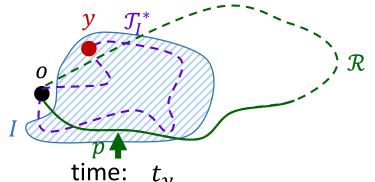
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...

$$|\mathcal{T}^{\text{PAH}}| \le t_y + |\mathcal{R}| - d(o, y) + |\mathcal{T}_I^*|$$



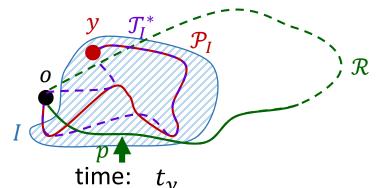
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_y, y) :
 - b) d(y,o) > d(p,o): ignore y ...

$$|\mathcal{T}^{\text{PAH}}| \le t_{v} + |\mathcal{R}| - d(o, y) + |\mathcal{T}_{I}^{*}|$$



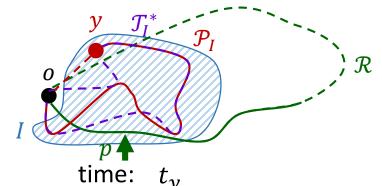
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...

$$|\mathcal{T}^{\text{PAH}}| \le t_y + |\mathcal{R}| - d(o, y) + |\mathcal{T}_I^*|$$

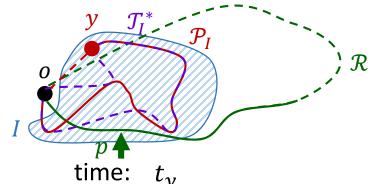


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

 $U \coloneqq \text{places yet to visit, } (\hat{t}, \hat{x}) \text{ last request, } y \text{ fst place in } I \text{ visited by OPT}$

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...



$$\begin{aligned} \left| \mathcal{T}^{\text{PAH}} \right| &\leq t_y + \left| \mathcal{R} \right| - d(o, y) + \underbrace{\left| \mathcal{T}_I^* \right|}_{I} \\ &\leq d(o, y) + \left| \mathcal{P}_I \right| \end{aligned}$$

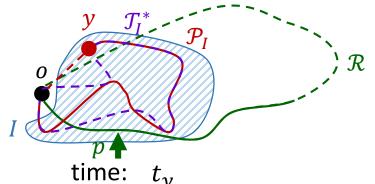
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...

$$\left|\mathcal{T}^{\text{PAH}}\right| \leq t_y + \left|\mathcal{R}\right| - d(o, y) + \left|\mathcal{T}_I^*\right|$$
 $\left|\mathcal{P}_I\right|$



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $\rho=2.5$	$\rho \geq 2$	

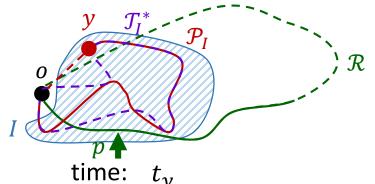
 $U \coloneqq \text{places yet to visit, } (\hat{t}, \hat{x}) \text{ last request, } y \text{ fst place in } I \text{ visited by OPT}$

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...

$$\left| \mathcal{T}^{\text{PAH}} \right| \le t_y + \left| \mathcal{R} \right| - d(o, y) + \left| \mathcal{T}_I^* \right|$$

$$\le t_y + \left| \mathcal{R} \right| + \left| \mathcal{P}_I \right|$$

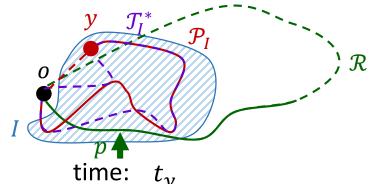


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

 $U \coloneqq \text{places yet to visit, } (\hat{t}, \hat{x}) \text{ last request, } y \text{ fst place in } I \text{ visited by OPT}$

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...



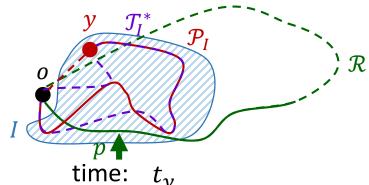
$$\begin{aligned} \left| \mathcal{T}^{\text{PAH}} \right| &\leq t_y + \left| \mathcal{R} \right| - d(o, y) + \left| \mathcal{T}_I^* \right| \\ &\leq t_y + \left| \mathcal{R} \right| + \left| \mathcal{P}_I \right| = t_y + \left| \mathcal{P}_I \right| + \left| \mathcal{R} \right| \end{aligned}$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

 $U \coloneqq \text{places yet to visit, } (\hat{t}, \hat{x}) \text{ last request, } y \text{ fst place in } I \text{ visited by OPT}$

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...



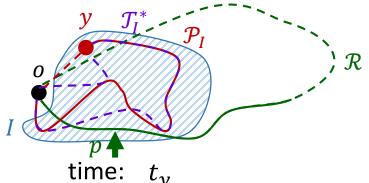
$$\begin{aligned} \left| \mathcal{T}^{\text{PAH}} \right| &\leq t_y + \left| \mathcal{R} \right| - d(o, y) + \left| \mathcal{T}_I^* \right| \\ &\leq t_y + \left| \mathcal{R} \right| + \left| \mathcal{P}_I \right| = \underbrace{t_y + \left| \mathcal{P}_I \right|} + \left| \mathcal{R} \right| \end{aligned}$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

 $U \coloneqq \text{places yet to visit, } (\hat{t}, \hat{x}) \text{ last request, } y \text{ fst place in } I \text{ visited by OPT}$

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...



$$\begin{aligned} \left| \mathcal{T}^{\text{PAH}} \right| &\leq t_y + \left| \mathcal{R} \right| - d(o, y) + \left| \mathcal{T}_I^* \right| \\ &\leq t_y + \left| \mathcal{R} \right| + \left| \mathcal{P}_I \right| = \underbrace{t_y + \left| \mathcal{P}_I \right|}_{\leq |\mathcal{T}^{\text{OPT}}|} + \left| \mathcal{R} \right| \end{aligned}$$

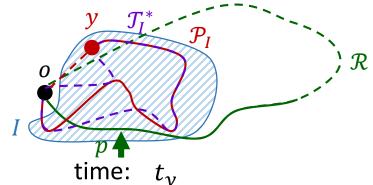
GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_v, y) :
 - b) d(y,o) > d(p,o): ignore y ...



$$\begin{aligned} \left| \mathcal{T}^{\text{PAH}} \right| &\leq t_y + \left| \mathcal{R} \right| - d(o, y) + \left| \mathcal{T}_I^* \right| \\ &\leq t_y + \left| \mathcal{R} \right| + \left| \mathcal{P}_I \right| = \underbrace{t_y + \left| \mathcal{P}_I \right|}_{\leq |\mathcal{T}^{\text{OPT}}|} + \left| \mathcal{T}^{\text{OPT}} \right| \end{aligned}$$

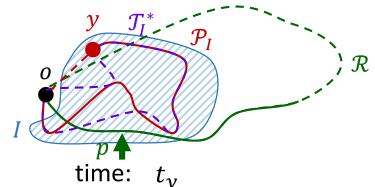
GOAL: PAH is 2

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_y, y) :
 - b) d(y,o) > d(p,o): ignore y ...



$$\begin{aligned} \left| \mathcal{T}^{\mathrm{PAH}} \right| &\leq t_y + \left| \mathcal{R} \right| - d(o, y) + \left| \mathcal{T}_I^* \right| \\ &\leq t_y + \left| \mathcal{R} \right| + \left| \mathcal{P}_I \right| = \underbrace{t_y + \left| \mathcal{P}_I \right|}_{} + \left| \mathcal{T}^{\mathrm{OPT}} \right| = 2 \cdot \left| \mathcal{T}^{\mathrm{OPT}} \right| \end{aligned}$$

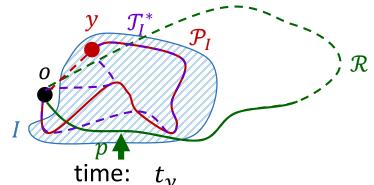
GOAL:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit, (\hat{t}, \hat{x}) last request, y fst place in I visited by OPT

I := ignored requests

- (2) For new request (t_y, y) :
 - b) d(y,o) > d(p,o): ignore y ...



$$\begin{aligned} \left| \mathcal{T}^{\text{PAH}} \right| &\leq t_y + \left| \mathcal{R} \right| - d(o, y) + \left| \mathcal{T}_I^* \right| \\ &\leq t_y + \left| \mathcal{R} \right| + \left| \mathcal{P}_I \right| = t_y + \left| \mathcal{P}_I \right| + \left| \mathcal{R} \right| \end{aligned}$$

$$\leq |\mathcal{T}^{OPT}| + |\mathcal{T}^{OPT}| = 2 \cdot |\mathcal{T}^{OPT}|$$

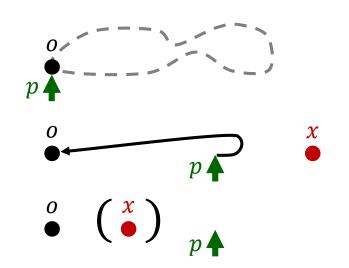
GOAL:



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	

U :=places yet to visit

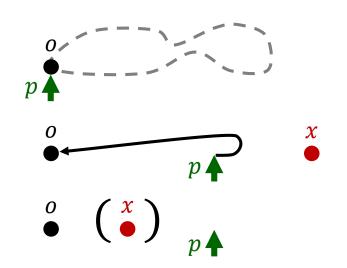
- (1) At *o*: start optimal tour through *U*
- (2) For new request (t, x):
 - a) If d(x, o) > d(p, o): go back to o
 - b) Else: ignore x until back at o



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP 🗢	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	

U :=places yet to visit

- (1) At *o*: start optimal tour through *U*
- (2) For new request (t, x):
 - a) If d(x, o) > d(p, o): go back to o
 - b) Else: ignore x until back at o

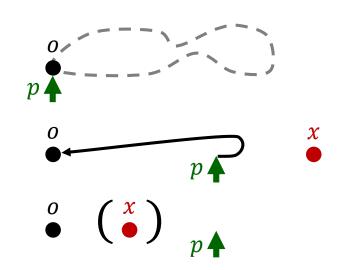


THEOREM: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP 🗢	PAH: $\rho=2$	$\rho \geq 2$	

U :=places yet to visit

- (1) At *o*: start optimal tour through *U*
- (2) For new request (t, x):
 - a) If d(x, o) > d(p, o): go back to o
 - b) Else: ignore x until back at o



THEOREM: PAH is 2-competitive for H-OLTSP

REMARK: PAH is optimal online algorithm for H-OLTSP

O Line TCD	L. Almanialana	II I D I.	III Del constellator tilono
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2,5$	$\rho \geq 2$	
H-OLTSP	$\rho \Delta H$ $\rho = 2$	o > 2	

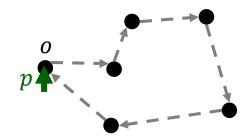
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H OITCD	DAII. 2 — 2	2 > 2	

Invariant: always on shortest path between points in *S*

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP \leftarrow	PAH: $\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in S

(1) At o: Find tour though $U \cup \{o\}$ with Christofides-Heuristic

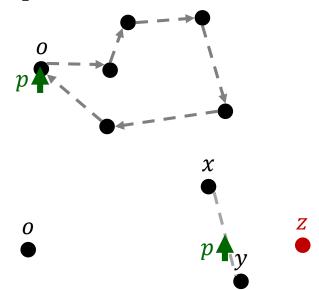


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2$,5	$\rho \geq 2$	

 $\rho \geq 2$

Invariant: always on shortest path between points in S

- (1) At o: Find tour though $U \cup \{o\}$ with Christofides-Heuristic
- (2) For new request (t, z) at time t and ALG between x and y:

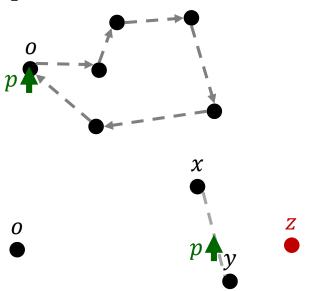


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	
	_		

 $\rho \geq 2$

Invariant: always on shortest path between points in S

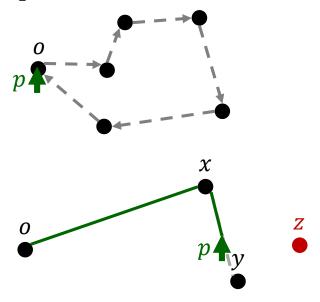
- (1) At o: Find tour though $U \cup \{o\}$ with Christofides-Heuristic
- (2) For new request (t, z) at time t and ALG between x and y:
 - Add z to U



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H OITCD	DALL	0 > 2	

Invariant: always on shortest path between points in S

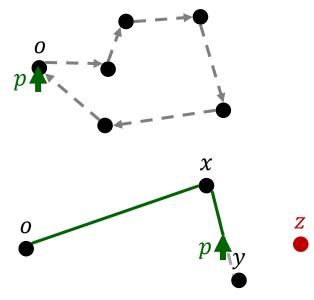
- (1) At o: Find tour though $U \cup \{o\}$ with Christofides-Heuristic
- (2) For new request (t, z) at time t and ALG between x and y:
 - Add z to U
 - go back to o via x or y
 (take shortest path)



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP \leftarrow	PAH: $\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in S

- (1) At o: Find tour though $U \cup \{o\}$ with Christofides-Heuristic
- (2) For new request (t, z) at time t and ALG between x and y:
 - Add z to U
 - go back to o via x or y
 (take shortest path)

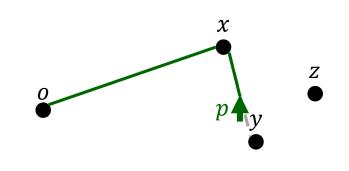


THEOREM: CHR is a polynomial (and correct).

Online-TSP	I. Algo	orithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Gree	dy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP	PAH:	$\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

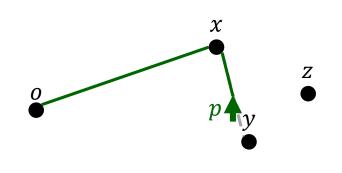
- (2) For last request (\hat{t}, z) at time \hat{t} and ALG between x and y:
 - go back to o via x or y
 (take shortest path)



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,	$\rho \geq 2$	
H-OLTSP $\overline{}$	PAH: $\rho=2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

- (2) For last request (\hat{t}, z) at time \hat{t} and ALG between x and y:
 - go back to o via x or y
 (take shortest path)



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} +$$

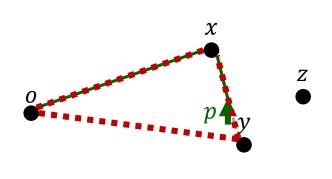
GOAL:

CHR is _-competitive.

Online-TSP	I. Algorithms		II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy	$\rho = 2,5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	PAH:	$\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

- (2) For last request (\hat{t}, z) at time \hat{t} and ALG between x and y:
 - go back to o via x or y
 (take shortest path)

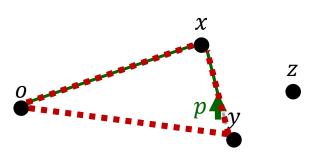


$$|\mathcal{T}^{CHR}| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\}$$

Online-TSP	I. Algorithms		II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy)): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP \leftarrow	PAH:	$\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic

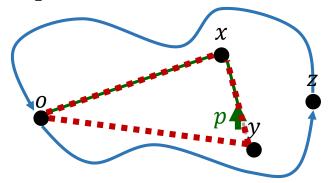


$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\}$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP $\overline{}$	PAH: $\rho=2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic

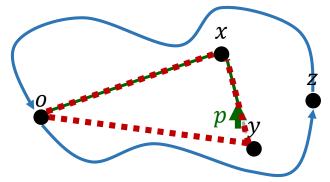


$$|\mathcal{T}^{CHR}| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + CHR(U)$$

Online-TSP	I. Algo	rithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greed	$(y): \rho = 2.5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	PAH:	$\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic



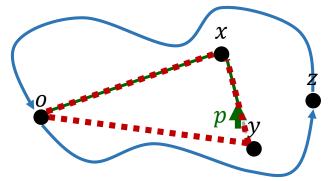
$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\text{OPT}}\right|$$

Online-TSP	I. Algo	rithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greed	$(y): \rho = 2.5$	$\rho \geq 2$	
H-OLTSP $\overline{}$	PAH:	$\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic



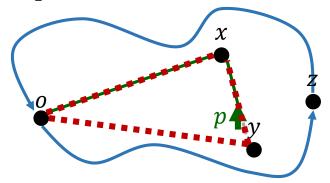
$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\text{OPT}}\right|$$

Online-TSP	I. Al	gorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Gre	edy): $ ho=2$,5	$\rho \geq 2$	
H-OLTSP	PAH:	$\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic



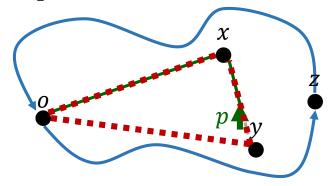
$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\text{OPT}}\right| + \frac{1}{2} \cdot \left|\mathcal{T}^{\text{OPT}}\right|$$

Online-TSP	I. Algo	rithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greed	ly): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP	PAH:	$\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic



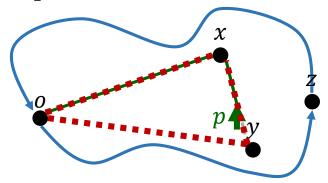
$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\text{OPT}}\right| + \frac{1}{2} \cdot \left|\mathcal{T}^{\text{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\text{OPT}}\right|$$

Online-TSP	I. Algor	ithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy	$(r): \rho = 2.5$	$\rho \geq 2$	
H-OLTSP	PAH:	$\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic

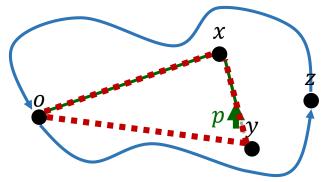


$$\begin{aligned} \left| \mathcal{T}^{\text{CHR}} \right| &= \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U) \\ &\leq \left| \mathcal{T}^{\text{OPT}} \right| + \frac{1}{2} \cdot \left| \mathcal{T}^{\text{OPT}} \right| + \frac{3}{2} \cdot \left| \mathcal{T}^{\text{OPT}} \right| = \end{aligned}$$

Online-TSP	I. Alg	orithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Gree	edy): $\rho = 2,5$	$\rho \geq 2$	
H-OLTSP \leftarrow	PAH:	$\rho = 2$	$\rho \geq 2$	

Invariant: always on shortest path between points in *S*

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic



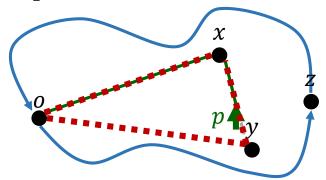
$$\left| \mathcal{T}^{\text{CHR}} \right| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$

$$\leq \left| \mathcal{T}^{\text{OPT}} \right| + \frac{1}{2} \cdot \left| \mathcal{T}^{\text{OPT}} \right| + \frac{3}{2} \cdot \left| \mathcal{T}^{\text{OPT}} \right| = 3 \cdot \left| \mathcal{T}^{\text{OPT}} \right|$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,	$\rho \geq 2$	
H-OLTSP \leftarrow	PAH: $\rho=2$	$\rho \geq 2$	

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic



$$\left| \mathcal{T}^{\text{CHR}} \right| = \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U)$$

$$\leq \left| \mathcal{T}^{\text{OPT}} \right| + \frac{1}{2} \cdot \left| \mathcal{T}^{\text{OPT}} \right| + \frac{3}{2} \cdot \left| \mathcal{T}^{\text{OPT}} \right| = 3 \cdot \left| \mathcal{T}^{\text{OPT}} \right|$$

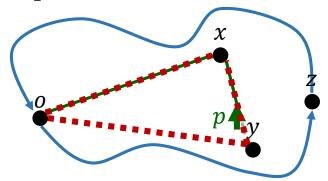
THEOREM: CHR is 3-competitive for H-OLTSP.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	
H-OLTSP /	$D\Delta H$ · $a = 2$	o > 2	CHR (Christofides): $a = 3$

Invariant: always on shortest path between points in *S*

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic

H-OLTSP



CHR (Christofides): $\rho = 3$

$$\begin{aligned} \left| \mathcal{T}^{\text{CHR}} \right| &= \hat{t} + \min\{d(o, x) + d(x, p), d(p, y) + d(y, o)\} + \text{CHR}(U) \\ &\leq \left| \mathcal{T}^{\text{OPT}} \right| + \frac{1}{2} \cdot \left| \mathcal{T}^{\text{OPT}} \right| + \frac{3}{2} \cdot \left| \mathcal{T}^{\text{OPT}} \right| = 3 \cdot \left| \mathcal{T}^{\text{OPT}} \right| \end{aligned}$$

REMARK: There is a 3-competitive algorithm for N-OLTSP.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithm	
N-OLTSP /	GTR (Greedy): $\rho=2.5$	$\rho \geq 2$	MST:	$\rho = 3$

 $\rho \geq 2$

 $\rho = 2$

Credits & References

 Based on Algorithms for the On-Line Travelling Salesman by G. Ausiello, E. Feuerstein, S. Leonardi, L. Stougie, and M. Talamo in Algorithmica (2001) 29: 560–581, DOI: 10.1007/s004530010071 (http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.8.5620)

Titlepage:

- Map: http://awoiaf.westeros.org/index.php/File:WorldoflceandFire.png
- Font by Charlie Samways:
 http://www.fonts4free.net/game-of-thrones-font.html

Online-TSP	I. Algo	orithms	II. Lower Bounds	III. Polynomi	ial Algorithms
N-OLTSP /	GTR (Gree	dy): $\rho = 2,5$	$\rho \geq 2$	MST:	$\rho = 3$
H-OLTSP $\overline{}$	PAH:	$\rho = 2$	$\rho \geq 2$	CHR (Christ	cofides): $\rho = 3$

Bonus-Slide

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms

N-OLTSP

GTR (Greedy): $\rho=2,5$

 $\rho \geq 2$

MST: $\rho = 3$

H-OLTSP

PAH: $\rho=2$

 $= 2 \qquad \qquad \rho \geq 2$

CHR (Christofides): $\rho=3$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms

H-OLTSP

PAH:

 $\rho = 2$

GTR (Greedy): $\rho = 2.5$

 $\rho \geq 2$

 $\rho \ge 2$

CHR (Christofides): $\rho = 3$

 $\rho = 3$

MST:

The lower bound of 2 was shown on \mathbb{R} . REMARK 1:

I. Algorithms III. Polynomial Algorithms Online-TSP II. Lower Bounds

H-OLTSP

N-OLTSP <

PAH:

,

 $\rho = 2$

GTR (Greedy): $\rho = 2.5$

 $\rho \geq 2$

MST:

 $\rho = 3$

 $\rho \geq 2$

CHR (Christofides): $\rho = 3$

REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

,

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms

H-OLTSP \angle

→ PAH

$$\rho = 2$$

GTR (Greedy): $\rho = 2.5$

 $\rho \geq 2$

 $\rho \geq 2$

CHR (Christofides): $\rho = 3$

MST:

 $\rho = 3$

REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

QUESTION: Better algorithm possible on \mathbb{R} ?

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms

H-OLTSP \angle

PAH:

GTR (Greedy): $\rho = 2.5$

$$\rho = 2$$

 $\rho \ge 2$

 $\rho \geq 2$

CHR (Christofides): $\rho = 3$

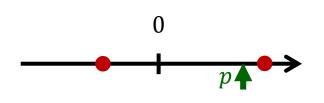
MST:

 $\rho = 3$

REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

QUESTION: Better algorithm possible on \mathbb{R} ?



 $U \coloneqq \mathsf{places} \; \mathsf{yet} \; \mathsf{to} \; \mathsf{visit} \; \mathsf{at} \; t$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms	
N-OLTSP /	GTR (Greedy): $\rho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

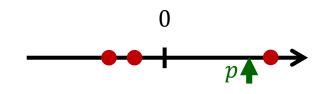
H-OLTSP $\rho = 2$ CHR (Christofides): $\rho = 3$

REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

QUESTION: Better algorithm possible on \mathbb{R} ?

For new request (t, z) at time t:



 $U \coloneqq \mathsf{places} \; \mathsf{yet} \; \mathsf{to} \; \mathsf{visit} \; \mathsf{at} \; t$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms	
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

H-OLTSP $\rho = 2$ CHR (Christofides): $\rho = 3$

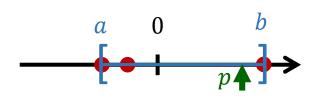
REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

QUESTION: Better algorithm possible on \mathbb{R} ?

For new request (t, z) at time t:

(1) let $U \subseteq [a, b]$ be minimal



 $U \coloneqq \mathsf{places} \; \mathsf{yet} \; \mathsf{to} \; \mathsf{visit} \; \mathsf{at} \; t$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms	
N-OLTSP /	GTR (Greedy): $\rho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

H-OLTSP $\rho = 2$ CHR (Christofides): $\rho = 3$

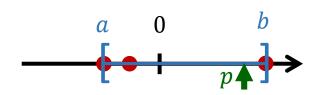
REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

QUESTION: Better algorithm possible on \mathbb{R} ?

For new request (t, z) at time t:

- (1) let $U \subseteq [a, b]$ be minimal
- (2) If d a, $0 \le d(b, 0)$: Go to a Else: Go to b



 $U \coloneqq \mathsf{places} \; \mathsf{yet} \; \mathsf{to} \; \mathsf{visit} \; \mathsf{at} \; t$

Offilite-13F	i. Algoritiiiis	II. LOWEL BOULIUS	III. POIYIIOIIII	iai Aigoritiiiis
N-OLTSP /	GTR (Greedy): $\rho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

H-OLTSP PAH: $\rho=2$ $\rho\geq 2$ CHR (Christofides): $\rho=3$

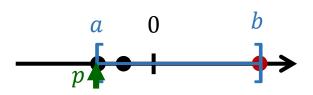
REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

QUESTION: Better algorithm possible on \mathbb{R} ?

For new request (t, z) at time t:

- (1) let $U \subseteq [a, b]$ be minimal
- (2) If d a, $0 \le d(b, 0)$: Go to a Else: Go to b



 $U \coloneqq \mathsf{places} \; \mathsf{yet} \; \mathsf{to} \; \mathsf{visit} \; \mathsf{at} \; t$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynor	nial Algorithms
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

H-OLTSP $\rho = 2$ $\rho \ge 2$ CHR (Christofides): $\rho = 3$

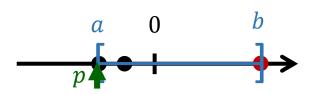
REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

QUESTION: Better algorithm possible on \mathbb{R} ?

For new request (t, z) at time t:

- (1) let $U \subseteq [a, b]$ be minimal
- (2) If d a, $0 \le d(b, 0)$: Go to a Else: Go to b
- (3) Start traversing [a, b]



 $U \coloneqq \mathsf{places} \; \mathsf{yet} \; \mathsf{to} \; \mathsf{visit} \; \mathsf{at} \; t$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms

N-OLTSP /	GTR (Gree	edy): $\rho=2,5$	$\rho \geq 2$	MST:	$\rho = 3$
H-OLTSP	PAH:	$\rho = 2$	$\rho \geq 2$	CHR (Christo	fides): $\rho = 3$

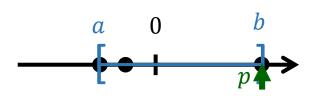
REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

QUESTION: Better algorithm possible on \mathbb{R} ?

For new request (t, z) at time t:

- (1) let $U \subseteq [a, b]$ be minimal
- (2) If d a, $0 \le d(b, 0)$: Go to a Else: Go to b
- (3) Start traversing [a, b]



 $U \coloneqq \mathsf{places} \; \mathsf{yet} \; \mathsf{to} \; \mathsf{visit} \; \mathsf{at} \; t$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms

N-OLTSP \checkmark GTR (Greedy): $\rho=2.5$

 $\rho \geq 2$

MST: $\rho = 3$

H-OLTSP

 $\rho = 2$

 $\rho \geq 2$

CHR (Christofides): $\rho = 3$

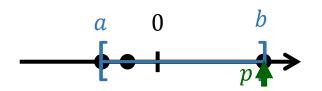
REMARK 1: The lower bound of 2 was shown on \mathbb{R} .

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on \mathbb{R} .

QUESTION: Better algorithm possible on \mathbb{R} ?

For new request (t, z) at time t:

- (1) let $U \subseteq [a, b]$ be minimal
- (2) If d a, $0 \le d(b, 0)$: Go to a Else: Go to b
- (3) Start traversing [a, b]



U :=places yet to visit at t

Extreme Nearest to the Origin first (ENO)

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynon	nial Algorithms
N-OLTSP	GTR (Greedy): $\rho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

H-OLTSP PAH:
$$\rho=2$$
 CHR (Christofides): $\rho=3$

For new request (t, z) at time t:

- (1) let $U \subseteq [a, b]$ be minimal
- (2) If $d(a, 0) \le d(b, 0)$: Go to a Else: Go to b
- (3) Start traversing [a, b]

a	0	b	
			\rightarrow
L	\overline{p}	J	

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms

N-OLTSP \checkmark GTR (Greedy): $\rho = 2.5$

 $\rho \geq 2$

MST: $\rho = 3$

H-OLTSP $\overline{}$

PAH: $\rho = 2$

 $\rho \geq 2$

CHR (Christofides): $\rho=3$

For new request (t, z) at time t:

- (1) let $U \subseteq [a, b]$ be minimal
- (2) If $d(a,0) \le d(b,0)$: Go to a Else: Go to b
- (3) Start traversing [a, b]

THEOREM: ENO is polynomial an correct.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynom	iai Aigorithms
N-OLTSP /	GTR (Greedy): $ ho=2.5$	$\rho \geq 2$	MST:	$\rho = 3$

For new request (t, z) at time t:

- (1) let $U \subseteq [a, b]$ be minimal
- (2) If $d(a, 0) \le d(b, 0)$: Go to a Else: Go to b
- (3) Start traversing [a, b]

THEOREM:	ENO is $\frac{7}{3}$ -competitive	$e\left(\frac{7}{3}\approx\right)$	2,3)
----------	-----------------------------------	------------------------------------	------

Offilitie 131	i. Algoritimis	II. LOWEI Doullus	iii. i OiyiiOiiii	ai Aigoritiiris
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

III Polynomial Algorithms

For last request (\hat{t}, z) at time \hat{t} :

- (2) If $d(a, 0) \le d(b, 0)$: Go to a (w.l.o.g.)
- (3) Start traversing [a, b]

THEOREM: ENO is $\frac{7}{3}$ -competitive ($\frac{7}{3} \approx 2.3$)

GTR (Greedy): $\rho = 2.5$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms

H-OLTSP \leftarrow

PAH:

 $\rho = 2$

 $\rho \geq 2$

 $\rho \geq 2$

CHR (Christofides): $\rho=3$

MST:

 $\rho = 3$

For last request (\hat{t}, z) at time \hat{t} :

- (2) If $d(a, 0) \le d(b, 0)$: Go to a (w.l.o.g.)
- (3) Start traversing [a, b]

Online-TSP

(w.l.o.g.) $\begin{array}{c} a & 0 & b \\ \hline l & p \end{array}$

II Lower Bounds III Polynomial Algorithms

leftmost request (all time) (0 if no request < 0)

THEOREM: ENO is $\frac{7}{3}$ -competitive ($\frac{7}{3} \approx 2.3$)

Offilite 131	1. 7 (1801)(111)	II. LOWEI Boarias	III. I CIYIICIII	idi / ligoriti ii ii
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

For last request (\hat{t}, z) at time \hat{t} :

- (2) If $d(a, 0) \le d(b, 0)$: Go to a (w.l.o.g.)
- (3) Start traversing [a, b]

Online-TSP

(w.l.o.g.) $\begin{array}{c} a & 0 & b \\ \hline l & p & \\ \hline \end{array}$

III Polynomial Algorithms

leftmost request (all time) (0 if no request < 0)

THEOREM: ENO is $\frac{7}{3}$ -competitive ($\frac{7}{3} \approx 2.3$)

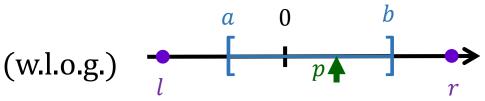
| Algorithms

Offilite 131	1. 7 (1801)(11)	II. LOWEI Doullas	iii. i Ciyiicii	mai / ligoritimiis
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

II Lower Bounds

For last request (\hat{t}, z) at time \hat{t} :

- (2) If $d(a, 0) \le d(b, 0)$: Go to a (w.l.o.g.)
- (3) Start traversing [a, b]



leftmost request (all time) (0 if no request < 0)

 $\Rightarrow l \leq a$, $b \leq r$ and $l \leq p(\hat{t}) \leq r$

THEOREM: ENO is $\frac{7}{3}$ -competitive ($\frac{7}{3} \approx 2.3$)

Online-ISP I. Algorithms		II. Lower Bounds	II. Lower Bounds III. Polynomiai Algo	
N-OLTSP /	GTR (Greedy): $ ho=2$,5	$\rho \geq 2$	MST:	$\rho = 3$

H-OLTSP $\rho = 2$ $\rho \ge 2$ CHR (Christofides): $\rho = 3$

For last request (\hat{t}, z) at time \hat{t} :

- (2) If $d(a, 0) \le d(b, 0)$: Go to a (w.l.o.g.)
- (3) Start traversing [a, b]
- Case-by-case analysis:

(w.l.o.g.) $\begin{array}{cccc} & a & 0 & b \\ & & & \\ l & & & \\ \end{array}$

leftmost request (all time) (0 if no request < 0)

 $\Rightarrow l \leq a$, $b \leq r$ and $l \leq p(\hat{t}) \leq r$

THEOREM: ENO is $\frac{7}{3}$ -competitive ($\frac{7}{3} \approx 2.3$)

Offine-13P 1. Algorithms		II. Lower Bourius	III. Polyflorfilai Algoritfilfis		
N-OLTSP /	GTR (Greedy): $ ho=2,5$	$\rho \geq 2$	MST:	$\rho = 3$	

H-OLTSP
$$\rho = 2$$
 CHR (Christofides): $\rho = 3$

For last request (\hat{t}, z) at time \hat{t} :

- (2) If $d(a, 0) \le d(b, 0)$: Go to a (w.l.o.g.)
 - (w.n.o.g.)
- $\begin{array}{cccc}
 a & 0 & b \\
 \hline
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & & \\
 & &$

(3) Start traversing [a, b]

leftmost request (all time) (0 if no request < 0)

Case-by-case analysis:

$$\Rightarrow l \leq a$$
, $b \leq r$ and $l \leq p(\hat{t}) \leq r$

1.
$$l \leq p \leq a$$



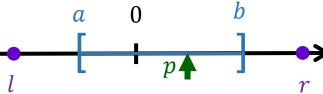
THEOREM: ENO is $\frac{7}{3}$ -competitive ($\frac{7}{3} \approx 2.3$)

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynor	mial Algorithms
N-OLTSP /	GTR (Greedy): $\rho=2.5$	$\rho \geq 2$	MST:	$\rho = 3$

H-OLTSP $\rho = 2$ $\rho \ge 2$ CHR (Christofides): $\rho = 3$

For last request (\hat{t}, z) at time \hat{t} :

- If $d(a,0) \le d(b,0)$: Go to a (w.l.o.g.)



(3) Start traversing [a, b]

leftmost request (all time) (0 if no request < 0)

Case-by-case analysis:

$$\Rightarrow l \leq a$$
, $b \leq r$ and $l \leq p(\hat{t}) \leq r$

1. $l \leq p \leq a$

2. $a \leq p \leq |a|$

ENO is $\frac{7}{2}$ -competitive ($\frac{7}{2} \approx 2.3$) THEOREM:

J-OITSP	\wedge	,

Online-TSP

I. Algorithms

II. Lower Bounds

III. Polynomial Algorithms

GTR (Greedy): $\rho = 2.5$

 $\rho \geq 2$

MST:

 $\rho = 3$

H-OLTSP

PAH:

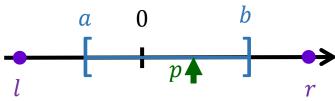
 $\rho = 2$

 $\rho \geq 2$

CHR (Christofides): $\rho = 3$

For last request (\hat{t}, z) at time \hat{t} :

- If $d(a, 0) \le d(b, 0)$: Go to a (w.l.o.g.)



Start traversing [a, b]

leftmost request (all time) (0 if no request < 0)

Case-by-case analysis:

$$\Rightarrow l \leq a$$
, $b \leq r$ and $l \leq p(\hat{t}) \leq r$

1. $l \leq p \leq a$

2. $a \leq p \leq |a|$

3. $|a| \le p \le r$

THEOREM:	ENO is $\frac{7}{3}$ -competitive	$\left(\frac{7}{3} \approx \right)$	2,3)
THEOREM.	2110 13 3 competitive	\ ₃	2 ,0,

N-OLTSP	,

Online-TSP

GTR (Greedy):
$$\rho = 2.5$$

I. Algorithms

$$\rho \geq 2$$

II. Lower Bounds

$$\rho = 3$$

PAH:

$$\rho = 2$$

$$\rho \geq 2$$

CHR (Christofides): $\rho = 3$

III. Polynomial Algorithms

For last request (\hat{t}, z) at time \hat{t} :

- (2) If $d(a, 0) \le d(b, 0)$: Go to a (w.l.o.g.)
- leftmost request (all time) (3) Start traversing [a, b]
- Case-by-case analysis:

$$\Rightarrow l \leq a$$
, $b \leq r$ and $l \leq p(\hat{t}) \leq r$

(0 if no request < 0)

- 1. $l \leq p \leq a$
- 2. $a \leq p \leq |a|$
- complicated... 3. $|a| \leq p \leq r$

ENO is $\frac{7}{2}$ -competitive ($\frac{7}{2} \approx 2.3$) THEOREM:

Online-TSP I. Algorithms		II. Lower Bounds	III. Polynomial Algorith		
N-OLTSP /	GTR (Greedy): $\rho = 2.5$	$\rho \geq 2$	MST:	$\rho = 3$	

H-OLTSP

PAH:

 $\rho \geq 2$

 $\rho \geq 2$

CHR (Christofides): $\rho = 3$

Credits & References

 Based on Algorithms for the On-Line Travelling Salesman by G. Ausiello, E. Feuerstein, S. Leonardi, L. Stougie, and M. Talamo in Algorithmica (2001) 29: 560–581, DOI: 10.1007/s004530010071 (http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.8.5620)

Titlepage:

- Map: http://awoiaf.westeros.org/index.php/File:WorldoflceandFire.png
- Font by Charlie Samways:
 http://www.fonts4free.net/game-of-thrones-font.html

Online-TSP	I. Alg	orithms	II. Lower Bounds	III. Polynom	nial Algorithms
N-OLTSP /	GTR (Gree	edy): $\rho = 2,5$	$\rho \geq 2$	MST:	$\rho = 3$
H-OLTSP $\overline{}$	PAH:	$\rho = 2$	$\rho \geq 2$	CHR (Chris	tofides): $\rho = 3$