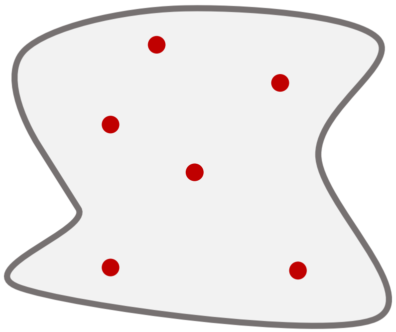
ONLINE - TSP


MILES
0 300 600 900 1200
The Shivering Sea
Thenn
Bay of Ice
Ib Nor
IBBEN
Port of Ibben
Far Ib
Ib Sar
Barrowton
White Harbor
The Bite
Braavos
Lorath
Morosh
New Ibbish
IFEQEVRON
JHOGWIN
Leviathan Sound
THE THOUSAND ISLANDS
MUSSOVY
Ironman's Bay
Gulltown
Saath
SARNOR
WESTEROS
Norvos
Vaes Dothrak
Stoney Sept
Duskendale
The Narrow Sea
Lannisport
Pentos
Qohor
Kayakayanaya
PLAINS OF THE JOGOS NHAI
Nefer
GREY WASTE
Tumbleton
King's Landing
Bitterbridge
DOTHRAKI SEA
Ashford
Tyrosh
Myr
Bleeding Sea
K'Dath
Samyriana
Trader Town
Bonetown
Oldtown
Sea of Dorne
Disputed Lands
Selhorys
Meereen
Hesh
Kosrak
Poison Sea
Valysar
Mantarys
Slaver's Bay
LHAZAR
DORNE
Stepstones
Volon Therys
Yunkai
Lhazosh
RED WASTE
Tiqui
Lys
Volantis
Elyria
Tolos
Bayasabhad
Astapor
City of the Winged Men
Qarkash
Qarth
Asabhad
Hidden Sea
Gulf of Grief
GHISCAR
Port Yhos
Straits of Qarth
YI TI
Carcosa
Jinqi
Valyria
Leng Yi
New Ghis
Faros
Yin
Last Lament
GREAT MORAQ
Vahar
Leng Ma
LENG
The Sunset Sea
Lotus Port
Tall Trees Town
The Summer Sea
Lesser Moraq
Turrani
SHADOW LANDS
THE SUMMER ISLANDS
Basilisk Isles
Cinnamon Straits
Port Moraq
Manticore Isles
Ulos
Indigo Straits
Naath
Mahraj
Saffron Straits
Zabhad
Asshai
SOTHORYOS

Online-TSP

(metric)

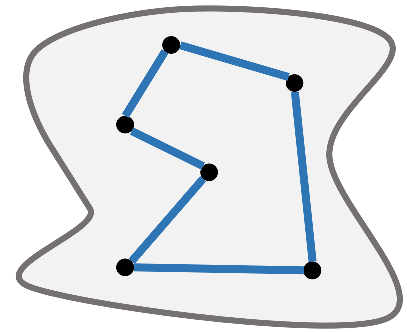
INPUT:



NP-hard!

ALG

OUTPUT:



- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

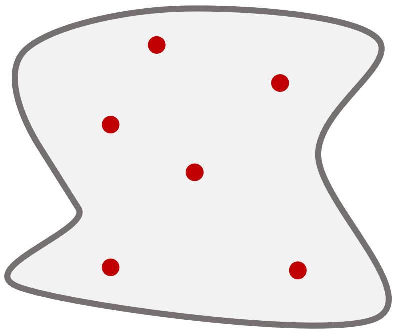
- Superpolynomial Alg.
- Approximation Alg.
  e.g. *Christofides*

Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

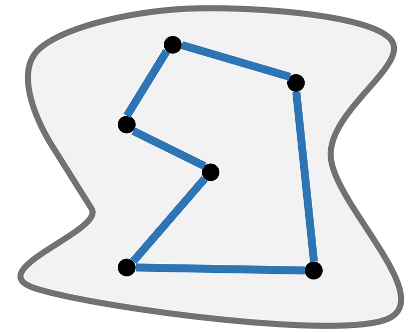
Input:



NP-hard!

ALG

Output:



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- places to visit: $\mathcal{S}$

- Superpolynomial Alg.
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Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

Christofides Algorithm:

(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

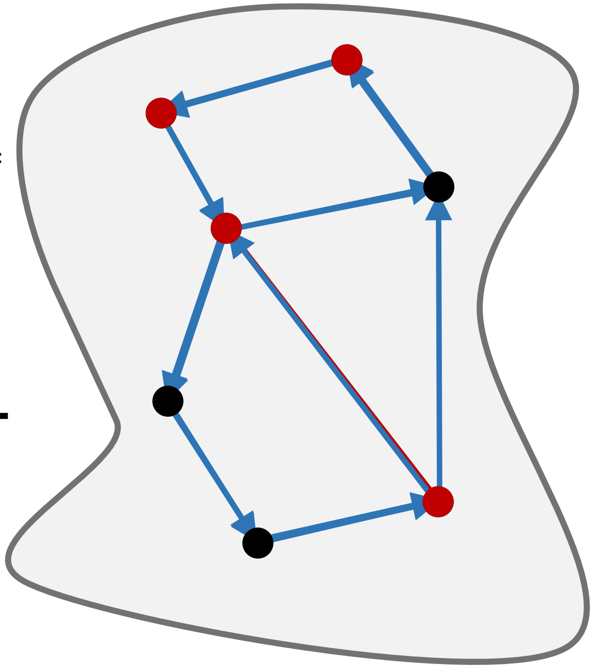
(2) minimum weighted perfect matching of odd vertices $\leq \frac{1}{2} \cdot \mathcal{T}^*$

(3) Euler tour $\leq \frac{3}{2} \cdot \mathcal{T}^*$

(4) Skip double visited vertices

$\Downarrow$

1,5-approximative solution



Online-TSP
(metric)

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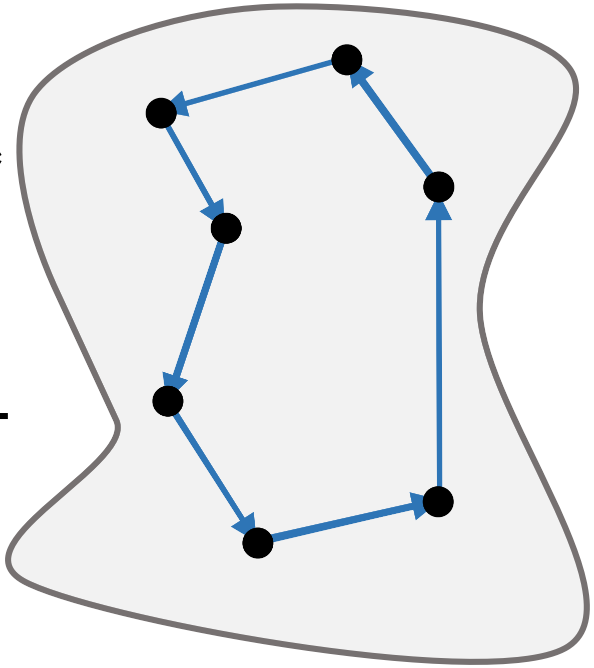
1,5-approximative solution



Online-TSP

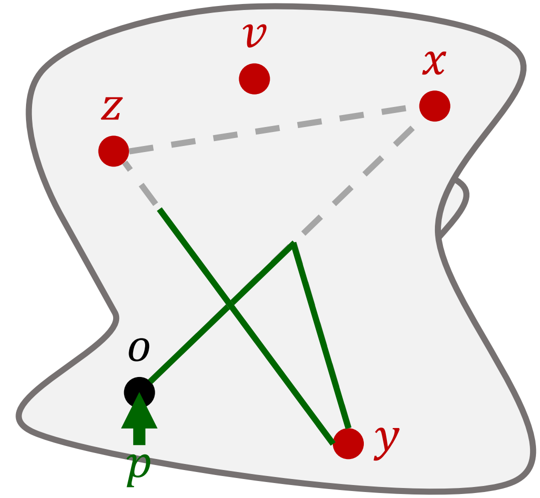
Input:

- metric space
- starting-point: o
- request-sequence σ:

$, (1, y), (1, z), \ldots$

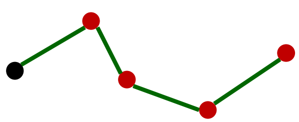
time: $3{,}5$

ALG



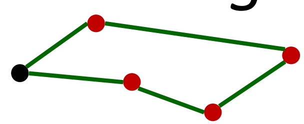
N-OLTSP

"nomadic"



H-OLTSP

"homing"



Def: ALG is *ρ-competitive*

$$\Leftrightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| \leq \rho \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

for all request- sequences

Goals

I. Find online-algorithms (superpolynomial)

II. Find lower bounds

III. Find *polynomial* online-algorithms

IV. Bonus: The real line

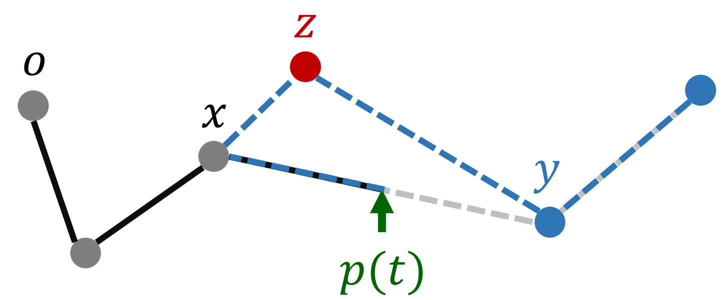
An algorithm for N-OLSTP

Invariant: always on shortest path between points in S

(1) New request (t, z) at time t and ALG between x and y

(2) Add z to U

(3) Follow shortest path through $\mathcal{U}$ beginning with x or y



$S \coloneqq$ places requested until t

$S \supseteq U \coloneqq$ places yet to visit at t

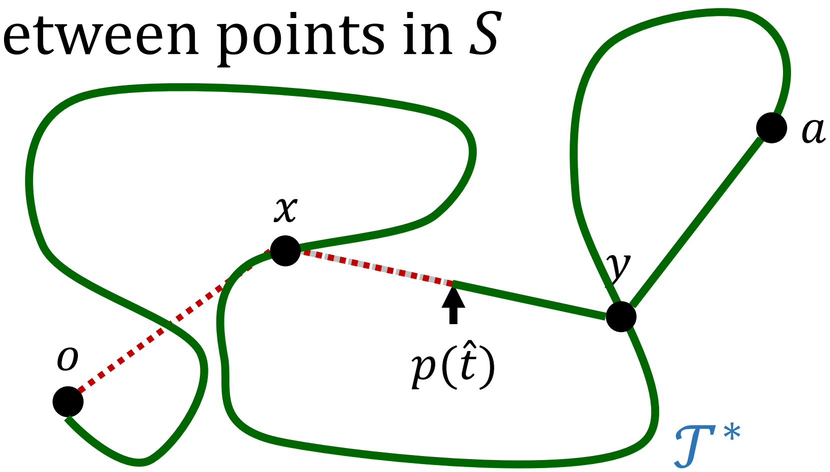
Greedily Travelling between Requests (GTR)

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

path found by opt. offline-ALG

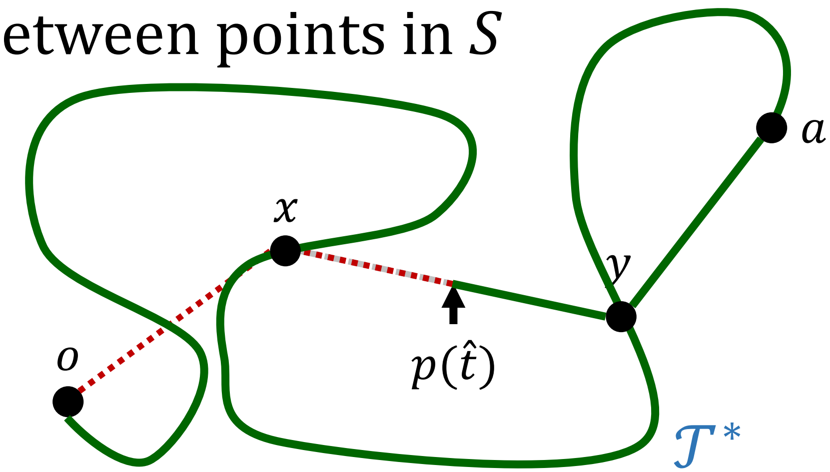
$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| = \frac{5}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



THEOREM: GTR is 2,5-competitive for N-OLTSP

REMARK: tightness

REMARK: GTR is also 2,5-competitive for H-OLTSP

$$|\mathcal{T}^{\mathrm{GTR}}| \leq |\mathcal{T}^{\mathrm{OPT}}| + \frac{3}{2} \cdot |\mathcal{T}^{\mathrm{OPT}}| = \frac{5}{2} \cdot |\mathcal{T}^{\mathrm{OPT}}|$$

Lower Bound for N-OLTSP

time, request

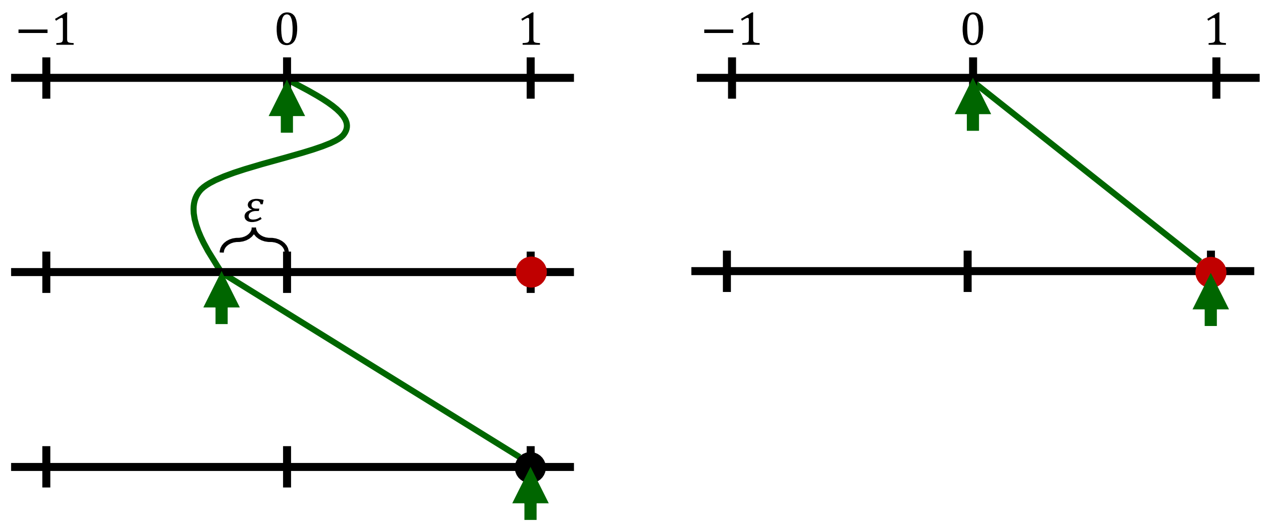
Online-ALG

Offline-ALG

0

$1,\quad 1$

$2+\varepsilon$



$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 2+\varepsilon$

$\Rightarrow \left|\mathcal{T}^{\mathrm{OFF}}\right| = 1$

THEOREM: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

Lower Bound for H-OLTSP

time, request

Online-ALG

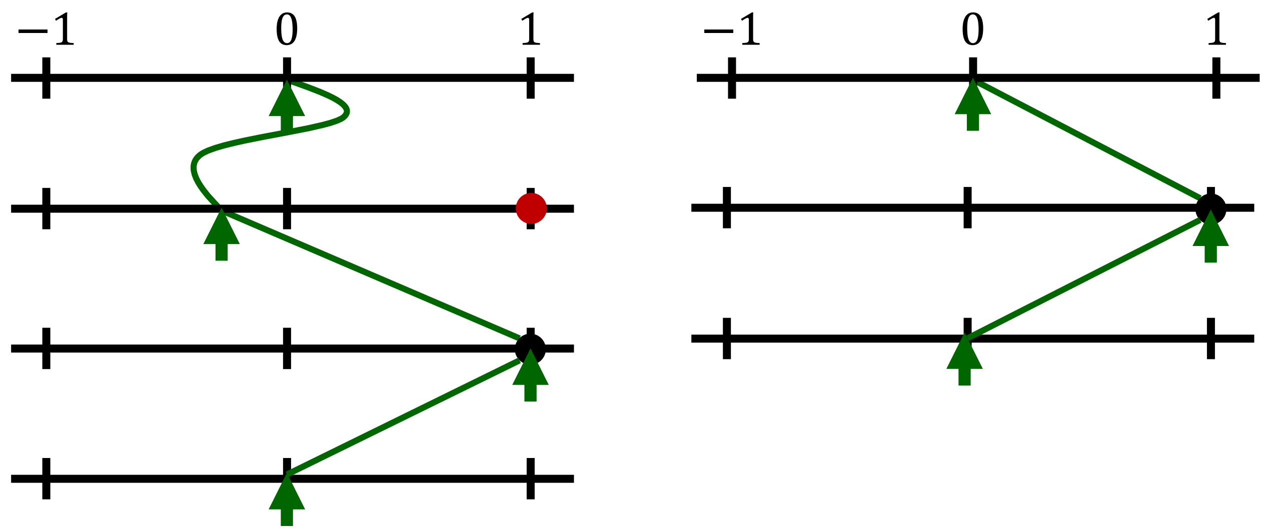
Offline-ALG

0

$1, \quad 1$

$2 + \varepsilon$

$3 + \varepsilon$



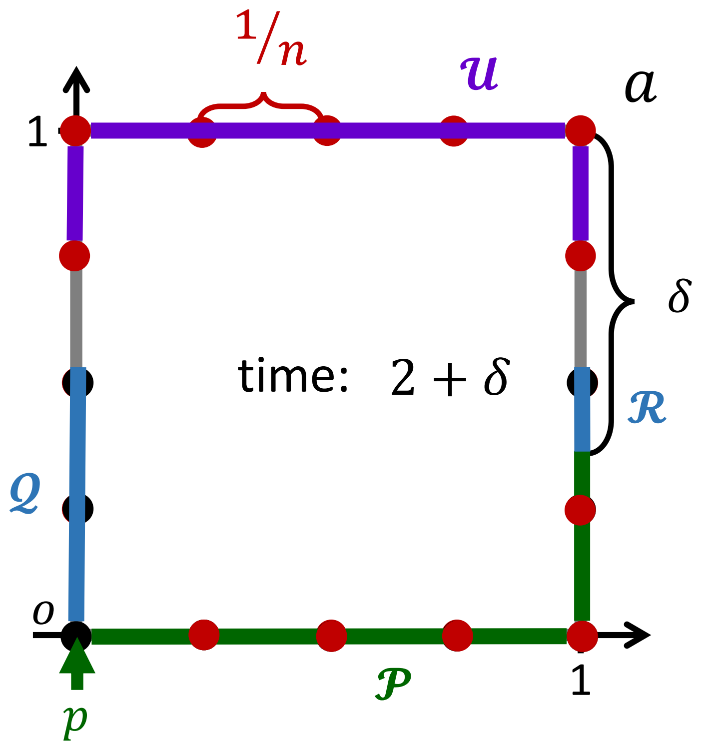
$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 3 + \varepsilon$

$\Rightarrow \left|\mathcal{T}^{\mathrm{OFF}}\right| = 2$

THEOREM: Any ρ-competitive ALG for H-OLTSP has $\rho \geq 1{,}5$.

Lower Bound for H-OLTSP

- requests at time 0
- wait until time $\mathrm{t} = 2$
- wait until $d(p, a) = t - 2$
- new requests on $\mathcal{P}$
- OPT finishes at $t = 4$
  ALG finishes at $t \geq 8 - \frac{4}{n}$



THEOREM: Any ρ-competitive ALG for H-OLTSP has $\rho \geq 2$.

A better algorithm for H-OLTSP

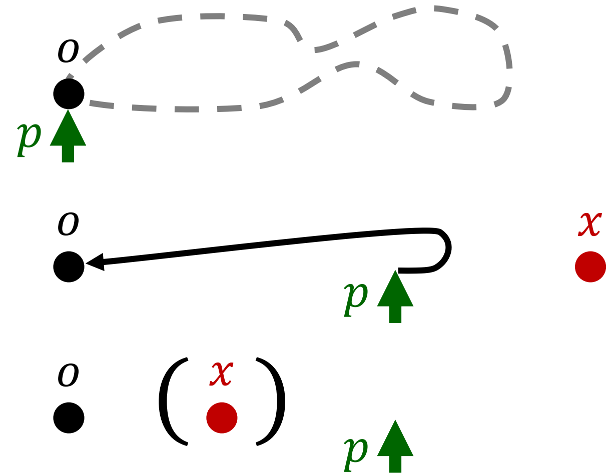
U := places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):

  a) If $d(x, o) > d(p, o)$: go back to o

  b) Else: ignore x until back at o



Plan At Home (PAH)

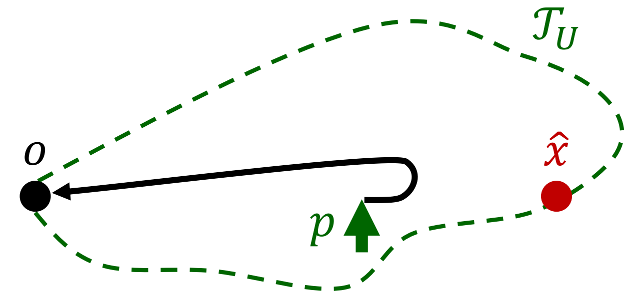
GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o



$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \underbrace{\hat{t} + d(\hat{x}, o)}_{\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|} + |\mathcal{T}_U|$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \quad \checkmark$$

GOAL: PAH is 2-competitive for H-OLTSP

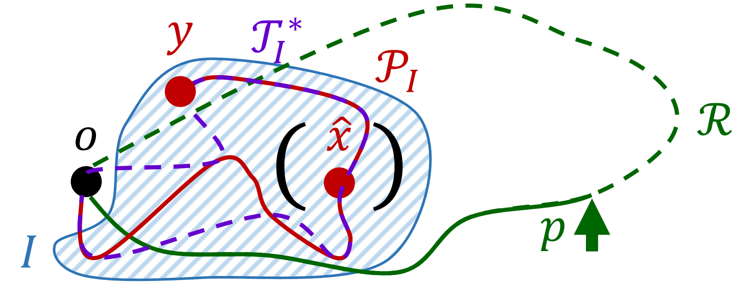
Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT

I := ignored requests

(2) For new request (t_y, y):

b) $d(y,o) > d(p,o)$: ignore y …



time: t_y

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \le t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

$$\le t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|} + |\mathcal{R}|$$

$$\le \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP ✓

Competitiveness of PAH

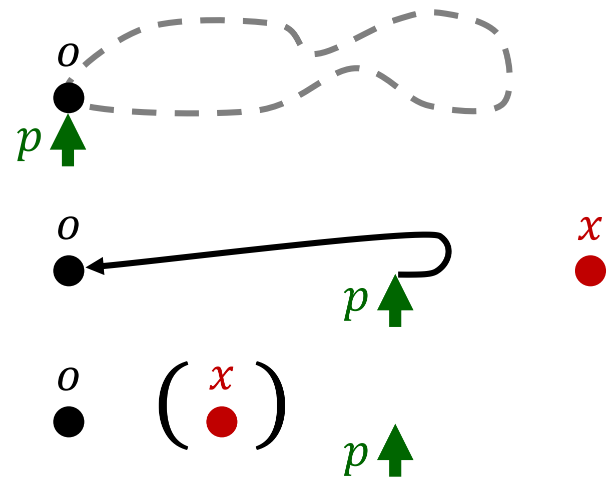
U := places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):

a) If $d(x, o) > d(p, o)$: go back to o

b) Else: ignore x until back at o

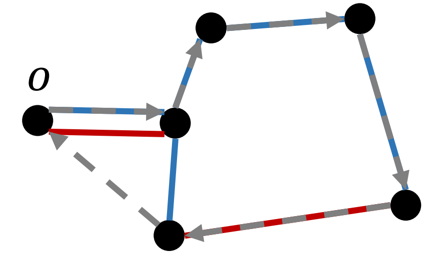


GOAL: PAH is 2-competitive for H-OLTSP r H-OLTSP

Polynomial Algorithm for H-OLTSP

Invariant: always on shortest path between points in S

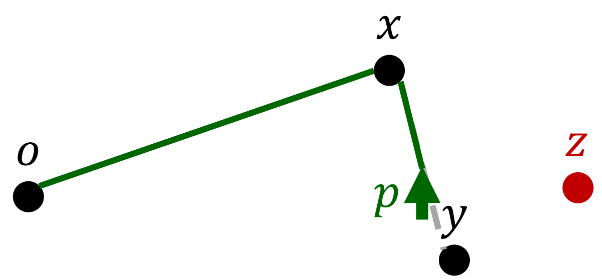
(1) At o: Find tour though $U \cup \{o\}$
with Christofides-Heuristic



(2) For new request (t, z) at time t
and ALG between x and y:

a) Add z to U

b) go back to o via x or y
(take shortest path)

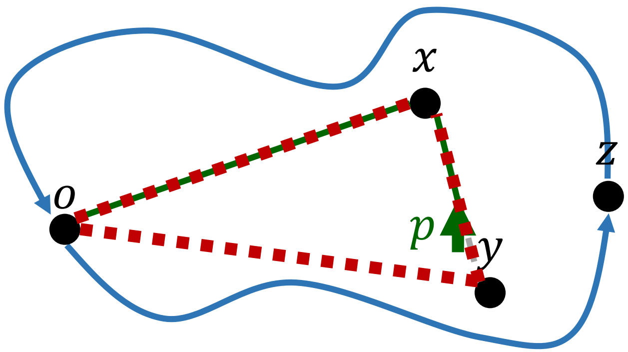


THEOREM: CHR is a polynomial (and correct).

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \underbrace{\min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\}} + \mathrm{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{1}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \;=\; 3 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

REMARK: There is a 3-competitive algorithm for N-OLTSP.

Credits & References

- Paper: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.8.5620
- Map: http://awoiaf.westeros.org/index.php/File:WorldofIceandFire.png
- Font: http://www.fonts4free.net/game-of-thrones-font.html