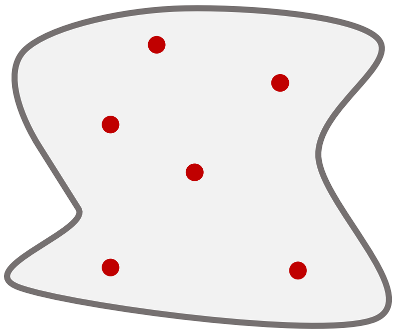
ONLINE - TSP


MILES
0
300
600
900
1200
Thenn
The Shivering Sea
Bay of Ice
Ib Nor
IBBEN
Port of Ibben
Far Ib
Ib Sar
Barrowton
White Harbor
The Bite
Braavos
Lorath
Morosh
New Ibbish
IFEQEVRON
JHOGWIN
Leviathan Sound
THE THOUSAND ISLANDS
MUSSOVY
Ironman's Bay
Gulltown
Saath
SARNOR
WESTEROS
Norvos
Vaes Dothrak
Stoney Sept
Duskendale
The Narrow Sea
Lannisport
Pentos
Qohor
Kayakayanaya
PLAINS OF THE JOGOS NHAI
Nefer
GREY WASTE
Tumbleton
King's Landing
Bitterbridge
DOTHRAKI SEA
Ashford
Tyrosh
Myr
Bleeding Sea
K'Dath
Samyriana
Trader Town
Bonetown
Selhorys
Disputed Lands
Meereen
Hesh
Kosrak
Poison Sea
Oldtown
Sea of Dorne
Valysar
Mantarys
Slaver's Bay
LHAZAR
Stepstones
Yunkai
RED WASTE
Tiqui
DORNE
Volon Therys
Lys
Volantis
Elyria
Tolos
Lhazosh
Bayasabhad
City of the Winged Men
Astapor
Gulf of Grief
Qarkash
Qarth
Asabhad
Hidden Sea
GHISCAR
Port Yhos
Straits of Qarth
YI TI
Carcosa
Jinqi
Valyria
Leng Yi
Last Lament
New Ghis
Faros
Yin
The Sunset Sea
GREAT MORAQ
Vahar
Leng Ma
Lotus Port
Tall Trees Town
The Summer Sea
LENG
SHADOW LANDS
Lesser Moraq
Turrani
THE SUMMER ISLANDS
Basilisk Isles
Cinnamon Straits
Port Moraq
Manticore Isles
Ulos
Indigo Straits
Naath
Mahraj
Saffron Straits
Zabhad
Asshai
SOTHORYOS

Online-TSP

(metric)

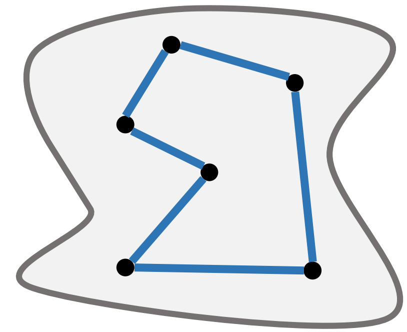
INPUT:



NP-hard!

ALG

OUTPUT:



- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

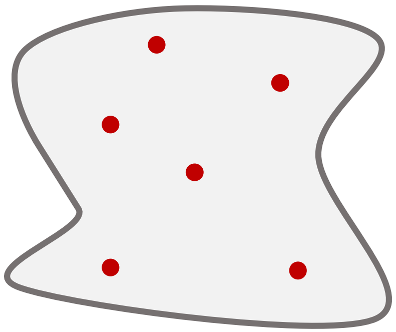
- Superpolynomial Alg.
- Approximation Alg. e.g. *Christofides*

Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

INPUT:



NP-hard!

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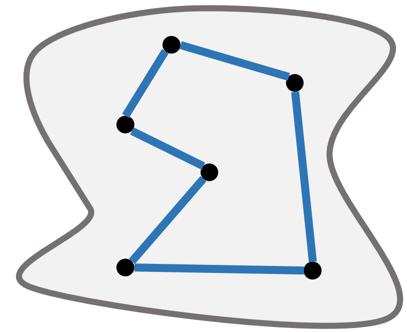

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  e.g. *Christofides*

Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP
(metric)

Christofides Algorithm:

(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

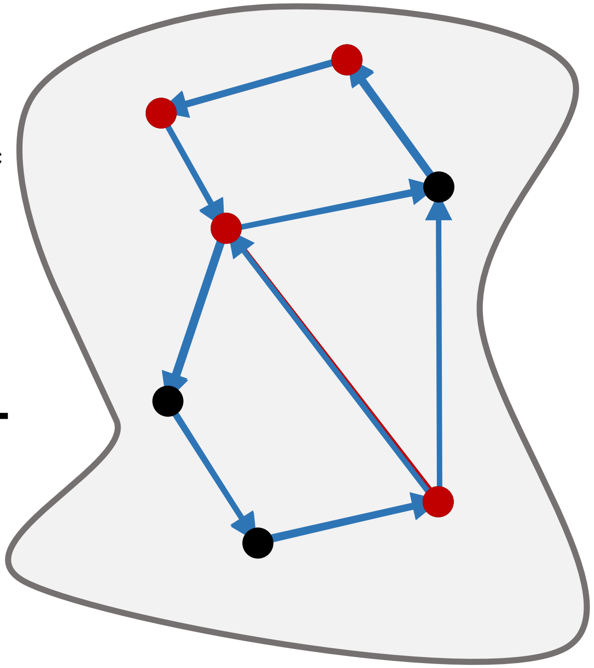
(2) minimum weighted matching of odd vertices $\leq \frac{1}{2} \cdot \mathcal{T}^*$

(3) Euler tour $\leq \frac{3}{2} \cdot \mathcal{T}^*$

(4) Skip double visited vertices

$\Downarrow$

1,5-approximative solution



Online-TSP

(metric)

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(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

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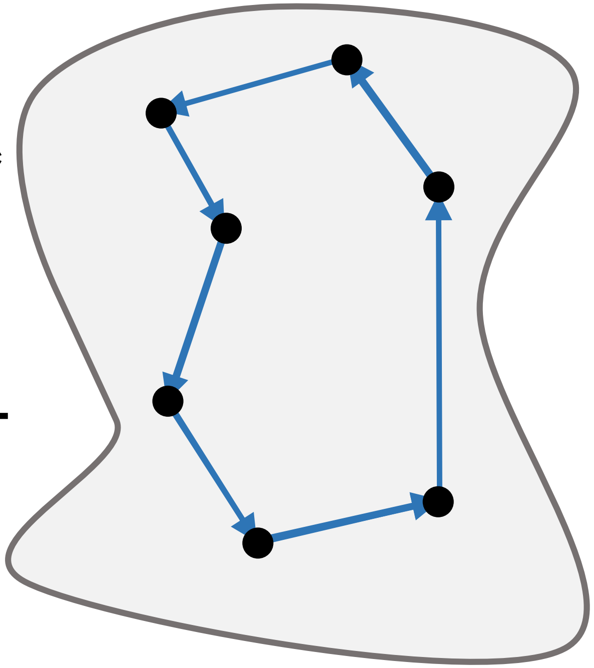


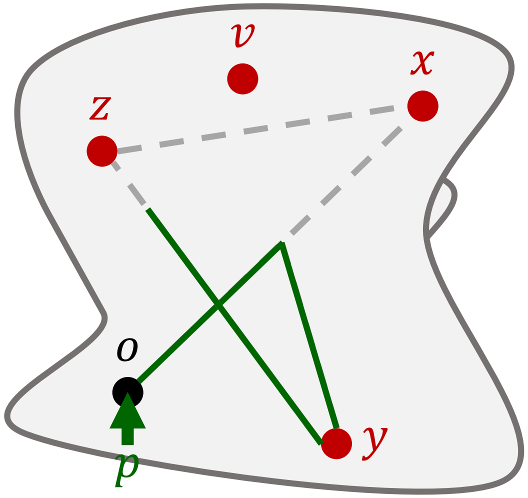
Online-TSP

INPUT:

- metric space
- starting-point: o
- request-sequence σ:
  $(0, x), (1, y), (1, z), \ldots$

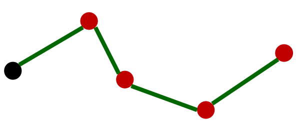
time: 3,5

ALG



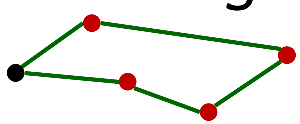
N-OLTSP

"nomadic"



H-OLTSP

"homing"



DEF: ALG is *ρ-competitive*

$$\Leftrightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| \leq \rho \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

for all request- sequences

Goals

I. Find online-algorithms (superpolynomial)

II. Find lower bounds

III. Find *polynomial* online-algorithms

IV. Bonus: The real line

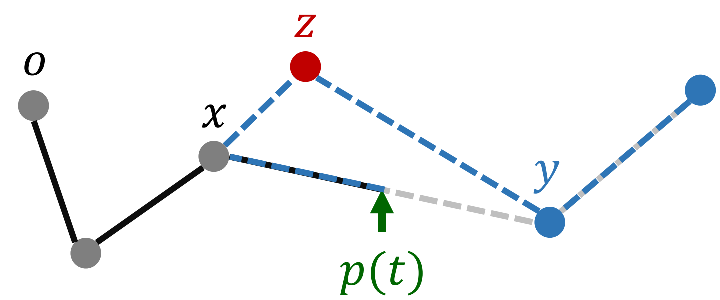
An algorithm for N-OLSTP

Invariant: always on shortest path between points in S

(1) New request (t, z) at time t and ALG between x and y

(2) Add z to U

(3) Follow shortest path through $\mathcal{U}$ beginning with x or y



$S :=$ places requested until t

$S \supseteq U :=$ places yet to visit at t

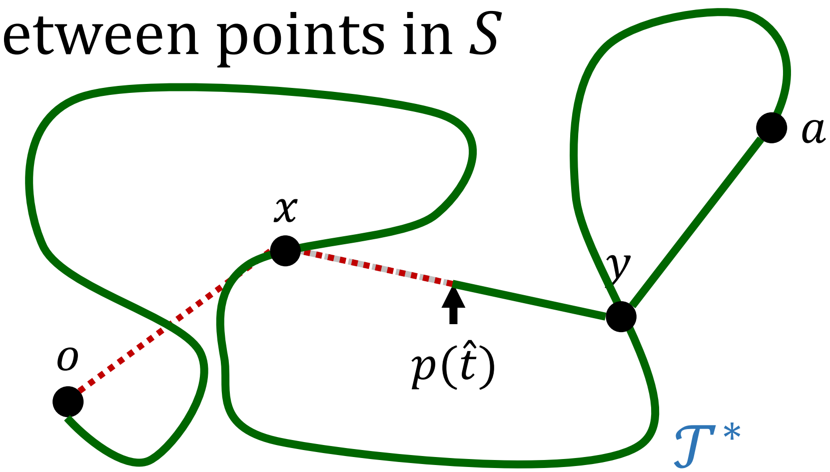
Greedily Travelling between Requests (GTR)

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

path found by opt. offline-ALG

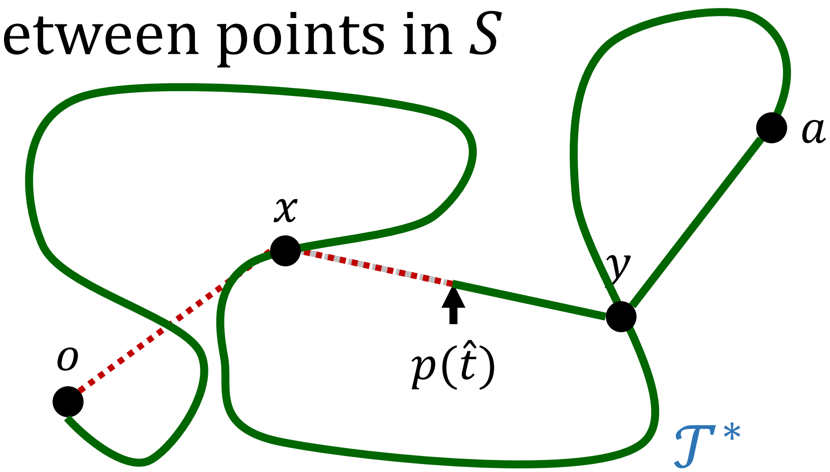
$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| = \frac{5}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



THEOREM: GTR is 2,5-competitive for N-OLTSP

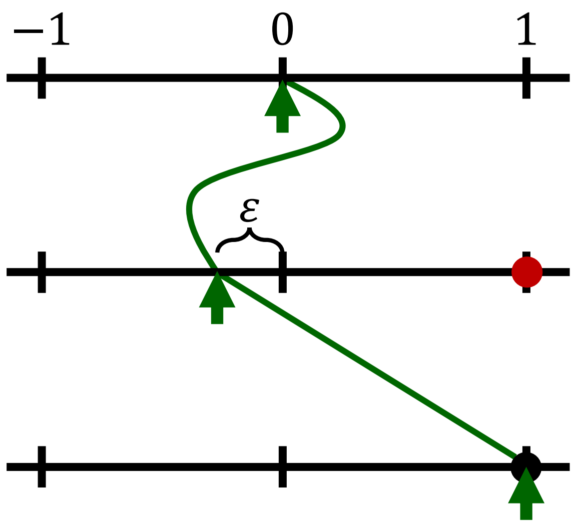
REMARK: tightness

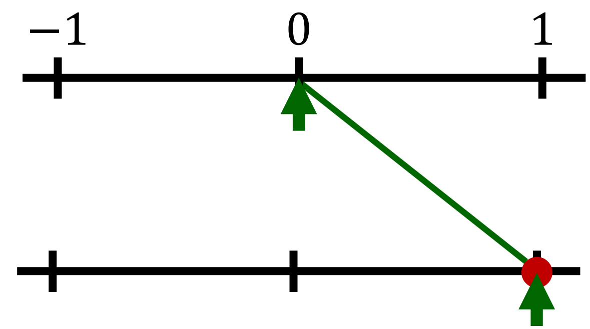
REMARK: GTR is also 2,5-competitive for H-OLTSP

$|\mathcal{T}^{\mathrm{GTR}}| \leq |\mathcal{T}^{\mathrm{OPT}}| + \frac{3}{2} \cdot |\mathcal{T}^{\mathrm{OPT}}| = \frac{5}{2} \cdot |\mathcal{T}^{\mathrm{OPT}}|$

Lower Bound for N-OLTSP

time, request		Online-ALG	Offline-ALG
0			
$1,$	1		
$2+\varepsilon$			





$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 2+\varepsilon$

$\Rightarrow \left|\mathcal{T}^{\mathrm{OFF}}\right| = 1$

THEOREM: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

Lower Bound for H-OLTSP

time, request

Online-ALG

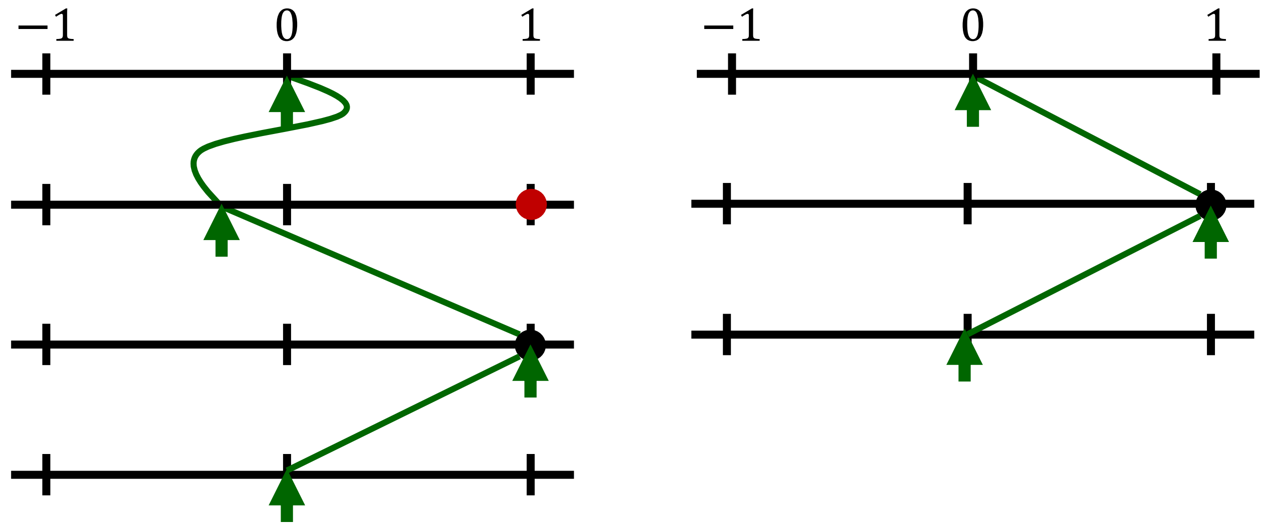
Offline-ALG

0

$1, \quad {\color{red}1}$

$2 + \varepsilon$

$3 + \varepsilon$



$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 3 + \varepsilon$

$\Rightarrow \left|\mathcal{T}^{\mathrm{OFF}}\right| = 2$

Theorem: Any ρ-competitive ALG for H-OLTSP has $\rho \geq 1{,}5$.

Lower Bound for H-OLTSP

A better algorithm for H-OLTSP

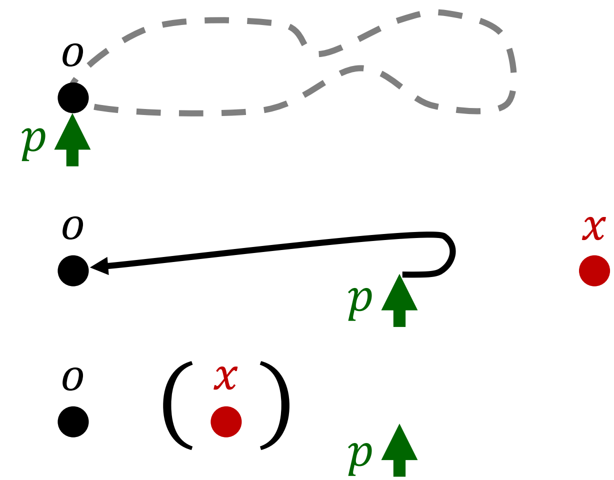
U := places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):

a) If $d(x, o) > d(p, o)$: go back to o

b) Else: ignore x until back at o



Plan At Home (PAH)

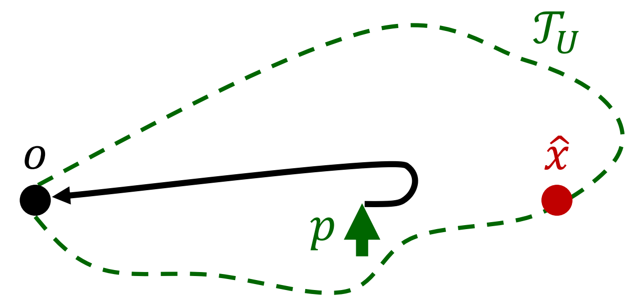
GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o



$$|\mathcal{T}^{\mathrm{PAH}}| = \underbrace{\hat{t} + d(\hat{x}, o)}_{\leq\ |\mathcal{T}^{\mathrm{OPT}}|} + |\mathcal{T}_U|$$

$$\leq |\mathcal{T}^{\mathrm{OPT}}| + |\mathcal{T}^{\mathrm{OPT}}| = 2 \cdot |\mathcal{T}^{\mathrm{OPT}}| \quad ✓$$

GOAL: PAH is 2-competitive for H-OLTSP

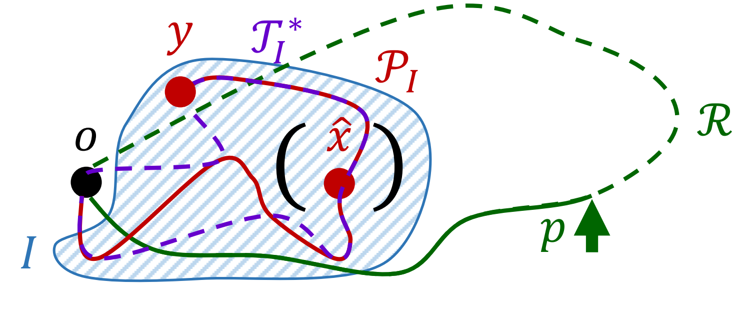
Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT

I := ignored requests

(2) For new request (t_y, y):

b) $d(y, o) > d(p, o)$: ignore y ...



$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \le t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

$$\le t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|} + |\mathcal{R}|$$

$$\le \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP ✓

Competitiveness of PAH

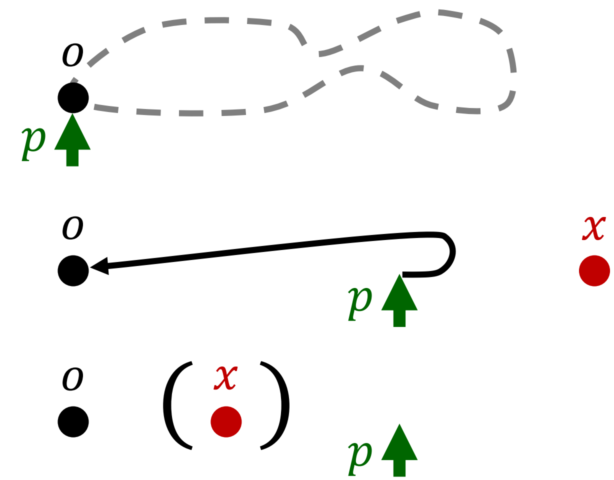
U := places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):

a) If $d(x, o) > d(p, o)$: go back to o

b) Else: ignore x until back at o



GOAL: PAH is 2-competitive for H-OLTSP r H-OLTSP

Polynomial Algorithm for H-OLTSP

proof

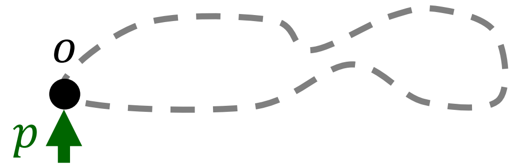
Credits & References

- Paper: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.8.5620
- Map: http://awoiaf.westeros.org/index.php/File:WorldofIceandFire.png
- Font: http://www.fonts4free.net/game-of-thrones-font.html

Competitiveness of PAH

U ≔ places yet to visit, I ≔ ignored requests, $(\hat{t}, \hat{x})$ last request

(1) At o: start optimal tour through U



$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \le \hat{t} + |\mathcal{T}^{*}| \le \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \qquad |\mathcal{T}^{*}|$$

GOAL: PAH is 2-competitive for H-OLTSP