ONLINE - TSP


MILES
0 300 600 900 1200
The Shivering Sea
Bay of Ice
Ib Nor
IBBEN
Port of Ibben
Far Ib
Ib Sar
New Ibbish
Barrowton
White Harbor
The Bite
Braavos
Lorath
Morosh
Ironman's Bay
Gulltown
Saath
SARNOR
IFEQEVRON
JHOGWIN
Leviathan Sound
THE THOUSAND ISLANDS
MUSSOVY
WESTEROS
Norvos
Vaes Dothrak
Stoney Sept
Duskendale
The Narrow Sea
Lannisport
Tumbleton
King's Landing
Pentos
Qohor
Kayakayanaya
PLAINS OF THE JOGOS NHAI
Nefer
GREY WASTE
Bitterbridge
Ashford
DOTHRAKI SEA
Bleeding Sea
K'Dath
Tyrosh
Myr
Samyriana
Trader Town
Bonetown
Oldtown
Sea of Dorne
Disputed Lands
Selhorys
Valysar
Meereen
Hesh
Kosrak
Poison Sea
Stepstones
Mantarys
Slaver's Bay
Yunkai
LHAZAR
Tiqui
DORNE
Lys
Volon Therys
Volantis
Elyria
Tolos
Lhazosh
RED WASTE
Bayasabhad
City of the Winged Men
Astapor
Gulf of Grief
GHISCAR
Port Yhos
Qarkash
Qarth
Asabahd
Hidden Sea
Carcosa
Straits of Qarth
YI TI
Jinqi
Valyria
Leng Yi
Last Lament
New Ghis
Faros
Yin
The Sunset Sea
Lotus Port
Tall Trees Town
The Summer Sea
Vahar
GREAT MORAQ
Leng Ma
LENG
SHADOW LANDS
THE SUMMER ISLANDS
Basilisk Isles
Lesser Moraq
Turrani
Indigo Straits
Naath
Cinnamon Straits
Port Moraq
Mahraj
Manticore Isles
Saffron Straits
Zabhad
Asshai
SOTHORYOS

Online-TSP

(w

Online-TSP

(metric)

(w

Online-TSP
(metric)

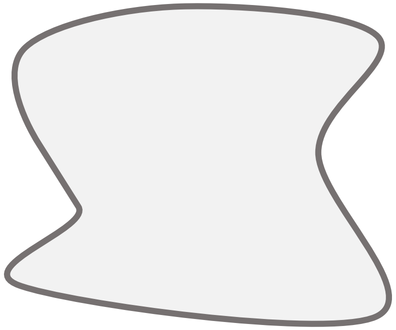
INPUT:

(w

Online-TSP

(metric)

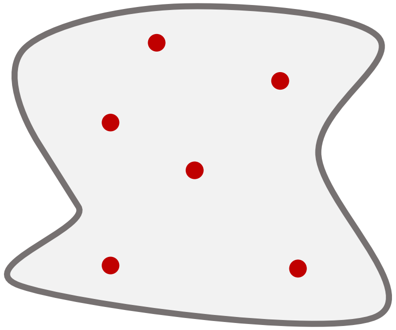
INPUT:



- metric space: M
  (with metric d)

Online-TSP
(metric)

INPUT:

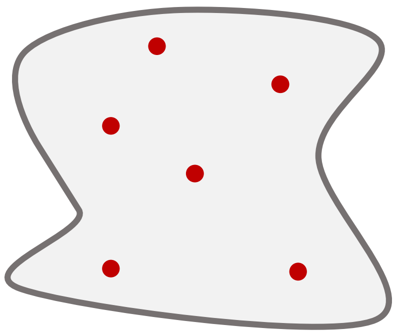


- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

Online-TSP

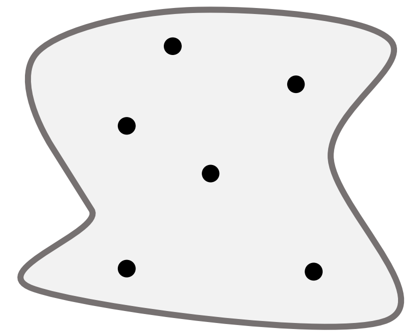
(metric)

INPUT:



- metric space: M
  (with metric d)
- places to visit: $\mathcal{S}$

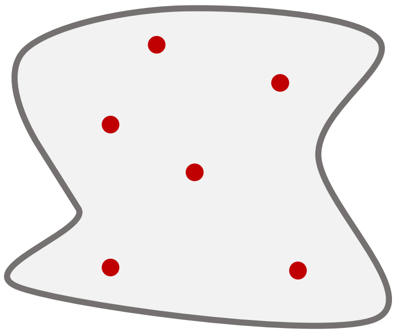
OUTPUT:



Online-TSP

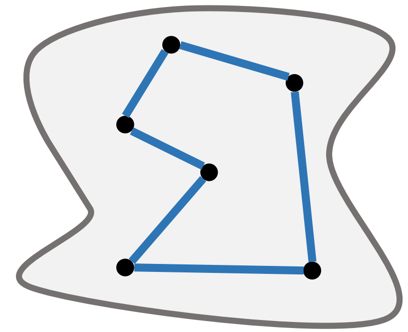
(metric)

INPUT:



- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

OUTPUT:

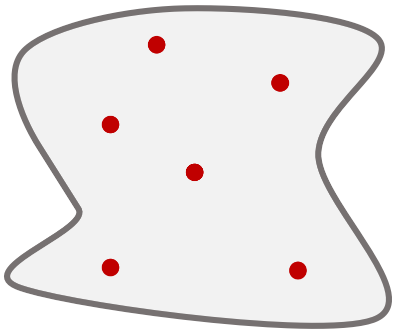


Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

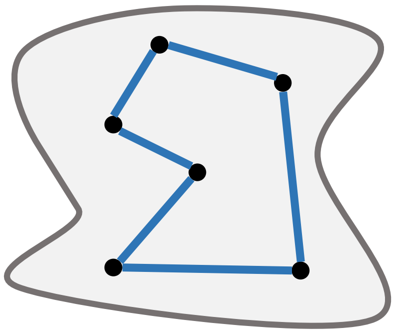
INPUT:



- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

NP-hard!

OUTPUT:

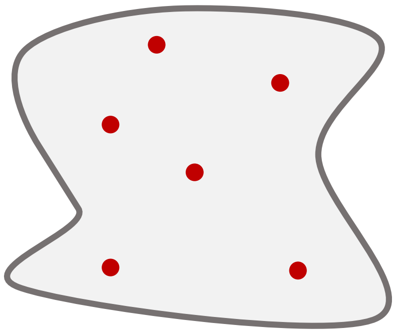


Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

INPUT:

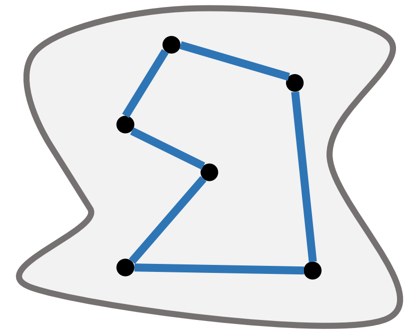


- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

NP-hard!

ALG

OUTPUT:

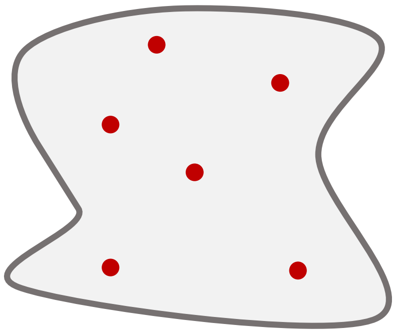


Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

INPUT:



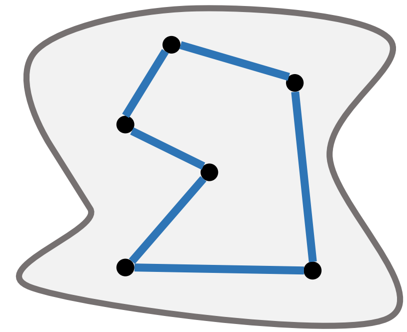
- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

NP-hard!

ALG

- Superpolynomial Alg.

OUTPUT:



Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

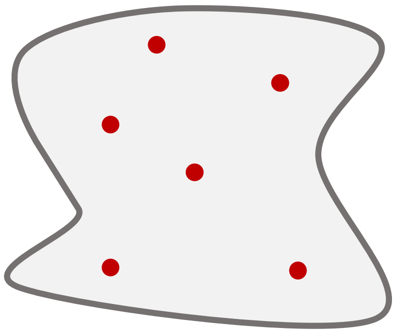
Online-TSP

(metric)

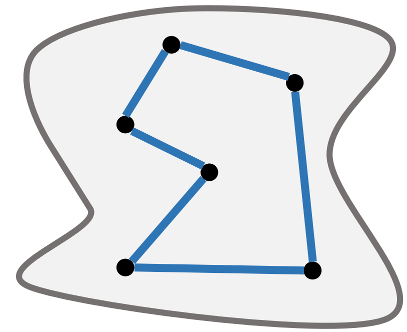
INPUT:

NP-hard!

OUTPUT:



ALG



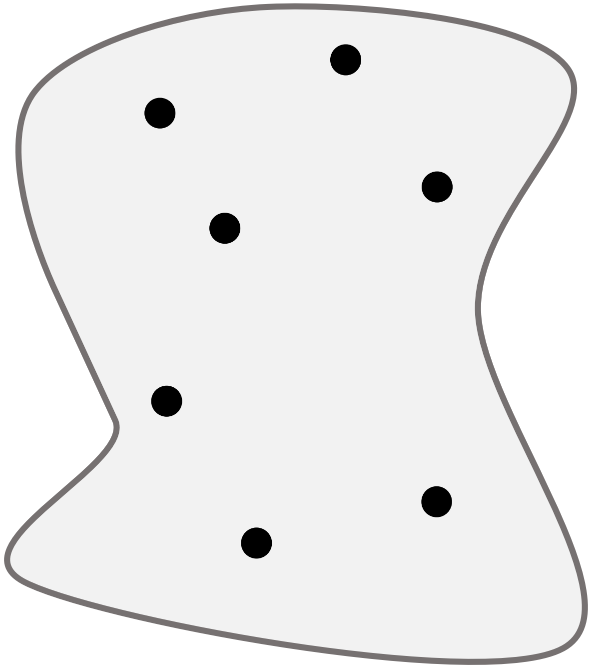
- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

- Superpolynomial Alg.
- Approximation Alg. e.g. *Christofides*

Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP
(metric)

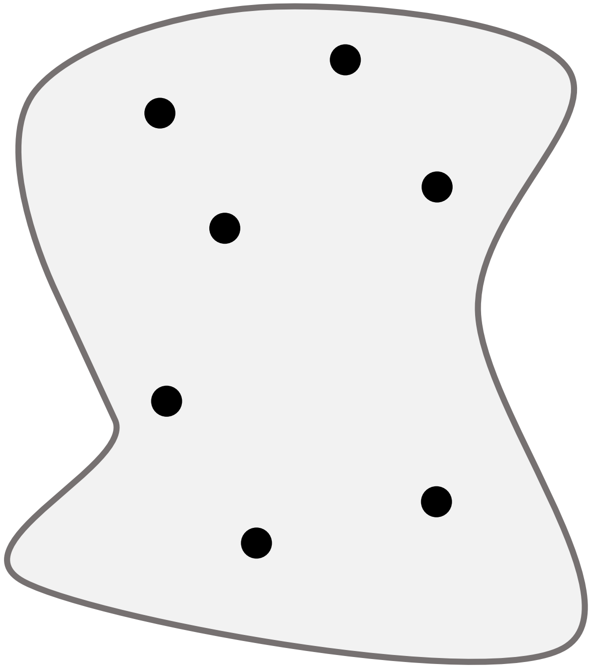
Christofides Algorithm:



Online-TSP

(metric)

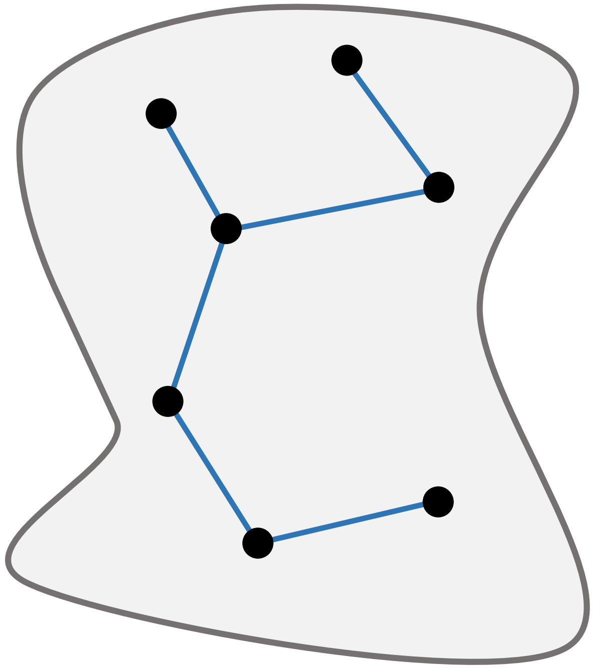
Christofides Algorithm:



Online-TSP
(metric)

Christofides Algorithm:

(1) minimal spanning tree

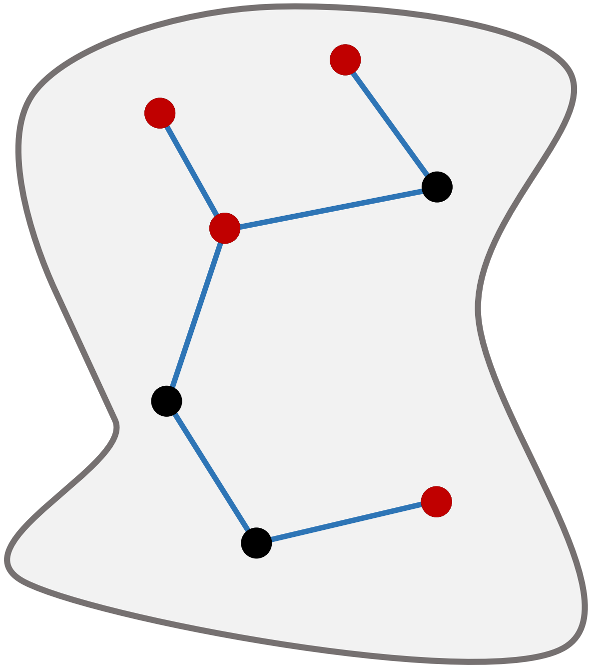


Online-TSP

(metric)

Christofides Algorithm:

(1) minimal spanning tree



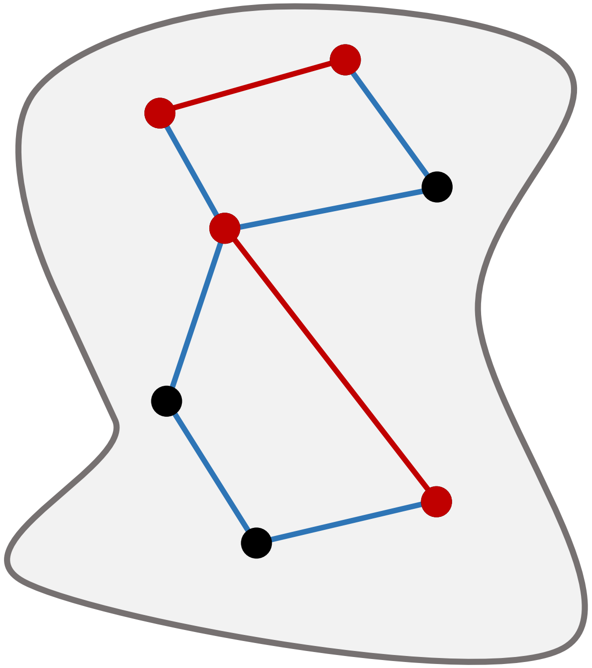
Online-TSP

(metric)

Christofides Algorithm:

(1) minimal spanning tree

(2) minimum weighted perfect matching of odd vertices



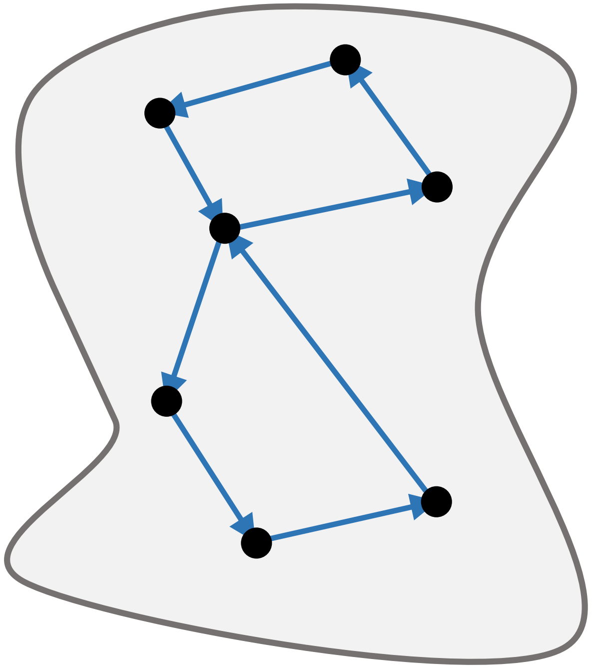
Online-TSP
(metric)

Christofides Algorithm:

(1) minimal spanning tree

(2) minimum weighted perfect matching of odd vertices

(3) Euler tour



Online-TSP

(metric)

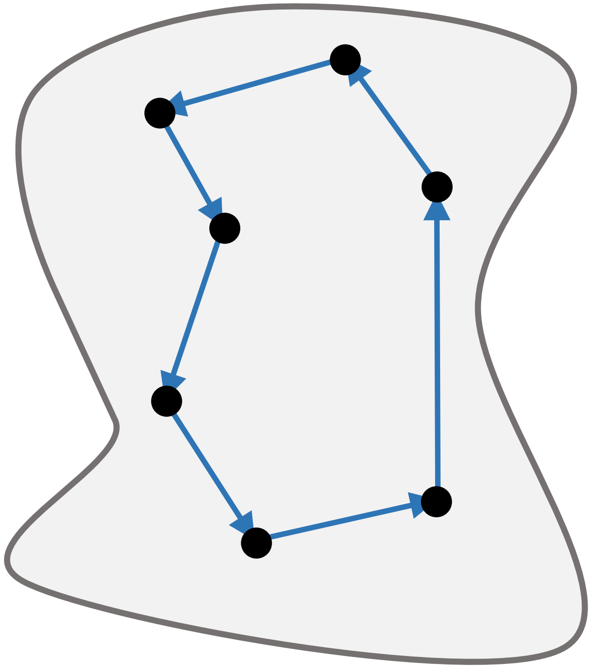
Christofides Algorithm:

(1) minimal spanning tree

(2) minimum weighted perfect matching of odd vertices

(3) Euler tour

(4) Skip double visited vertices



Online-TSP

(metric)

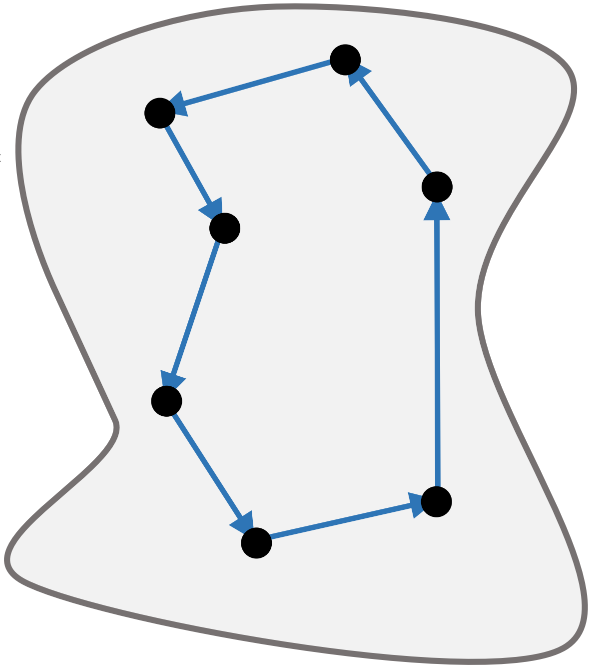
Christofides Algorithm:

(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

(2) minimum weighted perfect matching of odd vertices

(3) Euler tour

(4) Skip double visited vertices



Online-TSP

(metric)

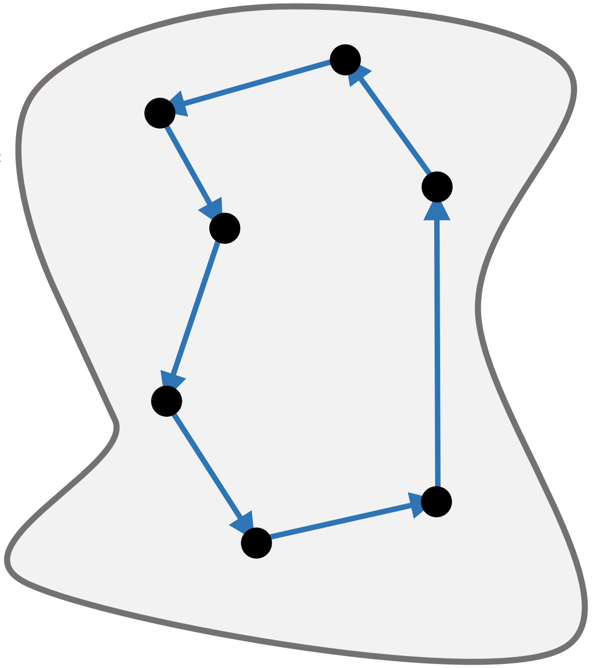
Christofides Algorithm:

(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

(2) minimum weighted perfect matching of odd vertices $\leq \frac{1}{2} \cdot \mathcal{T}^*$

(3) Euler tour

(4) Skip double visited vertices



Online-TSP

(metric)

Christofides Algorithm:

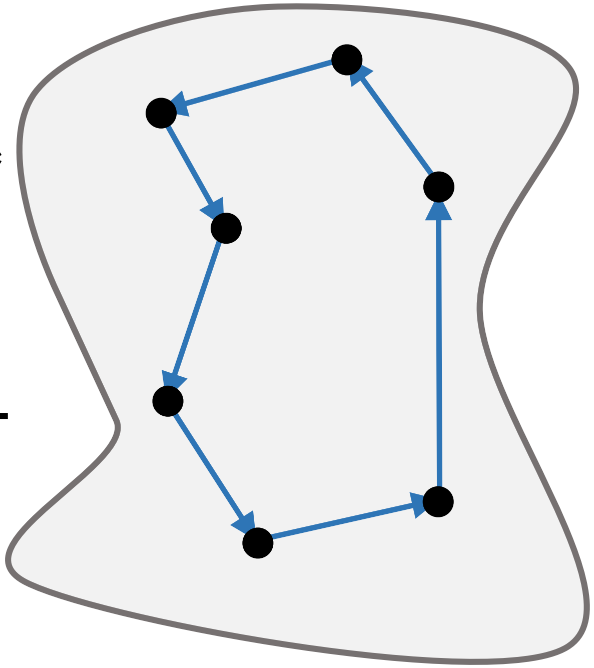
(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

(2) minimum weighted perfect matching of odd vertices $\leq \frac{1}{2} \cdot \mathcal{T}^*$

$\leq \frac{3}{2} \cdot \mathcal{T}^*$

(3) Euler tour

(4) Skip double visited vertices



Online-TSP
(metric)

Christofides Algorithm:

(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

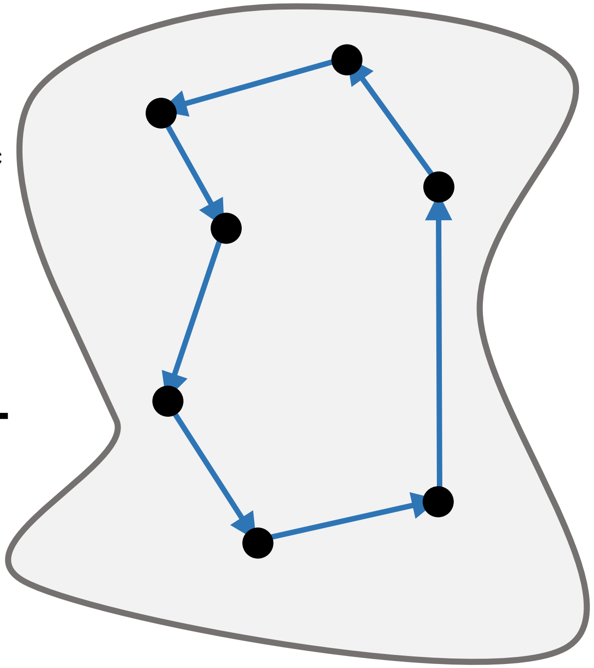
(2) minimum weighted perfect matching of odd vertices $\leq \frac{1}{2} \cdot \mathcal{T}^*$

(3) Euler tour $\leq \frac{3}{2} \cdot \mathcal{T}^*$

(4) Skip double visited vertices

$\Downarrow$

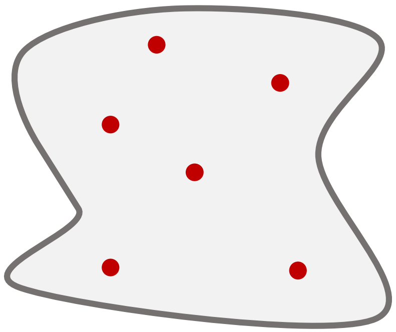
1,5-approximative solution



Online-TSP

(metric)

INPUT:



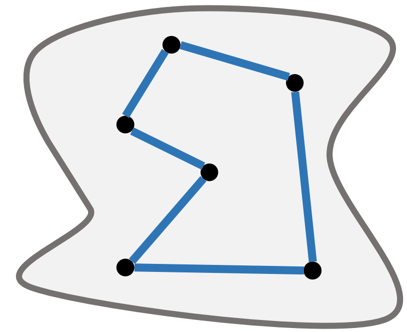
- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

NP-hard!

ALG

- Superpolynomial Alg.
- Approximation Alg.
  e.g. *Christofides*

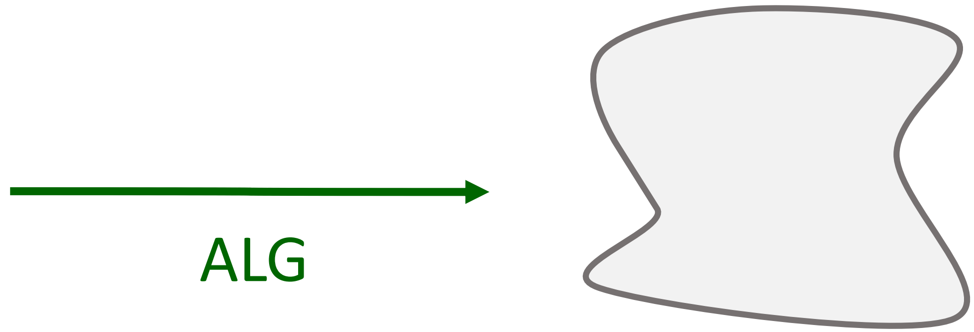
OUTPUT:



Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

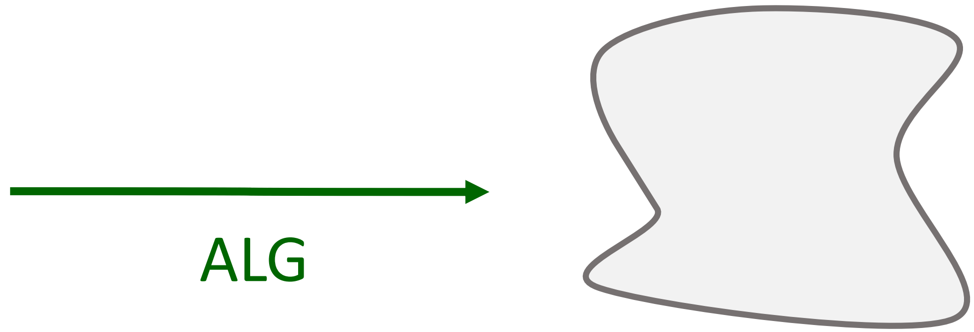
INPUT:



Online-TSP

INPUT:

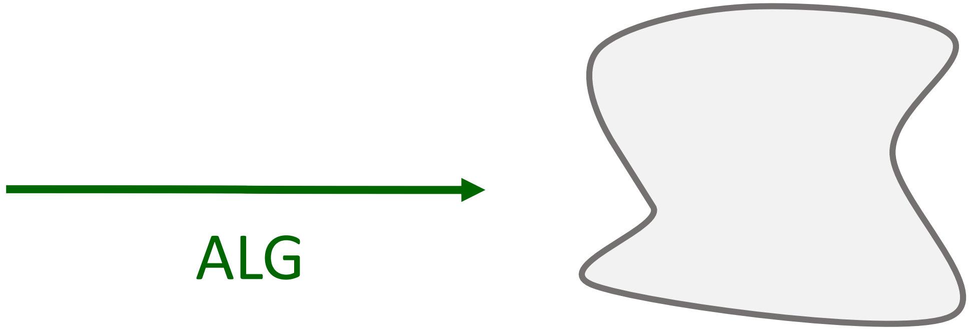
- metric space



Online-TSP

INPUT:

- metric space
- starting-point: o

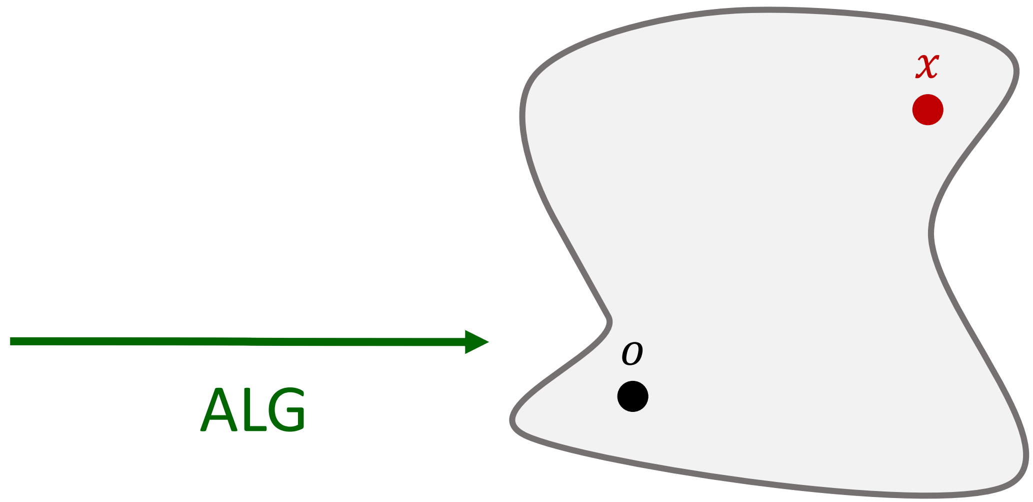


Online-TSP

INPUT:

- metric space
- starting-p,x
- ,x t: o
- request-sequence σ:

  $0, x$



Online-TSP

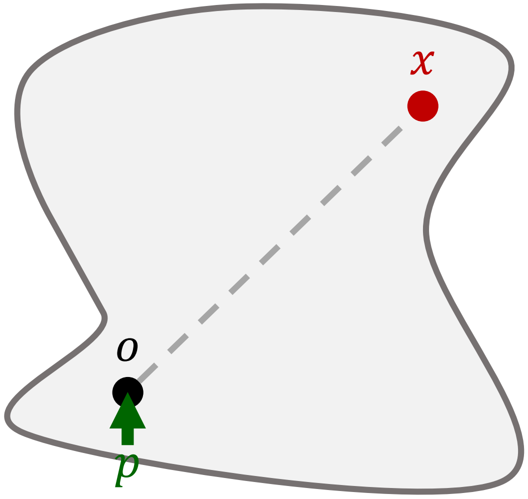
INPUT:

- metric space
- starting-p,x
- $,x$ t: o
- request-sequence σ:

  $0, x$

time: 0

ALG

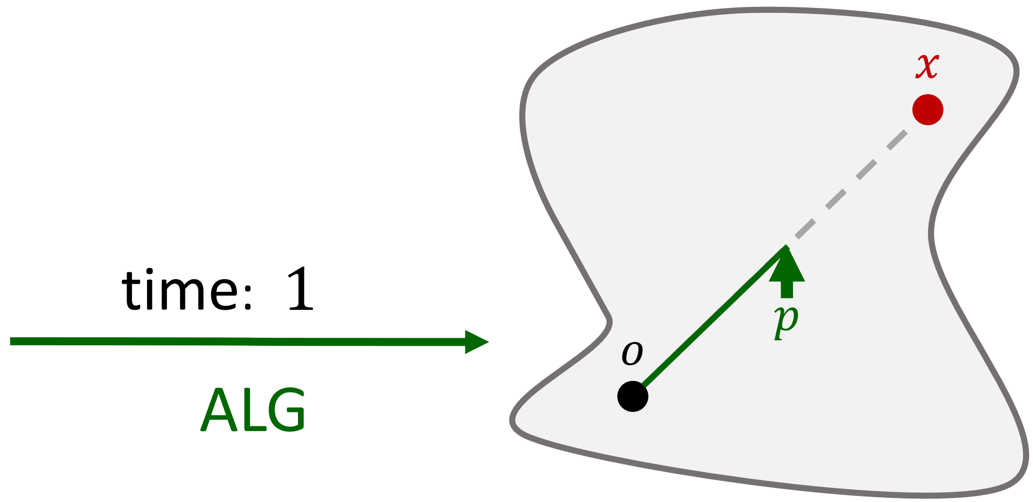


Online-TSP

INPUT:

- metric space
- starting-p,x
- ,x t: o
- request-sequence σ:

$0, x$

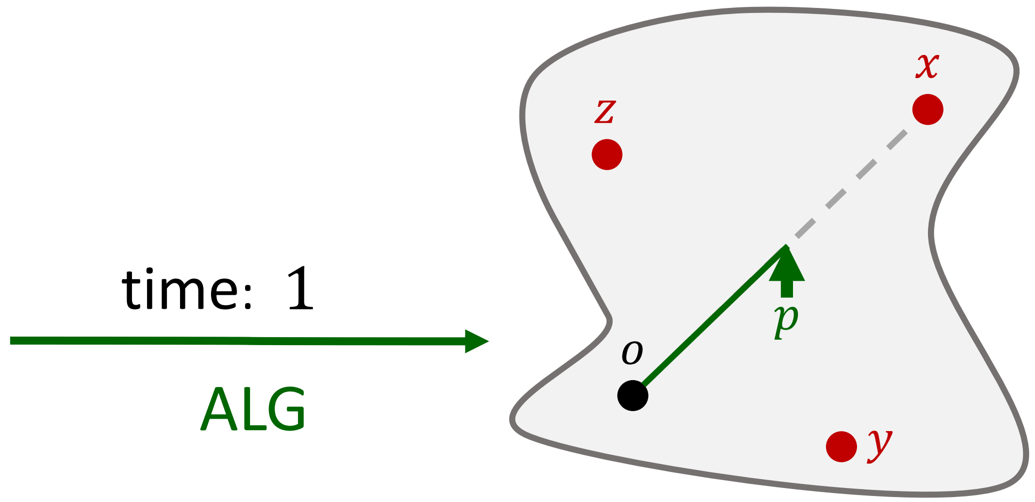


Online-TSP

INPUT:

- metric space
- starting-p,x
- $,x$ t: o
- request-sequence σ: $(1,y),(1,z)$

$0, x$

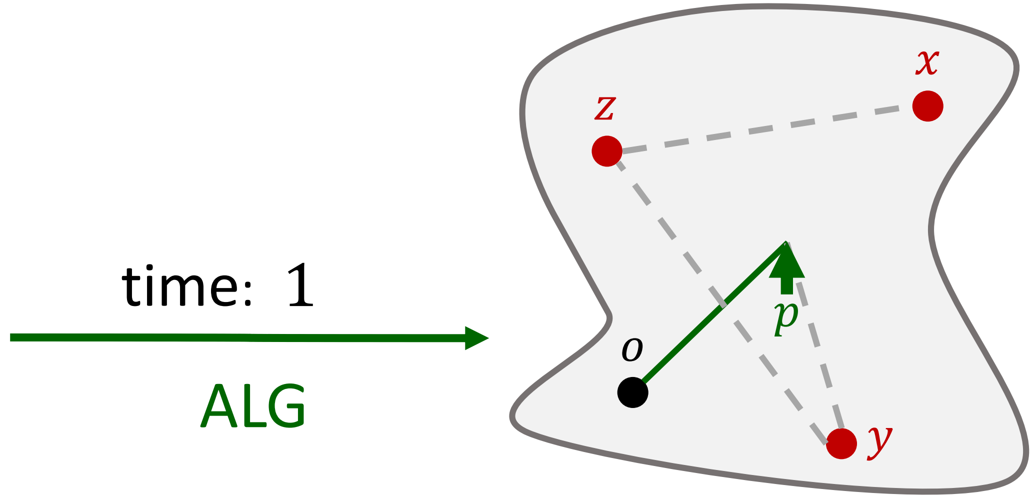


Online-TSP

INPUT:

- metric space
- starting-p,x
- $,x$ t: o
- request-sequence σ: $(1,y),(1,z)$

$0, x$

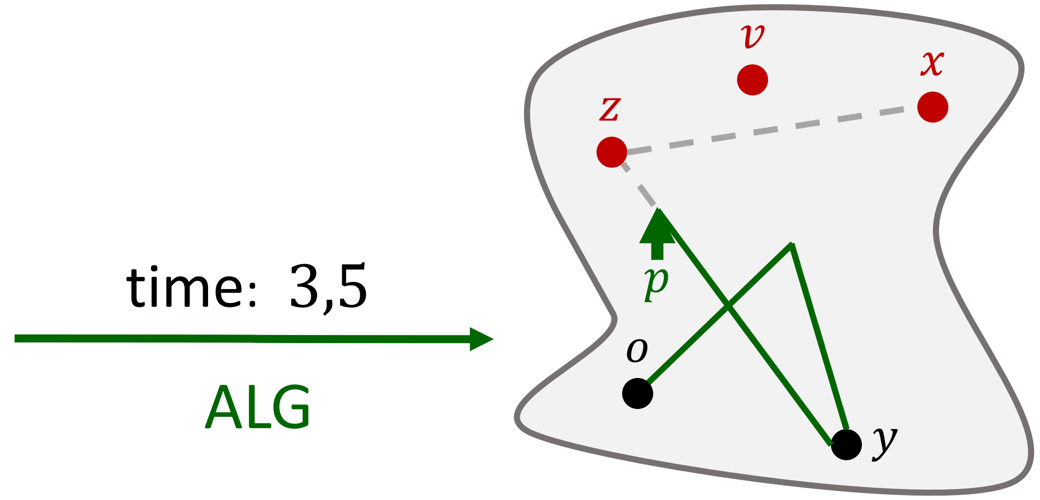


Online-TSP

INPUT:

- metric space
- starting-p,x
- $,x$ t: o
- request-sequence σ: $,(1,y),(1,z),\dots$

$0,x$



Online-TSP

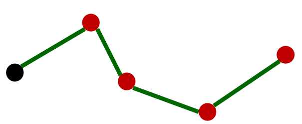
INPUT:

- metric space
- starting-p,x
- $,x$ t: o
- request-sequence σ: $, (1, y), (1, z), \ldots$

$0, x$

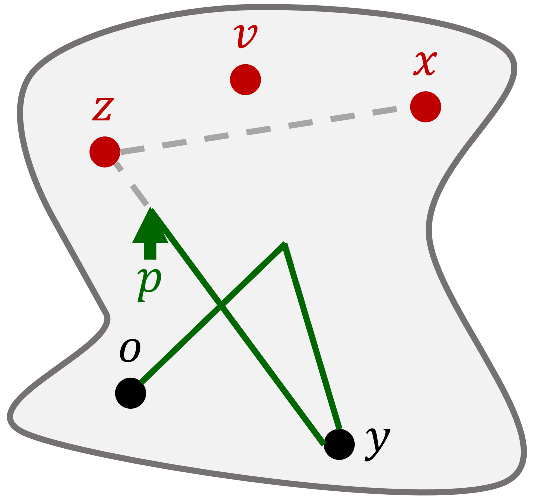
N-OLTSP

“nomadic”



time: 3,5

ALG



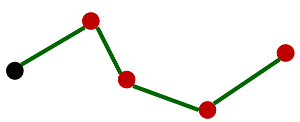
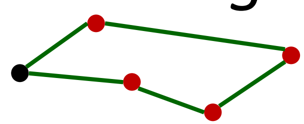
Online-TSP

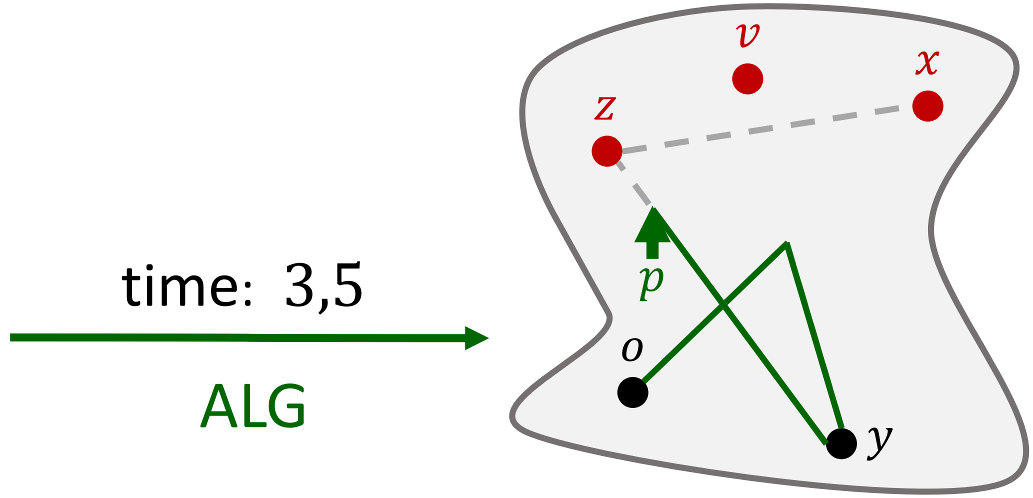
INPUT:

- metric space
- starting-p,x
- ,x t: o
- request-sequence σ: , $(1, y), (1, z), \ldots$

$0, x$

N-OLTSP	H-OLTSP
“nomadic”	*“homing”*

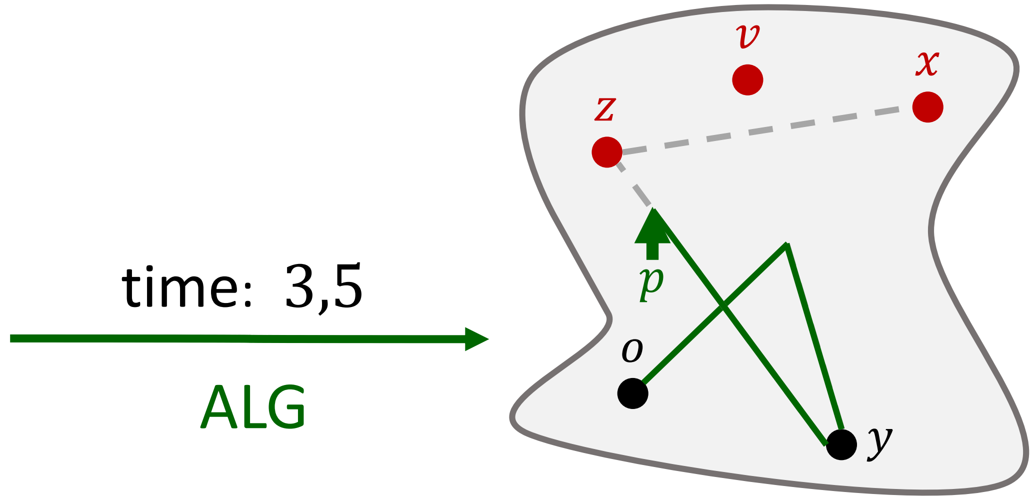


Online-TSP

INPUT:

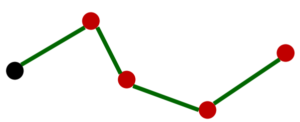
- metric space
- starting-p,x
- $,x$ t: o
- request-sequence σ: $, (1,y), (1,z), \ldots$

$0, x$



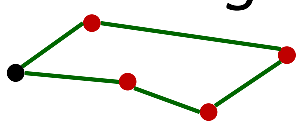
N-OLTSP

“nomadic”



H-OLTSP

“homing”



DEF: ALG is *ρ-competitive*

Online-TSP

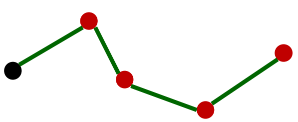
INPUT:

- metric space
- starting-p,x
- $,x$ t: o
- request-sequence σ: $,(1,y),(1,z),\ldots$

$0, x$

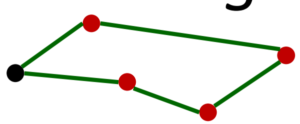
N-OLTSP

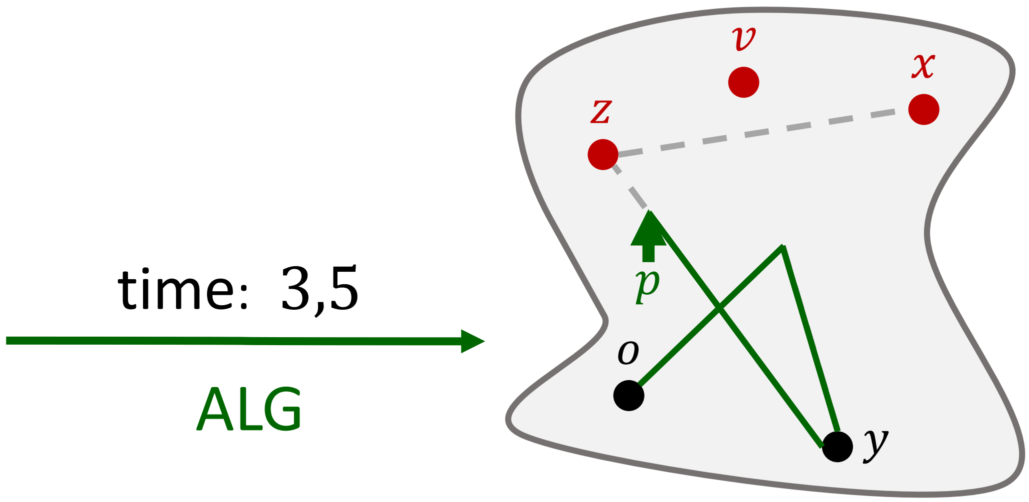
"nomadic"



H-OLTSP

"homing"





DEF: ALG is *ρ-competitive*

$$\Leftrightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| \leq \rho \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

Online-TSP

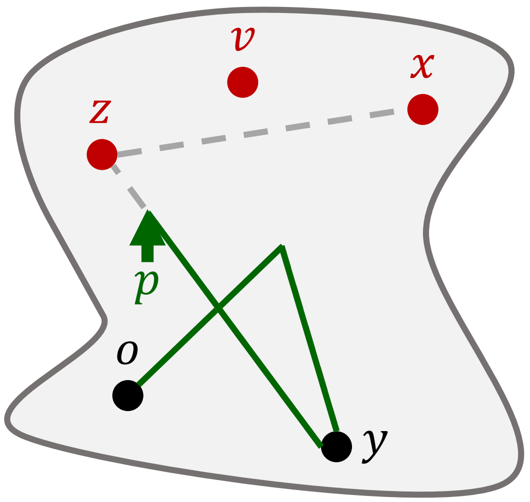
INPUT:

- metric space
- starting-p,x
- $,x$ t: o
- request-sequence σ: $,(1,y),(1,z),\ldots$

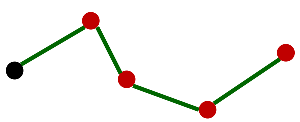
$0, x$

time: 3,5

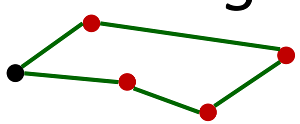
ALG



N-OLTSP *“nomadic”*



H-OLTSP *“homing”*



DEF: ALG is *ρ-competitive*

$$\Leftrightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| \leq \rho \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

for all request- sequences

Goals

Goals

I. Find online-algorithms

Goals

I. Find online-algorithms (superpolynomial)

Goals

I. Find online-algorithms (superpolynomial)

II. Find lower bounds

Goals

I. Find online-algorithms (superpolynomial)

II. Find lower bounds

III. Find *polynomial* online-algorithms

Goals

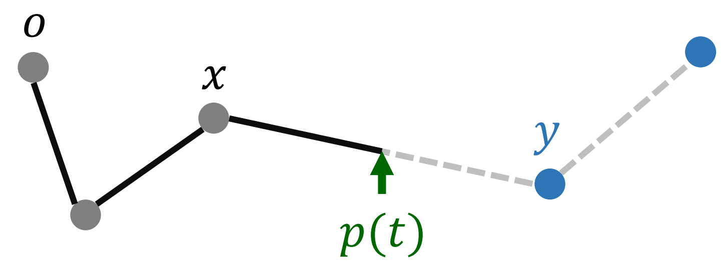
I. Find online-algorithms (superpolynomial)

II. Find lower bounds

III. Find *polynomial* online-algorithms

IV. Bonus: The real line

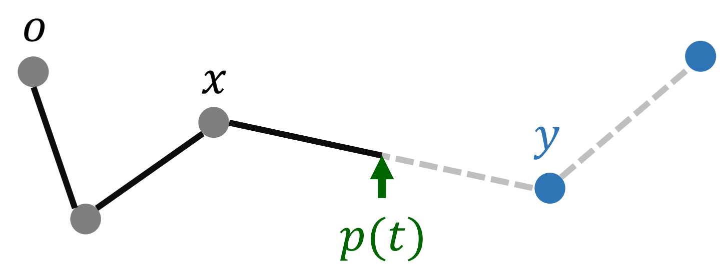
An algorithm for N-OLSTP



Greedily Travelling between Requests (GTR)

An algorithm for N-OLSTP

Invariant: always on shortest path between points in S

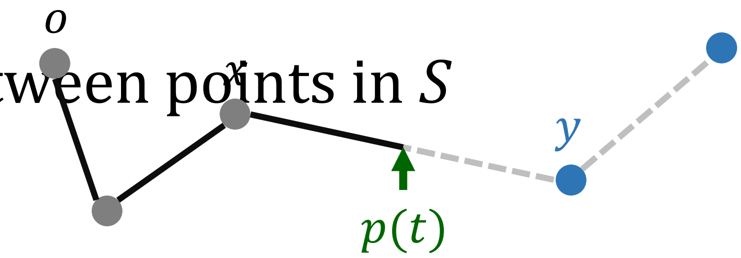


Greedily Travelling between Requests (GTR)

An algorithm for N-OLSTP

tt,zz) at time *tt*
and alg between *xx* and *yy*

Invariant*:* always on shortest path between points in S



(1) New request (t, z) at time t
and ALG between x and y

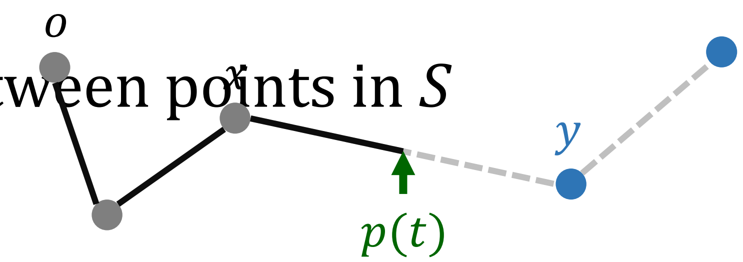
Greedily Travelling between Requests (GTR)

An algorithm for N-OLSTP

tt,zz) at time *tt*
and alg between *xx* and *yy*

Invariant*:* always on shortest path between points in S

(1) Add z to U



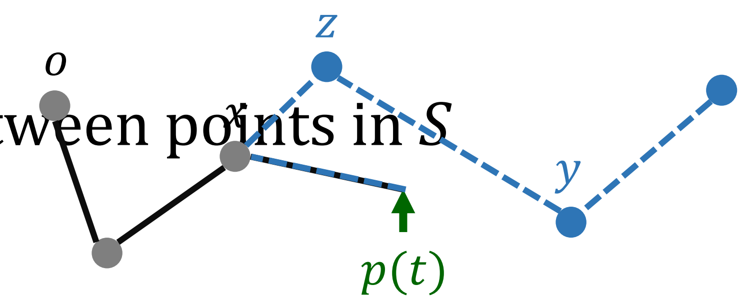
Greedily Travelling between Requests (GTR)

An algorithm for N-OLSTP

tt,zz) at time tt

and alg between xx and yy

Invariant*:* always on shortest path between points in S



(1) Add z to U

(2) Follow shortest path through $\mathcal{U}$ beginning with x or y

tt

$ta\ tisiv\ ot\ tey\ secalp\ \coloneqq UUtt$

$S \supseteq U \coloneqq$ places yet to visit at t

$S \supseteq U \coloneqq$ places yet to visit at t

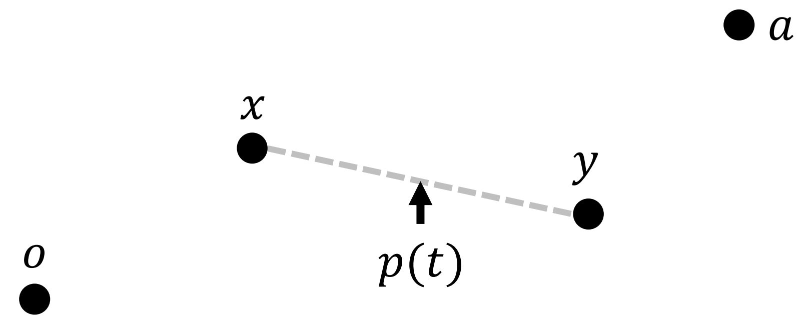
Greedily Travelling between Requests (GTR)

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) New request (t, z) at time t and ALG between x and y

(3) Follow shortest path through U beginning with x or y

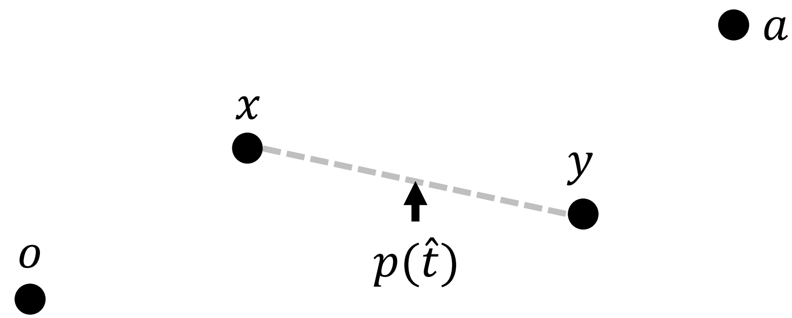


Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y

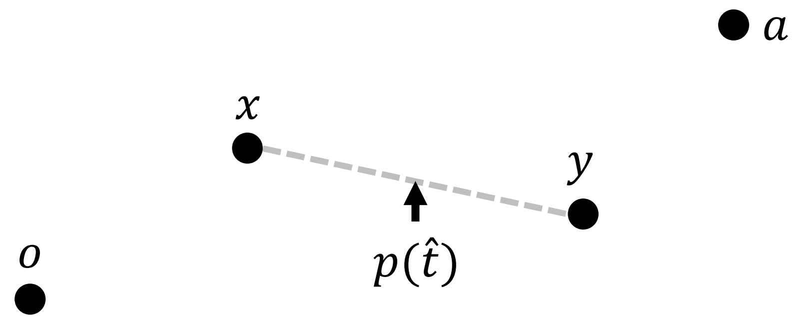


Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

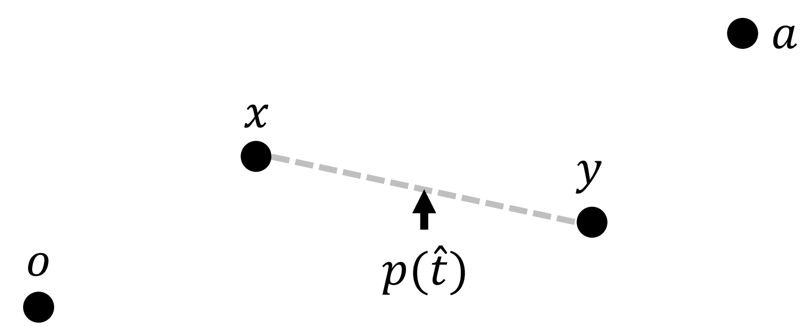
$$\left|\mathcal{T}^{\mathrm{GTR}}\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$
and ALG between x and y

(3) Follow shortest path through U
beginning with x or y



path found by GTR

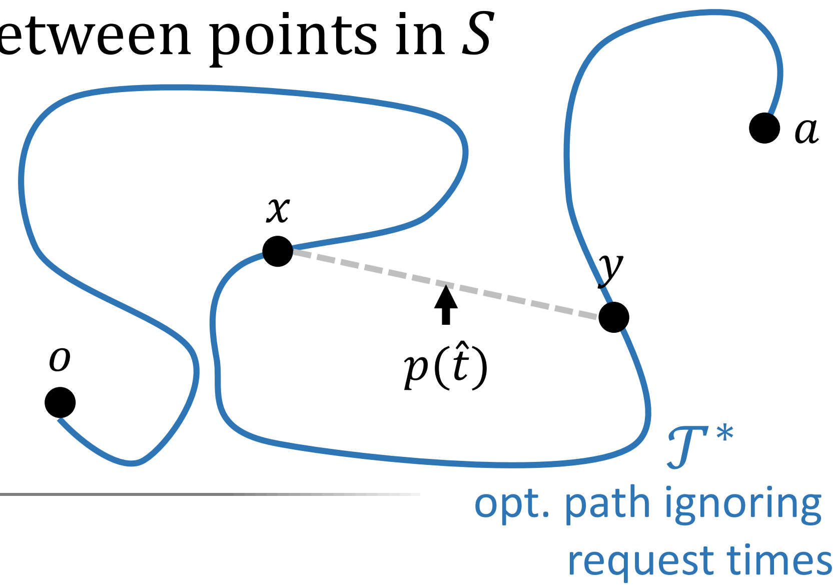
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \le \hat{t}$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

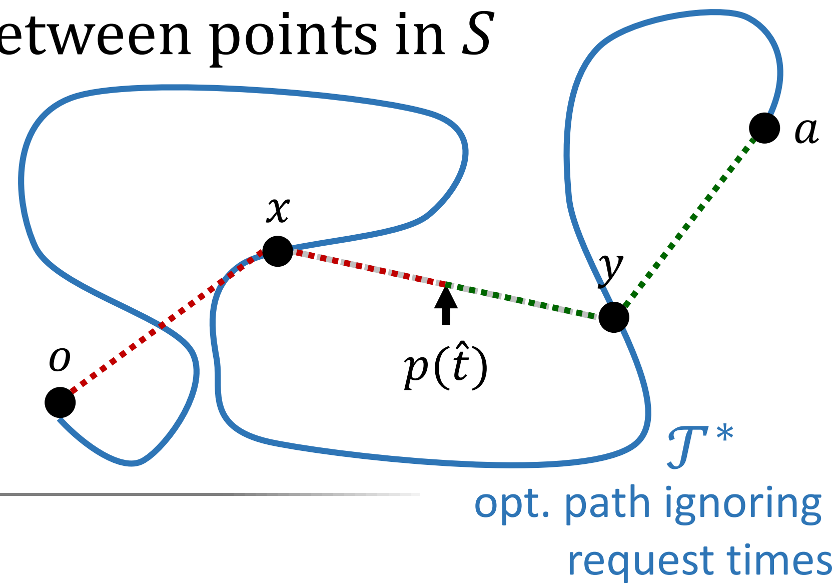
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t}$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

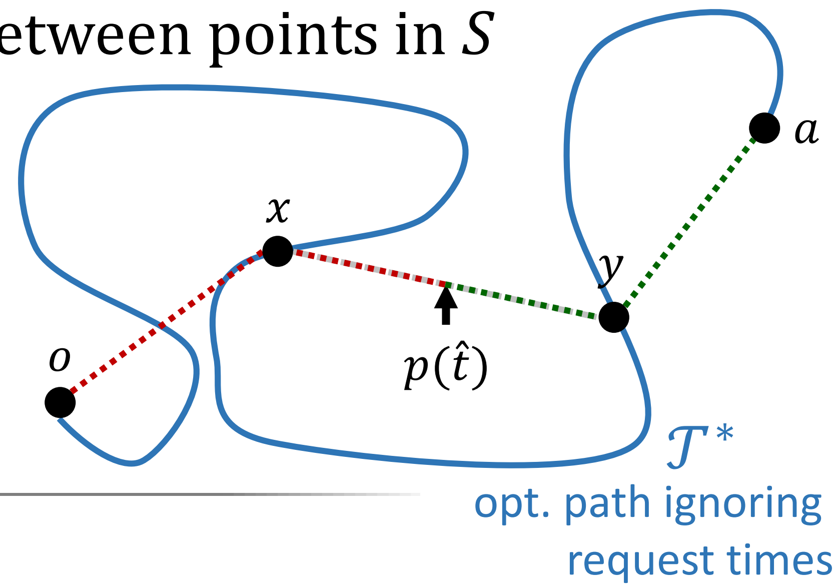
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t}$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

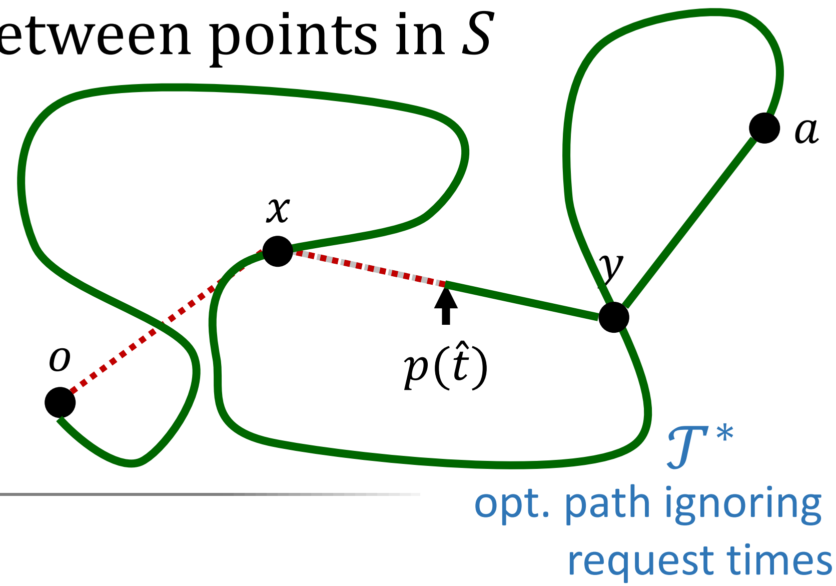
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

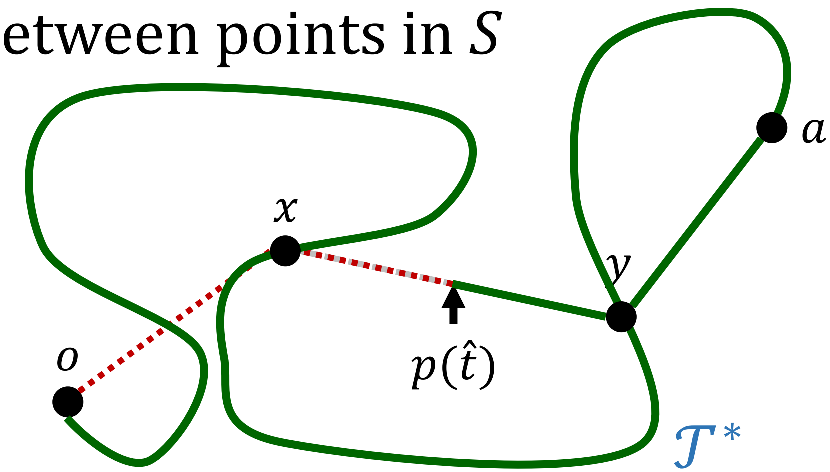
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$
$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

path found by opt. offline-ALG

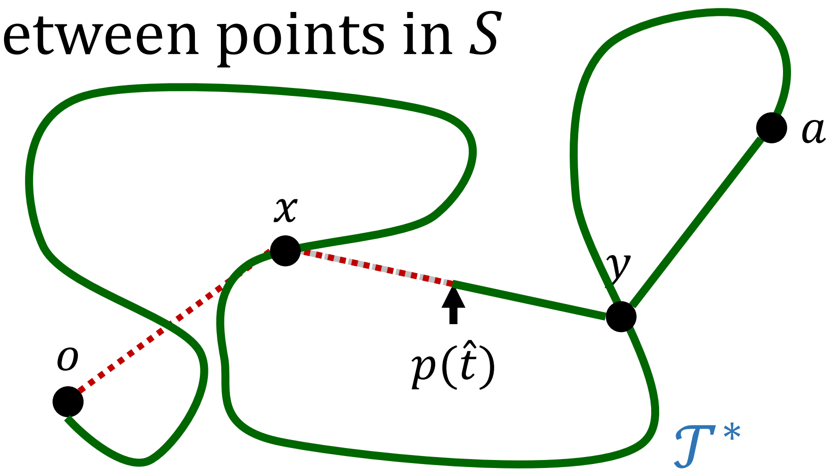
$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| +$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

path found by opt. offline-ALG

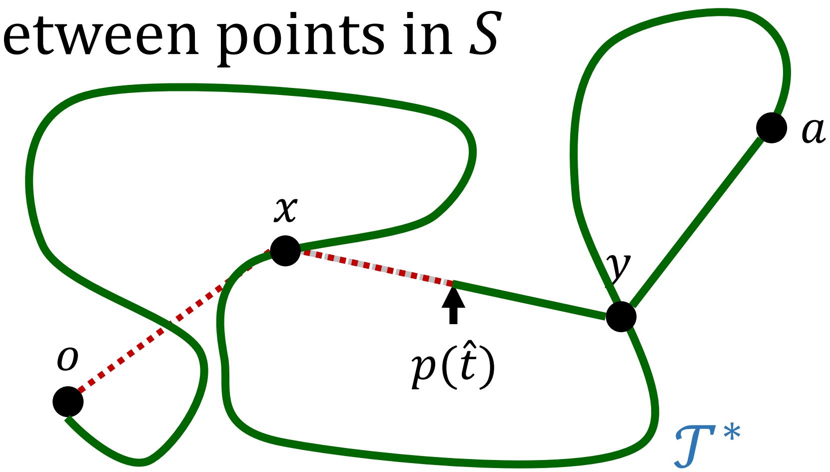
$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



opt. path ignoring request times

path found by GTR

$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

path found by opt. offline-ALG

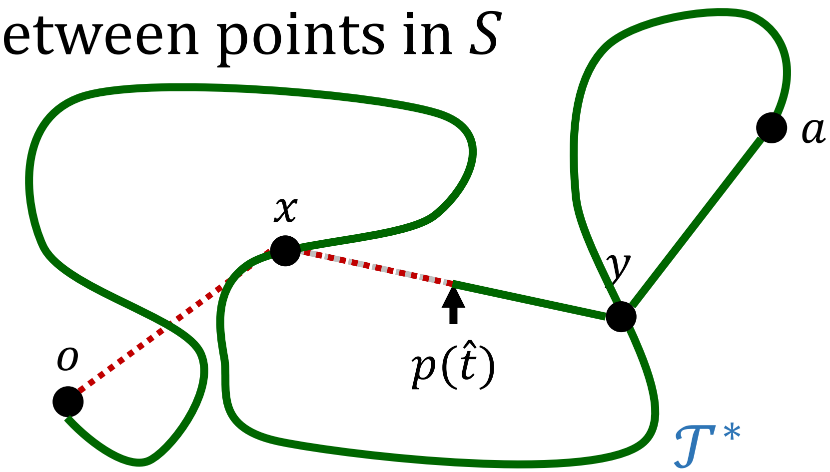
$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| = \frac{5}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



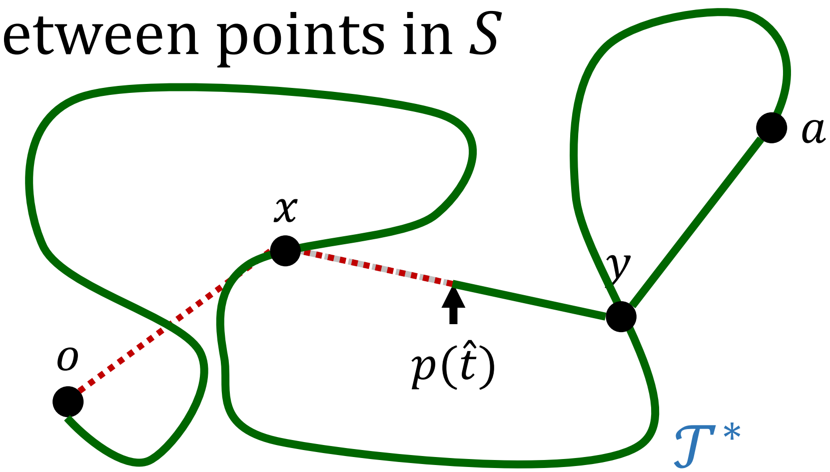
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \left|\mathcal{T}^{\mathrm{OPT}}\right| \quad + \quad \tfrac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \quad = \quad \tfrac{5}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



THEOREM: GTR is 2,5-competitive for N-OLTSP

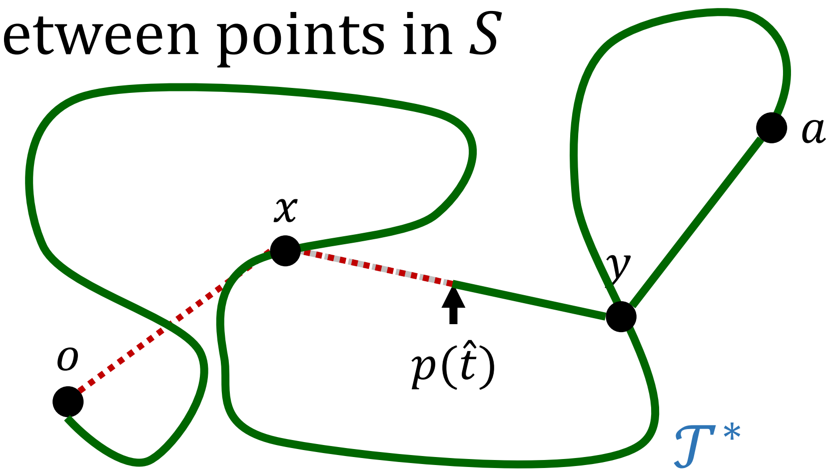
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| = \frac{5}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



THEOREM: GTR is 2,5-competitive for N-OLTSP

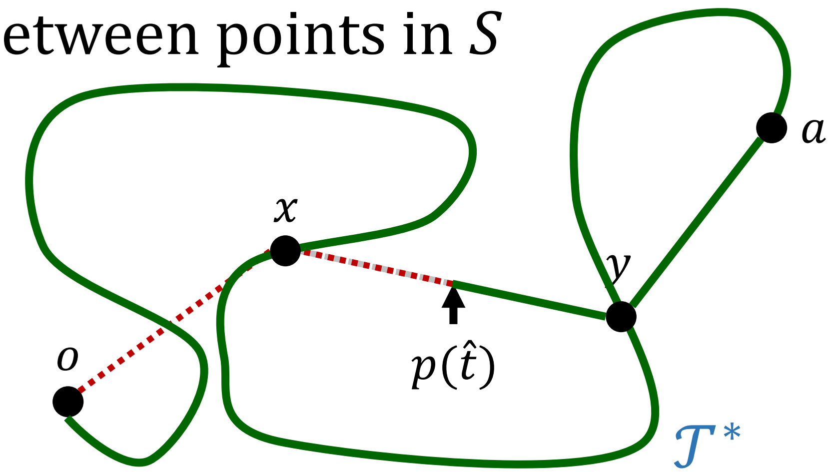
REMARK: tightness

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



THEOREM: GTR is 2,5-competitive for N-OLTSP

REMARK: tightness

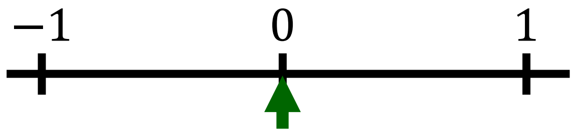
REMARK: GTR is also 2,5-competitive for H-OLTSP

Lower Bound for N-OLTSP

time, request

Online-ALG

0



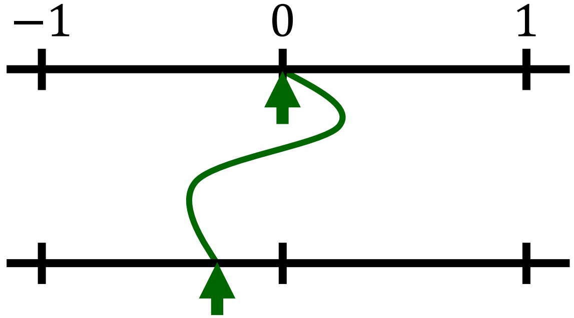
Lower Bound for N-OLTSP

time, request

Online-ALG

0

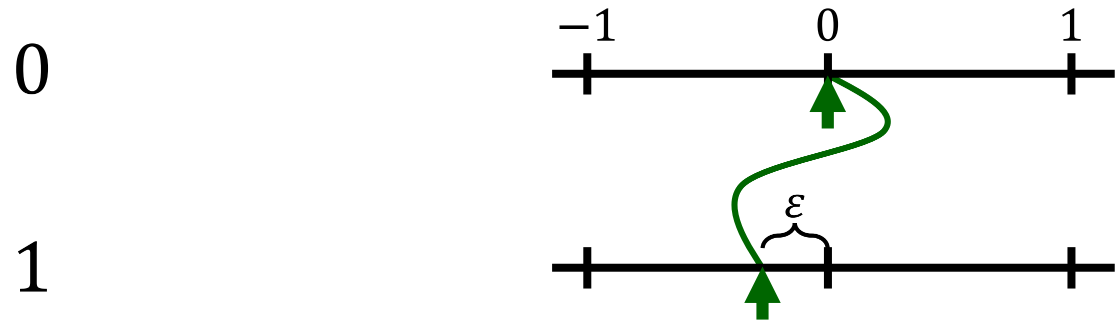
1



Lower Bound for N-OLTSP

time, request

Online-ALG



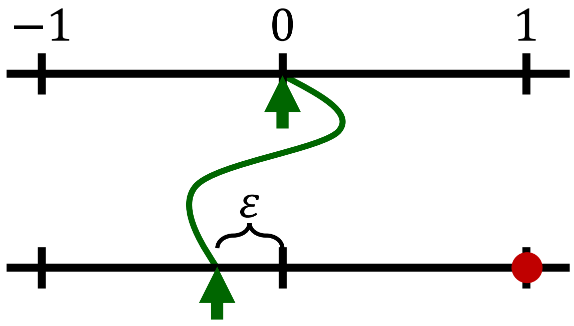
Lower Bound for N-OLTSP

time, request

0

1, 1

Online-ALG



Lower Bound for N-OLTSP

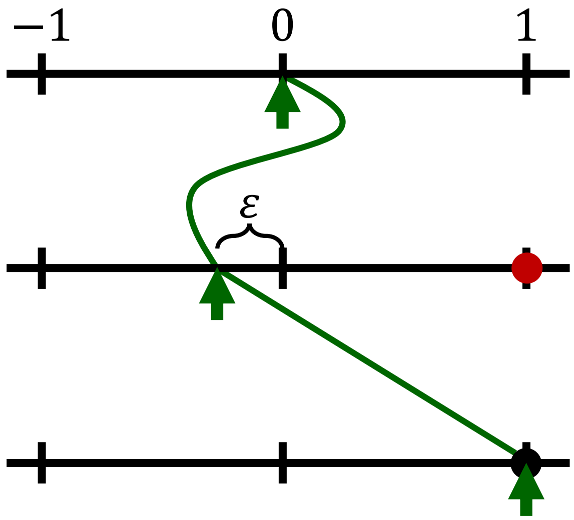
time, request

0

1, $\quad 1$

$2+\varepsilon$

Online-ALG



Lower Bound for N-OLTSP

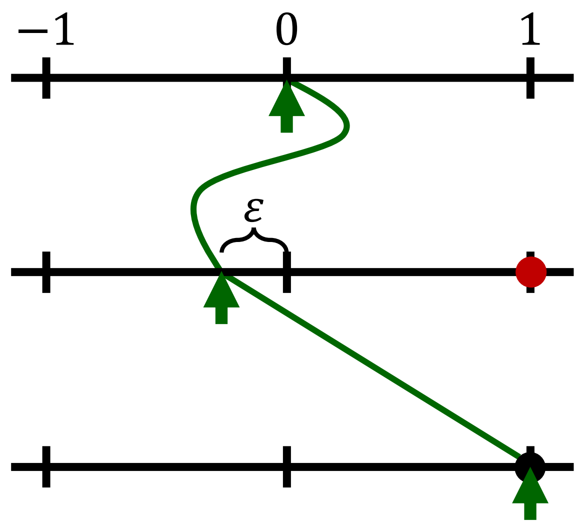
time, request

Online-ALG

0

$1,\quad 1$

$2 + \varepsilon$



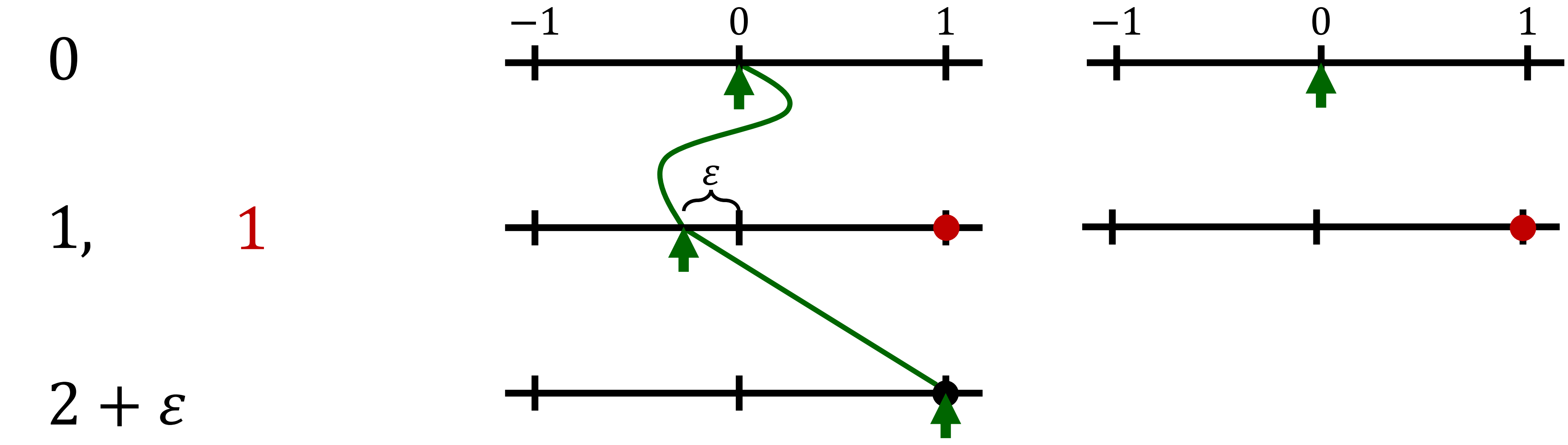
$$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 2 + \varepsilon$$

Lower Bound for N-OLTSP

time, request

Online-ALG

Offline-ALG



$$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 2 + \varepsilon$$

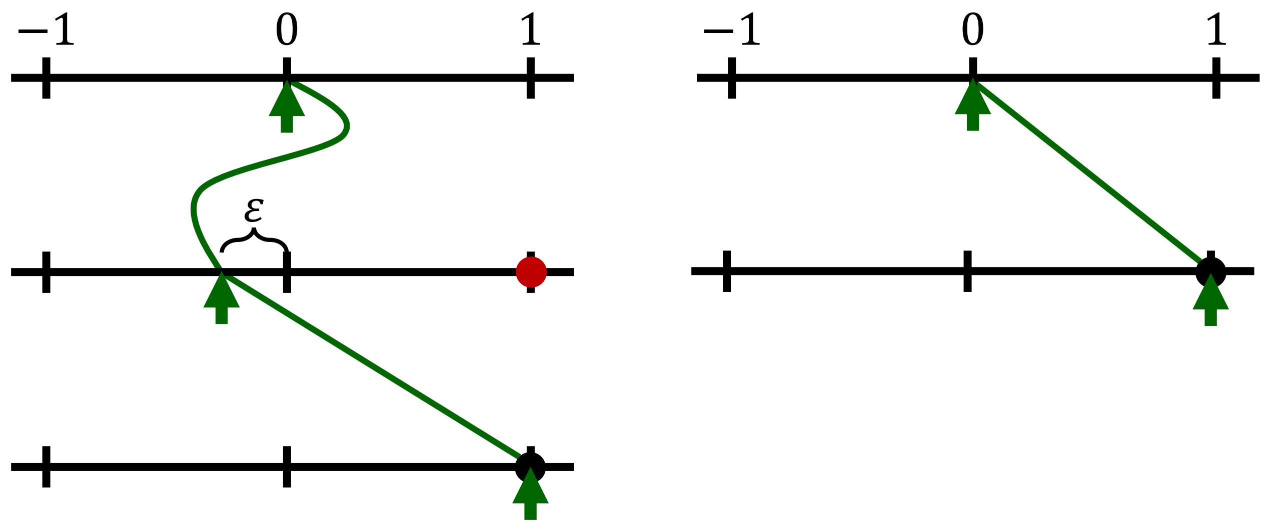
Lower Bound for N-OLTSP

time, request | Online-ALG | Offline-ALG

0

$1,\quad 1$

$2+\varepsilon$



$$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 2+\varepsilon$$

Lower Bound for N-OLTSP

time, request

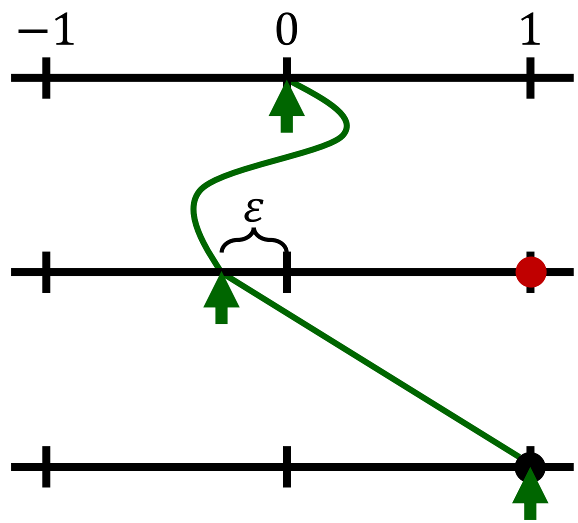
Online-ALG

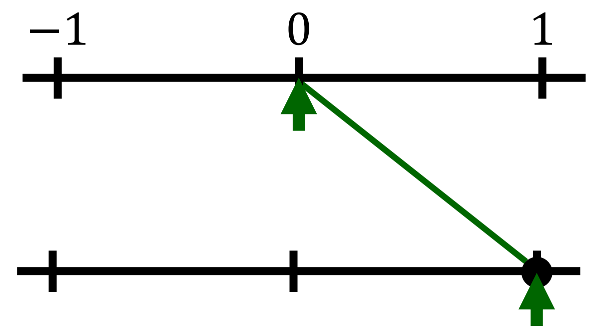
Offline-ALG

0

$1,\quad 1$

$2+\varepsilon$





$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 2+\varepsilon$

$\Rightarrow \left|\mathcal{T}^{\mathrm{OFF}}\right| = 1$

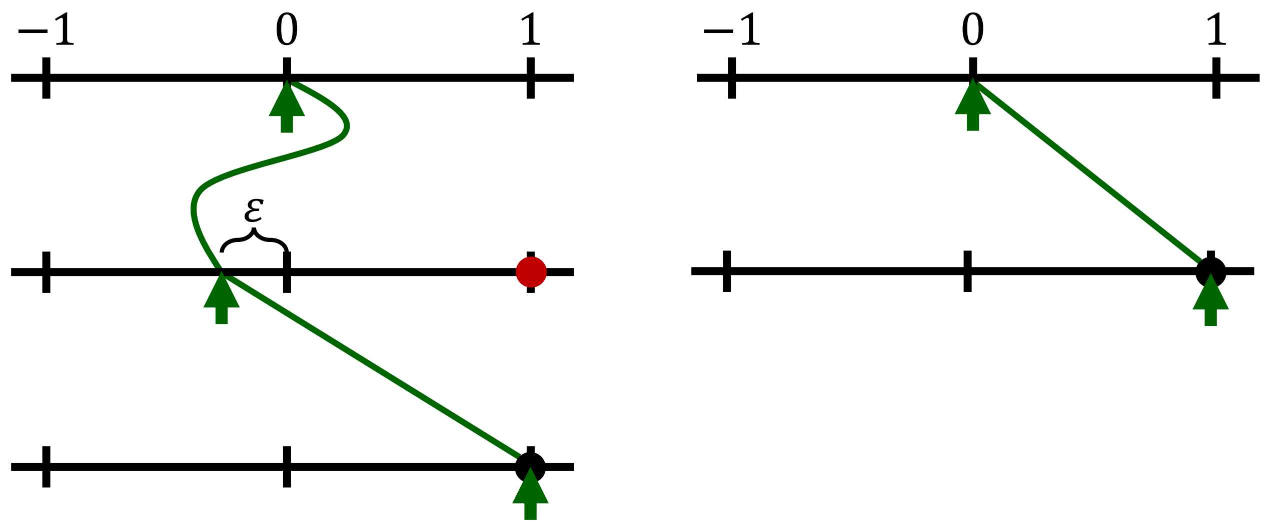
Lower Bound for N-OLTSP

time, request — Online-ALG — Offline-ALG

0

$1, \quad {\color{red}1}$

$2+\varepsilon$



$\Rightarrow |\mathcal{T}^{\mathrm{ON}}| = 2+\varepsilon$

$\Rightarrow |\mathcal{T}^{\mathrm{OFF}}| = 1$

THEOREM: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

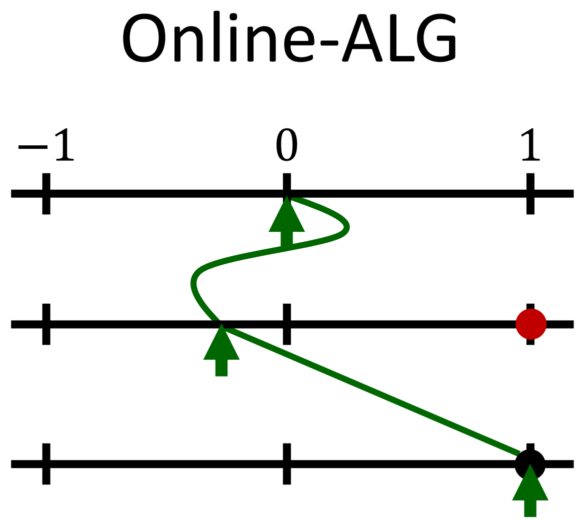
Lower Bound for H-OLTSP

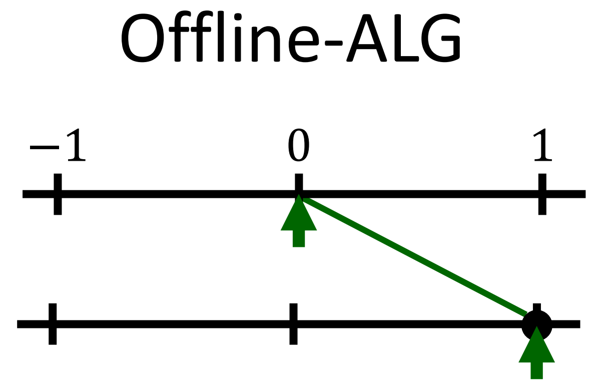
time, request

0

1, 1

$2 + \varepsilon$





$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 2 + \varepsilon$

$\Rightarrow \left|\mathcal{T}^{\mathrm{OFF}}\right| = 1$

THEOREM: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

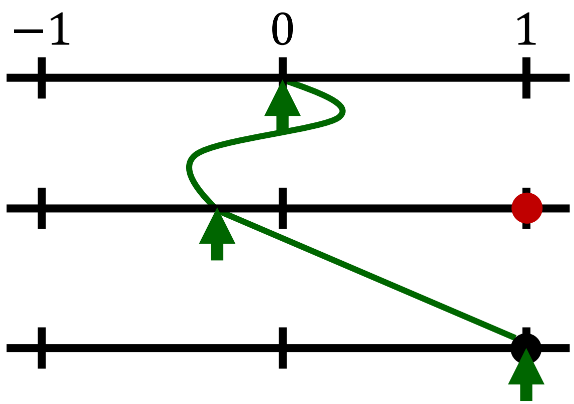
Lower Bound for H-OLTSP

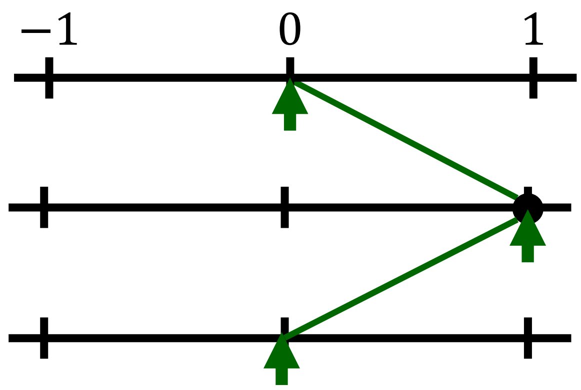
time, request | Online-ALG | Offline-ALG

0

1, $\quad 1$

$2 + \varepsilon$





$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 2 + \varepsilon$ $\qquad$ $\Rightarrow \left|\mathcal{T}^{\mathrm{OFF}}\right| = 2$

THEOREM: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

Lower Bound for H-OLTSP

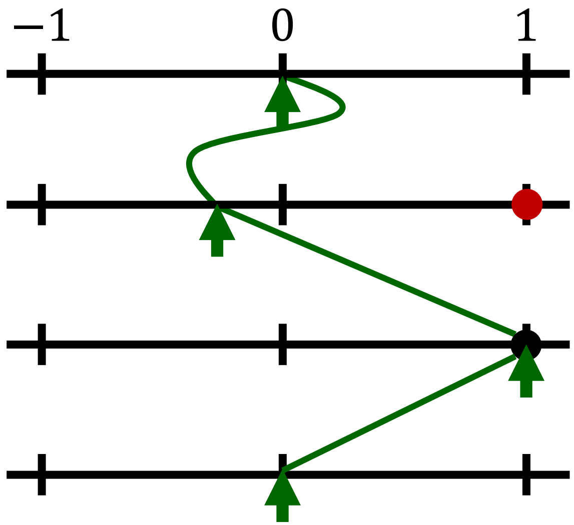
time, request | Online-ALG | Offline-ALG

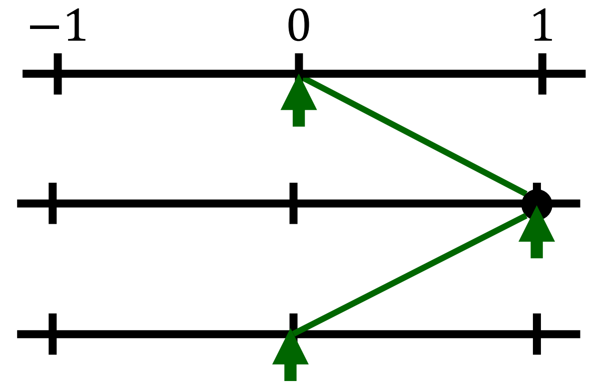
0

$1,\quad {\color{red}1}$

$2+\varepsilon$

$3+\varepsilon$





$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 3+\varepsilon$

$\Rightarrow \left|\mathcal{T}^{\mathrm{OFF}}\right| = 2$

THEOREM: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

Lower Bound for H-OLTSP

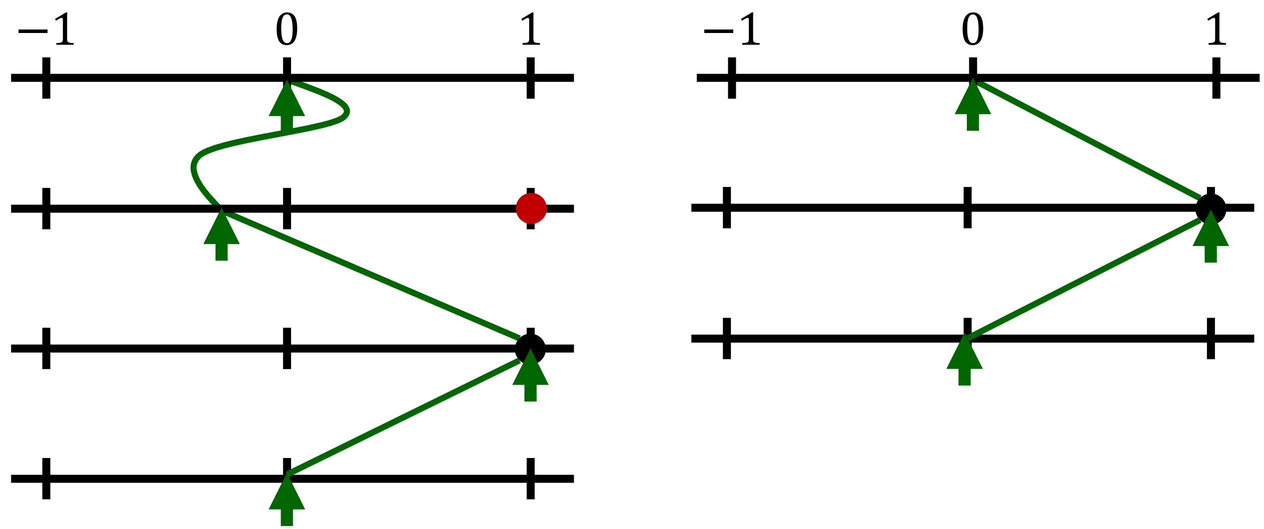
time, request — Online-ALG — Offline-ALG

0

$1, \quad 1$

$2 + \varepsilon$

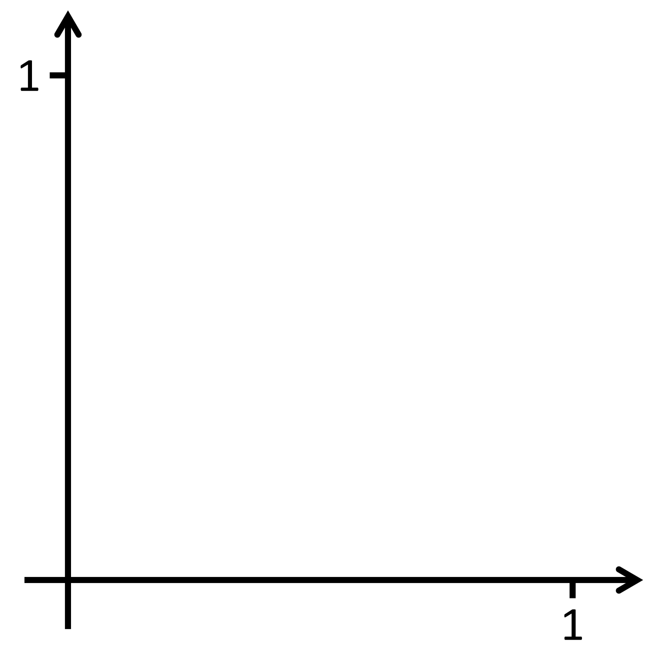
$3 + \varepsilon$



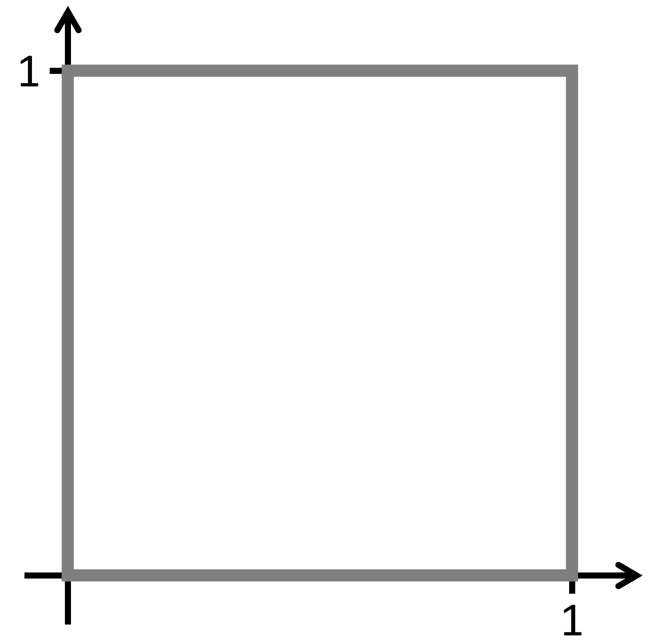
$\Rightarrow \left|\mathcal{T}^{\mathrm{ON}}\right| = 3 + \varepsilon$ $\qquad$ $\Rightarrow \left|\mathcal{T}^{\mathrm{OFF}}\right| = 2$

Theorem: Any ρ-competitive ALG for H-OLTSP has $\rho \geq 1{,}5$.

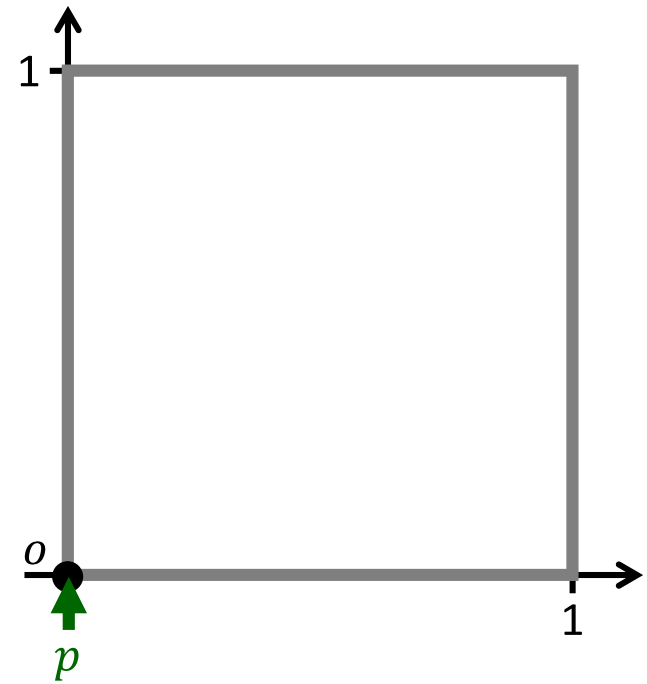
Lower Bound for H-OLTSP



Lower Bound for H-OLTSP

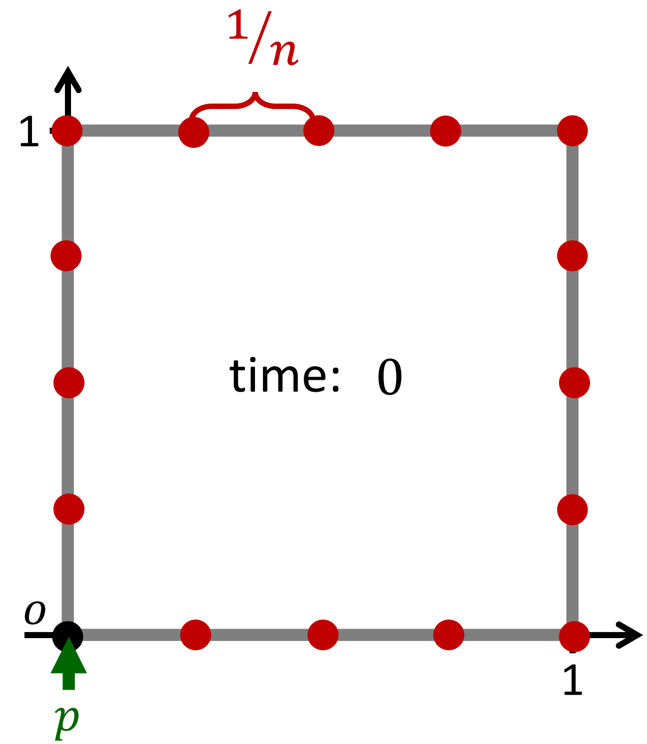


Lower Bound for H-OLTSP



Lower Bound for H-OLTSP

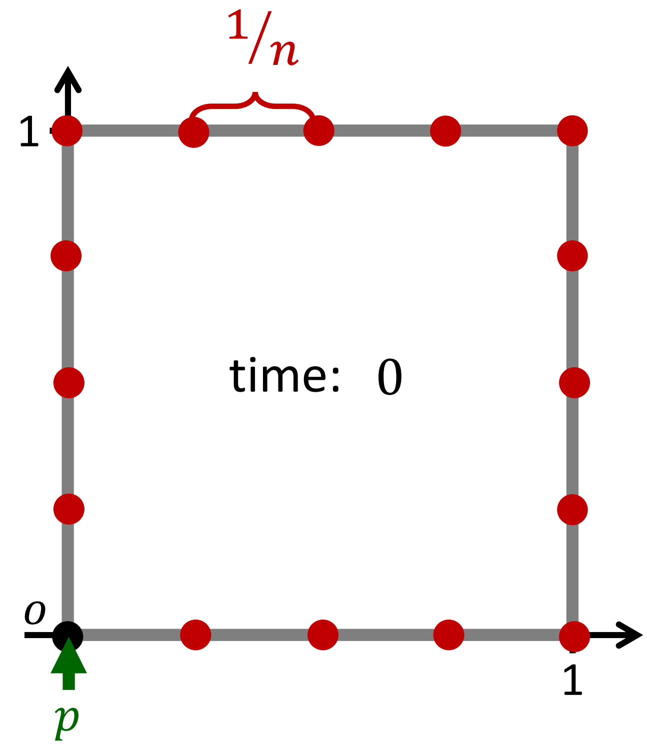
- requests at time 0



Lower Bound for H-OLTSP

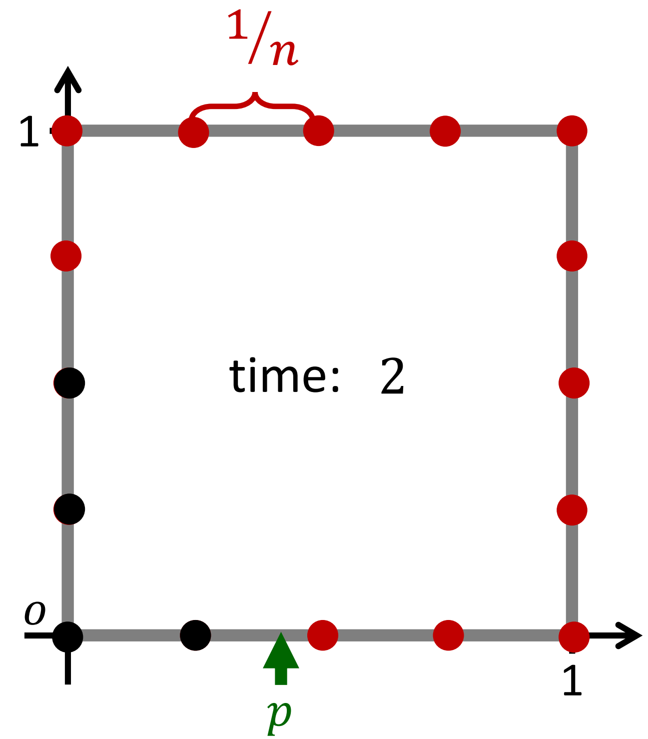
- $=2$

- wait until time $\mathrm{t} = 2$



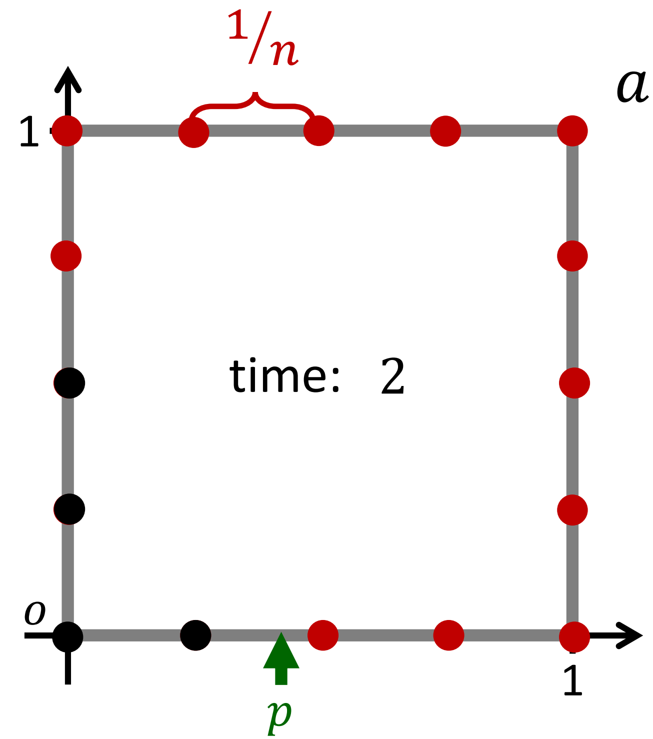
Lower Bound for H-OLTSP

- $=2$
- wait until time $\mathrm{t} = 2$



Lower Bound for H-OLTSP

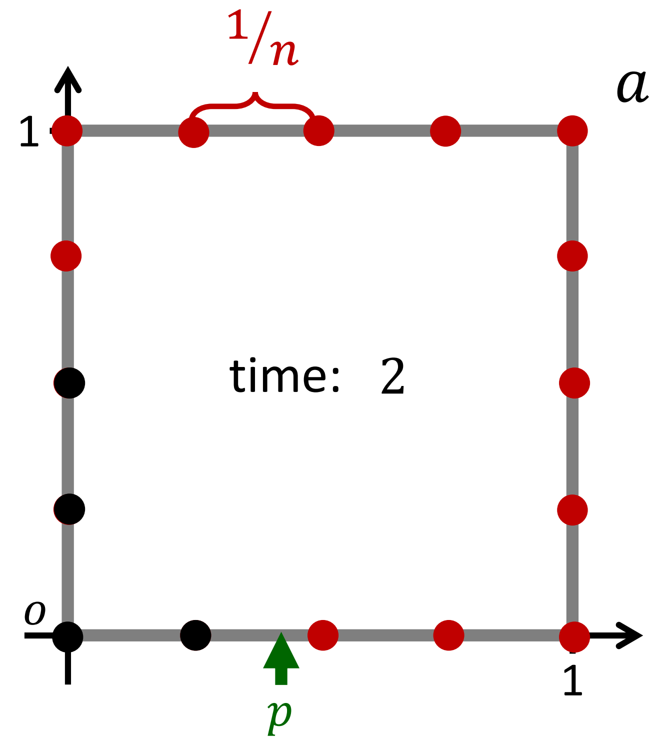
- $=2$
- wait until time $t = 2$



Lower Bound for H-OLTSP

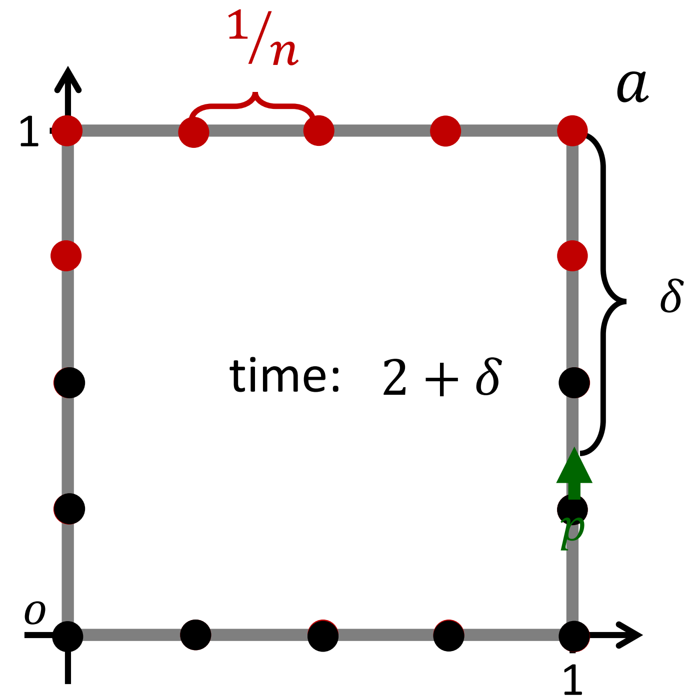
- $aa\ p,a\ = tt-2$
- $=2$

- wait until $d\ p, a\ = t - 2$



Lower Bound for H-OLTSP

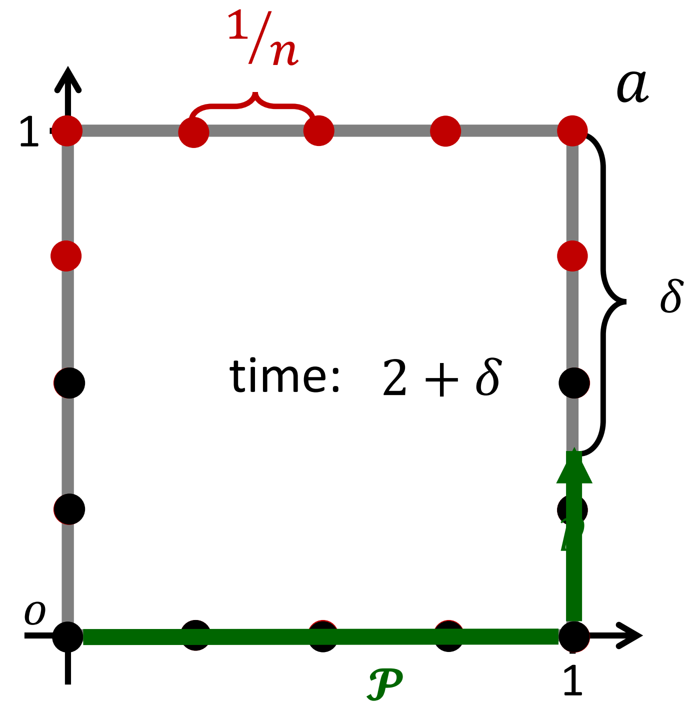
- $aa\ p{,}a\ = tt - 2$
- $=2$
- wait until $d\ p, a\ = t - 2$



Lower Bound for H-OLTSP

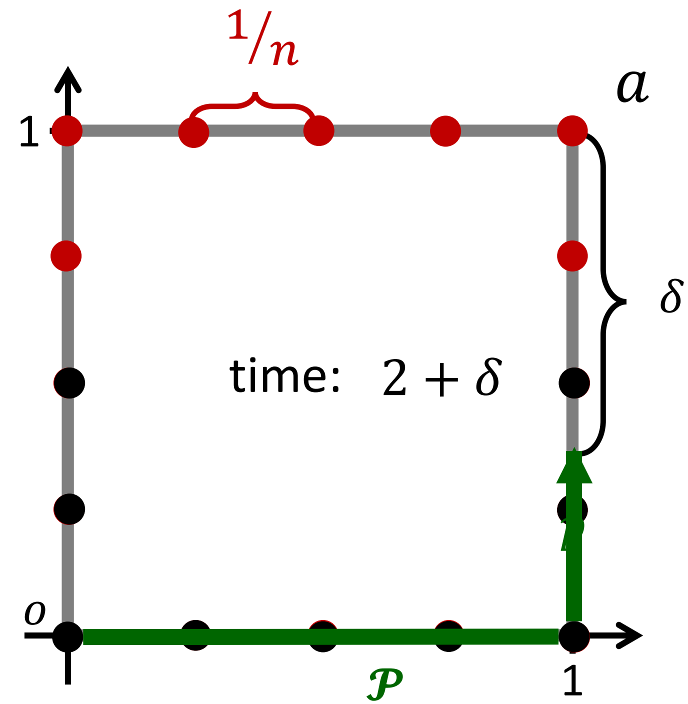
- $aa\ p{,}a\ {=}tt{-}2$
- ${=}2$

- wait until $d(p, a) = t - 2$



Lower Bound for H-OLTSP

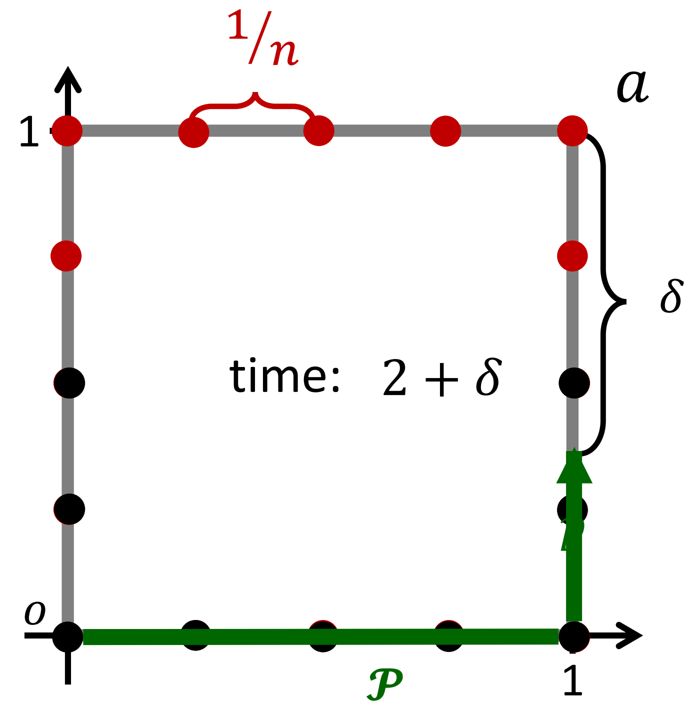
- $aa\ p,a\ = tt-2$
- $=2$
- wait until $d\ p,a\ = t-2$



$|\mathcal{P}|$ $=$

Lower Bound for H-OLTSP

- $aa\ p,a = tt - 2$
- $= 2$
- wait until $d\ p, a\ = t - 2$

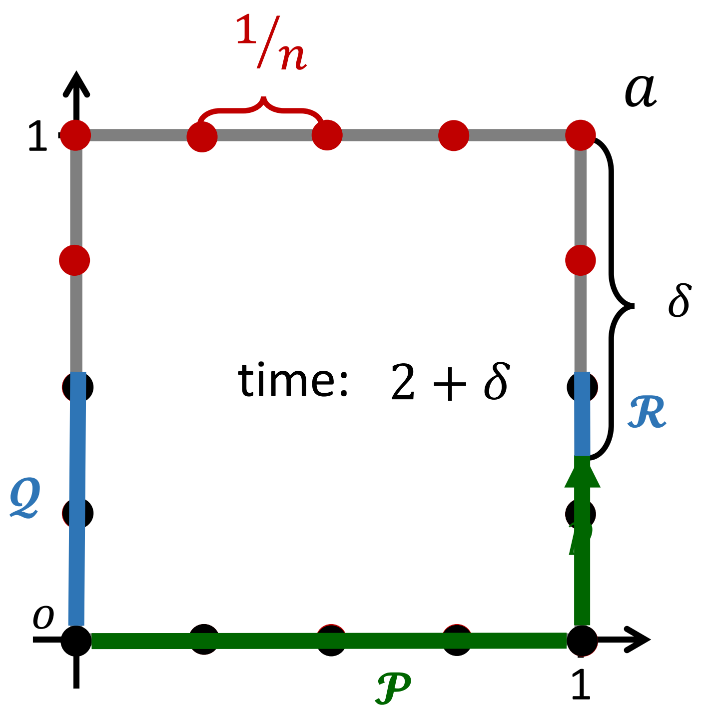


$|\mathcal{P}|$ $= 2 - \delta$

Lower Bound for H-OLTSP

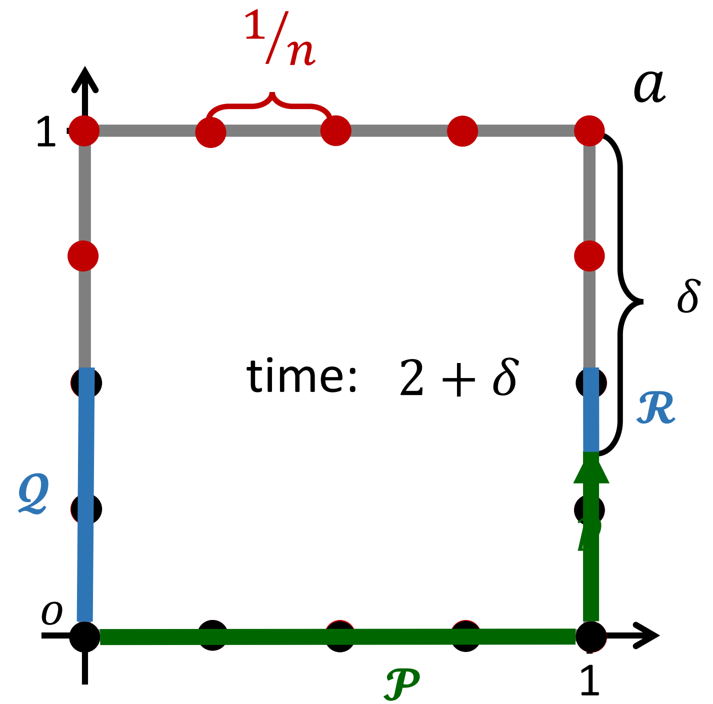
- $aa\ p,a\ =tt-2$
- $=2$
- wait until $d\ p,a\ = t-2$

$|\mathcal{P}|$ $= 2-\delta$



Lower Bound for H-OLTSP

- $aa\ p,a\ =tt-2$
- $=2$
- wait until $d\ p,a\ = t - 2$

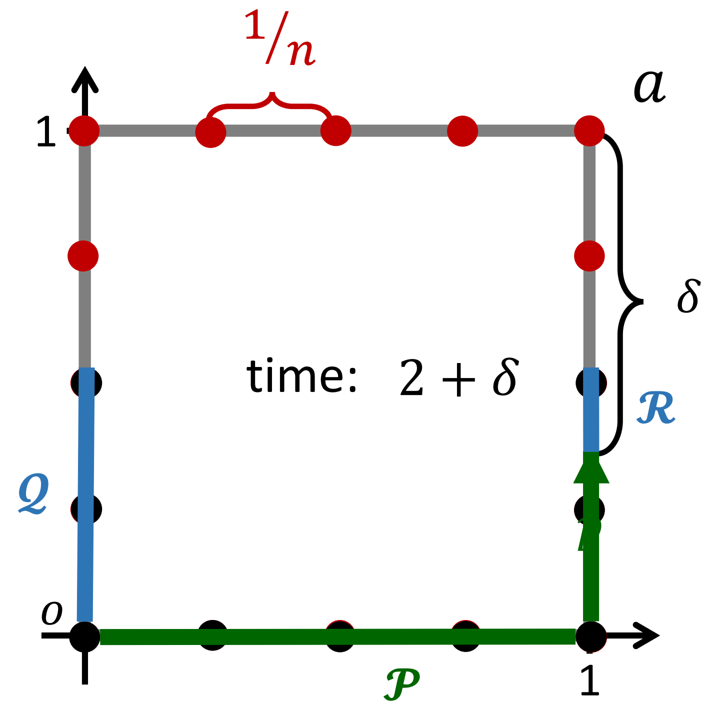


$$|\mathcal{P}| \qquad\qquad = 2-\delta$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq$$

Lower Bound for H-OLTSP

- $aa\ p,a = tt-2$
- $=2$
- wait until $d\ p,a\ = t-2$



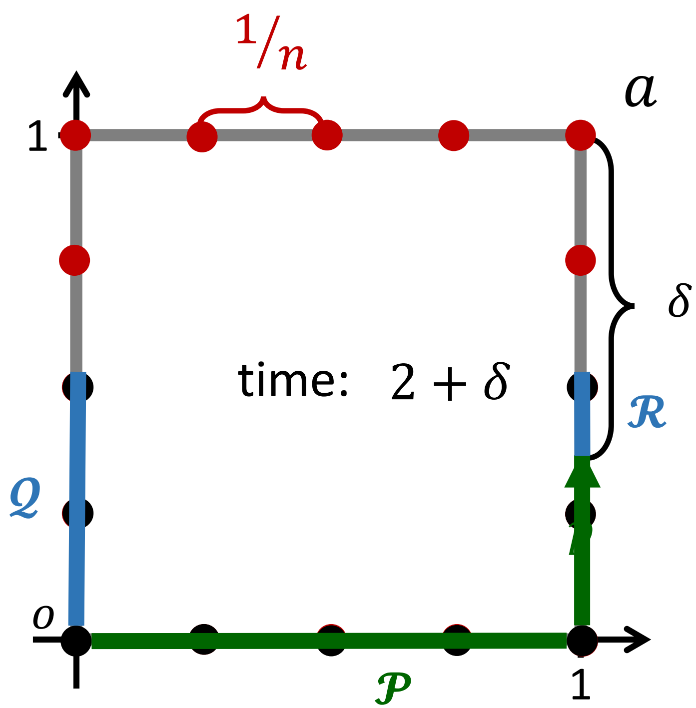
$$|\mathcal{P}| = 2-\delta$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \le 2+\delta$$

Lower Bound for H-OLTSP

- $aa\ p,a = tt - 2$
- $=2$

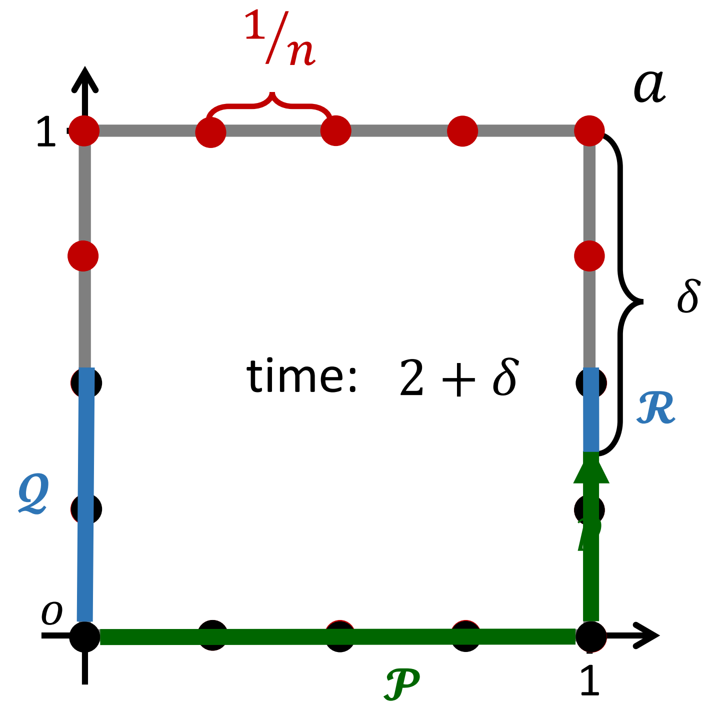
- wait until $d(p, a) = t - 2$



$$\left.\begin{aligned} |\mathcal{P}| &= 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| &\leq 2 + \delta \end{aligned}\right\} \quad |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}|$$

Lower Bound for H-OLTSP

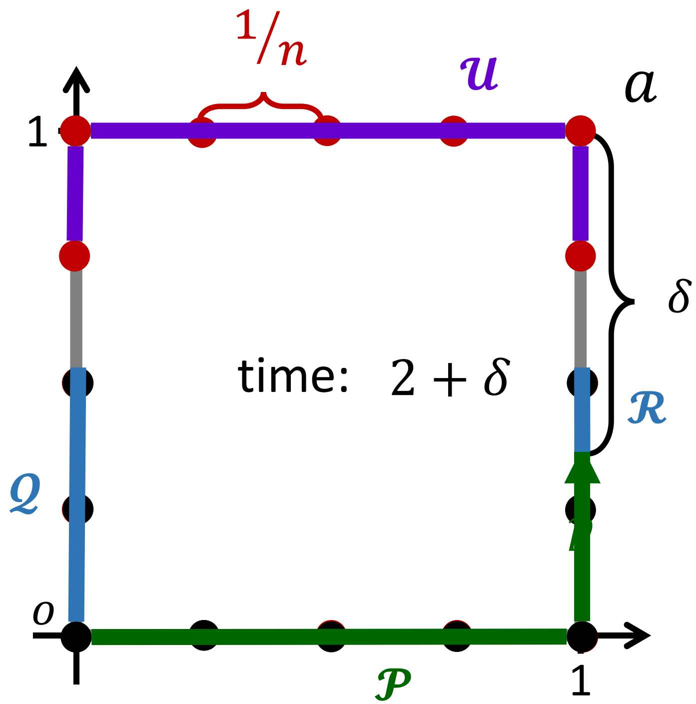
- $aa\ p,a\ = tt - 2$
- $= 2$
- wait until $d\ p, a\ = t - 2$



$$\left.\begin{aligned} |\mathcal{P}| &= 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| &\leq 2 + \delta \end{aligned}\right\} \ |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2$$

Lower Bound for H-OLTSP

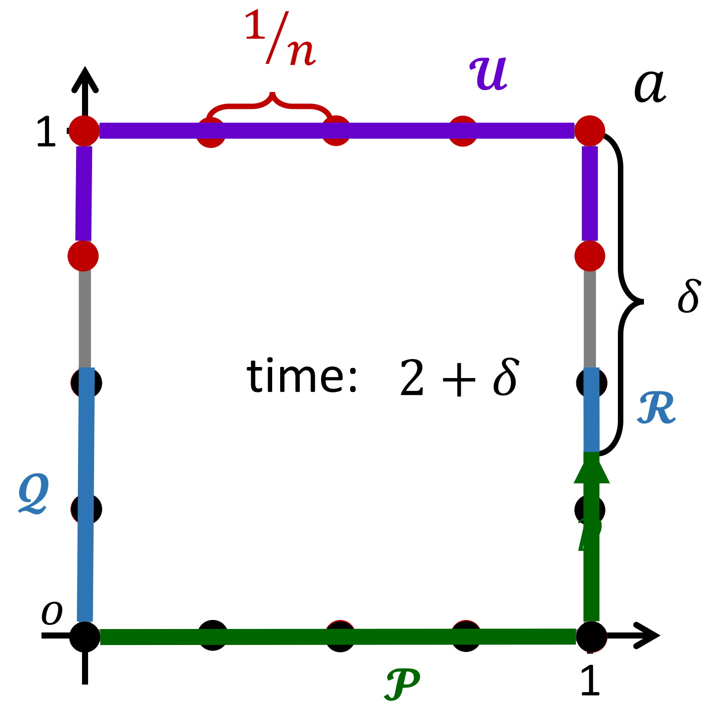
- $aa\ p,a\ = tt - 2$
- $=2$
- wait until $d\ p,a\ = t - 2$



$$\left.\begin{aligned} |\mathcal{P}| &= 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| &\le 2 + \delta \end{aligned}\right\} \begin{aligned} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| &\le 2 \\ \Rightarrow |\mathcal{U}| & \end{aligned}$$

Lower Bound for H-OLTSP

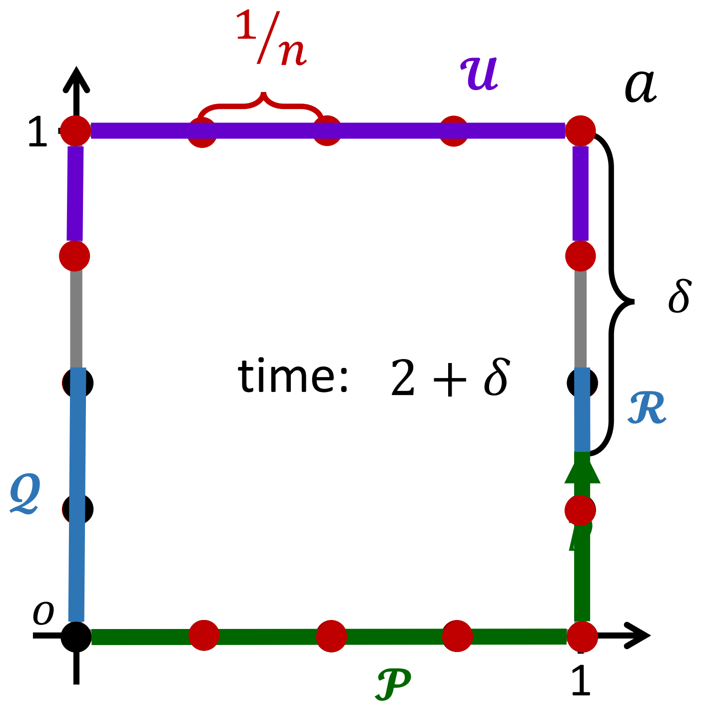
- $aa\ p{,}a\ {=}tt{-}2$
- ${=}2$
- wait until $d\ p, a\ = t - 2$



$$\left.\begin{aligned} |\mathcal{P}| &= 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| &\leq 2 + \delta \end{aligned}\right\} \quad \begin{aligned} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| &\leq 2 \\ \Rightarrow \ |\mathcal{U}| &\geq 2 - \frac{2}{n} \end{aligned}$$

Lower Bound for H-OLTSP

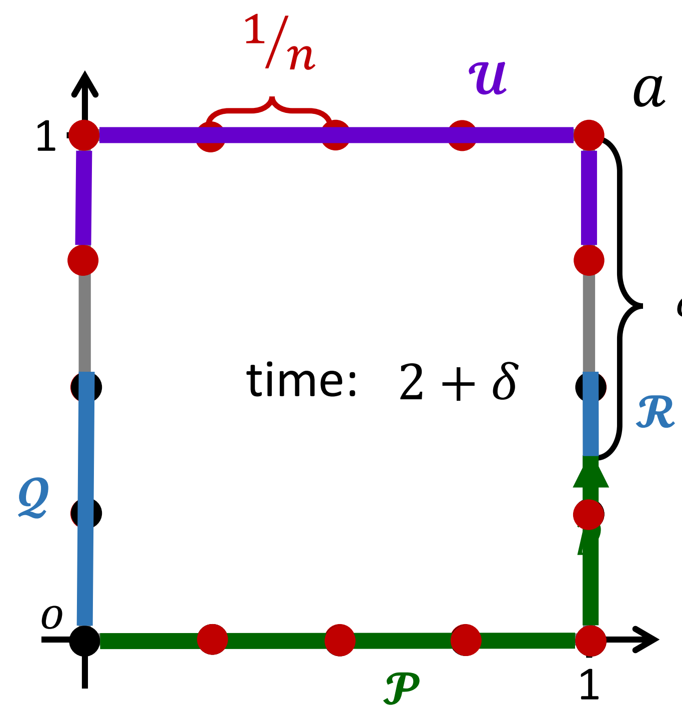
- $aa\ p,a\ =tt-2$
- $=2$
- new requests on $\mathcal{P}$



$$|\mathcal{P}| = 2-\delta$$
$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq 2+\delta$$
$$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2$$
$$\Rightarrow |\mathcal{U}| \geq 2-\frac{2}{n}$$

Lower Bound for H-OLTSP

- 4
- *aa p,a =tt−2*
- *=2*

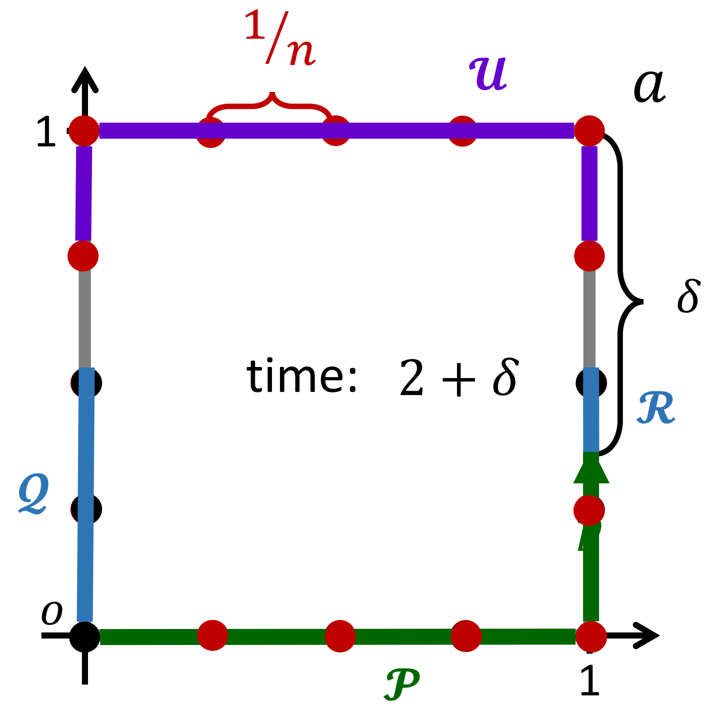


- new requests on $\mathcal{P}$
- OPT finishes at $t = 4$

$$\left.\begin{aligned} |\mathcal{P}| &= 2-\delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| &\leq 2+\delta \end{aligned}\right\} \quad |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2 \quad \Rightarrow |\mathcal{U}| \geq 2 - \frac{2}{n}$$

Lower Bound for H-OLTSP

- *4*
- *4*
- *aa p,a =tt−2*
- *=2*



- OPT finishes at $t = 4$
- OPT finishes at $t = 4$

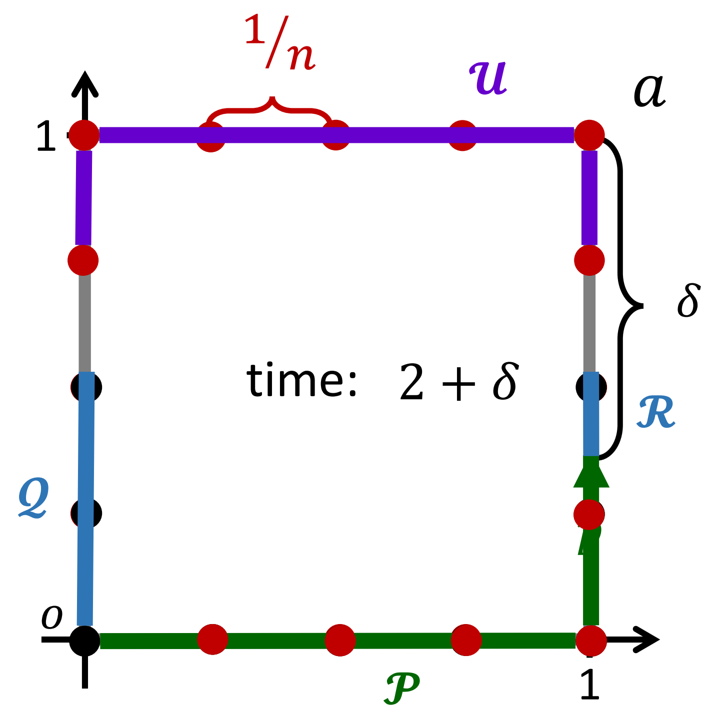
$$\left.\begin{aligned} |\mathcal{P}| &= 2-\delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| &\le 2+\delta \end{aligned}\right\} \quad |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \le 2$$

$$\Rightarrow |\mathcal{U}| \ge 2 - \frac{2}{n}$$

Lower Bound for H-OLTSP

- 4
- 4
- $aa\,p,a = tt-2$
- $=2$

- ALG finishes at $t \geq$
  OPT finishes at $t \geq 4$
- C

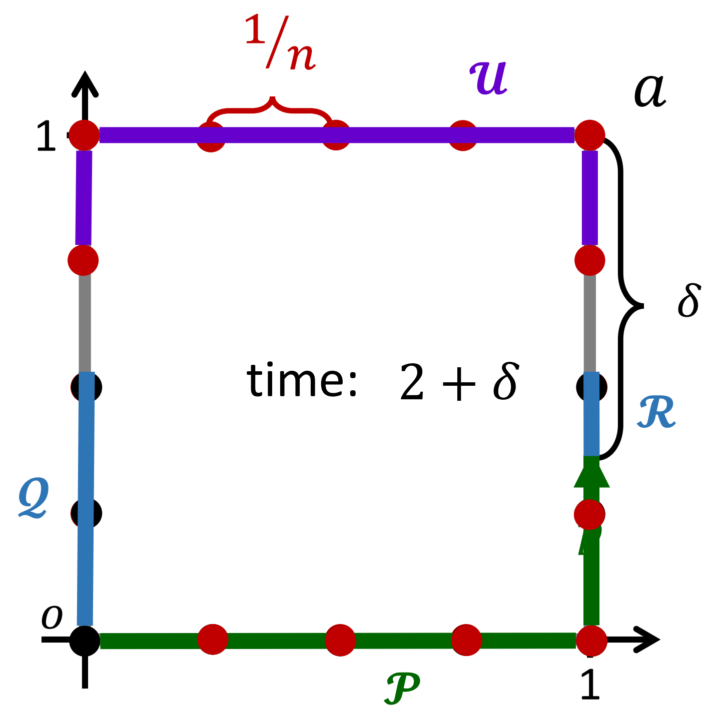


$$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \le 2$$

$$\Rightarrow |\mathcal{U}| \ge 2 - \frac{2}{n}$$

Lower Bound for H-OLTSP

- *4*
- *4*
- *aa p,a =tt−2*
- *=2*
- ALG finishes at $t \geq$
- OPT finishes at $t \geq 4$
- C



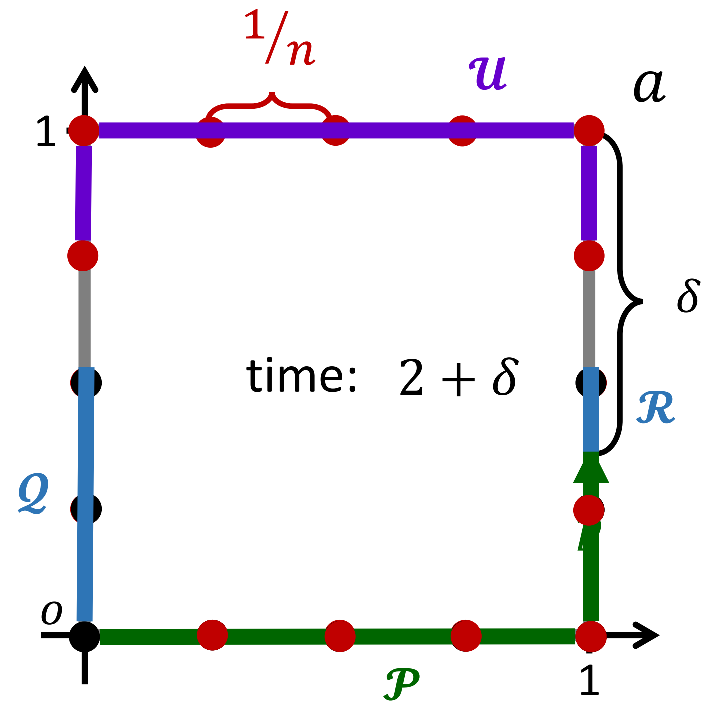
$$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2$$

$$\Rightarrow |\mathcal{U}| \geq 2 - \frac{2}{n}$$

Lower Bound for H-OLTSP

- *4*
- *4*
- $aa\ p{,}a = tt{-}2$
- $=2$

- ALG finishes at $t \geq 4$
- OPT finishes at $t = 4$
- C $\begin{cases} \delta\delta \\ -2\ 2\ n\ nn\ 2\ n\ \ 2\ n\ \ 2 - \ \ 2\ n \\ +\ \ 2\ n\ \ 2\ n\ 2 - \ 2 - \ \ 2\ n\ \ 2 \cdot 2\delta\delta \\ + + 2 \end{cases}$



$$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2$$

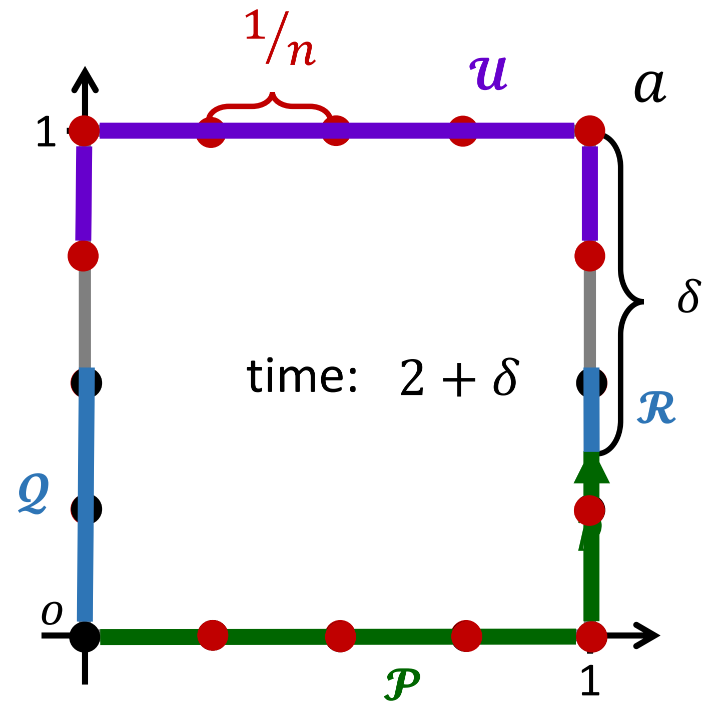
$$\Rightarrow |\mathcal{U}| \geq 2 - \frac{2}{n}$$

$2 - \frac{2}{n}$ $\quad 2 - \delta$

Lower Bound for H-OLTSP

- *4*
- *4*
- *aa p,a =tt−2*
- *=2*

- ALG finishes at $t \geq$
- OPT finishes at $t = 4$
- C$\begin{cases} 2 + \\ \delta + 2 - \ 2\ n\ + 22 + 2 - -\delta \\ -\delta\ n\ n\ n\ 2\ n\ \ 2 - \frac{2}{n} + 2 \\ + (2-\delta) \end{cases}$



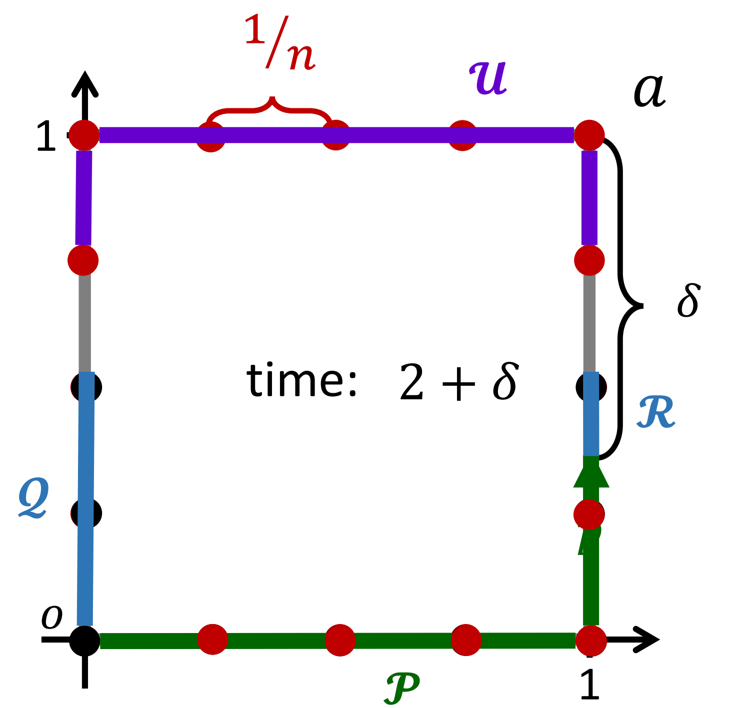
$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2$

$\Rightarrow\ |\mathcal{U}| \geq 2 - \frac{2}{n}$

Lower Bound for H-OLTSP

- *4*
- *4*
- *aa p,a =tt−2*
- *=2*

- ALG finishes at $t \geq 8 - \frac{4}{n}$
  OPT finishes at $t = 4$
- C $\begin{cases} 2 + \\ \delta + 2 - \;\; 2\,n \;\; + 22 + 2 - -\delta \\ -\delta \;\; n\; n\; n\; 2\; n \;\; 2 - \frac{2}{n} + 2 \\ + (2 - \delta) \end{cases}$

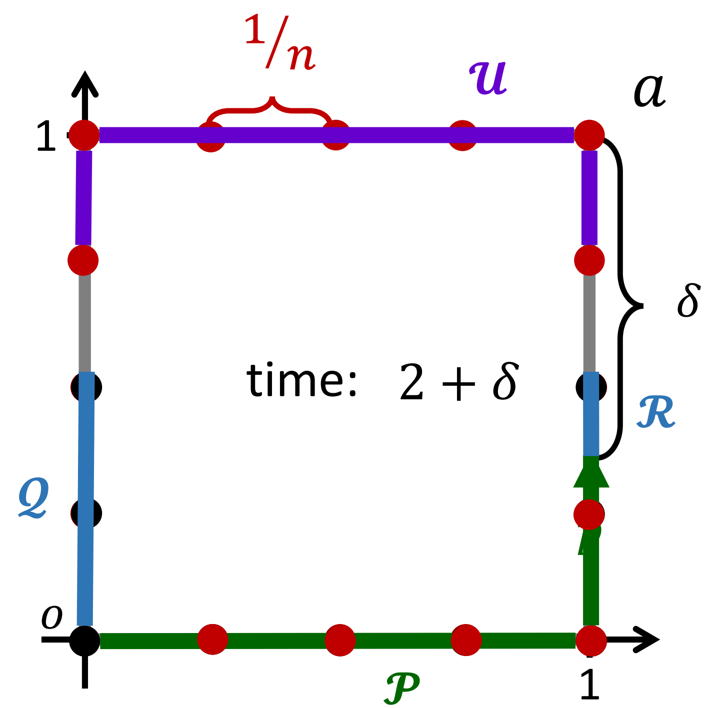


$$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \leq 2$$

$$\Rightarrow \; |\mathcal{U}| \geq 2 - \frac{2}{n}$$

Lower Bound for H-OLTSP

- *4*
- *4*
- *aa p,a =tt−2*
- *=2*
- ALG finishes at $t \geq 8 - \frac{4}{n}$
- OPT finishes at $t = 4$



THEOREM: Any ρ-competitive ALG for H-OLTSP has $\rho \geq 2$.

$-\delta$ n n n 2 n $2 - \frac{}{n} + 2$

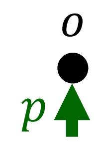
$+ (2 - \delta)$

A better algorithm for H-OLTSP

: start op

A better algorithm for H-OLTSP

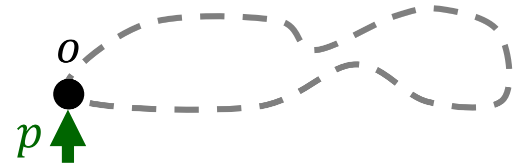
: start op



A better algorithm for H-OLTSP

U := places yet to visit

(1) At o: start optimal tour through U



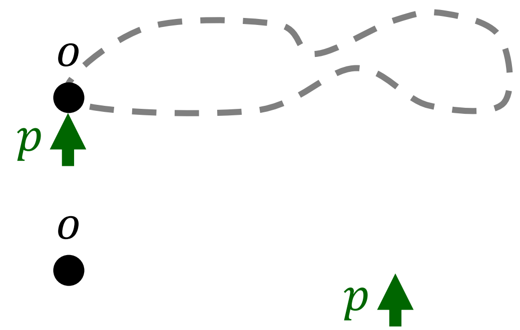
A better algorithm for H-OLTSP

U := places yet to visit

(1) tt,xx):

(2) At o: start optimal tour through U

(3) For new request (t, x):



A better algorithm for H-OLTSP

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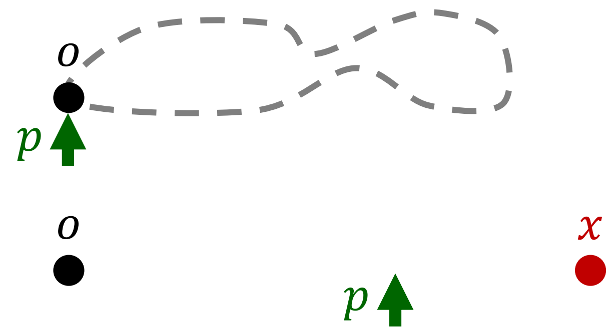
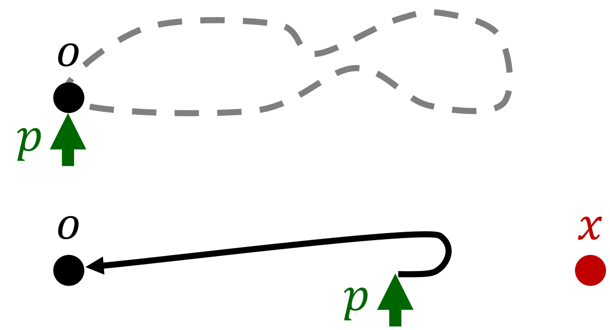


A better algorithm for H-OLTSP

U := places yet to visit



(1) *xx,oo)>dd(pp,oo): go back to oo*

(2) *tt,xx*):

(3) At o: start optimal tour through U

a) If $d(x, o) > d(p, o)$: go back to o

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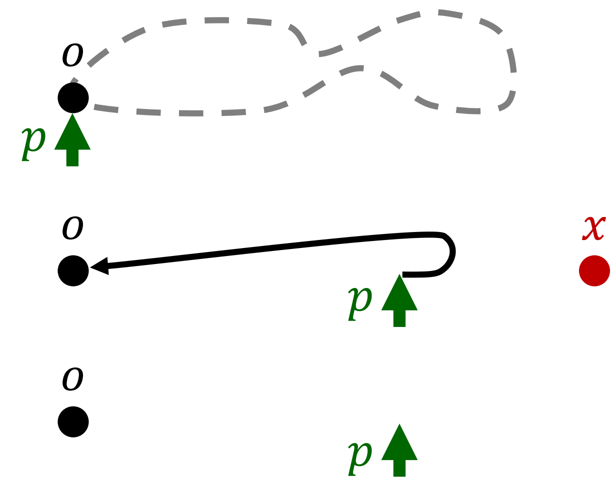

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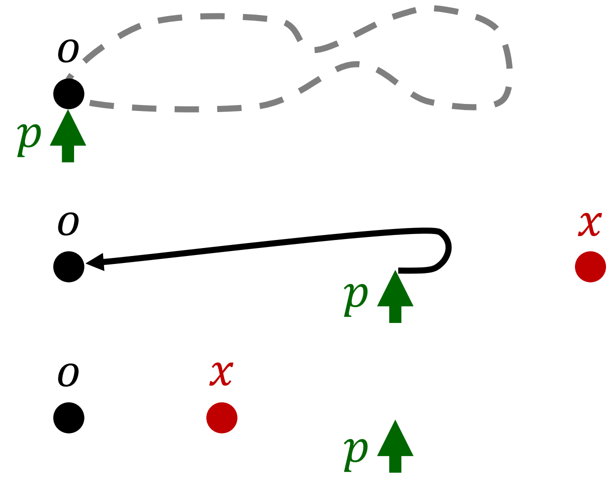

A better algorithm for H-OLTSP

U := places yet to visit

(1) $xx,oo)>dd(pp,oo)$: go back to oo

(2) $tt,xx)$:

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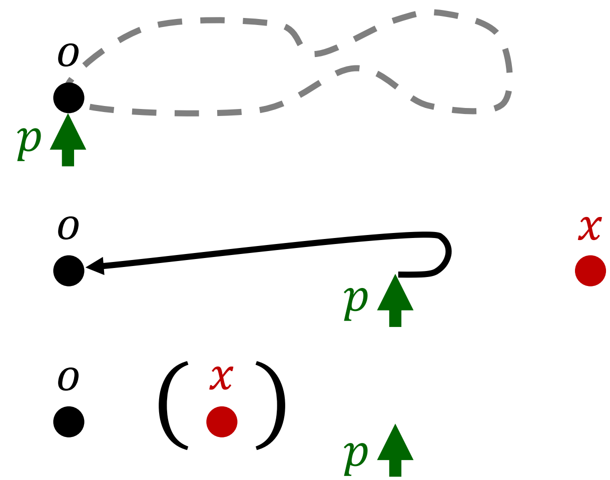
a) Else: ignore x until back at o



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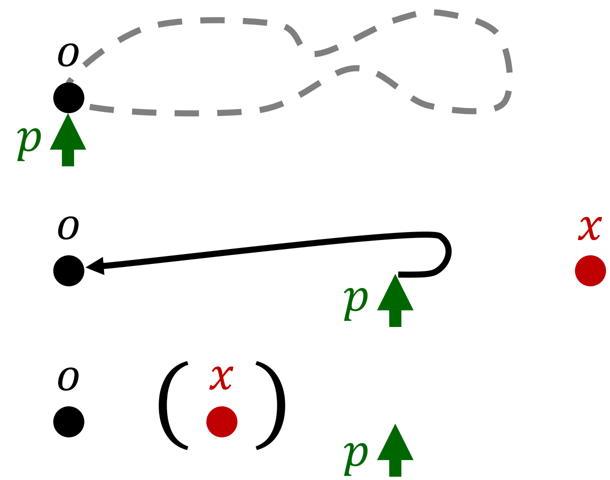

Plan At Home (PAH)

A better algorithm for H-OLTSP

U := places yet to visit

(1) *xx,oo)>dd(pp,oo): go back to oo*

(2) *tt,xx*):

(3) At o: start optimal tour through U

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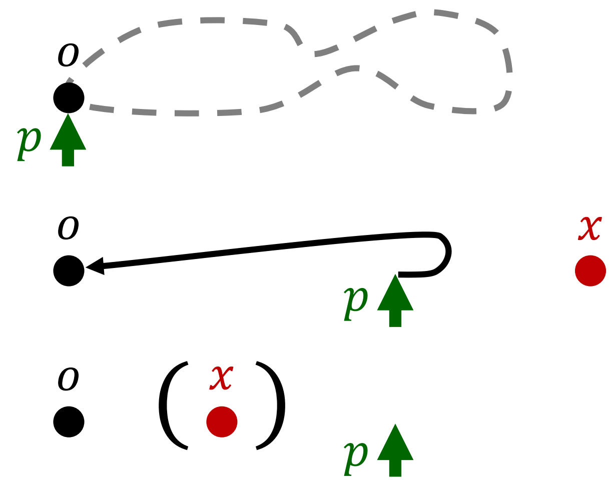

Plan At Home (PAH)

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

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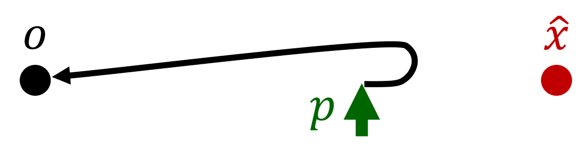

$|\mathcal{T}^{\text{PAH}}|$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

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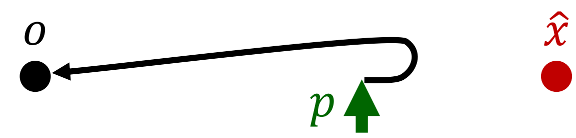


$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \hat{t} +$$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

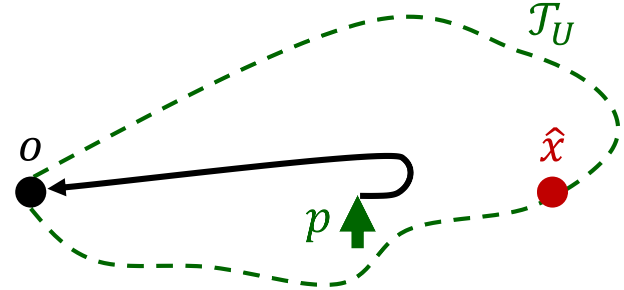
a) If $d(\hat{x}, o) > d(p, o)$: go back to o

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \hat{t} + d(p, o)$$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request



(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \hat{t} + d(p, o)$$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

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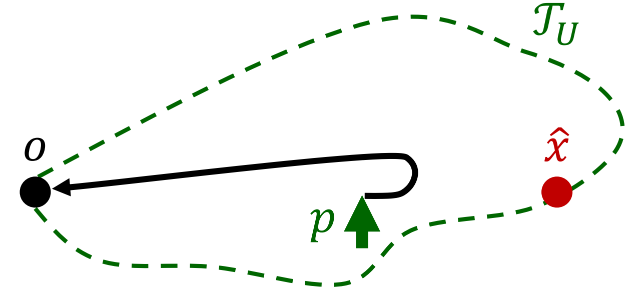

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \hat{t} + d(p,o) + |\mathcal{T}_U|$$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

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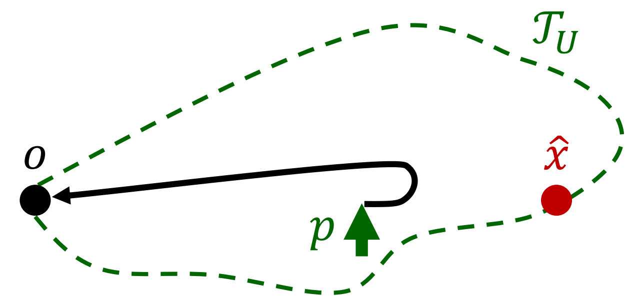

$$|\mathcal{T}^{\mathrm{PAH}}| = \hat{t} + d(p, o) + |\mathcal{T}_U|$$

$$\leq |\mathcal{T}^{\mathrm{OPT}}|$$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

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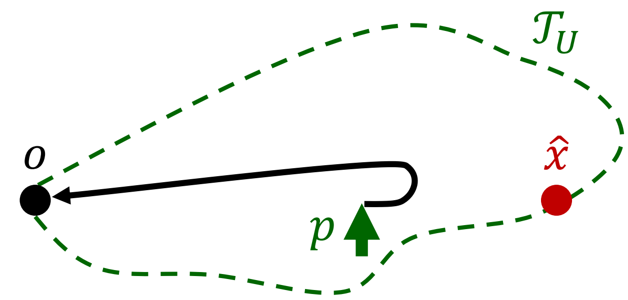


$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \hat{t} + d(\hat{x}, o) + |\mathcal{T}_U|$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

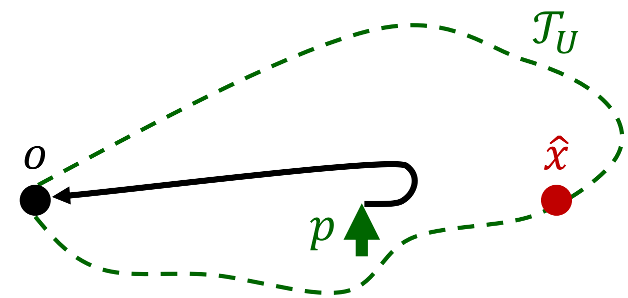
GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o



$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \underbrace{\hat{t} + d(\hat{x}, o)}_{} + |\mathcal{T}_U|$$

$$\leq \qquad\qquad\qquad \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

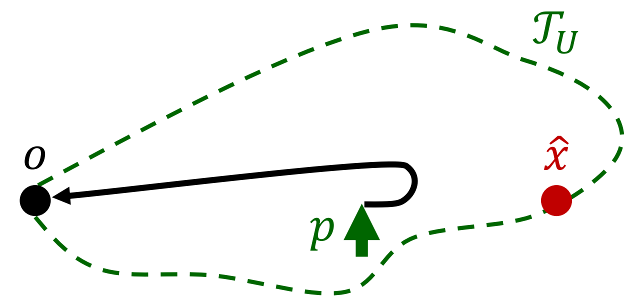
GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o



$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \underbrace{\hat{t} + d(\hat{x}, o)}_{\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|} + \left|\mathcal{T}_U\right|$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

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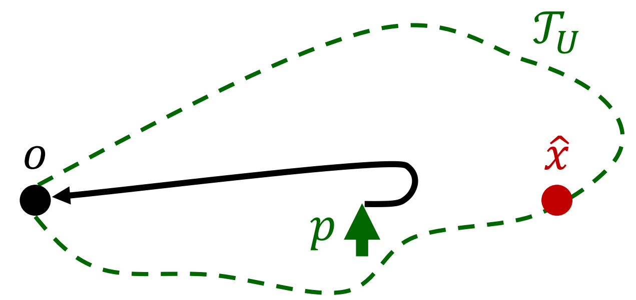

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \underbrace{\hat{t} + d(\hat{x}, o)}_{\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|} + |\mathcal{T}_U|$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP

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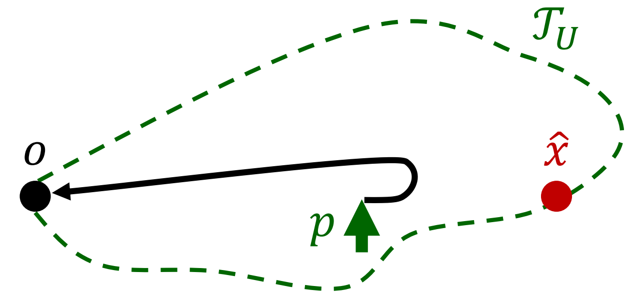

$$|\mathcal{T}^{\mathrm{PAH}}| = \underbrace{\hat{t} + d(\hat{x}, o)}_{\leq |\mathcal{T}^{\mathrm{OPT}}|} + |\mathcal{T}_U|$$

$$\leq |\mathcal{T}^{\mathrm{OPT}}| + |\mathcal{T}^{\mathrm{OPT}}| = 2 \cdot |\mathcal{T}^{\mathrm{OPT}}| \quad ✓$$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

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(2) For new request $(\hat{t}, \hat{x})$:

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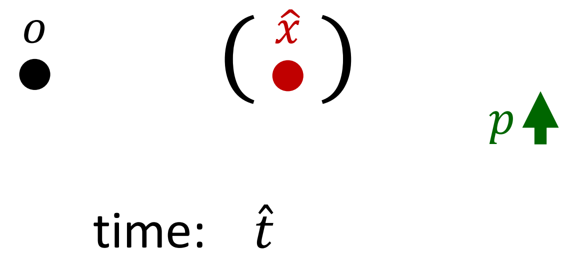

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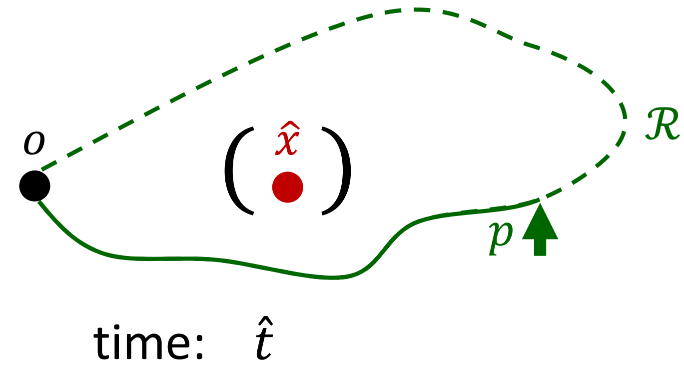

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request
I := ignored requests

(2) For new request $(\hat{t}, \hat{x})$:

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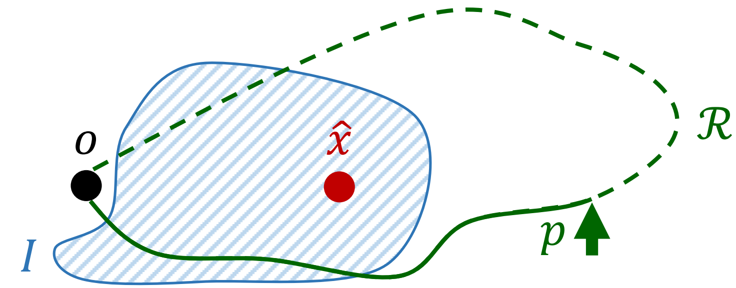

time: $\hat{t}$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

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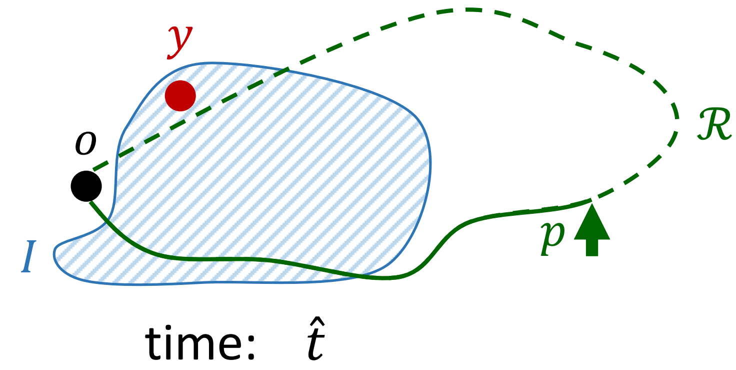

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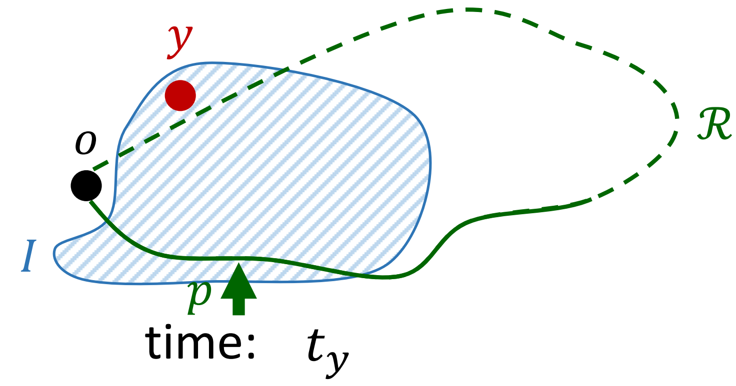

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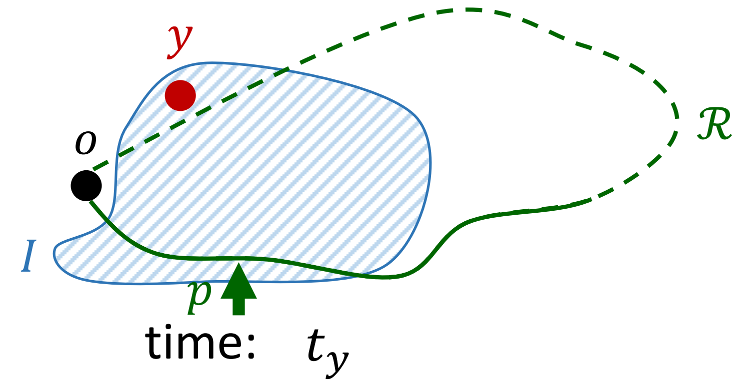

GOAL: PAH is 2-competitive for H-OLTSP

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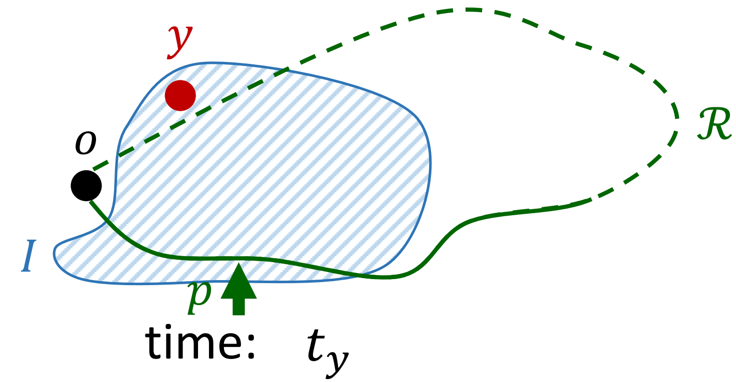

$$\left|\mathcal{T}^{\mathrm{PAH}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U ≔ places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT
I ≔ ignored requests

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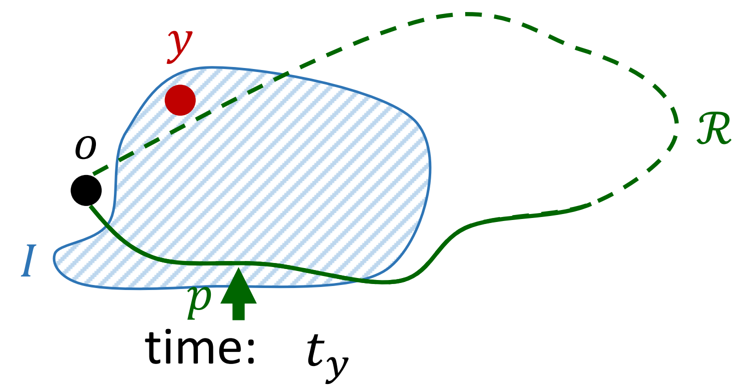

$$\left|\mathcal{T}^{\text{PAH}}\right| \leq t_y$$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

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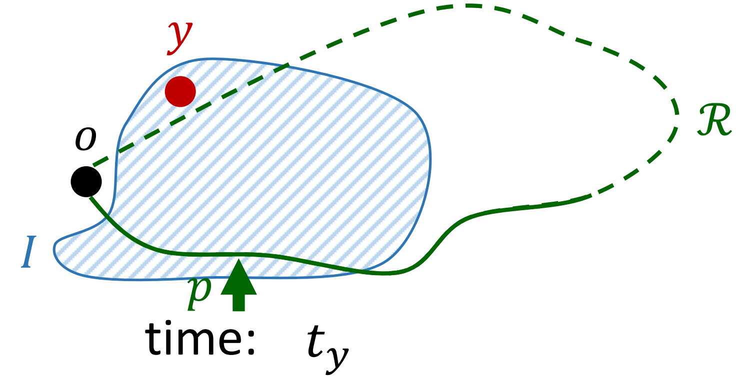

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}|$$

GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

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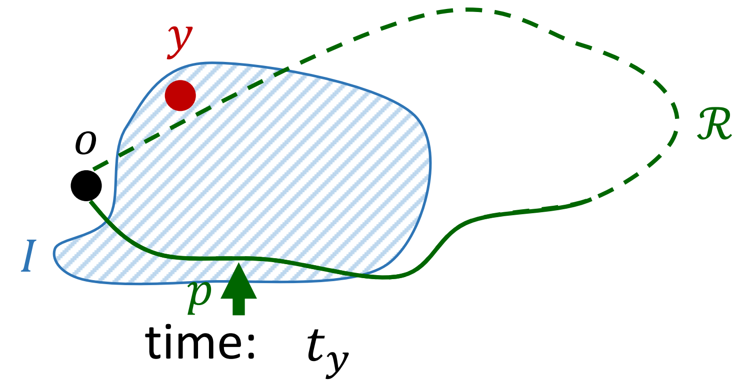

$$\left|\mathcal{T}^{\text{PAH}}\right| \leq t_y + |\mathcal{R}| - d(o, y)$$

GOAL: PAH is 2-competitive for H-OLTSP

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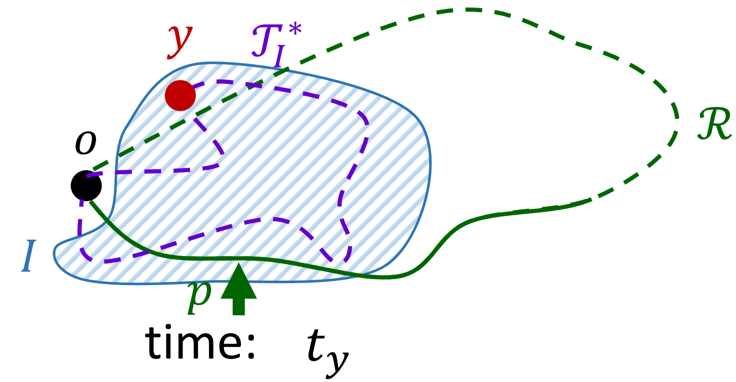

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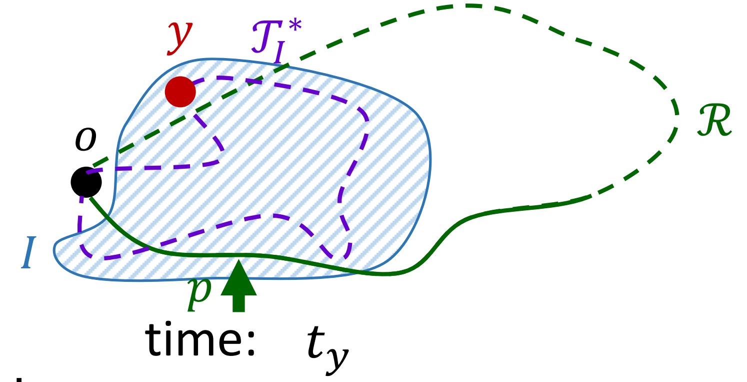

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}| - d(o, y) + |\mathcal{T}_I^*|$$

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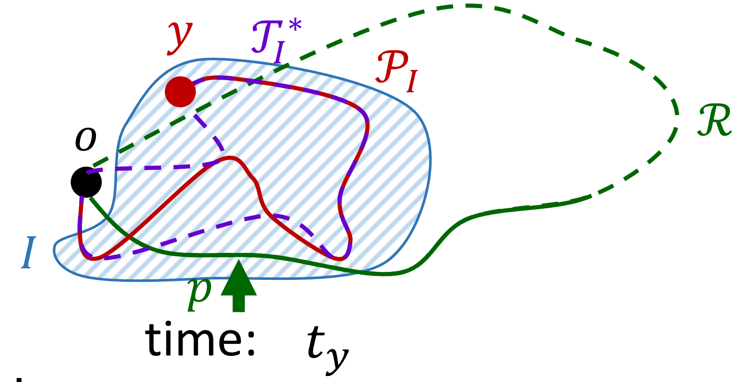

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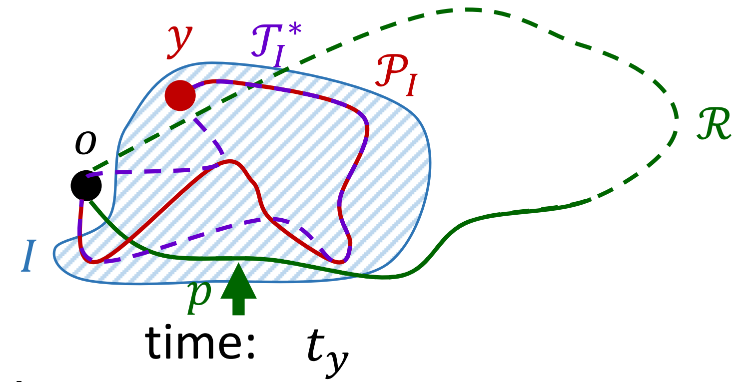

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \le t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

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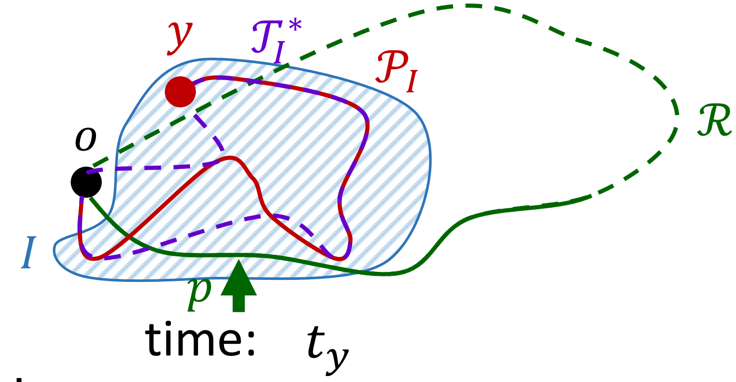

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}| \underbrace{- d(o, y) + |\mathcal{T}_I^*|}_{|\mathcal{P}_I|}$$

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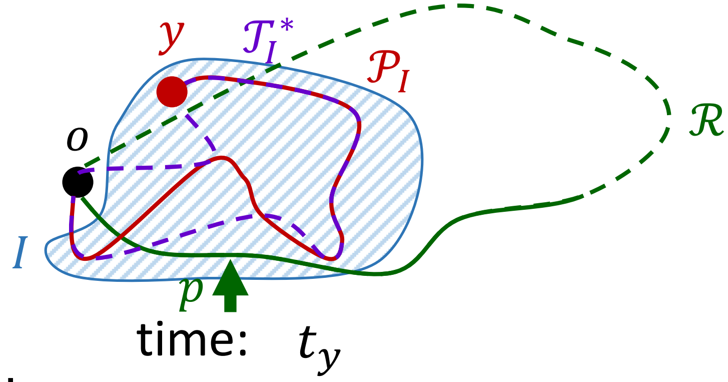

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}| \underbrace{- d(o, y) + |\mathcal{T}_I^*|}$$

$$\leq t_y + |\mathcal{R}| + \quad |\mathcal{P}_I|$$

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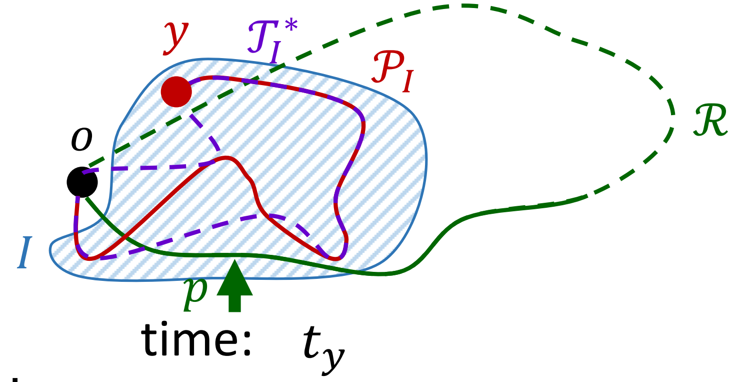

$$|\mathcal{T}^{\text{PAH}}| \leq t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

$$\leq t_y + |\mathcal{R}| + \quad |\mathcal{P}_I| \quad = \quad t_y + |\mathcal{P}_I| + |\mathcal{R}|$$

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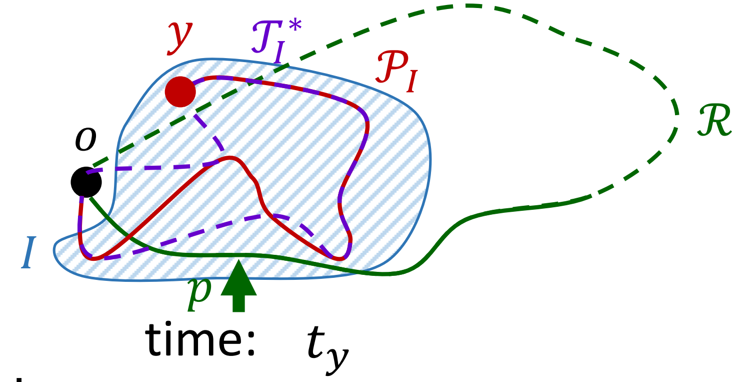

$$|\mathcal{T}^{\text{PAH}}| \leq t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

$$\leq t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|} + |\mathcal{R}|$$

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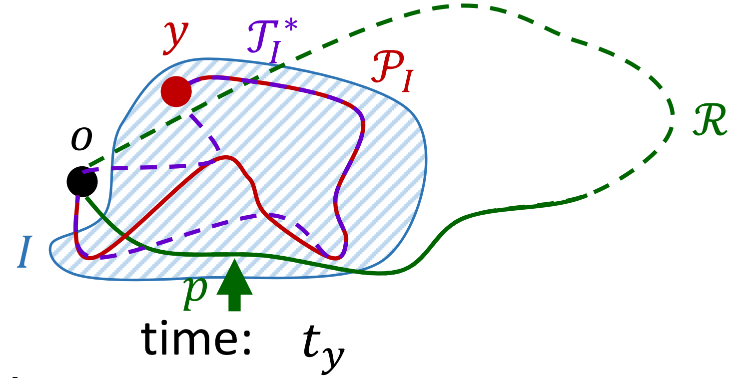

$$\left|\mathcal{T}^{\text{PAH}}\right| \le t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

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$$\le \quad \left|\mathcal{T}^{\text{OPT}}\right|$$

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Competitiveness of PAH

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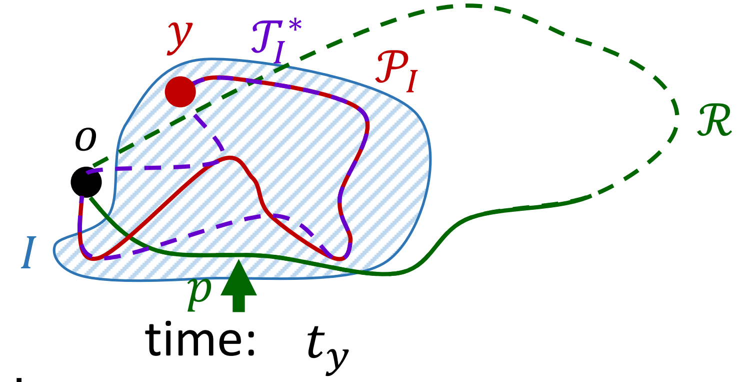

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

$$\leq t_y + |\mathcal{R}| + \quad |\mathcal{P}_I| \quad = \quad \underbrace{t_y + |\mathcal{P}_I|} + |\mathcal{R}|$$

$$\leq \quad \left|\mathcal{T}^{\mathrm{OPT}}\right| \quad + \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

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Competitiveness of PAH

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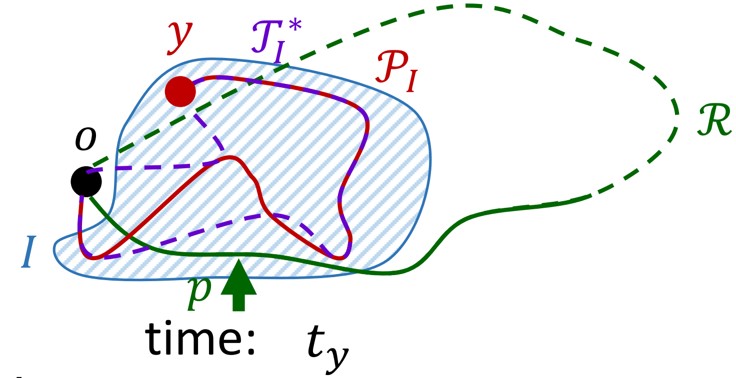

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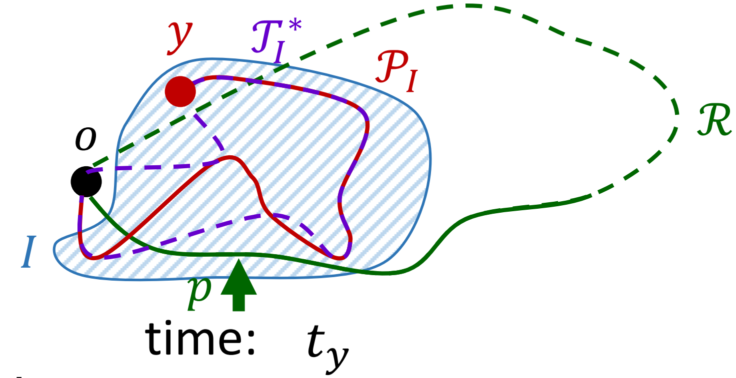


$$|\mathcal{T}^{\mathrm{PAH}}| \le t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

$$\le t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|} + |\mathcal{R}|$$

$$\le |\mathcal{T}^{\mathrm{OPT}}| + |\mathcal{T}^{\mathrm{OPT}}| = 2 \cdot |\mathcal{T}^{\mathrm{OPT}}|$$

GOAL: PAH is 2-competitive for H-OLTSP I

Competitiveness of PAH

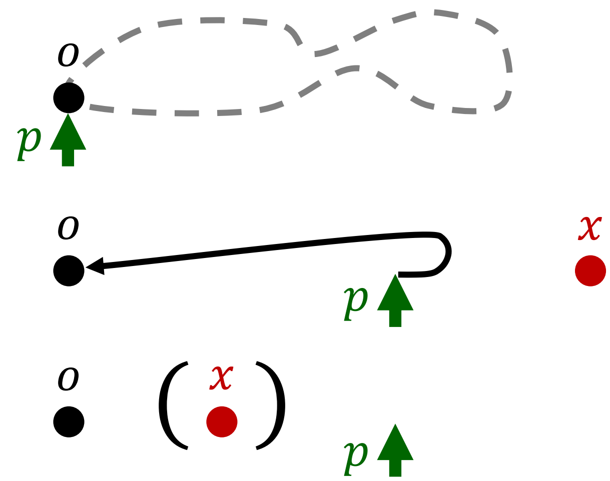
U := places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):

a) If $d(x,o) > d(p,o)$: go back to o

b) Else: ignore x until back at o



GOAL: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

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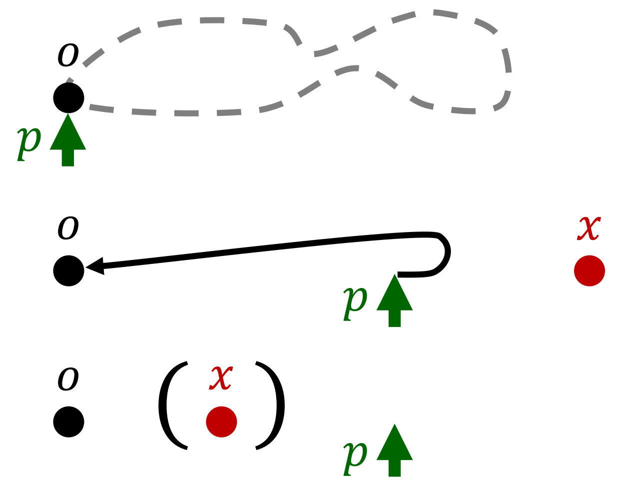

THEOREM: PAH is 2-competitive for H-OLTSP

Competitiveness of PAH

U := places yet to visit

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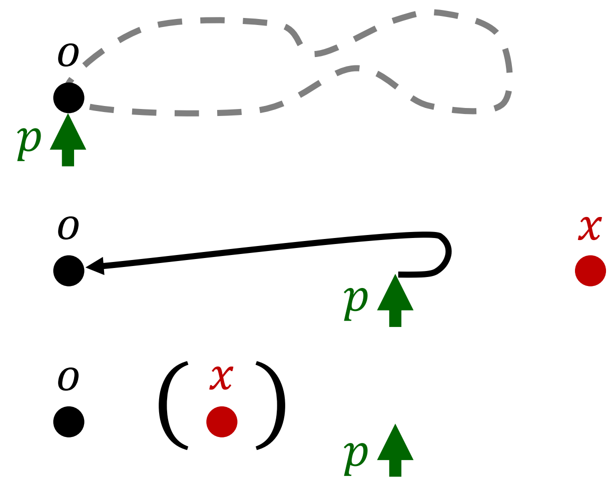


THEOREM: PAH is 2-competitive for H-OLTSP

REMARK: PAH is optimal online algorithm for H-OLTSP

Polynomial Algorithm for H-OLTSP

Polynomial Algorithm for H-OLTSP

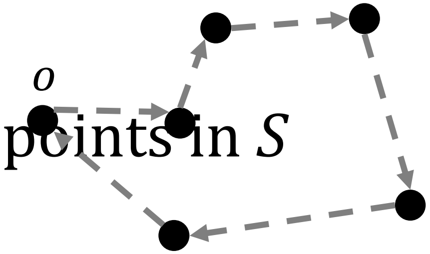
Invariant: always on shortest path between points in S

Polynomial Algorithm for H-OLTSP

o oo o

with Christofides-Heuristic

Invariant: always on shortest path between points in S



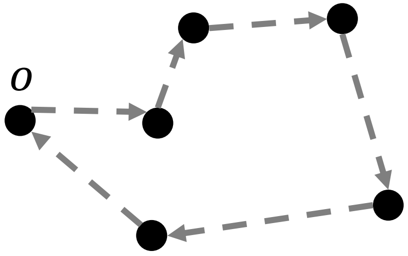
(1) At o: Find tour though $U \cup \{o\}$
with Christofides-Heuristic

Polynomial Algorithm for H-OLTSP

tt,zz) at time t
and alg between xx and yy:

0 00 0

with Christofides-Heuristic



Invariant: always on shortest path between points in S

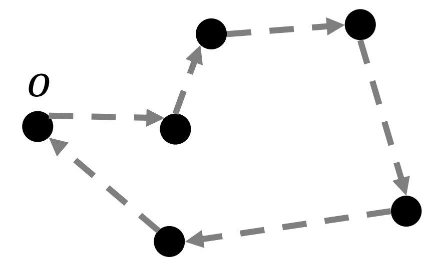
(1) For new request (t, z) at time t
and ALG between x and y:

Polynomial Algorithm for H-OLTSP

tt,zz) at time t
and alg between xx and yy:

o oo o

with Christofides-Heuristic



Invariant*:* always on shortest path between points in S

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Polynomial Algorithm for H-OLTSP

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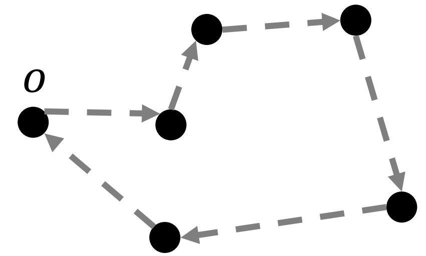

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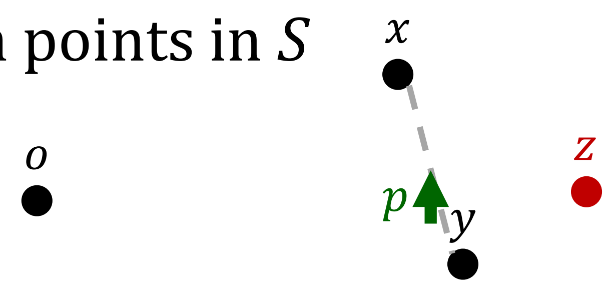

Polynomial Algorithm for H-OLTSP

tt,zz) at time t
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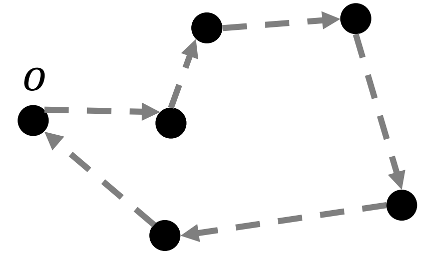
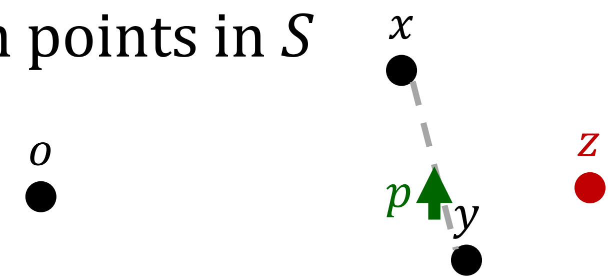
o oo o
with Christofides-Heuristic



Invariant*:* always on shortest path between points in S

a) Add z to U

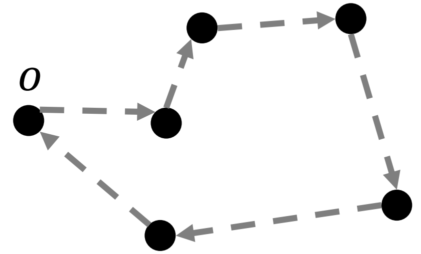


Polynomial Algorithm for H-OLTSP

tt,zz) at time t
and alg between xx and yy:

o oo o

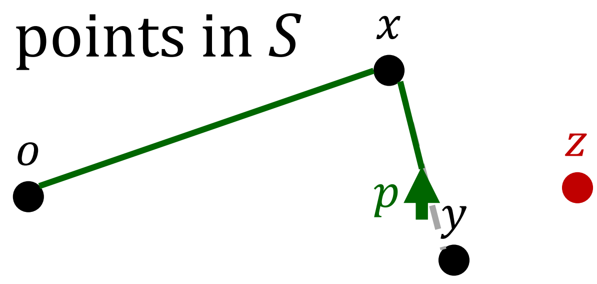
with Christofides-Heuristic



Invariant*:* always on shortest path between points in S

a) Add z to U

b) go back to o via x or y
(take shortest path)



Polynomial Algorithm for H-OLTSP

tt,zz) at time t
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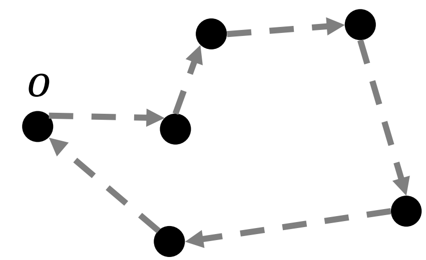

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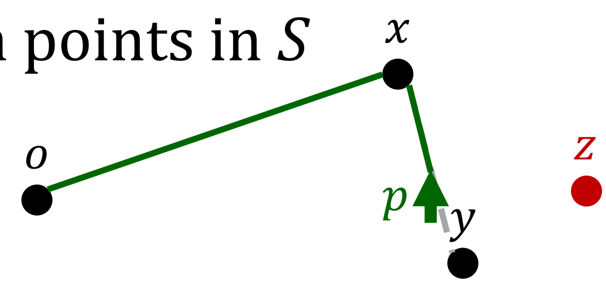
Invariant*:* always on shortest path between points in S



a) Add z to U

b) go back to o via x or y
(take shortest path)

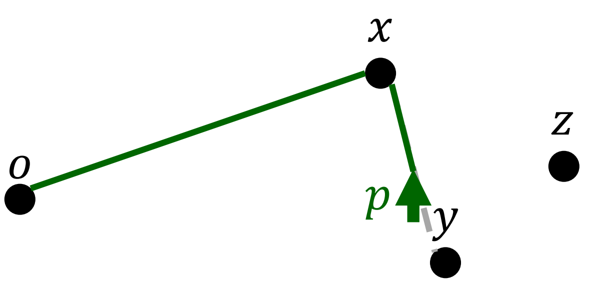
THEOREM: CHR is a polynomial (and correct).

Competitiveness of CHR

Invariant: always on shortest path between points in S

(2) For last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y:

b) go back to o via x or y (take shortest path)



GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(2) For last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y:

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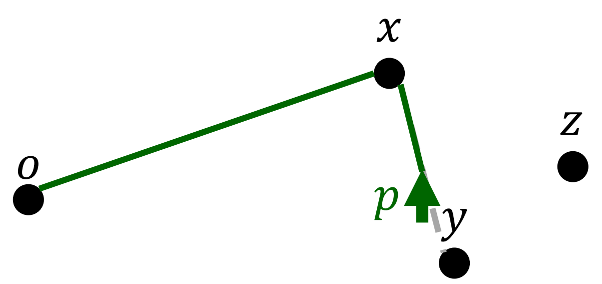

$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} +$$

GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(2) For last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y:

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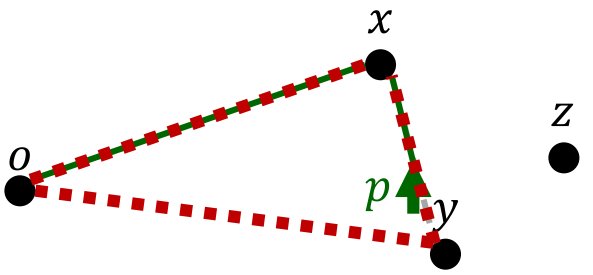


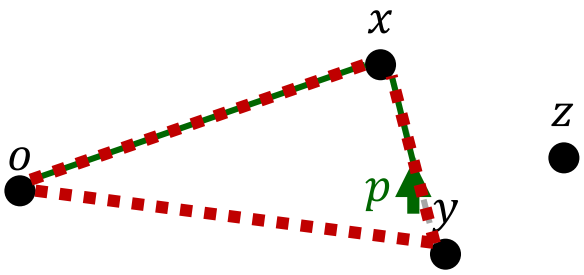
$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\}$$

GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\}$$

GOAL: CHR is _-competitive.

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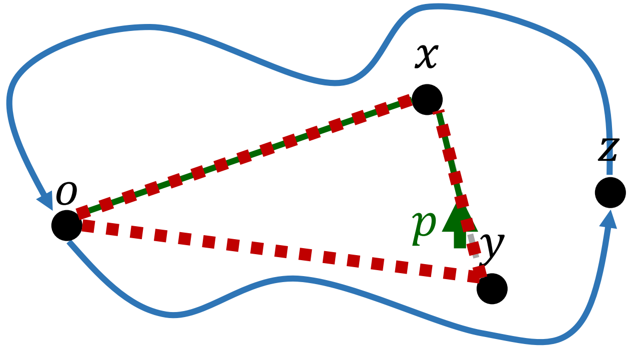

$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \mathrm{CHR}(U)$$

GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
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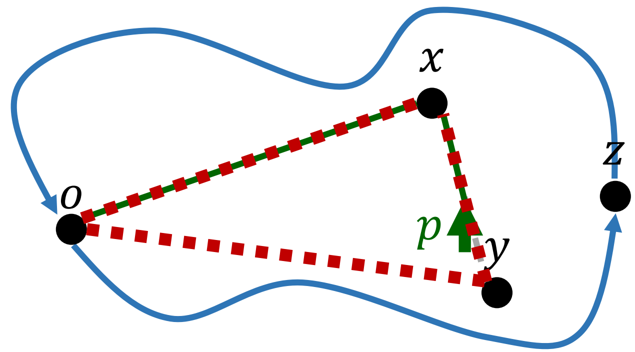

$$|\mathcal{T}^{\mathrm{CHR}}| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \mathrm{CHR}(U)$$

$$\leq |\mathcal{T}^{\mathrm{OPT}}|$$

GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

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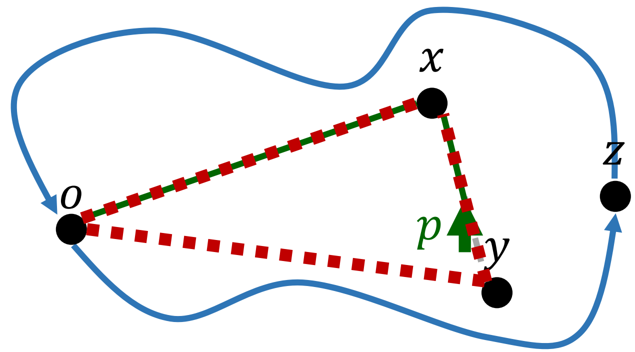

$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \underbrace{\min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\}} + \mathrm{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
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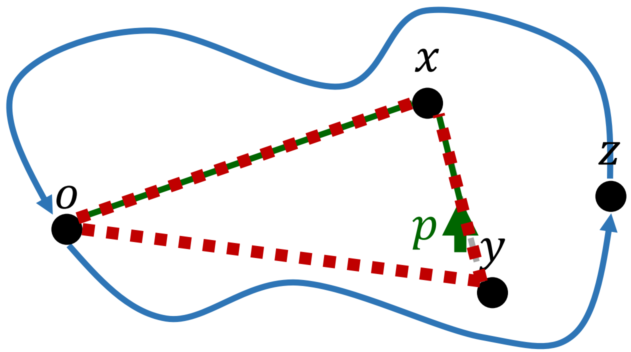


$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} + \underbrace{\min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\}} + \text{CHR}(U)$$

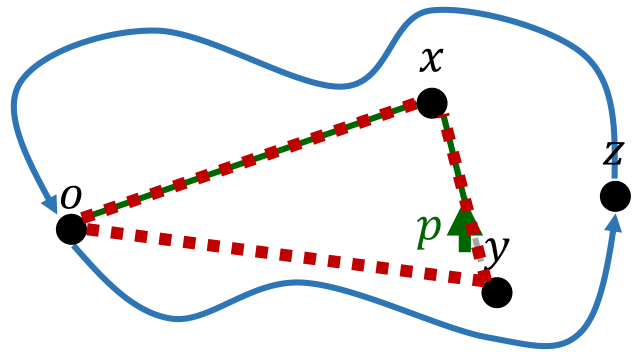
$$\leq \left|\mathcal{T}^{\text{OPT}}\right| \qquad + \qquad \tfrac{1}{2} \cdot \left|\mathcal{T}^{\text{OPT}}\right|$$

GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \underbrace{\min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\}} + \text{CHR}(U)$$

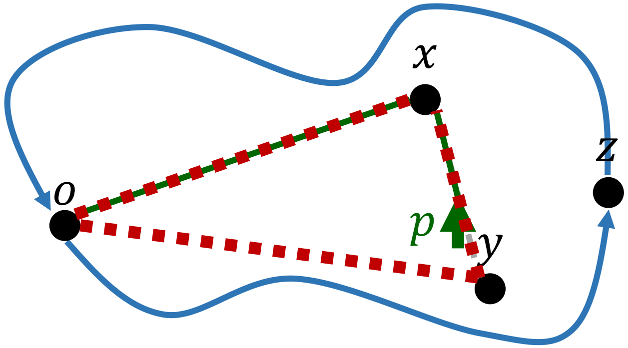
$$\leq |\mathcal{T}^{\text{OPT}}| \quad + \quad \tfrac{1}{2} \cdot |\mathcal{T}^{\text{OPT}}| \quad + \tfrac{3}{2} \cdot |\mathcal{T}^{\text{OPT}}|$$

GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \mathrm{CHR}(U)$$

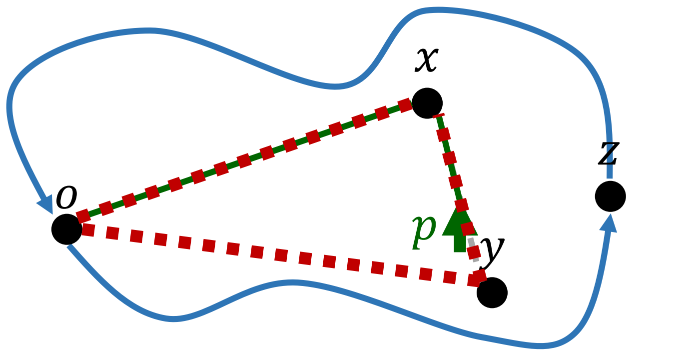
$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \tfrac{1}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| + \tfrac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \;\; =$$

GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \mathrm{CHR}(U)$$

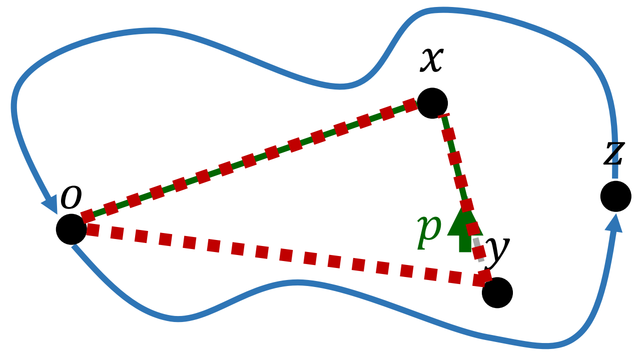
$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \tfrac{1}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| + \tfrac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \;=\; 3 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: CHR is _-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$ with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \mathrm{CHR}(U)$$

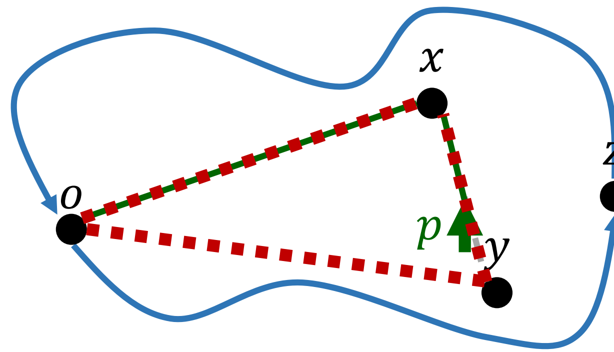
$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \tfrac{1}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| + \tfrac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \ = \ 3 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

THEOREM: CHR is 3-competitive.

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \text{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\text{OPT}}\right| + \frac{1}{2} \cdot \left|\mathcal{T}^{\text{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\text{OPT}}\right| \;=\; 3 \cdot \left|\mathcal{T}^{\text{OPT}}\right|$$

REMARK: There is a 3-competitive algorithm for N-OLTSP.

Credits & References

- Paper: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.8.5620
- Map: http://awoiaf.westeros.org/index.php/File:WorldofIceandFire.png
- Font: http://www.fonts4free.net/game-of-thrones-font.html