ONLINE — TSP

Presenting results from the paper „Algorithms for the On-Line Travelling Salesman“
by G. Ausiello, E. Feuerstein, S. Leonardi, L. Stougie, and M. Talamo

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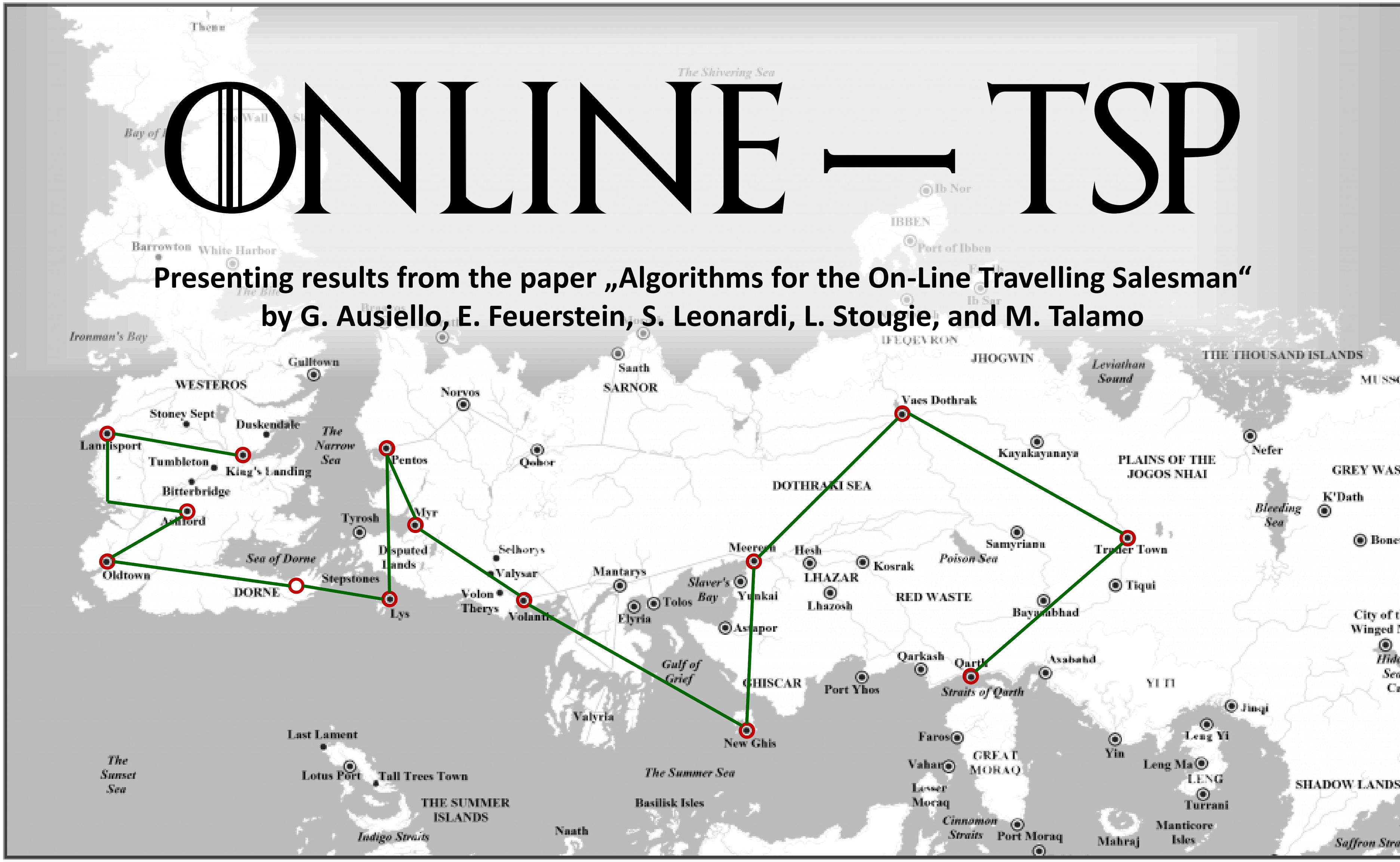

Online-TSP

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(metric)

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(metric)

INPUT:

Online-TSP

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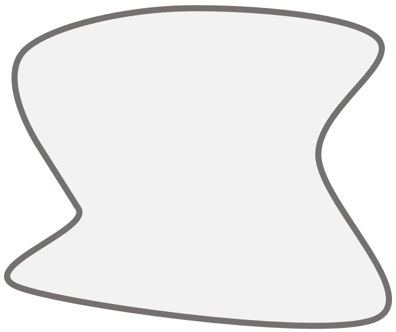
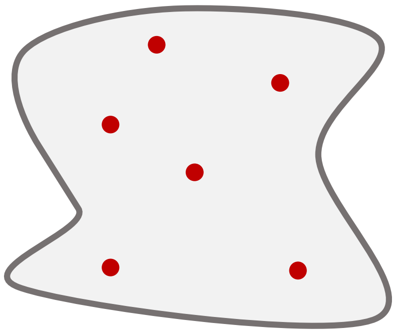


- metric space: M
  (with metric d)

Online-TSP

(metric)

INPUT:

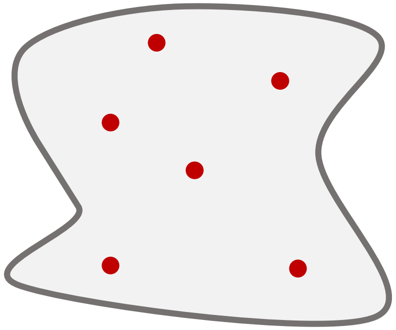


- metric space: M
  (with metric d)
- places to visit: $\mathcal{S}$

Online-TSP

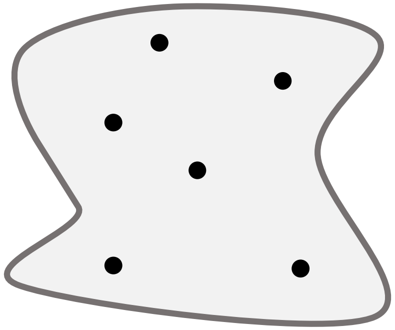
(metric)

INPUT:



- metric space: M (with metric d)
- places to visit: S

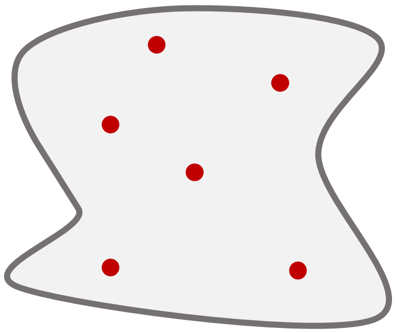
OUTPUT:



Online-TSP

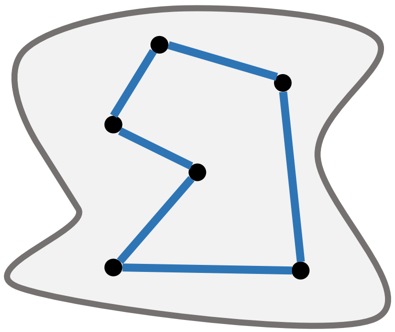
(metric)

INPUT:



- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

OUTPUT:

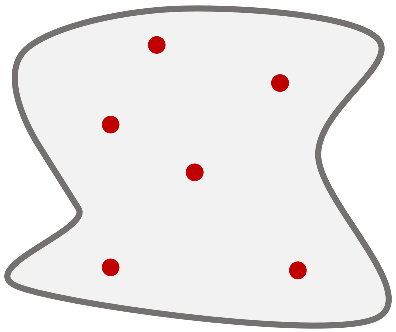


Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

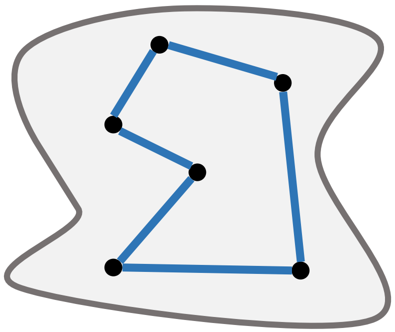
INPUT:



- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

NP-hard!

OUTPUT:



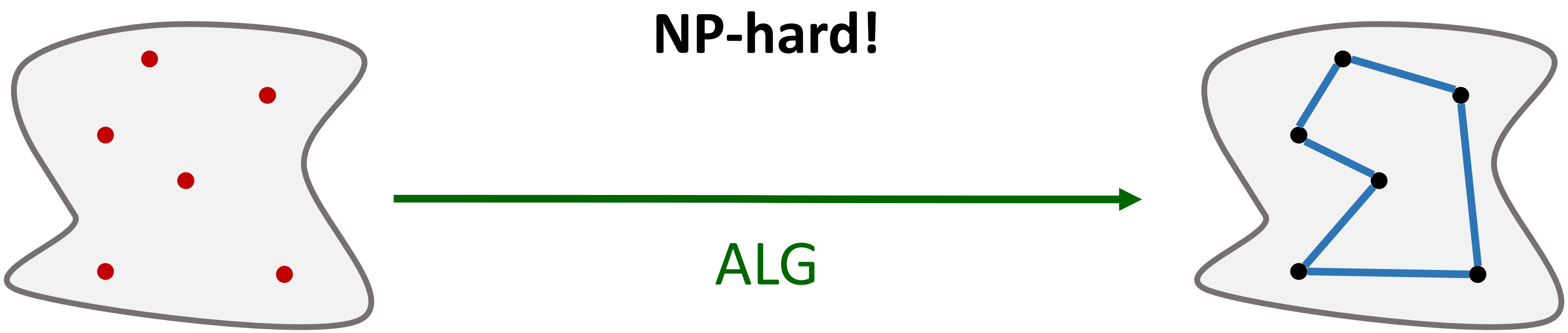
Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

INPUT:

OUTPUT:



- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

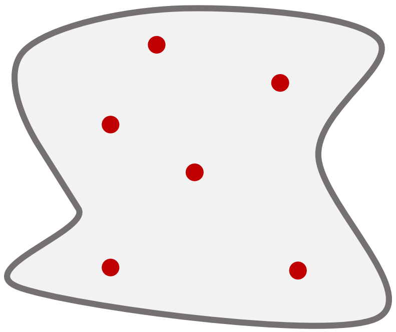
Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

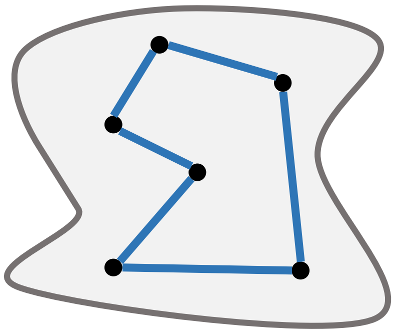
INPUT:

OUTPUT:



NP-hard!

ALG



- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

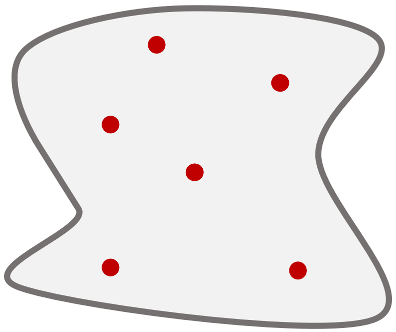
- Superpolynomial Alg.

Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

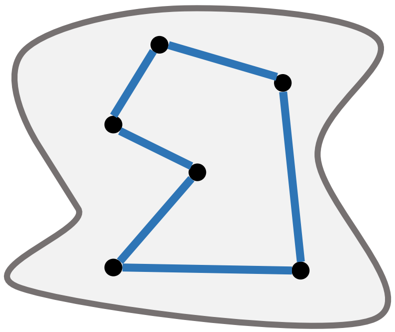
INPUT:



NP-hard!

ALG

OUTPUT:



- metric space: M (with metric d)
- places to visit: $\mathcal{S}$

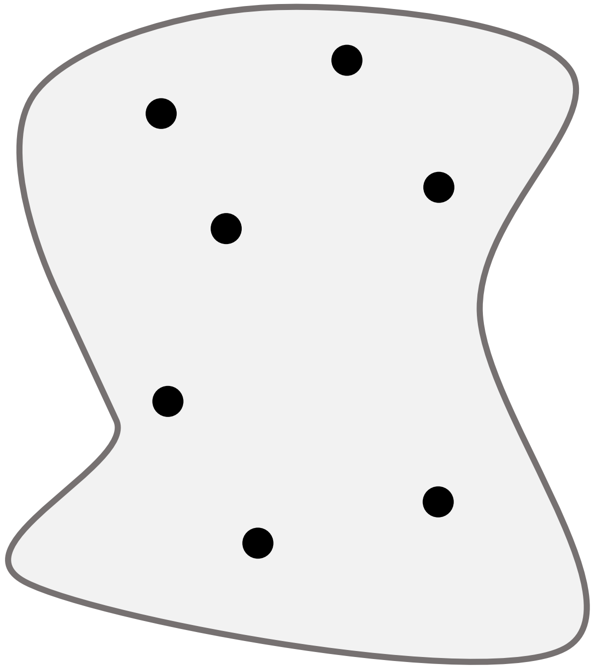
- Superpolynomial Alg.
- Approximation Alg.
  e.g. *Christofides*

Shortest tour $\mathcal{T}^*$ through $\mathcal{S}$

Online-TSP

(metric)

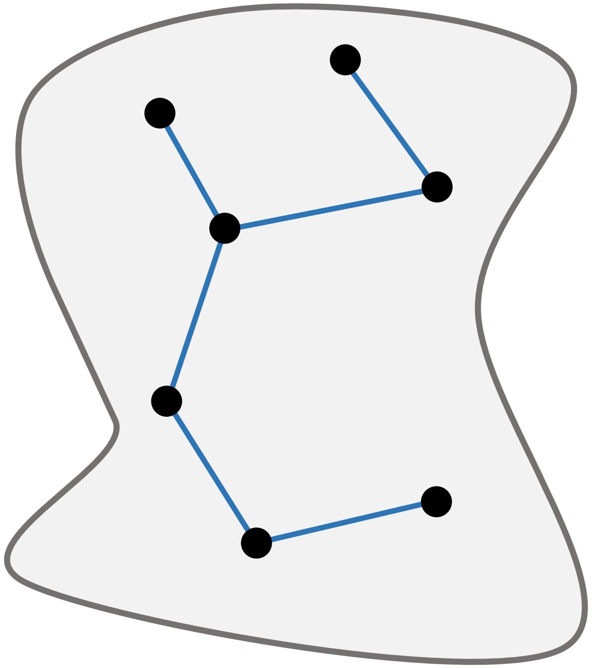
Christofides Algorithm:



Online-TSP
(metric)

Christofides Algorithm:

(1) minimal spanning tree

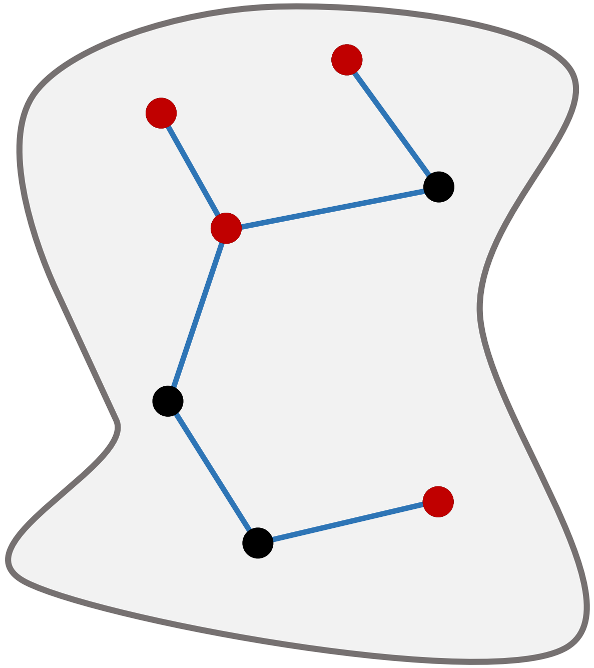


Online-TSP

(metric)

Christofides Algorithm:

(1) minimal spanning tree



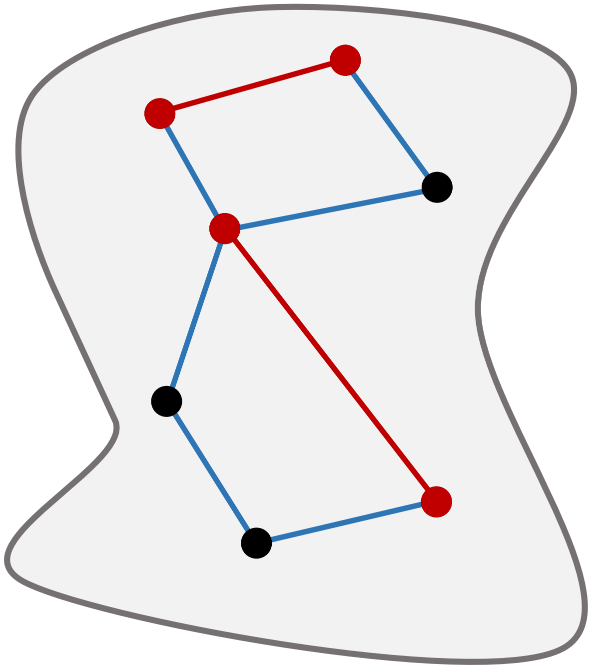
Online-TSP

(metric)

Christofides Algorithm:

(1) minimal spanning tree

(2) minimum weighted perfect matching of odd vertices



Online-TSP

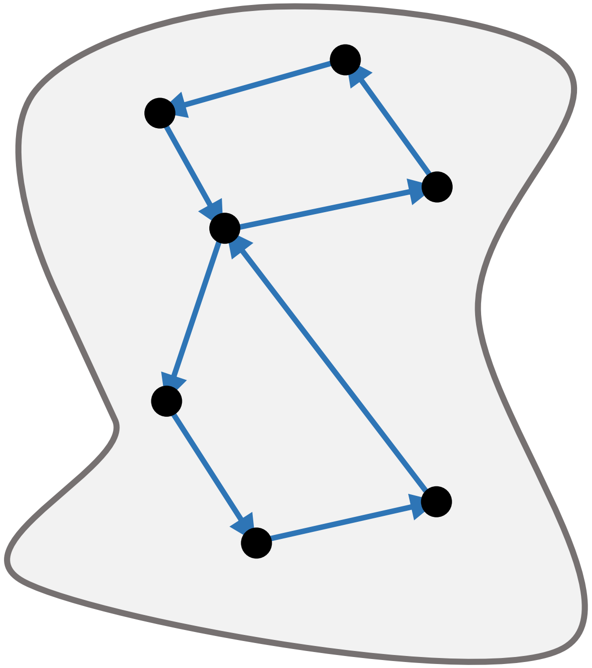
(metric)

Christofides Algorithm:

(1) minimal spanning tree

(2) minimum weighted perfect matching of odd vertices

(3) Euler tour



Online-TSP

(metric)

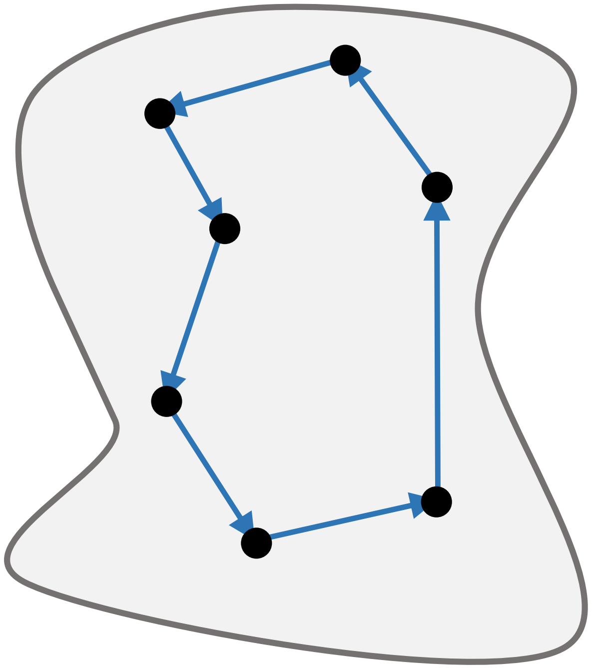
Christofides Algorithm:

(1) minimal spanning tree

(2) minimum weighted perfect matching of odd vertices

(3) Euler tour

(4) Skip double visited vertices



Online-TSP

(metric)

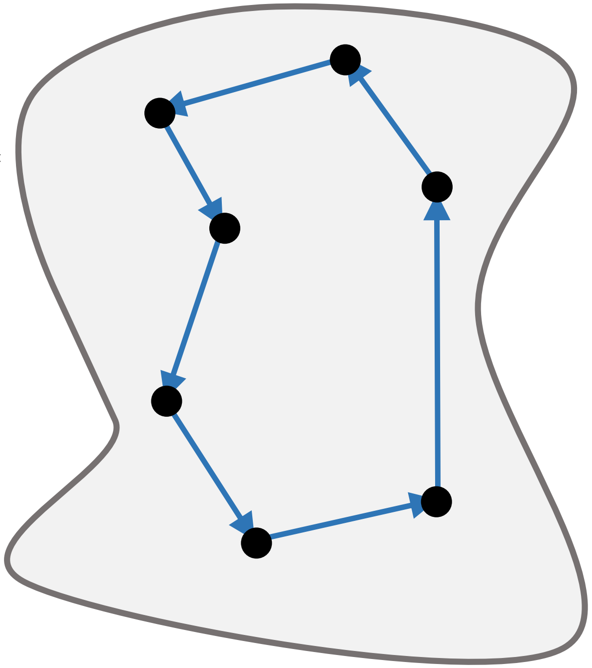
Christofides Algorithm:

(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

(2) minimum weighted perfect matching of odd vertices

(3) Euler tour

(4) Skip double visited vertices



Online-TSP

(metric)

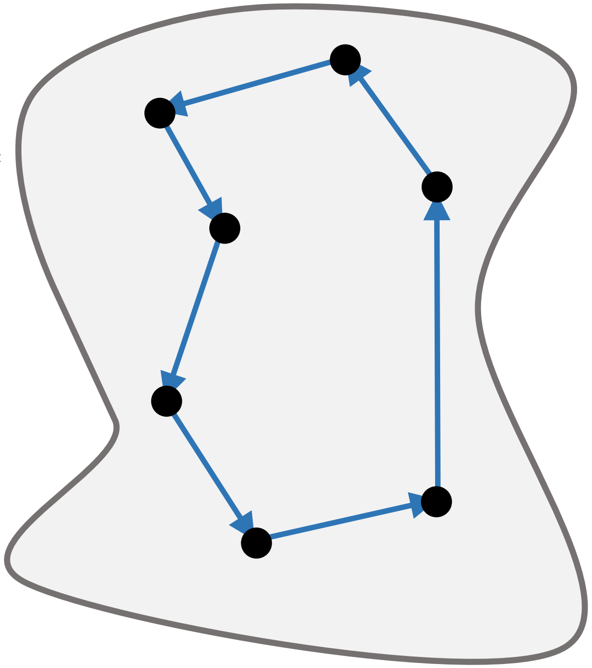
Christofides Algorithm:

(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

(2) minimum weighted perfect matching of odd vertices $\leq \frac{1}{2} \cdot \mathcal{T}^*$

(3) Euler tour

(4) Skip double visited vertices



Online-TSP

(metric)

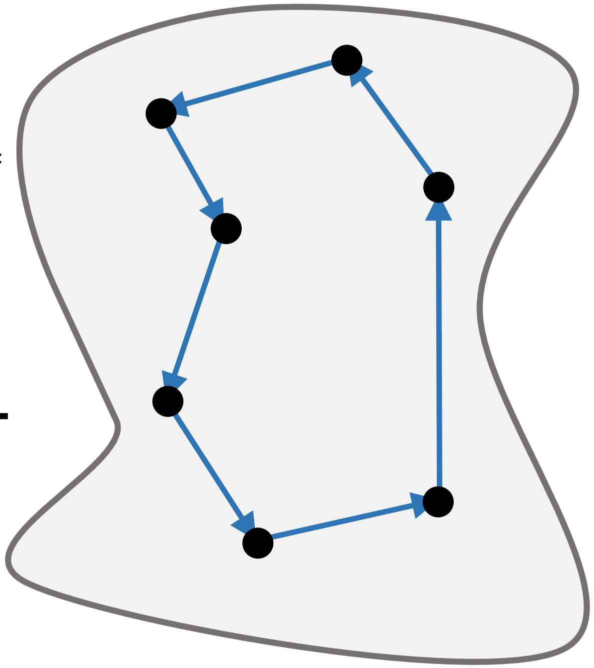
Christofides Algorithm:

(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

(2) minimum weighted perfect matching of odd vertices $\leq \frac{1}{2} \cdot \mathcal{T}^*$

(3) Euler tour $\leq \frac{3}{2} \cdot \mathcal{T}^*$

(4) Skip double visited vertices



Online-TSP

(metric)

Christofides Algorithm:

(1) minimal spanning tree $\leq 1 \cdot \mathcal{T}^*$

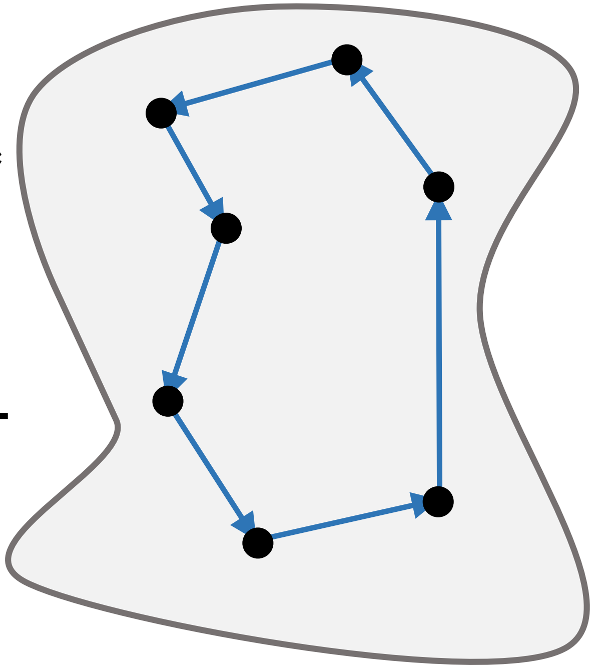
(2) minimum weighted perfect matching of odd vertices $\leq \frac{1}{2} \cdot \mathcal{T}^*$

(3) Euler tour $\leq \frac{3}{2} \cdot \mathcal{T}^*$

(4) Skip double visited vertices

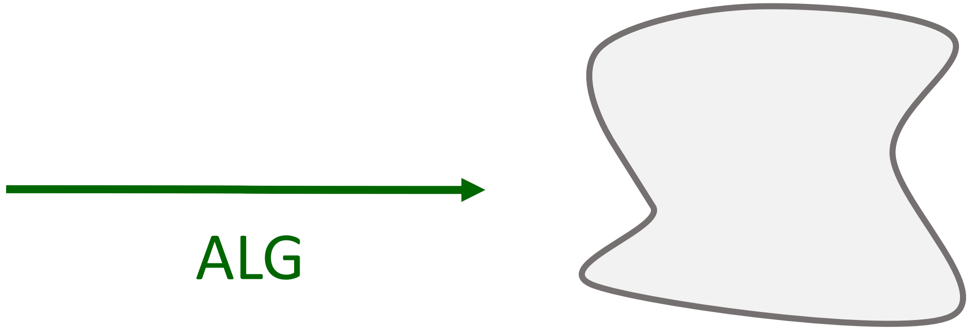
$\Downarrow$

1,5-approximative solution



Online-TSP

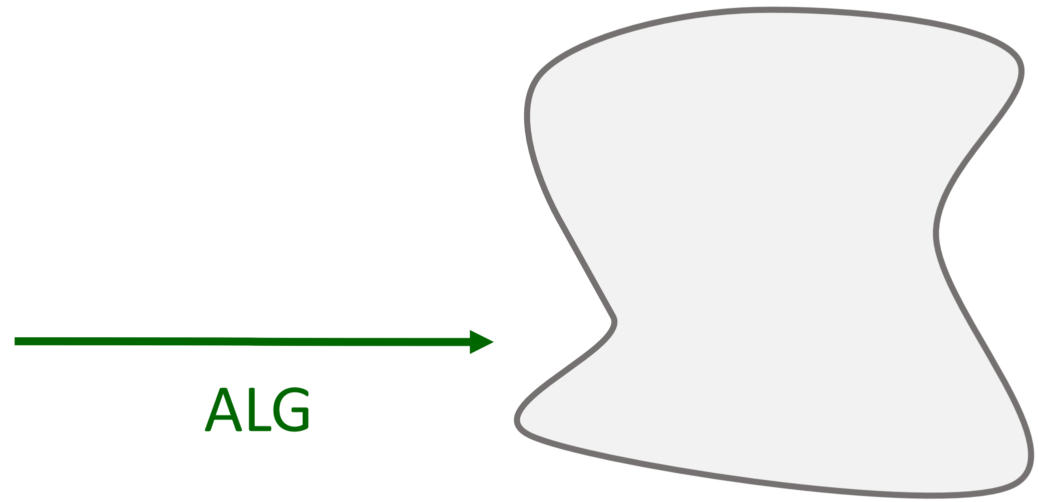
INPUT:



Online-TSP

INPUT:

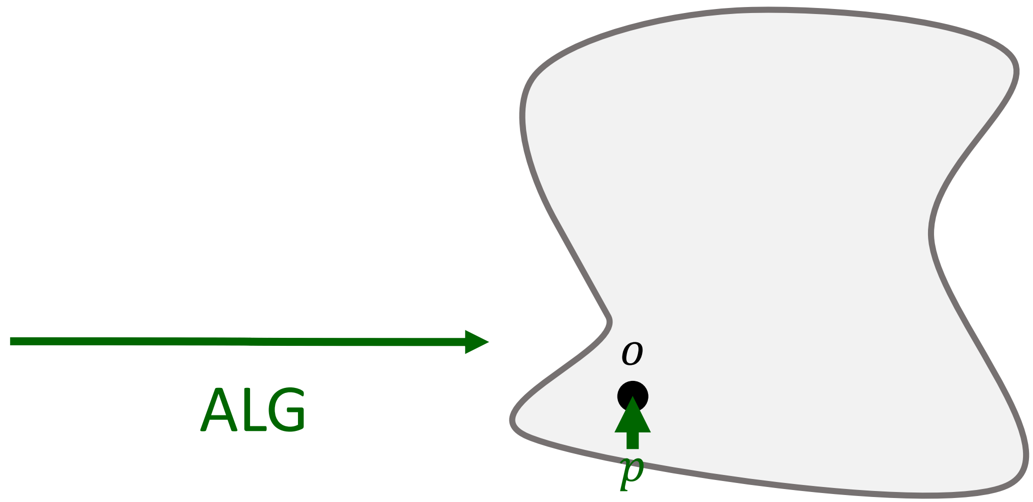
- metric space



Online-TSP

INPUT:

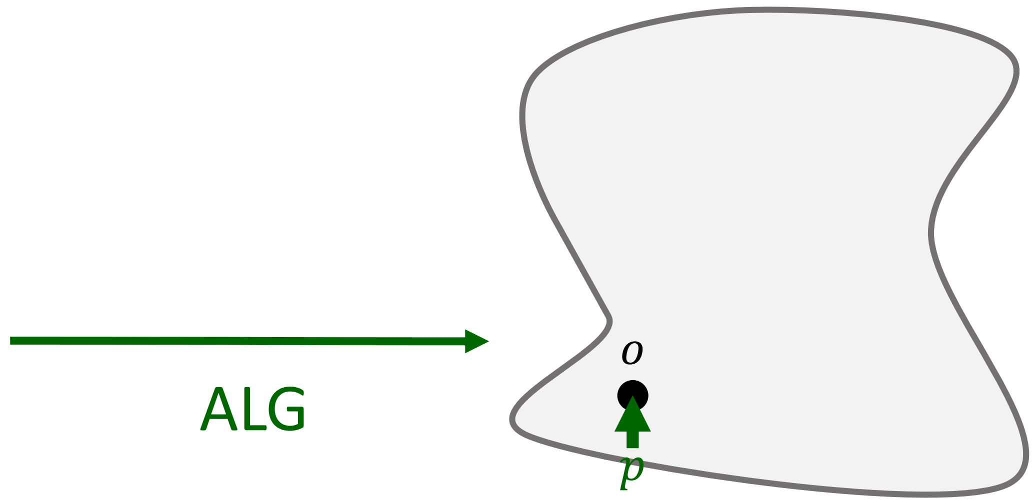
- metric space
- starting-point: o



Online-TSP

INPUT:

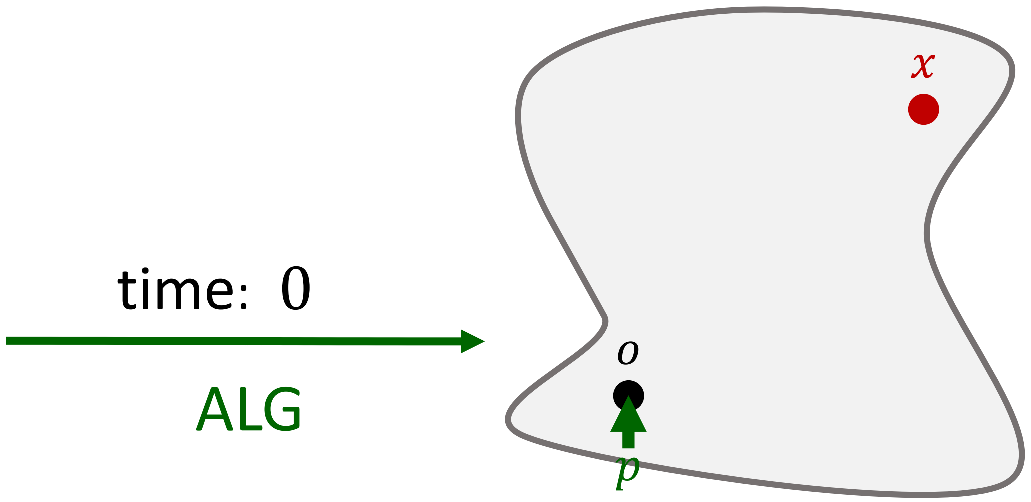
- metric space
- starting-point: o
- request-sequence σ:



Online-TSP

INPUT:

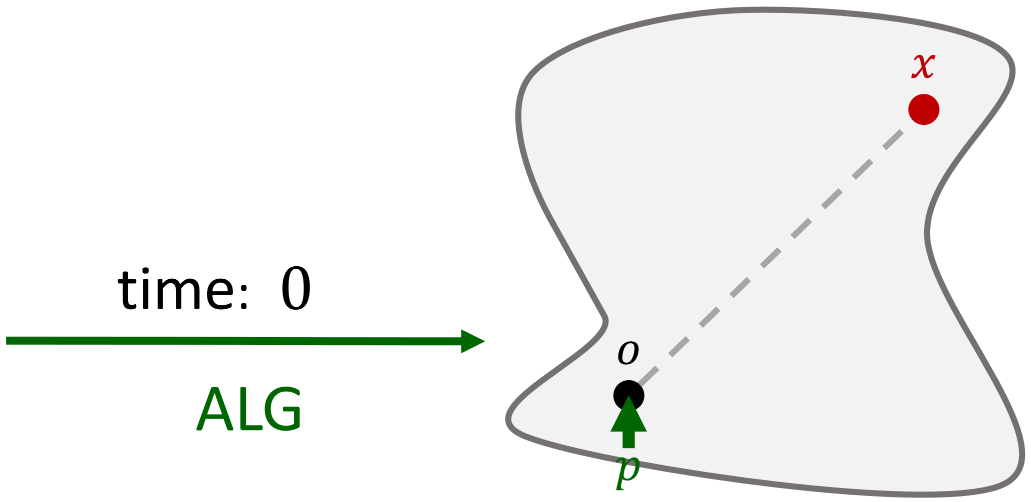
- metric space
- starting-point: o
- request-sequence σ: $(0, x)$



Online-TSP

INPUT:

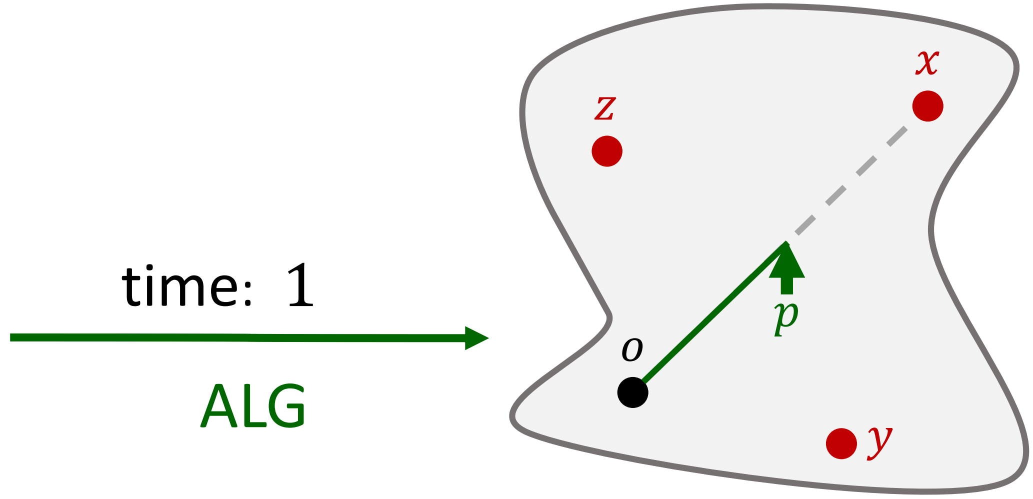
- metric space
- starting-point: o
- request-sequence σ:
  $(0, x)$



Online-TSP

INPUT:

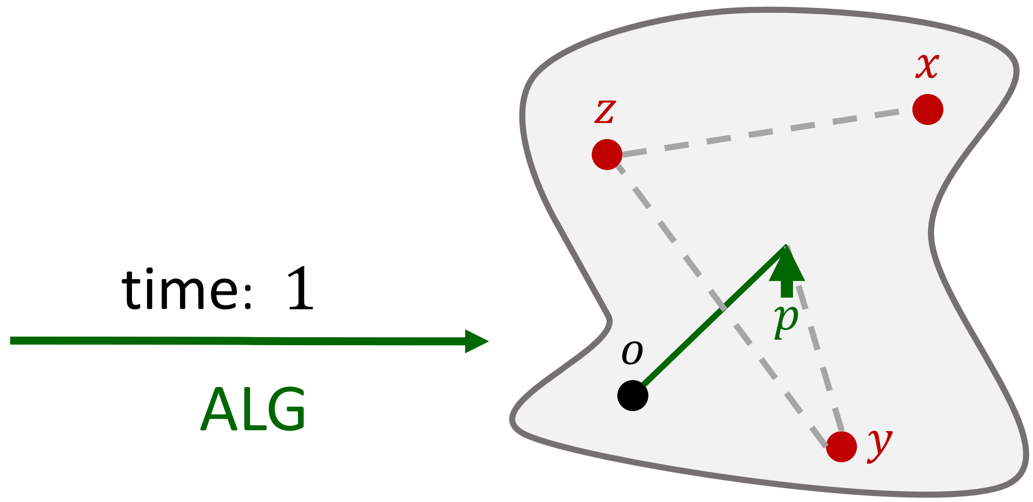
- metric space
- starting-point: o
- request-sequence σ: $(0, x), (1, y), (1, z)$



Online-TSP

INPUT:

- metric space
- starting-point: o
- request-sequence σ: $(0, x), (1, y), (1, z)$



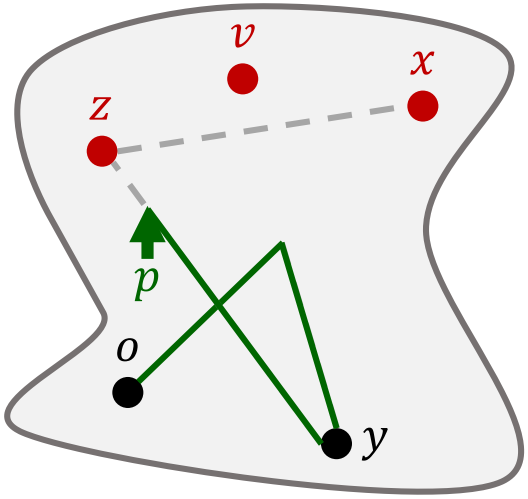
Online-TSP

INPUT:

- metric space
- starting-point: o
- request-sequence σ: $(0, x), (1, y), (1, z), \ldots$

time: 3,5

ALG



Online-TSP

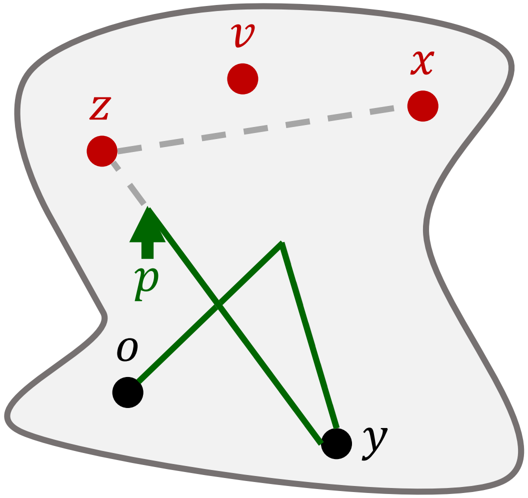
INPUT:

- metric space
- starting-point: o
- request-sequence σ: $(0, x), (1, y), (1, z), \ldots$

N-OLTSP
"nomadic"

time: 3,5

ALG



Online-TSP

INPUT:

- metric space
- starting-point: o
- request-sequence σ:
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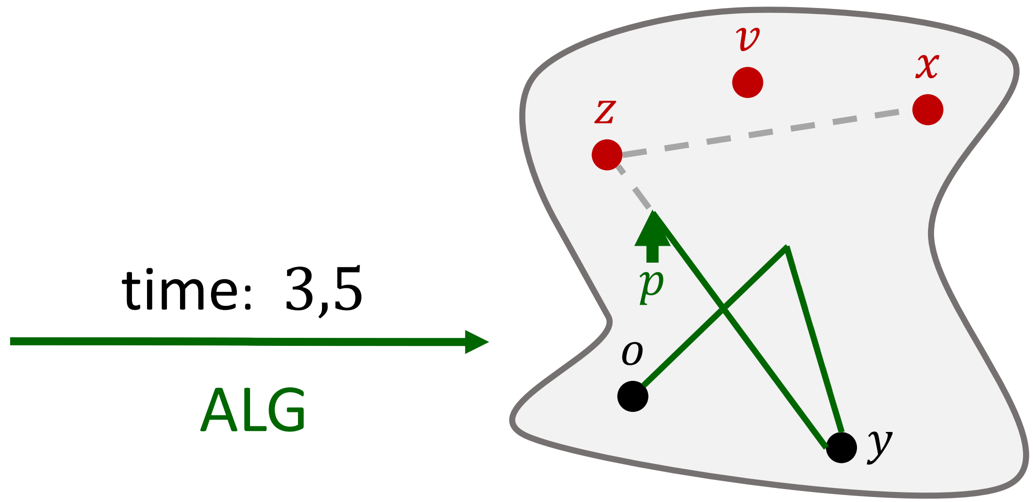
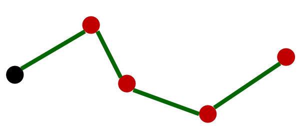


N-OLTSP

"nomadic"



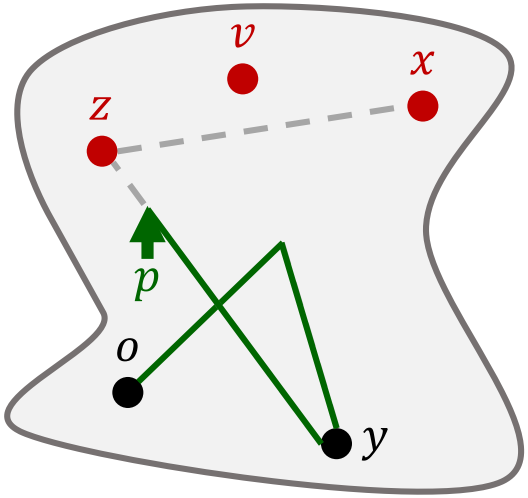
Online-TSP

INPUT:

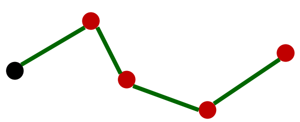
- metric space
- starting-point: o
- request-sequence σ:
  $(0, x), (1, y), (1, z), \ldots$

time: $3{,}5$

ALG



N-OLTSP
"nomadic"



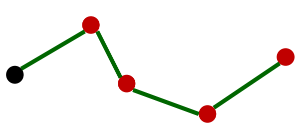
H-OLTSP
"homing"

Online-TSP

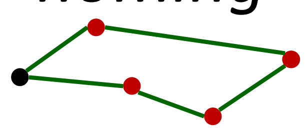
INPUT:

- metric space
- starting-point: o
- request-sequence σ: $(0, x), (1, y), (1, z),$ ...

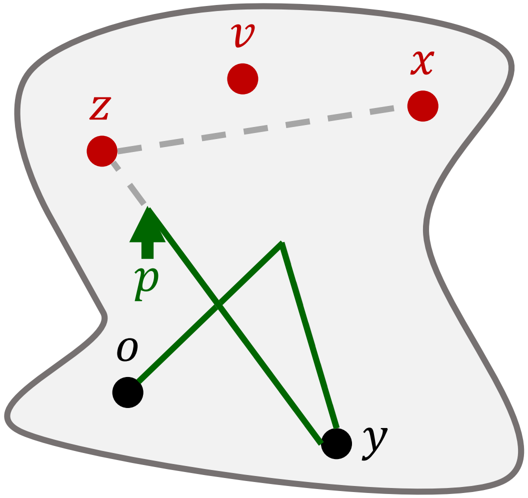
N-OLTSP *"nomadic"*



H-OLTSP *"homing"*



$\xrightarrow[\text{ALG}]{\text{time: } 3{,}5}$



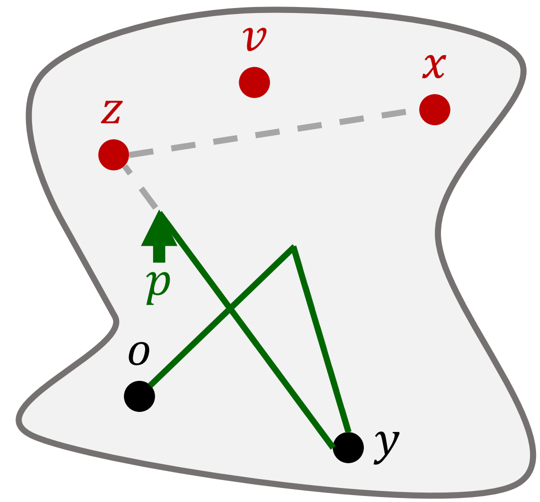
Online-TSP

INPUT:

- metric space
- starting-point: o
- request-sequence σ: $(0, x), (1, y), (1, z), \ldots$

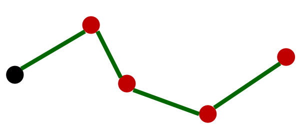
time: 3,5

ALG



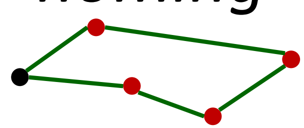
N-OLTSP

"nomadic"



H-OLTSP

"homing"



DEF: ALG is *ρ-competitive*

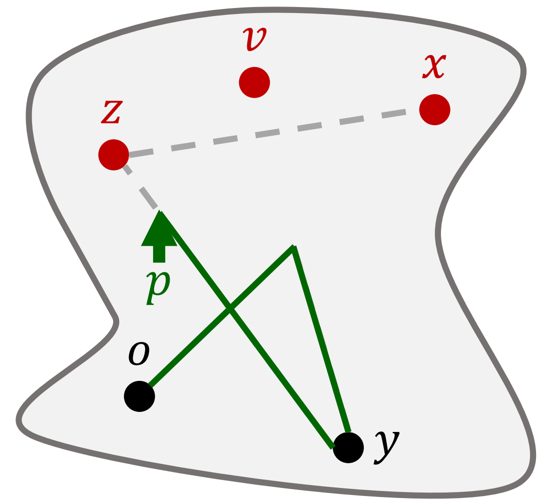
Online-TSP

INPUT:

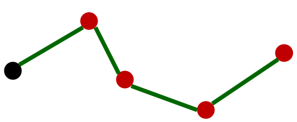
- metric space
- starting-point: o
- request-sequence σ: $(0, x), (1, y), (1, z), \ldots$

time: 3,5

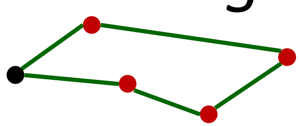
ALG



N-OLTSP *"nomadic"*



H-OLTSP *"homing"*



DEF: ALG is *ρ-competitive*

$$\Leftrightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| \leq \rho \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

for all request- sequences

Goals

Goals

I. Find online-algorithms

Goals

I. Find online-algorithms (superpolynomial)

Goals

I. Find online-algorithms (superpolynomial)

II. Find lower bounds

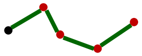
Goals

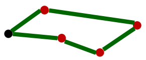
I. Find online-algorithms (superpolynomial)

II. Find lower bounds

III. Find *polynomial* online-algorithms

An algorithm for N-OLSTP

N-OLTSP 

H-OLTSP 

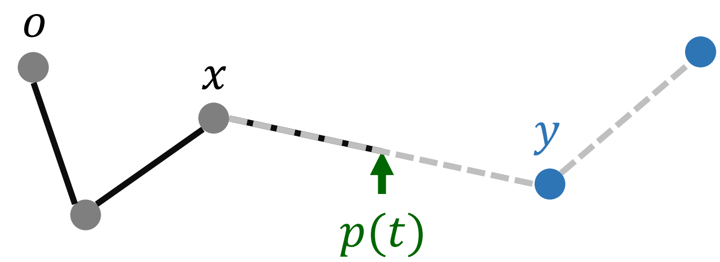
An algorithm for N-OLSTP

Invariant: always on shortest path between points in S

$S :=$ places requested until t

An algorithm for N-OLSTP

Invariant: always on shortest path between points in S

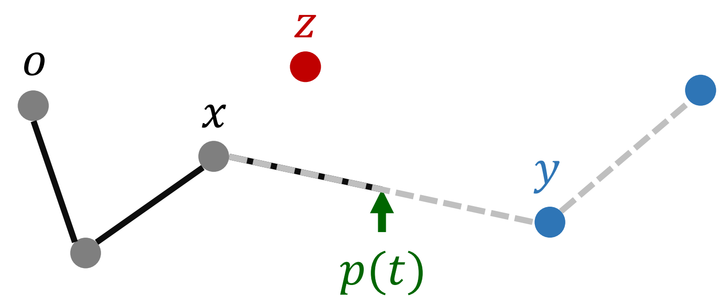


$S :=$ places requested until t

An algorithm for N-OLSTP

Invariant: always on shortest path between points in S

(1) New request (t, z) at time t and ALG between x and y

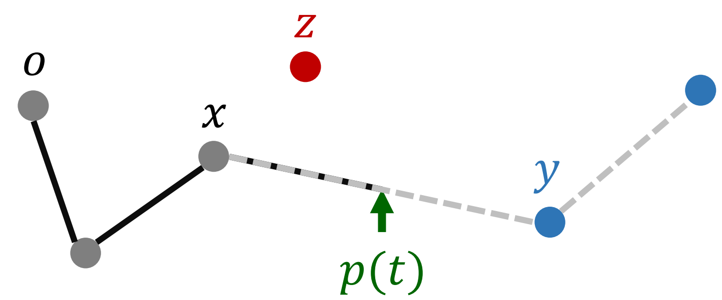


$S :=$ places requested until t

An algorithm for N-OLSTP

Invariant: always on shortest path between points in S

(1) Add z to U



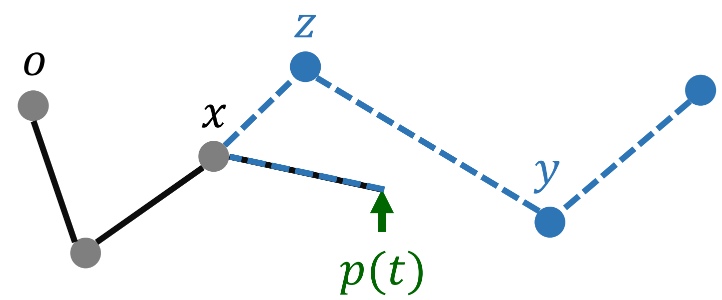
$S :=$ places requested until t
$S \supseteq U :=$ places yet to visit at t

An algorithm for N-OLSTP

Invariant*:* always on shortest path between points in S

(1) Add z to U

(2) Follow shortest path through $\mathcal{U}$ beginning with x or y



S $:=$ places requested until t

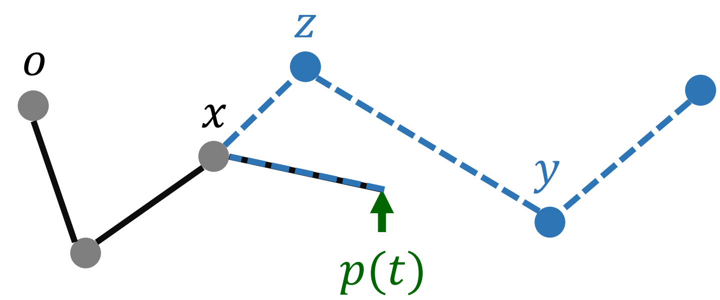
$S \supseteq U$ $:=$ places yet to visit at t

An algorithm for N-OLSTP

Invariant: always on shortest path between points in S

(1) Add z to U

(2) Follow shortest path through $\mathcal{U}$ beginning with x or y



$S :=$ places requested until t

$S \supseteq U :=$ places yet to visit at t

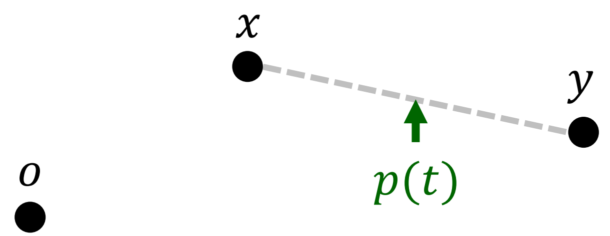
Greedily Travelling between Requests (GTR)

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) New request (t, z) at time t and ALG between x and y

(3) Follow shortest path through U beginning with x or y

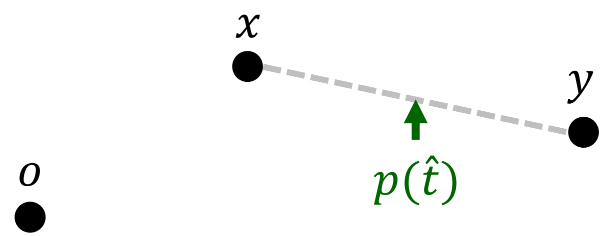


Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y

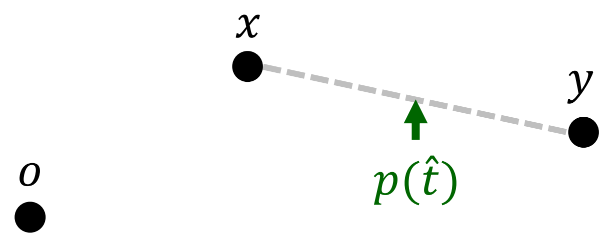


Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$
and ALG between x and y

(3) Follow shortest path through U
beginning with x or y



path found by GTR

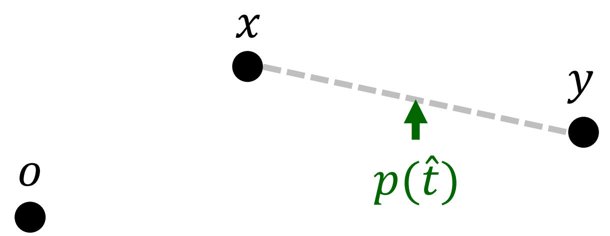
$$\left|\mathcal{T}^{\mathrm{GTR}}\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$
and ALG between x and y

(3) Follow shortest path through U
beginning with x or y



path found by GTR

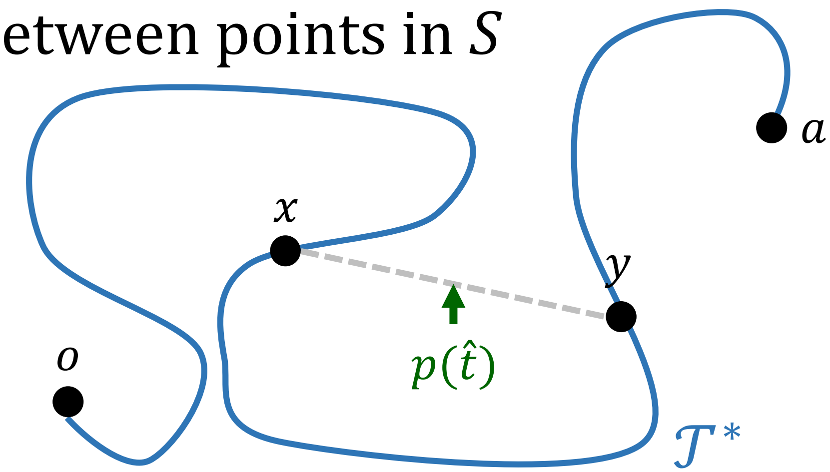
$$\left|\mathcal{T}^{\text{GTR}}\right| \leq \hat{t}$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

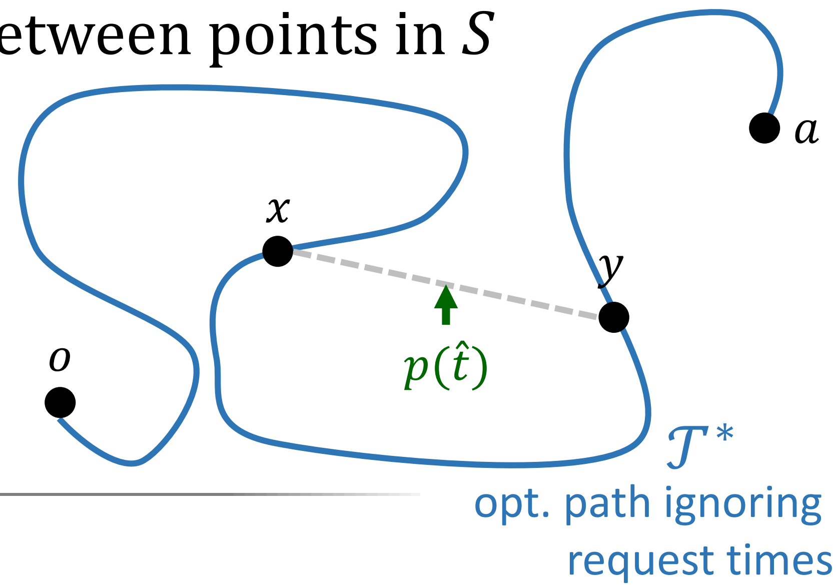
$$\left|\mathcal{T}^{\text{GTR}}\right| \leq \hat{t}$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



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path found by GTR

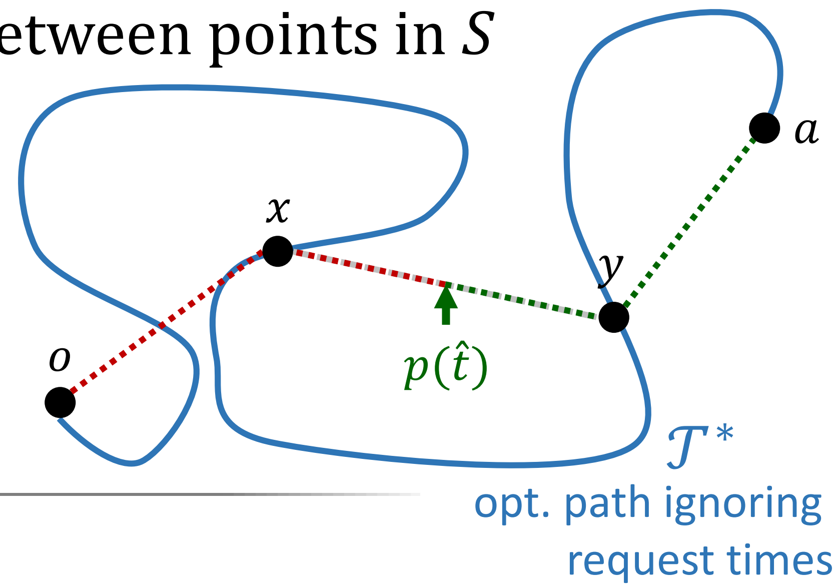
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} \qquad + \left|\mathcal{T}^*\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

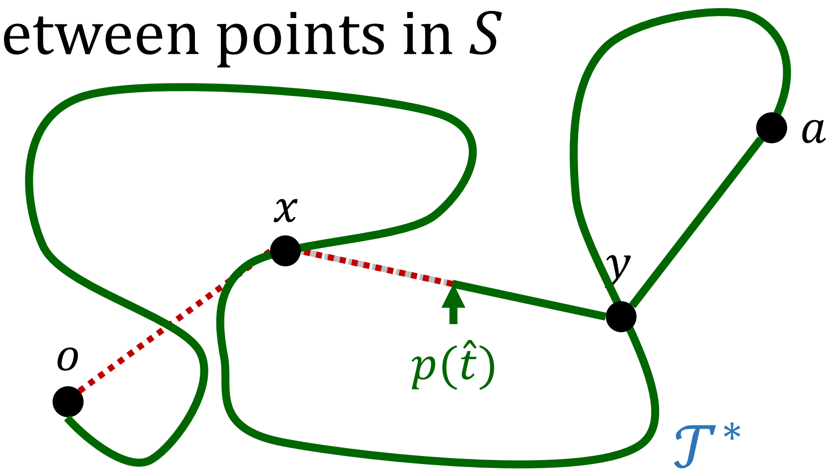
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



opt. path ignoring request times

path found by GTR

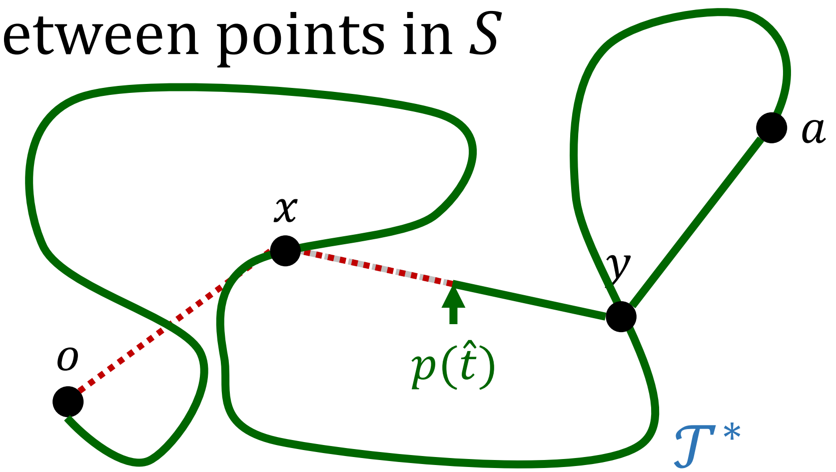
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$
$$\leq \hat{t} + \tfrac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



opt. path ignoring request times

path found by GTR

$$|\mathcal{T}^{\mathrm{GTR}}| \leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

path found by opt. offline-ALG

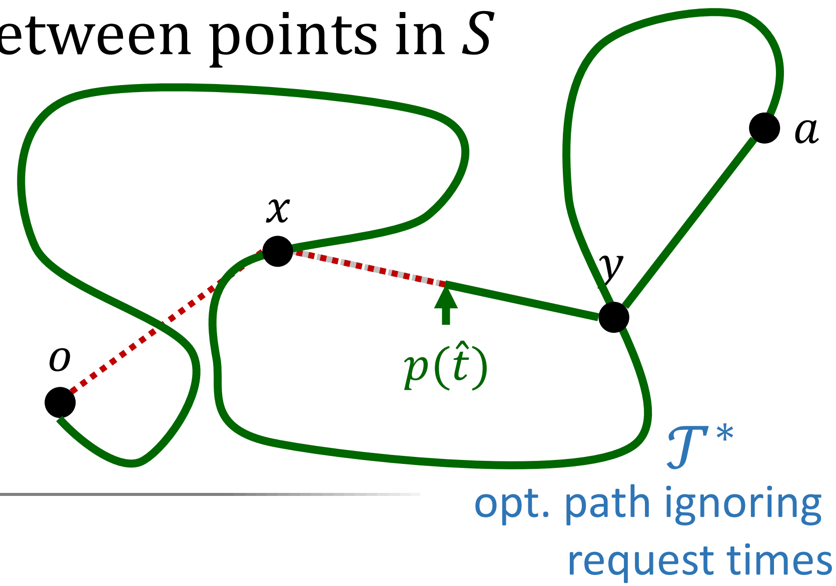
$$\leq |\mathcal{T}^{\mathrm{OPT}}| +$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

$$|\mathcal{T}^{\mathrm{GTR}}| \le \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*|$$

$$\le \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*|$$

path found by opt. offline-ALG

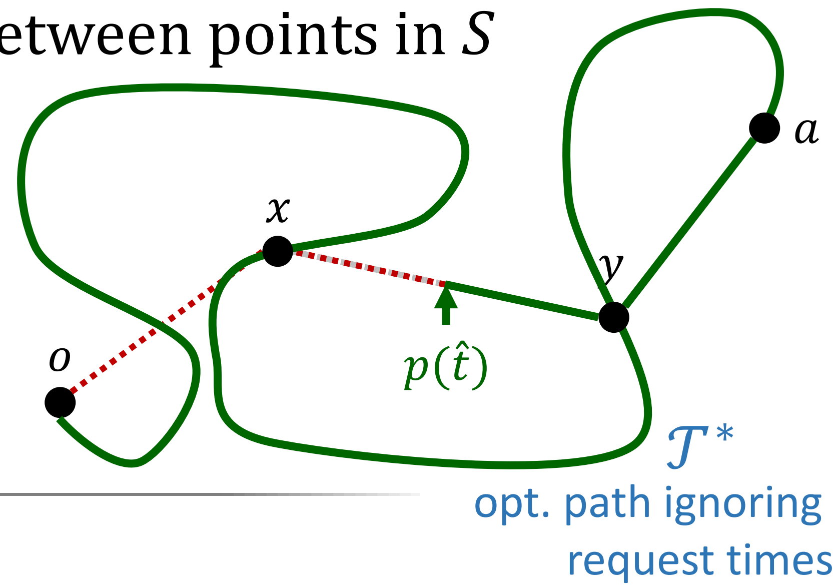
$$\le |\mathcal{T}^{\mathrm{OPT}}| + \frac{3}{2} \cdot |\mathcal{T}^{\mathrm{OPT}}|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



path found by GTR

path found by opt. offline-ALG

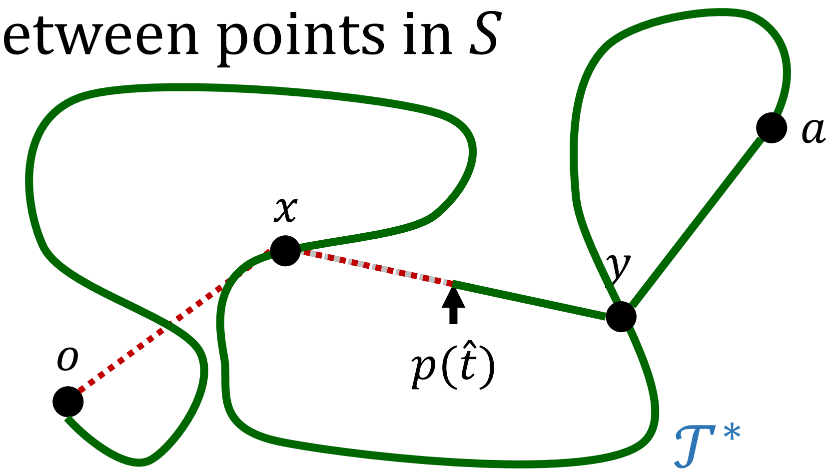
$$
\begin{aligned}
\left|\mathcal{T}^{\mathrm{GTR}}\right| &\leq \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,a)\} + |\mathcal{T}^*| \\
&\leq \hat{t} + \frac{1}{2} \cdot |\mathcal{T}^*| + |\mathcal{T}^*| \\
&\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| = \frac{5}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|
\end{aligned}
$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



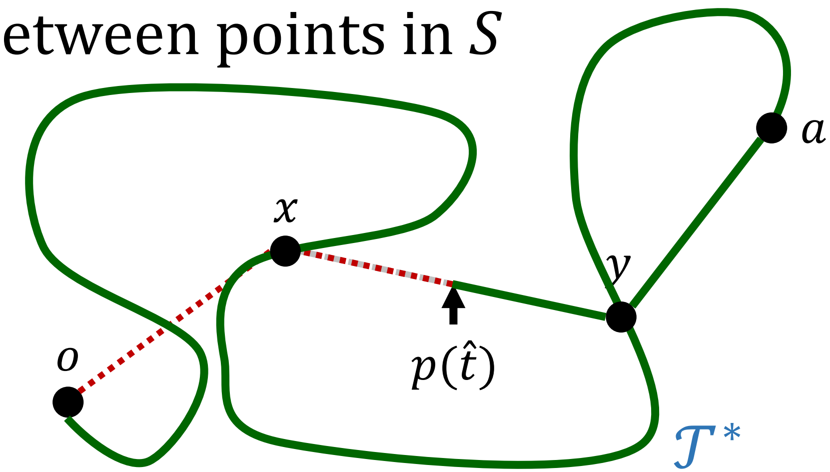
$$\left|\mathcal{T}^{\mathrm{GTR}}\right| \leq \left|\mathcal{T}^{\mathrm{OPT}}\right| \quad + \quad \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \quad = \quad \frac{5}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y



THEOREM: GTR is 2,5-competitive for N-OLTSP

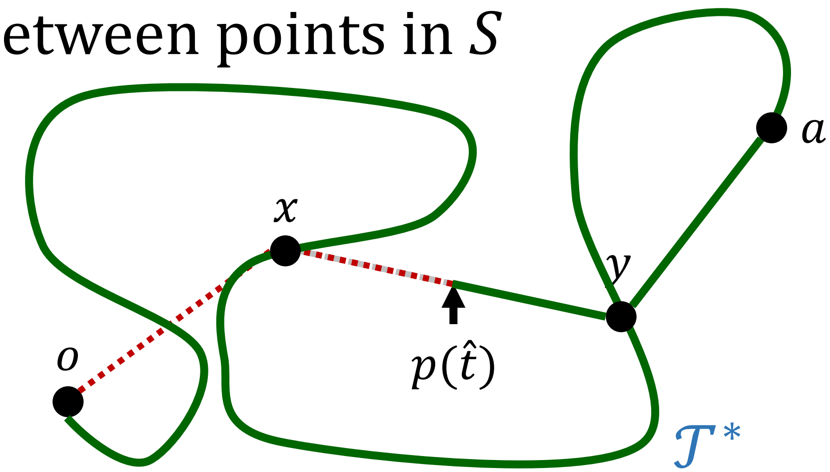
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$		
H-OLTSP			

Competitiveness of GTR

Invariant: always on shortest path between points in S

(1) Last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y

(3) Follow shortest path through U beginning with x or y

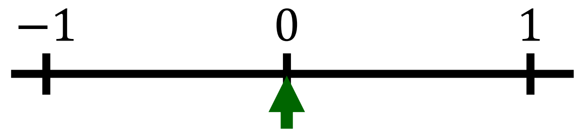


THEOREM: GTR is 2,5-competitive for N-OLTSP

REMARK: GTR is also 2,5-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	[redacted]	
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$		

Lower Bound for N-OLTSP

time, request	Online-ALG
0	

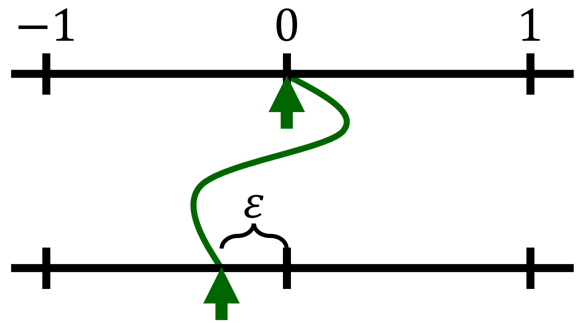
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	[redacted]	
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$		

Lower Bound for N-OLTSP

time, request | Online-ALG

0

1



N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	

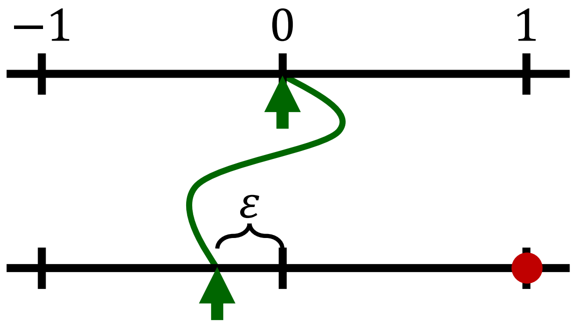
Lower Bound for N-OLTSP

time, request

Online-ALG

0

1, 1



N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	

Lower Bound for N-OLTSP

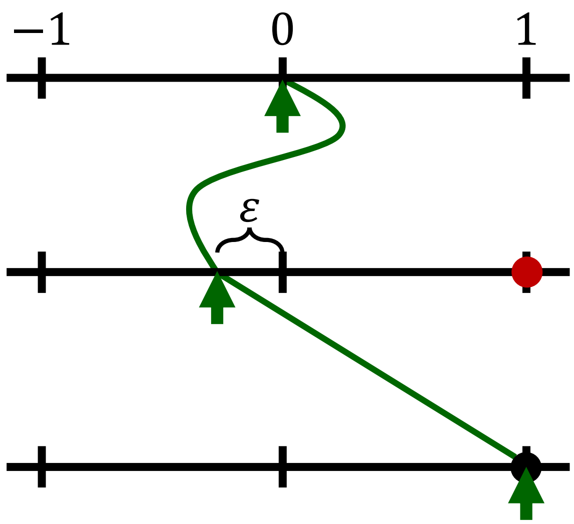
time, request

Online-ALG

0

$1, \quad 1$

$2 + \varepsilon$



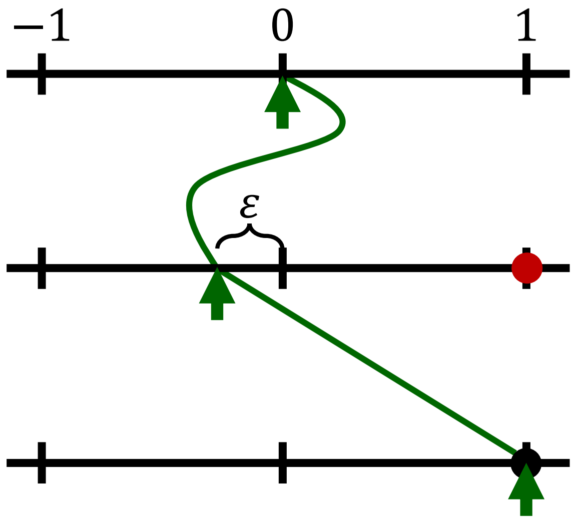
Lower Bound for N-OLTSP

time, request | Online-ALG

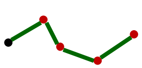
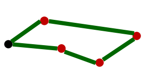
0

1, 1

$2 + \varepsilon$

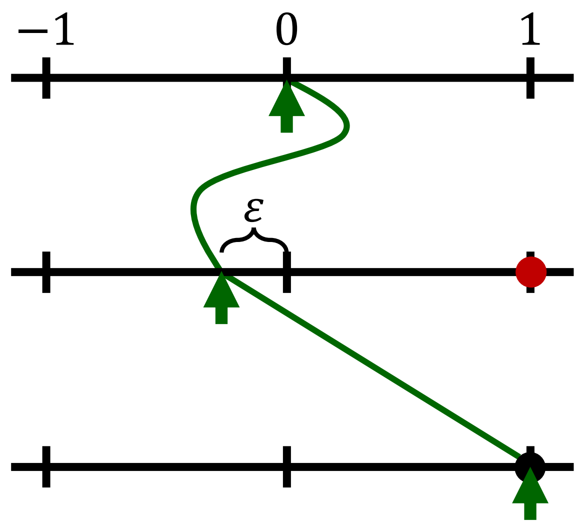


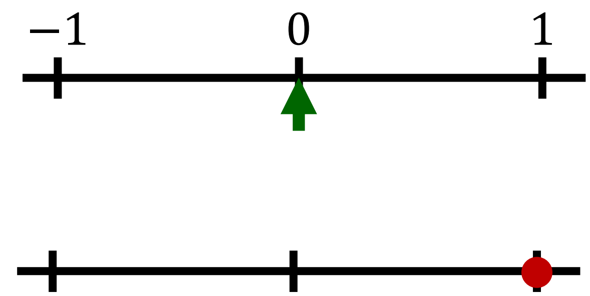
$$\Rightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| = 2 + \varepsilon$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$		
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$		

Lower Bound for N-OLTSP

time, request	Online-ALG	Opt. offline-ALG
0		
1, 1		
$2+\varepsilon$		

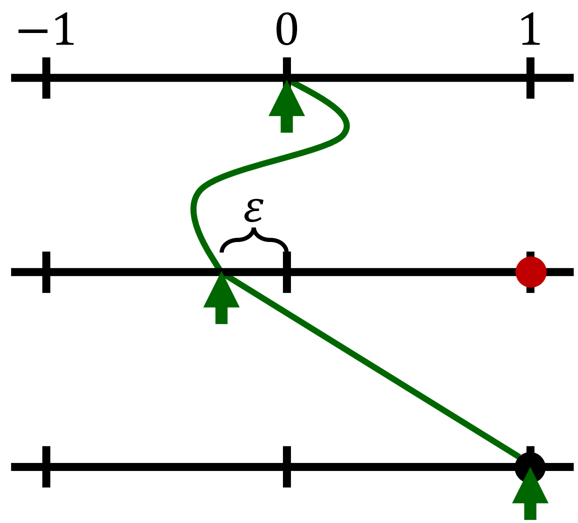


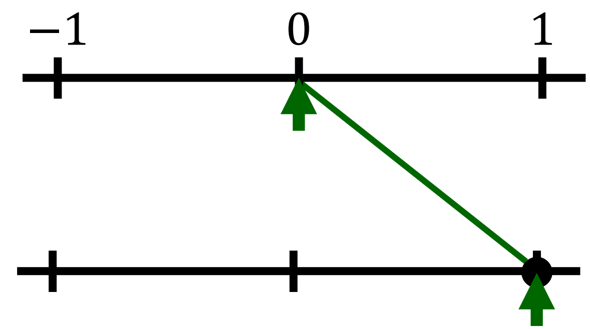


$$\Rightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| = 2+\varepsilon$$

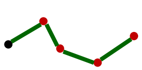
Lower Bound for N-OLTSP

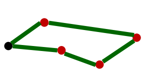
time, request	Online-ALG	Opt. offline-ALG
0		
$1, \quad 1$		
$2+\varepsilon$		





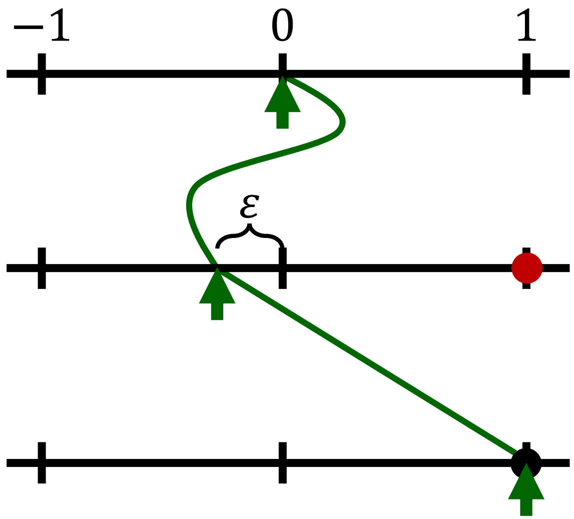
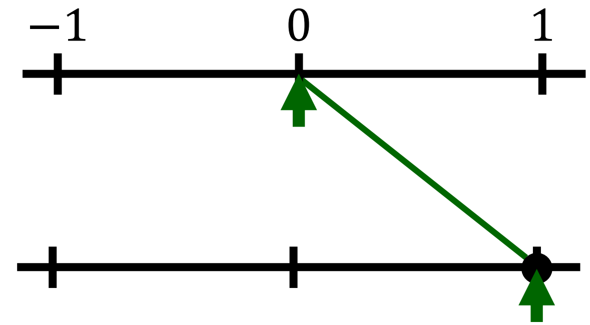
$$\Rightarrow \left|\mathcal{T}^{\text{ALG}}\right| = 2+\varepsilon$$

N-OLTSP  GTR (Greedy): $\rho = 2{,}5$

H-OLTSP  GTR (Greedy): $\rho = 2{,}5$

[redacted]

Lower Bound for N-OLTSP

time, request	Online-ALG	Opt. offline-ALG
0		
$1,\quad 1$		
$2+\varepsilon$		
	$\Rightarrow \left\vert \mathcal{T}^{\mathrm{ALG}} \right\vert = 2+\varepsilon$	$\Rightarrow \left\vert \mathcal{T}^{\mathrm{OPT}} \right\vert = 1$

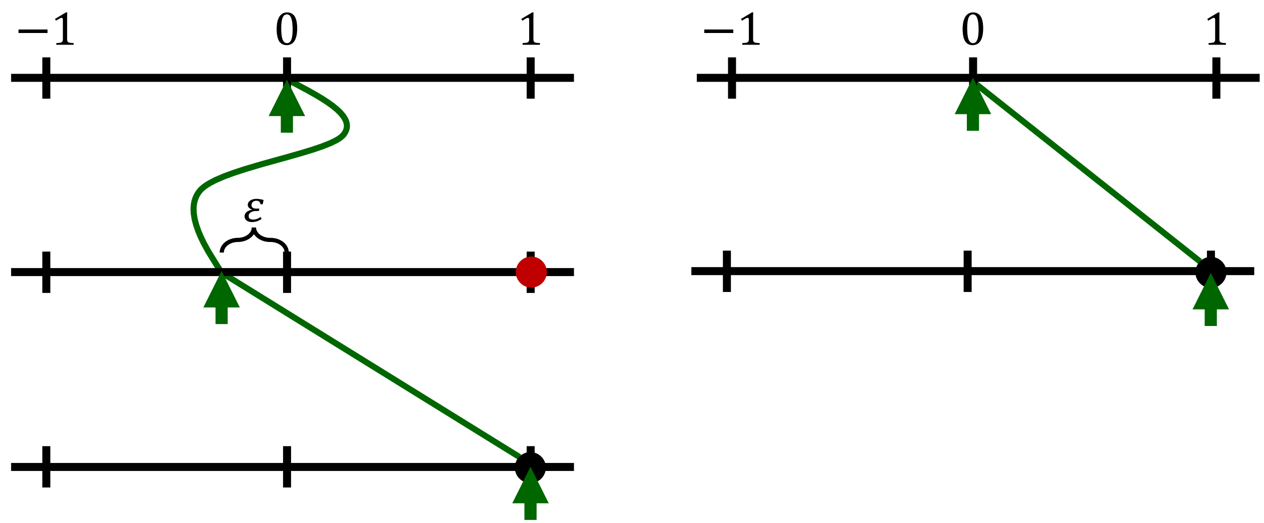
Lower Bound for N-OLTSP

time, request	Online-ALG	Opt. offline-ALG

0

$1,\quad 1$

$2 + \varepsilon$



$\Rightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| = 2 + \varepsilon$ $\qquad$ $\Rightarrow \left|\mathcal{T}^{\mathrm{OPT}}\right| = 1$

THEOREM: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$		

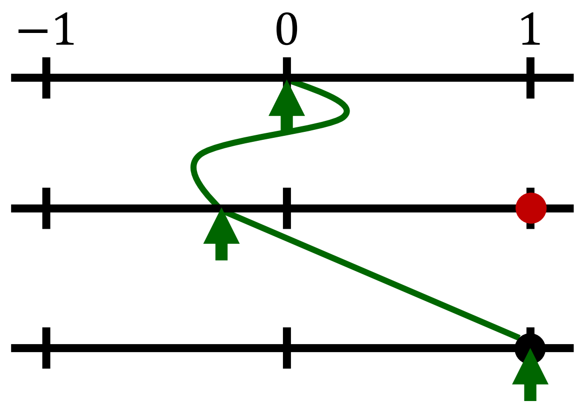
Lower Bound for H-OLTSP

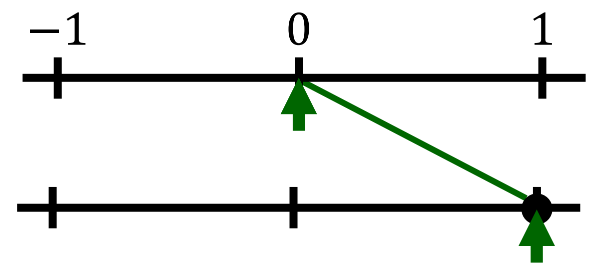
time, request	Online-ALG	Opt. offline-ALG

0

$1,\quad 1$

$2 + \varepsilon$





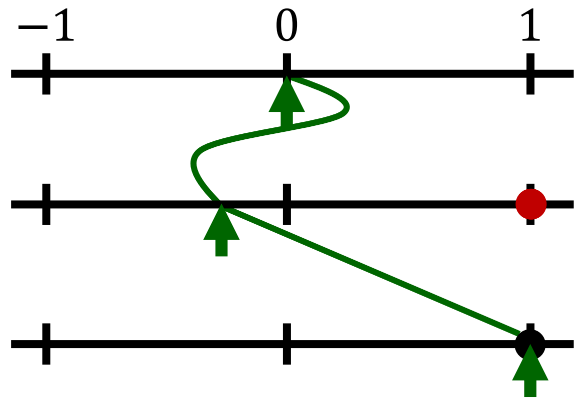
$\Rightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| = 2 + \varepsilon$ $\qquad$ $\Rightarrow \left|\mathcal{T}^{\mathrm{OPT}}\right| = 1$

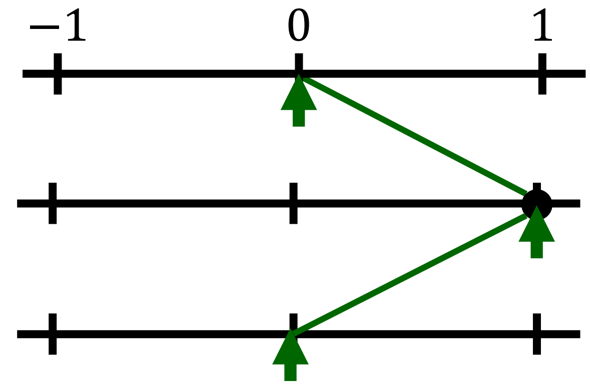
THEOREM: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$		

Lower Bound for H-OLTSP

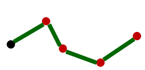
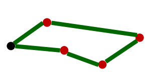
time, request	Online-ALG	Opt. offline-ALG
0		
$1, \quad 1$		
$2+\varepsilon$		





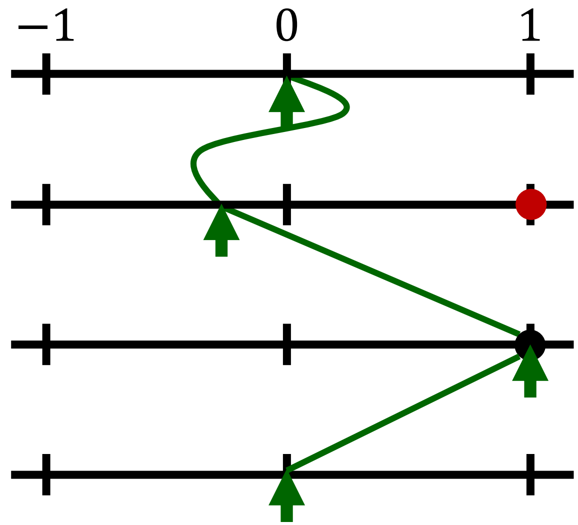
$\Rightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| = 2+\varepsilon$ $\qquad\qquad$ $\Rightarrow \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2$

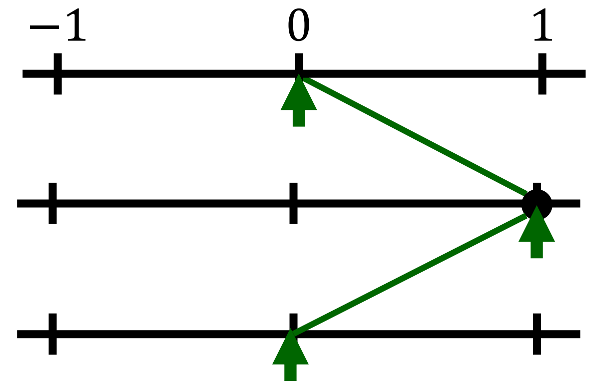
THEOREM: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$		

Lower Bound for H-OLTSP

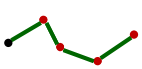
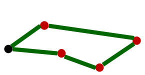
time, request		Online-ALG	Opt. offline-ALG
0			
$1,$	1		
$2+\varepsilon$			
$3+\varepsilon$			





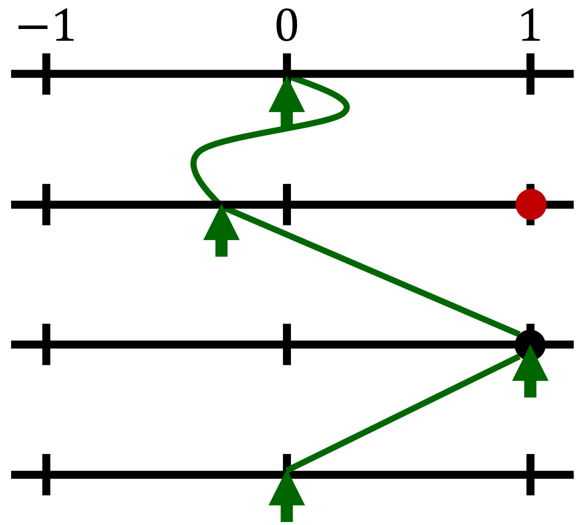
$\Rightarrow \left|\mathcal{T}^{\mathrm{ALG}}\right| = 3 + \varepsilon$ $\qquad$ $\Rightarrow \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2$

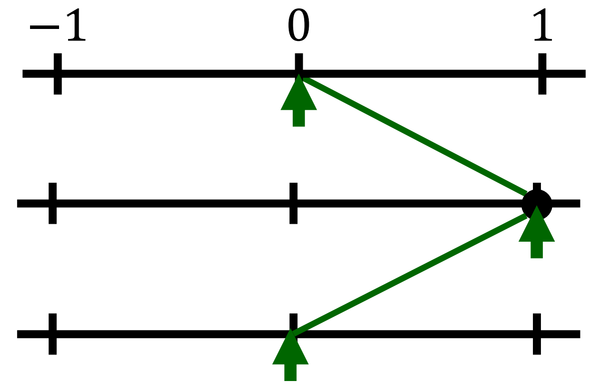
Theorem: Any ρ-competitive ALG for N-OLTSP has $\rho \geq 2$.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$		

Lower Bound for H-OLTSP

time, request	Online-ALG	Opt. offline-ALG
0		
1, 1		
$2+\varepsilon$		
$3+\varepsilon$		





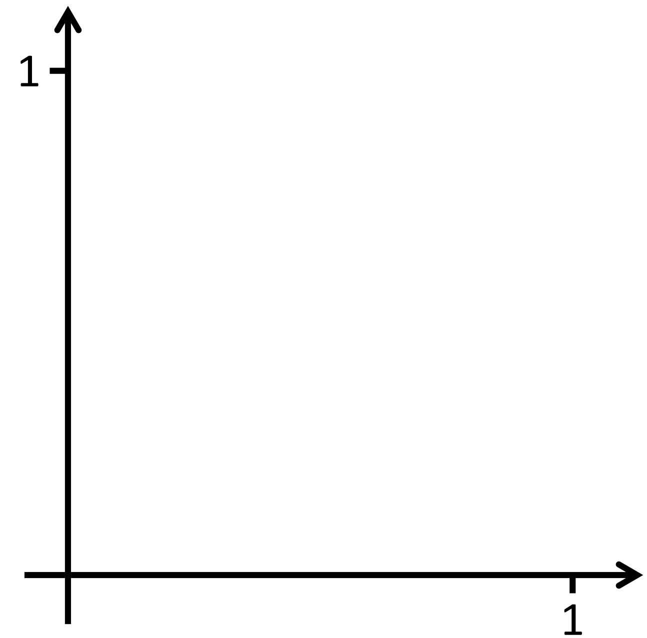
$\Rightarrow |\mathcal{T}^{\mathrm{ALG}}| = 3+\varepsilon$

$\Rightarrow |\mathcal{T}^{\mathrm{OPT}}| = 2$

THEOREM: Any ρ-competitive ALG for H-OLTSP has $\rho \geq 1{,}5$.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 1{,}5$	[redacted]

Lower Bound for H-OLTSP



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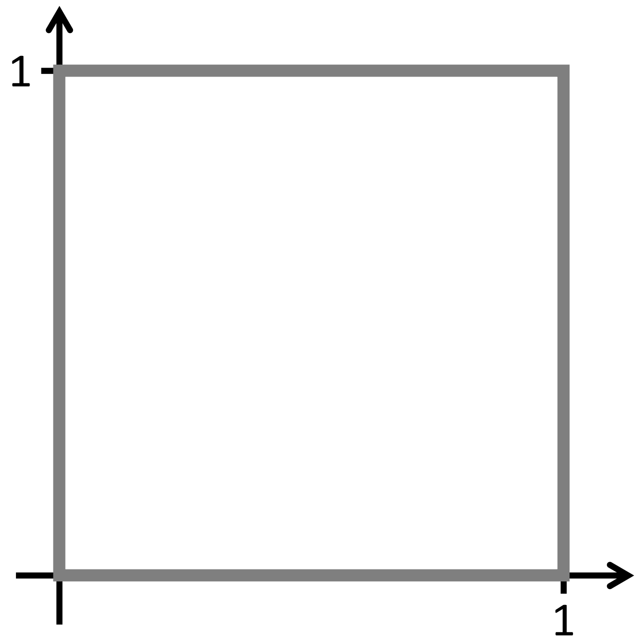

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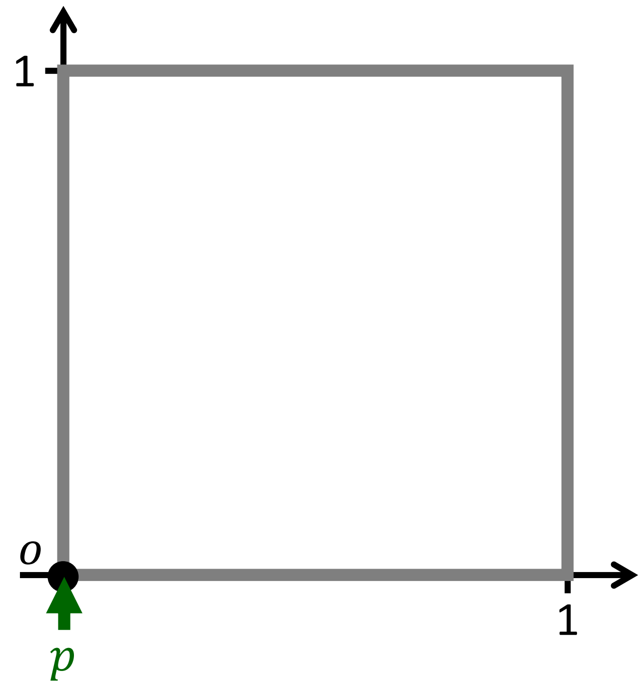

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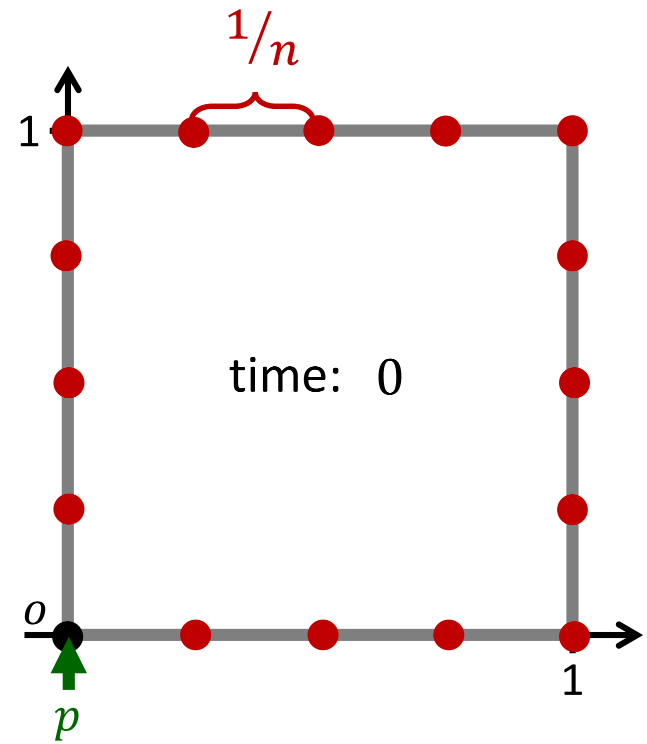
Lower Bound for H-OLTSP

- requests at time 0



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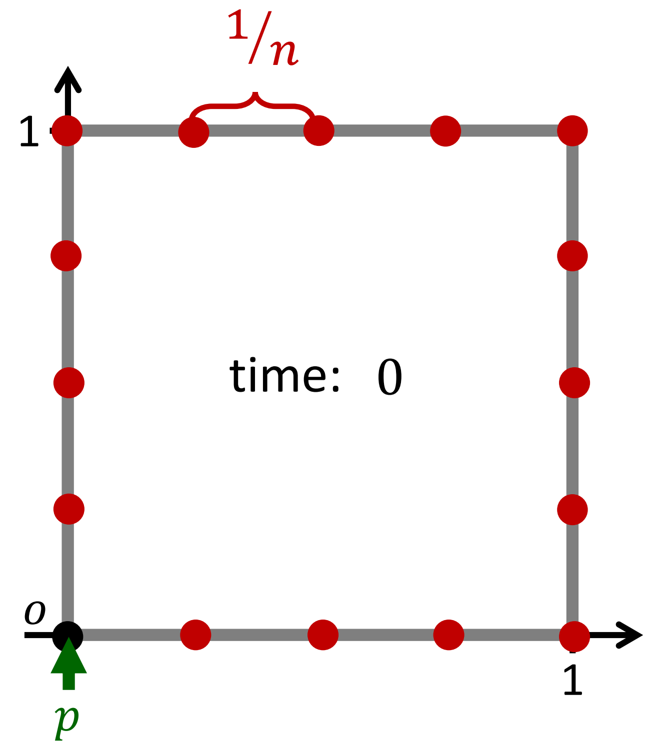
Lower Bound for H-OLTSP

- requests at time 0
- wait until time $\mathrm{t} = 2$



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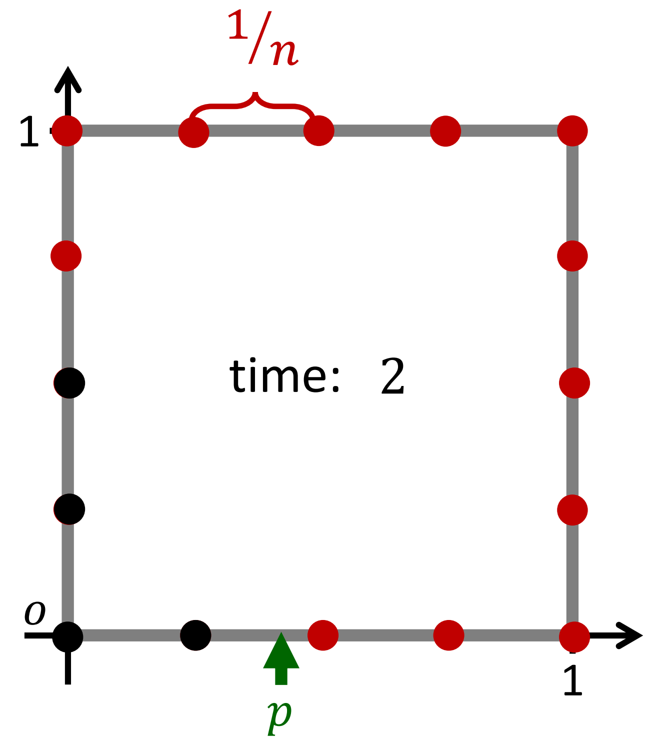

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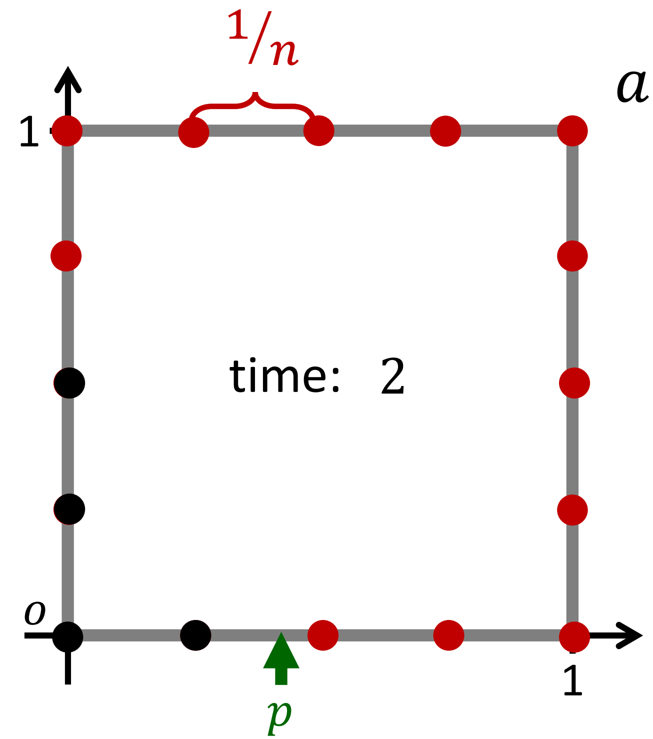

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Lower Bound for H-OLTSP

- requests at time 0
- wait until time t = 2
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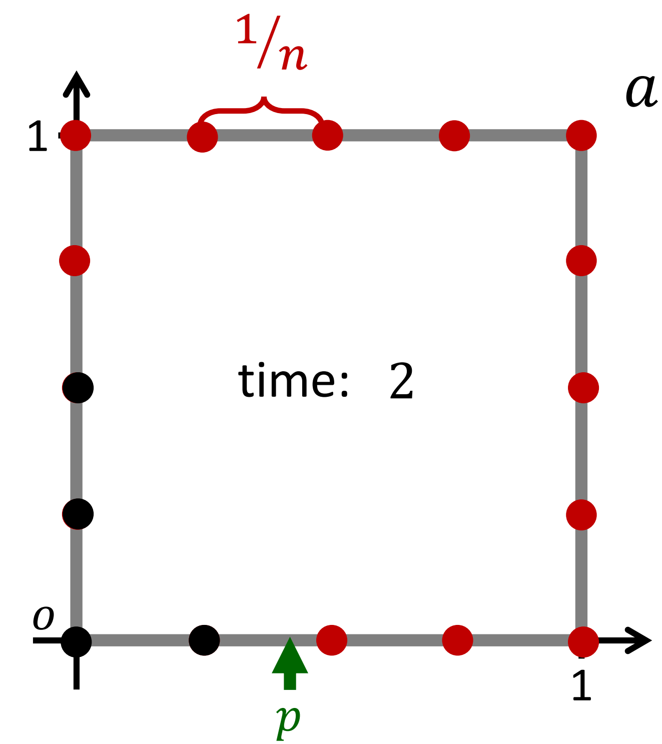

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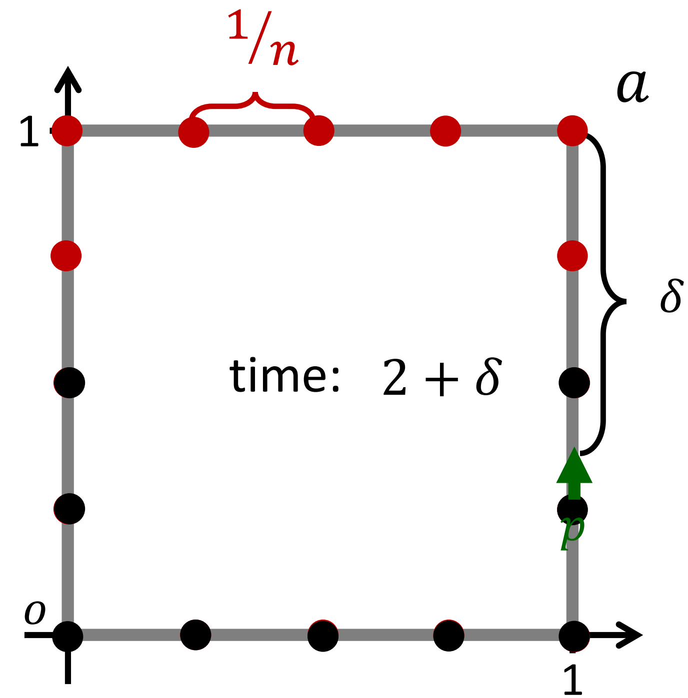

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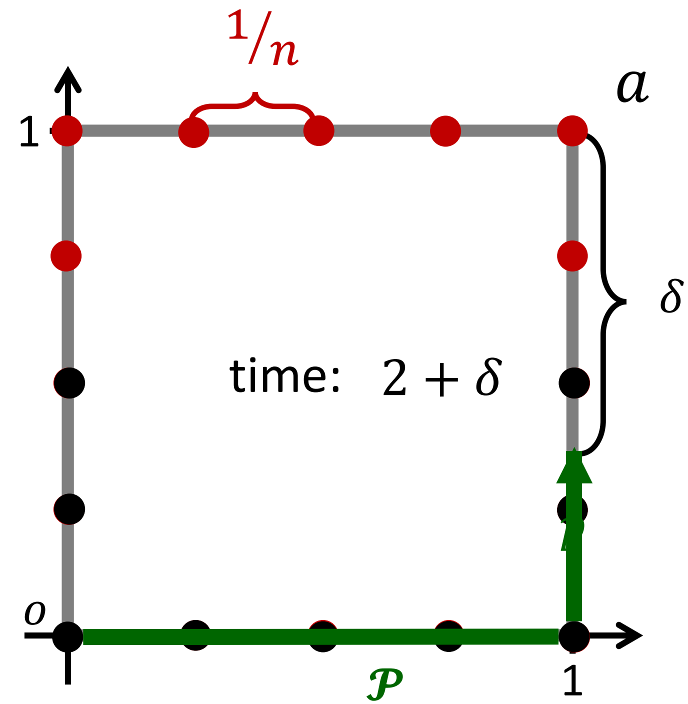

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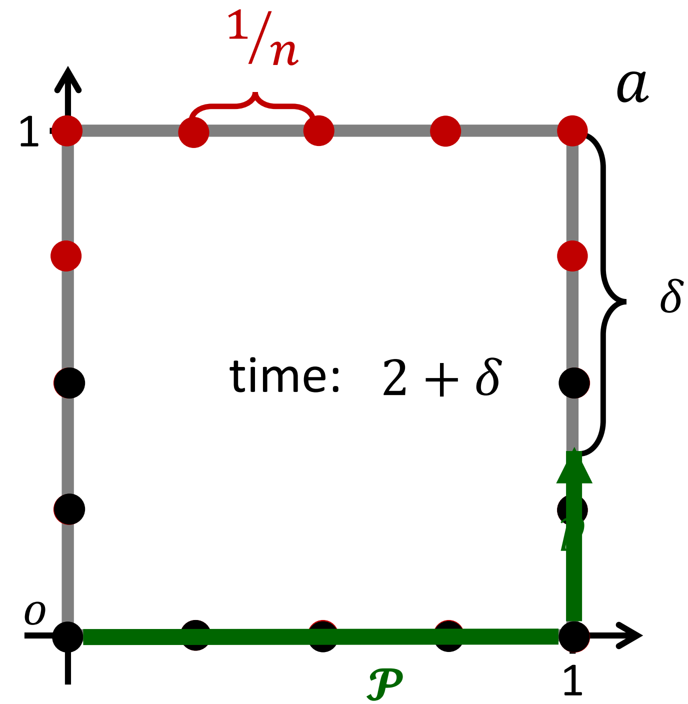

$$|\mathcal{P}| \qquad\qquad =$$

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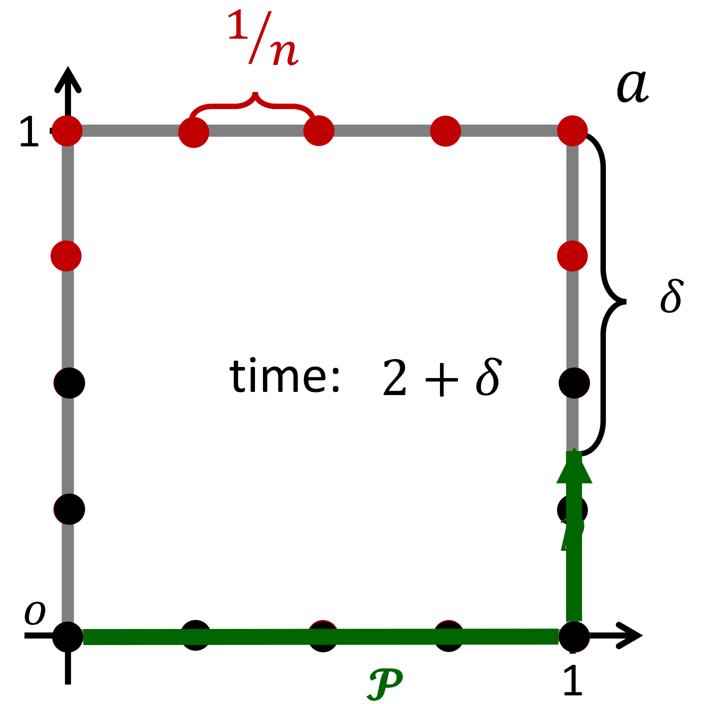
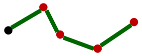
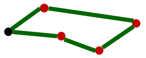


$$|\mathcal{P}| \qquad = 2 - \delta$$

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 1{,}5$	[redacted]

Lower Bound for H-OLTSP

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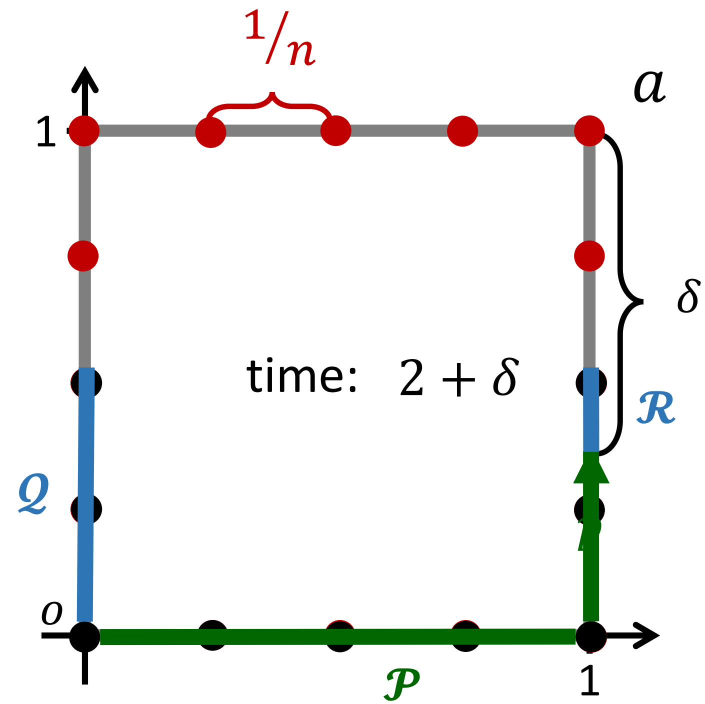

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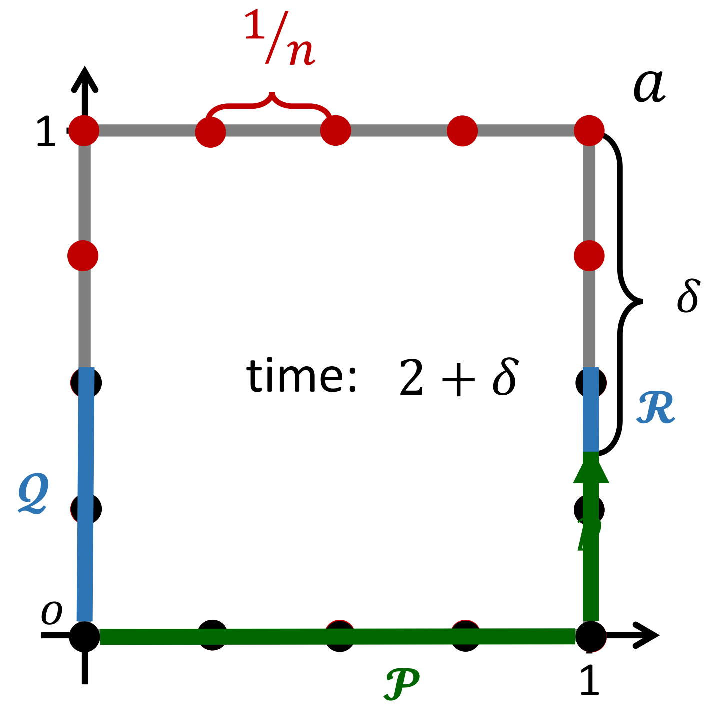

$$|\mathcal{P}| \qquad\qquad\qquad\qquad = \ 2 - \delta$$

$$|\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| \leq$$

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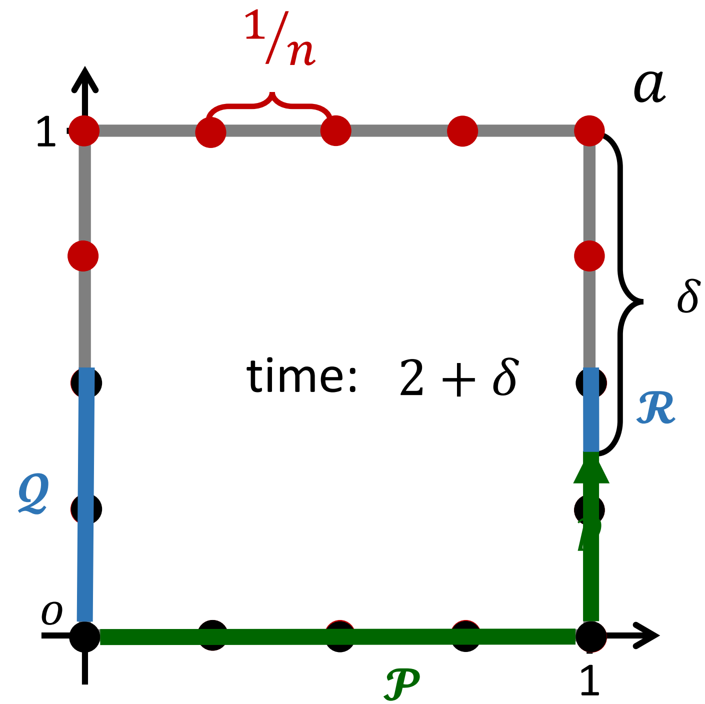

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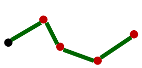
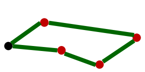
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 1{,}5$	[redacted]

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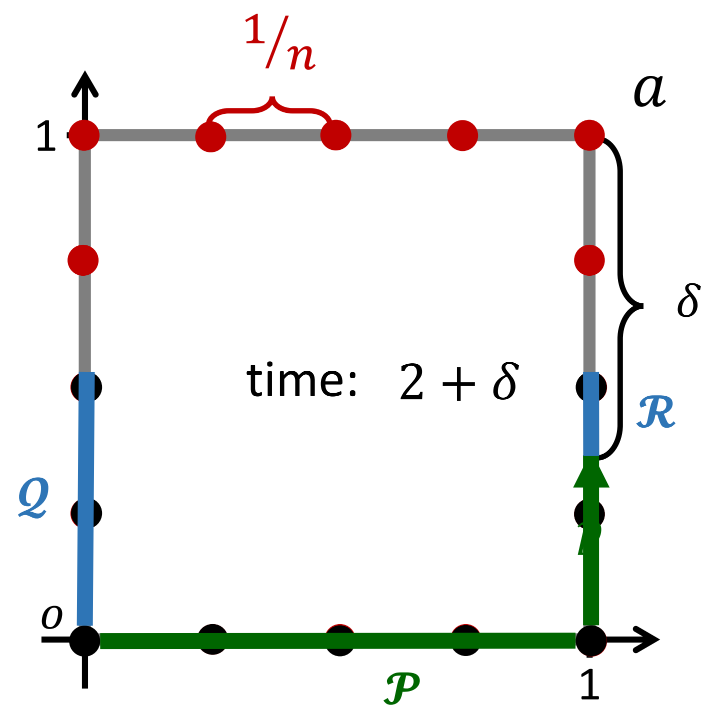

$$\left.\begin{aligned} |\mathcal{P}| &= 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| &\leq 2 + \delta \end{aligned}\right\} \quad |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}|$$

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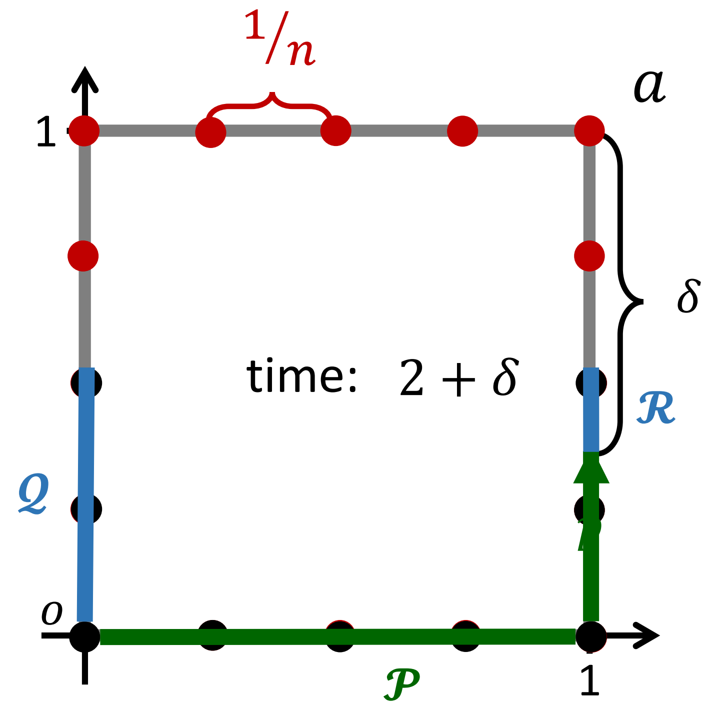

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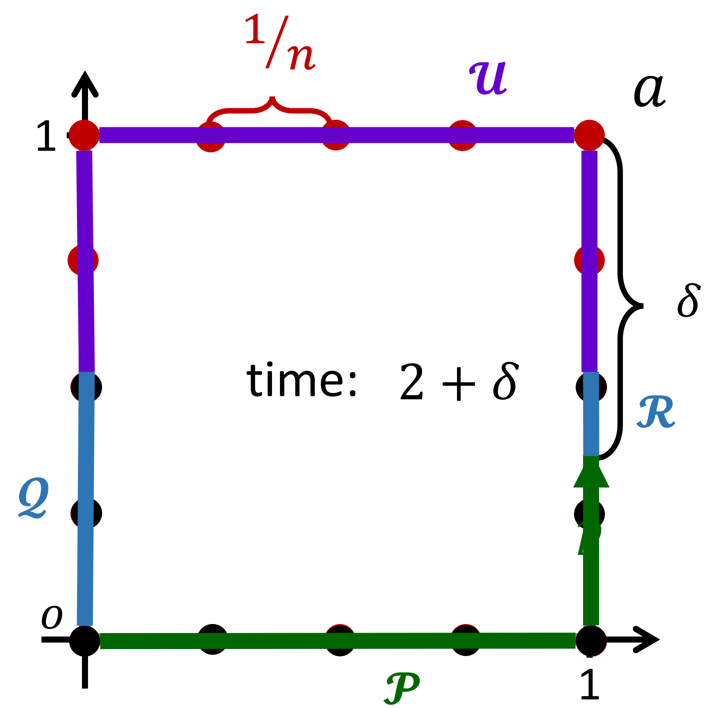

$$\left.\begin{aligned} |\mathcal{P}| \qquad\qquad\qquad\qquad &= 2-\delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| &\le 2+\delta \end{aligned}\right\} \begin{aligned} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \;\le 2 \\ \Rightarrow\; |\mathcal{U}| \end{aligned}$$

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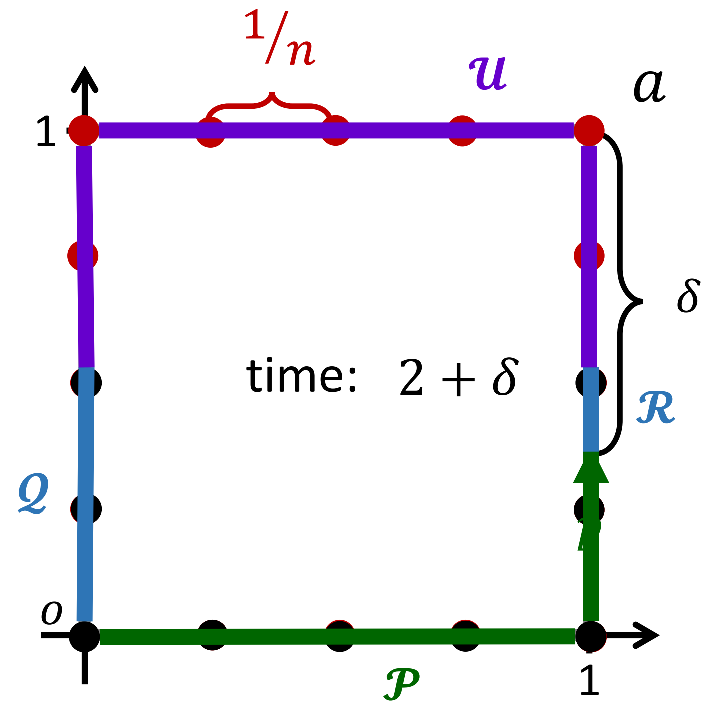

$$\left.\begin{aligned} |\mathcal{P}| &= 2 - \delta \\ |\mathcal{P}| + 2|\mathcal{Q}| + 2|\mathcal{R}| &\leq 2 + \delta \end{aligned}\right\} \quad \begin{aligned} |\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| &\leq 2 \\ \Rightarrow |\mathcal{U}| &\geq 2 \end{aligned}$$

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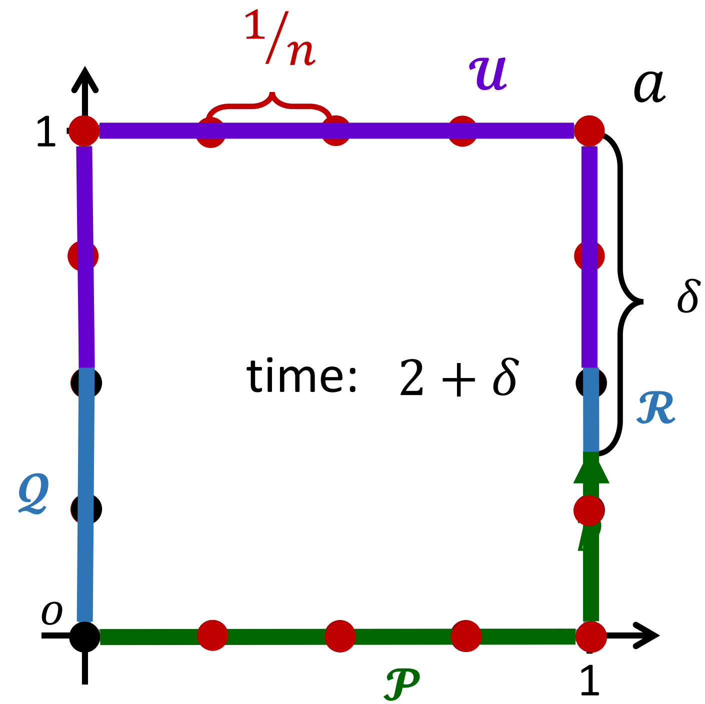
Lower Bound for H-OLTSP

- requests at time 0
- wait until time $t = 2$
- wait until $d(p, a) = t - 2$
- new requests on $\mathcal{P}$



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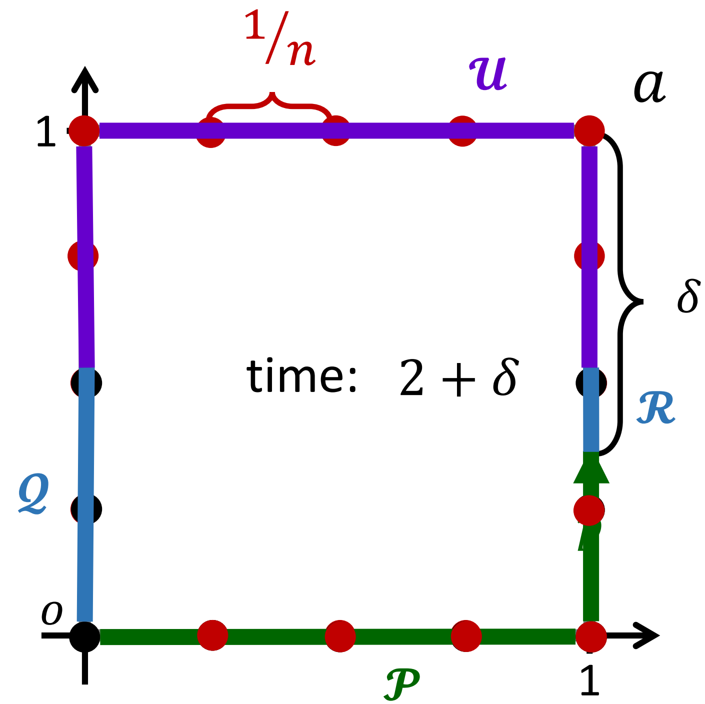
Lower Bound for H-OLTSP

- requests at time 0
- wait until time t $= 2$
- wait until $d(p, a) \; = t - 2$
- new requests on $\mathcal{P}$
- OPT finishes at $t = 4$



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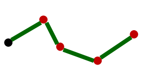
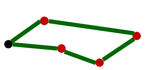
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 1{,}5$	

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- requests at time 0
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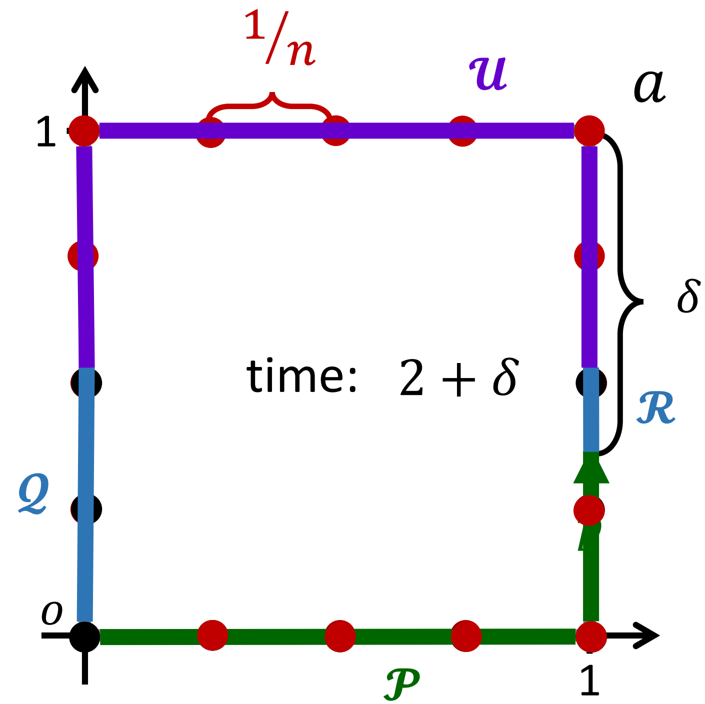
- new requests on $\mathcal{P}$
- OPT finishes at $t = 4$
  ALG finishes at $t \geq$



$$|\mathcal{Q}| + |\mathcal{P}| + |\mathcal{R}| \ \leq 2$$

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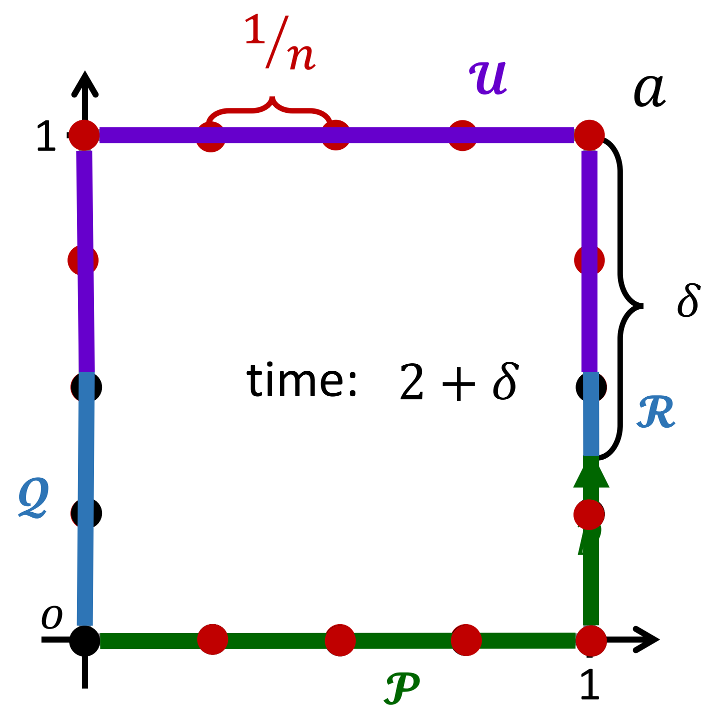

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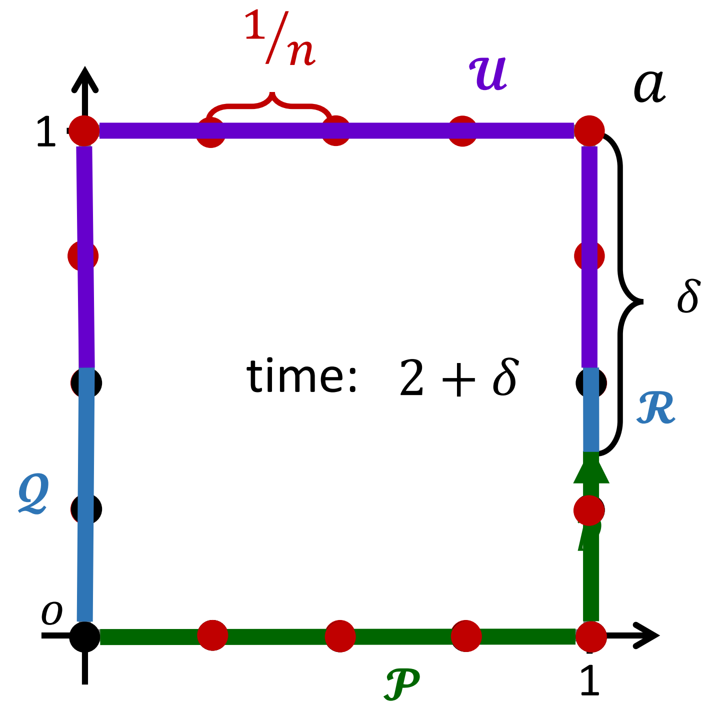
$$\begin{cases} \geq 2 + \delta + 2 \cdot \left(2 - \frac{1}{n}\right) + 2 - \delta \\ \geq 2 + \delta + 4 - \frac{1}{n} + (2 - \delta) - \frac{1}{n} \end{cases}$$



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- requests at time 0
- wait until time t $= 2$
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- new requests on $\mathcal{P}$
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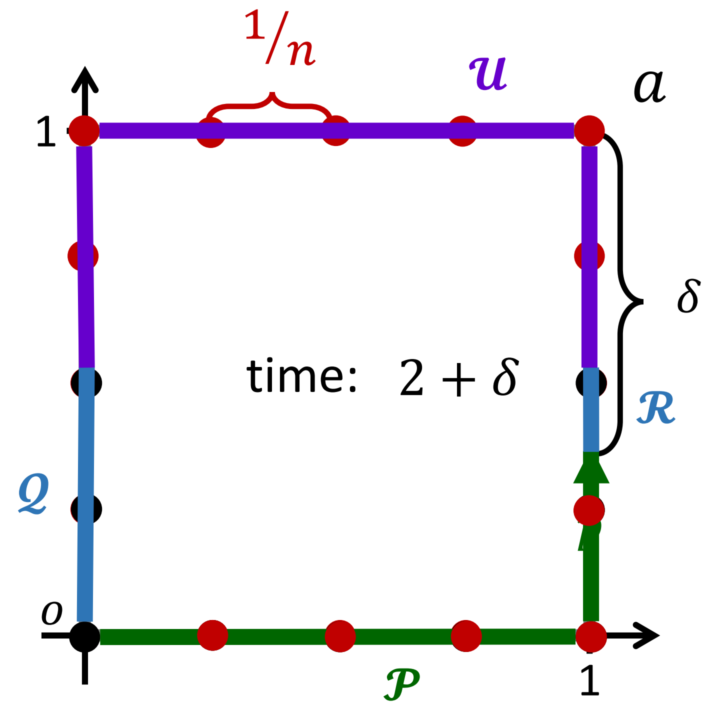
  ALG finishes at $t \geq 8 - \frac{2}{n}$

$$\begin{cases} \geq 2 + \delta + 2 \cdot \left(2 - \frac{1}{n}\right) + 2 - \delta \\ \geq 2 + \delta + 4 - \frac{1}{n} + (2 - \delta) - \frac{1}{n} \end{cases}$$



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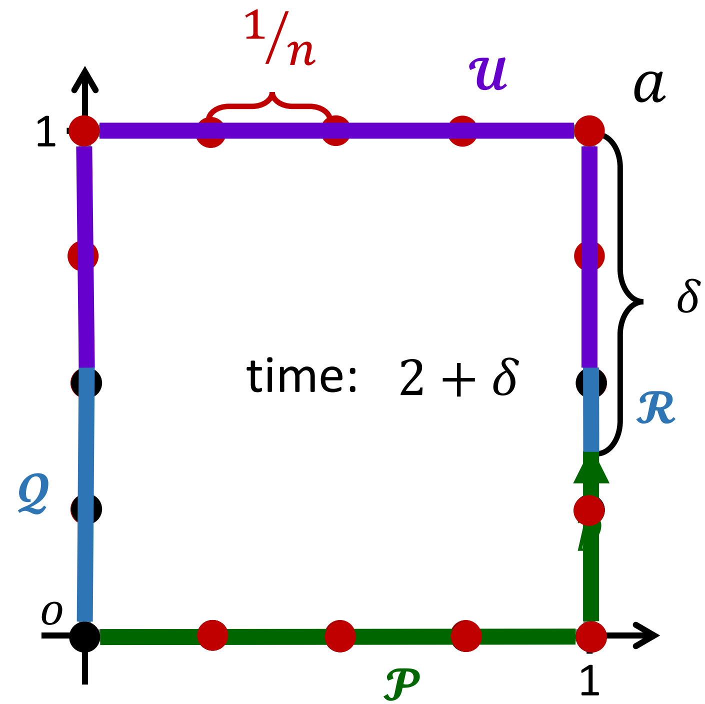

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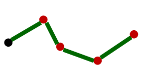
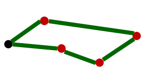
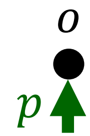
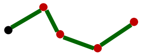
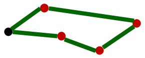
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	

A better algorithm for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]

A better algorithm for H-OLTSP

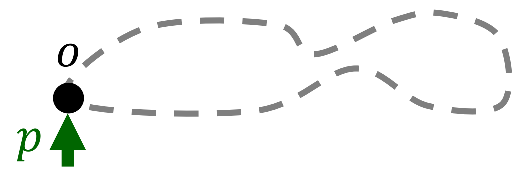


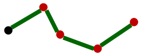
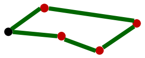
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]

A better algorithm for H-OLTSP

U := places yet to visit

(1) At o: start optimal tour through U



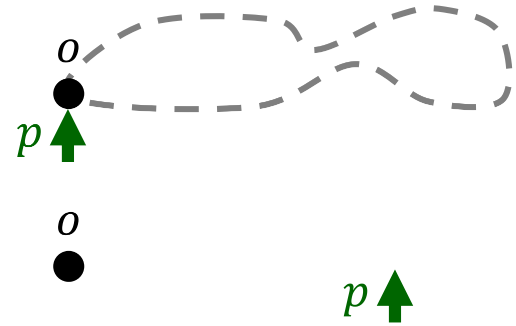
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	

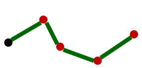
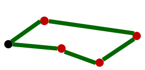
A better algorithm for H-OLTSP

U := places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	

A better algorithm for H-OLTSP

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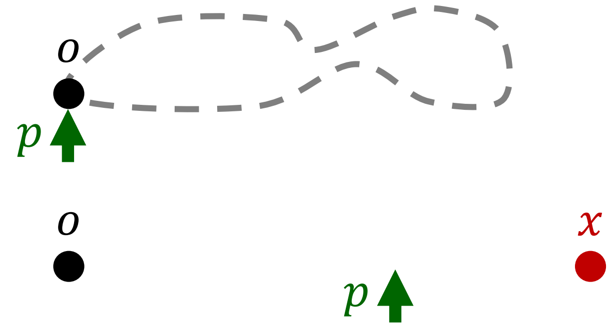

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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A better algorithm for H-OLTSP

U := places yet to visit

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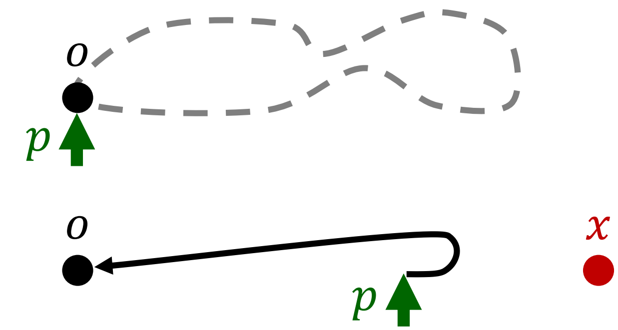
  a) If $d(x, o) > d(p, o)$: go back to o



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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A better algorithm for H-OLTSP

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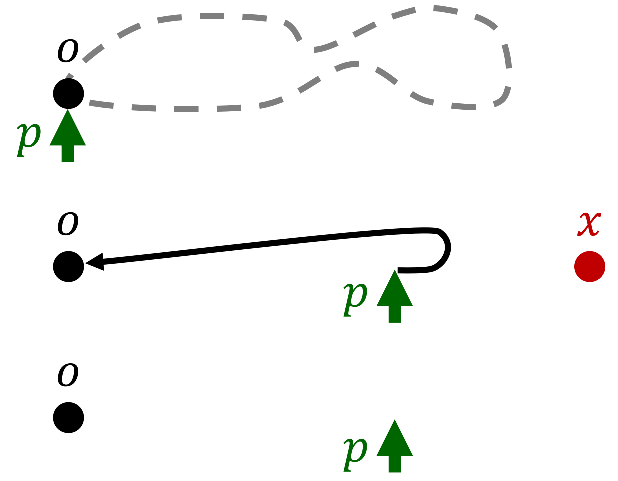
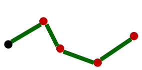
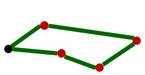


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	

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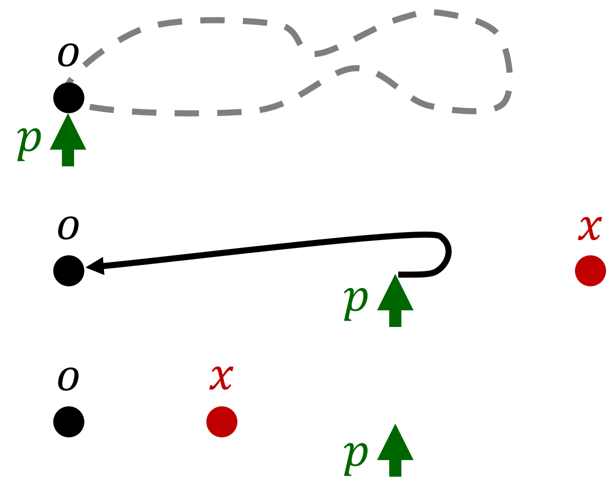

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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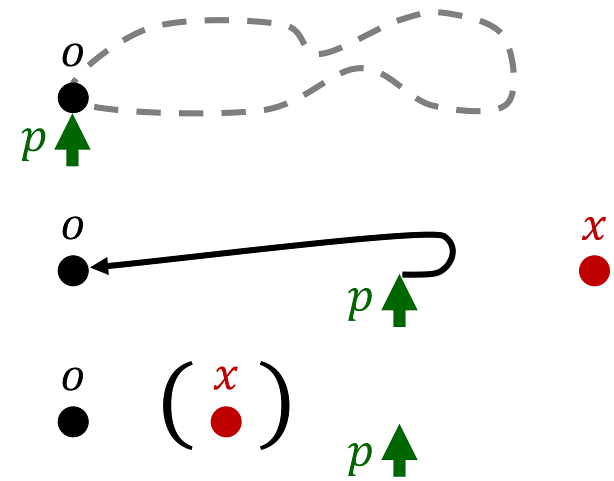

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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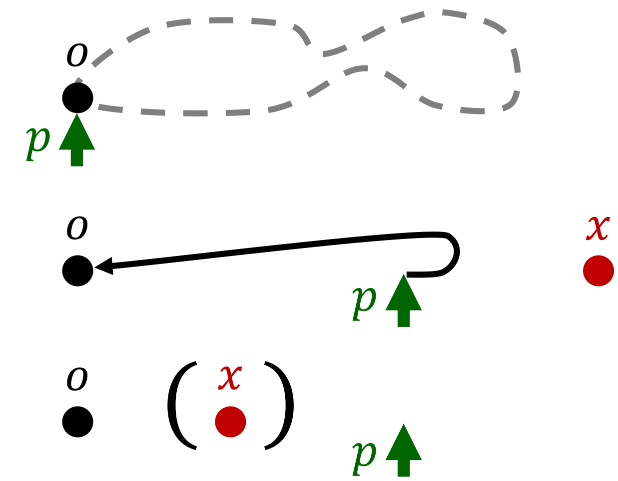

Plan At Home (PAH)

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]

A better algorithm for H-OLTSP

U := places yet to visit

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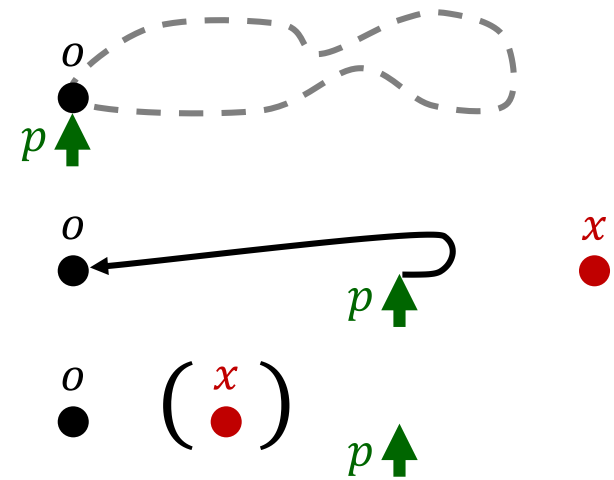


Plan At Home (PAH)

GOAL: PAH is 2-competitive for H-OLTSP

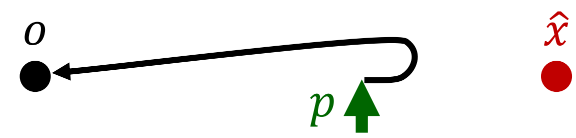
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o



GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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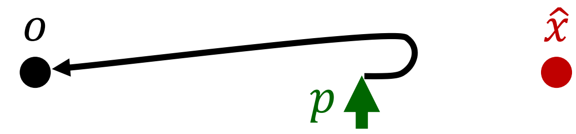

$|\mathcal{T}^{\mathrm{PAH}}|$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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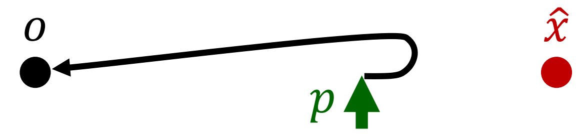

$$|\mathcal{T}^{\text{PAH}}| = \hat{t} +$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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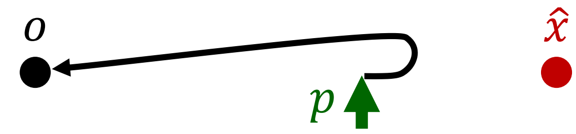
Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o



$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \hat{t} + d(p, o)$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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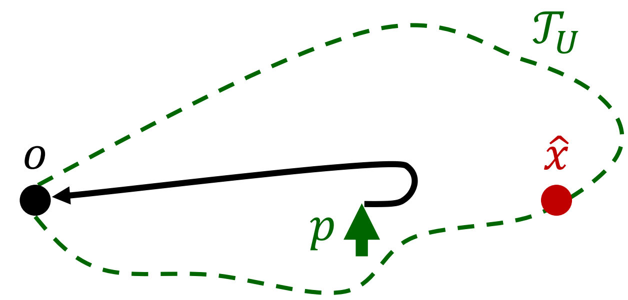
Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o



$$\left|\mathcal{T}^{\text{PAH}}\right| = \hat{t} + d(p, o) + |\mathcal{T}_U|$$

Goal: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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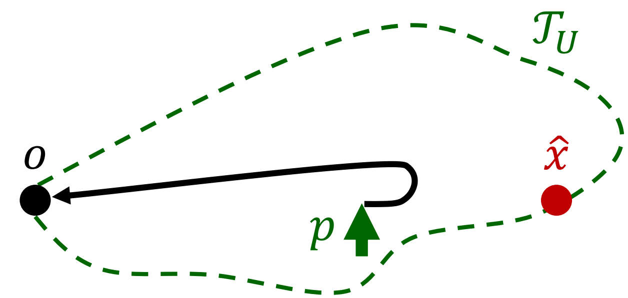
Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request



(2) For new request $(\hat{t}, \hat{x})$:

a) If $d(\hat{x}, o) > d(p, o)$: go back to o

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| = \hat{t} + d(p,o) + |\mathcal{T}_U|$$

$$\leq \qquad \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

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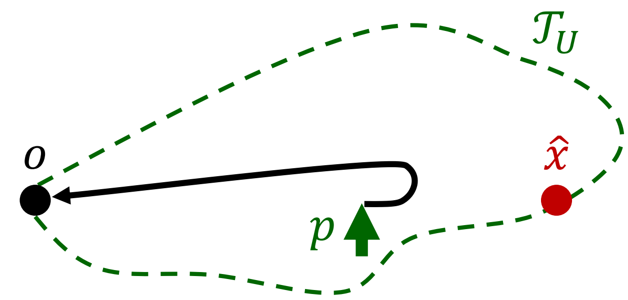

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq \hat{t} + d(\hat{x}, o) + |\mathcal{T}_U|$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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Competitiveness of PAH

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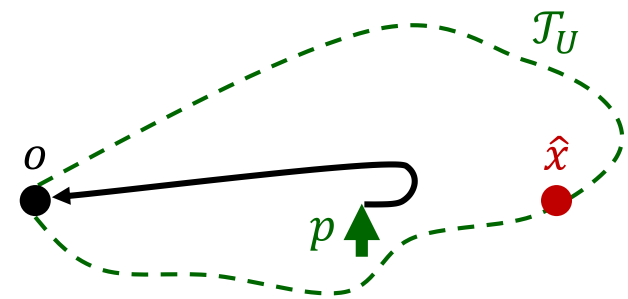

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq \underbrace{\hat{t} + d(\hat{x}, o)}_{} + |\mathcal{T}_U|$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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Competitiveness of PAH

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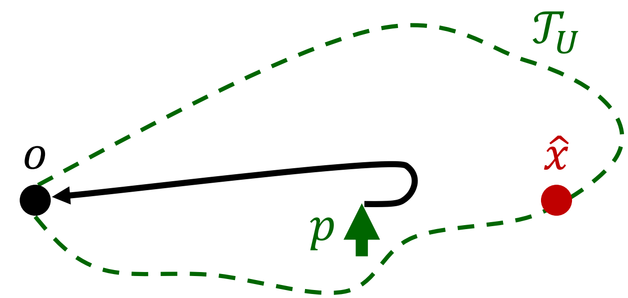
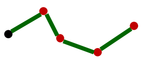
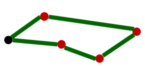


$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq \underbrace{\hat{t} \; + \; d(\hat{x}, o)} \; + \; |\mathcal{T}_U|$$

$$\leq \quad \left|\mathcal{T}^{\mathrm{OPT}}\right| \qquad + \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	

Competitiveness of PAH

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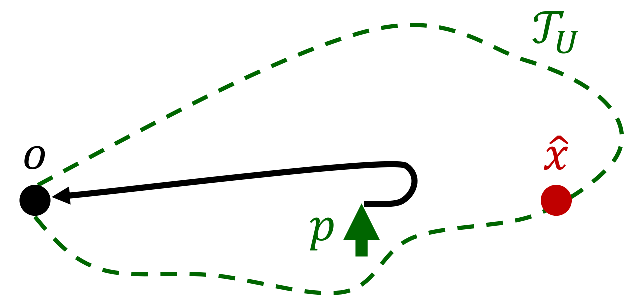

$$\left|\mathcal{T}^{\text{PAH}}\right| \leq \underbrace{\hat{t} + d(\hat{x}, o)}_{\leq\ \left|\mathcal{T}^{\text{OPT}}\right|} + |\mathcal{T}_U|$$

$$\leq \left|\mathcal{T}^{\text{OPT}}\right| + \left|\mathcal{T}^{\text{OPT}}\right| = 2 \cdot \left|\mathcal{T}^{\text{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

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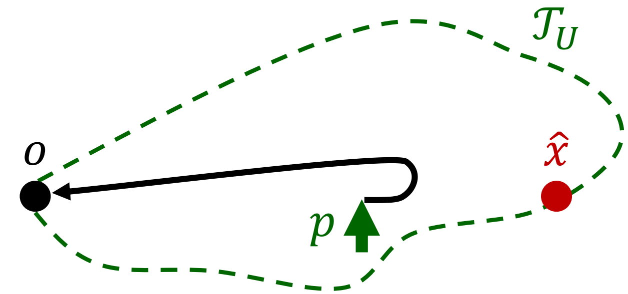

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq \hat{t} + d(\hat{x}, o) + |\mathcal{T}_U|$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \quad ✓$$

Goal: PAH is 2-competitive for H-OLTSP

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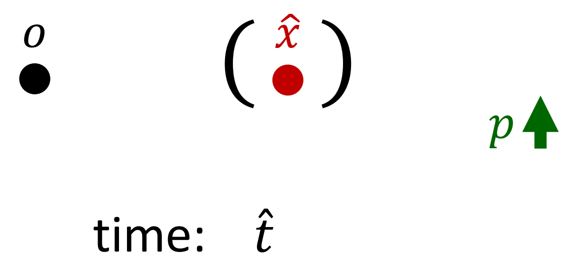
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

b) $d(\hat{x}, o) > d(p, o)$: ignore $\hat{x}$ ...



GOAL: PAH is 2-competitive for H-OLTSP

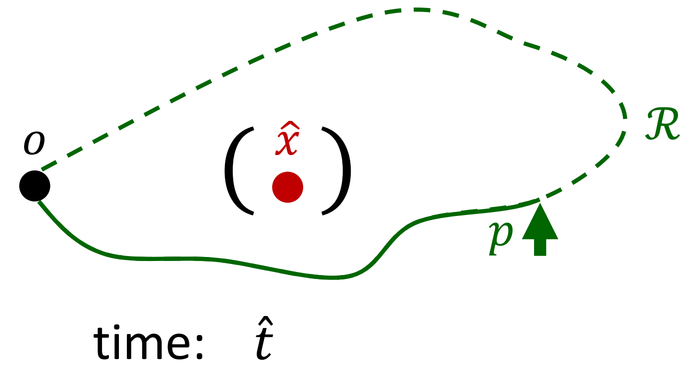
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]

Competitiveness of PAH

$U :=$ places yet to visit, $(\hat{t}, \hat{x})$ last request

(2) For new request $(\hat{t}, \hat{x})$:

b) $d(\hat{x}, o) > d(p, o)$: ignore $\hat{x}$ …



GOAL: PAH is 2-competitive for H-OLTSP

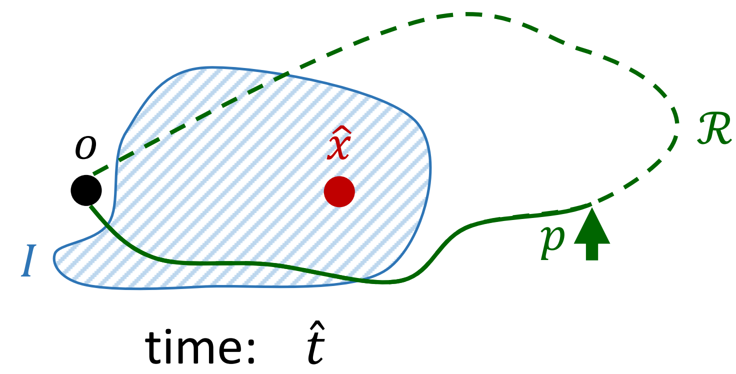
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request
I := ignored requests

(2) For new request $(\hat{t}, \hat{x})$:

b) $d(\hat{x}, o) > d(p, o)$: ignore $\hat{x}$ ...



GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	

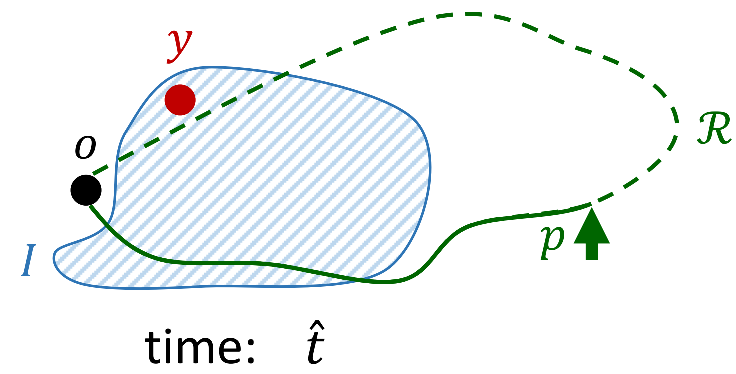
Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT

I := ignored requests

(2) For new request $(\hat{t}, \hat{x})$:

b) $d(\hat{x}, o) > d(p, o)$: ignore $\hat{x}$ ...



GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT

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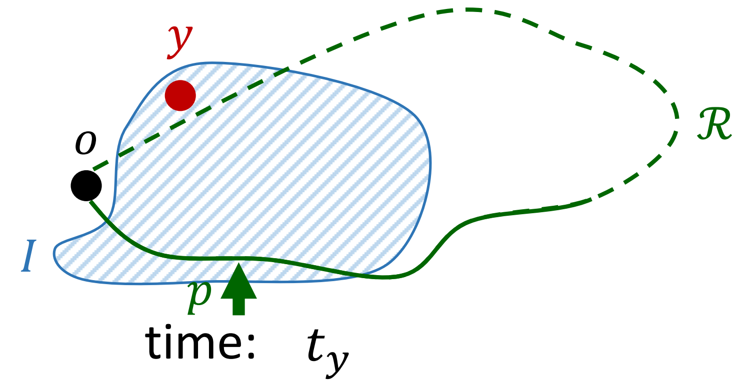

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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Competitiveness of PAH

U ≔ places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT
I ≔ ignored requests

(2) For new request (t_y, y):

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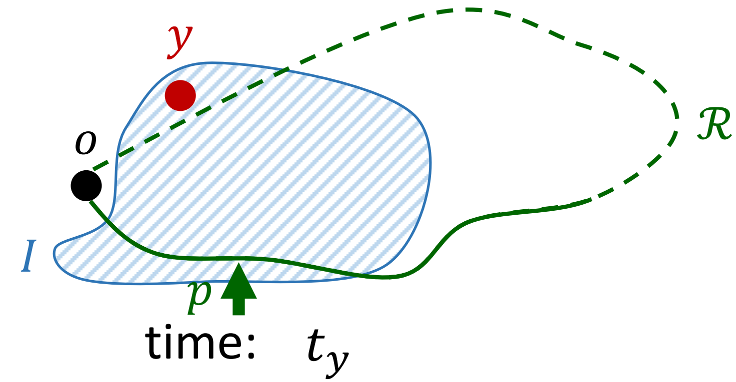

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Competitiveness of PAH

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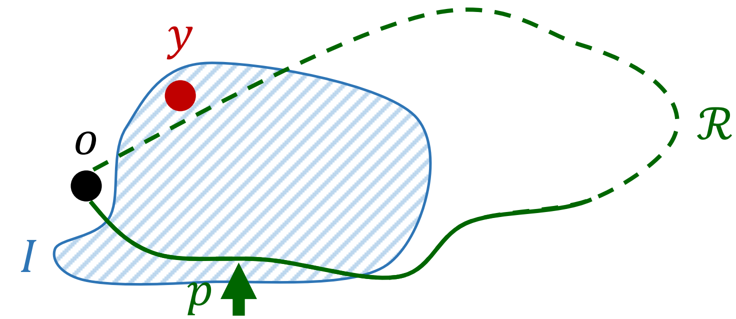

$|\mathcal{T}^{\text{PAH}}|$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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Competitiveness of PAH

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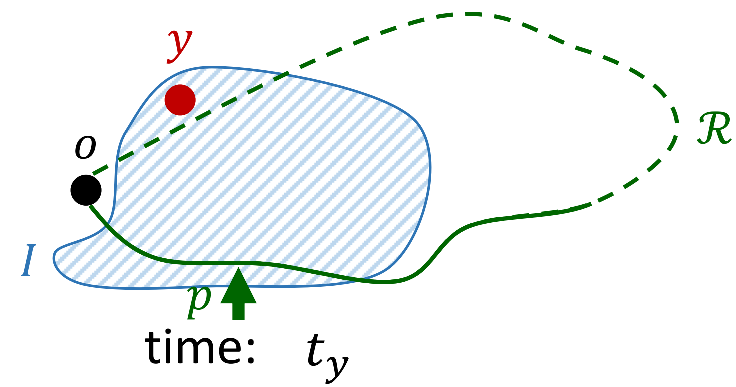

$$\left|\mathcal{T}^{\text{PAH}}\right| \leq t_y$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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Competitiveness of PAH

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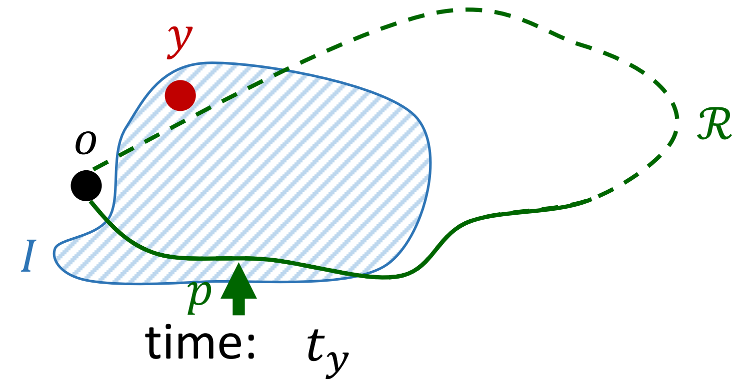

$$\left|\mathcal{T}^{\text{PAH}}\right| \leq t_y + |\mathcal{R}|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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Competitiveness of PAH

U ≔ places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT
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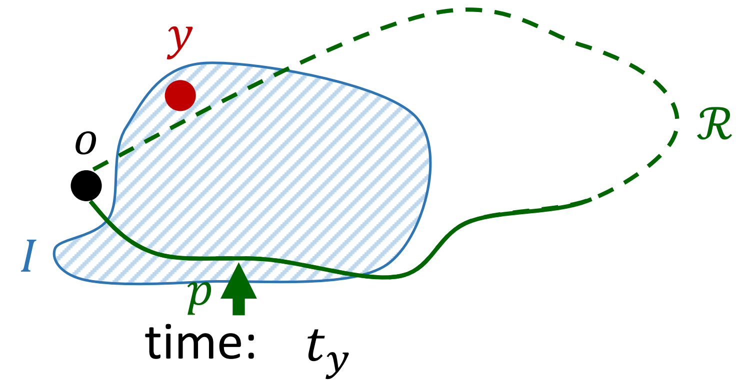

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}| - d(o, y)$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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Competitiveness of PAH

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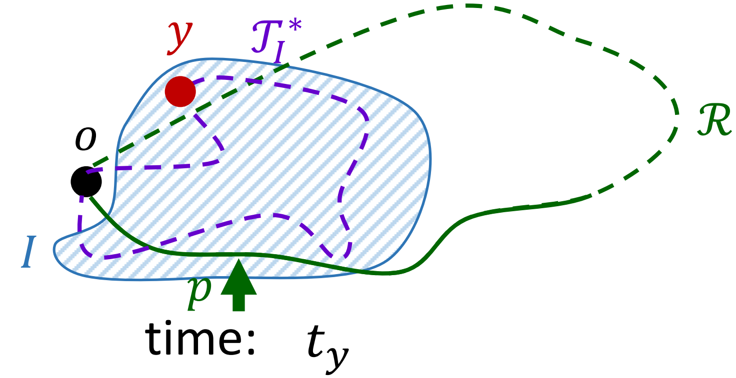

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}| - d(o, y)$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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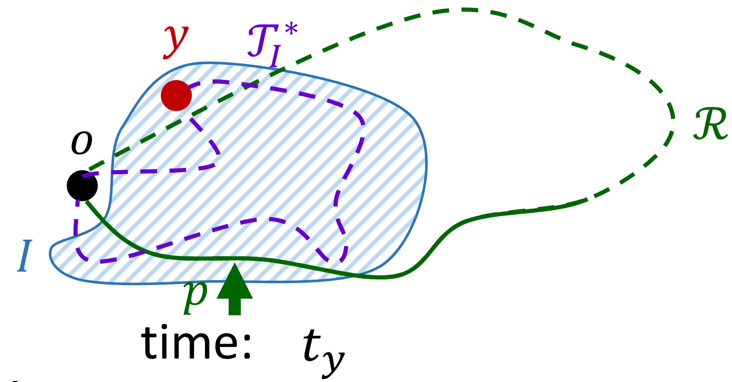
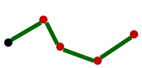
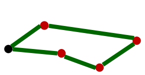


$$\left|\mathcal{T}^{\text{PAH}}\right| \le t_y + |\mathcal{R}| - d(o, y) + |\mathcal{T}_I^*|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \ge 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \ge 2$	[redacted]

Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT

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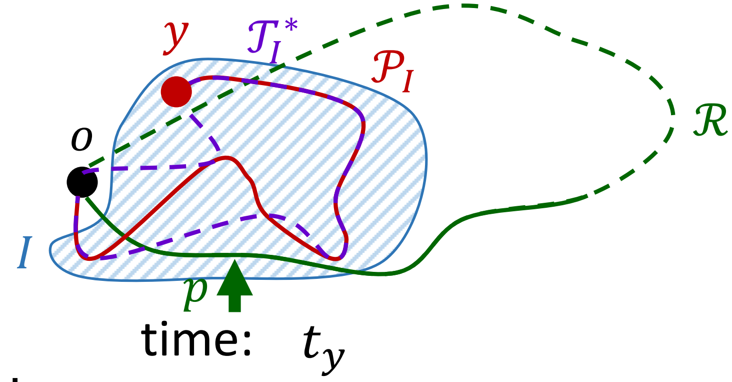

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \le t_y + |\mathcal{R}| - d(o, y) + |\mathcal{T}_I^*|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \ge 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \ge 2$	

Competitiveness of PAH

U ≔ places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT

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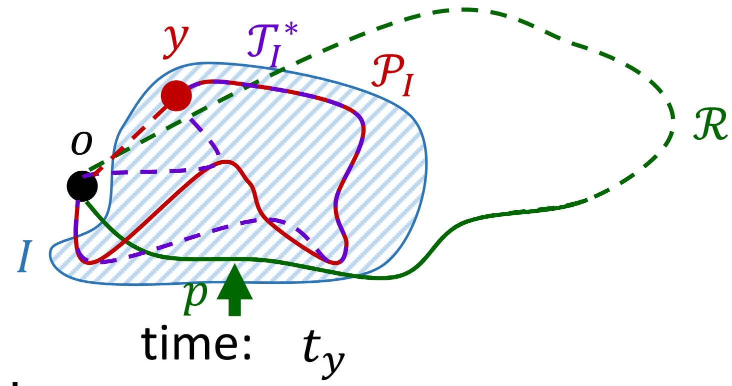

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}| - d(o, y) + |\mathcal{T}_I^*|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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Competitiveness of PAH

U := places yet to visit, $(\hat{t}, \hat{x})$ last request, y fst place in I visited by OPT

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(2) For new request (t_y, y):

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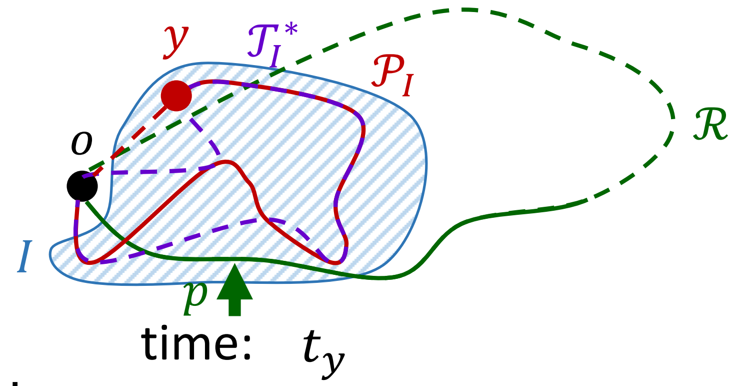

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}| - d(o,y) + \underbrace{|\mathcal{T}_I^*|}_{\leq\ d(o,y) + |\mathcal{P}_I|}$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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Competitiveness of PAH

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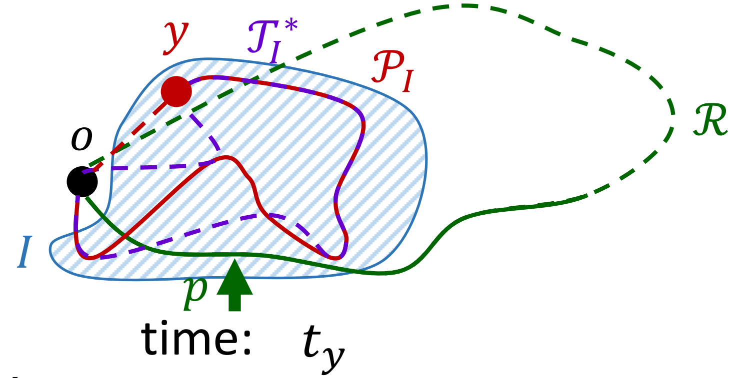
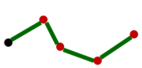
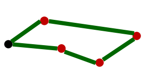


$$\left|\mathcal{T}^{\text{PAH}}\right| \leq t_y + |\mathcal{R}| \underbrace{- d(o, y) + |\mathcal{T}_I^*|}_{|\mathcal{P}_I|}$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]

Competitiveness of PAH

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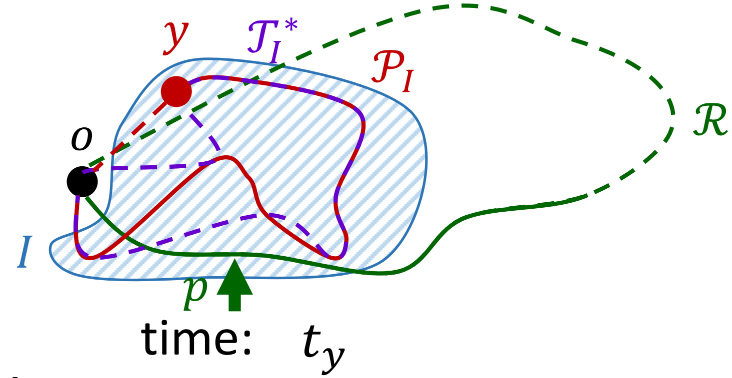

$$\begin{aligned} \left|\mathcal{T}^{\mathrm{PAH}}\right| &\le t_y + |\mathcal{R}| \underbrace{- d(o, y) + |\mathcal{T}_I^*|} \\ &\le t_y + |\mathcal{R}| + \quad |\mathcal{P}_I| \end{aligned}$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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Competitiveness of PAH

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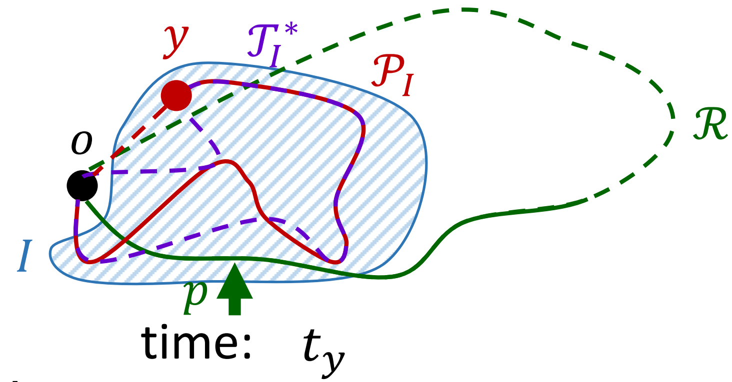

$$|\mathcal{T}^{\mathrm{PAH}}| \leq t_y + |\mathcal{R}| \underbrace{- d(o, y) + |\mathcal{T}_I^*|}$$

$$\leq t_y + |\mathcal{R}| + \quad |\mathcal{P}_I| \quad = \quad t_y + |\mathcal{P}_I| + |\mathcal{R}|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
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Competitiveness of PAH

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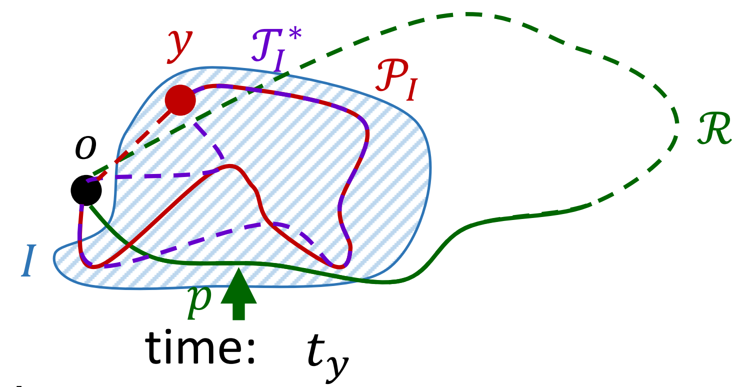

$$\left|\mathcal{T}^{\text{PAH}}\right| \leq t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

$$\leq t_y + |\mathcal{R}| + \quad |\mathcal{P}_I| \quad = \quad \underbrace{t_y + |\mathcal{P}_I|} + |\mathcal{R}|$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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Competitiveness of PAH

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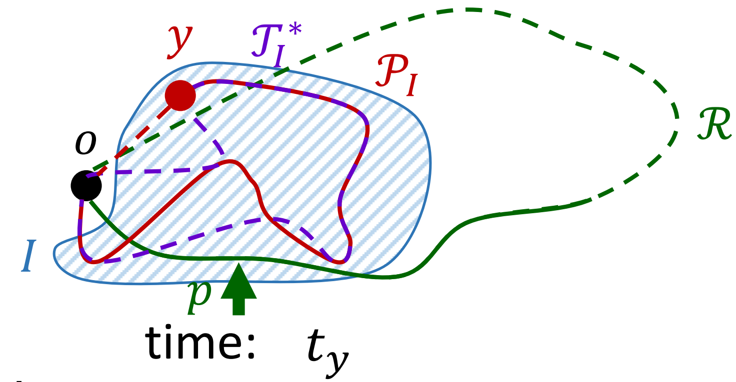
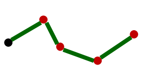
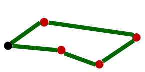


$$\begin{aligned} |\mathcal{T}^{\mathrm{PAH}}| &\le t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|} \\ &\le t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|}_{\le\ |\mathcal{T}^{\mathrm{OPT}}|} + |\mathcal{R}| \end{aligned}$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \ge 2$	[redacted]
H-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \ge 2$	[redacted]

Competitiveness of PAH

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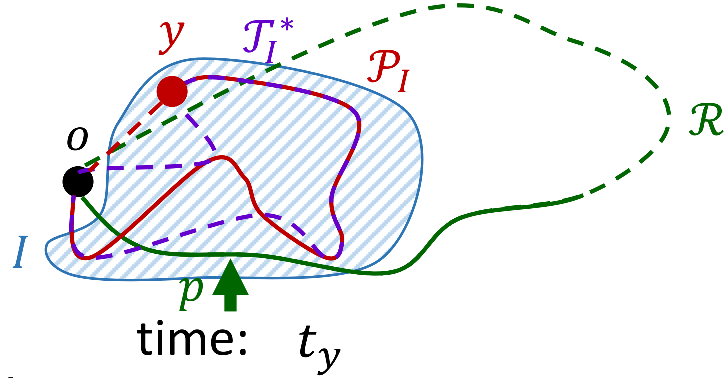

$$\begin{aligned}
\left|\mathcal{T}^{\mathrm{PAH}}\right| &\leq t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|} \\
&\leq t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|} + |\mathcal{R}| \\
&\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right|
\end{aligned}$$

GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
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Competitiveness of PAH

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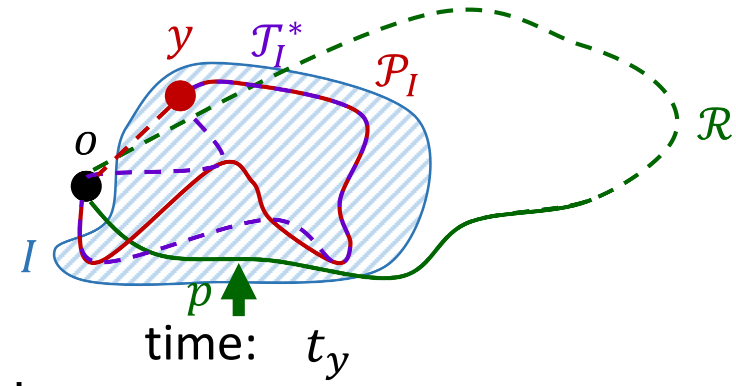

$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \le t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

$$\le t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|} + |\mathcal{R}|$$

$$\le \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP

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Competitiveness of PAH

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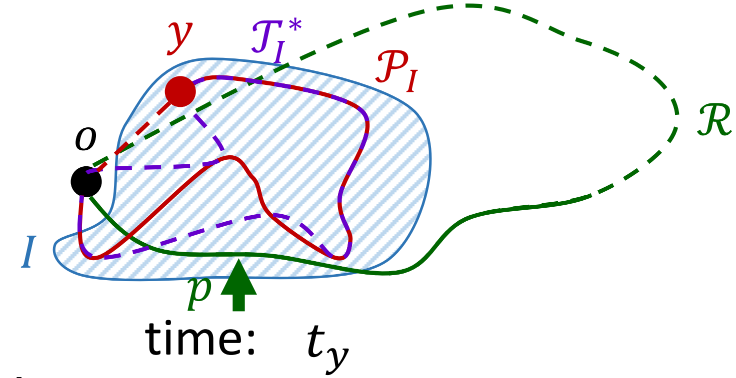


$$\left|\mathcal{T}^{\mathrm{PAH}}\right| \leq t_y + |\mathcal{R}| \underbrace{- d(o,y) + |\mathcal{T}_I^*|}$$

$$\leq t_y + |\mathcal{R}| + |\mathcal{P}_I| = \underbrace{t_y + |\mathcal{P}_I|} + |\mathcal{R}|$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \left|\mathcal{T}^{\mathrm{OPT}}\right| = 2 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

GOAL: PAH is 2-competitive for H-OLTSP ✓

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]

Competitiveness of PAH

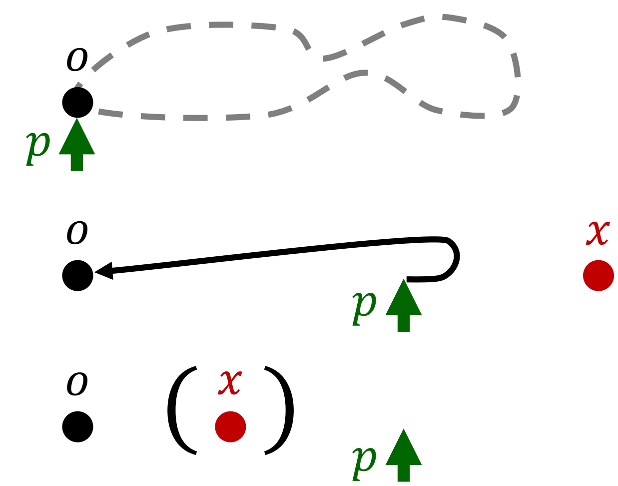
U := places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):

a) If $d(x, o) > d(p, o)$: go back to o

b) Else: ignore x until back at o



GOAL: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	

Competitiveness of PAH

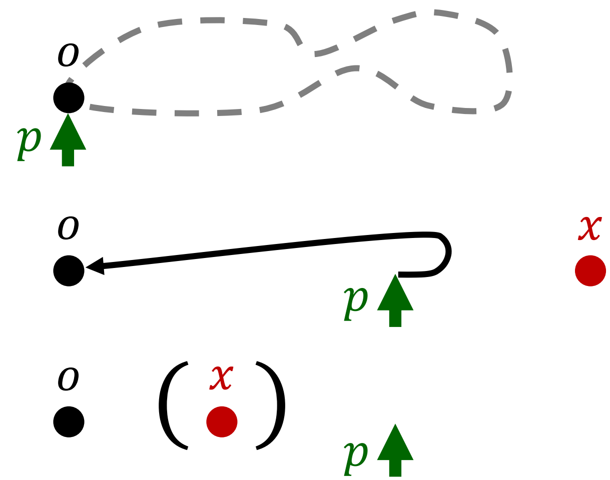
U := places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):

a) If $d(x, o) > d(p, o)$: go back to o

b) Else: ignore x until back at o



Theorem: PAH is 2-competitive for H-OLTSP

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

Competitiveness of PAH

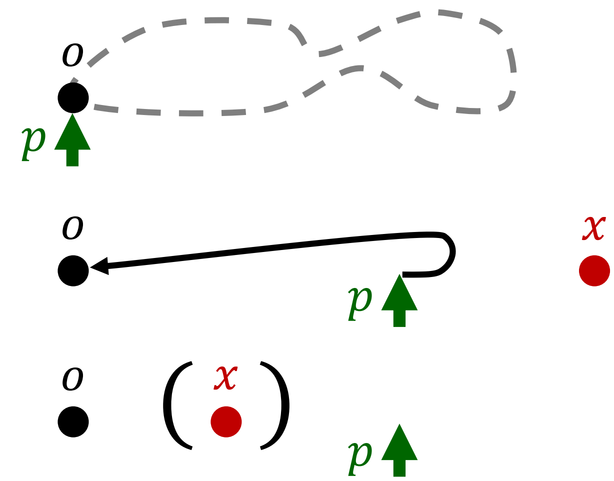
U := places yet to visit

(1) At o: start optimal tour through U

(2) For new request (t, x):

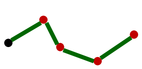
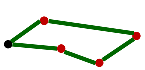
a) If $d(x, o) > d(p, o)$: go back to o

b) Else: ignore x until back at o

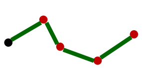
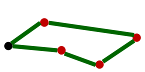


THEOREM: PAH is 2-competitive for H-OLTSP

REMARK: PAH is optimal online algorithm for H-OLTSP

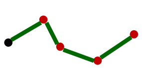
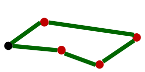
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	

Polynomial Algorithm for H-OLTSP

Online-TSP		I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP		GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP		PAH: $\rho = 2$	$\rho \geq 2$	

Polynomial Algorithm for H-OLTSP

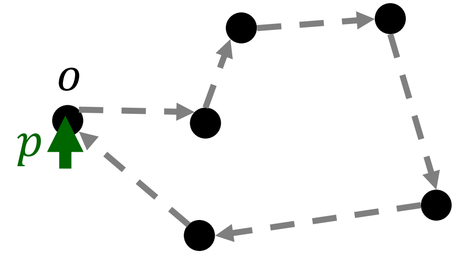
Invariant: always on shortest path between points in S

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	

Polynomial Algorithm for H-OLTSP

Invariant*:* always on shortest path between points in S

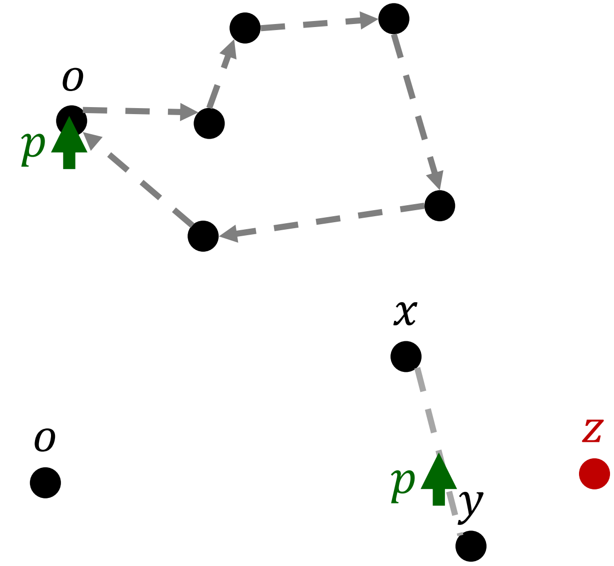
(1) At o: Find tour though $U \cup \{o\}$
with Christofides-Heuristic



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

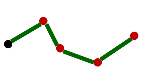
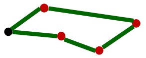
Polynomial Algorithm for H-OLTSP

Invariant*:* always on shortest path between points in S



(1) At o: Find tour though $U \cup \{o\}$
with Christofides-Heuristic

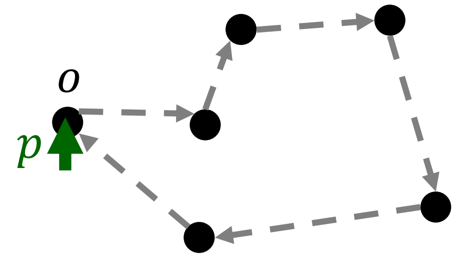
(2) For new request (t, z) at time t
and ALG between x and y:

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	

Polynomial Algorithm for H-OLTSP

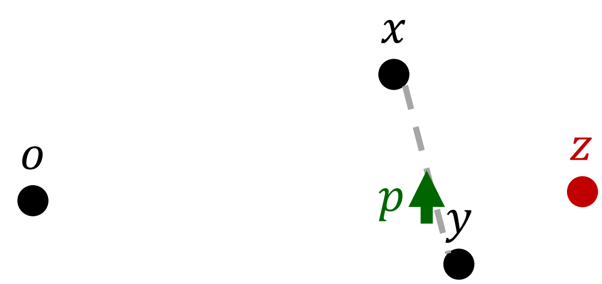
Invariant: always on shortest path between points in S

(1) At o: Find tour though $U \cup \{o\}$
with Christofides-Heuristic



(2) For new request (t, z) at time t
and ALG between x and y:

- Add z to U

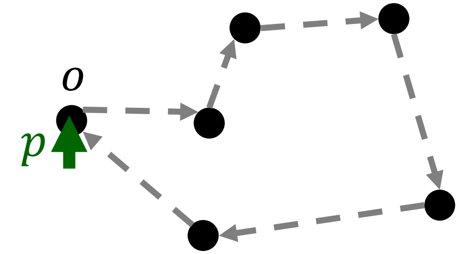


Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

Polynomial Algorithm for H-OLTSP

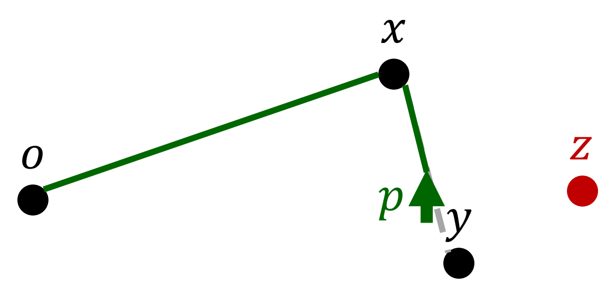
Invariant: always on shortest path between points in S

(1) At o: Find tour though $U \cup \{o\}$
with Christofides-Heuristic



(2) For new request (t, z) at time t
and ALG between x and y:

- Add z to U
- go back to o via x or y
(take shortest path)



Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

Polynomial Algorithm for H-OLTSP

Invariant: always on shortest path between points in S

(1) At o: Find tour though $U \cup \{o\}$
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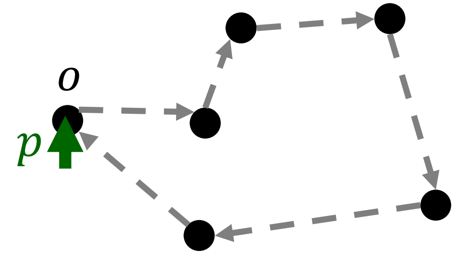
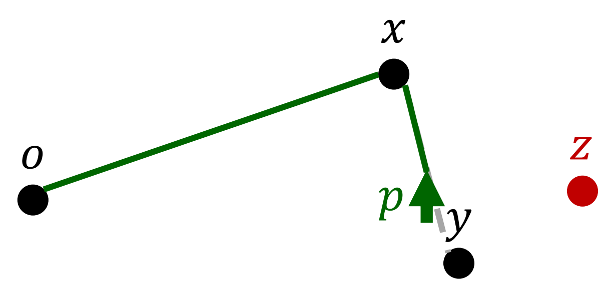
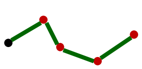
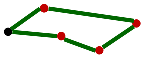


(2) For new request (t, z) at time t
and ALG between x and y:

- Add z to U
- go back to o via x or y
(take shortest path)



THEOREM: CHR is a polynomial (and correct).

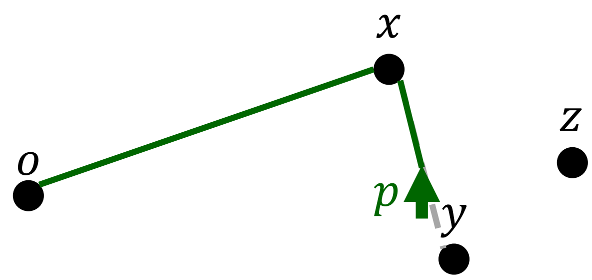
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	

Competitiveness of CHR

Invariant: always on shortest path between points in S

(2) For last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y:

- go back to o via x or y (take shortest path)



GOAL: CHR is _-competitive.

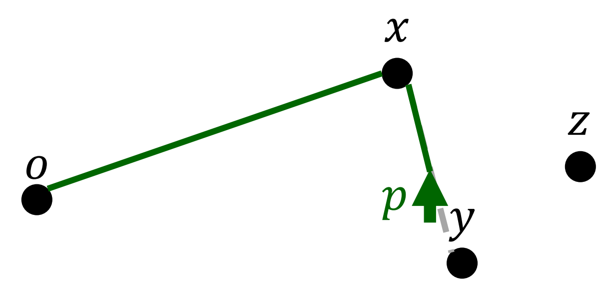
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	[redacted]

Competitiveness of CHR

Invariant: always on shortest path between points in S

(2) For last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y:

- go back to o via x or y (take shortest path)



$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} +$$

GOAL: CHR is _-competitive.

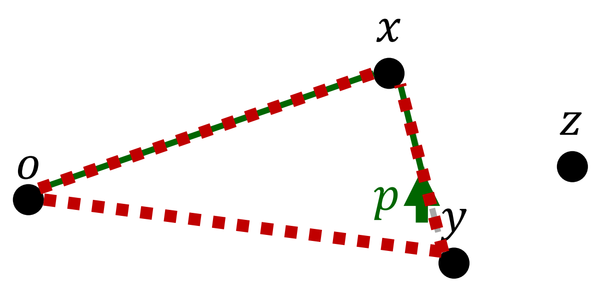
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

Competitiveness of CHR

Invariant: always on shortest path between points in S

(2) For last request $(\hat{t}, z)$ at time $\hat{t}$ and ALG between x and y:

- go back to o via x or y (take shortest path)



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\}$$

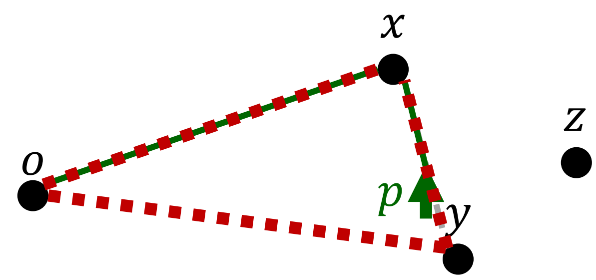
GOAL: CHR is _-competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	[redacted]

Competitiveness of CHR

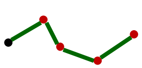
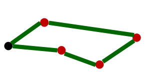
Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\}$$

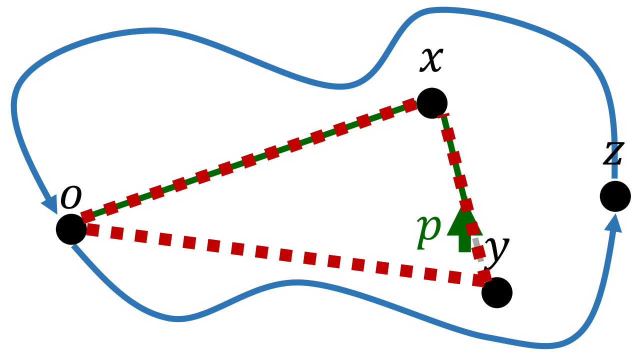
GOAL: CHR is _-competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \mathrm{CHR}(U)$$

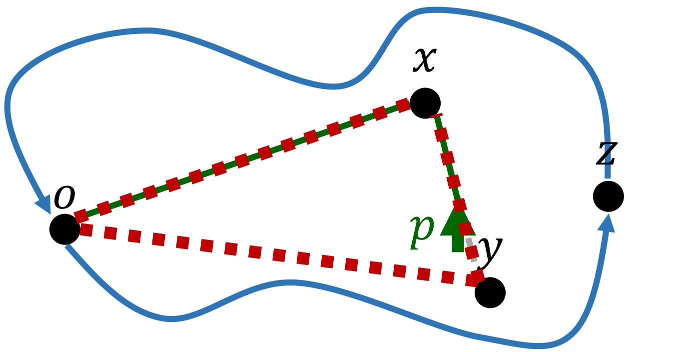
GOAL: CHR is _-competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \mathrm{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

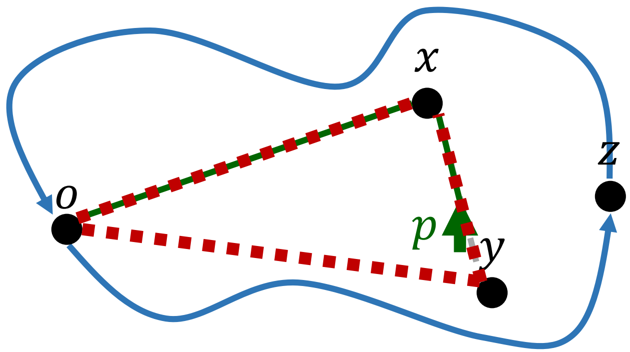
GOAL: CHR is _-competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	[redacted]

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \underbrace{\min\{d(o,x)+d(x,p), d(p,y)+d(y,o)\}}_{\leq \left|\mathcal{T}^{\mathrm{OPT}}\right|} + \mathrm{CHR}(U)$$

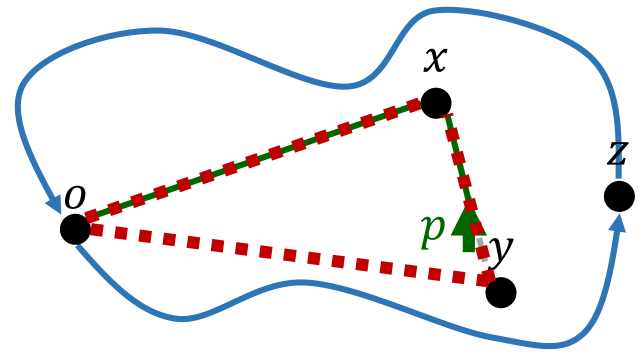
GOAL: CHR is _-competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	[redacted]

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \underbrace{\min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\}} + \mathrm{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| \qquad + \qquad \frac{1}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

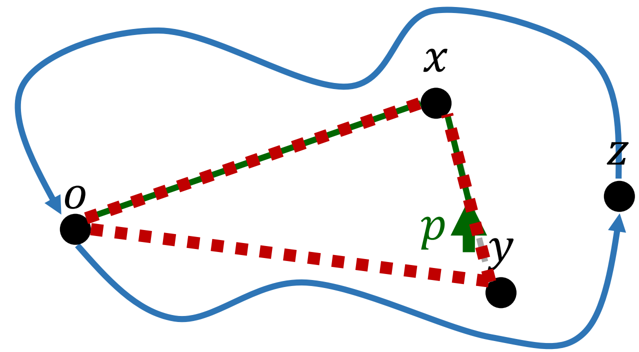
GOAL: CHR is _-competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

Competitiveness of CHR

Invariant: always on shortest path between points in S

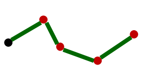
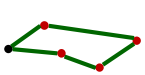
(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$|\mathcal{T}^{\text{CHR}}| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \text{CHR}(U)$$

$$\leq |\mathcal{T}^{\text{OPT}}| \quad + \quad \frac{1}{2} \cdot |\mathcal{T}^{\text{OPT}}| \quad + \frac{3}{2} \cdot |\mathcal{T}^{\text{OPT}}|$$

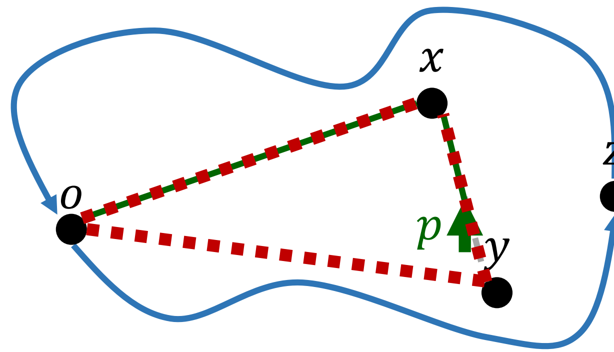
GOAL: CHR is _-competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	[redacted]

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\text{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \text{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\text{OPT}}\right| + \tfrac{1}{2} \cdot \left|\mathcal{T}^{\text{OPT}}\right| + \tfrac{3}{2} \cdot \left|\mathcal{T}^{\text{OPT}}\right| \;=$$

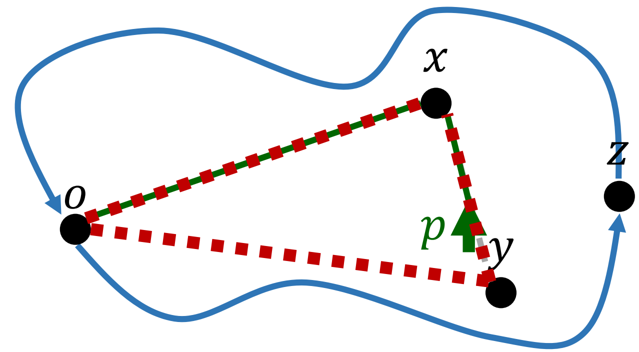
GOAL: CHR is _-competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \mathrm{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{1}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \ = \ 3 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

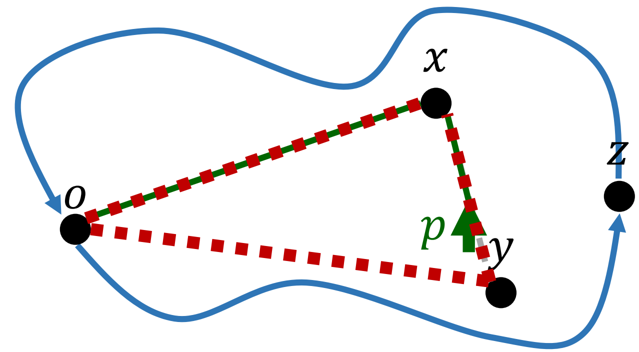
GOAL: CHR is _-competitive.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	[redacted]
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



$$\left|\mathcal{T}^{\mathrm{CHR}}\right| = \hat{t} + \min\{d(o,x) + d(x,p), d(p,y) + d(y,o)\} + \mathrm{CHR}(U)$$

$$\leq \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{1}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| + \frac{3}{2} \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right| \;=\; 3 \cdot \left|\mathcal{T}^{\mathrm{OPT}}\right|$$

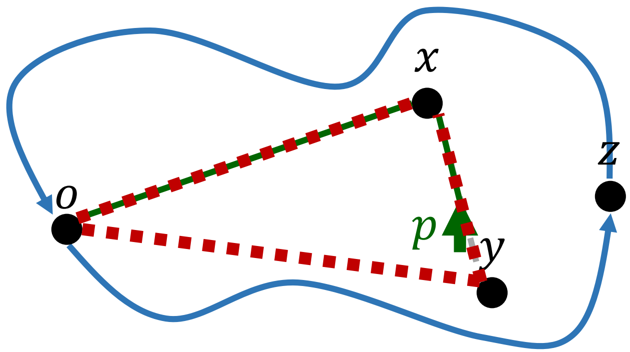
THEOREM: CHR is 3-competitive for H-OLTSP.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

Competitiveness of CHR

Invariant: always on shortest path between points in S

(1) At o: Find tour through $U \cup \{o\}$
with Christofides-Heuristic



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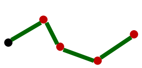
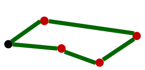
REMARK: There is a 3-competitive algorithm for N-OLTSP.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

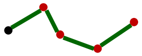
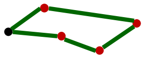
Credits & References

- Based on **Algorithms for the On-Line Travelling Salesman** by G. Ausiello, E. Feuerstein, S. Leonardi, L. Stougie, and M. Talamo in Algorithmica (2001) 29: 560–581, DOI: 10.1007/s004530010071 (http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.8.5620)

- Titlepage:
  - Map: http://awoiaf.westeros.org/index.php/File:WorldofIceandFire.png
  - Font by Charlie Samways: http://www.fonts4free.net/game-of-thrones-font.html

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

Bonus-Slide

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

N-OLTSP on the Real Line

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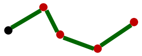
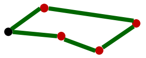
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

N-OLTSP on the Real Line

REMARK 1: The lower bound of 2 was shown on $\mathbb{R}$.

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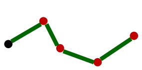
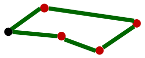
Online-TSP		I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP		GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP		PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

N-OLTSP on the Real Line

REMARK 1: The lower bound of 2 was shown on $\mathbb{R}$.

REMARK 2: Competitive ratio 2,5 of GTR is tight – even on $\mathbb{R}$.

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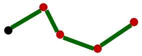
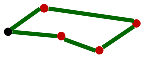
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

N-OLTSP on the Real Line

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QUESTION: Better algorithm possible on $\mathbb{R}$?

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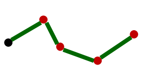
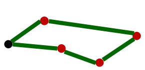
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

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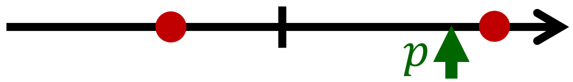

U := places yet to visit at t

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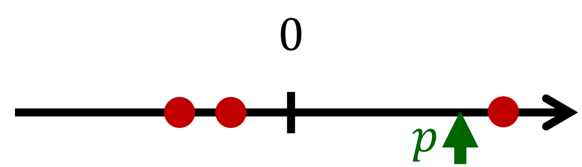

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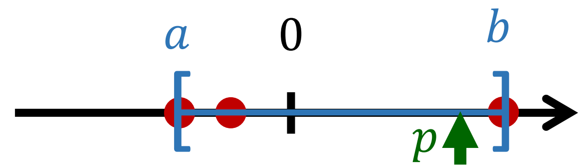

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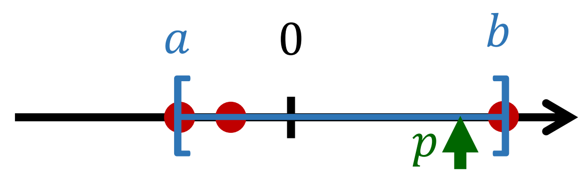

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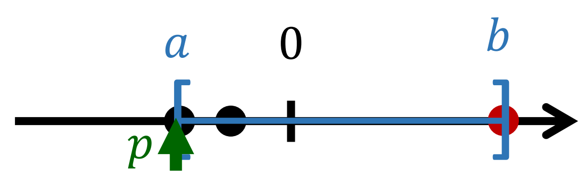

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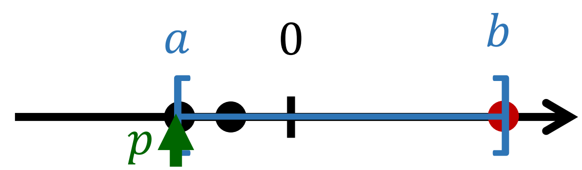

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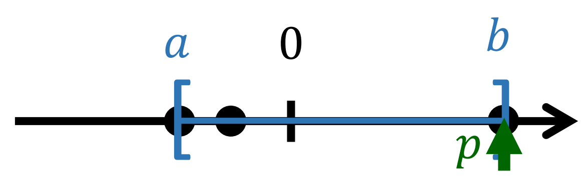

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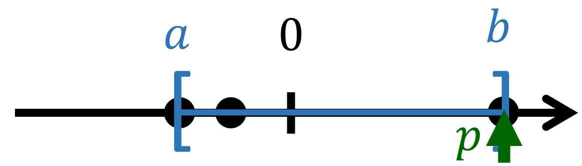

U := places yet to visit at t

Extreme Nearest to the Origin first (ENO)

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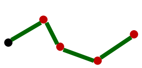
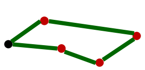
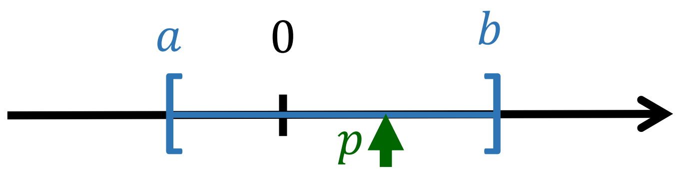
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

Competitiveness of ENO

For new request (t, z) at time t:



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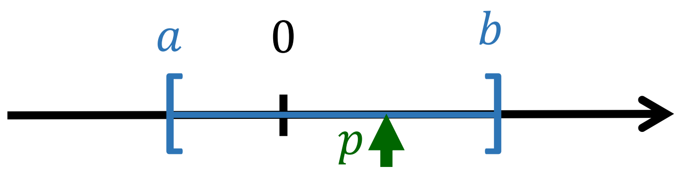

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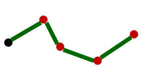
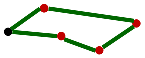
THEOREM: ENO is polynomial an correct.

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

Competitiveness of ENO

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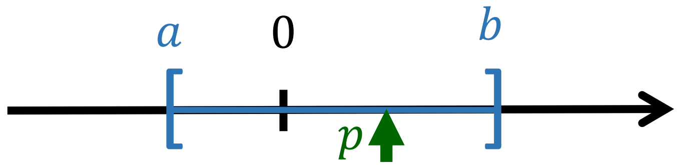

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Theorem: ENO is $\frac{7}{3}$-competitive ($\frac{7}{3} \approx 2{,}3$)

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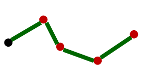
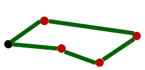
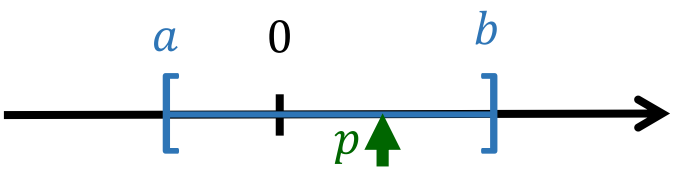
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

Competitiveness of ENO

For last request $(\hat{t}, z)$ at time $\hat{t}$:



(2) If $d(a,0) \leq d(b,0)$: Go to a (w.l.o.g.)

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THEOREM: ENO is $\frac{7}{3}$-competitive ($\frac{7}{3} \approx 2{,}3$)

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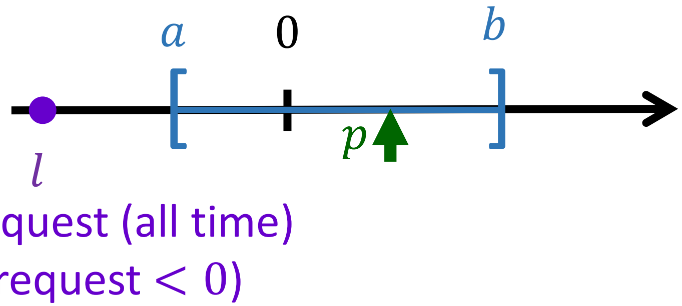

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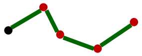
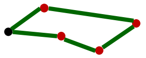
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

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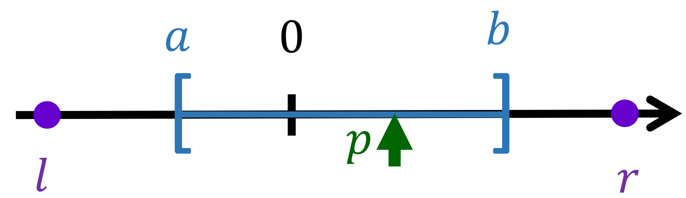
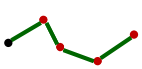
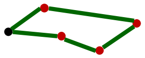


leftmost request (all time)
(0 if no request < 0)

THEOREM: ENO is $\frac{7}{3}$-competitive ($\frac{7}{3} \approx 2{,}3$)

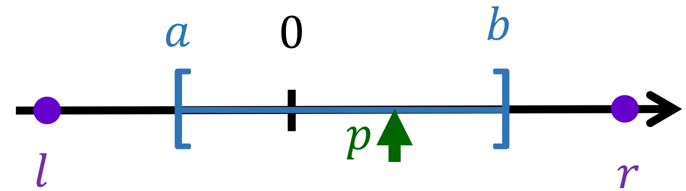
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

Competitiveness of ENO

For last request $(\hat{t}, z)$ at time $\hat{t}$:

(2) If $d(a,0) \leq d(b,0)$: Go to a (w.l.o.g.)

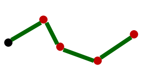
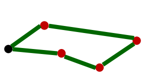
(3) Start traversing $[a,b]$



leftmost request (all time)
(0 if no request < 0)

$\Rightarrow l \leq a, \; b \leq r$ and $l \leq p(\hat{t}) \leq r$

THEOREM: ENO is $\frac{7}{3}$-competitive ($\frac{7}{3} \approx 2{,}3$)

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

Competitiveness of ENO

For last request $(\hat{t}, z)$ at time $\hat{t}$:

(2) If $d(a,0) \le d(b,0)$: Go to a (w.l.o.g.)

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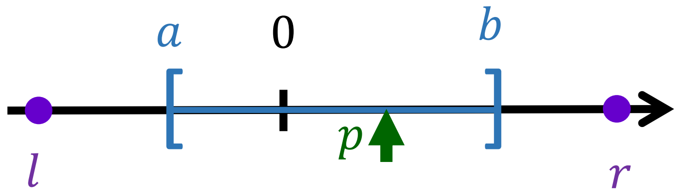

leftmost request (all time)
(0 if no request < 0)

Case-by-case analysis: $\Rightarrow l \le a,\ \ b \le r$ and $l \le p(\hat{t}) \le r$

THEOREM: ENO is $\frac{7}{3}$-competitive ($\frac{7}{3} \approx 2{,}3$)

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Competitiveness of ENO

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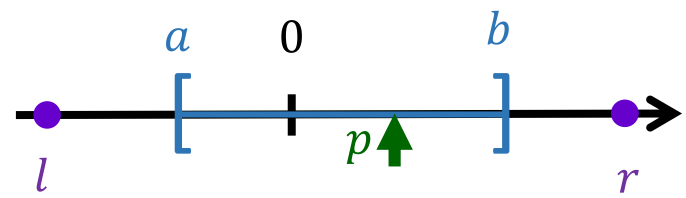
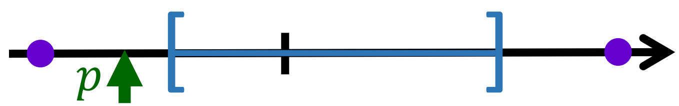


leftmost request (all time)
(0 if no request < 0)

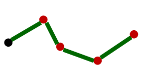
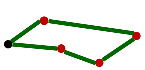
Case-by-case analysis: $\Rightarrow l \leq a,\ b \leq r$ and $l \leq p(\hat{t}) \leq r$

1. $l \leq p \leq a$



easy

Theorem: ENO is $\frac{7}{3}$-competitive ($\frac{7}{3} \approx 2{,}3$)

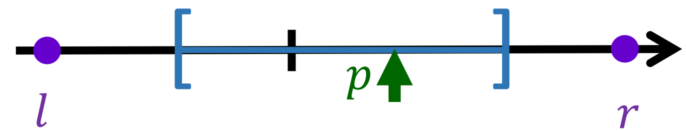
Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

Competitiveness of ENO

For last request $(\hat{t}, z)$ at time $\hat{t}$:

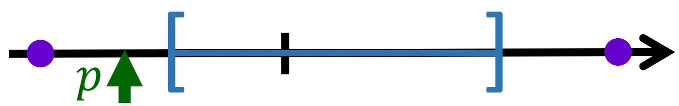
(2) If $d(a,0) \leq d(b,0)$: Go to a (w.l.o.g.)

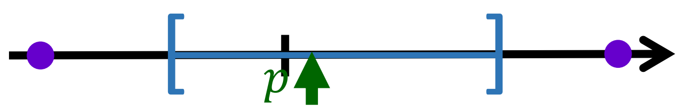
(3) Start traversing $[a, b]$



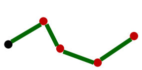
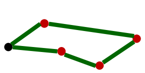
leftmost request (all time)
(0 if no request < 0)

Case-by-case analysis: $\Rightarrow l \leq a,\ \ b \leq r$ and $l \leq p(\hat{t}) \leq r$

1. $l \leq p \leq a$  easy

2. $a \leq p \leq |a|$  easy

THEOREM: ENO is $\frac{7}{3}$-competitive ($\frac{7}{3} \approx 2{,}3$)

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$

Competitiveness of ENO

For last request $(\hat{t}, z)$ at time $\hat{t}$:

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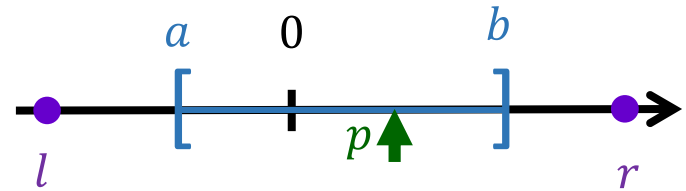

leftmost request (all time)
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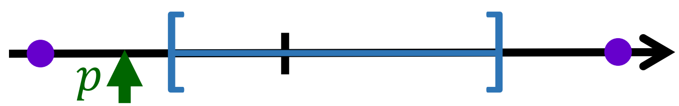
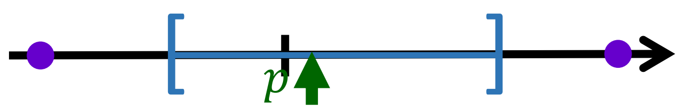
Case-by-case analysis: $\Rightarrow l \le a, \;\; b \le r$ and $l \le p(\hat{t}) \le r$

1. $l \le p \le a$



easy

2. $a \le p \le |a|$



easy

3. $|a| \le p \le r$

THEOREM: ENO is $\frac{7}{3}$-competitive ($\frac{7}{3} \approx 2{,}3$)

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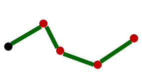
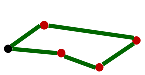
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \ge 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \ge 2$	CHR (Christofides): $\rho = 3$

Competitiveness of ENO

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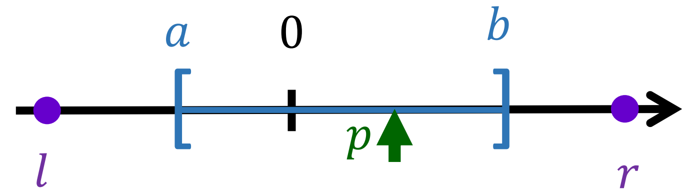

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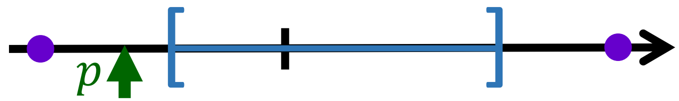
Case-by-case analysis: $\Rightarrow l \le a, \; b \le r$ and $l \le p(\hat{t}) \le r$

1. $l \le p \le a$ 

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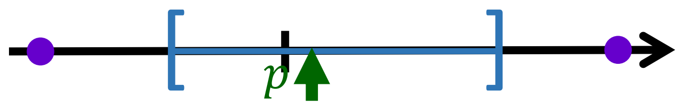
2. $a \le p \le |a|$ 

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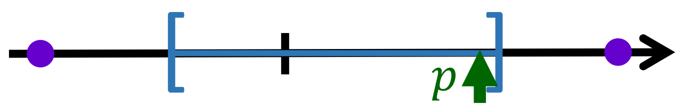
3. $|a| \le p \le r$ 

complicated…

THEOREM: ENO is $\frac{7}{3}$-competitive ($\frac{7}{3} \approx 2{,}3$)

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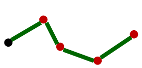
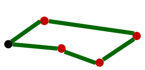
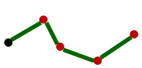
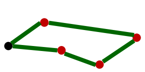
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \ge 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \ge 2$	CHR (Christofides): $\rho = 3$

Credits & References

- Based on **Algorithms for the On-Line Travelling Salesman** by G. Ausiello, E. Feuerstein, S. Leonardi, L. Stougie, and M. Talamo in Algorithmica (2001) 29: 560–581, DOI: 10.1007/s004530010071 (http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.8.5620)

- Titlepage:
  - Map: http://awoiaf.westeros.org/index.php/File:WorldofIceandFire.png
  - Font by Charlie Samways: http://www.fonts4free.net/game-of-thrones-font.html

Online-TSP	I. Algorithms	II. Lower Bounds	III. Polynomial Algorithms
N-OLTSP 	GTR (Greedy): $\rho = 2{,}5$	$\rho \geq 2$	MST: $\rho = 3$
H-OLTSP 	PAH: $\rho = 2$	$\rho \geq 2$	CHR (Christofides): $\rho = 3$