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Q3 a) $Cov(\hat{\beta}_0, \hat{\beta}_1) = \frac{-\bar{x}\sigma^2}{S_{xx}}$

$\bar{y} = \hat{\beta}_0 - \hat{\beta}_1 \bar{x}$

$E(\bar{y} - \hat{\beta}_1 \bar{x}) = E(\bar{y}) - E\hat{\beta}_1 \bar{x}$

$= \bar{y} - \beta_1 \bar{x}$

$$
\begin{aligned}
Cov(\hat{\beta}_0, \hat{\beta}_1) &= E[(\hat{\beta}_0 - E(\hat{\beta}_0))(\hat{\beta}_1 - E(\hat{\beta}_1))] \\
&= E[(\bar{y} - \hat{\beta}_1 \bar{x}) - (\bar{y} - \beta_1 \bar{x})(\hat{\beta}_1 - E(\hat{\beta}_1))] \\
&= E[(\bar{y} - \hat{\beta}_1 \bar{x} - \bar{y} + \hat{\beta}_1 \bar{x})(\hat{\beta}_1 - \beta_1)] \\
&= E[(-\hat{\beta}_1 \bar{x} + \beta_1 \bar{x})(\hat{\beta}_1 - \beta_1)] \\
&= E[(-\bar{x}(\hat{\beta}_1 - \beta_1)(\hat{\beta}_1 - \beta_1)] \\
&= E[-\bar{x}(\hat{\beta}_1 - \beta_1)^2] \\
&= -\bar{x} E(\hat{\beta}_1 - \beta_1)^2 \\
&= -\bar{x} Var(\hat{\beta}_1) \\
&= \frac{-\bar{x}\sigma^2}{S_{xx}}
\end{aligned}
$$