Q5 a) Show $F = \frac{R^2(n-2)}{1-R^2}$ for $H_0: \beta_1 = 0$

$$R^2 = \frac{SSR}{SST}$$

$$F = \frac{\frac{SSR}{SST}(n-2)}{1 - \frac{SSR}{SST}}$$

$$SSE = SST - SSR, \quad \frac{SSE}{SST} = \frac{SST}{SST} - \frac{SSR}{SST} = 1 - \frac{SSR}{SST}$$

$$= \frac{\frac{SSR}{SST}(n-2)}{\frac{SSE}{SST}}$$

$$= \frac{SSR(n-2)}{SSE} = \frac{SSR}{\frac{SSE}{n-2}} = \frac{MSR}{MSE} = F_{stat}$$

b) SLR w $n = 20$ at $\alpha = 0.05$, $df_T = n - 1 = 19$, $df_n = 1$, $df_p = 18$

$F_{threshold} = F_{(1,18,0.975)} = 5.9781$, $F_{obs} = \frac{R^2(n-2)}{1-R^2}$

$$F = \frac{R^2(20-2)}{1-R^2}$$

$$(1-R^2)F = R^2 18$$

$$(1-R^2)F - R^2 18 = 0$$

$$5.9781 - 5.9781R^2 - R^2 18 = 0$$

$$R^2 = \frac{5.9781}{23.9781} \cong 0.2493$$

$\therefore R^2$ must be greater than approx 0.2493