

Data Point	A	В	С	D
Year	2007	2008	2009	2010
Net Profit	14,065	17,681	14,596	18,760

Year	2011	2012	2013	2014
Net Profit	23,150	23,171	22,453	22,074

Problem 19:

- ♦ The table shows the net profit (in millions of dollars) for Microsoft from 2007 through 2014.
- ♦ A) Set up a system of equations to fit the data for the years 2007, 2008, 2009, and 2010 to a cubic model.
- ♦ B) Solve the system. Does the solution produce a reasonable model for determining net profits after 2010? Explain

Polynomial Curve Fitting

- One degree less than data points (cubic equation)
- ♦ Use equation:

$$\Rightarrow p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n$$

♦ Horizontal Shift (2007 = 0)

Equations for Data Points

Using equation:
$$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$p(a) = a_0 + a_1(0) + a_2(0)^2 + a_3(0)^3 = 14,065$$

$$p(b) = a_0 + a_1(1) + a_2(1)^2 + a_3(1)^3 = 17,681$$

$$p(c) = a_0 + a_1(2) + a_2(2)^2 + a_3(2)^3 = 14,569$$

$$p(d) = a_0 + a_1(3) + a_2(3)^2 + a_3(3)^3 = 18,760$$

Part A: Gauss/Jordan Elimination

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 14,065 \\ 1 & 1 & 1 & 1 & 17,681 \\ 1 & 2 & 4 & 8 & 14,569 \\ 1 & 3 & 9 & 27 & 18,760 \end{bmatrix} \quad \begin{matrix} R_2 - R_1 \to R_2 \\ R_3 - R_1 \to R_3 \\ R_4 - R_1 \to R_4 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 14,065 \\ 0 & 1 & 1 & 1 & 3,616 \\ 0 & 2 & 4 & 8 & 504 \\ 0 & 3 & 9 & 27 & 4,695 \end{bmatrix} \quad \begin{matrix} R_3 - 2R_2 \to R_3 \\ \frac{1}{2}R_3 \to R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 14,065 \\ 0 & 1 & 1 & 1 & 3,616 \\ 0 & 0 & 1 & 3 & -3,364 \\ 0 & 3 & 9 & 27 & 4,695 \end{bmatrix} \qquad \frac{1}{3}R_3 \to R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 14,065 \\ 0 & 1 & 1 & 1 & 3,616 \\ 0 & 0 & 1 & 3 & -3,364 \\ 0 & 1 & 3 & 9 & 1,565 \end{bmatrix} \quad R_4 - R_2 \to R_4$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 14,065 \\ 0 & 1 & 1 & 1 & 3,616 \\ 0 & 0 & 1 & 3 & -3,364 \\ 0 & 0 & 2 & 8 & -2,051 \end{bmatrix} \quad R_4 - 2R_3 \to R_4$$

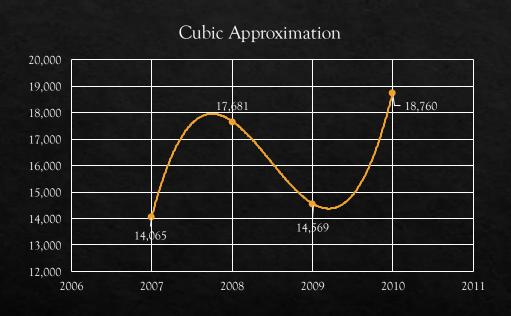
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 14,065 \\ 0 & 1 & 1 & 1 & 3,616 \\ 0 & 0 & 1 & 3 & -3,364 \\ 0 & 0 & 0 & 2 & 4,677 \end{bmatrix} \quad \frac{1}{2}R_4 \to R_4$$

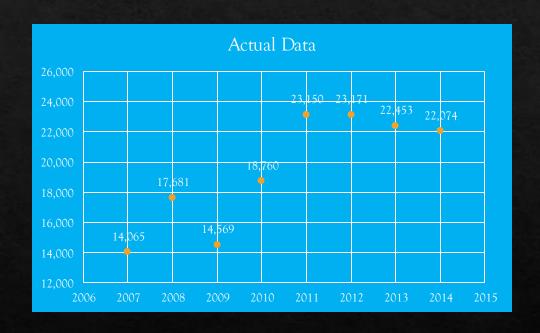
Part B: Back Substitution

$$a_3 = 2,338.5$$
 $a_2 = -10,379.5$
 $a_1 = 11,657$
 $a_0 = 14,065$

$$p(a) = 14,065 + 11,657(x - 2007) - 10,379.5(x - 2007)^2 + 2,338.5(x - 2007)^3$$

Graphing the Solution





$$p(2011) = 44285$$

 $p(2012) = 105175$
 $p(2013) = 215461$
 $p(2014) = 389174$