L-Università ta' Malta

CCE3206—Digital Signal Processing

Practical 3

Trevor Spiteri
trevor.spiteri@um.edu.mt

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Frequency response of an FIR filter

Objective

The objective of this practical is to obtain the frequency response of an FIR filter implemented as a transversal filter.

Tasks

1. Figure 1 shows a fifth-order finite-duration impulse response (FIR) filter described by the difference equation

$$y(n) = \sum_{k=0}^{5} h(k)x(n-k)$$

Write a Python function that takes a NumPy array as the input and outputs the response of the system in Figure 1 to this input with the impulse response defined as

$$h(n) = \{\underset{\uparrow}{-0.0147}, 0.173, 0.342, 0.342, 0.173, -0.0147\}$$

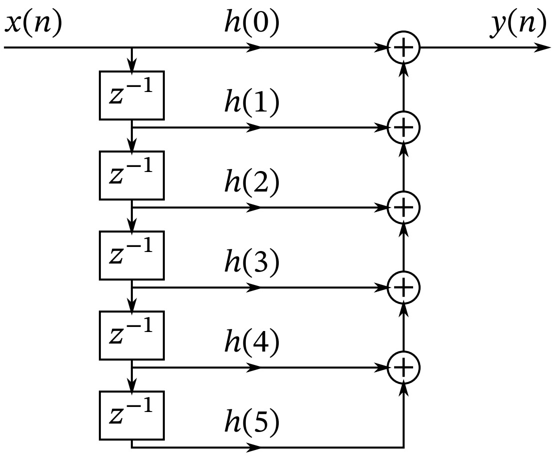

Figure 1: A fifth-order FIR system.

The output must be a NumPy array of the same length and type as the input.

The function can look something like this:

```
import numpy as np

def filter(x):
    # set y to have the same length and type as x
    y = np.zeros_like(x)
    # iterate over all input samples
    for n, x_n in enumerate(x):
        # TODO: compute y(n) and store it in y[n]
```

Hint: At the beginning of iteration n, you need to have $x(n-1)$, $x(n-2), \dots, x(n-5)$ in memory.

(10 marks)

2. Plot the sample response of the filter. This can be achieved as follows.

   (a) Generate an input sequence $x(n) = \delta(n)$ for a suitable length. You can create a zero array using `np.zeros` and then set its first sample to one.

   (b) Produce an output sequence $y(n)$ by passing the input sequence to the filter from Task 1.

   (c) Plot the output using a stem plot, with code similar to that used in

Practical 2.

(10 marks)

3. Confirm that the output $y(n)$ of the filter when the input is the unit impulse $x(n) = \delta(n)$ is indeed $h(n)$.

4. Plot the magnitude response $|H(\omega)|$ and the phase response $\sphericalangle H(\omega)$ of the filter. Since $H(\omega)$ is the Fourier transform of $h(n)$, this can be achieved by performing a Fourier transform on the impulse response generated in Task 2b.

   To plot the spectrum, you can use something like

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.fft import fft, fftfreq, fftshift

# use a 256-point DFT
H = fft(h, 256)
# frequencies ω corresponding to the output of fft
w = fftfreq(256) * 2 * np.pi
# shift so that ω = 0 is at the center
H = fftshift(H)
w = fftshift(w)

fig, ax = plt.subplots()
ax.plot(w, abs(H))
ax.set_xlabel('$\\omega$')
ax.set_ylabel('$|H(\\omega)|$')
ax.grid(True)

# TODO: similarly plot ∢H(ω) using np.angle(H)

plt.show()
```

(10 marks)

5. (a) Determine the type of filter implemented (whether it is low pass, or high pass, or something else).

   (b) Determine the filter's cut-off frequency, known as the 3 dB point. The 3 dB point is the frequency at which the power $|H(\omega)|^2$ is attenuated by 3 dB, that is the power is attenuated by a factor of $10^{3/10} = 2$. This requires $|H(\omega)|$ to be $\sqrt{1/2} = 0.7071$.

(c) Determine the frequency of any null points. Null points are points where $H(\omega)$ is zero.

(d) Comment on the phase response.

(10 marks)

6. Determine the magnitude and phase response of the system analytically. This can be performed using the following steps.

   (a) Evaluate the Fourier transform of $h(n)$ given by

   $$H(\omega) = \sum_{n=-\infty}^{\infty} h(n)\mathrm{e}^{-\mathrm{j}\omega n}$$

   where $h(n)$ is the same as in Task 1. Expand the summation, such that $H(\omega)$ is expressed as a sum of six terms of the form $c\mathrm{e}^{-\mathrm{j}\omega n}$.

   (b) Group the terms in pairs that have equal coefficients. You should have three pairs.

   (c) Two terms with equal coefficients can be expressed as

   $$\begin{aligned} c(\mathrm{e}^{-\mathrm{j}\omega l} + \mathrm{e}^{-\mathrm{j}\omega m}) &= c\mathrm{e}^{-\mathrm{j}\omega(l+m)/2}(\mathrm{e}^{-\mathrm{j}\omega(l-m)/2} + \mathrm{e}^{-\mathrm{j}\omega(m-l)/2}) \\ &= 2c\mathrm{e}^{-\mathrm{j}\omega(l+m)/2}\cos\frac{\omega(l-m)}{2} \end{aligned}$$

   Apply this to all the pairs with equal coefficients.

   (d) Rewrite the whole expression such that $H(\omega)$ is expressed in the form

   $$H(\omega) = a(\omega)\mathrm{e}^{-\mathrm{j}b(\omega)}$$

   where $a(\omega)$ and $b(\omega)$ are both real functions.

(10 marks)

7. Plot the expression $H(\omega)$ obtained in Task 6 and compare to the magnitude and phase response obtained experimentally and plotted in Task 4. Comment on any possible differences. **(10 marks)**

8. Change the input source $x(n)$ to be 32 samples of a sinusoid with a frequency equal to smallest null point frequency determined in Task 5c.

```python
# w0 is the frequency of the null point
x = np.cos(np.arange(32) * w0)
```

Plot the response using a stem plot and comment on the observed output. **(10 marks)**

9. The impulse response $h(n)$ of the filter is changed by multiplying by the sequence $\{\underset{\uparrow}{1}, -1, 1, -1, \ldots\}$ such that now it is

$$h'(n) = \{-0.\underset{\uparrow}{0}147, -0.173, 0.342, -0.342, 0.173, 0.0147\}$$

Repeat Tasks 1–5 for the new impulse response. **(20 marks)**

10. Compare the filter with the new impulse response $h'(n)$ to the original filter $h(n)$. **(10 marks)**

Report

- If there are answers that require some calculations on plot readings, the report must show both the actual readings and the calculated answers.
- Include any general observations and comments in your report.

Acknowledgements

This practical is based on a similar practical by Victor Buttigieg.