CCE3206—Digital Signal Processing

Tutorial sheet 1

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1. Sketch the sequences
   (a) $x(n) = \mathrm{U}(n)$
   (b) $x(n) = \mathrm{U}(n-3)$
   (c) $x(n) = \mathrm{U}(n) - \mathrm{U}(n-3)$
   (d) $x(n) = (-1)^n$
   (e) $x(n) = (-1)^n\big(\mathrm{U}(n) - \mathrm{U}(n-5)\big)$
2. The output sequence $y(n)$ of an LTI system is given by

$$y(n) = x(n) * h(n)$$

   where $x(n)$ is the input sequence and $h(n)$ is the impulse response of the system.

   (a) Determine $y(n)$ when
      i. $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$ $\quad h(n) = \{1/4, \underset{\uparrow}{1/2}, 1/4\}$
      ii. $x(n) = \{\underset{\uparrow}{1}, -1, 1, -1, 1\}$ $\quad h(n) = \{1/4, \underset{\uparrow}{1/2}, 1/4\}$
      iii. $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$ $\quad h(n) = \{-1/4, \underset{\uparrow}{1/2}, -1/4\}$
      iv. $x(n) = \{\underset{\uparrow}{1}, -1, 1, -1, 1\}$ $\quad h(n) = \{-1/4, \underset{\uparrow}{1/2}, -1/4\}$

v. $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$ $\quad h(n) = \{\underset{\uparrow}{4/7}, 2/7, 1/7\}$

vi. $x(n) = \{\underset{\uparrow}{1}, -1, 1, -1, 1\}$ $\quad h(n) = \{\underset{\uparrow}{4/7}, 2/7, 1/7\}$

(b) Determine the energy $E_x = \sum |x(n)|^2$ of the four input sequences in Part (a).

(c) Determine the energy $E_y = \sum |y(n)|^2$ of the four output sequences in Part (a).

3. The output $y(n)$ of an LTI system is given by

$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$$

where $x(n)$ is the input sequence, and b_0, b_1 and b_2 are constants.

(a) Write down the impulse response $h(n)$.

(b) Assuming that the input is causal, that is $x(n) = 0$ for all $n < 0$, write down expressions for $y(n)$ for the range $0 \leq n \leq 4$. You should have five expressions for $y(0), y(1), \ldots$

(c) Write down an expression for $y(n)$ in terms of $h(0), h(1), h(2)$ and $x(n)$.

(d) Write down an expression for $y(n)$ in terms of $h(n)$ and $x(n)$. The expression should be in the form of a summation.

4. The impulse response of an LTI system is given by

$$h(n) = \{1, \underset{\uparrow}{4}, 1.5, 0.5\}$$

The system is excited by the input

$$x(n) = \cos(0.7n)\mathrm{U}(n)$$

Calculate the output $y(n)$ when $n = 7$, that is $y(7)$.

5. A system is described by the difference equation

$$y(n) = 0.9y(n-1) - 0.2y(n-2) + 2x(n)$$

(a) Determine the first four values of the impulse response: $h(0), h(1), h(2)$ and $h(3)$.
Hint: To find the impulse response, find the output $y(n)$ assuming that the system is relaxed at $n = 0$, that is $y(-1) = y(-2) = 0$, and that the input $x(n) = \delta(n)$.

(b) The system has a homogenous solution

$$y_{\mathrm{h}}(n) = \big(C_1(0.4)^n + C_2(0.5)^n\big)\mathrm{U}(n)$$

where C_1 and C_2 are constants. This has the effect that the impulse response will be of the form

$$h(n) = \big(C_1(0.4)^n + C_2(0.5)^n\big)\mathrm{U}(n)$$

Using this equation for $h(n)$ and your answer to Part (a), determine the constants C_1 and C_2, and hence write an expression for $h(n)$ in terms of n.

6. Using a similar method to Question 5, determine the impulse response $h(n)$ to the system described by the difference equation

$$y(n) = 0.9y(n-1) - 0.2y(n-2) + Kx(n)$$

where K is a constant. The only difference from the system in Question 5 is that here K is used instead of the constant 2. Again, the impulse response will be of the form

$$h(n) = \big(C_1(0.4)^n + C_2(0.5)^n\big)\mathrm{U}(n)$$

Your answer should be an expression in terms of n and K.

7. From the answer to Question 6, deduce the impulse response of the systems described by the difference equations

   (a) $y(n) = 0.9y(n-1) - 0.2y(n-2) - x(n)$

   (b) $y(n) = 0.9y(n-1) - 0.2y(n-2) + 3x(n)$

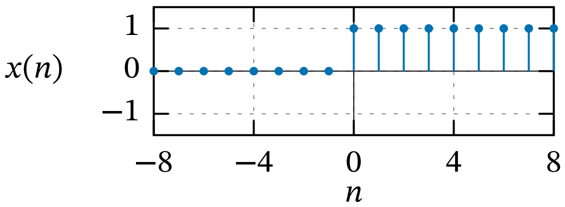
   (c) $y(n) = 0.9y(n-1) - 0.2y(n-2) - x(n-1)$
   Hint: Here, the impulse response should be the same as the impulse response of Part (a) delayed by one sample.

   (d) $y(n) = 0.9y(n-1) - 0.2y(n-2) - x(n-2)$

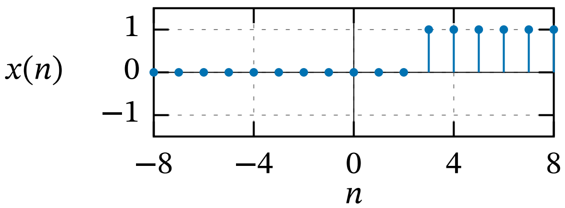
   (e) $y(n) = 0.9y(n-1) - 0.2y(n-2) + 2x(n) - x(n-2)$

Answers

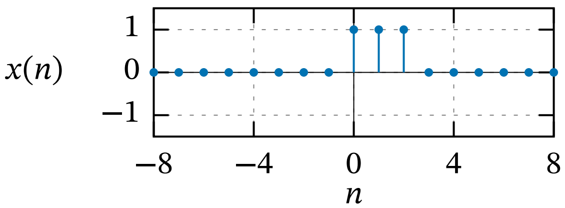
1. (a)



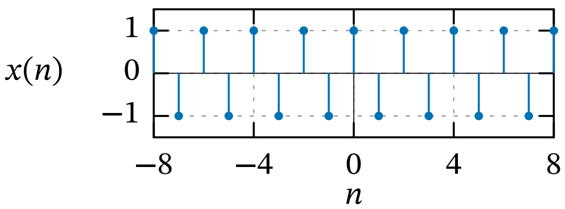
(b)



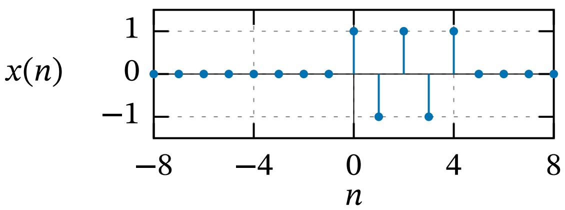
(c)



(d)



(e)



2. (a) i. $y(n) = \{\underset{\uparrow}{1/4}, 3/4, 1, 1, 1, 3/4, 1/4\}$

ii. $y(n) = \{\underset{\uparrow}{1/4}, 1/4, 0, 0, 0, 1/4, 1/4\}$

iii. $y(n) = \{\underset{\uparrow}{-1/4}, 1/4, 0, 0, 0, 1/4, -1/4\}$

iv. $y(n) = \{\underset{\uparrow}{-1/4}, 3/4, -1, 1, -1, 3/4, -1/4\}$

v. $y(n) = \{\underset{\uparrow}{4/7}, 6/7, 1, 1, 1, 3/7, 1/7\}$

vi. $y(n) = \{\underset{\uparrow}{4/7}, -2/7, 3/7, -3/7, 3/7, 1/7, 1/7\}$

(b) i. 5

ii. 5

iii. 5

iv. 5

v. 5

vi. 5

(c) i. 4.25

ii. 0.25

iii. 0.25

iv. 4.25

v. 4.265

vi. 1

3. (a) $h(n) = \{\underset{\uparrow}{b_0}, b_1, b_2\}$

(b) $y(0) = b_0 x(0)$

$y(1) = b_0 x(1) + b_1 x(0)$

$y(2) = b_0 x(2) + b_1 x(1) + b_2 x(0)$

$y(3) = b_0 x(3) + b_1 x(2) + b_2 x(1)$

$y(4) = b_0 x(4) + b_1 x(3) + b_2 x(2)$

(c) $y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2)$

(d) $y(n) = \sum_{k=0}^{2} h(k)x(n-k)$

4. 0.318

5. (a) $h(0) = 2$

$h(1) = 1.8$

$h(2) = 1.22$

$h(3) = 0.738$

(b) $C_1 = 8$

$C_2 = 10$

$h(n) = [-8(0.4)^n + 10(0.5)^n]\mathrm{U}(n)$

6. $h(n) = [-4K(0.4)^n + 5K(0.5)^n]\mathrm{U}(n)$

7. (a) $h(n) = [4(0.4)^n - 5(0.5)^n]\mathrm{U}(n)$

(b) $h(n) = [-12(0.4)^n + 15(0.5)^n]\mathrm{U}(n)$

(c) $h(n) = [10(0.4)^n - 10(0.5)^n]\mathrm{U}(n-1)$

(d) $h(n) = [25(0.4)^n - 20(0.5)^n]\mathrm{U}(n-2)$

(e) $h(n) = 2\delta(n) + 1.8\delta(n-1) + [17(0.4)^n - 10(0.5)^n]\mathrm{U}(n-2)$