CCE2503_Assignment_2024-2025

May 6, 2025

1 CCE2503 Search and Optimisation

1.1 Assignment

1.1.1 Year 2024-2025 - Semester II

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- Minor edits: Johann A. Briffa, 2025

1.1.2 Write your name, ID Card Number and class below.

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- ID Number: 0352804L
- Class:

1.2 Instructions:

In this assignment you will be looking for a minimiser for the 2D Griewangk function using the following methods.

1. Standard Gradient descent
2. Random search
3. Random Search + Gradient descent
4. Simulated annealing

You will also draw convergence plots and compare the methods with each other.

You may be using some of the functions you had developed during the preparatory stage, e.g. standard gradient descent and python boilerplate code for plotting contour surfaces, 3D wireframes, timing code, counting function calls etc.

1.3 Notes:

1. With the exception of Gradient Descent, for which you shall modify the one in the preparatory exercises notebook, program all optimisation algorithms from scratch.

2. You can only make use of the python packages included in the notebook in its original form, i.e. ('matplotlib.pyplot', 'mpl_toolkits.mplot3d', 'numpy' and 'time')

3. Cite any blogs, code repositories and/or generative AI tools (e.g. ChatGPT) used in completing this assignment. In the case of generative AI tools, explain how these tools were used.
4. This work is to be attempted individually. It is essential that the work you eventually submit and present for your assignment consists only of your own work; use of copied material will be treated as plagiarism. Discussion is only permitted on general issues, and it is absolutely forbidden to discuss specific details with anyone and/or share results.
5. Please sign the plagiarism form that can be found here: https://www.um.edu.mt/ict/students/formsguidelines/
6. Your submission consists of two components:
   - The python notebook, with all cells executed.
   - The signed plagiarism form.

1.4 Preparation:

[1]:
```
# We first load the required modules
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm
import numpy as np
import time
%matplotlib inline
```

The Griewangk function is:

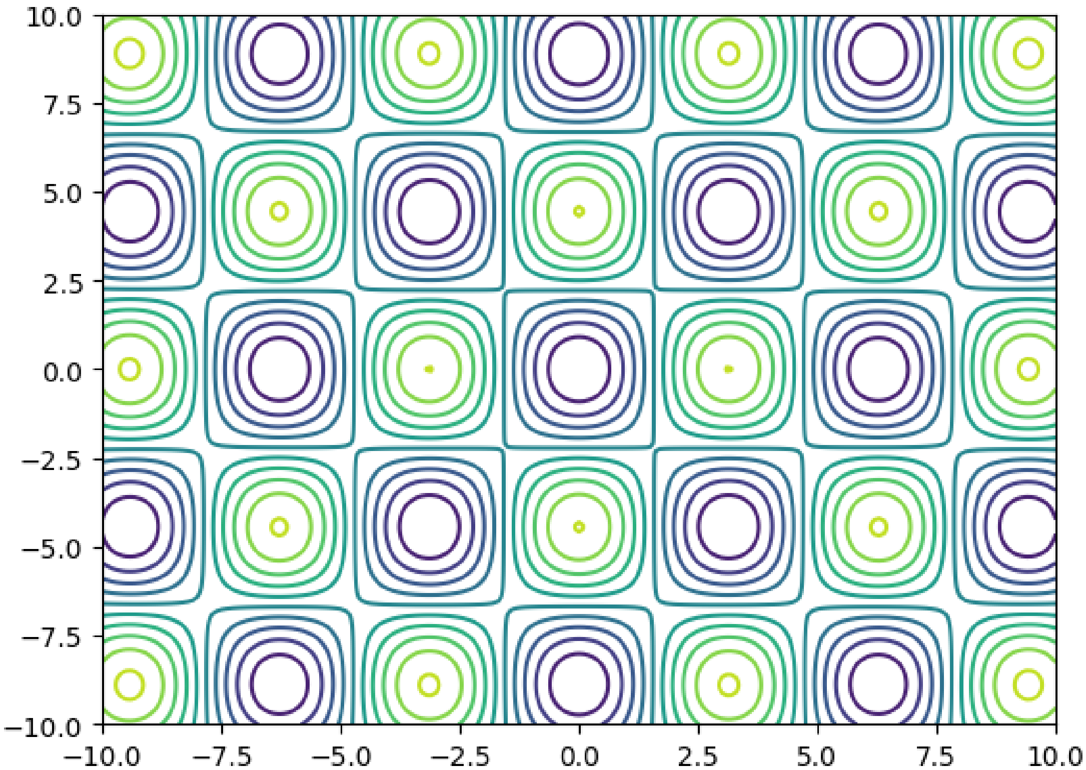
$G(\mathbf{x}) = 1 + \frac{1}{4000}x_1^2 + \frac{1}{4000}x_2^2 - cos(x_1)cos(\frac{\sqrt(2)}{2}x_2)$

[2]:
```
# Define objective function to minimize
def Griewangk(x, a=1, b=5):
    """
    Rosenbrock's function is a 2D uni-modal function
    This implementation follows the definition in Kochenderfer & Wheeler, 2019
    x : x is a numpy array of dimensions [2,m], where m is the number of 2D␣
↪points
    """
    return 1+ x[0]**2/4000 + x[1]**2/4000 - np.cos(x[0])*np.cos(0.5*x[1]*np.
↪sqrt(2))
```

[3]:
```
# Plot contour plot in the search space (xmin=-10, ymin=-10), (xmin=10, ymin=10)
N=201
limit=10
xx = np.linspace(-limit, limit, N)
yy = np.linspace(-limit, limit, N)
X = np.repeat(xx,N).reshape(N,N)
```

```
Y = np.tile(yy,N).reshape(N,N)
#
x = np.column_stack((X.flatten(),Y.flatten())).T
Z = Griewangk(x).reshape(N,N)
#
plt.contour(X,Y,Z, levels=10)
```

[3]: <matplotlib.contour.QuadContourSet at 0x7fcb40c30fd0>

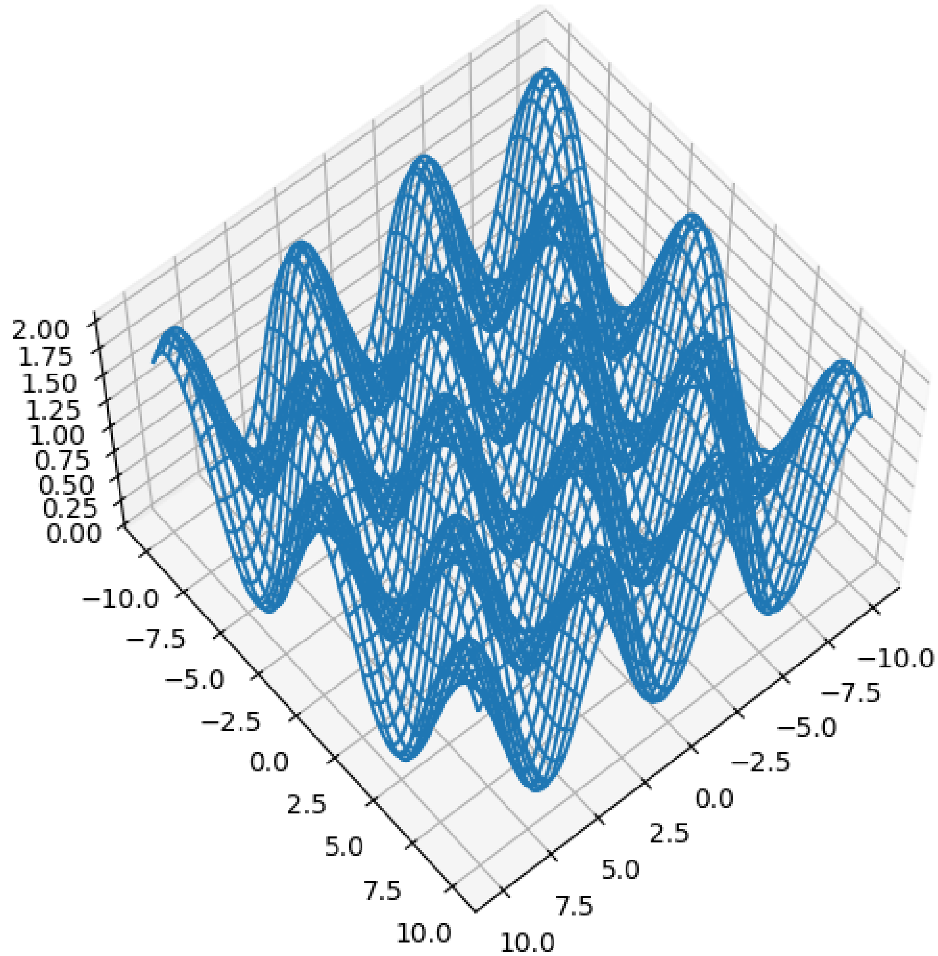


[4]:
```
# Find the location of the minimum value in Z
print(Z.shape)
idx = np.unravel_index(np.argmin(Z, axis=None), Z.shape)
print(idx, '(',X[idx], ', ', Y[idx],')', Z[idx])
print('Therefore global minimum is at (0.0, 0.0)')
```

```
(201, 201)
(np.int64(100), np.int64(100)) ( 0.0 ,  0.0 ) 0.0
Therefore global minimum is at (0.0, 0.0)
```

[5]:
```
# Plot a 3D wireframe of the objective function
#
fig = plt.figure(figsize=(6, 6))
```

```
ax = fig.add_subplot(111, projection='3d')
ax.plot_wireframe(X, Y, Z, rstride=4, cstride=4, cmap=cm.jet)
#ax.plot_wireframe(X, Y, np.log(Z), rstride=2, cstride=2)
#ax.view_init(0, 90) # elevation, azimuth
ax.view_init(60, 50) # elevation, azimuth
```



[6]:
```
# Random seed for reproducibility
# Comment out for different results, to see the nature of stochastic algorithms
np.random.seed(0)
#np.random.seed(42)
#np.random.seed(99)
```

1.5 Q1

Modify the gradient descent algorithm with the derivate of the $G(\mathbf{x})$ and starting from a random position in the search space $(xmin = -10, ymin = -10)$, $(xmax = 10, ymax = 10)$, find a minimiser of $G(\mathbf{x})$.

Execute the process 100 times and plot the average value of $G(\mathbf{x})$ at each iteration. In addition compute the standard deviation of the final values.

1.5.1 [15 marks]

$$G(\mathbf{x}) = 1 + \frac{1}{4000}x_1^2 + \frac{1}{4000}x_2^2 - \cos(x_1)\cos\left(\frac{\sqrt{2}}{2}x_2\right)$$

- x_1 and x_2 are the two input variables of the function.
- Let $G(\mathbf{x})$ be the function to minimize.

x_1 partial derivative:

$$\frac{\partial G}{\partial x_1} = \frac{1}{2000}x_1 + \sin(x_1)\cos\left(\frac{\sqrt{2}}{2}x_2\right)$$

x_2 partial derivative:

$$\frac{\partial G}{\partial x_2} = \frac{1}{2000}x_2 + \cos(x_1)\cdot\frac{\sqrt{2}}{2}\cdot\sin\left(\frac{\sqrt{2}}{2}x_2\right)$$

Gradient vector:

$$\nabla G(\mathbf{x}) = \begin{bmatrix} \frac{1}{2000}x_1 + \sin(x_1)\cos\left(\frac{\sqrt{2}}{2}x_2\right) \\ \frac{1}{2000}x_2 + \frac{\sqrt{2}}{2}\cos(x_1)\sin\left(\frac{\sqrt{2}}{2}x_2\right) \end{bmatrix}$$

This gradient can now be used in the gradient descent algorithm to iteratively minimize $G(\mathbf{x})$.

[7]:
```
# Now we can encapsulate the gradient step from the calculation of the gradient
def gradient_step(x, alpha):

    # Gradient step for the Griewangk function
    dx = (1/2000)*x[0] + np.sin(x[0]) * np.cos((np.sqrt(2)/2) * x[1])
    dy = (1/2000)*x[1] + (np.sqrt(2)/2) * np.cos(x[0]) * np.sin((np.sqrt(2)/2)␣
↪* x[1])

    #Gradient is a 2D vector
    grad = np.array([dx, dy])

    # Update x using gradient descent
    return x - alpha * grad
```

[8]:
```
# Updated gradient descent algorithm to use the respective gradient
def grad_descent(alpha=0.1, iterations=20):
```

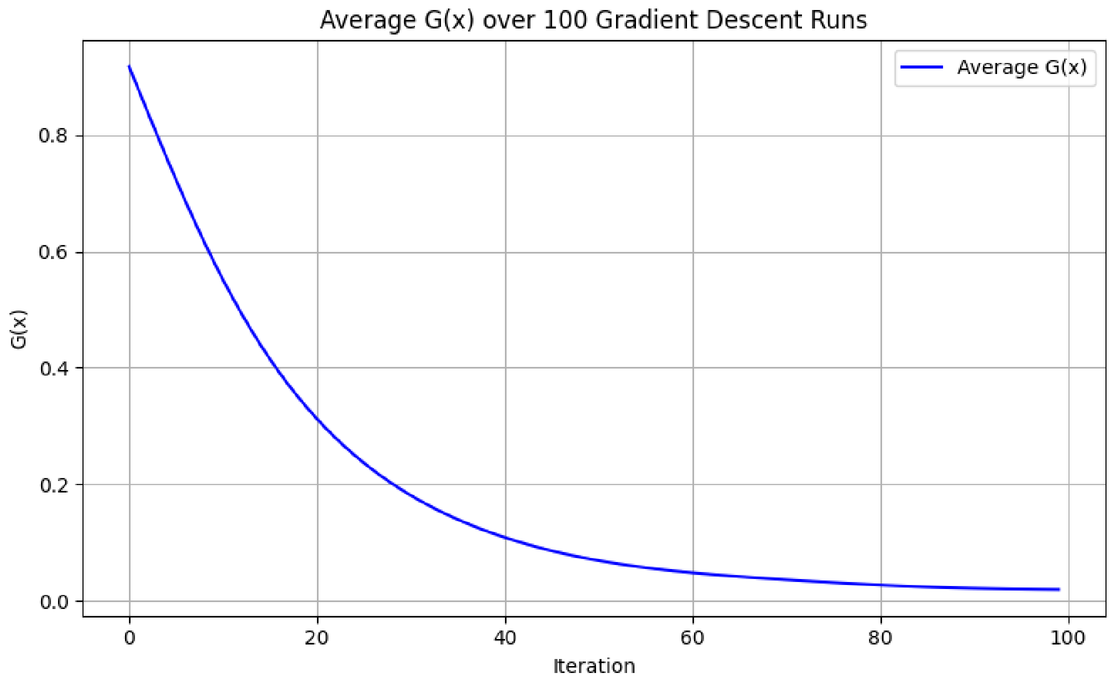
```python
    # Randomly initialize x0 in the range [-10, 10] (once per run)
    x= np.random.uniform(-10, 10, size=2)

    # Initialize history of objective values
    history = np.zeros(iterations)

    # Iterate over the number of iterations
    for i in range(iterations):

        # Perform gradient step
        x = gradient_step(x, alpha)

        # Store objective value making use of the predefined function
        history[i] = Griewangk(x)

    return history
```

[9]:

```python
runs = 100 # Executing the process for 100 times
iterations = 100 # Increased iterations as 20 gave early convergence
alpha = 0.1 # Same alpha as before

# Initialize an array to store the results of each run
gd_runs = np.zeros((runs, iterations))


# Execute the gradient descent algorithm for 100 runs
for r in range(runs):

    # Run gradient descent and store the result
    gd_runs[r] = grad_descent(alpha, iterations)

# Average G(x) values over all runs
avg_gx = np.mean(gd_runs, axis=0)

# Standard deviation of final G(x) values
std_gd = np.std(gd_runs[:, -1])
print(f"Standard deviation of final values after {iterations} iterations:␣
 ↪{std_gd:.5f}")

# Plot the average G(x) over 100 runs
plt.figure(figsize=(8, 5))
plt.plot(avg_gx, label='Average G(x)', color='blue')
plt.title('Average G(x) over 100 Gradient Descent Runs')
plt.xlabel('Iteration')
plt.ylabel('G(x)')
plt.grid(True)
plt.legend()
```

```
plt.tight_layout()
plt.show()
```

```
Standard deviation of final values after 100 iterations: 0.01260
```



This implementation applies gradient descent using the manually derived gradient of the Griewangk function. A fixed learning rate is used, and the optimizer is run 100 times from random initial positions to observe average convergence behavior. While efficient in reducing the objective function, the method can be seen to be sensitive to initial conditions and may converge to local minima.

1.6 Q2

Implement a random search algorithm to find a minimiser of $G(\mathbf{x})$ within the search space $(xmin = -10, ymin = -10), (xmax = 10, ymax = 10)$.

Execute the process 100 times and plot the average value of $G(\mathbf{x})$ at each iteration. In addition compute the standard deviation of the final values.

1.6.1 [15 marks]

[10]:
```
# Random search for minimizing Griewangk function
def random_search(iterations=100):
    # Initialize best value tracker and history array
    best_val = float('inf')
```

```python
    history = np.zeros(iterations)

    # Iterate over the number of iterations
    for i in range(iterations):

        # Generate random x in the range [-10, 10] (once per iteration)
        x = np.random.uniform(-10, 10, size=2)

        # Evaluate the objective function
        val = Griewangk(x)

        # Update best value if current value is lower
        if val < best_val:
            best_val = val

        # Store the best value in history
        history[i] = best_val

    return history
```

[11]:
```python
runs = 100 # Constant number of runs
iterations = 100 # Constant number of iterations

# Initialize an array to store the results of each run
rs_runs = np.zeros((runs, iterations))

for r in range(runs):
    rs_runs[r] = random_search(iterations=iterations)

# Average and std of G(x) values over all runs
avg_gx = np.mean(rs_runs, axis=0)
std_rs = np.std(rs_runs[:, -1])

# Print the standard deviation of final G(x) values
print(f"Standard deviation of final values after {iterations} iterations:␣
 ↪{std_rs:.5f}")

# Plot the average G(x) over 100 runs
plt.figure(figsize=(8, 5))
plt.plot(avg_gx, label='Average Best G(x)', color='orange')
plt.title('Random Search: Average Best G(x) over 100 Runs')
plt.xlabel('Iteration')
plt.ylabel('Best-so-far G(x)')
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.show()
```

```
Standard deviation of final values after 100 iterations: 0.02543
```

This implementation uses a pure random search strategy to minimize the Griewangk function. At each iteration, a new random point in the search space is sampled, and the best-so-far value is tracked. The process is repeated 100 times to obtain a robust estimate of average performance. Although simple and easy to implement, the method relies heavily on chance and shows high variability in results, as seen in the final standard deviation of **0.02543**.

1.7 Q3

Implement a two step approach:

1. In the first step carry out a random search and find minimiser $\mathbf{x}^*_{initial}$, within the search space $(xmin = -10, ymin = -10), (xmax = 10, ymax = 10)$.
2. In the second step use gradient descent and starting from $\mathbf{x}^*_{initial}$ find minimiser $\mathbf{x}^*_{final}$ within the search space $(xmin = -10, ymin = -10), (xmax = 10, ymax = 10)$.

Execute the process 100 times and plot the average value of $G(\mathbf{x})$ at each iteration. In addition compute the standard deviation of the final values.

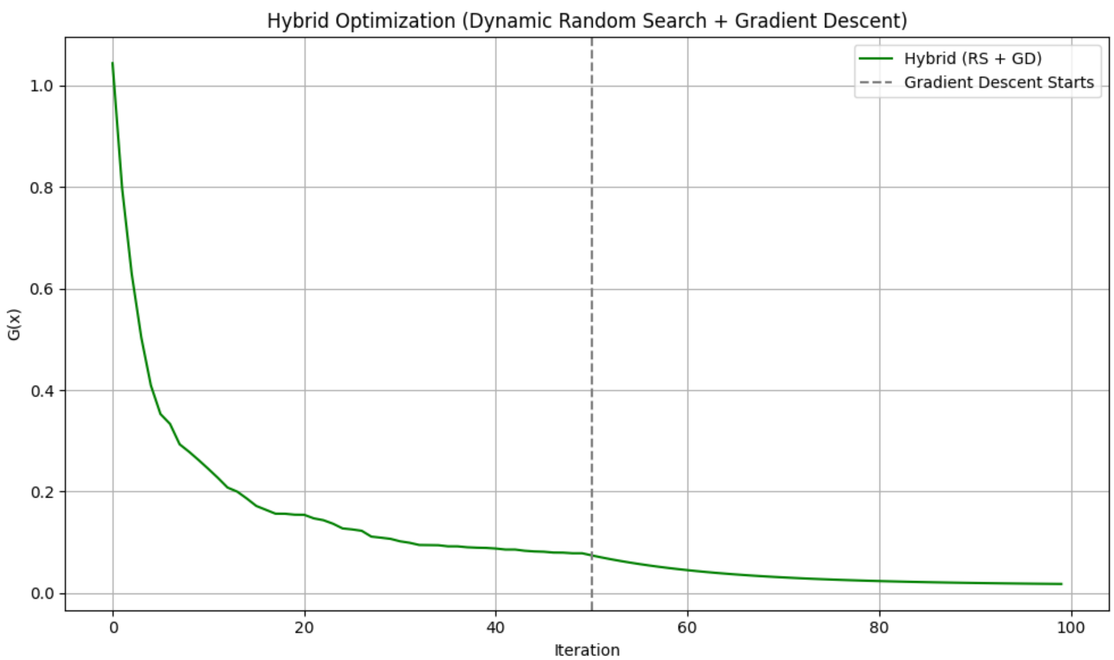
1.7.1 [10 marks]

[12]:
```
# Hybrid optimizer: Random Search + Gradient Descent
def hybrid_optimizer(random_steps=50, gd_steps=50, alpha=0.05):
    # Initialize best values
    best_val = float('inf')
    best_x = None

    # Initialize history of objective values
    history = np.zeros(random_steps + gd_steps)

    # Conduct random search
    for i in range(random_steps):

        # Generate random x in the range [-10, 10] (once per iteration)
        x = np.random.uniform(-10, 10, size=2)

        # Evaluate the objective function
        val = Griewangk(x)

        # Update best value if current value is lower
        if val < best_val:
            # Update best value
            best_val = val
            # Update best x
            best_x = x

        # Store the best value in history
        history[i] = best_val  # track best value at each RS step

    # We do not need to reinitialize x here for gradient descent as it is taken␣
↪from the best x found in random search
    # Copy best x found during random search to x for gradient descent
    x = best_x.copy()
    # Perform gradient descent from the best x found
    for i in range(gd_steps):
        # Perform gradient step
        x = gradient_step(x, alpha)
        # Store objective value
        history[random_steps + i] = Griewangk(x)

    return history
```

[13]:
```
runs = 100 # Total number of runs remains 100
rs_steps = 50 # Random search steps
gd_steps = 50 # Gradient descent steps
total_steps = rs_steps + gd_steps
```

```
# Initialize an array to store the results of each run
hy_runs = np.zeros((runs, total_steps))

# Execute the hybrid optimizer for 100 runs
for r in range(runs):
    hy_runs[r] = hybrid_optimizer(rs_steps, gd_steps, alpha=0.05)

# Average and std of G(x) values over all runs
avg_gx = np.mean(hy_runs, axis=0)
std_hy = np.std(hy_runs[:, -1])

# Standard deviation of final G(x) values
print(f"Standard deviation of final values after {total_steps} iterations: 
 ↪{std_hy:.5f}")

# Plot the average G(x) over 100 runs
plt.figure(figsize=(10, 6))
plt.plot(avg_gx, label='Hybrid (RS + GD)', color='green')
# Plot a dividing line between random search and gradient descent
plt.axvline(rs_steps, linestyle='--', color='gray', label='Gradient Descent 
 ↪Starts')
plt.title('Hybrid Optimization (Dynamic Random Search + Gradient Descent)')
plt.xlabel('Iteration')
plt.ylabel('G(x)')
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.show()
```

```
Standard deviation of final values after 100 iterations: 0.01103
```



This implementation combines random search with gradient descent in a two-phase hybrid optimizer. The algorithm first explores the search space randomly to locate a promising starting point, then switches to gradient descent for local refinement. The result is a method that balances exploration and exploitation effectively. The hybrid method achieved a final standard deviation of **0.01103**, making it one of the most consistent and reliable approaches overall.

1.8 Q4

Implement a simulated annealing search algorithm to find a minimiser of $G(\mathbf{x})$ within the search space $(xmin = -10, ymin = -10), (xmax = 10, ymax = 10)$.

Execute the process 100 times and plot the average value of $G(\mathbf{x})$ at each iteration. In addition compute the standard deviation of the final values.

1.8.1 [30 marks]

[ ]:
```
# Simulated Annealing for minimizing Griewangk function
def simulated_annealing(iterations=100, temp_start=1.0, temp_end=0.01):

    # Randomly initialize x0 in the range [-10, 10] (once per run)
    x = np.random.uniform(-10, 10, size=2)

    # Initialize current value and best value
    current_val = Griewangk(x)
    best_val = current_val
```

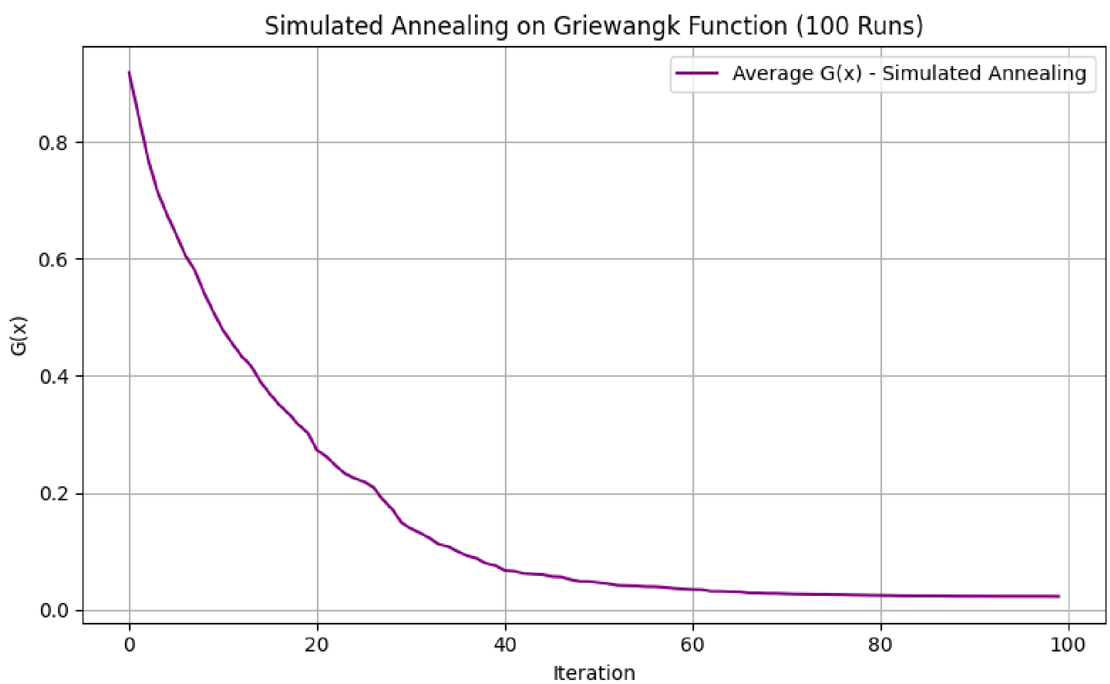
```
    # Initialize history of objective values
    history = np.zeros(iterations)

    # Iterate over the number of iterations
    for i in range(iterations):

        # Cooling schedule (exponential decay)
        temp = temp_start * (temp_end / temp_start) ** (i / iterations)

        # Small random step (perturbation)
        x_new = x + np.random.normal(0, 0.5, size=2)
        x_new = np.clip(x_new, -10, 10)

        # Evaluate the objective function at the new point
        new_val = Griewangk(x_new)

        # Calculate the change in value
        delta = new_val - current_val

        # Accept new point if better or with probability
        if delta < 0 or np.random.rand() < np.exp(-delta / temp):
            x = x_new
            current_val = new_val

        # Update best if needed
        if current_val < best_val:
            best_val = current_val

        # Store the best value in history
        history[i] = best_val

    return history
```

```
  Cell In[18], line 44
    """
    ^
SyntaxError: incomplete input
```

[15]:
```
runs = 100 # Number of runs
iterations = 100 # Number of iterations

# Initialize an array to store the results of each run
sa_runs = np.zeros((runs, iterations))
```

```
# Execute the simulated annealing algorithm for 100 runs
for r in range(runs):
    # Run simulated annealing and store the result
    sa_runs[r] = simulated_annealing(iterations=iterations)

# Average and std of G(x) values over all runs
avg_gx = np.mean(sa_runs, axis=0)
std_sa = np.std(sa_runs[:, -1])
# Print the standard deviation of final G(x) values
print(f"Standard deviation of final values after {iterations} iterations:␣
 ↪{std_sa:.5f}")

# Plot the average G(x) over 100 runs
plt.figure(figsize=(8, 5))
plt.plot(avg_gx, label='Average G(x) - Simulated Annealing', color='purple')
plt.title('Simulated Annealing on Griewangk Function (100 Runs)')
plt.xlabel('Iteration')
plt.ylabel('G(x)')
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.show()
```

```
Standard deviation of final values after 100 iterations: 0.00993
```



This implementation uses simulated annealing to optimize the Griewangk function, gradually reducing the temperature to control exploration. The algorithm occasionally accepts worse solutions to escape local minima, especially early on. Executed over 100 runs, the method achieved a low final standard deviation of **0.00993**, indicating reliable performance and effective convergence through probabilistic search.

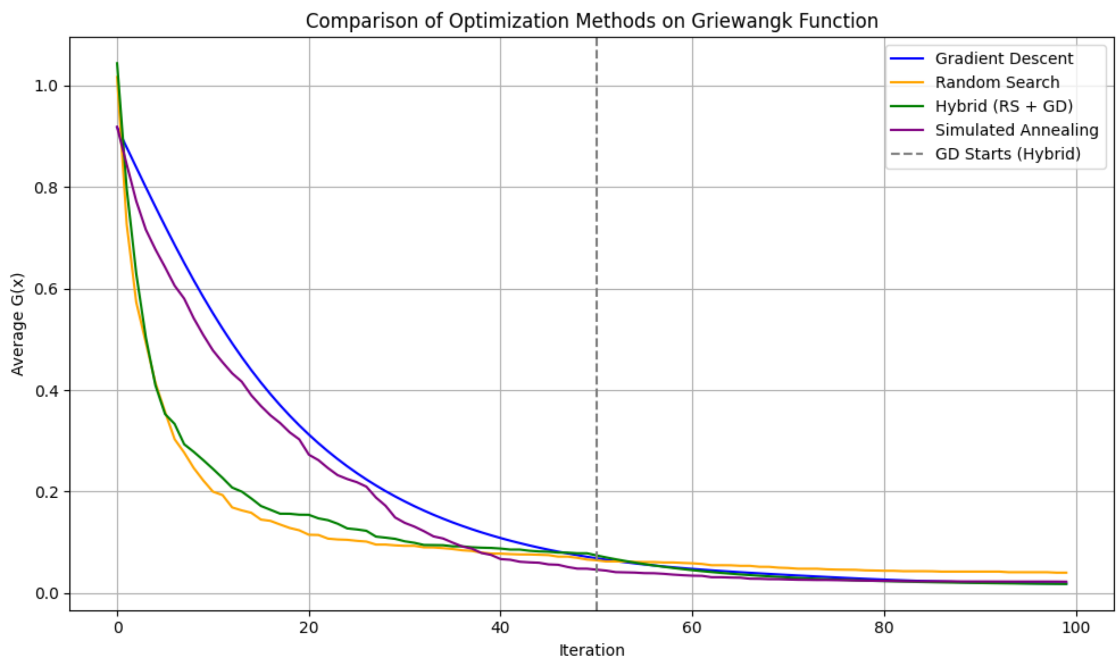
1.9 Q5

Superimpose all plots from all four optimisation methods and compare the algorithms in terms of rate of convergence, effectiveness and robustness.

1.9.1 [30 marks]

[16]:
```python
# Compute average curves from each optimizer
avg_gd = np.mean(gd_runs, axis=0)
avg_rs = np.mean(rs_runs, axis=0)
avg_hy = np.mean(hy_runs, axis=0)
avg_sa = np.mean(sa_runs, axis=0)

# Print all the final standard deviations of each method
print(f"Gradient Descent final std: {std_gd:.5f}")
print(f"Random Search final std: {std_rs:.5f}")
print(f"Hybrid final std: {std_hy:.5f}")
print(f"Simulated Annealing final std: {std_sa:.5f}")

# Plot all in one figure
plt.figure(figsize=(10, 6))
plt.plot(avg_gd, label='Gradient Descent', color='blue')
plt.plot(avg_rs, label='Random Search', color='orange')
plt.plot(avg_hy, label='Hybrid (RS + GD)', color='green')
plt.plot(avg_sa, label='Simulated Annealing', color='purple')
plt.axvline(x=50, color='gray', linestyle='--', label='GD Starts (Hybrid)')
plt.title('Comparison of Optimization Methods on Griewangk Function')
plt.xlabel('Iteration')
plt.ylabel('Average G(x)')
plt.grid(True)
plt.legend()
plt.tight_layout()
plt.show()
```

```
Gradient Descent final std: 0.01260
Random Search final std: 0.02543
Hybrid final std: 0.01103
Simulated Annealing final std: 0.00993
```



1.9.2 Stochastic Optimization Random Natrue

Due to the randomized nature of all the optimization algorithms, final performance metrics may vary slightly with each full run of the notebook. This variability is expected in stochastic optimization and highlights the importance of analyzing performance over many runs and trials.

To illustrate this, we ran each optimizer using three different fixed random seeds: `0`, `42`, and `99`.

Seed	Gradient Descent	Random Search	Hybrid (RS + GD)	Simulated Annealing
0	0.01260	0.02543	0.01103	0.00993
42	0.01149	0.02666	0.01161	0.01225
99	0.01614	0.02330	0.01044	0.01170

- **Seed 0**:
  Simulated Annealing had the lowest standard deviation (0.00993), indicating very stable performance across runs. The hybrid method followed closely (0.01103), while Random Search showed the most fluctuation (0.02543), consistent with its blind search nature.
- **Seed 42**:
  Here, Gradient Descent produced the most stable results (0.01149), slightly outperforming the hybrid. Interestingly, Simulated Annealing exhibited a slight increase in variance (0.01225) compared to seed 0, possibly due to a less favorable initialization path.
- **Seed 99**:
  The Hybrid method achieved the lowest deviation (0.01044), reinforcing its consistency. Gradient Descent and Simulated Annealing showed moderate variance, while Random Search remained the least stable.

1.9.3 Overall Evaluation

Across all seeds, the **Hybrid (RS + GD)** method consistently demonstrated the lowest or near-lowest standard deviation, making it the most robust and reliable optimizer in this study. Its combination of global exploration (via random search) and local refinement (via gradient descent) allows it to avoid poor local minima while still converging efficiently.

Simulated Annealing showed competitive performance, particularly under certain seeds like `0`, where it even outperformed the hybrid method. This confirms its strength in escaping local minima thanks to its probabilistic acceptance of worse solutions during early iterations.

Gradient Descent, while simple and fast, showed greater sensitivity to initialization. Its performance varied more noticeably across seeds, indicating a higher risk of convergence to suboptimal local minima. However, its convergence curve was the smoothest, visually reflecting a stable and consistent optimization path once a direction is found.

Random Search, although effective on occasion, had the highest variability across all trials, further reinforcing its nature as a brute-force method with no directional learning.

Overall, the evaluation confirms that combining stochastic exploration with informed gradient-based exploitation yields the most reliable performance on complex functions like Griewangk.