Learn Physics with Functional Programming - Scott N. Walck - Chapter 4: Exercises

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4.3 For $f(x) = x^3$, we have $f'(x) = 3x^2$, so that the relative error is defined by

$$\begin{aligned}
\text{err}(a) &= \left| \frac{f(x + a/2) - f(x - a/2)}{a} - 3x^2 \right| \\
&= \left| \frac{(x + a/2)^3 - (x - a/2)^3}{a} - 3x^2 \right| \\
&= \left| \frac{[x^3 + (3x^2a)/2 + (3xa^2)/4 + a^3/8] - [x^3 - (3x^2a)/2 + (3xa^2)/4 - a^3/8]}{a} - 3x^2 \right| \\
&= \left| \frac{3x^2a + a^3/4 - 3x^2a}{a} \right| \\
&= \left| \frac{a^3}{4} \right|.
\end{aligned}$$

Thus we have an error of 1 percent if

$$\begin{aligned}
\text{err}(a) &= 0.01 \\
\Leftrightarrow \left| \frac{a^3}{4} \right| &= 0.01 \\
\Leftrightarrow \frac{a^3}{4} &= \pm 0.01 \\
\Leftrightarrow a^3 &= \pm 0.04 \\
\Leftrightarrow a &= \pm 0.04^{(1/3)},
\end{aligned}$$

which is valid for $x = 4$ and $x = 0.1$, since $\text{err}(a)$ does not depend on x.