

Chapter 4 - Exercises

Graham Strickland

July 24, 2024

4.3 For $f(x) = x^3$, we have $f'(x) = 3x^2$, so that the relative error is defined by

$$\begin{aligned}\text{err}(a) &= \left| \frac{f(x+a/2) - f(x-a/2)}{a} - 3x^2 \right| \\ &= \left| \frac{(x+a/2)^3 - (x-a/2)^3}{a} - 3x^2 \right| \\ &= \left| \frac{[x^3 + (3x^2a)/2 + (3xa^2)/4 + a^3/8] - [x^3 - (3x^2a)/2 + (3xa^2)/4 - a^3/8]}{a} - 3x^2 \right| \\ &= \left| \frac{3x^2a + a^3/4 - 3x^2a}{a} \right| \\ &= \left| \frac{a^3}{4} \right|.\end{aligned}$$

Thus we have an error of 1 percent if

$$\begin{aligned}\text{err}(a) &= 0.01 \\ \Leftrightarrow \left| \frac{a^3}{4} \right| &= 0.01 \\ \Leftrightarrow \frac{a^3}{4} &= \pm 0.01 \\ \Leftrightarrow a^3 &= \pm 0.04 \\ \Leftrightarrow a &= \pm 0.04^{(1/3)},\end{aligned}$$

which is valid for $x = 4$ and $x = 0.1$, since $\text{err}(a)$ does not depend on x .