Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry, and Engineering 2nd Edition

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2 Flows on the Line

2.1 A Geometric Way of Thinking

2.1.1

For a fixed point of the flow $\dot{x} = \sin x$ on the line, we have

$$\dot{x} = 0 \Rightarrow \sin x = 0 \Rightarrow x = n\pi, \quad n \in \mathbb{Z}.$$

Thus all fixed points are given by $x = n\pi$, $n \in \mathbb{Z}$.

2.1.2

The points x for which the flow has the greatest velocity to the right are those for which $\dot{x} > 0$ and x is a local maximum. These are given by

$$x = \frac{(4n+1)\pi}{2}, \quad n \in \mathbb{Z}.$$

2.1.3

(a) We have

$$\dot{x} = \sin x,$$

so that

$$\begin{aligned}\ddot{x} &= \frac{d}{dt}(\sin x)\\ &= \dot{x}\cos x\\ &= \sin x \cos x\\ &= \frac{1}{2}\sin(2x).\end{aligned}$$

(b) The maximum positive acceleration is given by the local maxima of $\ddot{x} = (1/2)\sin(2x)$, i.e., where

$$2x = \frac{(4n+1)\pi}{2} \Rightarrow x = \frac{(4n+1)\pi}{4}, \quad n \in \mathbb{Z}.$$

2.1.4

(a)

(b)