

Informath:

Interlingual Autoformalization and Informalization
with Dedukti and GF

INRIA/ENS Saclay 10 April 2025

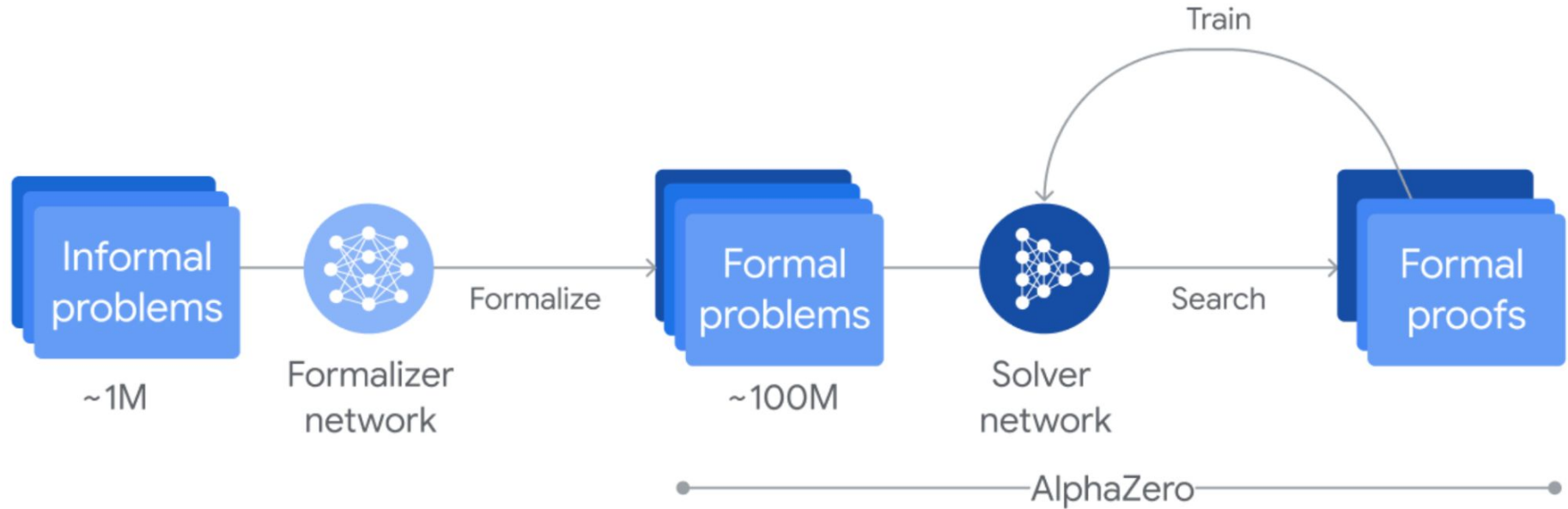
Aarne Ranta

aarne.ranta@cse.gu.se

Prologue:

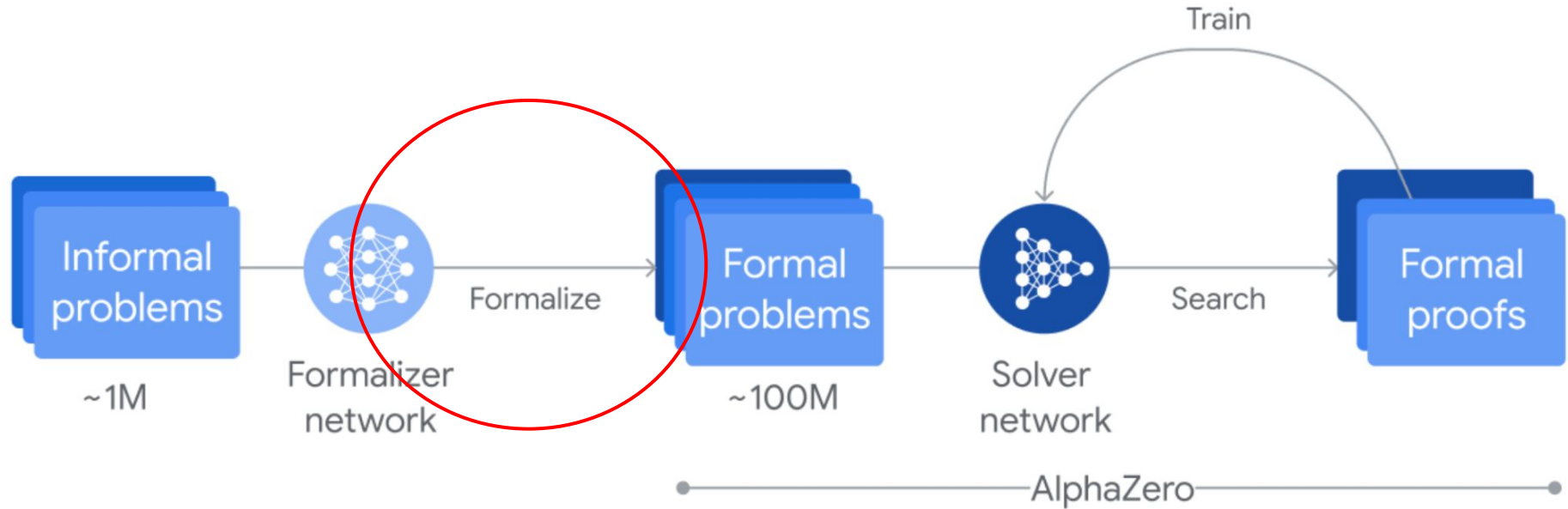
AlphaProof and Autoformalization

AI achieves silver-medal standard solving International Mathematical Olympiad problems



<https://deepmind.google/discover/blog/ai-solves-imo-problems-at-silver-medal-level/>

AI achieves silver-medal standard solving International Mathematical Olympiad problems



"First, the problems were manually translated into formal mathematical language for our systems to understand."

<https://deepmind.google/discover/blog/ai-solves-imo-problems-at-silver-medal-level/>

(In)formalization, symbolic CNL

Coscoy, Kahn & Théry 1994: Coq proofs to text

Wenzel 1999: Isabelle-Izar

Hallgren & Ranta 2000: GF-Alfa (Agda)

Paskevich 2007: ForTheL

Cramer, Koepke & al 2009: Naproche

Humayoun & Raffalli 2011: MathNat

Pathak 2023: GF-Lean

Massot 2024: Verbose-Lean4

Autoformalization, neural

Wang, Brown, Kaliczyk & Urban 2020: NMT and Mizar

Wu, Jiang, Li, Rabe, Staats, Jamnik & Szegedy 2022 : autoformalization with LLM

MULTILINGUAL MATHEMATICAL AUTOFORMALIZATION

Albert Q. Jiang

University of Cambridge
qj213@cam.ac.uk

Wenda Li

University of Edinburgh
wli8@ed.ac.uk

Mateja Jamnik

University of Cambridge
mj201@cam.ac.uk

- "informalisation is much easier than formalisation"
- uses an GPT-4 to produce the dataset MMA to fine-tune LLaMA
 - ~70% more or less acctable
- resulting autoformalization:
 - 16-18% "acceptable with minimal corrections"
- ~~symbolic informalization~~

"symbolic informalisation tools

- result in natural language content that lacks the inherent diversity and flexibility in expression: they are rigid and not natural-language-like.
- symbolic informalisation tools are hard to design and implement
- They also differ a lot for different formal languages, hence the approach is not scalable for multiple formal languages. "

Informath

The goal of Informath

Symbolic informalization that

has

- results in natural language content that ~~lacks~~ the inherent diversity and flexibility in expression: they are ~~rigid and not~~ natural-language-like.

feasible

- symbolic informalisation tools are ~~hard~~ to design and implement *with proper methods*

can be shared

- They ~~also differ a lot~~ for different formal languages, hence the approach is ~~not~~ scalable for multiple formal languages. *And even for multiple natural languages.*

Agda:

```
postulate prop110 :  
  (a : Int) -> (c : Int) ->  
    and (odd a) (odd c) -> all Int (\ b ->  
      even (plus (times a b) (times b c)))
```

Coq:

```
prop110 : forall a : Int, forall c : Int,  
  (odd a /\ odd c -> forall b : Int,  
    even (a * b + b * c)) .
```

Lean:

```
prop110 (a c : Int) (x : odd a ∧ odd c)  
:  
  ∀ b : Int, even (a * b + b * c)
```

Prop110. Let $a, c \in \mathbb{Z}$. Assume that both a and c are odd. Then $ab + bc$ is even for all integers b .

Prop110. Soient $a, c \in \mathbb{Z}$. Supposons que a et c sont impairs. Alors $ab + bc$ est pair pour tous les entiers b .

Prop110. Låt $a, c \in \mathbb{Z}$. Anta att både a och c är udda. Då är $ab + bc$ jämnt för alla heltal b .

Agda:

postulate prop110 :

```
(a : Int) -> (c : Int) ->  
  and (odd a) (odd c) -> all Int (\ b ->  
    even (plus (times a b) (times b c)))
```

Coq:

```
prop110 : forall a : Int, forall  
  (odd a /\ odd c -> forall b  
    even (a * b + b * c)) .
```

Dedukti:

```
prop110 : (a : Elem Int) ->  
  (c : Elem Int) ->  
    Proof (and (odd a)  
      (odd c)) ->  
      Proof (forall Int  
        (b => even (plus  
          (times a b) (times b c)))).
```

Lean:

```
prop110 (a c : Int) (x : odd a ∧ odd c)  
:  
  ∀ b : Int, even (a * b + b * c)
```

Prop110. Let $a, c \in \mathbb{Z}$. Assume that both a and c are odd. Then $ab + bc$ is even for all integers b .

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Agda:

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```

Coq:

```
prop110 : forall a : Int, forall  
  (odd a /\ odd c -> forall b  
    even (a * b + b * c)) .
```

Lean:

```
prop110 (a c : Int) (x : odd a ∧ odd c)  
:  
  ∀ b : Int, even (a * b + b * c)
```

Dedukti:

```
prop110 : (a : Elem Int) ->  
  (c : Elem Int) ->  
    Proof (and (odd a)  
      (odd c)) ->  
      Proof (forall Int  
        (b => even (plus  
          (times a b) (times b c)))).
```

Prop110. Let $a, c \in \mathbb{Z}$. Assume that both a and c are odd. Then $ab + bc$ is even for all integers b .

GF:

```
AxiomJmt (StrLabel "prop110")  
(ConsHypo (LetFormulaHypo (FElem  
(ConsTerm (TIdent (StrIdent "a"))  
(BaseTerm (TIdent (StrIdent "c")))))  
(SetTerm integer_Set))) (ConsHypo  
(PropHypo (AdjProp odd_Adj (AndExp  
(BaseExp (TermExp (TIdent (StrIdent  
"a")))) (TermExp (TIdent (StrIdent  
"c")))))))) BaseHypo)) (PostQuantProp  
(AdjProp even_Adj (TermExp  
(AppOperTerm plus_Oper (TTimes (TIdent  
(StrIdent "a")) (TIdent (StrIdent  
"b")))) (TTimes (TIdent (StrIdent "b"))  
(TIdent (StrIdent "c"))))))  
(AllIdentsKindExp (BaseIdent (StrIdent  
"b")) (SetKind integer_Set)))
```

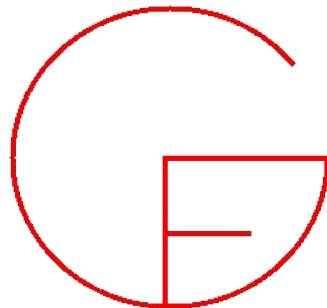
Let $a, c \in \mathbb{Z}$. Supposons que a et c sont impairs. Alors $ab + bc$ est pair pour tous entiers b .

Prop110. Låt $a, c \in \mathbb{Z}$. Anta att både a och c är udda. Då är $ab + bc$ jämnt för alla heltal b .

Interlude: GF

GF = Grammatical Framework

GF = Logical Framework + Grammar



First release 1998 at Xerox Research Centre Europe, Grenoble

Based on earlier work with ALF (Another LF, predecessor of Agda) 1992

<https://www.grammaticalframework.org/>

Abstract and concrete syntax: judgements

```
-- abstract syntax = LF
```

```
cat C  $\Gamma$ 
```

```
fun f : T
```

```
def t = u
```

```
-- concrete syntax
```

```
lincat C = L
```

```
lin f = t
```

```
param P = C | ... | C
```

```
oper h : T = t
```


Abstract and concrete syntax: examples

```
-- abstract syntax = LF

cat Prop ; Term

fun commutative : Term -> Prop

def commutative f =
  forall Obj (\x, y ->
    Id Obj (f x y) (f y x)
```

```
-- concrete syntax

lincat Prop, Term = Str

lin commutative x =
  x ++ "is commutative"
```

Concrete syntax: parameters and operations

```
-- abstract syntax = LF  
  
cat Prop ; Term  
  
fun commutative : Term -> Prop
```

```
-- concrete syntax for English  
  
lincat  
  Prop = Str  
  Term = {s : Str ; n : Number}  
  
lin commutative x = x.s ++  
  copula ! x.n ++ "commutative"  
  
param  
  Number = Sg | Pl  
  
oper  
  copula : Number => Str =  
    table {Sg => "is" ; Pl => "are"}
```

Concrete syntax: parameters and operations

```
-- abstract syntax = LF

cat Prop ; Term

fun commutative : Term -> Prop
```

```
-- concrete syntax for French

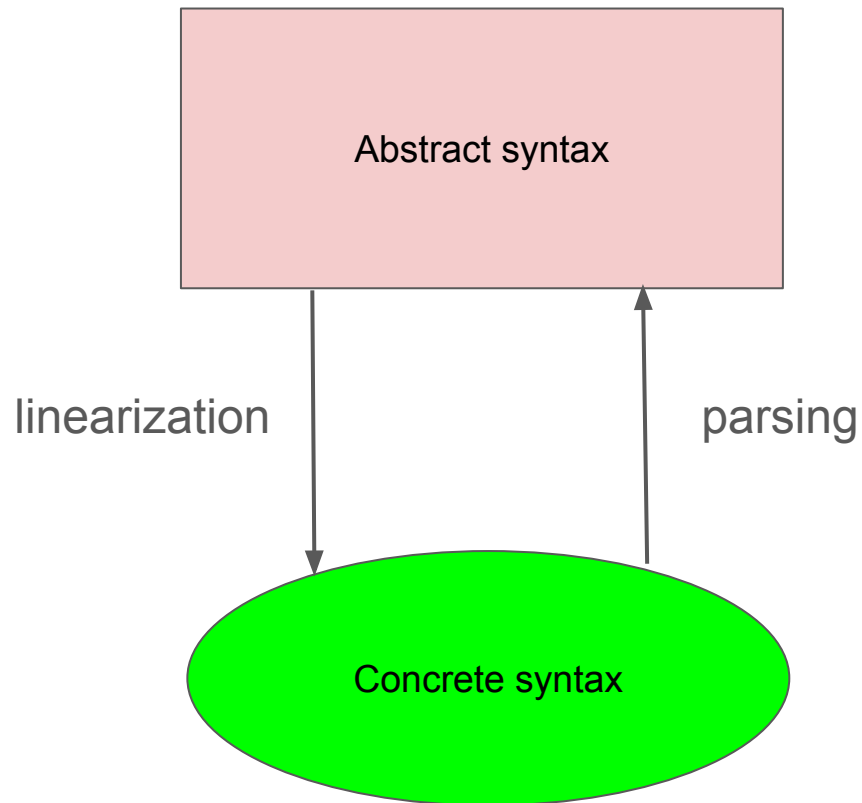
lincat
  Prop = Mood => Str
  Term = {s : Str ; g : Gender ; n : Number}

lin commutative x = \\m => x.s ++
  copula ! m ! n ++
  mkA "commutatif" ! x.g ! x.n

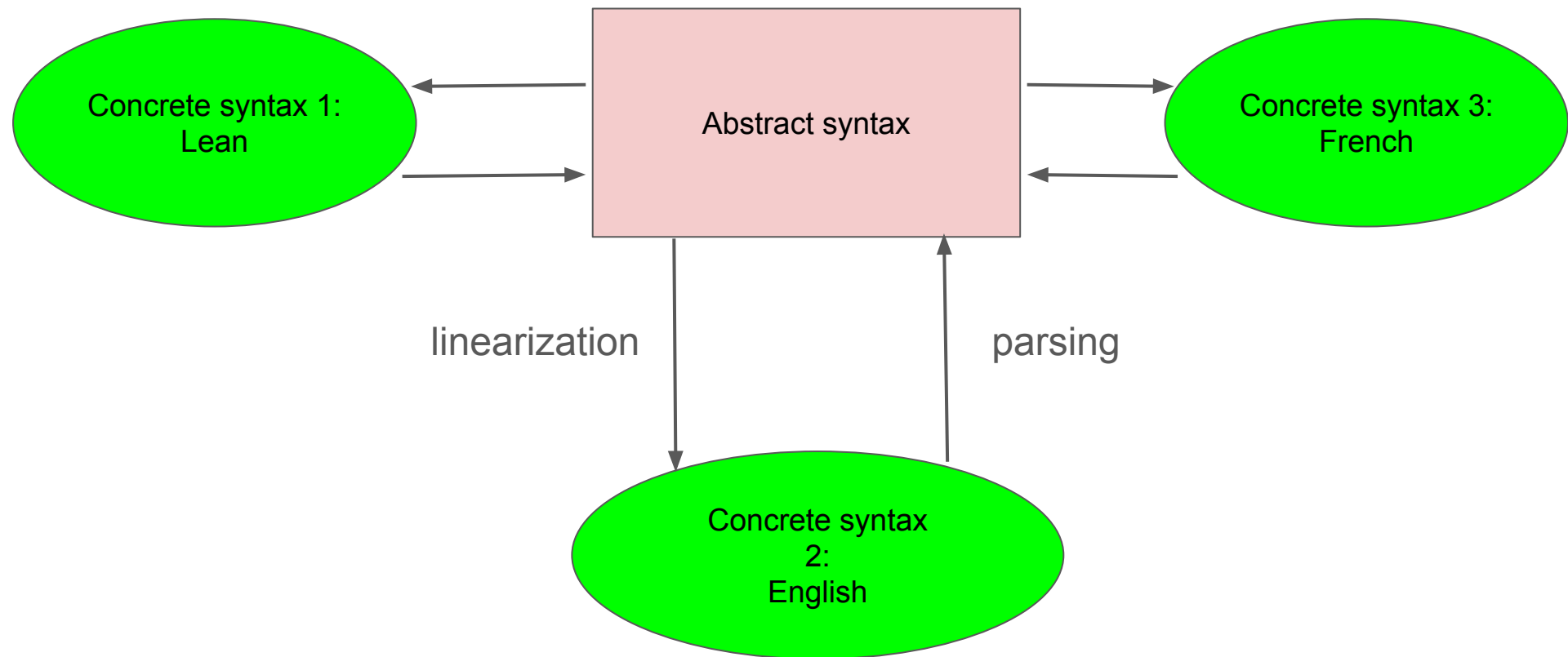
param
  Number = Sg | Pl
  Gender = Masc | Fem
  Mood = Ind | Subj

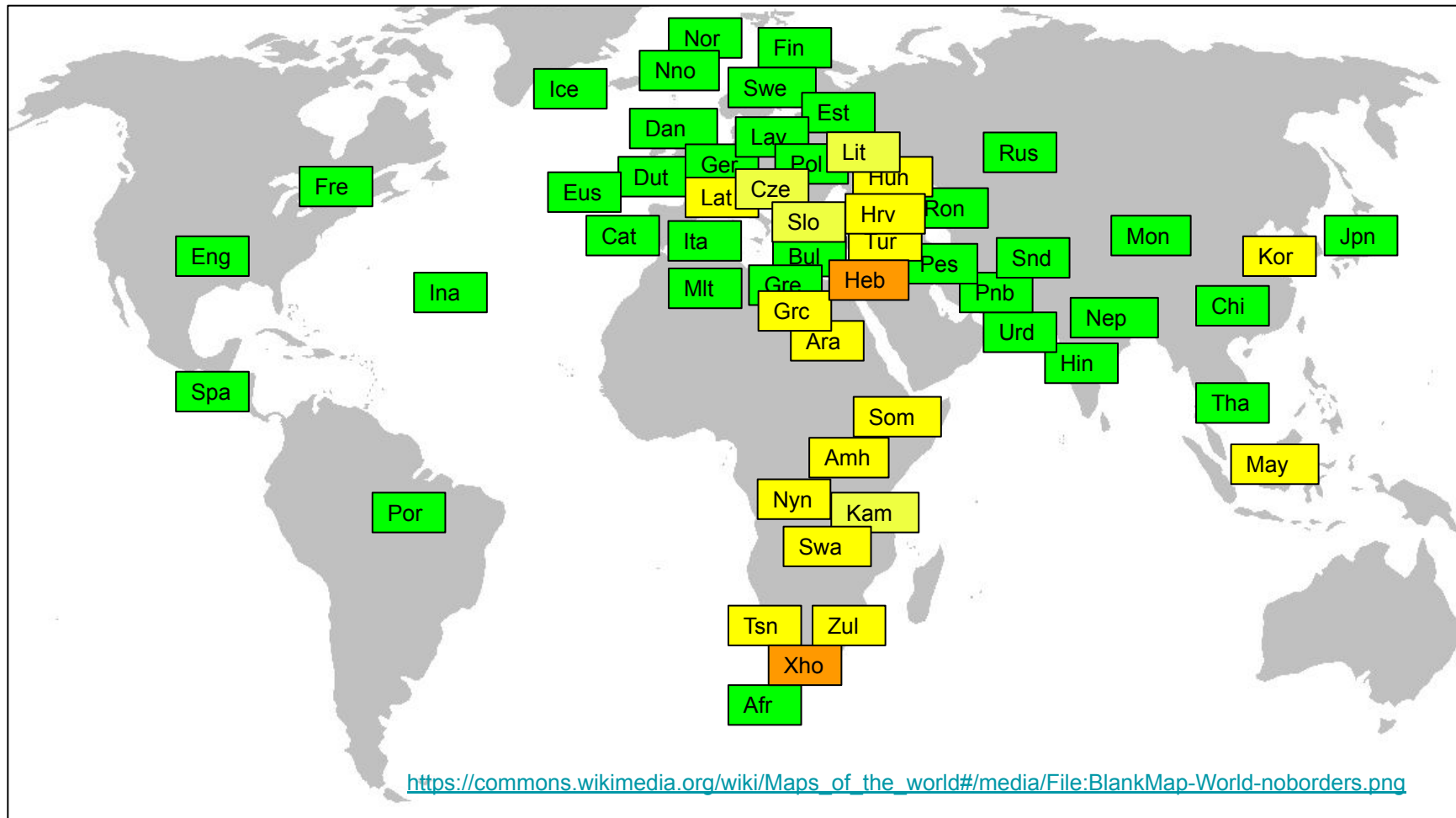
oper
  mkA : Str -> Gender => Number = Str = ...
  copula : Mood => Number => Str = ...
```

Reversible mappings



Multilingual grammars





RGL = Resource Grammar Library

morphology and
syntax for ~50
languages

```
-- inflection of French adjectives, slightly simplified

mkA : Str -> A = \adj ->
  case adj of {
    _ + "eux"=> <adj, init adj + "se", adj, init adj + "ses"> ;
    _ + "al"  => <adj, adj + "e", init adj + "ux", adj + "es"> ;
    _ + "en"  => <adj, adj + "ne", adj + "s", adj + "nes"> ;
    _ + "el"  => <adj, adj + "le", adj + "s", adj + "les"> ;
    x + "er"  => <adj, x + "ère", adj + "s", x + "ères"> ;
    _ + "if"  => <adj, init adj + "ve", adj + "s", init adj + "ves"> ;
    _ + "s"   => <adj, adj + "e", adj, adj + "es"> ;
    _ + "e"   => <adj, adj, adj + "s", adj + "s"> ;
    _         => <adj, adj + "e", adj + "s", adj + "es">
  } ;
```

RGL

syntactic combination
API

shared by all
languages in the
library

usable as functor
interface + instances

<http://www.grammaticalframework.org/lib/doc/synopsis/>

mkCl	NP -> V2Q -> NP -> QS -> CI	she asks him who sleeps
mkCl	NP -> V2V -> NP -> VP -> CI	she begs him to sleep
mkCl	NP -> VPSlash -> NP -> CI	she begs him to sleep here
mkCl	NP -> A -> CI	she is old
mkCl	NP -> A -> NP -> CI	she is old
mkCl	NP -> A2 -> NP -> CI	she is old
mkCl	NP -> AP -> CI	she is old
mkCl	NP -> NP -> CI	she is old
mkCl	NP -> N -> CI	she is old
mkCl	NP -> CN -> CI	she is old
mkCl	NP -> Adv -> CI	she is old
mkCl	NP -> VP -> CI	she is old
mkCl	N -> CI	there is old
mkCl	CN -> CI	there is old
mkCl	NP -> CI	there is old
mkCl	NP -> RS -> CI	it is she
mkCl	Adv -> S -> CI	it is here
mkCl	V -> CI	it rains
mkCl	VP -> CI	it rains
mkCl	CG -> VP -> CI	it rains

- API: mkUtt (mkCl she_NP old_A)
- Afr: sy is oud
- Ara: هي قديمة
- Bul: тя е стара
- Cat: ella és vella
- Chi: 她是老的
- Cze: je stará
- Dan: hun er gammel
- Dut: zij is oud
- Eng: she is old
- Est: tema on vana
- Eus: hura zaharra da
- Fin: hän on vanha
- Fre: elle est vieille
- Ger: sie ist alt
- Gre: αυτή είναι παλιά
- Hin: वह बूढ़ी है
- Ice: constant not found: old_A
- Ita: lei è vecchia
- Jpn: 彼女は古い
- Lat: vetus est
- Lav: viņa ir veca
- Mlt: hi hija qadima
- Mon: түүний хуучин байдаг нь
- Nep: उनी बुढी छिन्
- Nno: ho er gammal

Concrete syntax: functor over the RGL

```
-- abstract syntax code
```

```
cat Prop ; Term  
fun commutative : Term -> Prop
```

```
-- shared functor code
```

```
lincat  
  Prop = Cl  
  Term = NP
```

```
lin  
  commutative x =  
    mkCl x commutative_A
```

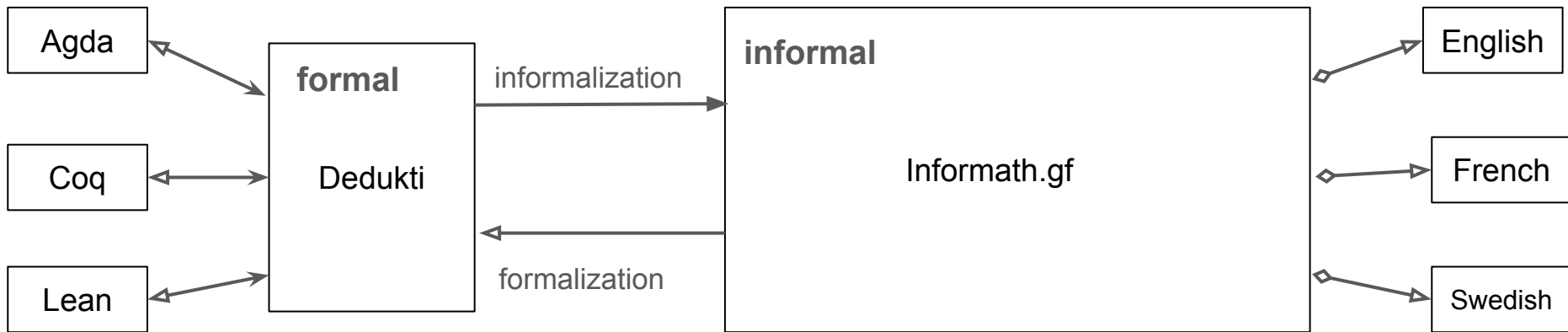
```
-- added code for each language
```

```
-- Eng  
  commutative_A =  
    mkA "commutative"
```

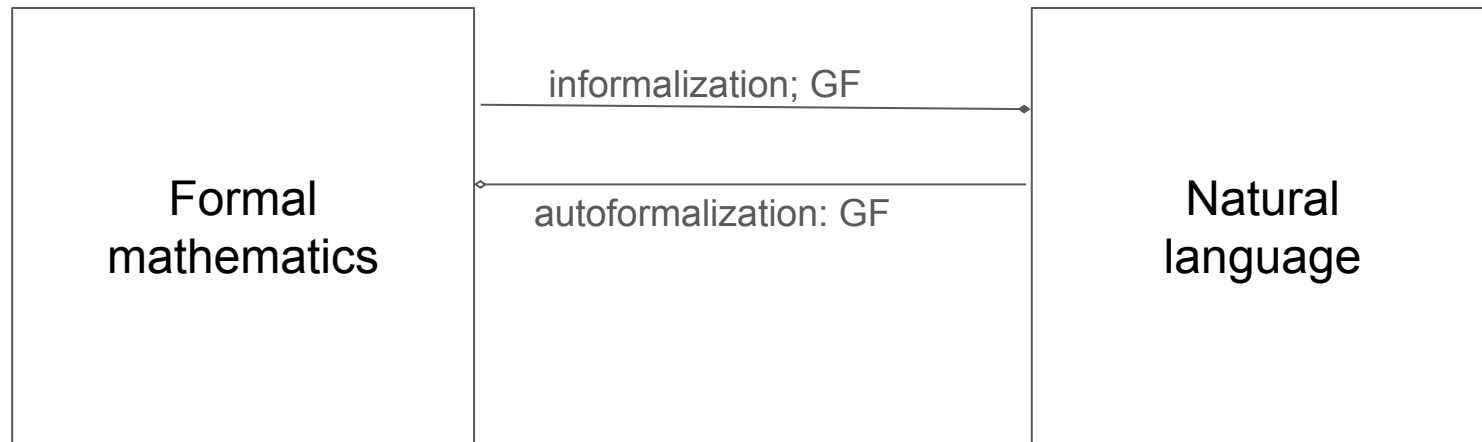
```
-- Fre  
  commutative_A =  
    mkA "commutatif"
```

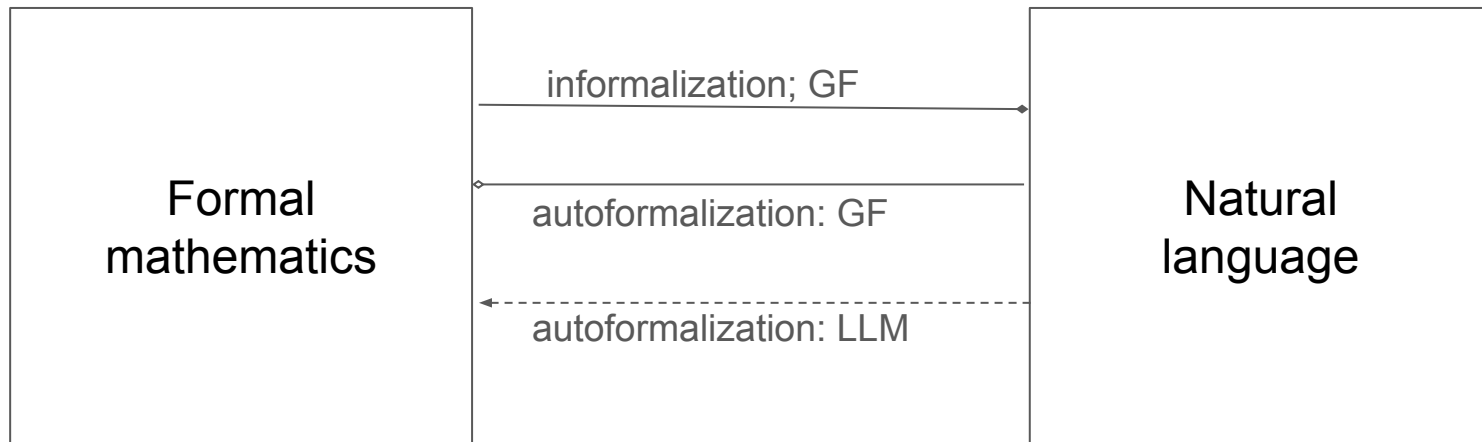
```
-- Fin  
  commutative_A =  
    mkA "kommutatiivinen"
```

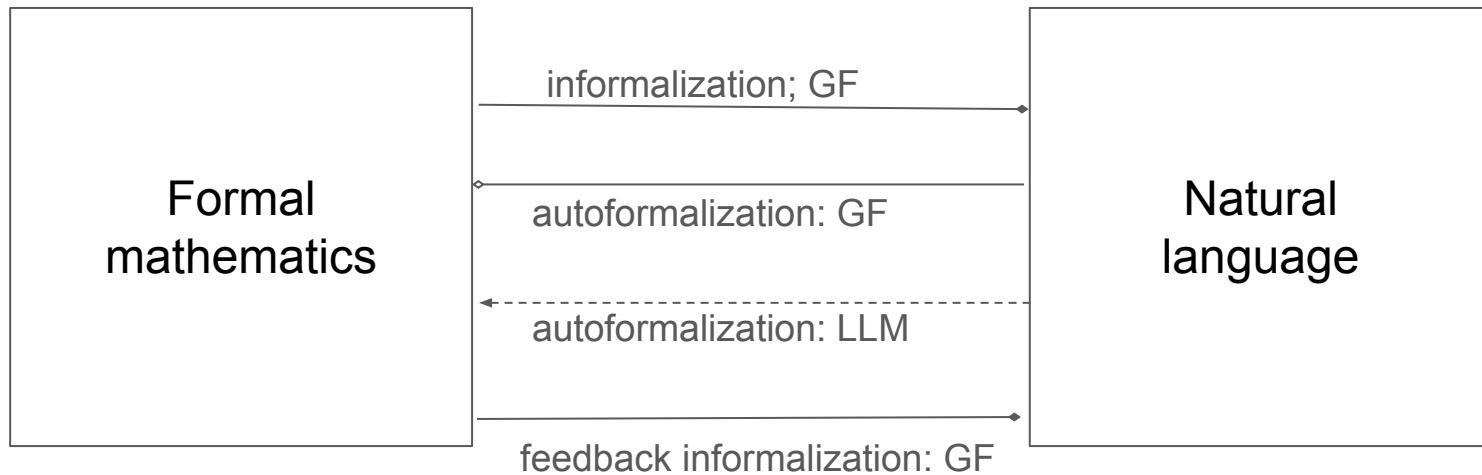
Back to Informath

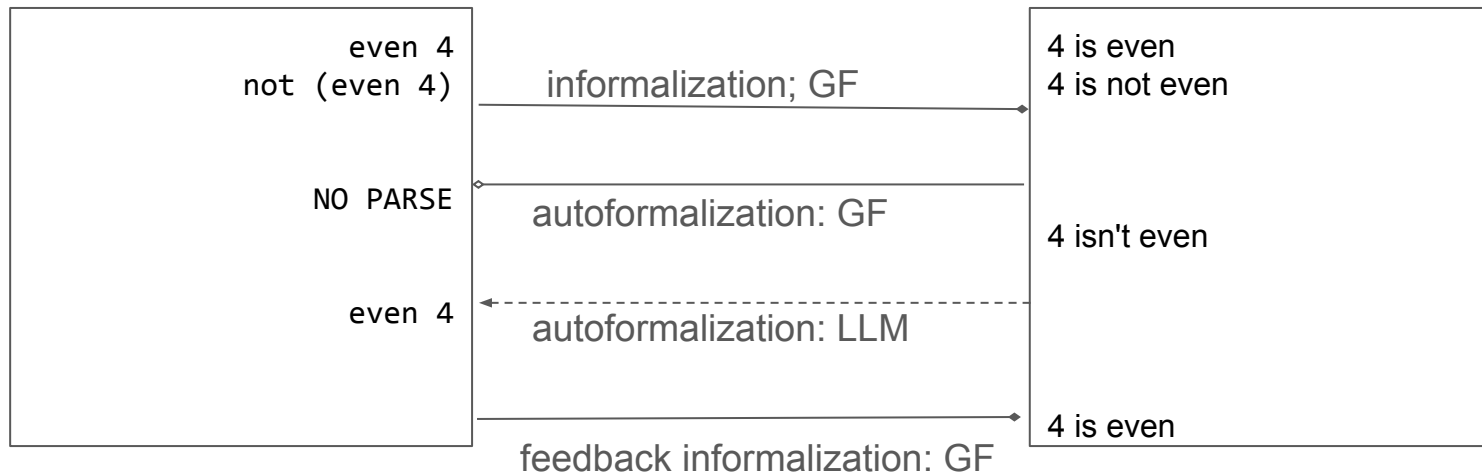


	to one	to many
total	\longrightarrow	$\longrightarrow\blacktriangleright$
partial	$\longrightarrow\triangleright$	$\longrightarrow\diamond$









Mapping between Dedukti and GF


```

-- Dedukti.bnf

MJmts. Module ::= [Jmt] ;

terminator Jmt "" ;

comment "(" ";" ")" ;
comment "#" ; ----

JStatic. Jmt ::= QIdent ":" Exp "." ;
JDef. Jmt ::= "def" QIdent MTyp MExp "." ;
JInj. Jmt ::= "inj" QIdent MTyp MExp "." ;
JThm. Jmt ::= "thm" QIdent MTyp MExp "." ;
JRules. Jmt ::= [Rule] "." ;

RRule. Rule ::= "[" [Pattbind] "]" Patt "-->" Exp ;
separator nonempty Rule "" ;

separator Pattbind "," ;

MTNone. MTyp ::= ;
MTExp. MTyp ::= ":" Exp ;

MENone. MExp ::= ;
MEExp. MExp ::= ":"=" Exp ;

EIdent. Exp9 ::= QIdent ;
EApp. Exp5 ::= Exp5 Exp6 ;
EAbs. Exp2 ::= Bind "=>" Exp2 ;
EFun. Exp1 ::= Hypo "->" Exp1 ;

coercions Exp 9 ;

-- plus some rules for Hypo and Bind

token QIdent (letter | digit | '_' | '!' | '?' | '\')+
('.' (letter | digit | '_' | '!' | '?' | '\'))? ;

```

```
-- Dedukti.bnf
```

```
MJmts. Module ::= [Jmt] ;
```

```
terminator Jmt "" ;
```

```
comment "(:" ";)" ;
```

```
comment "#" ; ----
```

```
JStatic. Jmt ::= QIdent ":" Exp "." ;
```

```
JDef. Jmt ::= "def" QIdent MTyp MExp "." ;
```

```
JInj. Jmt ::= "inj" QIdent MTyp MExp "." ;
```

```
JThm. Jmt ::= "thm" QIdent MTyp MExp "." ;
```

```
JRules. Jmt ::= [Rule] "." ;
```

```
RRule. Rule ::= "[" [Pattbind] "]" Patt "-->" Exp ;
```

```
separator nonempty Rule "" ;
```

```
separator Pattbind "," ;
```

```
MTNone. MTyp ::= ;
```

```
MTExp. MTyp ::= ":" Exp ;
```

```
MENone. MExp ::= ;
```

```
MEExp. MExp ::= ":" Exp ;
```

```
EIdent. Exp9 ::= QIdent ;
```

```
EApp. Exp5 ::= Exp5 Exp6 ;
```

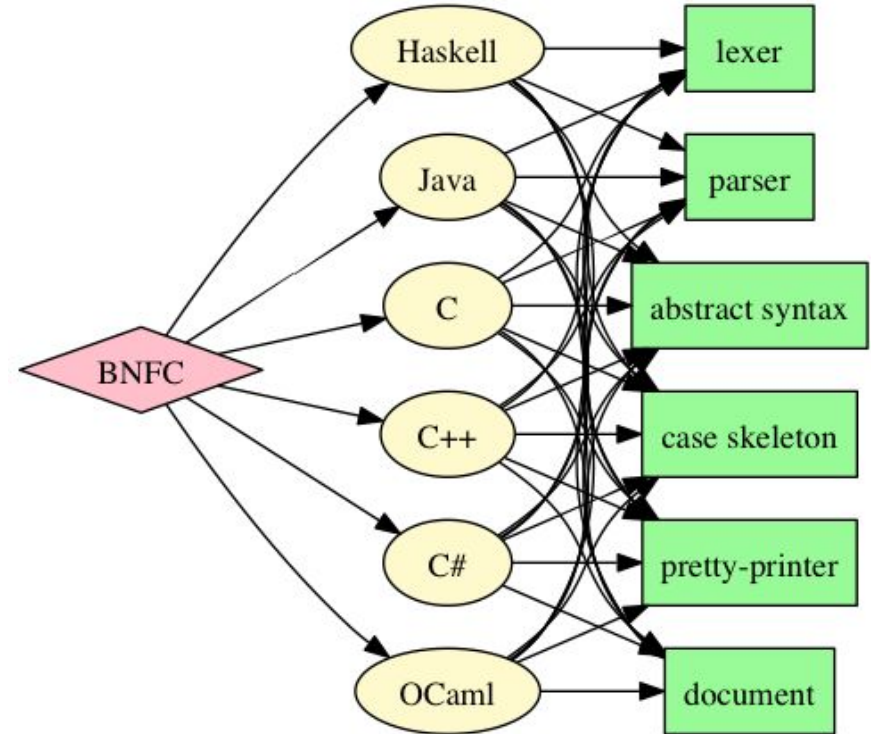
```
EAbs. Exp2 ::= Bind "=>" Exp2 ;
```

```
EFun. Exp1 ::= Hypo "->" Exp1 ;
```

```
coercions Exp 9 ;
```

```
-- plus some rules for Hypo and Bind
```

```
token QIdent (letter | digit | '_' | '!' | '?' | '\\')+  
( '.' (letter | digit | '_' | '!' | '?' | '\\') )? ;
```



<https://bnfc.digitalgrammars.com/>

```
-- Dedukti.bnf

MJmts. Module ::= [Jmt] ;

terminator Jmt "" ;

comment "(:" ";)" ;
comment "#" ; ----

JStatic. Jmt ::= QIdent ":" Exp "." ;
JDef. Jmt ::= "def" QIdent MTyp MExp "." ;
JInj. Jmt ::= "inj" QIdent MTyp MExp "." ;
JThm. Jmt ::= "thm" QIdent MTyp MExp "." ;
JRules. Jmt ::= [Rule] "." ;

RRule. Rule ::= "[" [Pattbind] "]" Patt "-->" Exp ;
separator nonempty Rule "" ;

separator Pattbind "," ;

MTNone. MTyp ::= ;
MTExp. MTyp ::= ":" Exp ;

MENone. MExp ::= ;
MEExp. MExp ::= "!=" Exp ;

EIdent. Exp9 ::= QIdent ;
EApp. Exp5 ::= Exp5 Exp6 ;
EAbs. Exp2 ::= Bind "=>" Exp2 ;
EFun. Exp1 ::= Hypo "-->" Exp1 ;

coercions Exp 9 ;

-- plus some rules for Hypo and Bind

token QIdent (letter | digit | '_' | '!' | '?' | '\')+
('.' (letter | digit | '_' | '!' | '?' | '\'))? ;
```

```
-- MathCore.gf

abstract MathCore =
  Terms, UserConstants
  ** {
cat
  Jmt ;
  Exp ;
  Exps ;
  Prop ;
  Kind ;
  Hypo ;
  [Hypo] ;
  Proof ;
  Label ;
  -- plus more categories

fun
  ThmJmt : Label -> [Hypo] -> Prop -> Proof -> Jmt ;
  AxiomJmt : Label -> [Hypo] -> Prop -> Jmt ;
  DefPropJmt : Label -> [Hypo] -> Prop -> Prop -> Jmt ;
  DefKindJmt : Label -> [Hypo] -> Kind -> Kind -> Jmt ;
  DefExpJmt : Label -> [Hypo] -> Exp -> Kind -> Exp -> Jmt ;
  AxiomPropJmt : Label -> [Hypo] -> Prop -> Jmt ;
  AxiomKindJmt : Label -> [Hypo] -> Kind -> Jmt ;
  AxiomExpJmt : Label -> [Hypo] -> Exp -> Kind -> Jmt ;

  AppExp : Exp -> Exps -> Exp ;
  AbsExp : [Ident] -> Exp -> Exp ;
  TermExp : Term -> Exp ;
  KindExp : Kind -> Exp ;
  TypedExp : Exp -> Kind -> Exp ;

  AndProp : [Prop] -> Prop ;
  OrProp : [Prop] -> Prop ;
  IfProp : Prop -> Prop -> Prop ;
  IffProp : Prop -> Prop -> Prop ;
  NotProp : Prop -> Prop ;
  -- plus many more functions
```

```
-- Dedukti.bnf
```

```
MJmts. Module ::= [Jmt] ;
```

```
terminator
```

```
module AbsDedukti where
```

```
comment "(C
```

```
comment "#
```

```
JStatic.
```

```
JDef.
```

```
JInj.
```

```
JThm.
```

```
JRules.
```

```
RRule. Ru
```

```
separator
```

```
separator
```

```
MTNone. MT
```

```
MTEExp. MT
```

```
MENone. ME
```

```
MEEExp. ME
```

```
EIdent. E
```

```
EApp. E
```

```
EAbs. E
```

```
EFun. E
```

```
coercions
```

```
-- plus so
```

```
token QId
```

```
('.' (letter | digit | '_' | '!' | '?' | '\\')+)? ;
```

```
data Tree (a :: Tag) where
```

```
  MJmts :: [Jmt] -> Tree 'Module_
```

```
  JStatic :: QIdent -> Exp -> Tree 'Jmt_
```

```
  JDef :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
```

```
  JInj :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
```

```
  JThm :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
```

```
  JRules :: [Rule] -> Tree 'Jmt_
```

```
  RRule :: [Pattbind] -> Patt -> Exp -> Tree 'Rule_
```

```
  MTNone :: Tree 'MTyp_
```

```
  MTEExp :: Exp -> Tree 'MTyp_
```

```
  MENone :: Tree 'MExp_
```

```
  MEEExp :: Exp -> Tree 'MExp_
```

```
  EIdent :: QIdent -> Tree 'Exp_
```

```
  EApp :: Exp -> Exp -> Tree 'Exp_
```

```
  EAbs :: Bind -> Exp -> Tree 'Exp_
```

```
  EFun :: Hypo -> Exp -> Tree 'Exp_
```

```
  BVar :: QIdent -> Tree 'Bind_
```

```
  BTyped :: QIdent -> Exp -> Tree 'Bind_
```

```
  PBVar :: QIdent -> Tree 'Pattbind_
```

```
  PBTTyped :: QIdent -> Exp -> Tree 'Pattbind_
```

```
  HExp :: Exp -> Tree 'Hypo_
```

```
  HVarExp :: QIdent -> Exp -> Tree 'Hypo_
```

```
  HParVarExp :: QIdent -> Exp -> Tree 'Hypo_
```

```
  PVar :: QIdent -> Tree 'Patt_
```

```
  PBracket :: Patt -> Tree 'Patt_
```

```
  PApp :: Patt -> Patt -> Tree 'Patt_
```

```
  PBind :: Bind -> Patt -> Tree 'Patt_
```

```
  QIdent :: String -> Tree 'QIdent_
```

```
-- MathCore.gf
```

```
abstract MathCore =
```

```
  Terms, UserConstants
```

```
  ** {
```

```
cat
```

```
  Jmt ;
```

```
  Exp ;
```

```
  Exps ;
```

```
  Prop ;
```

```
  Kind ;
```

```
  Hypo ;
```

```
  [Hypo] ;
```

```
  Proof ;
```

```
  Label ;
```

```
  -- plus more categories
```

```
fun
```

```
  ThmJmt : Label -> [Hypo] -> Prop -> Proof -> Jmt ;
```

```
  AxiomJmt : Label -> [Hypo] -> Prop -> Jmt ;
```

```
  DefPropJmt : Label -> [Hypo] -> Prop -> Prop -> Jmt ;
```

```
  DefKindJmt : Label -> [Hypo] -> Kind -> Kind -> Jmt ;
```

```
  DefExpJmt : Label -> [Hypo] -> Exp -> Kind -> Exp -> Jmt ;
```

```
  AxiomPropJmt : Label -> [Hypo] -> Prop -> Jmt ;
```

```
  AxiomKindJmt : Label -> [Hypo] -> Kind -> Jmt ;
```

```
  AxiomExpJmt : Label -> [Hypo] -> Exp -> Kind -> Jmt ;
```

```
  AppExp : Exp -> Exps -> Exp ;
```

```
  AbsExp : [Ident] -> Exp -> Exp ;
```

```
  TermExp : Term -> Exp ;
```

```
  KindExp : Kind -> Exp ;
```

```
  TypedExp : Exp -> Kind -> Exp ;
```

```
  AndProp : [Prop] -> Prop ;
```

```
  OrProp : [Prop] -> Prop ;
```

```
  IfProp : Prop -> Prop -> Prop ;
```

```
  IffProp : Prop -> Prop -> Prop ;
```

```
  NotProp : Prop -> Prop ;
```

```
  -- plus quite a few more functions
```

```
-- Dedukti.bnf
```

```
MJmts. Module ::= [Jmt] ;
```

```
terminator
```

```
comment "(
```

```
comment "#
```

```
JStatic.
```

```
JDef.
```

```
JInj.
```

```
JThm.
```

```
JRules.
```

```
RRule. Ru
```

```
separator
```

```
separator
```

```
MTNone. MT
```

```
MTEExp. MT
```

```
MENone. ME
```

```
MEEExp. ME
```

```
EIdent. E
```

```
EApp. E
```

```
EAbs. E
```

```
EFun. E
```

```
coercions
```

```
-- plus so
```

```
token QId
```

```
('.' (letter | digit | '_' | '!' | '?' | '\\')+)? ;
```

```
module AbsDedukti where
```

```
data Tree (a :: Tag) where
```

```
  MJmts :: [Jmt] -> Tree 'Module_
```

```
  JStatic :: QIdent -> Exp -> Tree 'Jmt_
```

```
  JDef :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
```

```
  JInj :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
```

```
  JThm :: QIdent -> MTyp -> MExp -> Tree 'Jmt_
```

```
  JRules :: [Rule] -> Tree 'Jmt_
```

```
  RRule :: [Pattpbind] -> Patt -> Exp -> Tree 'Rule_
```

```
  MTNone :: Tree 'MTyp_
```

```
  MTEExp :: Exp -> Tree 'MTyp_
```

```
  MENone :: Tree 'MExp_
```

```
  MEEExp :: Exp -> Tree 'MExp_
```

```
  EIdent :: QIdent -> Tree 'Exp_
```

```
  EApp :: Exp -> Exp -> Tree 'Exp_
```

```
  EAbs :: Bind -> Exp -> Tree 'Exp_
```

```
  EFun :: Hypo -> Exp -> Tree 'Exp_
```

```
  BVar :: QIdent -> Tree 'Bind_
```

```
  BTyped :: QIdent -> Exp -> Tree 'Bind_
```

```
  PBVar :: QIdent -> Tree 'Pattpbind_
```

```
  PBTTyped :: QIdent -> Exp -> Tree 'Pattpbind_
```

```
  HExp :: Exp -> Tree 'Hypo_
```

```
  HVarExp :: QIdent -> Exp -> Tree 'Hypo_
```

```
  HParVarExp :: QIdent -> Exp -> Tree 'Hypo_
```

```
  PVar :: QIdent -> Tree 'Patt_
```

```
  PBracket :: Patt -> Tree 'Patt_
```

```
  PApp :: Patt -> Patt -> Tree 'Patt_
```

```
  PBind :: Bind -> Patt -> Tree 'Patt_
```

```
  QIdent :: String -> Tree 'QIdent_
```

```
-- this is all
```

```
-- MathCore.gf
```

```
abstract MathCore =  
  Terms, UserConstants
```

```
module Informath where
```

```
data Tree :: * -> * where
```

```
  GAndAdj :: GListAdj -> Tree GAdj_
```

```
  GComparAdj :: GCompar -> GExp -> Tree GAdj_
```

```
  GOrAdj :: GListAdj -> Tree GAdj_
```

```
  GReladjAdj :: GReladj -> GExp -> Tree GAdj_
```

```
  LexAdj :: String -> Tree GAdj_
```

```
  GIdentArgKind :: GKind -> GListIdent -> Tree GArgKind_
```

```
  GKindArgKind :: GKind -> Tree GArgKind_
```

```
  LexCompar :: String -> Tree GCompar_
```

```
  LexComparnoun :: String -> Tree GComparnoun_
```

```
  LexConst :: String -> Tree GConst_
```

```
  GComparEqsign :: GCompar -> Tree GEqsign_
```

```
  GComparnounEqsign :: GComparnoun -> Tree GEqsign_
```

```
  GEBinary :: GEqsign -> GTerm -> GTerm -> Tree GEquation_
```

```
  GAbsExp :: GListIdent -> GExp -> Tree GExp_
```

```
  GAllIdentKindExp :: GListIdent -> GKind -> Tree GExp_
```

```
  GAllKindExp :: GKind -> Tree GExp_
```

```
  GAndExp :: GListExp -> Tree GExp_
```

```
  GAppExp :: GExp -> GExps -> Tree GExp_
```

```
  GCoercionExp :: GCoercion -> GExp -> Tree GExp_
```

```
  GConstExp :: GConst -> Tree GExp_
```

```
  GEveryIdentKindExp :: GIdent -> GKind -> Tree GExp_
```

```
  GEveryKindExp :: GKind -> Tree GExp_
```

```
  GFunListExp :: GFun -> GExps -> Tree GExp_
```

```
  GIndefIdentKindExp :: GIdent -> GKind -> Tree GExp_
```

```
  GIndefKindExp :: GKind -> Tree GExp_
```

```
  GIndexedTermExp :: GInt -> Tree GExp_
```

```
-- plus quite a few more
```

```
NotProp : Prop -> Prop ;
```

```
-- plus more functions
```

```
-- Dedukti.bnf
```

```
MJmts. Module ::= [Jmt]
```

```
terminator
```

```
comment "(C
```

```
comment "#
```

```
JStatic.
```

```
JDef.
```

```
JInj.
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JThm.
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```
JRules.
```

```
RRule. Ru
```

```
separator
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```

```
MTNone. MT
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MTEExp. MT
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MENone. ME
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MEEExp. ME
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EIdent. E
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EApp. E
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EAbs. E
```

```
EFun. E
```

```
coercions
```

```
-- plus so
```

```
token QIde
```

```
('.' (letter | digit |
```

```
module AbsDe
```

```
data Tree (a
```

```
MJmts ::
```

```
JStatic
```

```
JDef ::
```

```
JInj ::
```

```
JThm ::
```

```
JRules ::
```

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RRule ::
```

```
MTNone ::
```

```
MTEExp ::
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```
MENone ::
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```
MEEExp ::
```

```
EIdent ::
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EApp ::
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EAbs ::
```

```
EFun ::
```

```
BVar ::
```

```
BTyped ::
```

```
PBVar ::
```

```
PBTTyped
```

```
HExp ::
```

```
HVarExp
```

```
HParVarE
```

```
PVar ::
```

```
PBracket
```

```
PApp ::
```

```
PBind ::
```

```
QIdent ::
```

```
module Dedukti2Core where
```

```
import Dedukti.AbsDedukti
```

```
import Informath
```

```
import DeduktiOperations
```

```
jmt2jmt :: Jmt -> GJmt
```

```
jmt2jmt jmt = case jmt of
```

```
  JDef ident (MTEExp typ) meexp ->
```

```
    let mexp = case meexp of
```

```
      MEEExp exp -> Just exp
```

```
      _ -> Nothing
```

```
    in case (splitType typ, guessCat ident typ) of
```

```
      ((hypos, kind), c) | elem c ["Noun", "Set"] ->
```

```
        (maybe (GAxiomKindJmt axiomLabel)
```

```
          (\exp x y -> GDefKindJmt definitionLabel x y (exp2kind exp)) mexp)
```

```
        (GListHypo (hypos2hypos hypos))
```

```
        (ident2kind ident)
```

```
      ((hypos, kind), c) | elem c ["Name", "Const", "Unknown"] ->
```

```
        (maybe (GAxiomExpJmt axiomLabel)
```

```
          (\exp x y z -> GDefExpJmt definitionLabel x y z (exp2exp exp)) mexp)
```

```
        (GListHypo (hypos2hypos hypos))
```

```
        (ident2exp ident)
```

```
        (exp2kind kind)
```

```
...
```

```
exp2kind :: Exp -> GKind
```

```
exp2prop :: Exp -> GProp
```

```
exp2exp :: Exp -> GExp
```

```
exp2proof :: Exp -> GProof
```

```
GAdj_
```

```
GAdj_
```

```
-> Tree GArgKind_
```

```
-
```

```
rnoun_
```

```
gn_
```

```
ee GEqsign_
```

```
-> Tree GEquation_
```

```
GExp_
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```
nd -> Tree GExp_
```

```
mt ;
```

```
ree GExp_
```

```
-> Tree GExp_
```

```
Exp_
```

```
-> Tree GExp_
```

```
t ;  
t ;  
-> Jmt ;
```

Dedukti Exp	GF category	linearization	linguistic category
union A B	Exp	<i>the union of A and B</i>	noun phrase
Nat	Kind	<i>natural number</i>	common noun
divisible 9 3	Prop	<i>9 is divisible by 3</i>	sentence
oddS 0 evenZ	Proof	<i>0 is even. Therefore 1 is odd.</i>	text

```
-- Dedukti.bnf
```

```
MJmts. Module ::= [Jmt]
```

```
terminator
```

```
module AbsDe
```

```
comment "(C
```

```
comment "#
```

```
JStatic.
```

```
JDef.
```

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JInj.
```

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JThm.
```

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JRules.
```

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RRule. Ru
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separator
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separator
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```
MTNone. MT
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```

```
EIdent. E
```

```
EApp. E
```

```
EAbs. E
```

```
EFun. E
```

```
coercions
```

```
-- plus so
```

```
token QIde
```

```
('.' (letter | digit |
```

```
module Dedukti2Core where
```

```
import
```

```
import
```

```
import
```

```
jmt2.
```

```
jmt2.
```

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```

```
module Core2Dedukti where
```

```
import Dedukti.AbsDedukti
```

```
import Informath
```

```
import DeduktiOperations
```

```
prop2dedukti :: GProp -> Exp
```

```
prop2dedukti prop = case prop of
```

```
  GProofProp p -> EApp (EIdent (QIdent "Proof")) (prop2dedukti p)
```

```
  GFalseProp -> propFalse
```

```
  GIdentProp ident -> EIdent (ident2ident ident)
```

```
  GAndProp (GListProp props) -> foldl1 propAnd (map prop2dedukti props)
```

```
kind2dedukti :: GKind -> Exp
```

```
kind2dedukti kind = case kind of
```

```
  GElemKind k -> EApp (EIdent (QIdent "Elem")) (kind2dedukti k)
```

```
  GTermKind (GTIdent ident) -> EIdent (ident2ident ident)
```

```
  GSetKind (LexSet s) -> EIdent (QIdent (s))
```

```
exp2dedukti :: GExp -> Exp
```

```
exp2dedukti exp = case exp of
```

```
  GTermExp (GTNumber (GInt n)) -> int2exp n
```

```
  GTermExp (GTIdent ident) -> EIdent (ident2ident ident)
```

```
  GAppExp exp exps ->
```

```
    foldl1 EApp (map exp2dedukti (exp : (exps2list exps)))
```

```
  GAbsExp (GListIdent ids) exp ->
```

```
    foldr
```

```
      (\x y -> EAbs (BVar (ident2ident x)) y)
```

```
      (exp2dedukti exp)
```

```
      ids
```

```
ident
```

```
ident
```

```
QI
```

```
QI
```

```
QI
```

```
QI
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QI
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QI
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QI
```

```
e GArgKind_
```

```
sign_
```

```
e GEquation_
```

```
Tree GExp_
```

```
xp_
```

```
e GExp_
```

```
e GExp_
```

```
t ;  
t ;  
-> Jmt ;
```

```
mt ;
```


Dealing with identifiers

```

-- BaseConstants.dk

Set : Type.
Prop : Type.

(; ignored in Dedukti2Core ;)
Elem : Set -> Type.
Proof : Prop -> Type.

(; logical operators, hard-coded in MathCore ;)
false : Prop.
and : (A : Prop) -> (B : Prop) -> Prop.
or : (A : Prop) -> (B : Prop) -> Prop.
if : Prop -> Prop -> Prop.
forall : (A : Set) -> (Elem A -> Prop) -> Prop.
exists : (A : Set) -> (Elem A -> Prop) -> Prop.

def not : Prop -> Prop := A => if A false.
def iff : Prop -> Prop -> Prop :=
  A => B => and (if A B) (if B A).

(; constants defined in a lexicon ;)

def Nat : Set := Num.
def Int : Set := Num.
def Rat : Set := Num.
def Real : Set := Num.

Eq : Elem Real -> Elem Real -> Prop.
Lt : Elem Real -> Elem Real -> Prop.
Gt : Elem Real -> Elem Real -> Prop.
Neq : Elem Real -> Elem Real -> Prop.
Leq : Elem Real -> Elem Real -> Prop.
Geq : Elem Real -> Elem Real -> Prop.

plus : (x : Elem Real) -> (y : Elem Real) -> Elem Real.
minus : Elem Real -> Elem Real -> Elem Real.
times : Elem Real -> Elem Real -> Elem Real.

```

```

(; BaseConstants.dk ;)

Set : Type.
Prop : Type.

(; ignored in Dedukti2Core ;)
Elem : Set -> Type.
Proof : Prop -> Type.

(; logical operators, hard-coded in MathCore ;)
false : Prop.
and : (A : Prop) -> (B : Prop) -> Prop.
or : (A : Prop) -> (B : Prop) -> Prop.
if : Prop -> Prop -> Prop.
forall : (A : Set) -> (Elem A -> Prop) -> Prop.
exists : (A : Set) -> (Elem A -> Prop) -> Prop.

def not : Prop -> Prop := A => if A false.
def iff : Prop -> Prop -> Prop :=
  A => B => and (if A B) (if B A).

(; constants defined in a lexicon ;)

Nat : Set.
Int : Set.
Rat : Set.
Real : Set.

Eq : Elem Real -> Elem Real -> Prop.
Lt : Elem Real -> Elem Real -> Prop.
Gt : Elem Real -> Elem Real -> Prop.

even : Elem Int -> Prop.
def odd : Elem Int -> Prop := n => not (even n).

plus : (x : Elem Real) -> (y : Elem Real) -> Elem Real.
minus : Elem Real -> Elem Real -> Elem Real.
times : Elem Real -> Elem Real -> Elem Real.

```

```

# constant_data.dkgf

# for translating between Dedukti and GF abstract syntax

Nat BASE Set natural_Set
Int BASE Set integer_Set
Rat BASE Set rational_Set
Real BASE Set real_Set

Eq BASE Compar Eq_Compar
Lt BASE Compar Lt_Compar
Gt BASE Compar Gt_Compar

plus BASE Oper plus_Oper
minus BASE Oper minus_Oper
times BASE Oper times_Oper

even BASE Adj even_Adj
odd BASE Adj odd_Adj

# for generating GF linearization rules

#LIN Eng natural_Set = mkSet "N" "natural" number_N
#LIN Fre natural_Set = mkSet L.natural_Set "naturel" nombre_N
#LIN Swe natural_Set = mkSet L.natural_Set "naturlig" tal_N

#LIN Eng Lt_Compar = mkCompar "<" "less" "than"
#LIN Fre Lt_Compar = mkCompar "<" (mkAP (mkA "inférieur")) dative
#LIN Swe Lt_Compar = mkCompar "<" "mindre" "än"

#LIN Eng even_Adj = mkAdj "even"
#LIN Fre even_Adj = mkAdj "pair"
#LIN Swe even_Adj = mkAdj "jämn"

# for converting identifiers from third-party projects

le ALIAS matita Leq

```

```
abstract BaseConstants = {
```

-- GF cat	usage	example
Noun ;	-- Kind	-- set
Fam ;	-- Kind -> Kind	-- list of integers
Adj ;	-- Exp -> Prop	-- even
Verb ;	-- Exp -> Exp	-- converge
Reladj ;	-- Exp -> Exp -> Prop	-- divisible by
Relverb ;	-- Exp -> Exp -> Prop	-- divide
Relnoun ;	-- Exp -> Exp -> Prop	-- root of
Name ;	-- Exp	-- contradiction
Fun ;	-- [Exp] -> Exp	-- radius of
Label ;	-- Exp	-- theorem 1
Set ;	-- Kind Term	-- integer, Z
Const ;	-- Exp Term	-- the empty set, \emptyset
Oper ;	-- Exp -> Exp -> Exp Term	-- the sum of, +
Compar ;	-- Exp -> Exp -> Prop Formula	-- greater than, >
Comparnoun ;	-- Exp -> Exp -> Prop Formula	-- a subset of, \sub

```
def sphenic : Nat -> Prop
  := ...
  (; GF: sphenic number ;)
```

lexical rule extraction

```
# from Wikidata

{"Q638185": {
  "pl": "Liczby sfeniczne",
  "de": "sphenische Zahl",
  "en": "sphenic number",
  "es": "número esfénico",
  "fr": "nombre sphénique",
  "zh": "楔形数",
  "sv": "sfeniskt tal",
  "ta": "ஸ்டீனிக் எண்",
}}
```

```
sphenic NEW number_theory Adj sphenic_Adj

#LIN Eng sphenic_Adj = mkAdj "sphenic"
#LIN Fre sphenic_Adj = mkAdj "sphénique"
#LIN Swe sphenic_Adj = mkAdj "sfenisk"
```

AR, Building Grammar Libraries for Mathematics and
Avoiding Manual Work.. Presentation at Hausdorff
Center for Mathematics, 2024,
https://www.youtube.com/watch?v=EQ-k_JQ7fDM&t=5s

From MathCore to full Informath

has

natural language content that ~~lacks~~
the inherent diversity and flexibility in
expression: they are ~~rigid and not~~
natural-language-like.

has
natural language content that ~~lacks~~
the inherent diversity and flexibility in
expression: they are ~~rigid and not~~
natural-language-like.

Mohan Ganesalingam

LNC5 7805

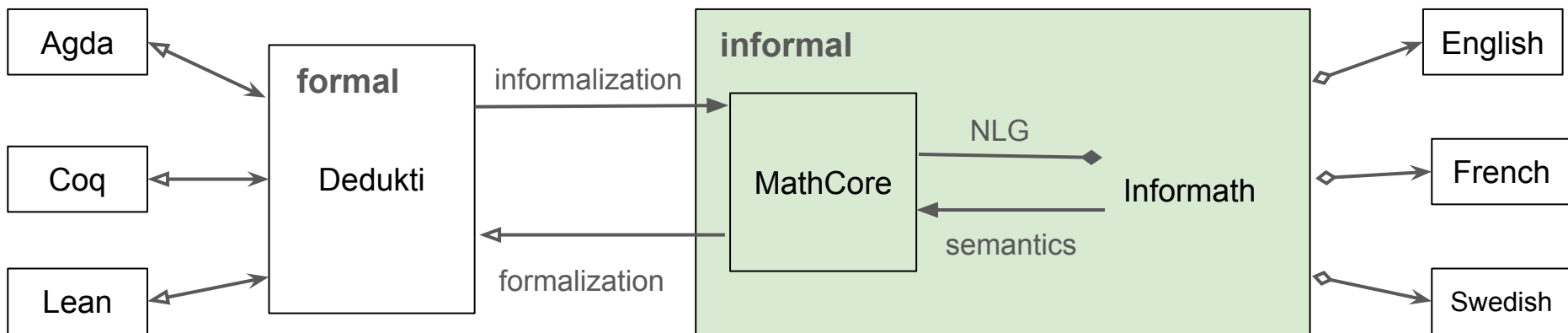
The Language of Mathematics

A Linguistic and Philosophical Investigation

If $K \leq G$ and there are inclusions $g \leq K$ and $g^{-1} \leq K$, then $g \leq K$ for every $g \in G$, and this gives the reverse inclusion $K \leq G$: replacing K by g^{-1} , we have the inclusion $g \leq K^{-1}$. The kernel K of a homomorphism $f: G \rightarrow H$ is a normal subgroup of G , i.e. $f(g) = e_H$ for all $g \in K$. If f is a monomorphism, then $f(K) = \{e_H\}$. If f is an isomorphism, then $f(K) = K$.

 Springer





	to one	to many
total	\longrightarrow	$\longrightarrow\blacklozenge$
partial	$\longrightarrow\triangleright$	$\longrightarrow\diamond$

```

abstract Informath = MathCore ** {

cat
  [Adj] {2} ;
  [Exp] {2} ;

fun
  -- to use symbolic expressions whenever possible
  FormulaProp : Formula -> Prop ;
  SetTerm : Set -> Term ;
  ConstTerm : Const -> Term ;
  ComparEqsign : Compar -> Eqsign ;

  -- to remove parentheses around complex propositions
  SimpleAndProp : [Prop] -> Prop ;

  -- to aggregate adjectives and expressions
  AndAdj : [Adj] -> Adj ;
  OrAdj : [Adj] -> Adj ;

  AndExp : [Exp] -> Exp ;
  OrExp : [Exp] -> Exp ;

  -- in situ quantifiers

  AllKindExp : Kind -> Exp ;
  AllIdentsKindExp : [Ident] -> Kind -> Exp ;

  SomeKindExp : Kind -> Exp ;
  SomeIdentsKindExp : [Ident] -> Kind -> Exp ;

  NoIdentsKindExp : [Ident] -> Kind -> Exp ;
  NoKindExp : Kind -> Exp ;

  -- miscellaneous alternative expressions

  PostQuantProp : Prop -> Exp -> Prop ;
}

```

```

prop110 : (a : Elem Int) -> (c : Elem Int) ->
  Proof (and (odd a) (odd c)) -> Proof (forall
    Int (b => even (plus (times a b) (times b c)))).

```

Prop110. For all instances a and c of integers, if we can prove that a is odd and c is odd, then we can prove that for all integers b , the sum of the product of a and b and the product of b and c is even.

```

abstract Informath = MathCore ** {

cat
  [Adj] {2} ;
  [Exp] {2} ;

fun
-- to use symbolic expressions whenever possible
  FormulaProp : Formula -> Prop ;
  SetTerm : Set -> Term ;
  ConstTerm : Const -> Term ;
  Compareqsign : Compar -> Eqsign ;

-- to remove parentheses around complex propositions
  SimpleAndProp : [Prop] -> Prop ;

-- to aggregate adjectives and expressions
  AndAdj : [Adj] -> Adj ;
  OrAdj : [Adj] -> Adj ;

  AndExp : [Exp] -> Exp ;
  OrExp : [Exp] -> Exp ;

-- in situ quantifiers

  AllKindExp : Kind -> Exp ;
  AllIdentsKindExp : [Ident] -> Kind -> Exp ;

  SomeKindExp : Kind -> Exp ;
  SomeIdentsKindExp : [Ident] -> Kind -> Exp ;

  NoIdentsKindExp : [Ident] -> Kind -> Exp ;
  NoKindExp : Kind -> Exp ;

-- miscellaneous alternative expressions

  PostQuantProp : Prop -> Exp -> Prop ;
}

```

```

prop110 : (a : Elem Int) -> (c : Elem Int) ->
  Proof (and (odd a) (odd c)) -> Proof (forall
    Int (b => even (plus (times a b) (times b c)))).

```

Prop110. For all instances a and c of integers, if we can prove that a is odd and c is odd, then we can prove that for all integers b , the sum of the product of a and b and the product of b and c is even.

Prop110. Let $a, c \in \mathbb{Z}$. Assume that both a and c are odd. Then for all integers b , $ab + bc$ is even.

Prop110. Let $a, c \in \mathbb{Z}$. Assume that both a and c are odd. Then $ab + bc$ is even for all integers b .

```

abstract Informath = MathCore ** {

cat
  [Adj] {2} ;
  [Exp] {2} ;

fun
  -- to use symbolic expressions whenever possible
  FormulaProp : Formula -> Prop ;
  SetTerm : Set -> Term ;
  ConstTerm : Const -> Term ;
  ComparEqsign : Compar -> Eqsign ;

  -- to remove parentheses around complex propositions
  SimpleAndProp : [Prop] -> Prop ;

  -- to aggregate adjectives and expressions
  AndAdj : [Adj] -> Adj ;
  OrAdj : [Adj] -> Adj ;

  AndExp : [Exp] -> Exp ;
  OrExp : [Exp] -> Exp ;

  -- in situ quantifiers

  AllKindExp : Kind -> Exp ;
  AllIdsKindExp : [Ident] -> Kind -> Exp ;

  SomeKindExp : Kind -> Exp ;
  SomeIdsKindExp : [Ident] -> Kind -> Exp ;

  NoIdsKindExp : [Ident] -> Kind -> Exp ;
  NoKindExp : Kind -> Exp ;

  -- miscellaneous alternative expressions

  PostQuantProp : Prop -> Exp -> Prop ;
}

```

```

prop50 : Proof (forall Nat
  (n => not (and (even n) (odd n)))).

```

Prop50. We can prove that for all natural numbers n , it is not the case that n is even and n is odd.

Prop50. For all natural numbers n , n is not both even and odd.

Prop50. No natural number n is both even and odd.

Prop50. No natural number is both even and odd.

```

module Core2Informath where

import Informath

nlg :: Opts -> Tree a -> [Tree a]
nlg opts tree = concatMap variations [t, ut, ft, aft, iaft, viaft]
  where
    t = unparenth tree
    ut = uncoerce t
    ft = formalize ut
    aft = aggregate (flatten ft)
    iaft = insitu aft
    viaft = varless iaft

insitu :: Tree a -> Tree a
insitu t = case t of
  GAllProp (GListArgKind [argkind]) (GAdjProp adj exp) -> case subst argkind exp of
    Just (x, kind) -> GAdjProp adj (GAllIdentsKindExp (GListIdent [x]) kind)
    _ -> t
  GAllProp (GListArgKind [argkind]) (GNotAdjProp adj exp) -> case subst argkind exp of
    Just (x, kind) -> GAdjProp adj (GNoIdentsKindExp (GListIdent [x]) kind)
    _ -> t
  GExistProp (GListArgKind [argkind]) (GAdjProp adj exp) -> case subst argkind exp of
    Just (x, kind) -> GAdjProp adj (GSomeIdentsKindExp (GListIdent [x]) kind)
    _ -> t
  _ -> composOp insitu t

varless :: Tree a -> Tree a
varless t = case t of
  GAllIdentsKindExp (GListIdent [_]) kind -> GAllKindExp kind
  GNoIdentsKindExp (GListIdent [_]) kind -> GNoKindExp kind
  GSomeIdentsKindExp (GListIdent [_]) kind -> GSomeKindExp kind
  _ -> composOp varless t

```

NLG (Natural Language Generation) is a combination of selected **almost compositional operations**.

Example: **in situ quantification** is possible if the variable x occurs exactly once in the body.

The variable can optionally be omitted.

B Bringert and A. Ranta, A pattern for almost compositional functions. Journal of Functional Programming 18 (5-6), 567-598, 2008.

```

module Informath2Core where

import Informath

data SEnv = SEnv {varlist :: [String]}

initSEnv = SEnv {varlist = []}

newVar :: SEnv -> (GIdent, SEnv)

sem :: SEnv -> Tree a -> Tree a
sem env t = case t of

    GAdjProp (GAndAdj (GListAdj adjs)) x ->
        let sx = sem env x
        in GAndProp (GListProp [GAdjProp adj sx | adj <- adjs])

    GAdjProp adj (GEveryKindExp kind) ->
        let (x, env') = newVar env
        in sem env'
            (GAllProp (GListArgKind [GIdentArgKind kind (GListIdent [x])])
                (GAdjProp adj (GTermExp (GIdent x))))

```

The opposite direction is, again, simpler:

- deterministic conversion of Informath extensions to MathCore
- like logical semantics (since MathCore is an unambiguous syntax for logic)
- fresh variables must be created for varless in situ quantifiers

Order is important:

every number is even or odd

→ *for all numbers x , x is (even or odd)*

→ *for all numbers x , (x is even or x is odd)*

~~→ *every number is even or every number is odd*~~

~~→ *(for all numbers x , x is even) or (for all numbers x , x is odd)*~~

Demos


```
all: Informath.pgf Dedukti Agda Coq Lean RunInformath
```

```
Informath.pgf:
```

```
    cd grammars ; gf --make -output-format=haskell -haskell=lexical --haskell=gadt  
-lexical=Name,Noun,Fam,Adj,Rel,Fun,Label,Const,Oper,Compar,Set,Coercion,Relverb,Relno  
un,Reladj,Comparnoun,Verb --probs=Informath.probs InformathEng.gf InformathFre.gf  
InformathSwe.gf
```

```
Dedukti:
```

```
    cd typetheory ; bnfc -m -p Dedukti --haskell-gadt Dedukti.bnf ; make
```

```
Agda:
```

```
    cd typetheory ; bnfc -m -p Agda --haskell-gadt Agda.bnf ; make
```

```
Lean:
```

```
    cd typetheory ; bnfc -m -p Lean --haskell-gadt Lean.bnf ; make
```

```
Coq:
```

```
    cd typetheory ; bnfc -m -p Coq --haskell-gadt Coq.bnf ; make
```

```
RunInformath:
```

```
    ghc -package gf RunInformath.hs
```

make

demo:

```
./RunInformath -lang=Fre test/exx.dk  
./RunInformath -lang=Swe test/exx.dk  
./RunInformath -lang=Eng test/exx.dk  
./RunInformath -lang=Eng test/exx.dk >exx.txt  
./RunInformath -lang=Eng exx.txt  
./RunInformath -lang=Eng test/gflean-data.txt
```

```
cat BaseConstants.dk test/exx.dk >bexx.dk  
dk check bexx.dk
```

```
./RunInformath -to-agda test/exx.dk >exx.agda  
agda --prop exx.agda
```

```
./RunInformath -to-coq test/exx.dk >exx.v  
cat BaseConstants.v exx.v >bexx.v
```

```
coqc bexx.v  
./RunInformath -to-lean test/exx.dk >exx.lean
```

```
cat BaseConstants.lean exx.lean >bexx.lean  
lean bexx.lean
```

```
./RunInformath -to-latex-file -variations test/top100.dk >out/top100.tex  
echo "consider pdflatex out/top100.tex"
```

```
./RunInformath -to-latex-file -variations test/sets.dk >out/sets.tex  
echo "consider pdflatex out/sets.tex"
```

make demo

Generating synthetic data

For those who are interested just in the generation of synthetic data, the following commands (after building Informath with make) can do it: assuming that you have a .dk file available, build a .jsonl file with all conversions of each Dedukti judgement:

```
$ ./RunInformath -parallel <file>.dk > <file>.jsonl
```

After that, select the desired formal and informal languages to generate a new .jsonl data with just those pairs:

```
$ python3 test/jsonltest.py <file.jsonl> <formal> <informal>
```

The currently available values of <formal> and <informal> are the keys in <file>.jsonl - for example, agda and InformathEng, respectively.

<https://github.com/aarneranta/informath/>

Preview

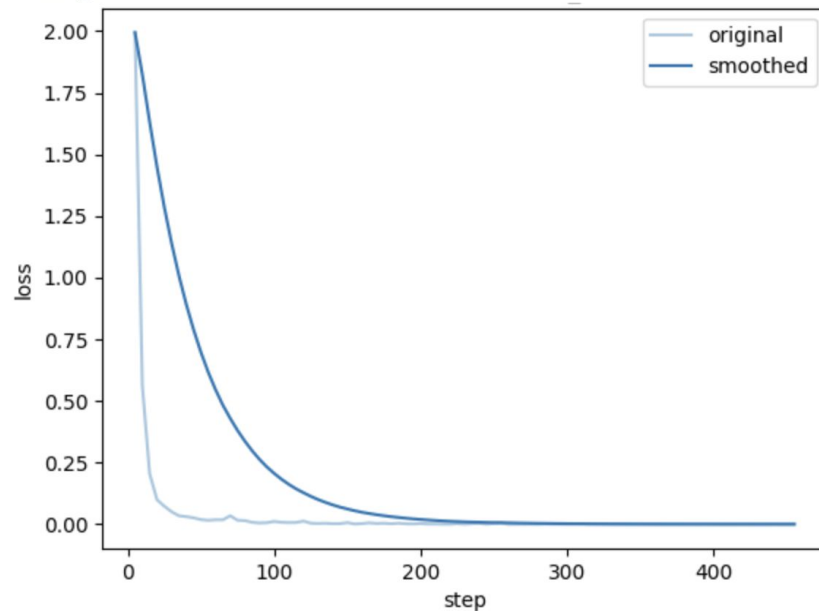
Code

Blame

267 lines (252 loc) · 12 KB

- **Checked Dataset with Last English Expression as Testset (2460)**

Training



Evaluation

On checked_test_eng_lastsplit (328):

"predict_bleu-4": 96.97752286585367,

"predict_model_preparation_time": 0.0026,

"predict_rouge-1": 98.09061829268293,

Pei Huang, MSc project
at Chalmers, 2025

Conclusion

Symbolic informalization can be

- natural and fluent
 - by extending CNL towards the full language of mathematics
- feasible to develop
 - by Dedukti, GF, and rule extraction
- shared by different formal and informal languages
 - by Dedukti and GF interlinguas
- inverted to autoformalization
 - by fine-tuned LLM + feedback informalization

Questions to work on here

Collect Dedukti formalizations

- from Agda
- from Lean
- 100 theorems

Communicate with Dedukti in type checking

- choose from ambiguous parse results

Resolve hidden arguments

- by lambda-pi?
- overloading
- coercions, number types

FIN



<https://github.com/aarneranta/informath>