Assignment 2

This exercises concern topics that were covered in Lectures 5, 6, 7 and 8. Before solving the assignment, study the corresponding topics. Unless otherwise specified, the significance level is $\alpha = 0.05$.

If you are asked to perform a test, do not only give the conclusion of your test, but report:

- *the hypotheses in terms of the population parameter of interest,*
- *the significance level,*
- *the test statistic and its distribution under the null hypothesis,*
- *the observed value of the test statistic,*
- *the p-value or the critical value(s),*
- *the conclusion.*

Theoretical exercises

For the theoretical exercises use Tables 2 and 3 from the Appendix in the book (also given on Canvas) to find critical values (do not use R), you need to learn this for the exam.

Exercise 1. You have been employed by an airline and given the task of estimating the percentage of flights that arrive on time, i.e. no later than 20 minutes after the scheduled arrival time. How many flights should you survey in order to be 95% confident that your estimate is within two percentage points of the true population percentage? Solve this problem under each of the following assumptions:

a) nothing is known about the percentage of on-time flights of your airline;

b) for a previous year, 90% of all flights of your airline were on time (based on an official statistic from the government).

Exercise 2. Consider a study about the brain volumes (in cm^3) of 30 pairs of twins. The aim of the study was to quantify the difference in the population mean volumes of first- and second-born twins. The average volume of the 30 first-born twins was $\bar{x}_1 = 1124.3$, and that of the 30 second-borns was $\bar{x}_2 = 1118.1$. Some more statistics that you may or may not use are: $s_1 = 130.5, s_2 = 124.7, s_d = 57.8$. Construct a 90% confidence interval for the difference of the population means.

Exercise 3. Consider a study about the brain volumes (in cm^3) of 25 first-borns and 20 (unrelated) second-borns. The aim of the study was to test whether there is a difference in the population mean volumes of first- and second-born babies. The average volume of the 25 first-borns was $\bar{x}_1 = 1131.3$, and that of the 20 second-borns was $\bar{x}_2 = 1123.8$. Some more statistics that you may or may not use are: $s_1 = 129.0, s_2 = 127.2, s_p = 128.2$. Conduct the statistical test of interest and use $\alpha = 5\%$. Do we need to make specific assumptions in order to be allowed to use the test?

R-exercises

Hints concerning R:

- Use `read.table(file="file.txt",header=T)` to read the data (with a header) from file `file.txt`.
- The R-commands `pt(...,df=k)` and `qt(...,df=k)` can be used to compute probabilities and quantiles of a t-distributed random variable with k degrees of freedom.
- The R-command `t.test` can be used for computing a confidence interval based on a t-distribution and for performing t-tests. For example, to perform a t-test with default significance level 5% for testing the null hypothesis $\mu = 10$ against the alternative hypothesis $\mu > 10$, using the data set `example` use the command `t.test(example,mu=10,alt="greater")`. For arguments of `t.test` study `help(t.test)`.
- Use `t.test(...)$p.value` to access the p-value of the t-test without printing the whole output of `t.test`.

- The R-command `t.test` can also be used for *two*-sample problems: put the values of the two samples into two vectors, `x` and `y`, say; the command `t.test(x,y, ...)` performs a two-sample test and computes a confidence interval as well, with default significance level $\alpha = 5\%$. Values of other arguments, like `"paired = TRUE"` to perform a paired t-test, can be specified.
- The R-command `prop.test` can be used for testing about proportions; study `help(prop.test)`. When relating to material in the lecture, pay attention whether the test is performed with continuity correction or not: option `correct=F` in command `prop.test`.

Exercise 4. The data set `birthweight.txt` contains the birthweights (in grams) of 188 newborn babies. Denote the underlying mean birthweight by μ. CI stands for confidence interval.

a) Check normality of the data. Assuming normality (irrespective of your conclusion about normality), construct a bounded 96%-CI for μ. Evaluate the sample size needed to provide that the length of the 96%-CI is at most 100.

b) An expert claims that the mean birthweight is bigger than 2800 gram. Verify this claim by using a relevant t-test, explain the meaning of the CI in the R-output for this test.

c) Let p be the probability that birthweight of a newborn baby is less than 2600 gram. Using asymptotic normality, the expert computed the left end $\hat{p}_l = 0.25$ of the confidence interval $[\hat{p}_l, \hat{p}_r]$ for p. Recover the whole confidence interval and its confidence level.

Exercise 5. Alice and Bob work evening shifts in a supermarket. Alice has complained to the manager that she works, on average, much more than Bob. The manager claims that on average they both work the same amount of time, i.e. the competing claim is that the average working hours are different. After a short discussion between the manager and Alice, the manager randomly selected 50 evenings when Alice and Bob both worked. The columns `Alice` and `Bob` in the data file `Alice.txt` contain the number of hours they have worked per evening.

a) Give an estimate and also a 90% confidence interval for the difference of mean working time per evening of Alice and Bob.

b) Investigate the manager's claim with a suitable test, with significance level $\alpha = 0.1$.

c) Now investigate Alice's claim with a suitable test, with significance level $\alpha = 0.01$.

Alice has another concern. By contract the employees are supposed to work 3.8 hours per evening. Alice claims that the proportion of evenings on which she worked more than 3.8 hours is larger than the proportion of evenings during which Bob worked more than 3.8 hours. *(In contrast to the previous situation, we now suppose that the data given in `Alice.txt` were all collected on different evenings.)*

d) Investigate Alice's claim with a suitable test, with significance level $\alpha = 0.01$.