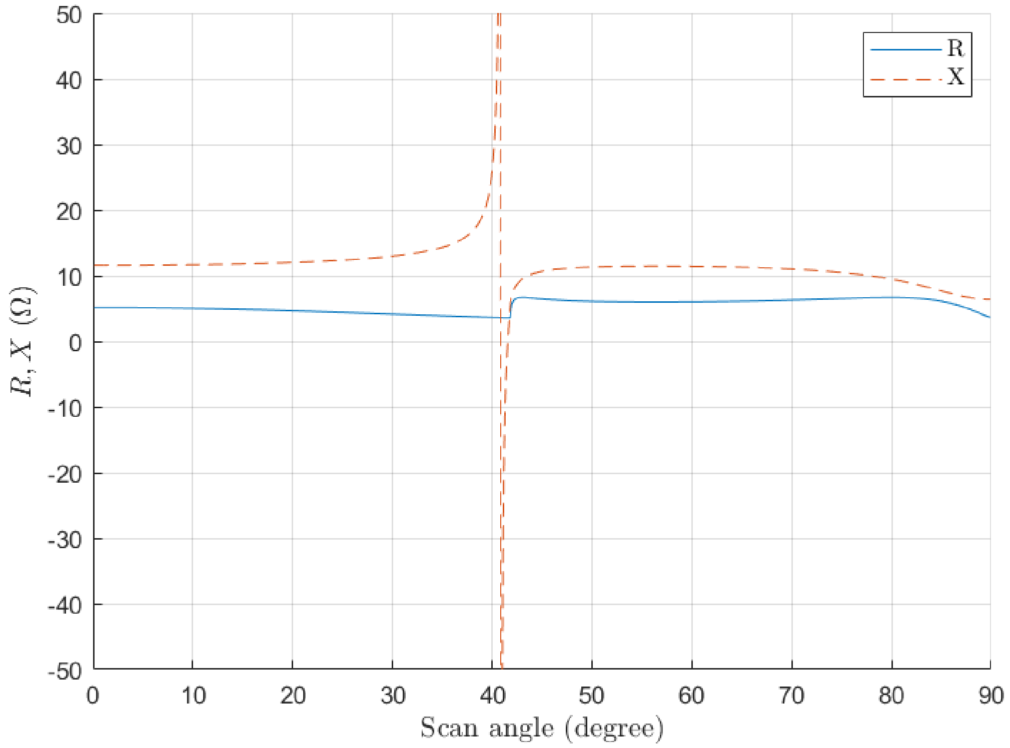
$$\vec{I}(x,y)=\hat{x}\sum_i\sum_j f(x-x_{ij},y-y_{ij})\exp(-\mathrm{j}k_{x0}x_{ij}-\mathrm{j}k_{y0}y_{ij}),$$

$$\text{where } f(x,y)=\begin{cases} I_0\sin\left(k_0\left(\frac{l}{2}+x\right)\right)\delta(y), & -\frac{l}{2}\le x\le 0\\ I_0\sin\left(k_0\left(\frac{l}{2}-x\right)\right)\delta(y), & 0\le x\le\frac{l}{2}\\ 0, & \text{otherwise}\end{cases}$$

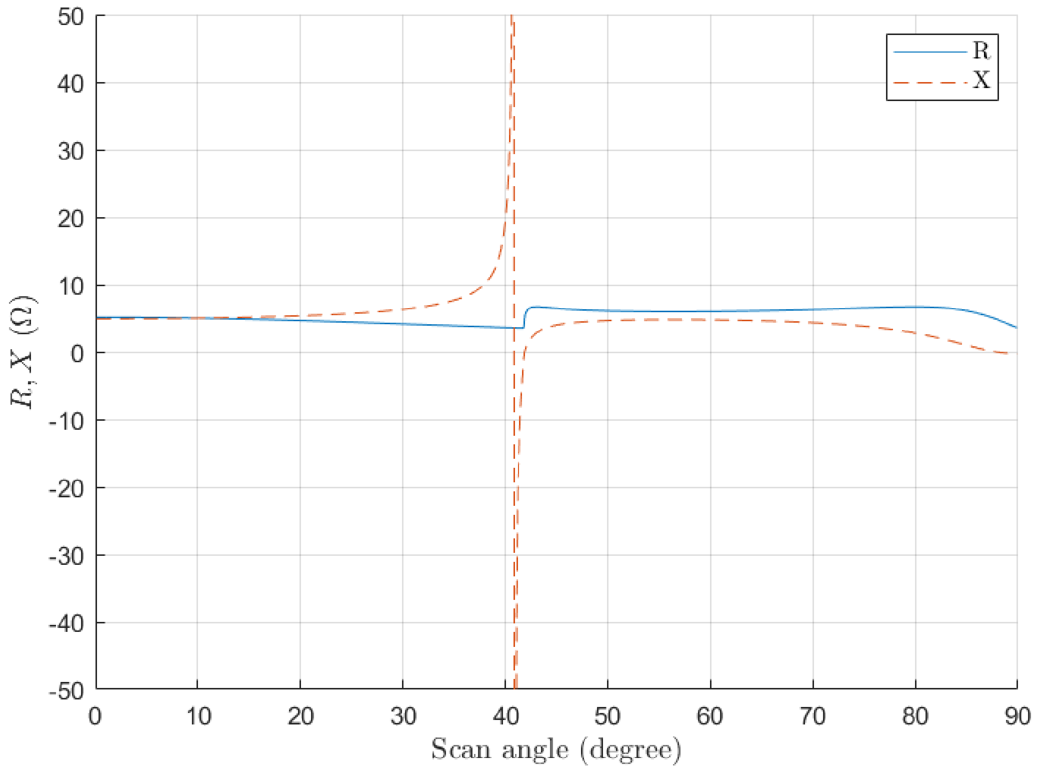
$$\begin{aligned}
\tilde{f}(k_{xmn},k_{ymn}) &= \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp(\mathrm{j}k_{xmn}x+\mathrm{j}k_{ymn}y)\,\mathrm{d}x\,\mathrm{d}y\\
&= \frac{1}{4\pi^2}\left(\int_{-\frac{l}{2}}^{0} I_0\sin\left(k_0\left(\frac{l}{2}+x\right)\right)\exp(\mathrm{j}k_{xmn}x)\,\mathrm{d}x+\int_0^{\frac{l}{2}} I_0\sin\left(k_0\left(\frac{l}{2}-x\right)\right)\exp(\mathrm{j}k_{xmn}x)\,\mathrm{d}x\right)\\
&\quad\cdot\int_{-\infty}^{\infty}\delta(y)\exp(\mathrm{j}k_{ymn}y)\,\mathrm{d}y\\
&= \frac{1}{4\pi^2}\left(\int_{-\frac{l}{2}}^{0} I_0\sin\left(k_0\left(\frac{l}{2}+x\right)\right)\exp(\mathrm{j}k_{xmn}x)\,\mathrm{d}x+\int_0^{\frac{l}{2}} I_0\sin\left(k_0\left(\frac{l}{2}-x\right)\right)\exp(\mathrm{j}k_{xmn}x)\,\mathrm{d}x\right)\\
&= \frac{1}{4\pi^2}\left(\int_{-\frac{l}{2}}^{0} I_0\sin\left(k_0\left(\frac{l}{2}+x\right)\right)\exp(\mathrm{j}k_{xmn}x)\,\mathrm{d}x+\int_{-\frac{l}{2}}^{0} I_0\sin\left(k_0\left(\frac{l}{2}+u\right)\right)\exp(-\mathrm{j}k_{xmn}u)\,\mathrm{d}u\right)\\
&= \frac{2I_0}{4\pi^2}\int_{-\frac{l}{2}}^{0}\sin\left(k_0\left(\frac{l}{2}+x\right)\right)\cos(k_{xmn}x)\,\mathrm{d}x\\
&= \frac{I_0}{4\pi^2}\int_{-\frac{l}{2}}^{0}\sin\left((k_0+k_{xmn})x+\frac{l}{2}k_0\right)+\sin\left((k_0-k_{xmn})x+\frac{l}{2}k_0\right)\mathrm{d}x\\
&= \frac{I_0}{4\pi^2}\cdot\left(-\left.\frac{\cos\left((k_0+k_{xmn})x+\frac{l}{2}k_0\right)}{k_0+k_{xmn}}\right|_{-\frac{l}{2}}^{0}-\left.\frac{\cos\left((k_0-k_{xmn})x+\frac{l}{2}k_0\right)}{k_0-k_{xmn}}\right|_{-\frac{l}{2}}^{0}\right)\\
&= \frac{I_0}{2\pi^2}\frac{k_0}{k_0^2-k_{xmn}^2}\left[\cos\left(\frac{k_{xmn}}{2}l\right)-\cos\left(\frac{k_0}{2}l\right)\right]
\end{aligned}$$

$$\begin{aligned}
Z^{\mathrm{FL}}(k_{x0},k_{y0}) &= \frac{1}{I_0^2}\sum_m\sum_n\left[\frac{|I_{mn}^{\mathrm{TE}}|^2}{y_{mn}^{\mathrm{TE}}}+\frac{|I_{mn}^{\mathrm{TM}}|^2}{y_{mn}^{\mathrm{TM}}}\right]\\
&= \frac{1}{I_0^2}\sum_m\sum_n\left[\frac{k_{ymn}^2}{y_{mn}^{\mathrm{TE}}}+\frac{k_{xmn}^2}{y_{mn}^{\mathrm{TM}}}\right]\frac{ab}{k_0^2-k_{zmn}^{+2}}I_{xmn}^2\\
&= \frac{1}{I_0^2}\sum_m\sum_n\left[\frac{k_{ymn}^2}{y_{mn}^{\mathrm{TE}}}+\frac{k_{xmn}^2}{y_{mn}^{\mathrm{TM}}}\right]\frac{ab}{k_0^2-k_{zmn}^{+2}}\left(\frac{4\pi^2}{ab}\right)^2\tilde{f}^2(k_{xmn},k_{ymn})\\
&= \frac{4k_0^2}{ab}\sum_m\sum_n\left[\frac{k_{ymn}^2}{y_{mn}^{\mathrm{TE}}}+\frac{k_{xmn}^2}{y_{mn}^{\mathrm{TM}}}\right]\frac{1}{k_0^2-k_{zmn}^{+2}}\left(\frac{1}{k_0^2-k_{xmn}^2}\left[\cos\left(\frac{k_{xmn}}{2}l\right)-\cos\left(\frac{k_0}{2}l\right)\right]\right)^2
\end{aligned}$$

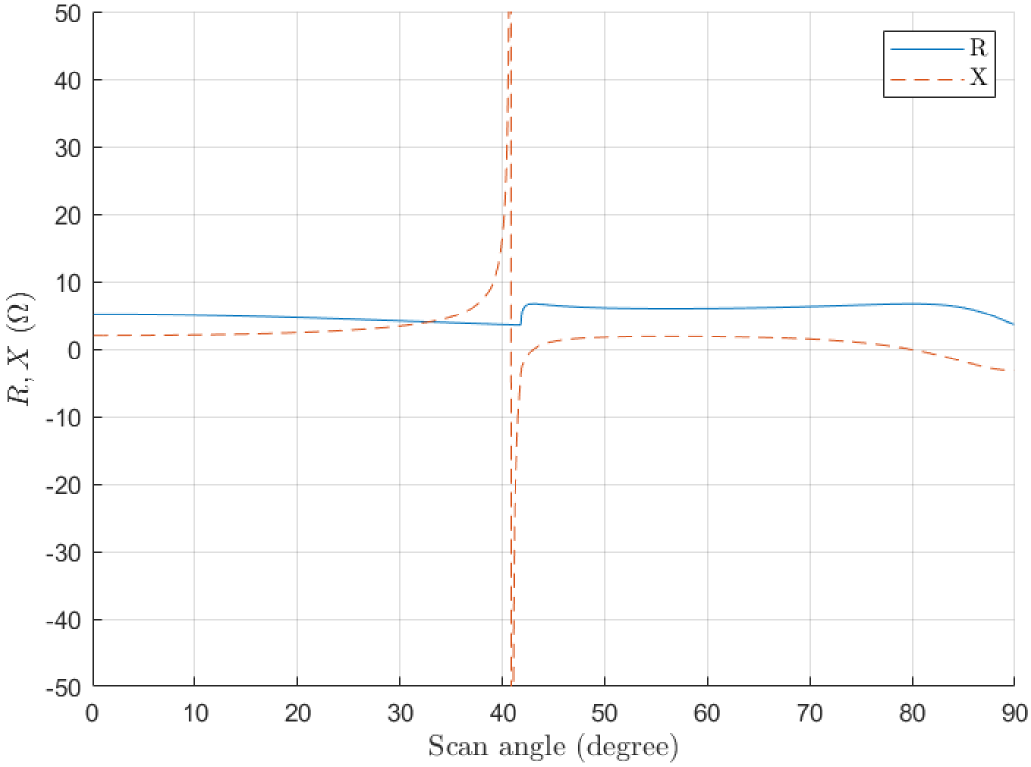
m=n=-100:1:100




m=n=-500:1:500




m=n=-1000:1:1000



R
X
50
40
30
20
10
0
-10
-20
-30
-40
-50
0
10
20
30
40
50
60
70
80
90
$R, X$ ($\Omega$)
Scan angle (degree)