~~##~~ MAT_cv 10 2.5.2024

Diferenciální rovnice

Dokažte, že $y(x)$, která je ~~[illegible]~~ daná implicitně rovnicí: $x^2+y^2=1$ je na $(-1;1)$ maximálním řešením dif. rovnice $x+yy'=0$ při počát. podmínce $y(0)=1$

$$x^2+y^2=1$$

1der. $2x+2yy'=0$

$$y'=\frac{-x}{y}$$

$L=x+y\cdot\left(-\frac{x}{y}\right)=0$

$P=0$ $\qquad \underline{\underline{L=P}}$

řešte dif. rci:

$$\frac{y'}{x}=\frac{1}{1+x^2}$$

$$y'=\frac{x}{1+x^2}$$

$$\frac{dy}{dx}=\frac{x}{1+x^2} \;/dx$$

$$\int dy=\int\frac{x}{1+x^2}dx \qquad \begin{aligned} s&=1+x^2\\ ds&=2x\,dx\\ dx&=\frac{1}{2x}ds\end{aligned}$$

$$y=\frac{1}{2}\int\frac{2x}{1+x^2}dx$$

$$y=\frac{1}{2}\ln|1+x^2|+C$$

1 SQUARE = ____________

$$2y^{\frac{1}{2}} + c_1 = 2x\ln x - 2x + 2c_2$$

$$y^{\frac{1}{2}} = x\ln x - x + c$$

$$y = (x\ln x - x + c)^2$$

$$1 = (e \cdot \ln e - e + c)^2$$

$$1 = (e - e + c)^2$$

$$1 = c^2$$

$$c = 1 \qquad \underline{y = (x \cdot \ln x - x + 1)^2}$$

4) $y' = \dfrac{x+y}{y-x}$; $y(1) = 0$

$$y' = \frac{\cancel{x} \cdot (1 + \frac{y}{x})}{\cancel{x} \cdot (\frac{y}{x} - 1)} \qquad u = \frac{y}{x} \Leftrightarrow y = ux$$

$$y' = u'x + u$$

$$u'x + u = \frac{1+u}{u-1}$$

$$u'x = \frac{1+u}{u-1} - u$$

$$u'x = \frac{-u^2 + 2u + 1}{u-1}$$

$$\frac{dx}{du} \cdot \frac{1}{x} = \frac{u-1}{-u^2+2u+1}$$

$$\int \frac{1}{x} dx = \int \frac{u-1}{-u^2+2u+1} du$$

$$\ln|x| + c = -\frac{1}{2} \ln(u^2 - 2u - 1)$$

1 SQUARE = ____________

2) $y' = \cos x \quad ; \quad y(0) = 1$

$$\frac{dy}{dx} = \cos x$$

$$dy = \cos x \, dx$$

$$\int dy = \int \cos x \, dx$$

$$y = \sin x + C$$

$$1 = \sin(0) + C \qquad \underline{\underline{y = \sin(x) + 1}}$$

$$C = 1$$

3) $y' = 2\sqrt{y} \ln x \quad , \quad y(e) = 1$

$$\frac{dy}{dx} = 2\sqrt{y} \ln x$$

$$\frac{dy}{\sqrt{y}} = 2 \ln x \cdot dx$$

$$\int y^{-\frac{1}{2}} dy = 2 \cdot \int \ln x \, dx$$

$$\frac{y^{\frac{1}{2}}}{\frac{1}{2}} + C_1 = 2 \cdot \int 1 \cdot \ln x \, dx \Rightarrow \left| \begin{matrix} u' = 1 & u = x \\ v = \ln x & v' = \frac{1}{x} \end{matrix} \right| = x \cdot \ln x - \int \frac{x}{x} dx =$$

$$x \cdot \ln x - x + C_2$$

1 SQUARE =