MAT2_cv05

21.3.2024

přes přímky:

způsob 1) $\lim\limits_{(x,y)\to(2,3)} \dfrac{y-3}{x+y-5}$ $\qquad y = k(x-2)+3$

vzorec přímky

$\downarrow$

$y = kx+q$

$$\lim_{x\to 2}\frac{k(x-2)+3-3}{x+k(x-2)+3-5} = \lim_{x\to 2}\frac{k(x-2)}{x-2+k(x-2)} = \lim_{x\to 2}\frac{k}{1+k}$$

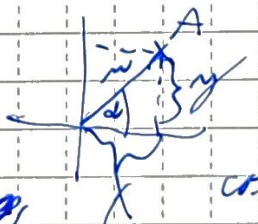
$\to$ lim závisí na k $\to$ původní limita neexistuje

způsob:

2) $\lim\limits_{\substack{x\to 2\\ y=3}} \dfrac{y-3}{x+y-5} = \lim\limits_{x\to 2}\dfrac{0}{x-2} = 0$

$$\lim_{\substack{x=2\\ y\to 3}}\frac{y-3}{x+y-5} = \lim_{y\to 3}\frac{y-3}{y-3} = 1$$

3) $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy}{x^2+y^2} =$



$\cos\alpha = \dfrac{x}{r}$

$\sin\alpha = \dfrac{y}{r}$

$$\lim_{r\to 0^+}\frac{2\cdot(r\cdot\cos\alpha)\cdot(r\cdot\sin\alpha)}{r^2\cos^2\alpha + r^2\cdot\sin^2\alpha} = \lim_{r\to 0^+}\frac{r^2\cdot 2\cos\alpha\sin\alpha}{r^2(\cos^2\alpha\sin^2\alpha)}$$

$$\lim_{r\to 0^+}\frac{2\cos\alpha\cdot\sin\alpha}{1} = \lim_{r\to 0^+}\sin 2\alpha = \sin 2\alpha$$

$\downarrow$

neexistuje, nezávisí na r

$$\boxed{\begin{aligned} x &= r\cdot\cos\alpha \\ y &= r\cdot\sin\alpha \end{aligned}}$$

$\sin^2\alpha + \cos^2\alpha = 1$

$2\cos\alpha\sin\alpha = \sin 2\alpha$

1 SQUARE = ZPíčes ☺

Parciální derivace

4) $f(x,y) = x^2 y + \frac{y^3}{x^4}$ $D_f \{[x,y] \in \mathbb{R}^2 : x \neq 0\}$

parciá. derivace 1. řádu

$$\frac{\partial f}{\partial x} = 2xy - 5y^3 x^{-5}$$

$$\frac{\partial f}{\partial y} = x^2 + 3y^2 x^{-4}$$

5) $f(x,y) = x^y$ $x > 0$

$(x^a)' = a \cdot x^{a-1}$

$(a^x)' = a^x \cdot \ln a$

$$\frac{\partial f}{\partial x} = y \cdot x^{y-1}$$

$$\frac{\partial f}{\partial y} = x^y \cdot \ln x$$

6 $f(x,y) = \operatorname{arctg}\left(\frac{x}{y}\right)$

$\operatorname{arctg}(x)' = \frac{1}{1+x^2}$

$$\frac{\partial f}{\partial x} = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot y^{-1} = \frac{1}{\frac{y^2+x^2}{y^2}} \cdot \frac{1}{y} = \frac{y^{\cancel{2}}}{y^2+x^2} \cdot \frac{1}{\cancel{y}} = \frac{y}{y^2+x^2}$$

$$\frac{\partial f}{\partial y} = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot x \cdot (-1 y^{-2}) = \frac{\cancel{y^2}}{y^2+x^2} \cdot \frac{-x}{\cancel{y^2}} = -\frac{x}{y^2+x^2}$$

1 SQUARE =

$f(x,y,z) = x^2 \cdot \sin(2y-z)$

$\frac{\partial f}{\partial x} = 2x \cdot \sin(2y-z)$

$\frac{\partial f}{\partial y} = x^2 \cdot \cos(2y-z) \cdot 2 = 2x^2 \cdot \cos(2y-z)$

$\frac{\partial f}{\partial z} = x^2 \cdot \cos(2y-z) \cdot (-1) = -x^2 \cdot \cos(2z-z)$

$f(x,y) = x - e^{xy^2}$ $\quad u = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\quad v\ (0,1)$

1) gradiant ... $\nabla f(x,y)$

2) směrové derivace

1) $\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \left(1 - e^{xy^2} \cdot y^2, -e^{xy^2} \cdot 2yx\right)$

$\frac{\partial f}{\partial x} = 1 - e^{xy^2} \cdot y^2$ $\quad \nabla f(0,0) = (1, 0)$

$\nabla f(2,1) = (1 - e^2 \cdot 1, -e^2 \cdot 4)$

$\frac{\partial f}{\partial y} = x - e^{xy^2} \cdot 2y = -e^{xy^2} \cdot 2yx$

1 SQUARE = ______________

$\frac{\partial f}{\partial u}(0,0)$, $\frac{\partial f}{\partial v}(2,1)$ normy:

$\frac{\partial f}{\partial u}(2,1)$

$\|u\| = \sqrt{\left(\frac{\sqrt2}{2}\right)^2 + \left(\frac{\sqrt2}{2}\right)^2} = 1$

$\|v\| = \sqrt{0^2 + (-1)^2} = 1$

pokud norma = 1, tak je to ok, jinak se musí udělat tohle: $\frac{\partial f}{\partial u}(0,0) = \nabla f(0,0) \cdot \frac{u}{\|u\|}$

$$\frac{\partial f}{\partial u}(0,0) = (\nabla f(0,0)) \cdot u = (1,0) \cdot \left(\frac{\sqrt2}{2}; \frac{\sqrt2}{2}\right) = \frac{\sqrt2}{2} + 0 = \frac{\sqrt2}{2}$$

$$\frac{\partial f}{\partial u}(2,1) = (\nabla f(2,1)) \cdot u = (1-e^2, -e^2 \cdot 4) \cdot \left(\frac{\sqrt2}{2}; \frac{\sqrt2}{2}\right) =$$

$$\frac{\sqrt2}{2} - \frac{\sqrt2}{2}e^2 + \left(-\frac{\sqrt2}{2}e^2 \cdot 4\right) = \frac{\sqrt2}{2}\left(1 - e^2 - e^2 \cdot 4\right)$$

$$\frac{\partial f}{\partial v}(2,1) = (1-e^2, -4e^2) \cdot (0,-1) = 0 + (4 \cdot e^2) = 4 \cdot e^2$$

1 SQUARE =