1.1 $$\sum_{n=1}^{\infty} \frac{(x-2)^n \cdot 2^{1-n}}{n} = \lim_{n\to\infty} \left| \frac{\frac{(x-2)^{n+1} \cdot 2^{1-(n+1)}}{n+1}}{\frac{(x-2)^n \cdot 2^{1-n}}{n}} \right| = \lim_{n\to\infty} \frac{(x-2)^{n+1} \cdot 2^{1-n-1}}{n+1} \cdot \frac{n}{\cancel{(x-2)^n} \cdot \cancel{2^{1-n}}} =$$

$$\lim_{n\to\infty} \frac{(x-2)\cdot 2^{-1}}{n+1} \cdot \frac{n}{1} = \frac{|x-2|}{2} \cdot \lim_{n\to\infty} \underbrace{\left|\frac{n}{n+1}\right|}^{1} \Rightarrow -1 < \frac{x-2}{2} < 1$$

$$-2 < x-2 < 2$$

$$0 < x < 4 \qquad x \in (0,4)$$

1.5 $$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^{(2n-1)} \cdot 2^{(1-n)}}{2n-1} \Rightarrow \lim_{n\to\infty} \left| \frac{\cancel{(-1)}^{n+1} \cdot (x-2)^{2(n+1)-1} \cdot \cancel{2}^{1-n-1}}{2(n+1)-1} \cdot \frac{2n-1}{\cancel{-1}^n \cdot \cancel{(x-2)}^{2n-1} \cdot \cancel{2^{1-n}}} \right| =$$

$$\lim_{n\to\infty} \left| \frac{-1\cdot (x-2)^2 \cdot 2^{-1}}{2n+1} \cdot \frac{2n-1}{1} \right| = -\frac{(x-2)^2}{2} \cdot \lim_{n\to\infty} \underbrace{\left|\frac{2n-1}{2n+1}\right|}^{1} =$$

$$-\frac{(x-2)^2}{2} < 1$$

$$-(x-2)^2 < 2$$

$$-(x-2) < \pm\sqrt{2}$$

$$x > 2 \pm \sqrt{2}$$

$$x \in (2-\sqrt{2}; 2+\sqrt{2})$$

DATE

1.9 $\sum_{n=1}^{\infty} \frac{(x-2)^n \cdot 2^{(1-n)}}{n^2} = \lim_{n\to\infty} \left| \frac{(x-2)^{n+1} \cdot 2^{1-n-1}}{(n+1)^2} \cdot \frac{n^2}{(x-2)^n \cdot 2^{1-n}} \right| =$

n^2+2n+1

$\lim_{n\to\infty} \left| \frac{\frac{x-2}{2}}{n^2+2n+1} \cdot \frac{n^2}{1} \right| = \left| \frac{x-2}{2} \right| \cdot \lim_{n\to\infty} \frac{\overbrace{n^2}^{1}}{n^2+2n+1} \Rightarrow$

$-1 < \frac{x-2}{2} < 1$

$-2 < x-2 < 1$

$0 < x < 4 \quad x \in (0,4)$

1.19 $\sum_{n=1}^{\infty} \frac{(x+4)^n}{(n+1)^2 \cdot 2^{n-2}} = \lim_{n\to\infty} \left| \frac{(x+4)^{x+1}}{(n+2)^2 \cdot 2^{n-1}} \cdot \frac{(n+1)^2 \cdot 2^{n-2}}{(x+4)^n} \right| = |x+4| \cdot \lim_{n\to\infty} \left| \frac{1}{(n+2)^2} \cdot \frac{(n+1)^2}{2} \right| =$

$\left| \frac{x+4}{2} \right| \cdot \lim_{n\to\infty} \left| \frac{\overbrace{(n+1)^2}^{1}}{(n+2)^2} \right| = \left| \frac{x+4}{2} \right| < 1 \qquad -1 < \frac{x+4}{2} < 1$

$-2 < x+4 < 2$

$-6 < x+4 < -2$

$x \in (-6, -2)$

1 SQUARE = ____

$1.6 \quad \sum_{n=1}^{\infty} (x-2)^n \cdot 2^{(1-2n)} \cdot (n+1)^2 = \lim_{n\to\infty} \left| \frac{(x-2)^{n+1} \cdot 2^{(1-2n-2)} \cdot (n+2)^2}{\cancel{(x-2)^n} \cdot 2^{1-2n} \cdot (n+1)^2} \right| =$

$\lim_{n\to\infty} \left| \frac{\frac{x-2}{2^2} \cdot (n+2)^2}{(n+1)^2} \right| = \left| \frac{x-2}{4} \right| \cdot \lim_{n\to\infty} \left| \overbrace{\frac{(n+2)^2}{(n+1)^2}}^{1} \right|$

$-1 < \frac{x-2}{4} < 1$

$-4 < x-2 < 4$

$-2 < x < 6 \Rightarrow x \in (-2, 6)$

3.9

$f(x,y) = e^{x^2 y} + \operatorname{arctg}(2y) + \frac{y}{2x}$ v bodě $[2,0]$ $= 1+0+0=1$

$\frac{\partial f}{\partial x} = e^{x^2 y} \cdot 2xy + 0 + y \cdot (-2 \cdot x^{-2}) = 1 \cdot 0 + 0 + 0 = 0$

$\frac{\partial f}{\partial y} = e^{x^2 y} \cdot x^2 + 2 \cdot (1+(2y)^2)^{-1} + \frac{1}{2x} = 1 \cdot 4 + 2 + \frac{1}{4} = \frac{25}{4}$

$\frac{\partial^2 f}{\partial x^2} = e^{x^2 y} \cdot (2xy)^2 + e^{x^2 y} \cdot 2y + y(-4) \cdot x^{-3} = 1 \cdot 0 \cdot 0 + 1 \cdot 0 + 0 = 0$

1 SQUARE = ____________

$$\frac{\partial^2 f}{\partial y^2} = e^{x^2 y}\cdot x^2\cdot x^2 + e^{x^2\cdot y}\cdot 0 - \frac{2\cdot(1+(2y)^2)'}{(1+(2y)^2)^2} + 0 = 1\cdot 4\cdot 4 + 1\cdot 0 = 16$$

$$\frac{\partial^2 f}{\partial x\partial y} = e^{x^2 y}\cdot x^2\cdot 2xy + e^{x^2 y}\cdot 2x + (-2)\cdot x^{-2} = 1\cdot 4\cdot 0 + 1\cdot 4 + \left(-\frac{1}{8}\right) = \frac{31}{8}$$

$$f(x,y) + \frac{\frac{\partial f}{\partial x}\cdot(x-x_0) + \frac{\partial f}{\partial y}\cdot(y-y_0)}{1!} + \frac{\frac{\partial^2 f}{\partial x^2}\cdot(x-x_0)^2 + 2\cdot\frac{\partial^2 f}{\partial x\partial y}(x-x_0)\cdot(y-y_0) + \frac{\partial^2 f}{\partial y^2}(y-y_0)^2}{2!}$$

$$1 + \frac{0\cdot(x-2) + \frac{25}{4}\cdot(y-0)}{1} + \frac{0\cdot(x-2)^2 + 2\cdot\frac{31}{8}\cdot(x-2)\cdot(y-0) + \cancel{16}^{\,8}\cdot(y-0)^2}{\cancel{2!}} =$$

$$1 + \frac{25}{4}y + \frac{31}{8}(x-2)\cdot y + 8y^2$$

3.1 $[0;1]$

$$f(x,y) = e^{xy+x} + \operatorname{arcctg}(2x) + \frac{\sin(xy)}{y} = 1 + \frac{\pi}{2} + \frac{0}{1} = 1 + \frac{\pi}{2}$$

$$\frac{\partial f}{\partial x} = e^{xy+x}\cdot(y+1) + \left(-\frac{1}{1+(2x)^2}\right)\cdot 2 + \frac{\cos xy\cdot y}{y} = 1\cdot 2 - 2 + 1 = 1$$

$$\frac{\partial f}{\partial y} = e^{xy+x}\cdot x + 0 + \left(\cos(xy)\cdot x\cdot\frac{1}{y} + \sin(xy)\cdot\left(-\frac{1}{y^2}\right)\right) = x^{xy+x}\cdot x + \left(\frac{\cos(xy)\cdot xy - \sin(xy)}{y^2}\right) = 0$$

$$\frac{\partial^2 f}{\partial x\partial y} = x^{xy+x}\cdot x\cdot(y+1) + e^{xy+x}\cdot 1 + 0 + (-\sin(xy)\cdot x) = 1\cdot 0\cdot 2 + 1 + 0 = 1$$

$$\frac{\partial^2 f}{\partial x^2} = e^{xy+x}\cdot(y+1)^2 + e^{xy+x}\cdot 0 + \left(\frac{((1+(2x)^2)^2)'\cdot 2}{(1+(2x)^2)^2}\right) - \sin xy\cdot y =$$

$$e^{xy+x}\cdot(y+1)^2 + \frac{16x}{(1+(2x)^2)^2} - \sin xy\cdot y = 1\cdot 4 + 1\cdot 0 + 0 - 0 = 4$$

1 SQUARE =

1.6 $\sum_{n=1}^{\infty} (x-2)^n \cdot 2^{(1-2n)} \cdot (n+1)^2 = \lim_{n\to\infty} \left| \frac{(x-2)^{n+1} \cdot 2^{(1-2n-2)} \cdot (n+2)^2}{\cancel{(x-2)^n} \cdot 2^{1-2n} \cdot (n+1)^2} \right| =$

$\lim_{n\to\infty} \left| \frac{\frac{x-2}{2^2} \cdot (n+2)^2}{(n+1)^2} \right| = \left| \frac{x-2}{4} \right| \cdot \lim_{n\to\infty} \left| \underbrace{\frac{(n+2)^2}{(n+1)^2}}_{1} \right|$

$-1 < \frac{x-2}{4} < 1$

$-4 < x-2 < 4$

$-2 < x < 6 \Rightarrow x \in (-2,6)$

v bodě $[2,0]$

3.9

$f(x,y) = e^{x^2y} + \operatorname{arctg}(2y) + \frac{y}{2x}$ $= 1+0+0=1$

$\frac{\partial f}{\partial x} = e^{x^2y} \cdot 2xy + 0 + y \cdot (-2 \cdot x^{-2}) = 1 \cdot 0 + 0 + 0 = 0$

$\frac{\partial f}{\partial y} = e^{x^2y} \cdot x^2 + 2 \cdot (1+(2y)^2)^{-1} + \frac{1}{2x} = 1 \cdot 4 + 2 + \frac{1}{4} = \frac{25}{4}$

$\frac{\partial^2 f}{\partial x^2} = e^{x^2y} \cdot (2xy)^2 + e^{x^2y} \cdot 2y + y(-4) \cdot x^{-3} = 1 \cdot 0 \cdot 0 + 1 \cdot 0 + 0 = 0$

1 SQUARE = ______________

$16y$

$$\frac{d^2f}{dy^2} = e^{x^2y}\cdot x^2\cdot x^2 + e^{x^2\cdot y}\cdot 0 - \frac{2\cdot(1+(2y)^2)'}{(1+(2y)^2)^2} + 0 = 1\cdot 4\cdot 4 + 1\cdot 0 = 16$$

$$\frac{d^2f}{dx\,dy} = e^{x^2y}\cdot x^2\cdot 2xy + e^{x^2y}\cdot 2x + (-2)\cdot x^{-2} = 1\cdot 4\cdot 0 + 1\cdot 4 + \left(-\frac{1}{8}\right) = \frac{31}{8}$$

$$f(x,y) + \frac{\frac{df}{dx}\cdot(x-x_0) + \frac{df}{dy}\cdot(y-y_0)}{1!} + \frac{\frac{d^2f}{dx^2}\cdot(x-x_0)^2 + 2\cdot\frac{d^2f}{dx\,dy}(x-x_0)\cdot(y-y_0) + \frac{d^2f}{dy^2}(y-y_0)^2}{2!}$$

$$1 + \frac{0\cdot(x-2) + \frac{25}{4}\cdot(y-0)}{1} + \frac{0\cdot(x-2)^2 + \cancel{2}\cdot\frac{31}{8}\cdot(x-2)\cdot(y-0) + \overset{8}{\cancel{16}}\cdot(y-0)^2}{\cancel{2!}} =$$

$$1 + \frac{25}{4}y + \frac{31}{8}\cdot(x-2)\cdot y + 8y^2$$

3.1 $[0;1]$

$$f(x,y) = e^{xy+x} + \operatorname{arcctg}(2x) + \frac{\sin(xy)}{y} = 1 + \frac{\pi}{2} + \frac{0}{1} = 1 + \frac{\pi}{2}$$

$$\frac{df}{dx} = e^{xy+x}\cdot(y+1) + \left(-\frac{1}{1+(2x)^2}\right)\cdot 2 + \frac{\cos xy\cdot \cancel{y}}{\cancel{y}} = 1\cdot 2 - 2 + 1 = 1$$

$$\frac{df}{dy} = e^{xy+x}\cdot x + 0 + \left(\cos(xy)\cdot x\cdot\frac{1}{y} + \sin(xy)\cdot\left(-\frac{1}{y^2}\right)\right) = x^{xy+x}\cdot x + \left(\frac{\cos(xy)\cdot xy - \sin(xy)}{y^2}\right) = 0$$

$$\frac{d^2f}{dx\,dy} = x^{xy+x}\cdot x\cdot(y+1) + e^{xy+x}\cdot 1 + 0 + (-\sin(xy)\cdot x) = 1\cdot 0\cdot 2 + 1 + 0 = 1$$

$$\frac{d^2f}{dx^2} = e^{xy+x}\cdot(y+1)^2 + e^{xy+x}\cdot 0 + \left(\frac{(1+(2x)^2)'\cdot 2}{(1+(2x)^2)^2}\right) - \sin xy\cdot y =$$

$$e^{xy+x}\cdot(y+1)^2 + \frac{16x}{(1+(2x)^2)^2} - \sin xy\cdot y = 1\cdot 4 + 1\cdot 0 + 0 - 0 = 4$$

1 SQUARE = ____________

$$\frac{d f^2}{d y^2} = e^{xy+x} \cdot x^2 + \left(\frac{-\sin(xy) \cdot y \cdot xy + \cancel{\cos xy \cdot y - \cos xy \cdot xy}}{y^2} \right) + \frac{\cos(xy) \cdot xy - \sin xy(-2)}{y^3}$$

$$= e^{xy+x} \cdot x^2 - \frac{\sin xy \cdot \cancel{y}}{y^{\cancel{2}}} + \frac{\cos xy \cdot xy + 2 \sin xy}{y^3} = 1 \cdot 0 + 0 + 0 = 0$$

$$1 + \frac{\pi}{2} + \frac{1 \cdot (x-0) + 0 \cdot (y-1)}{1!} + \frac{4 \cdot (x-0)^2 + 2 \cdot (x-0) \cdot (y-1) + 0 \cdot (y-1)^2}{2!} = 1 + \frac{\pi}{2} + x + \frac{4x^2 + 2x(y-1)}{2}$$

4.16

$$f(x,y) = x^4 - 3xy + y^4 + e^{3x-2y-1} = 0 \qquad [1;1]$$

$$\frac{d f}{d x} = 4x^3 - 3y(x) - 3x y(x)' + 4y^3(x) \cdot y(x)' + e^{3x-2y-1} \cdot (3 - 2 \cdot (y(x)')$$

$x = 1$

$y(x) = 1$

$4 - 3 - 3y(x)' + 4y(x)' + 1 \cdot (3 - 2y(x)') = 0$

$1 + y(x)' + 3 - 2y(x)' = 0$ do vzorce přímky

$4 = y(x)'$

$y - y_a = a \cdot (x - x_a)$

$y - 1 = 4 \cdot (x - 1)$

$y = 4x - 3$

$$\frac{d^2 f}{d x^2}\ 12x^2 - 3y(x)' - 3y'(x) - 3x y(x)'' + 12 y^2(x) \cdot y(x)' \cdot y(x)' + 4y^3(x) \cdot y(x)'' +$$

$$e^{3x-2y-1} \cdot (3 - 2y(x)')^2 + e^{3x-2y-1} \cdot (-2y(x)'')$$

$$\cancel{12} - \cancel{12} - 12 - 3y(x)'' + \overset{192}{12 \cdot 16} + 4 \cdot y(x)'' + 1 \cdot (3-8)^2 + 1 \cdot (-2y(x)'') = 0$$

1 SQUARE = $-3y(x)'' + 180 + 4 \cdot y(x)'' + 25 - 2y(x)'' = 0$

$205 = y(x)''$ $\quad 205 > 0$ leží nad přímkou

4.18

$f(x,y)$ $2x^2-5xy+2y^2+e^{x-y-2}=10 \Rightarrow 2x^2-5xy+2y^2+e^{x-y-2}-10=0$

$\frac{df}{dx}=4x-5y(x)-5xy(x)'+4y(x)\cdot y(x)'+e^{x-y-2}\cdot(1-y(x)')=0$

$x=1$

$y(1)=-1$

$4+5-5y(x)'-4y(x)'+1\cdot(1-y(x)')=0$

$10-10y(x)'=0$

$y(x)'=1$

$y-y_a=a\cdot(x-x_a)$

$y+1=1\cdot(x-1)$

$\underline{\underline{y=x-2}}$

$\frac{d^2f}{dx^2}=4-5y(x)'-5y(x)'-5xy(x)''+4(y(x)')^2+4y(x)\cdot y(x)''+e^{x-y-2}\cdot(1-y(x)')^2+$
$e^{x-y-2}\cdot(-y(x)'')=0$

$4-5-5-5y(x)''+4-4y(x)''+1\cdot(1-1)^2+1\cdot(-y(x)'')=0$

$-6-5y(x)''+4-4y(x)''-y(x)''=0$

$-2-10y(x)''=0$

$\underline{\underline{y(x)''=-\frac{1}{5}}}$

$-\frac{1}{5}<0 \Rightarrow$ leží pod priamkou ✓

1 SQUARE = ____

5.3

$$f(x,y) = (y-3x)^{2x+1} + \arcsin(xy)$$

$$u = (1,1) \quad \frac{\partial f}{\partial u} = (0,1) \quad \nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \downarrow$$

$$\frac{\partial f}{\partial x} = (y-3x)^{2x+1} \cdot \left(\frac{1}{y-3x} \cdot (-3) \cdot (2x+1) + \ln(y-3x) \cdot 2\right) + \frac{1}{\sqrt{1-(xy)^2}} \cdot y =$$

$$(y-3x)^{2x+1} \cdot \left(\frac{-6x-3}{y-3x} + 2\ln(y-3x)\right) + \frac{y}{\sqrt{1-(xy)^2}}$$

$$\frac{\partial f}{\partial y} = (y-3x)^{2x+1} \cdot \left(\frac{1}{y-3x} \cdot 1 \cdot (2x+1)\right) + \frac{x}{\sqrt{1-(xy)^2}} = (y-3x)^{2x} \cdot (2x+1) + \frac{x}{\sqrt{1-(xy)^2}}$$

$$\nabla f(0;1) = \left((1-0)^1 \cdot \left(2\ln(1) - \frac{3}{1}\right) + \frac{1}{\sqrt{1}} ; (1)^0 \cdot (1) + \frac{0}{\sqrt{1}}\right) = (-2;1)$$

$$\|u\| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\frac{\partial f}{\partial u}(0,1) = \nabla f(0,1) \cdot \frac{u}{\|u\|} = (-2;1) \cdot \frac{(1,1)}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

1 SQUARE = ____________

5.4

$$u = 2x+3y \qquad \frac{\partial^2 f}{\partial x^2}(x,y) + \frac{4\partial^2 f}{9\partial y^2}(x,y) = 0$$
$$v = 2x-3y$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} = \frac{\partial f}{\partial u}\cdot 2 + \frac{\partial f}{\partial v}\cdot 2$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} = \frac{\partial f}{\partial u}\cdot 3 + \frac{\partial f}{\partial v}\cdot(-3)$$

$$\frac{\partial^2 f}{\partial x^2} = \left(\frac{\partial f}{\partial x}\right)^2 = 4\frac{\partial^2 f}{\partial u^2} + 8\cdot\frac{\partial^2 f}{\partial u \partial v} + 4\frac{\partial^2 f}{\partial v^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \left(\frac{\partial f}{\partial y}\right)^2 = 9\cdot\left(\frac{\partial^2 f}{\partial u^2} - 2\cdot\frac{\partial^2 f}{\partial u \partial v} + \frac{\partial^2 f}{\partial v^2}\right)$$

dosazení:

$$4\frac{\partial^2 f}{\partial u^2} + \cancel{8\frac{\partial^2 f}{\partial u\partial v}} + 4\frac{\partial^2 f}{\partial v^2} + \frac{4}{\cancel{9}}\left(\cancel{9}\cdot\left(\frac{\partial^2 f}{\partial u^2} - \cancel{2\cdot\frac{\partial^2 f}{\partial u\partial v}} + \frac{\partial^2 f}{\partial v^2}\right)\right) \Rightarrow$$

$$\underline{\underline{8\frac{\partial^2 f}{\partial u^2}(u,v) + 8\frac{\partial^2 f}{\partial v^2}(u,v) = 0}}$$

1 SQUARE = ____________

1) $\sum\limits_{n=1}^{\infty} (x-2)^{n} \cdot 2^{1-n} \cdot n^{-1}$

podiel. krit.

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}\cdot 2^{1-n-1}\cdot (n+1)^{-1}}{\cancel{(x-2)^{n}}\cdot \cancel{2^{1-n}}\cdot n^{-1}}\right| = \lim_{n\to\infty}\left|\frac{(x-2)\cdot 2^{-1}\cdot n}{(n+1)}\right| =$$

$$\left|\frac{x-2}{2}\right|\cdot \lim_{n\to\infty}\underbrace{\left|\frac{n}{n+1}\right|}_{1} \Rightarrow \quad -1 < \frac{x-2}{2} < 1$$

$$-2 < x-2 < 2$$

$$0 < x < 4$$

0: $\quad \underbrace{(-1)^{n}\cdot(2)^{n}}$

$$\sum_{n=1}^{\infty} (0-2)^{n}\cdot 2^{1-n}\cdot n^{-1}$$

Leibniz: $\lim\limits_{n\to\infty}\left|2\cdot 2^{1-n}\cdot n^{-1}\right| = \lim\limits_{n\to\infty}\frac{2}{n} = 0 \rightarrow$ konverguje

4:

$$\sum_{n=1}^{\infty} (2)^{n}\cdot 2^{1-n}\cdot n^{-1} \Rightarrow \text{podiel:} \quad \lim_{n\to\infty}\left|\frac{2^{n+1}\cdot 2^{1-n-1}\cdot (n+1)^{-1}}{\cancel{2^{n}}\cdot \cancel{2^{1-n}}\cdot n^{-1}}\right| = \left|\frac{2}{2}\right|\cdot\lim_{n\to\infty}\left|\frac{n}{n+1}\right| =$$

∞

$\downarrow$

diverguje

$\underline{x \in \langle 0;4)}$

1 SQUARE = ______

2) $$\lim_{(x,y)\to(-1,1)} \frac{4x-y+5}{(1+2x+y)^2} = \lim_{r^+\to 0} \frac{4\cdot r\cos(\theta) - \cancel{4} - r\cdot\sin(\theta) - \cancel{1} + \cancel{5}}{(\cancel{1} + 2r\cdot\cos(\theta) - \cancel{2} + r\sin(\theta) + \cancel{1})^2} =$$

$x = r\cos(\theta) - 1$

$y = r\cdot\sin(\theta) + 1$

$$\lim_{r^+\to 0} \frac{4r\cdot\cos(\theta) - r\cdot\sin(\theta)}{(r\cdot(2\cos(\theta)+\sin(\theta)))^2} = \lim_{r^+\to 0} \frac{r\cdot(4\cos(\theta) - \sin(\theta))}{r^2\cdot(2\cos(\theta)+\sin(\theta))^2} =$$

$$\frac{4\cos(\theta) - \sin(\theta)}{(2\cos(\theta)+\sin(\theta))^2} \cdot \lim_{r^+\to 0} \frac{1}{r} \to$$ blíží se k 0 zprava, tudíž $0, \frac{\pi}{2}$

0:

$$\lim_{r^+\to 0} \frac{4\cos(0) - \sin(0)}{r\cdot(2\cos(0)+\sin(0))^2} = \lim_{r\to 0} \frac{4-0}{r\cdot 4} = \infty$$

$\frac{\pi}{2}$

$$\lim_{r^+\to 0} \frac{4\cos\frac{\pi}{2} - \sin\frac{\pi}{2}}{r\cdot(2\cos\frac{\pi}{2} + \sin\frac{\pi}{2})^2} = \lim_{r^+\to 0} \frac{4\cdot 0 - 1}{r\cdot(2\cdot 0+1)^2} = \lim_{r^+\to 0} \frac{-1}{r} = -\infty$$

limita neexistuje

1 SQUARE = ____________

3) $f(x,y) = x^3 + 3x^2y + 3xy^2 + y^3 = 0+0+0+0=0$

$[0,0]$

1. der. $\dfrac{\partial f}{\partial x} = 3x^2 + 6xy + 3y^2 = 0$

$\dfrac{\partial f}{\partial y} = 3x^2 + 6xy + 3y^2 = 0$

2. der.

$\dfrac{\partial^2 f}{\partial x^2} = 6x + 6y = 0$

$\dfrac{\partial^2 f}{\partial y^2} = 6x + 6y = 0$

$\dfrac{\partial^2 f}{\partial x \partial y} = 6x + 6y = 0$

3. der.

$\dfrac{\partial^3 f}{\partial x^3} = 6 = 6$

$\dfrac{\partial^3 f}{\partial y^3} = 6 = 6$

$\dfrac{\partial^3 f}{\partial x^2 \partial y} = 6 = 6$

$\dfrac{\partial^3 f}{\partial y^2 \partial x} = 6 = 6$

$P = 0 + \dfrac{0\cdot(x-0) + 0\cdot(y-0)}{1!} + \dfrac{0\cdot(x-0)^2 + 2\cdot 0\cdot(x-0)\cdot(y-0) + 0\cdot(y-0)^2}{2!} + \dfrac{\cancel{6}\cdot(x-0)^3 + 3\cdot\cancel{6}\cdot(x-0)^2\cdot(y-0) +}{\cancel{3!}\ \cancel{6}}$

$\underline{3\cdot\cancel{6}(x-0)\cdot(y-0)^2 + \cancel{6}\cdot(y-0)^3} = x^3 + 3x^2y + 3xy^2 + y^3 =$

$x^3 + y^3 + 3x^2y + 3xy^2$

1 SQUARE = ______________

5)

$F(x,y) = x^4 + 2xy + y^4 = 0$

bod: [-1; 1]

$\frac{df}{dx} = 4x^3 + 2y(x) + 2x\,y(x)' + 4y^3(x)\cdot y(x)'$

$x = -1$

$y(x) = 1$

$-4 + 2 + (-2)y(x)' + 4y(x)' = 0$

$2y(x)' = 2$ $\qquad y - y_a = a\cdot(x - x_a)$

$y(x)' = 1$ $\qquad y - 1 = 1\cdot(x+1)$

$\underline{\underline{y = x + 2}}$

$\frac{d^2f}{dx^2} = 12x^2 + 2y(x)' + 2y(x)' + 2x\,y(x)'' + 12y^2(x)\cdot(y(x)')^2 + 4y^3(x)\cdot y(x)'' = 0$

$12 + 2 + 2 + (-2)\cdot y(x)'' + 12 + 4y(x)'' = 0$

$16 + 12 + 2y(x)'' = 0$

$\underline{\underline{y(x)'' = -14}}$ $\rightarrow$ bod [-1; 1] leží pod tečnou

$u = 2x + 3y$

$v = 2x - 3y$

$$\frac{\partial^2 f}{\partial x^2}(x,y) - \frac{4}{9}\frac{\partial^2 f}{\partial y^2}(x,y) = 0$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} = \frac{\partial f}{\partial u}\cdot 2 + \frac{\partial f}{\partial v}\cdot 2$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} = \frac{\partial f}{\partial u}\cdot 3 + \frac{\partial f}{\partial v}\cdot(-3)$$

$$\frac{\partial^2 f}{\partial x^2} = \left(\frac{\partial f}{\partial x}\right)^2 = 4\frac{\partial^2 f}{\partial u^2} + 8\cdot\frac{\partial^2 f}{\partial u \partial v} + 4\frac{\partial^2 f}{\partial v^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \left(\frac{\partial f}{\partial y}\right)^2 = 9\frac{\partial^2 f}{\partial u^2} - 18\frac{\partial^2 f}{\partial u \partial v} + 9\frac{\partial^2 f}{\partial v^2}$$

dosazení

$$\cancel{4\frac{\partial^2 f}{\partial u^2}} + 8\frac{\partial^2 f}{\partial u \partial v} + \cancel{4\frac{\partial^2 f}{\partial v^2}} - \frac{4}{\cancel{9}}\cdot\cancel{9}\cdot\left(\cancel{\frac{\partial^2 f}{\partial u^2}} - 2\cdot\frac{\partial^2 f}{\partial u \partial v} + \cancel{\frac{\partial^2 f}{\partial v^2}}\right) = \underline{16\frac{\partial^2 f}{\partial u \partial v}(u,v) = 0}$$

1 SQUARE =

$$f(x,y,z) = x^2 \cdot \sin(2y-z)$$

$$\frac{\partial f}{\partial x} = 2x \cdot \sin(2y-z)$$

$$\frac{\partial f}{\partial y} = x^2 \cdot \cos(2y-z) \cdot 2 = 2x^2 \cdot \cos(2y-z)$$

$$\frac{\partial f}{\partial z} = x^2 \cdot \cos(2y-z) \cdot (-1) = -x^2 \cdot \cos(2y-z)$$

$f(x,y) = x - e^{xy^2}$ $\quad u = \left(\frac{\sqrt{2}}{2} ; \frac{\sqrt{2}}{2}\right)$ v $(0,1)$

1) gradient ... $\nabla f(x,y)$

2) směrové derivace

1) $\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \left(1 - e^{xy^2} \cdot y^2, -e^{xy^2} \cdot 2yx\right)$

$\frac{\partial f}{\partial x} = 1 - e^{xy^2} \cdot y^2$ $\quad \nabla f(0,0) = (1,0)$

$\nabla f(2,1) = (1 - e^2 \cdot 1, -e^2 \cdot 4)$

$\frac{\partial f}{\partial y} = x - e^{xy^2} \cdot 2y = -e^{xy^2} \cdot 2yx$

$$\frac{\partial f}{\partial u}(0,0),\ \frac{\partial f}{\partial v}(2,1)$$

$$\frac{\partial f}{\partial u}(2,1)$$

normy:

$$\|u\| = \sqrt{\left(\tfrac{\sqrt{2}}{2}\right)^2 + \left(\tfrac{\sqrt{2}}{2}\right)^2} = 1$$

$$\|v\| = \sqrt{0^2 + (-1)^2} = 1$$

pokud norma = 1, tak je to ok, jinak se musí udělat tohle:

$$\frac{\partial f}{\partial u}(0,0) = \nabla f(0,0) \cdot \frac{u}{\|u\|}$$

$$\frac{\partial f}{\partial u}(0,0) = (\nabla f(0,0)) \cdot u = (1,0) \cdot \left(\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} + 0 = \frac{\sqrt{2}}{2}$$

$$\frac{\partial f}{\partial u}(2,1) = (\nabla f(2,1)) \cdot u = (1-e^2, -e^2 \cdot 4) \cdot \left(\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}\right) =$$

$$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} e^2 + \left(-\frac{\sqrt{2}}{2} e^2 \cdot 4\right) = \frac{\sqrt{2}}{2}\left(1 - e^2 - e^2 \cdot 4\right)$$

$$\frac{\partial f}{\partial v}(2,1) = (1-e^2, -4e^2) \cdot (0,-1) = 0 + (4 \cdot e^2) = 4 \cdot e^2$$

5.7 $\frac{\partial^2 f}{\partial x^2}(x,y) + 4\frac{\partial^2 f}{\partial y^2}(x,y) = 0$

$u = 2x + y$
$v = 2x - y$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot 2 + \frac{\partial f}{\partial v}\cdot 2 \quad \frac{\partial^2 f}{\partial x^2} = 4\cdot\frac{\partial^2 f}{\partial u^2} + 8\frac{\partial^2 f}{\partial u \partial v} + 4\frac{\partial^2 f}{\partial v^2}$

$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\cdot 1 + \frac{\partial f}{\partial v}\cdot(-1) \quad \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial u^2} - 2\cdot\frac{\partial^2 f}{\partial u \partial v} + \frac{\partial^2 f}{\partial v^2}$

dosazení:

$4\frac{\partial^2 f}{\partial u^2} + \cancel{8\frac{\partial^2 f}{\partial u \partial v}} + 4\frac{\partial^2 f}{\partial v^2} + 4\cdot\frac{\partial^2 f}{\partial u^2} - \cancel{8\cdot\frac{\partial^2 f}{\partial u \partial v}} + 4\frac{\partial^2 f}{\partial v^2} = 7$

$8\frac{\partial^2 f}{\partial u^2}(u,v) + 8\cdot\frac{\partial^2 f}{\partial v^2}(u,v)$

5.6 $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial x} = 0$

$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{1}{y} + \frac{\partial f}{\partial v}\cdot 0$

$u = \frac{x}{y} \quad x = u\cdot y = u\cdot v$

$y = v$

$v = y$

$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{-x}{y^2} + \frac{\partial f}{\partial v}\cdot 1$

dosazení:

$u\cancel{y}\cdot\left(\frac{\partial f}{\partial u}\cdot\frac{1}{\cancel{v}}\right) + \cancel{v}\left(\frac{\partial f}{\partial u}\cdot\frac{-u\cancel{v}}{\cancel{v^2}} + \frac{\partial f}{\partial v}\right) =$

1 SQUARE = ____________

$u\frac{\partial f}{\partial u}(u,v) - u\frac{\partial f}{\partial u}(u,v) + v\frac{\partial f}{\partial v}(u,v)$