MAT2 - cv 06

28.3.2024

$y = \operatorname{tg} z(\ )$

$$\left(y''(x) - 1\right) \cdot \left(1 + y^2(x)\right) = 2 \cdot \left(1 + y(x)\right) \cdot \left(y'(x)\right)^2$$

$z \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$(y)'$

$(\operatorname{tg} z)' = \frac{1}{\cos^2 z}$

$\cos z(x) = -\sin(z(x)) \cdot z'(x)$

$\operatorname{tg}'(z(x))$

$y'(x) = \frac{1}{\cos^2 z(x)} \cdot z'(x)$

$y(x)'' = \frac{-2}{\cos^3 z(x)} \cdot (-\sin(z(x)) \cdot z'(x) \cdot z'(x) + \frac{1}{\cos^2(z(x))} \cdot z''(x) =$

$$\frac{2\sin(z(x))}{\cos^3(z(x))} \cdot (z'(x))^2 + \frac{1}{\cos^2(z(x))} \cdot z''(x)$$

$$\left(\frac{2\sin(z(x))}{\cos^3(z(x))} \cdot (z'(x))^2 + \frac{1}{\cos^2(z(x))} \cdot z''(x) - 1\right) \cdot \left(1 + \operatorname{tg}^2(z(x))\right) - 2 \cdot \left(1 + \operatorname{tg}(z(x))\right) \cdot$$

$$\left(\frac{1}{\cos^2 z(x)} \cdot z'(x)\right)^2 = \left(\frac{2\sin s}{\cos^3(s)} \cdot (s')^2 + \frac{1}{\cos^2 s} \cdot s'' - 1\right) \cdot (1 + \operatorname{tg}^2 s) - 2 \cdot (1 + \operatorname{tg} s) \cdot$$

$s = z(x)$

$\left(\frac{1}{\cos^2 s} \cdot s'\right)$

$(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g(x)'$

výsledek:

$$\frac{z''(x) - 2(z'(x))^2 - \cos^2(z(x)))}{\cos^4(z(x))}$$

1 SQUARE =

$$(1+x^2)^2 \cdot y''(x) + 2x(1+x^2)y^2(x) + y(x)$$

$x = tg\, t$

$t = arctg\, x$ $\qquad y'(x) = \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dt} \cdot \left(\frac{1}{1+x^2}\right)$

$dt = \frac{1}{1+x^2}\, dx$

$\frac{dt}{dx} = \frac{1}{1+x^2} \qquad \frac{dx}{dt} = \frac{1+x^2}{1}$

1 SQUARE = ______________

26.2.2024

Taylorovi polynomi

Najděte Taylorův rozvoj v určitém oboru konvergence:

$\sin x$ v bodě $x=0$

Taylorova řada v bodě x_0

$$f(x_0)+\frac{f'_{(x_0)}(x-x_0)}{1!}+\frac{f''_{(x_0)}(x-x_0)^2}{2!}\cdots\cdots+\frac{f^{n}_{(x_0)}(x-x_0)^n}{n!}+R_n(x)$$

v bodě 0:

$f=\sin x \rightarrow 0$

$f'=\cos x \rightarrow 1$

$f''=-\sin x \rightarrow 0$

$f'''=-\cos x \rightarrow -1$

$f''''=\sin x \rightarrow 0$

$$T(0)=0+\frac{1\cdot x}{1!}+\frac{0\cdot x^2}{2!}+\frac{(-1)\cdot x^3}{3!}+\frac{0\cdot x^4}{4!}+\frac{1\cdot x^5}{5!}+\cdots$$

$$+\frac{(-1)^{n-1}\cdot x^{2n-1}}{(2n-1)!}=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\cdot x^{2n-1}}{(2n-1)!}$$

podíl krit.:

$$\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1-1}\cdot x^{2(n+1)-1}}{(2(n+1)-1)!}}{\frac{(-1)^{n-1}\cdot x^{2n-1}}{(2n-1)!}}\right|=\lim_{n\to\infty}\left|\frac{\cancel{(-1)^{n}}\cdot x^{\cancel{2n}+1}}{(2n+1)!}\cdot\frac{(2n-1)!}{\cancel{(-1)^{n-1}}\cdot x^{\cancel{2n}-1}}\right|=$$

$$\lim_{n\to\infty}\left|\frac{x^1}{(2n+1)\cdot 2n}\cdot\frac{1}{x^{-1}}\right|=\lim_{n\to}\left|\frac{x^2}{4n^2+2n}\right|=x^2\cdot\underbrace{\lim_{n\to\infty}\left|\frac{1}{4n^2+2n}\right|}_{0}\Rightarrow$$

konvergujo pro jakékoliv x

1 SQUARE =