MA2 - cv 3

7.3.2024

Obory konvergence - pro jaké x řada konverguje

Př: $\sum_{n=1}^{\infty} n x^n$

podíl. kritérium

$$\lim_{n\to\infty} \left| \frac{(n+1)\cdot x^{n+1}}{n x^n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)\cdot \cancel{x^n}\cdot x^1}{n\, \cancel{x^n}} \right| = \lim_{n\to\infty} \underbrace{\frac{n+1}{n}}_{1} \cdot |x| = |x|$$

bude konvergovat pokud $|x| < 1$

$x \in (-1;1) \to$ konverguje

$x = -1$

$\sum_{n=1}^{\infty} n(-1)^n \to$ diverguje

$x = 1$

$\sum_{n=1}^{\infty} n \to$ nesplňuje podmínku konvergence $\to$ diverguje

$\lim_{n\to\infty} n \neq 0$

Př: $\sum_{n=1}^{\infty} \frac{x^{n+1}}{2^n(2n-1)} \Rightarrow$ podíl. krit. $= \lim_{n\to\infty} \left| \frac{\frac{x^{n+2}}{2^{n+1}(2(n+1)-1)}}{\frac{x^{n+1}}{2^n(2n-1)}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{x^{n+2}}^{\,x} \cdot \cancel{2^n}\cdot(2n-1)}{\cancel{2^{n+1}}_{2}(2n+1)\,\cancel{x^{n+1}}} \right| =$

$$\lim_{n\to\infty} \left| \frac{x}{2\cdot(2n+1)} \cdot \frac{2n-1}{1} \right| = \lim_{n\to\infty} \left| \frac{x\cdot(2n-1)}{2\cdot(2n+1)} \right| = \lim_{n\to\infty} \frac{|x|\cdot(2n-1)}{2\cdot(2n+1)} =$$

$$\left|\frac{x}{2}\right| \cdot \lim_{n\to\infty} \frac{(2n-1)}{(2n+1)} = \left|\frac{x}{2}\right|$$

$\left|\frac{x}{2}\right| < 1 \to x \in (-2;2)$ - bude konvergovat v tomto intervalu

$$\lim_{n\to\infty} \frac{2n-1}{2n+1} = \lim_{n\to\infty} \frac{n\cdot(2-\frac{1}{n})}{n\cdot(2+\frac{1}{n})} = 1$$

$x = -2$

$$\sum_{n=1}^{\infty} \frac{-2^{n+1}}{2^n(2n-1)}$$

$x = 2$

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{2^n(2n-1)}$$

1 SQUARE =

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 2^{n+1}}{2^n(2n-1)} = \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{2^{n+1}}{2^n(2n-1)} \xrightarrow{\text{pod. krit}} \lim_{n\to\infty} \frac{1}{(2n-1)} = 0$$

lim. je. klesající

konverguje

$$\sum_{n=1}^{\infty} \frac{2^{n+1}}{2^n(2n-1)} \qquad \sum_{n=1}^{\infty} \frac{2}{2n-1} \xrightarrow{\text{int krit}} \int_1^{\infty} \frac{2}{2x-1}\,dx = \left[\ln|2x-1|\right]_1^{\infty}$$

$$\infty - 0 = \infty$$

nekonverguje

konverguje pro: $x \in \langle -2;2)$

$$\sum_{n=1}^{\infty} \frac{(x-6)^{n+1}}{4^n} \rightarrow \text{podíl. krit} = \lim_{n\to\infty} \left| \frac{\frac{(x-6)^{n+2}}{4^{n+1}}}{\frac{(x-6)^{n+1}}{4^n}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(x-6)}^{n+2}}{4^{n+1}} \cdot \frac{\cancel{4^n}}{\cancel{(x-6)^{n+1}}} \right| =$$

$$\lim_{n\to\infty} \left| \frac{(x-6)}{4} \right| = \lim_{n\to\infty} \frac{|x-6|}{4} = \frac{|x-6|}{4}$$

$$\frac{|x-6|}{4} < 1$$

$$|x-6| < 4 \rightarrow \underline{\underline{x \in (2;10)}}$$

$x=2$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 4^{n+1}}{4^n} = \sum_{n=1}^{\infty} (-1)^{n+1} \cdot 4$$

nekonverguje

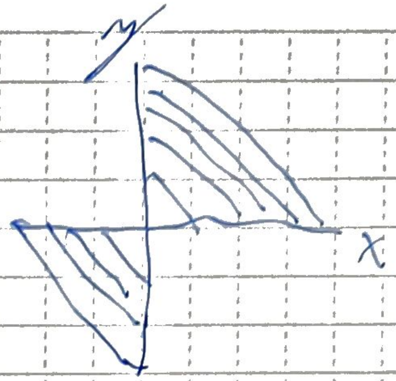
$x=10$

$$\sum_{n=1}^{\infty} \frac{4^{n+1}}{4^n} = \sum_{n=1}^{\infty} 4$$

nekonverguje

1 SQUARE =

$f(x,y) = \sqrt{xy} = (xy)^{\frac{1}{2}}$



$xy \geq 0$

$(x \geq 0 \wedge y \geq 0) \quad \vee \quad (x \leq 0 \wedge \cancel{x}\, y \leq 0)$

$D_f : \{[x,y] \in \mathbb{R}^2 : xy \geq 0 \vee xy \leq 0$

$\vee$

$D_f : \{[x,y] \in \mathbb{R}^2 : (x \geq 0 \wedge y \geq 0) \vee (x \leq 0 \wedge y \leq 0)\}$

1 SQUARE = ______________