MAT 2 - cv 12

$y' = \dfrac{-2x+3y+7}{-7y+x-3}$ → substituce $x = u + A$ $dx = du$

$y = v + B$ $dy = dv$

$\dfrac{dy}{dx} = \dfrac{dv}{du}$

$y' = 2yx + 2xe^{x^2}$, $y(0) = 4$

homogenní LDR 1. řádu

$(y' - 2yx = 0)$ → homogenní

1) y_H ... řešení homog. rce:

$y' = 2yx$

$\dfrac{dy}{dx} = 2yx$

$\dfrac{dy}{y} = 2x \cdot dx$

$\int \frac{1}{y} dy = \int 2x \, dx$

$\ln|y| = x^2 + C$

$e^{\ln|y|} = e^{x^2} \cdot e^{C} \Rightarrow |y| = e^{x^2} \cdot K(x)$

$\underline{y_H = e^{x^2} \cdot K}$ $K \in R$ (protože y = nějaký náš...)

2) y_p ... → variace konstanty

$y_p = K(x) \cdot e^{x^2}$

$y_p' = K'(x) \cdot e^{x^2} + K(x) \cdot e^{x^2} \cdot 2x$

dosazení do zadání

$\underbrace{K'(x) \cdot e^{x^2} + \cancel{K(x) \cdot e^{x^2} \cdot 2x}}_{y'} = 2 \cdot \underbrace{\cancel{K(x) e^{x^2}}}_{y} \cdot x + 2x e^{x^2}$

$K'(x) e^{x^2} = 2x e^{x^2}$

$K'(x) = 2x$

$\underline{\underline{K(x) = x^2}}$ → $y_p = x^2 \cdot e^{x^2}$

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3) $y = y_H + y_p$

$y = K e^{x^2} + x^2 \cdot e^{x^2}$

$y = e^{x^2} \cdot (K + x^2)$

$4 = e^0 \cdot (K + 0)$

$4 = K \rightarrow \underline{\underline{y = e^{x^2}(4 + x^2)}}$

Bernoulliho rce:

$y' = -2y + y^2 e^x \rightarrow$ vydělíme y^n y^2, substituce $z = y^{1-n}$

$\frac{y'}{y^2} = -\frac{2}{y} + e^x$

der. podle x $\left\{ \begin{array}{l} z = y^{-1} \\ z' = -1 y^{-2} \cdot y' = \frac{-y'}{y^2} \end{array} \right.$

1) homogenní rce

$-z' = -2z$

$\frac{dz}{dx} = 2z$

$\frac{dz}{z} = 2dx$

$\int \frac{dz}{z} = \int 2dx \rightarrow \ln|z| = 2x + C$

2) partikulární řešení

$-z' = -2z + e^x$

$z_p = e^{2x} \cdot K(x)$

$z'_p = e^{2x} \cdot 2 \cdot K(x) + K(x)' \cdot e^{2x}$

$\underbrace{-K(x)' \cdot e^{2x} - \cancel{2 e^{2x} \cdot K(x)}}_{z'_p} = \underbrace{-\cancel{2 K(x) e^{2x}}}_{z_p} + e^x$

$z_h = e^{2x} \cdot K(x)$

$K'(x) = -e^{-x}$

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$k(x) = \int -e^{-x} dx \quad s = -x$

$\quad ds = -1 dx$

$k(x) = -\int e^{s} \cdot (-1) \cdot ds \quad dx = -1 ds$

$k(x) = e^{s}$

$k(x) = e^{-x} \rightarrow z_p = e^{-x} \cdot e^{2x} = e^{x}$

$z = z_H + z_p = k \cdot e^{2x} + e^{x} \qquad y = \frac{1}{z}$

$$y = \frac{1}{k(x) \cdot e^{2x} + e^{x}}$$

exaktní rovnice

$$\underbrace{(2x^3 + xy^2)}_{g} dx + \underbrace{(x^2y + 2y^3)}_{h} dy = 0$$

$\frac{dg}{dy} = \frac{dh}{dx}$

$2yx = 2yx$

$\cancel{x^2y} + c'(y) = \cancel{x^2y} + 2y^3$

1) $H = \int \underbrace{2x^3 + xy^2}_{g} dx = 2\frac{x^4}{4} + \frac{x^2}{2}y^2 + c(y)$

$c'(y) = 2y^3$

$c(y) = \frac{y^4}{2}$

2) zderivujeme H podle $y \Rightarrow 0 + \frac{x^2}{2} \cdot 2y + c'(y)$

a položíme rovno h

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rce diferen. funkce H

→ řešení rce $H(x,y) = C$

$$\frac{x^4}{2} + \frac{x^2 y^2}{2} + \frac{y^4}{2} = C \quad / \cdot 2$$

$K = 2C$

$$K = x^4 + x^2 y^2 + y^4$$

Př: $y'' = \sin x + e^{-2x}$

$y' = \int \sin x + e^{-2x} \, dx$

$y' = -\cos x + \int e^{t} \cdot \left(-\frac{1}{2}\right) dt$ $\quad t = -2x$, $dt = -2\,dx$, $dx = -\frac{1}{2}\,dt$

$y' = -\cos x - \frac{1}{2} \cdot e^{-2x} + C$

$y = \int -\cos x - \frac{1}{2} e^{-2x} + c_1 \, dx$

$y = -\sin x + \frac{1}{4} e^{-2x} + c_1 x + c_2$

Př: $y'' + a y' + b y = f(x)$

1) $f(x) = 0$ $\quad y = e^{kx}$

homog. rovnice $\quad y' = e^{kx} \cdot k$

$y'' = e^{kx} \cdot k^2$

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$e^{kx} \cdot k^2 + a e^{kx} \cdot k + b e^{kx} = 0$

$k^2 + ak + b = 0$

1 SQUARE =