MAT 2 cv 9

Př 1) Najděte lokální extrémy fce $f(x,y) = xy$ na množině $x+y=1$

$F = f(x, 1-x) = x\cdot(1-x) = x - x^2$

$F' = 1-2x$

$$\left.\begin{array}{l} x = \frac{1}{2} \\ 1-\frac{1}{2} = \frac{1}{2} \\ y = \frac{1}{2} \end{array}\right\} S\left[\frac{1}{2}; \frac{1}{2}\right]$$

$F'' = -2 \Rightarrow F'' < 0 \rightarrow$ F je konkávní $\Rightarrow$ v S je MAXIMUM

Př 2)

$L = xy + \lambda(x+y-1) = xy + \lambda x + \lambda y - \lambda$

$\frac{dL}{dx} = 0 \qquad y + \lambda = 0 \rightarrow \lambda = -y$

$\Rightarrow y = x$

$\frac{dL}{dy} = 0 \qquad x + \lambda = 0 \rightarrow \lambda = -x$

$x+y-1=0 \qquad x+y-1=$

$$\left.\begin{array}{l} 2y-1=0 \\ y = \frac{1}{2} \\ 2x-1=0 \\ x = \frac{1}{2} \end{array}\right\} S\left[\frac{1}{2}; \frac{1}{2}\right]$$

$\lambda = -\frac{1}{2}$

1 SQUARE = ________

$$\begin{pmatrix} \frac{dL}{dx^2} & \frac{d^2L}{dx\,dy} \\ \frac{dL}{dx\,dy} & \frac{d^2L}{dy^2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$\frac{dL}{dx} = y - \frac{1}{2}$

$\frac{dL}{dy} = x - \frac{1}{2}$

$f(x,y) = x + 2y - 1$ mnozina $x^2 + y^2 = 1$

$L = x + 2y - 1 + \lambda(x^2 + y^2 - 1) = x + 2y - 1 + \lambda x^2 + \lambda y^2 - \lambda$

$\frac{dL}{dx} = 2\lambda x + 1 \rightarrow x = -\frac{1}{2\lambda}$

$x_1 = -\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

$x_2 = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\frac{dL}{dy} = 2\lambda y + 2 \rightarrow y = -\frac{1}{\lambda}$

$S_1\left[-\frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}\right]$

$S_2\left[\frac{\sqrt{5}}{5}, \frac{2\sqrt{5}}{5}\right]$

$x^2 + y^2 = 1$

$\left(-\frac{1}{2\lambda}\right)^2 + \left(-\frac{1}{\lambda}\right) = 1$ $\quad \lambda^2 = \frac{5}{4}$

$\frac{1}{4\lambda^2} + \frac{1}{\lambda^2} = 1$ $\quad \lambda = \pm\sqrt{\frac{5}{4}} = \pm\frac{\sqrt{5}}{2}$

1 SQUARE = __________

absolutní (globální) extrémy

$f(x,y) = \sqrt{x^2+y^2}$ na množině $x^2+y^2 \leq 9$

$$\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} = 0 \to x = 0$$

$S[0;0] \to$ stac. bod

$$\frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2}} = 0 \to y = 0$$

1 SQUARE = ________________