MA 2 - cv 1

22.2.2024

$$\int \frac{x^3-2x+1}{x^3}\, dt = \int \frac{x^3}{x^3} - \frac{2x}{x^3} + \frac{1}{x^3}\, dx = \int 1\, dx + \int \frac{-2x}{x^3}\, dx + \int \frac{1}{x^3}\, dx =$$

$$x - 2\int x^{-2}\, dx + \int x^{-3}\, dx = x - 2\cdot\left(\frac{-1}{x}\right) + \left(\frac{-1}{2x^2}\right) = x + \frac{2}{x} - \frac{1}{2x^2} + C$$

$$\int \frac{\ln x}{x^2}\, dx = \int \ln x \cdot \frac{1}{x^2}\, dx = \left| \begin{matrix} u = \ln x & u' = \frac{1}{x} \\ v' = \frac{1}{x^2} & v = -\frac{1}{x} \end{matrix} \right| = \ln x \cdot \left(-\frac{1}{x}\right) - \int \frac{1}{x} \cdot \left(-\frac{1}{x}\right) dx =$$

$$\ln x \left(-\frac{1}{x}\right) + \int \frac{1}{x^2}\, dx = \ln x \cdot \left(-\frac{1}{x}\right) - \frac{1}{x} + C$$

per partes:

$$\boxed{\int u v' = \left| \begin{matrix} u & u' \\ v' & v \end{matrix} \right| = u \cdot v - \int u' v}$$

substituce

$$\int \frac{2x}{(1+x^2)^2}\, dx = \int \frac{\cancel{2x}}{t^2} \cdot \frac{1}{\cancel{2x}}\, dt = \int \frac{1}{t^2}\, dt = -\frac{1}{t} + C =$$

$t = 1+x^2$

$dx = \frac{1}{2x} \cdot dt$

$$-\frac{1}{1+x^2} + C$$

parciálky:

$$\int \frac{2x}{(1+x)^3}\, dx = \int \frac{2}{(1+x)^2} - \frac{2}{(1+x)^3}\, dx = 2\cdot\left[\int \frac{1}{t^2}\, dt - \int \frac{1}{t^3}\, dt\right] = 2\cdot\left(-\frac{1}{t} + \frac{1}{2t^2}\right) + C =$$

$t = 1+x$

$dx = \frac{1}{1}\, dt$

$$\frac{2x}{(1+x)^3} = \frac{A}{(1+x)} + \frac{B}{(1+x)^2} + \frac{C}{(1+x)^3} \quad / \cdot (1+x)^3$$

$$2x = A\cdot(1+x)^2 + B\cdot(1+x) + C$$

$$-\frac{2}{1+x} + \frac{\cancel{2}}{\cancel{2}\cdot(1+x)^2} + C$$

$$-\frac{2}{1+x} + \frac{1}{(1+x)^2} + C$$

1 SQUARE =

$$2x = A + A2x + Ax^2 + B + Bx + C$$

$$2x = Ax^2 + A2x + Bx + A + B + C$$

$x^2: 0 = A \qquad A = 0$

$x^1: 2 = 2A + B \rightarrow B = 2$

$x^0: 0 = A + B + C \qquad C = -2$

$$\int \frac{2}{x^2+2x+2}\,dx = 2\cdot\int \frac{1}{(x+1)^2+1}\,dx = 2\cdot\int \frac{1}{t^2+1}\,dt = 2\cdot \operatorname{arctg}(t) + c$$

$$2\cdot \operatorname{arctg}(x+1) + c$$

$t = x+1$

$dx = 1\,dt$

$$\int_0^4 \frac{x-1}{x+1}\,dx = \int_0^4 1 - \frac{2}{x+1}\,dx = \int_0^4 1\,dx - 2\int_0^4 \frac{1}{x+1}\,dx = \Big[x - 2\cdot \ln|x+1|\Big]_0^4 =$$

$(x-1):(x+1) = 1 - \frac{2}{x+1} \qquad 4 - 2\cdot\ln 5 - 0 + 2\ln 1 = 4 - 2\cdot \ln 5$

$$\int_1^\infty \frac{3}{\sqrt{x^5}}\,dx = 3\cdot\int_1^\infty x^{-\frac{5}{2}}\,dx = 3\cdot\left[\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}}\right]_1^\infty = 3\cdot\left[-\frac{2}{3}\cdot x^{-\frac{3}{2}}\right]_1^\infty =$$

$$\left[-2\cdot\frac{1}{\sqrt{x^3}}\right]_1^\infty = \lim_{x\to\infty}\Big(\underbrace{\frac{-2}{\sqrt{x^3}}}_{\text{jde k } 0}\Big) + \frac{2}{\sqrt{1}} = 0 + 2 = \underline{\underline{2}}$$

1 SQUARE = ____________