MAT2 - cv 4

14.3.2024

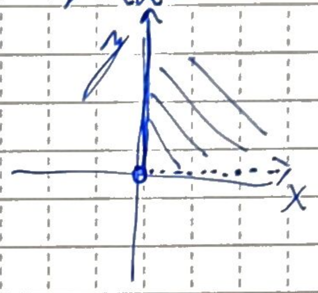
fce více proměnných

$f(x,y) = \sqrt{3x} - \frac{2}{\sqrt{y}}$

$3x \geq 0 \quad \land \quad \underbrace{\sqrt{y} \neq 0 \quad \land y \geq 0}_{y > 0}$

$x \geq 0$

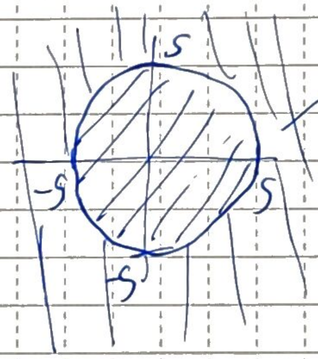
$D_f \{[x,y] \in \mathbb{R}^2 : x \geq 0 \land y > 0\}$



$f(x,y) = \frac{1}{25 - x^2 - y^2} = \frac{1}{5^2 - x^2 - y^2}$

$5^2 - x^2 - y^2 \neq 0 \quad \land \quad x^2 + y^2 \neq 5^2$

$D_f \{[x,y] \in \mathbb{R}^2 : x^2 + y^2 \neq 25\}$



vše krom hrany kružnice

$f(x,y) = \frac{\pi}{6} y^2 \sqrt{x^2 - y^2}$

$x^2 - y^2 \geq 0$

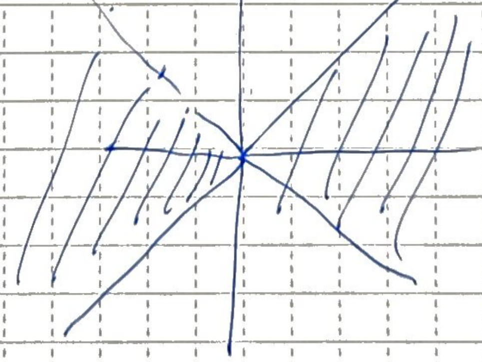
$x^2 \geq y^2$

$|x| \geq |y|$

$y \leq 0$ | $y \geq 0$

$-y \leq |x|$ | $y \leq |x|$

$y \geq -|x|$



$D_f \{[x,y] \in \mathbb{R}^2 : |x| \geq |y|\}$

1 SQUARE = ____________

$f(x) = \ln(y \cdot \ln(y-x))$

$y - x > 0 \quad \wedge \quad y \cdot \ln(y-x) > 0$

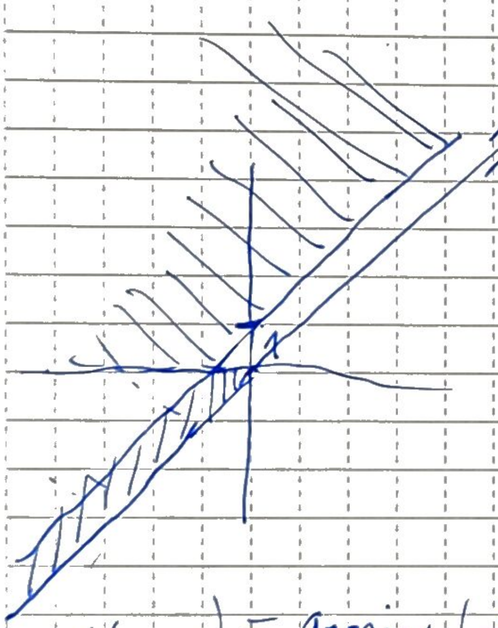
$y > x$

$y > 0 \wedge \ln(y-x) > 0 \quad \vee \quad y < 0 \wedge \ln(y-x) < 0$

$y - x > e^0$

$y - x > 1$

$y > 0 \wedge y > x+1 \quad \vee \quad y < 0 \wedge y < 1+x$



$f(x,y) = \arcsin\left(\frac{x}{y^2}\right) + \arcsin(1-y)$

$-1 \leq \frac{x}{y^2} \leq 1 \quad \wedge \quad -1 \leq 1-y \leq 1$

$-2 \leq -y \leq 0$

$2 \geq y \geq 0$

$y^2 \neq 0$

$y \neq 0$

$f(x) \quad \frac{2xy}{x^2+y^2}$

$x^2+y^2=0$

$x^2 = -y^2$

nespojitá v bodě $[0,0]$

1 SQUARE = ______________

$f(x,y) = e^{x+y} \cdot \sqrt{x^2+y^2} + \cos(x-y)$

$x^2+y^2 \geq 0$

$\hookrightarrow$ platí vždy – spojitá vždy na $\mathbb{R}^2$

$$\lim_{(x,y)\to(2,1)} (x^2+xy+y^2) = \underline{7}$$

Př:

$$\lim_{(x,y)\to(4,4)} \frac{x^3-y^3}{x^4-y^4} = \frac{0}{0} = \lim_{(x,y)\to(4,4)} \frac{\cancel{(x-y)}\cdot(x^2+xy+y^2)}{(x^2+y^2)\cdot\underbrace{\cancel{(x^2-y^2)}}_{x+y}} = \lim_{(x,y)\to(4,4)} \frac{x^2+xy+y^2}{(x^2+y^2)\cdot(x+y)} =$$

$$\frac{\cancel{48}^{6}}{32\cdot\cancel{8}_{1}} = \frac{6}{32} = \frac{3}{16}$$

Př:

$$\lim_{(x,y)\to(0,0)} \frac{3\cdot(x^2+y^2)}{\sqrt{x^2+y^2+4}-2} = \frac{0}{0} = \lim_{(x,y)\to(0,0)} \frac{3\cdot(x^2+y^2)}{\sqrt{x^2+y^2+4}-2}\cdot\frac{\sqrt{x^2+y^2+4}+2}{\sqrt{x^2+y^2+4}+2} =$$

$$\lim_{(x,y)\to(0,0)} \frac{\cancel{(3x^2+3y^2)}\cdot(\sqrt{x^2+y^2+4}+2)}{x^2+y^2\cancel{+4}\cancel{-4}} = \lim_{(x,y)\to(0,0)} 3\cdot(\sqrt{x^2+y^2+4}+2) = \underline{12}$$

Př:

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{x-y}$$

$\lim_{\substack{x\to 0\\ y=0}} \frac{x}{x} = 1$

$\lim_{\substack{y\to 0\\ x=0}} \frac{y}{-y} = -1$

různé – nemá v bodě $[0,0]$ limitu

$$\lim_{(x,y)\to[2,1]} \frac{x+3}{2x-y+7} = \frac{5}{10} = \frac{1}{2}$$

1 SQUARE =