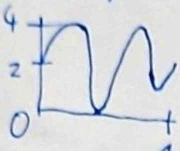
Vlastnosti analog. signál

$x(t) = A\cdot\cos(2\pi\cdot f\cdot t+\varphi)+M$

A - amplituda
f - frekvence
φ - fáze
M - stejnosměrná složka



$A=2$, $f=2$, $M=2$

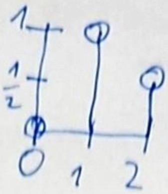
$\sin(2\pi\cdot f\cdot t)=\cos(2\pi\cdot f\cdot t-\frac{\pi}{2})$

$\cos(2\pi\cdot f\cdot t)=\sin(2\pi\cdot f\cdot t+\frac{\pi}{2})$

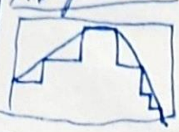
Číslicový diskrétní signál

$x[n]=[0;1;\frac{1}{2};\dots]$

$x[0]=0$
$x[1]=1$
$x[2]=\frac{1}{2}$



AD převod:



vstup
dolní propust
vzorkování
kvantování
kódování
výstup

vzorkování

$t[n]=n\cdot T_s$

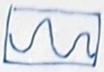
$F_s=8\,Hz$ $\quad t[n]=[0;\frac{1}{8};\frac{2}{8};\dots$

$T_s=\frac{1}{8}\,s$ $\quad x(t)=4\cos(2\pi\cdot2\cdot t\dots$

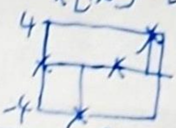
$T=\frac{1}{2}\,s$ $\quad x(0)=4\cdot\cos(2\pi\cdot2\cdot0+\frac{\pi}{2})$

$N=4$ $\quad x(1)=4\cdot\cos(2\pi\cdot2\cdot1+\frac{\pi}{2})=$

$x(t)=4\cdot\cos(2\pi\cdot2\cdot t+\frac{\pi}{2})$



$x[n]=[0;-4;0;4\dots$



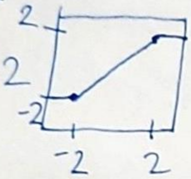
Energie:

$E=\sum_{n=0}^{N-1}x[n]^2$

$-2+2i=\sqrt{8}\,e^{i\frac{3}{4}\pi}$

Operace se signály:

$x(t)=\begin{cases}-2, & t\le-2\\ t, & -2<t<2\\ 2, & t\ge2\end{cases}$



$y(t)=2\cdot x(-t)+1$

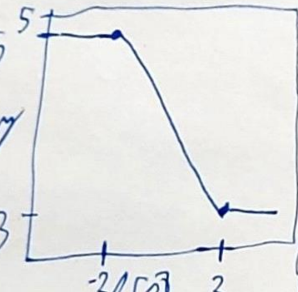
$y(-2)=2\cdot x(2)+1=5$

$y(-1)=2\cdot x(1)+1=3$

$y(0)=1$

$y(1)=-1$

$y(2)=-3$



jednotkový impulz:

$\delta[n]=\begin{cases}1, & n=0\\ 0, & n\neq0\end{cases}$

jednotkový skok

$u[n]=\begin{cases}1, & n\ge0\\ 0, & n<0\end{cases}$

jednot. skoky

$x[n]=u[n-2]+u[n-4]+u[n\dots$
$-3\cdot u[n-8]$

jednot. impulz

$x[n]=\delta[n-2]+\delta[n-3]+2\cdot(\delta[n-4]\dots$
$[n-5])+3\cdot(\delta[n-6]+\delta[n-7])$

diference:

$\Delta f[n]=f[n]-f[n-1] \Rightarrow$ zpětná

$\Delta f[n]=f[n+1]-f[n] \Rightarrow$ dopředná

$\Delta f[n]=f[n]-f[n-1]$

$x[n]=[1,0,2,2,3,0]=\delta[n]+2\cdot(\delta[n-2]+\delta[n-3])+3\cdot\delta[n-4]$

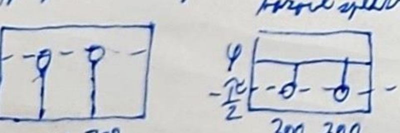
$x[n-1]=[0,1,0,2,2,3]=\delta[n-1]+2\cdot(\delta[n-3]+\delta[n-4])+3\cdot\delta[n-5]$

$\Delta x[n]=x[n]-x[n-1]=[1,-1,2,0,1,-3]=\delta[n]-\delta[n-1]+2\delta[n-2]+\delta[n-4]-3\delta[n-5]$

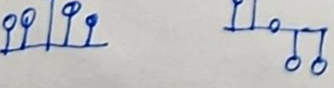
Frekvenční spektrum:

$x(t)=1+\cos(2\pi\cdot300\cdot t-\frac{\pi}{2})+\sin(2\pi\cdot700\cdot t)=$
$1+\cos(2\pi\cdot300\cdot t-\frac{\pi}{2})+\cos(2\pi\cdot700\cdot t-\frac{\pi}{2})$

magnitudové spekt. / fázové spektrum



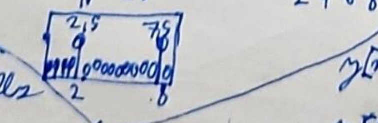
symetrické / antisymetrické



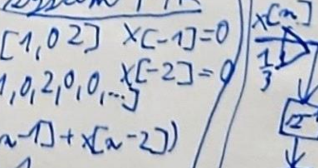
spektrum

$F_s=8\,Hz$ ⇒ vzork. frek.

$x(t)=\cos(2\pi\cdot2t-\frac{\pi}{2})+\cos(2\pi\cdot7t+\frac{\pi}{2})$



Typy filtrů:



fáz. posun
dolní propust
horní propust
pásmová propust
pásmová zádrž

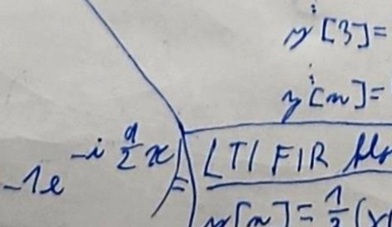
rozmazání spektra

$N=10$

$x(t)=\cos(2\pi f t)$

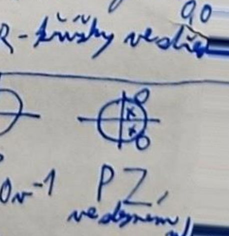
$F_s=10$ $\quad f=2{,}5\,Hz$

$N=20$



LTI IIR

$y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2])-\frac{1}{2}y[n-1]$



DFT

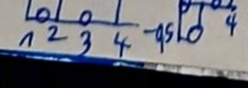
$k=[0,2,4,6]$

$x[n]=[0,1,0,-1], N=4, F_s=100\,Hz$

$X[k]=\frac{1}{N}\sum_{n=0}^{N-1}x[n]e^{-\frac{i2\pi nk}{N}}$

$X[3]=\frac{1}{4}\cdot(0e^{-\frac{i2\pi\cdot0\cdot3}{4}}+1e^{-\frac{i2\pi\cdot1\cdot3}{4}}+$
$0e^{-\frac{i2\pi\cdot2\cdot3}{4}}-1e^{-\frac{i2\pi\cdot3\cdot3}{4}})=\frac{1}{4}(0+1e^{-\frac{3}{2}i\pi}+0-1e^{-i\frac{9}{2}\pi})$
$=\frac{1}{4}(0+i+0+i)=\frac{i}{2}$

$k\cdot\frac{F_s}{N}=3\cdot\frac{100}{4}$ $\quad X[k]=[0;-\frac{i}{2};0;\frac{i}{2}]$



LTI systém FIR

$x[n]=[1,0,2]$ $\quad x[-1]=0$ $\quad x[-2]=0$

$x[n]=[0,0,1,0,2,0,0,\dots]$

$y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2])$

$y[0]=\frac{1}{3}(1+0+0)=\frac{1}{3}$

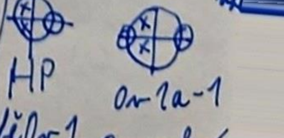
$y[3]=\frac{1}{3}(0+2+0)=\frac{2}{3}$

$y[n]=(\frac{1}{3},\frac{1}{3},1,\frac{2}{3},\frac{2}{3})$

LTI FIR blokové schéma

$y[n]=\frac{1}{3}(x[n]+x[n-1]+x[n-2])$

IIR - kořeny mimo
FIR - kořeny vestřed



DP
PZ
HP

Konvoluce: c) násobení polynomů: $x \cdot h = (1+2x^{-2}) \cdot (\frac{1}{3}+\frac{1}{3}x^{-1}+\frac{1}{3}x^{-2})$

a) posun. průběhy

$x[n]=[1,0,2]$ $\quad h[n]=[\frac{1}{3},\frac{1}{3},\frac{1}{3}]$

$$\begin{array}{ccccc} & & 1 & 0 & 2 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & \\ \hline & & 0 & 0 & 0\ 0\ 0 \end{array} \rightarrow \begin{array}{c} 1\,0\,2 \\ \frac{1}{3}\frac{1}{3}\frac{1}{3} \\ \hline \frac{1}{3} \end{array} \rightarrow \begin{array}{c} 1\,0\,2 \\ \frac{1}{3}\frac{1}{3}\frac{1}{3} \\ \hline \frac{1}{3}\ 0\ \frac{2}{3} \\ \hline 1 \end{array} \rightarrow \dots$$

b) superposice

$$\begin{array}{lccccc} 1 \cdot h[n] & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & \\ 0 \cdot h[n] & & 0 & 0 & 0 & \\ 2 \cdot h[n] & & & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ \hline & \frac{1}{3} & \frac{1}{3} & 1 & \frac{2}{3} & \frac{2}{3} \end{array}$$

$x[0] \cdot h[n]$

$x[1] \cdot h[n]$

$x[2] \cdot h[n]$

$y[n]=[\frac{1}{3},\frac{1}{3},1,\frac{2}{3},\frac{2}{3}]$

amplituda a fáze DFT

$A_k = \sqrt{\text{Im}\cdot\text{část}^2 + \text{Re}\cdot\text{část}^2}$

$\phi_k = \text{arctg} \frac{\text{Im}(\text{část})}{\text{Re}(\text{část})}$

stejnosměrná součást

$$\frac{x[0]+x[1]\dots+x[n]}{n}$$

LTI IIR impulzní odezva – musí vycházet z blok. schém. – nekonečný rozvoj

$x[n]=[1]$ $\quad x[-1]=0$

$x[-2]=0$

$h[n]$ – impulzní odezva $\quad y[-1]=0$

$h[0]=\frac{1}{3}(1+0+0)-0=\frac{1}{3}$

$h[1]=\frac{1}{3}(0+1+0)-\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$

metoda nejmenší vzdálenosti

010	100	101
000	000	000
010	010	001
X	T1	T2
	2	5

počet neshod

Z-transformace $\quad z \in C$

$$X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

LTI systémů

$x[n]=[1,3,5] \rightarrow X(z)=1+3z^{-1}+5z^{-2}$

$x[n-1]=[0,1,3,5] \rightarrow X(z)=1z^{-1}+3z^{-2}+5z^{-3}$

FIR

$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]$

$h[n]=[\frac{1}{3},\frac{1}{3},\frac{1}{3}]$ $\quad x[n]=[1,0,2]$

$\mathcal{H}(z)=\frac{1}{3}+\frac{1}{3}z^{-1}+\frac{1}{3}z^{-2}$ $\quad X(z)=1+2z^{-2}$

$Y(z)=X(z)\cdot\mathcal{H}(z)=(1+2z^{-2})\cdot(\frac{1}{3}+\frac{1}{3}z^{-1}+\frac{1}{3}z^{-2})=$

$\frac{1}{3}+\frac{1}{3}z^{-1}+z^{-2}+\frac{2}{3}z^{-3}+\frac{2}{3}z^{-4}$

$y[n]=[\frac{1}{3},\frac{1}{3},1,\frac{2}{3},\frac{2}{3}]$

IIR

$y[n]=\frac{1}{3}x[n]+\frac{1}{3}x[n-1]+\frac{1}{3}x[n-2]-\frac{1}{2}y[n-1]$

$\mathcal{H}(z)=\frac{\frac{1}{3}+\frac{1}{3}z^{-1}+\frac{1}{3}z^{-2}}{1+\frac{1}{2}z^{-1}}$ $\quad X(z)=1+2z^{-2}$

$Y(z)=\frac{(\frac{1}{3}+\frac{1}{3}z^{-1}+\frac{1}{3}z^{-2})\cdot(1+z^{-2})}{1+\frac{1}{2}z^{-1}}$

---

nuly a póly – frekv. charakteristika

$H(z)=\frac{Y(z)}{H(z)}=\frac{B_0+B_1z^{-1}+B_2z^{-2}}{1-A_1z^{-1}-A_2z^{-2}}$ – nuly / póly

$y[n]=\frac{1}{2}(x[n]+x[n-1])$

$\mathcal{H}(z)=\frac{1}{2}+\frac{1}{2}z^{-2}=\frac{1+z^{-2}}{2}=\frac{z^2+1}{2z^2}$

$z^2+1=0$

$z^2=-1$

$z=i$

nuly $[i,-i]$

$2z^2=0$

$z^2=0$

$z=0$

póly $[0;0]$

potlačená frekvence $H(L)=0$

$H(z)=\frac{1+z^{-2}}{2}$

$H(e^{iw})=\frac{1+e^{-i2w}}{2}$

$\frac{1+e^{-i2w}}{2}=0$

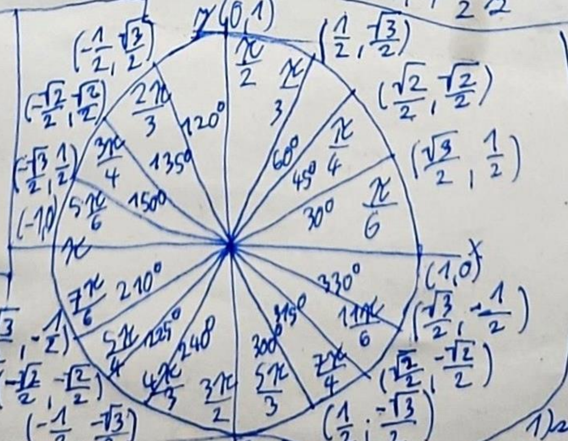
$-i2w=-1$

$e^{-i2w}=e^{-i\pi}$

$-i2w=-i\pi$

$w=\frac{\pi}{2} \rightarrow 2\pi F=\frac{\pi}{2}$

$F=\frac{1}{4}$



sigmoidy – počítání do jaké třídy patří

$$\begin{array}{c} \quad w_1\ w_2\ b\ \text{nep} \\ W=\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \end{array}$$

$X=[4,4]$

1) $\text{sigm}(1\cdot4+2\cdot4+0)=\text{sigm}(12)$

2) $\text{sigm}(2\cdot4+3\cdot4+1)=\text{sigm}(21)$

3) $\text{sigm}(3\cdot4+1\cdot4+2)=\text{sigm}(18)$

---

konvoluční filtr

$k=1$

```
000000
012120
021120
033620
014520
00000
```

$$w=\boxed{\begin{matrix} 1 & -1 & 0 \\ 2 & 0 & 2 \\ 3 & 2 & 1 \end{matrix}}$$

$$U_{3,3}=\begin{bmatrix} 1 & 1 & 2 \\ 3 & 6 & 2 \\ 4 & 5 & 2 \end{bmatrix}\cdot\begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}=$$

$1\cdot1+1\cdot(-1)+2\cdot0+3\cdot2+6\cdot0+2\cdot2+4\cdot3+2\cdot5+2\cdot1+\overset{1}{k}=35$