

dsa HW2

1

2

可以

概念是從preorder 按照順序放入節點，透過inorder 判斷位置，每當inorder list節點以左的元素都放入成功，即可拋棄該節點inorder list以左的資訊，新的節點放在該節點的右子節點

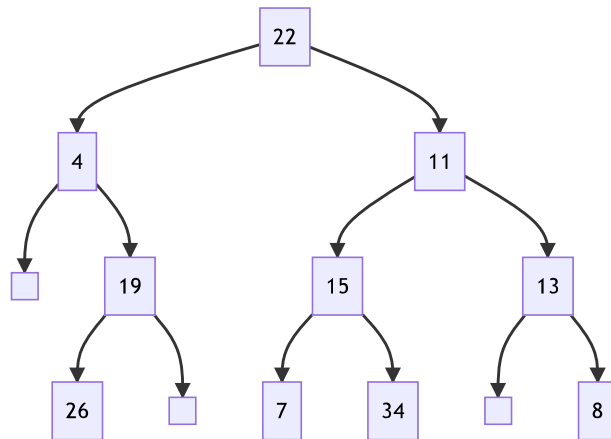
步驟是首先看preorder的第一個元素得知為22

得知tree head是22

有了第一個點後，看inorder 22 的位置得知在其左邊的node 都在22 左子樹

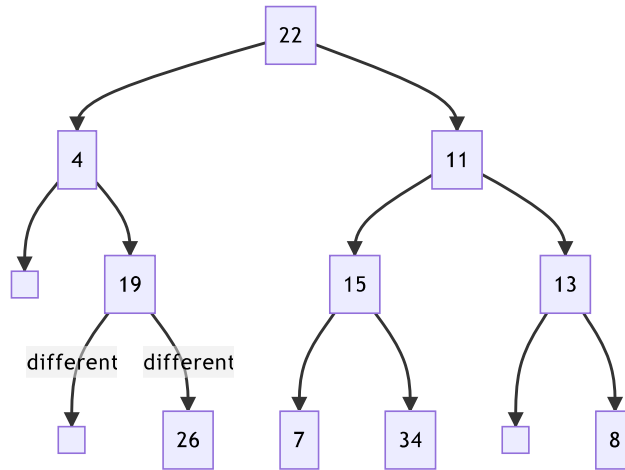
接著將4放到22左子樹，看4在inorder 之中的位置，4以前的值是左子樹，4以右到父節點22之前是右子樹，發現4沒有左子樹，可以得知19是4的右節點，接下來確認19只有左子樹，26放到19左子樹

接下來以此類推，透過觀察preorder節點在inorder list 中父節點的右邊還是左邊決定是放在左子節點還是右子節點



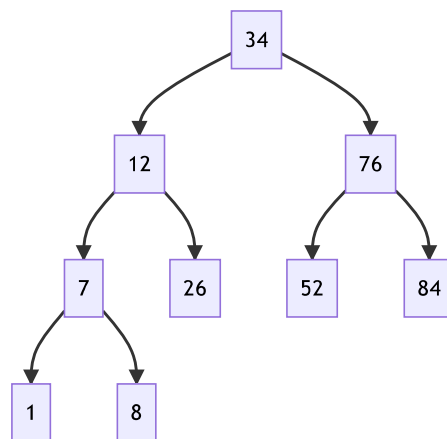
3

不行，會使很多左子樹跟右子樹位置不明確，以圖證明，第二題的tree 跟第三題的tree在微調node 26 位置後都符合題三的treversal result，因此可能有多種正確結果如果只根據preorder 跟 postorder



4

[1, 7, 8, 12, 26, 34, 52, 76, 84]



5

```

function construct_shortest_BST(arr, s, e)

    if s > e:
        return NIL
    mid := ceil((e + s) / 2)
    node := new node address
    node->num := arr[mid]
    node->left := construct_shortest_BST(arr, s, mid-1)
    node->right := construct_shortest_BST(arr, mid+1, e)
    return node

```

6

time complexity = pigeon_search line 2 for loop N *
 pigeon_search line 4 for loop M * pigeon_search line 6 for
 loop P

$O(N * M * P)$

extra space complexity = pigeon_store line 1 two

dimension array MP array + *pigeon_search* line 1 one
 dimension array N
 $O(N + MP)$

7

time complexity = Spotteddove_search line 2 for loop $N * P$
 Spotteddove_search line 6 for loop P
 $O(N * P)$

extra space complexity = extra space complexity for
 storing bugs root is $\min(26, P) * 26 + \text{Spotteddove_search}$
 line 1 one dimension array N
 $O(N)$

8

I'll choose Spotteddove method, it's obvious that it's time
 complexity and extra-space complexity spend less than
 previous one.

Although it take more extra-space when M is
 small. However, I prefer wasting some space when the task
 is simple rather than spending enormous time and space
 when the task is complicate.

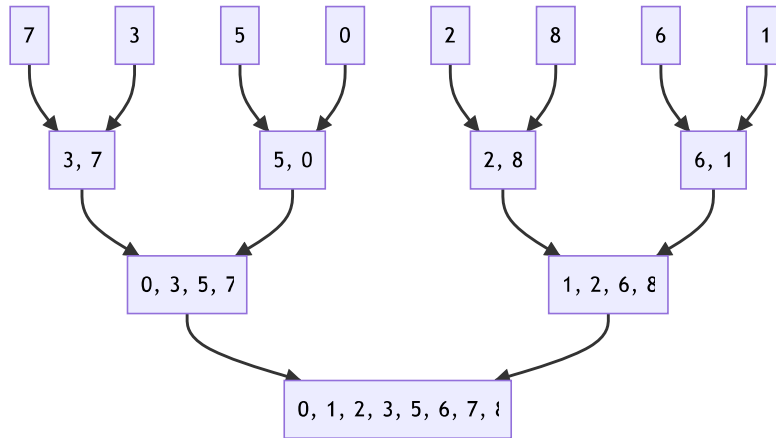
2

1

```

1  function Button-up_merge_sort(arr, n)
2      n := size of arr
3      for i:=1; i<n; i*=2
4          for j:=1; j<=n; j+=2i
5
6              mid := min(n, j+i)
7              end := min(n, j+2i)
8              s1, s2, e1, e2 := j, mid, mid, end
9              tmp := array with size [end - j]
10             idx := 1
11             while(s1 < e1 and s2 < e2)
12                 if arr[s1] <= arr[s2]:
13                     tmp[idx] := arr[s1]
14                     s1++
15                 else
16                     tmp[idx] := arr[s2]
17                     s2++
18             while(s2 < e2)
19                 tmp[idx] := arr[s2]
20                 s2++
21             while(s1 < e1)
22                 tmp[idx] := arr[s1]
23                 s1++
24             for k := 0 to end - j - 1
25                 arr[j+k] := tmp[k + 1]
26         return arr
27

```



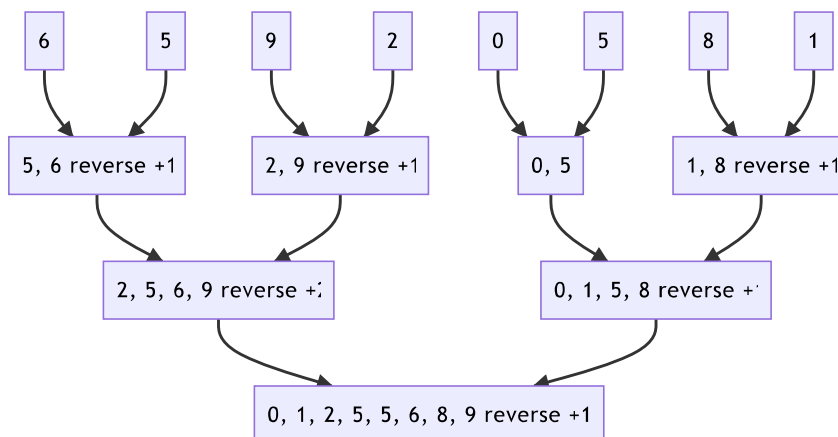
2

```

1  function count_number_of_reversion(arr, n)
2      n := size of arr
3      cnt := number of reversion
4      for i:=1; i<n; i*=2
5          for j:=1; j<=n; j+=2i
6
7              mid := min(n, j+i)
8              end := min(n, j+2i)
9              s1, s2, e1, e2 := j, mid, mid, end
10             tmp := array with size [end - j]
11             idx := 1
12             while(s1 < e1 and s2 < e2)
13                 if arr[s1] <= arr[s2]:
14                     tmp[idx] := arr[s1]
15                     s1++
16                 else
17                     tmp[idx] := arr[s2]
18                     s2++
19                     cnt += e1 - s1
20             while(s2 < e2)
21                 tmp[idx] := arr[s2]
22                 s2++
23             while(s1 < e1)
24                 tmp[idx] := arr[s1]
25                 s1++
26             for k := 0 to end - j - 1
27                 arr[j+k] := tmp[k + 1]
28         return cnt
29

```

在merger sort過程，每發成一次先加入後面陣列的元素，
Reversion cnt += 前面陣列剩餘的元素



total reversion = 17

3

```

1  funtion count_number_of_reversion(arr, n)
2      cnt := number of reversion
3      for i:=1; i<n; i*=2
4          for j:=1; j<=n; j+=2i
5              mid := min(n, j+i)
6              end := min(n, j+2i)
7              s1, s2, e1, e2 := j, mid, mid, end
8              tmp := array with size [end - j]
9              idx := 1
10             while(s1 < e1 and s2 < e2)
11                 if arr[s1] <= arr[s2]:
12                     tmp[idx] := arr[s1]
13                     s1++
14                 else
15                     tmp[idx] := arr[s2]
16                     s2++
17                     cnt += e1 - s1
18             while(s2 < e2)
19                 tmp[idx] := arr[s2]
20                 s2++
21             while(s1 < e1)
22                 tmp[idx] := arr[s1]
23                 s1++
24             for k := 0 to end - j - 1
25                 arr[j+k] := tmp[k + 1]
26         return cnt
27

```

time complexity = line 4 for loop $[log_2 N]$ * line 5 for loop $[n/2i]$ * while loop from line 10 to line 23 $[2i]$

$O(N \log_2 N)$

extra space complexity = line 8 one dimension array with at most size N

$O(N)$

4

```

1  funtion count_number_of_candidate(arr, n)
2      arr := Button-up_merge_sort(arr, n) // overload operator that arr[i] <= arr[j]
3      KD := K/D ratio list of arr
4      cnt := (n - 1)n/2 // number of candidate
5      cnt -= count_number_of_reversion(KD, n)
6      return cnt

```

time complexity = Button-up_merge_sort $O(N \log_2 N)$ + count_number_of_reversion $O(N \log_2 N)$

$O(N \log_2 N)$

extra space complexity = Button-up_merge_sort $O(N)$ + count_number_of_reversion $O(N)$

$O(N)$

$O(N)$

5

```

1  function binary_search_k-th(arr1, arr2, n, k)
2      s1, e1, s2, e2 = 1, n, 1, n
3      while 2n <= k
4          m1, m2 = floor(s1 + e1)/2, floor(s2 + e2)/2
5          if (arr[m1] + arr[m2] > k)
6              if m1 > m2
7                  e1 = m1 - 1
8              else
9                  e2 = m2 - 1
10         else if (arr[m1] + arr[m2] < k)
11             if m1 < m2
12                 s1 = m1 + 1
13             else
14                 s2 = m2 + 1
15         else
16             return max(m1, m2)
17     return cnt
18

```

time complexity = while loop $2\log_2 N$

$O(\log_2 N)$

extra space complexity = s1, m1, e1, s2, m2, e2 (6)

$O(1)$

6

time complexity of partition function is $2N$. However, pi that partition return is always be latest - 1.

Hence, NeonSort will call recursive function with parameter (match, oldest, latest - 1), and (match, latest, latest), only NEONSort with parameter (match, oldest, latest - 1) will continually call partition function.

Considering all the above, the time complexity of NeonSort is $2N \cdot (N)/2 = N^2$

$O(N^2)$

3

1

result = [74, 16, 10, 11, 6, 7, 4, 6, 8, 3]

2

result = [74, 16, 10, 11, 6, 7, 8, 4, 6, 3]

3

21964800

用程式把每種可能計算，同時進行剪枝減少進行時間，array由前至後插入值0-13，如果插入值>於其index/2的值或該值已被插入過，中斷for迴圈，否則填入新值，只有一路填完15格的case納入計算，max_heap可能性應等於min_heap可能性

```

1  cnt = 0
2  def count (arr, deph):
3      cnt = 0
4      for i in range(14):
5          past = arr.copy()
6          if i in past[0:deph]: continue
7          if deph == 15 :
8              if i > past[deph//2]:
9                  return 0
10             else:
11                 past.append(i)
12                 return 1
13             if i > past[deph//2]:
14                 continue
15             past.append(i)
16             cnt += count(past, deph+1)
17         return cnt
18  print(count([-1, 14], 2))

```

4

min heap

for 1-15 計算每個點大於其後面的值的數量(inverse)

第一個子樹無庸置疑 沒有inverse

[1]

第二個值

最大可以是9，小於整個右子樹中的值，有 2^3-1 個inverse

而另一個便只能是2-noinverse

總共7

[1, 9, 2]

第三層

可以是[1, 9, 2, 13, 10, 6, 3]

inverse 數分別是 小於3到1個子樹 = $3(2^2-1) + 2(2^2-1) + 1(2^2-1)$

總共18

最後一層

可以是[1, 9, 2, 13, 10, 6, 3, 15, 14, 12, 11, 9, 8, 5, 4]

以此類推

$7*(2^1-1) + 6(2^1-1) + \dots + 1(2^1-1)$

總共 7

加總為32

5

```

1  function minimum_cost_fusing(arr)
2      cost := 0
3      build arr to min_heap          O(nlogn)
4      for i := 1; i < arr.length; i++: O(n)
5          a := REMOVE-SMALLEST(arr)  O(1)
6          b := REMOVE-SMALLEST(arr)  O(1)
7          cost += a + b
8          arr.insert(a + b)          O(logn)
9

```

the only extra-space the algorithm is cost which is $O(1)$
 and the time complexity is as mention in pseudo code

$$is = n\log n(\text{build min heap}) + n(\text{for loop in line 4}) * (\log n(\text{heap.insert}) + 1(\text{heap.remove smallest}))$$

$$= O(n\log n)$$

6

```

1  funtion Button-up_merge_sort(array_list, M, N, new)
2
3      for i:=N; i<MN; i*=2
4          for j:=1; j<=MN; j+=2i
5
6              mid := min(MN, j+i)
7              end := min(MN, j+2i)
8              s1, s2, e1, e2 := j, mid, mid, end
9              idx := 1
10             while(s1 < e1 and s2 < e2)
11                 if array_list[s1/M+1][s1%M] <= array_list[s2/M+1][s2%M]:
12                     new[idx] := array_list[s1/M+1][s1%M]
13                     s1++
14                 else
15                     new[idx] := array_list[s2/M+1][s2%M]
16                     s2++
17             while(s2 < e2)
18                 new[idx] := array_list[s2/M+1][s2%M]
19                 s2++
20             while(s1 < e1)
21                 new[idx] := array_list[s1/M+1][s1%M]
22                 s1++
23             for k := 0 to end - j - 1
24                 array_list[(j+k)/M+1][(j+k)%M] := new[k + 1]
25         for i:=0; i<M; i+=1
26             for j:=1; j<=N; j+=1:
27                 new[i*M+j] = array_list[i][j]
28
29         return new
30

```

it is equivalent to bottom which have been process $\log N$ round

Hence BigO = $O(mn (\log (mn) - \log (n))) = O(mn (\log (m)))$
 and extra space = $O(1)$ because new is not included in the extra space

