

Net present value.

1. Time is discrete

period 0, 1, 2, 3, ...

2. discount factor β

interest rate r

$$\Rightarrow \beta = \frac{1}{1+r}$$

periods 0 $\xrightarrow{\beta}$ 1 $\xrightarrow{\beta}$ 2 $\xrightarrow{\beta}$ 3

assuming constant dis. fac. β

Example 1:

stock: pay d every period, starting from 0

β

$$\begin{aligned} \text{PV of this stock? } PV &= d + \beta \cdot d + \beta^2 \cdot d + \dots \\ &= \sum_{t=0}^{\infty} d \cdot \beta^t = d \cdot \frac{1}{1-\beta} \end{aligned}$$

PV of this stock?

assume: pays I

$$\begin{aligned} PV &= d + \beta \cdot d + \dots + \beta^T \cdot d \\ &= \sum_{t=0}^T d \cdot \beta^t = d \cdot \frac{1-\beta^{T+1}}{1-\beta} \end{aligned}$$

Example 2

purchase a bond

$$\beta = \frac{1}{1+r}$$

period 0 1 2 3 4
pay to buy price P

$$PV = \beta^0 \cdot c + \beta^1 \cdot c + \beta^2 \cdot c + \beta^3 \cdot c + \beta^4 \cdot (c + P)$$

$$PV \text{ at period 2? } = c + \beta \cdot c + \beta^2 \cdot (c + P)$$

Example 3

stock 0 $\xrightarrow{\beta}$ 1 2 3 4 5 ...
period d_1 d_3 d_5 ...

$$PV? = \beta \cdot d_1 + \beta^3 \cdot d_3 + \beta^5 \cdot d_5 + \dots$$

Example 4

stock 0 $\xrightarrow{\beta_1}$ 1 $\xrightarrow{\beta_2}$ 2 $\xrightarrow{\beta_3}$ 3 $\xrightarrow{\beta_4}$ 4 $\xrightarrow{\beta_5}$ 5
period d_1 d_3 d_5

$$PV_0 = \beta_1 \cdot d_1 + \beta_1 \cdot \beta_2 \cdot \beta_3 \cdot d_3 + \left(\prod_{t=1}^5 \beta_t \right) \cdot d_5$$