Bayesian Reasoning

Zhihan Fang

Rutgers University

zhihan.fang@cs.rutgers.edu

March 23, 2016

Overview

1. Probability Basics
   - Conditional Probability
   - Product Rule
2. Bayesian Rule
   - Bayesian Reasoning
   - Bayesian Examples
3. Implement Bayesian in R
   - write a function in R
   - call function in R

Probability Basics

Definition of conditional probability:

Theorem (Conditional Probability)

$P(A|B) = \frac{P(A,B)}{P(B)}$

Example: Professor Imielinski gave two quizzes in his class, 70% of students passed both quizzes and 80% students in his class passed the first quiz. What percent of student who passed the first quiz also passed the second quiz?

$$P(pass_2|pass_1) = \frac{P(pass_1, pass_2)}{P(pass_1)} = \frac{70\%}{80\%} = 87.5\% \tag{1}$$

Product Rule

From conditional probability,

$$P(A|B) = \frac{P(A,B)}{P(B)} \Rightarrow P(A,B) = P(A|B)P(B)$$
$$P(B|A) = \frac{P(B,A)}{P(A)} \Rightarrow P(A,B) = P(B|A)P(A) \tag{2}$$

We can get product rule,

$$P(A,B) = P(A|B)P(B) = P(B|A)P(A) \tag{3}$$

Example

Example: 80% student passed the first quiz and 87.5% of them also passes the second quiz. For all student in the class, 70% passed the second quiz, students who passes the second quiz all passed the first quiz. What percent of students passes both quiz?

$$P(pass_1, pass_2) = P(pass_1)P(pass_2|pass_1) = 80\% \times 87.5\% = 70\%$$

or

$$P(pass_1, pass_2) = P(pass_2)P(pass_1|pass_2) = 70\% \times 100\% = 70\%$$

Bayesian Reasoning

Bayesian rule is derived from the product rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

for the previous problem, if we are given

$$P(pass_1) = 80\% \; P(pass_2) = 70\% \quad P(pass_2|pass_1) = 87.5\%$$

then we can get the percentage who passed the second quiz also passed the first quiz.

$$P(pass_2|pass_1) = \frac{P(pass_1, pass_2)}{P(pass_1)} = \frac{P(pass_1|pass_2)P(pass_2)}{P(pass_1)}$$
$$= \frac{80\% \times 87.5\%}{70\%} = 100\%$$

Bayesian Example: Rain or Not

Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding.

$$
\begin{aligned}
P(r|f) &= \frac{P(f|r)P(r)}{P(f)} \\
&= \frac{P(f|r)P(r)}{P(f|r)P(r) + P(f|\neg r)P(\neg r)} \\
&= \frac{0.9 \times \frac{5}{365}}{0.9 \times \frac{5}{365} + 0.1 \times \frac{365-5}{365}} = 0.11
\end{aligned}
$$

Bayesian example: Drug Test

Suppose a drug test is 99% sensitive and 99% specific. That is, the test will produce 99% true positive results for drug users and 99% true negative results for non-drug users. Suppose that 0.5% of people are users of the drug. If a randomly selected individual tests positive, what is the probability he or she is a drug user?
$\Rightarrow$ Posterior to be computed: $P(user|positive)$

$$\begin{aligned} P(user|positive) &= \frac{P(positive|user)P(user)}{P(positive)} \\ &= \frac{P(positive|user)P(user)}{P(positive|user)P(user) + P(positive|\neg user)P(\neg user)} \\ &= \frac{0.99 \times 0.005}{0.99 \times 0.005 + 0.01 \times 0.995} = 0.33 \end{aligned}$$

Define your own function

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\neg A)P(\neg A)}$$

Example (Bayesian Function)

```
bayesian.rule <- function(P_A,P_B_A,P_B_nA){
  P_A_B = (P_B_A * P_A)/(P_B_A*P_A+P_B_nA*(1-P_A))
  return(P_A_B)
}
```

Call function

Example (call function)

```
> bayesian.rule(5/365,0.9,0.1)
[1] 0.1111111
>bayesian.rule(0.005,0.99,0.01)
[1] 0.3322148
```