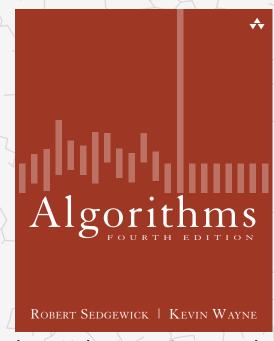
## Algorithms



http://algs4.cs.princeton.edu

## 5.4 REGULAR EXPRESSIONS

- regular expressions
- REs and NFAs
- ▶ NFA simulation
- ▶ NFA construction
- applications

## 5.4 REGULAR EXPRESSIONS

regular expressions

NEAS

NFA simulation

NFA construction

applications

# Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

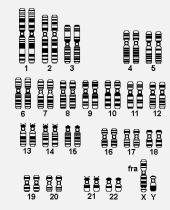
## Pattern matching

Substring search. Find a single string in text.

Pattern matching. Find one of a specified set of strings in text.

#### Ex. [genomics]

- Fragile X syndrome is a common cause of mental retardation.
- A human's genome is a string.
- It contains triplet repeats of CGG or AGG, bracketed by GCG at the beginning and CTG at the end.
- Number of repeats is variable and is correlated to syndrome.



pattern GCG(CGG|AGG)\*CTG

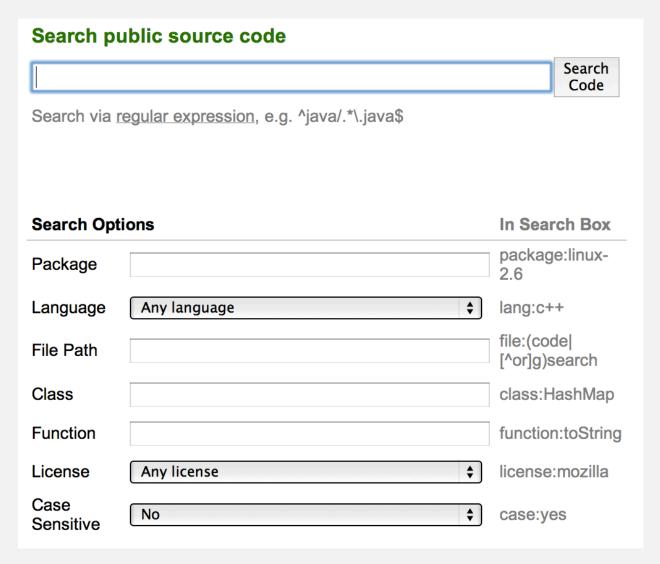
text GCGGCGTGTGTGCGAGAGAGTGGGTTTAAAGCTGGCGCGGAGGCGGCTGGCGCGGAGGCTG

## Syntax highlighting

```
/****************
  Compilation: javac NFA.java
* Execution: java NFA regexp text
  Dependencies: Stack.java Bag.java Digraph.java DirectedDFS.java
* % java NFA "(A*B|AC)D" AAAABD
* true
* % java NFA "(A*B|AC)D" AAAAC
* false
******************
public class NFA {
   private Digraph G;  // digraph of epsilon transitions
   private String regexp; // regular expression
   private int M;  // number of characters in regular expression
   // Create the NFA for the given RE
   public NFA(String regexp) {
      this.regexp = regexp;
      M = regexp.length();
      Stack<Integer> ops = new Stack<Integer>();
      G = new Digraph(M+1);
```

input	output
Ada	HTML
Asm	XHTML
Applescript	LATEX
Awk	MediaWiki
Bat	ODF
Bib	TEXINFO
Bison	ANSI
C/C++	DocBook
C#	
Cobol	
Caml	
Changelog	
Css	
D	
Erlang	
Flex	
Fortran	
GLSL	
Haskell	
Html	
Java	
Javalog	
Javascript	
Latex	
Lisp	
Lua	

## Google code search



http://code.google.com/p/chromium/source/search

## Pattern matching: applications

#### Test if a string matches some pattern.

- Scan for virus signatures.
- Process natural language.
- Specify a programming language.
- Access information in digital libraries.
- Search genome using PROSITE patterns.
- Filter text (spam, NetNanny, Carnivore, malware).
- Validate data-entry fields (dates, email, URL, credit card).

. . .

## Parse text files.

- Compile a Java program.
- · Crawl and index the Web.
- Read in data stored in ad hoc input file format.
- Create Java documentation from Javadoc comments.

Form Validation





. . .

## Regular expressions

A regular expression is a notation to specify a set of strings.



operation	order	example RE	matches	does not match
concatenation	3	AABAAB	AABAAB	every other string
or	4	AA   BAAB	AA BAAB	every other string
closure	2	AB*A	AA ABBBBBBBBA	AB ABABA
parentheses 1	1	A(A B)AAB	AAAAB ABAAB	every other string
	I	(AB)*A	A ABABABABABA	AA ABBA

## Regular expression shortcuts

Additional operations are often added for convenience.

operation	example RE	matches	does not match
wildcard	.U.U.U.	CUMULUS JUGULUM	SUCCUBUS TUMULTUOUS
character class	[A-Za-z][a-z]*	word Capitalized	camelCase 4illegal
at least 1	A(BC)+DE	ABCDE ABCBCDE	ADE BCDE
exactly k	[0-9]{5}-[0-9]{4}	08540-1321 19072-5541	11111111 166-54-111

Ex. [A-E]+ is shorthand for (A|B|C|D|E)(A|B|C|D|E)\*

## Regular expression examples

RE notation is surprisingly expressive.

regular expression	matches	does not match
.*SPB.* (substring search)	RASPBERRY CRISPBREAD	SUBSPACE SUBSPECIES
[0-9]{3}-[0-9]{2}-[0-9]{4} (U. S. Social Security numbers)	166-11-4433 166-45-1111	11-5555555 8675309
[a-z]+@([a-z]+\.)+(edu com) (simplified email addresses)	wayne@princeton.edu rs@princeton.edu	spam@nowhere
[\$_A-Za-z][\$_A-Za-z0-9]* (Java identifiers)	ident3 PatternMatcher	3a ident#3

REs play a well-understood role in the theory of computation.

## Illegally screening a job candidate

```
"[First name]! and pre/2 [last name] w/7
bush or gore or republican! or democrat! or charg!
or accus! or criticiz! or blam! or defend! or iran contra
or clinton or spotted owl or florida recount or sex!
or controvers! or fraud! or investigat! or bankrupt!
or layoff! or downsiz! or PNTR or NAFTA or outsourc!
or indict! or enron or kerry or iraq or wmd! or arrest!
or intox! or fired or racis! or intox! or slur!
or controvers! or abortion! or gay! or homosexual!
or gun! or firearm! "
```



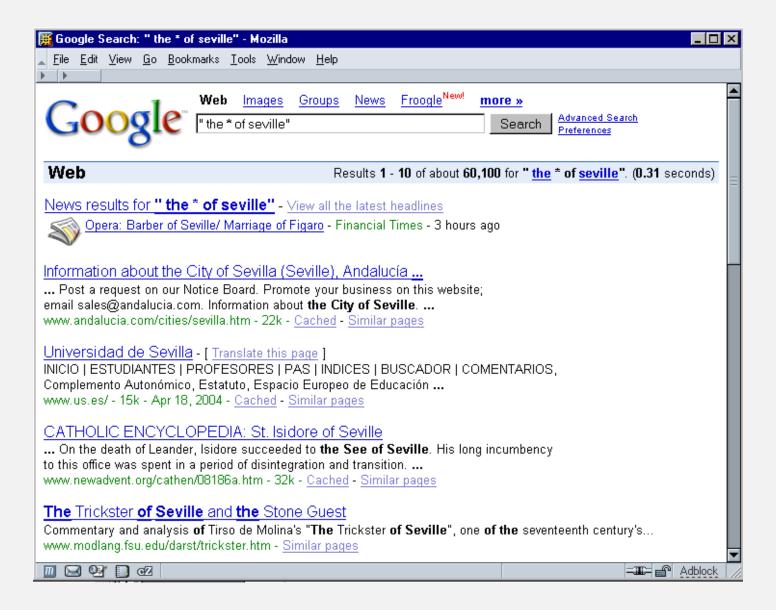
 LexisNexis search string used by Monica Goodling to illegally screen candidates for DOJ positions



http://www.justice.gov/oig/special/s0807/final.pdf

## Can the average web surfer learn to use REs?

Google. Supports \* for full word wildcard and | for union.



## Regular expressions to the rescue



http://xkcd.com/208

## Can the average programmer learn to use REs?

#### Perl RE for valid RFC822 email addresses

 $(?:(?:|r\n)?[ \t]) * (?:(?:[^() < \emptyset,;: \t]) + (?:(?:(r\n)?[ \t]) + |\Z|(?=[["() < \emptyset,;: \t]))|"(?:[^\"\r\]| | \.|(?:(?:(r\n)?[ \t])) * "(?:(?:(r\n)?[ \t]))|"(?:[^\"\r\]|)| | (?:(?:(r\n)?[ \t])) * "(?:(?:(r\n)?[ \t]))|"(?:[^\"\r\]|)| | (?:(?:(r\n)?[ \t])|"(?:(?:(r\n)?[ \t]))|"(?:[^\"\r\]|)| | (?:(?:(r\n)?[ \t]))|"(?:(?:(r\n)?[ \t])|"(?:(?:(r\n)?[ \t])|"(?:(?:(r\n)?[ \t])|"(?:(?:(r\n)?[ \t])|"(?:(?:(r\n)?[ \t])|"(?:(?:(r\n)?[ \t])|"(?:(?:(r\n)?[ \t])|"(?:(?:(r\n)?[ \t])|"(?:(?:(r\n)?[ \t])|"(?:(r\n)?[ \t])|"(?:($  $| (?:(?:(r))?[ \t]) (?:(?:(r))?[ \t]) (?:[(?:(r))?[ \t]) (?:[(?:(r))?[ \t]) (?:[(?:(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t]) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t]) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t]) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t])) (?:[(r))?[ \t]) (?:[(r))?[ \t])) (?:[(r))$ \t]))\*"(?:(?:\r\n)?[\t])\*))\*@(?:(?:\r\n)?[\t])\*(?:[^()<@,;:\\".\[\])\(]([^\[\])\)\([([^\[\]]\\.)\*\  $(?: \r\n)?[ \t])*)|*(?:[^() \Leftrightarrow \emptyset,;: \t])|*(?:[^() \Leftrightarrow \emptyset,;: \r\n)?[ \t]))|*(?:[^() \Leftrightarrow \emptyset,;: \t])|*(?:[^() \Leftrightarrow \emptyset,;::[^() \Leftrightarrow \emptyset,;::[^() \Leftrightarrow \emptyset,;::[^() \Leftrightarrow \emptyset,;::[^() \Leftrightarrow \emptyset,;::[^() \Leftrightarrow \emptyset,;:[^() \Leftrightarrow \emptyset,;:[$  $$$ \frac{(?:(?:(r\cdot n)?[ \t])*(?:[^()<@,;:\''.\[] \t])+(?:(?:(r\cdot n)?[ \t])+(?[(["()<@,;:\".\[]]))|([[^{[]}r\)]+(?:(?:(r\cdot n)?[ \t])+(?))} $$$ )\*))\*(?:,@(?:(?:\r\n)?[ \t])\*(?:[^() $\diamond$ @,;:\\".\[\] \000-\031]+(?:(?:\r\n)?[ \t])+\\Z\(?=[\["() $\diamond$ @,;:\\".\[\]]))\\[([^\[\]\\.)\*\](?:(?:\r\n)?[ \t])\*  $)(?:\.(?:(?:\r\n)?[\t])*(?:[^()<\emptyset,;:\''.\[\]\))|([(^{(),r\)}(?:(?:\r\n)?[\t])*))*)$ \*:(?:(r\n)?[\t])\*)?(?:[^()<>@,;:\\".\[\]\000-\031]+(?:(?:(?:\r\n)?[\t])+|\Z|(?=[\["()<>@,;:\\".\[\]]))|"(?:[^\\"\r\\]|\\.|(?:(?:\r\n)?[\t]))\*"(?:(?:\r\n)?[\t])  $\label{eq:linear_continuous} $$ n)^{(::(::(r\cdot n)?[ \t])^{(::(:(:\cdot r\cdot n)?[ \t])^{(::(:\cdot r\cdot n$  $?:(?:(r\n)?[\t])*)(?:(?:(?:(r\n)?[\t]))|([(^{[)]r\]})|(?:(?:(r\n)?[\t]))|([(^{[)]r\]})|(?:(?:(r\n)?[\t]))|([(^{[)]r\]})|(?:(?:(r\n)?[\t]))|([(^{[)]r\]})|(?:(?:(r\n)?[\t]))|([(^{[)]r\]})|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|([(^{[)]r\]})|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|([(^{[)]r\n}])|(?:(?:(r\n)?[\t]))|([(^{[)]r\n}])|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t])|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t])|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t])|(?:(r\n)?[\t]))|(?:(r\n)?[\t])|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t])|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\n)?[\t]))|(?:(r\$  $(?:[^\"\r]) \cdot (?:(?:\r)?[\t])) *"(?:(?:\r)?[\t])) *"(?:(?:\t])) *"(?$  $". 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## Regular expression caveat

#### Writing a RE is like writing a program.

- Need to understand programming model.
- Can be easier to write than read.
- Can be difficult to debug.



"Some people, when confronted with a problem, think 'I know I'll use regular expressions.' Now they have two problems."

- Jamie Zawinski (flame war on alt.religion.emacs)

Bottom line. REs are amazingly powerful and expressive, but using them in applications can be amazingly complex and error-prone.

## 5.4 REGULAR EXPRESSIONS

regular expressions

NEAS

NFA simulation

NFA construction

applications

# Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu



regular expressions

▶ NFAs

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Algorithms

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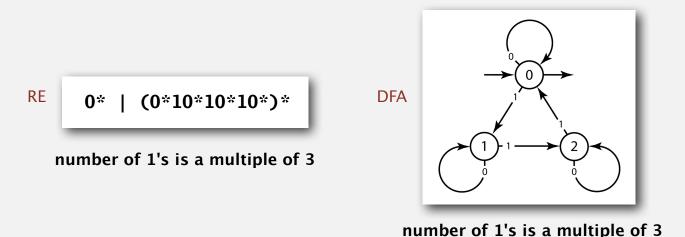
## Duality between REs and DFAs

RE. Concise way to describe a set of strings.

DFA. Machine to recognize whether a given string is in a given set.

#### Kleene's theorem.

- For any DFA, there exists a RE that describes the same set of strings.
- For any RE, there exists a DFA that recognizes the same set of strings.





Stephen Kleene Princeton Ph.D. 1934

## Pattern matching implementation: basic plan (first attempt)

#### Overview is the same as for KMP.

- No backup in text input stream.
- Linear-time guarantee.

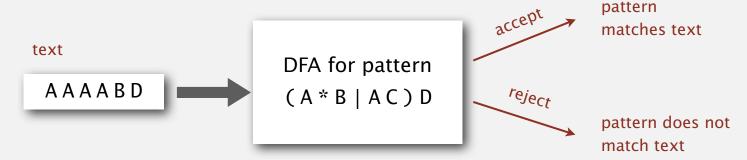


Ken Thompson Turing Award '83

Underlying abstraction. Deterministic finite state automata (DFA).

#### Basic plan. [apply Kleene's theorem]

- Build DFA from RE.
- · Simulate DFA with text as input.



Bad news. Basic plan is infeasible (DFA may have exponential # of states).

## Pattern matching implementation: basic plan (revised)

#### Overview is similar to KMP.

- No backup in text input stream.
- Quadratic-time guarantee (linear-time typical).

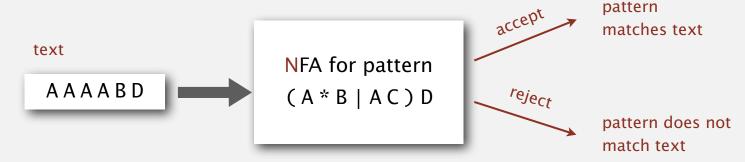


Ken Thompson Turing Award '83

Underlying abstraction. Nondeterministic finite state automata (NFA).

#### Basic plan. [apply Kleene's theorem]

- Build NFA from RE.
- Simulate NFA with text as input.



#### Q. What is an NFA?

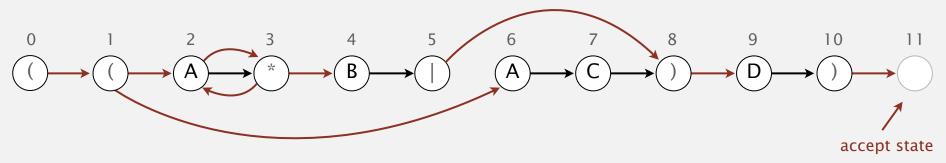
#### Regular-expression-matching NFA.

- RE enclosed in parentheses.
- One state per RE character (start = 0, accept = M).
- Red ε-transition (change state, but don't scan text).
- Black match transition (change state and scan to next text char).
- Accept if any sequence of transitions ends in accept state.

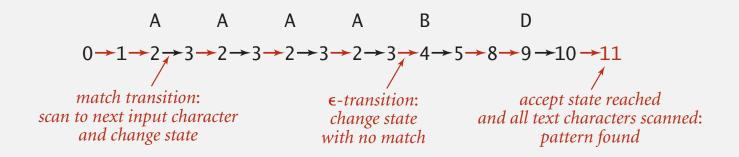
after scanning all text characters

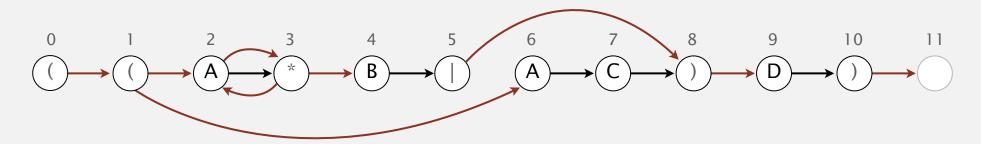
#### Nondeterminism.

- One view: machine can guess the proper sequence of state transitions.
- Another view: sequence is a proof that the machine accepts the text.



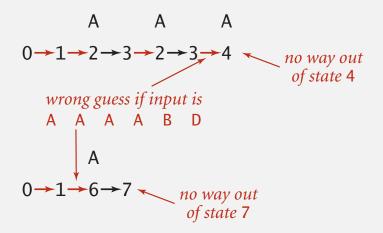
- Q. Is AAAABD matched by NFA?
- A. Yes, because some sequence of legal transitions ends in state 11.

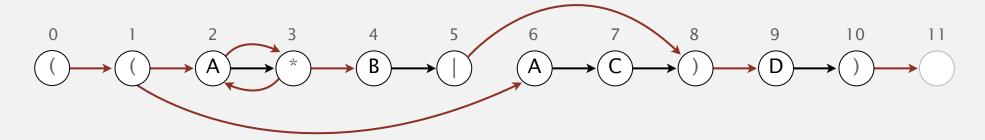




- Q. Is AAAABD matched by NFA?
- A. Yes, because some sequence of legal transitions ends in state 11.

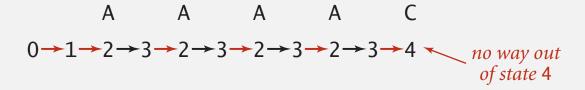
[ even though some sequences end in wrong state or stall ]

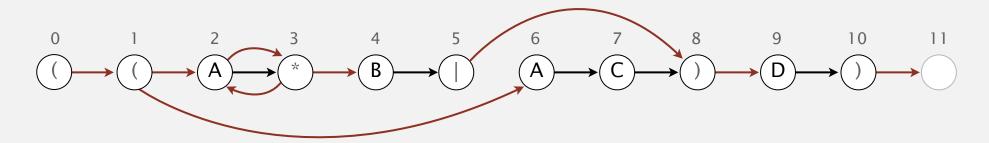




- Q. Is AAAC matched by NFA?
- A. No, because no sequence of legal transitions ends in state 11.

[ but need to argue about all possible sequences ]





#### Nondeterminism

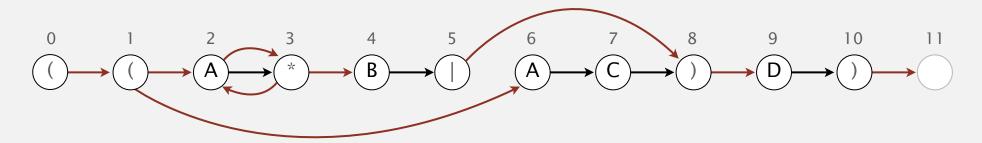
Q. How to determine whether a string is matched by an automaton?

DFA. Deterministic  $\Rightarrow$  easy because exactly one applicable transition.

NFA. Nondeterministic  $\Rightarrow$  can be several applicable transitions; need to select the right one!

Q. How to simulate NFA?

A. Systematically consider all possible transition sequences.





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## NFA representation

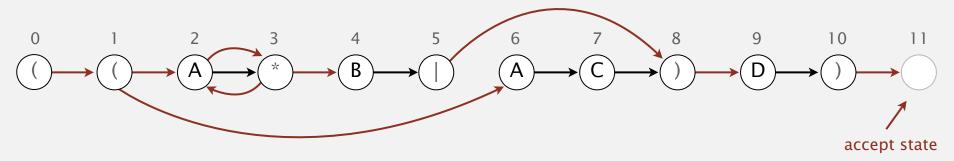
State names. Integers from 0 to *M*.

number of symbols in RE

Match-transitions. Keep regular expression in array re[].

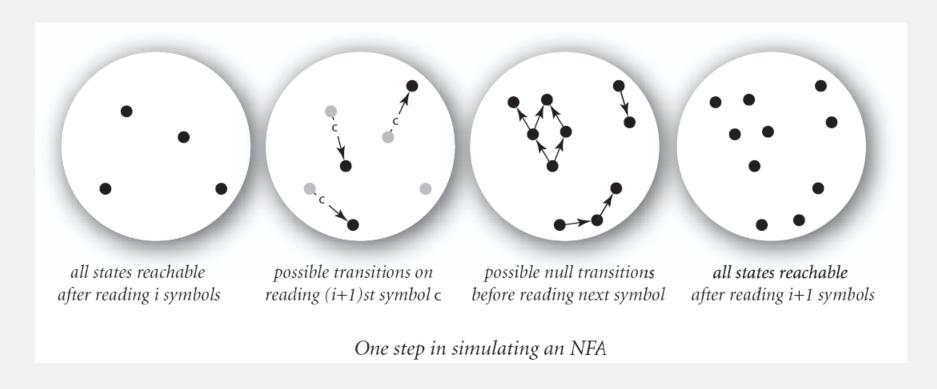
 $\epsilon$ -transitions. Store in a digraph G.

$$0 \rightarrow 1, 1 \rightarrow 2, 1 \rightarrow 6, 2 \rightarrow 3, 3 \rightarrow 2, 3 \rightarrow 4, 5 \rightarrow 8, 8 \rightarrow 9, 10 \rightarrow 11$$



#### NFA simulation

- Q. How to efficiently simulate an NFA?
- A. Maintain set of all possible states that NFA could be in after reading in the first *i* text characters.



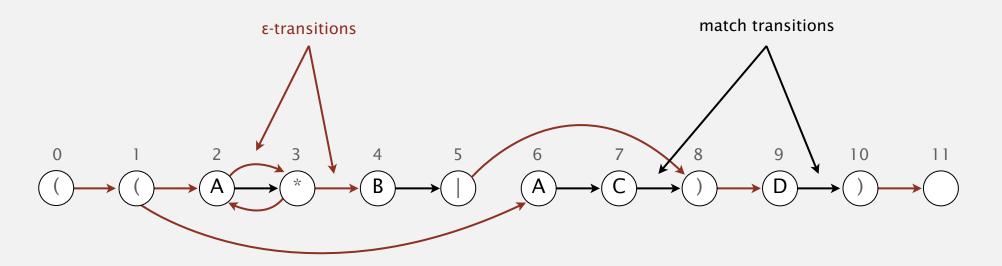
Q. How to perform reachability?

## NFA simulation demo

Goal. Check whether input matches pattern.



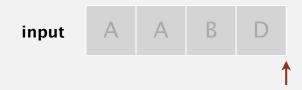


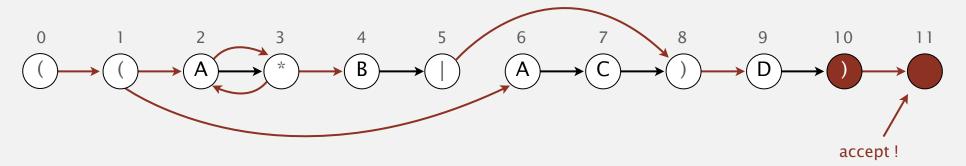


### NFA simulation demo

#### When no more input characters:

- Accept if any state reachable is an accept state.
- Reject otherwise.





set of states reachable: { 10, 11 }

## Digraph reachability

Digraph reachability. Find all vertices reachable from a given source or set of vertices.

recall Section 4.2

Solution. Run DFS from each source, without unmarking vertices. Performance. Runs in time proportional to E + V.

## NFA simulation: Java implementation

```
public class NFA
   private char[] re;  // match transitions
   private Digraph G;  // epsilon transition digraph
   private int M;  // number of states
   public NFA(String regexp)
     M = regexp.length();
     re = regexp.toCharArray();
                                                               stay tuned (next segment)
     G = buildEpsilonTransitionsDigraph();
   }
   public boolean recognizes(String txt)
   { /* see next slide */ }
   public Digraph buildEpsilonTransitionDigraph()
   { /* stay tuned */ }
```

## NFA simulation: Java implementation

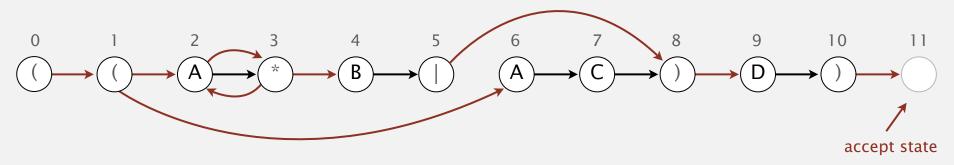
```
public boolean recognizes(String txt)
   Bag<Integer> pc = new Bag<Integer>();
                                                                    states reachable from
   DirectedDFS dfs = new DirectedDFS(G, 0);
                                                                    start by ε-transitions
   for (int v = 0; v < G.V(); v++)
      if (dfs.marked(v)) pc.add(v);
   for (int i = 0; i < txt.length(); i++)
      Bag<Integer> match = new Bag<Integer>();
                                                                    states reachable after
      for (int v : pc)
                                                                    scanning past txt.charAt(i)
         if (v == M) continue;
         if ((re[v] == txt.charAt(i)) || re[v] == '.')
             match.add(v+1);
      dfs = new DirectedDFS(G, match);
      pc = new Bag<Integer>();
                                                                    follow ε-transitions
      for (int v = 0; v < G.V(); v++)
         if (dfs.marked(v)) pc.add(v);
   for (int v : pc)
      if (v == M) return true:
   return false;
                                                                    accept if can end in state M
```

## NFA simulation: analysis

Proposition. Determining whether an N-character text is recognized by the NFA corresponding to an M-character pattern takes time proportional to MN in the worst case.

Pf. For each of the N text characters, we iterate through a set of states of size no more than M and run DFS on the graph of  $\varepsilon$ -transitions.

[The NFA construction we will consider ensures the number of edges  $\leq 3M$ .]



NFA corresponding to the pattern ( (  $A * B \mid A C$  ) D )

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States. Include a state for each symbol in the RE, plus an accept state.



Concatenation. Add match-transition edge from state corresponding to characters in the alphabet to next state.

Alphabet. A B C D Metacharacters. ( ) . \* |

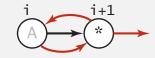


Parentheses. Add  $\epsilon$ -transition edge from parentheses to next state.

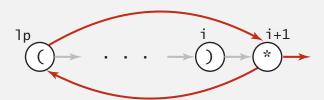


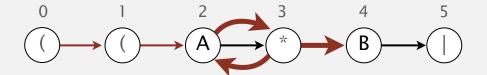
Closure. Add three  $\varepsilon$ -transition edges for each \* operator.

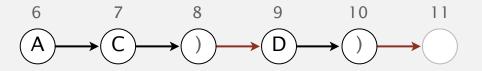
#### single-character closure



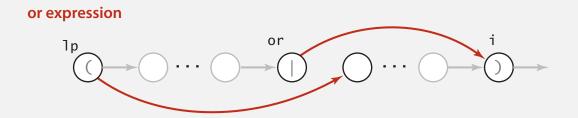
#### closure expression

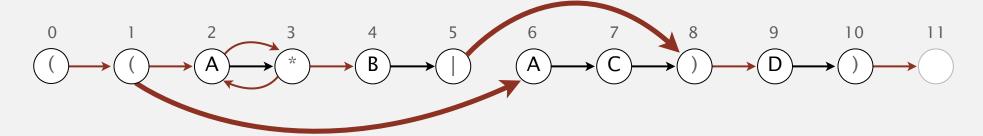






Or. Add two  $\varepsilon$ -transition edges for each | operator.





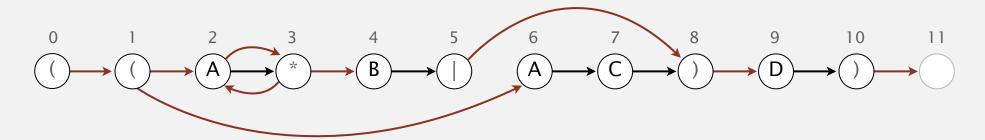
# NFA construction: implementation

Goal. Write a program to build the  $\varepsilon$ -transition digraph.

Challenges. Remember left parentheses to implement closure and or; remember | to implement or.

Solution. Maintain a stack.

- (symbol: push (onto stack.
- | symbol: push | onto stack.
- ) symbol: pop corresponding ( and any intervening  $\mid$ ; add  $\epsilon$ -transition edges for closure/or.



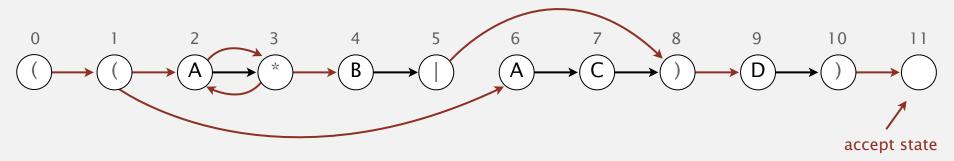
# NFA construction demo



stack

# NFA construction demo





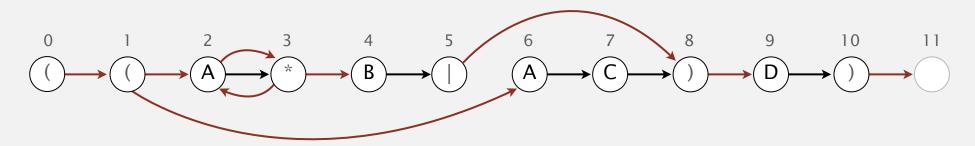
# NFA construction: Java implementation

```
private Digraph buildEpsilonTransitionDigraph() {
   Digraph G = \text{new Digraph}(M+1);
   Stack<Integer> ops = new Stack<Integer>();
   for (int i = 0; i < M; i++) {
      int lp = i;
      if (re[i] == '(' || re[i] == '|') ops.push(i);
                                                                    left parentheses and |
      else if (re[i] == ')') {
         int or = ops.pop();
         if (re[or] == '|') {
                                                                    or
            lp = ops.pop();
            G.addEdge(lp, or+1);
            G.addEdge(or, i);
         else lp = or:
                                                                    closure
      if (i < M-1 \&\& re[i+1] == '*') {
                                                                    (needs 1-character lookahead)
         G.addEdge(lp, i+1);
         G.addEdge(i+1, lp);
      if (re[i] == '(' || re[i] == '*' || re[i] == ')') ←
                                                                    metasymbols
         G.addEdge(i, i+1);
   return G;
```

# NFA construction: analysis

Proposition. Building the NFA corresponding to an M-character RE takes time and space proportional to M.

Pf. For each of the M characters in the RE, we add at most three  $\epsilon$ -transitions and execute at most two stack operations.



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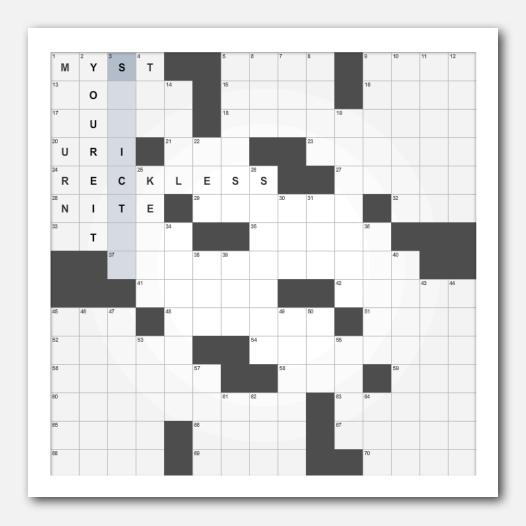
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# Generalized regular expression print

Grep. Take a RE as a command-line argument and print the lines from standard input having some substring that is matched by the RE.

Bottom line. Worst-case for grep (proportional to MN) is the same as for brute-force substring search.

# Typical grep application: crossword puzzles



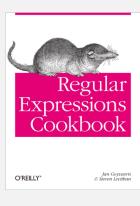
```
% more words.txt
a
aback
abacus
abacus
abalone
abandon
...

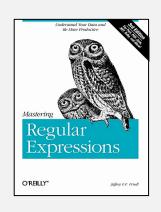
% grep "s..ict.." words.txt
constrictor
stricter
stricture
```

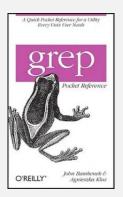
# Industrial-strength grep implementation

#### To complete the implementation:

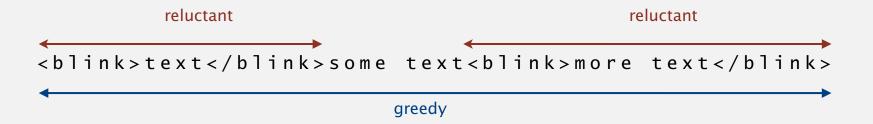
- Add character classes.
- Handle metacharacters.
- Add capturing capabilities.
- Extend the closure operator.
- Error checking and recovery.
- Greedy vs. reluctant matching.







Ex. Which substring(s) should be matched by the RE <bli>k>.\*</blink>?



# Regular expressions in other languages

#### Broadly applicable programmer's tool.

- Originated in Unix in the 1970s.
- Many languages support extended regular expressions.
- Built into grep, awk, emacs, Perl, PHP, Python, JavaScript, ...

#### PERL. Practical Extraction and Report Language.

## Regular expressions in Java

Validity checking. Does the input match the re?

Java string library. Use input.matches(re) for basic RE matching.

```
public class Validate
{
   public static void main(String[] args)
   {
     String regexp = args[0];
     String input = args[1];
     StdOut.println(input.matches(re));
   }
}
```

```
% java Validate "[$_A-Za-z][$_A-Za-z0-9]*" ident123
true

% java Validate "[a-z]+@([a-z]+\.)+(edu|com)" rs@cs.princeton.edu
true

% java Validate "[0-9]{3}-[0-9]{2}-[0-9]{4}" 166-11-4433

Social Security number
true
```

# Harvesting information

Goal. Print all substrings of input that match a RE.

```
% java Harvester "gcg(cgg|agg)*ctg" chromosomeX.txt
gcgcgggcggcggcggcggctg
gcgctg
gcgctg
gcgctg
harvest patterns from DNA
gcgcggcggcggaggcggaggcggctg

harvest links from website

### W java Harvester "http://(\\w+\\.)*(\\w+)" http://www.cs.princeton.edu
http://www.princeton.edu
http://www.google.com
http://www.cs.princeton.edu/news
```

# Harvesting information

RE pattern matching is implemented in Java's java.util.regexp.Pattern and java.util.regexp.Matcher classes.

```
import java.util.regex.Pattern;
import java.util.regex.Matcher;
                                                                     compile() creates a
public class Harvester
                                                                     Pattern (NFA) from RE
   public static void main(String[] args)
                                                                     matcher() creates a
      String regexp = args[0];
                                                                     Matcher (NFA simulator)
      In in
                       = new In(args[1]);
                                                                     from NFA and text
      String input = in.readAll();
      Pattern pattern = Pattern.compile(regexp);
      Matcher matcher = pattern.matcher(input);
                                                                     find() looks for
      while (matcher.find())
                                                                     the next match
         StdOut.println(matcher.group());
                                                                     group() returns
                                                                     the substring most
                                                                     recently found by find()
```

# Algorithmic complexity attacks

#### Warning. Typical implementations do not guarantee performance!



#### SpamAssassin regular expression.

```
% java RE "[a-z]+@[a-z]+([a-z\.]+\.)+[a-z]+" spammer@x.......
```

- Takes exponential time on pathological email addresses.
- Troublemaker can use such addresses to DOS a mail server.

## Not-so-regular expressions

#### Back-references.

- \1 notation matches subexpression that was matched earlier.
- Supported by typical RE implementations.

#### Some non-regular languages.

- Strings of the form ww for some string w: beriberi.
- Unary strings with a composite number of 1s: 111111.
- Bitstrings with an equal number of 0s and 1s: 01110100.
- Watson-Crick complemented palindromes: atttcggaaat.

Remark. Pattern matching with back-references is intractable.

#### Context

#### Abstract machines, languages, and nondeterminism.

- Basis of the theory of computation.
- Intensively studied since the 1930s.
- Basis of programming languages.

Compiler. A program that translates a program to machine code.

- KMP string  $\Rightarrow$  DFA.
- grep RE  $\Rightarrow$  NFA.
- javac Java language ⇒ Java byte code.

	КМР	grep	Java
pattern	string	RE	program
parser	unnecessary	check if legal	check if legal
compiler output	DFA	NFA	byte code
simulator	DFA simulator	NFA simulator	JVM

# Summary of pattern-matching algorithms

#### Programmer.

- Implement substring search via DFA simulation.
- Implement RE pattern matching via NFA simulation.



#### Theoretician.

- RE is a compact description of a set of strings.
- NFA is an abstract machine equivalent in power to RE.
- DFAs, NFAs, and REs have limitations.



You. Practical application of core computer science principles.

#### Example of essential paradigm in computer science.

- · Build intermediate abstractions.
- Pick the right ones!
- Solve important practical problems.



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NFA simulation

NFA construction

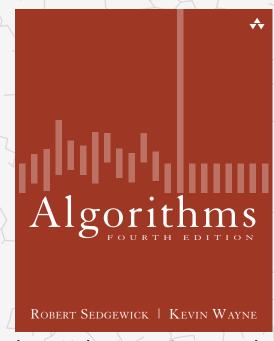
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- NFA simulation
- ▶ NFA construction
- applications