Maxwell's Equations

Introduction

Maxwell's equations are a set of four partial differential equations that form the foundation of classical electromagnetism, classical optics, and electric circuits. These equations describe how electric and magnetic fields are generated by charges, currents, and changes of each other. They were first published by James Clerk Maxwell between 1861 and 1862.

The Four Equations

Gauss's Law for Electricity:

$$\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$$

The electric field diverges from electric charges.

Gauss's Law for Magnetism:

$$\nabla \cdot \mathbf{B} = 0$$

Magnetic field lines form closed loops; there are no magnetic monopoles.

Faraday's Law of Induction:

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$

A changing magnetic field creates an electric field.

Ampère's Law with Maxwell's Addition:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \partial \mathbf{E}/\partial t$$

Electric currents and changing electric fields create magnetic fields.

Implications

Maxwell's equations have profound implications for our understanding of electromagnetism and light:

1. Electromagnetic Waves

Maxwell's equations predict the existence of electromagnetic waves that propagate at the speed of light. This led to the understanding that light itself is an electromagnetic wave.

2. Speed of Light

The speed of electromagnetic waves in vacuum can be derived from Maxwell's equations as $c = 1/\sqrt{(\varepsilon_0\mu_0)}$, which is approximately 3×10^8 m/s.

3. Electromagnetic Spectrum

Maxwell's equations apply to all electromagnetic waves, from radio waves to gamma rays, differing only in frequency and wavelength.

4. Technological Applications

Maxwell's equations form the foundation for numerous technologies, including radio, television, radar, wireless communications, and optical fiber communications.