Homework 2 - BI-Objective Periodic Review

Stefano Tonini*, Lorenzo Sciotto, Jacopo Dallafior

*Group representative

At least 1000 words per homework (without code)

1. Problem description

The problem presented requires minimizing the total inventory cost and obtaining the number and the relative quantity of orders. The total cost is given by the sum of the holding cost, i.e. the cost associated with storing materials in the warehouse, and the reorder cost, i.e. the cost linked to the reorder of a certain quantity of material. The minimization of the total cost varies depending on the following aspects.

Initially, it was asked to evaluate the minimization of the cost by focusing on performance from an economic point of view and then to evaluate it from an environmental point of view. It was also asked to evaluate the improvements and/or worsening in terms of costs and emissions when moving from economic to environmental optimization. Analyzing these two aspects give the anchor points, i.e. the extremes of the pareto front. By then varying the various weights that decide whether to give more or less importance to the economic or environmental aspect, we can populate the Pareto front. The Pareto- efficient solutions are defined as the one that do not perform worse in the set of considered objective functions. The final task involves manipulating input factors and visualizing the Pareto front for varying factor values. This step aims to explore the sensitivity of solutions to changes in input parameters.

2. Mathematical model

The mathematical model used for the first part, aims to minimize the Total Inventory Cost, to do this the formulation of the 'Periodic Review Mixed Integer Problem' (MIP) was used.

The formulation of the objective function is as follows:

$$\text{minimize} \quad \sum_{t=1}^{N} h_t \cdot I_t + \sum_{t=1}^{N} k_t \cdot y_t$$

and, as said before the formula aim is to minimize the total cost given by the holding cost (the first sum part of the formula) plus the reorder cost

In order to understand it better, its elements must be defined:

- ht holding costs (for each item in stock at the end of a period)
- kt reorder costs
- yt is a binary variable, that is 1 if a order is issued at period t and is 0 otherwise
- It is the leftover stock outs at period t and is defined as:

$$I_t = \sum_{j=1}^{t}(x_j - D_j)$$

- xt is a variable that tell the reorder quantity

This formula can be used when we want to focus on economic performance. If the goal is to focus on environmental performance, we have to use the same formulation as before but with some changes in order to achieve the minimum emission.
The only elements that change are:

- hc is substituted with he holding emissions (for each item in stock at the end of a period)
- kc in substituted with ke reorder emissions

The constraints to take in considerations are the same in all the optimizations done:

- Demand must be satisfied for every period.

$$I_t = \sum_{j=1}^{t}(x_j - D_j) \geq 0 \quad \forall\, t \in \{1, \ldots, N\}$$

- Whenever xt > 0, a cost k is incurred. M (a large enough value) allows to link logically the variables yt and xt.

$$x_t \leq M\, y_t \quad \forall\, t \in \{1, \ldots, N\}$$

- Constraint for integrability and to have a reorder quantity equal or major than 0.

For the second part of the homework the formulation used to solve the problem is a new objective function that is able to consider both economic and environmental performance together. We use the MIP formulation for the economic evaluation multiplied for a factor “percentage” between 0 and 1 (that will be used to populate the pareto front) summed with the MIP formulation for the environmental evaluation, multiplied for the complement to one of the factor “percentage”. In this way we obtain a single objective function that can evaluate the two different aspects together. The constraints didn’t change from before.

3. Main code components

At the beginning every library used in the code with the respective abbreviation must be imported and also the solver CPLEX in order to use them later.
The first step is to import the data from the file and define them as parameters:

```
# Parameters

D = [12, 4, 1, 5, 9, 2, 1, 15, 51, 2, 13, 9, 11] # Demand in every period
N = len(D)  # Numeber of period
kc = 200     # Reorder cost
hc = 1  # Holding cost
ke = 100     # Reorder emission
he = 6  # Holding emission

M = 10000
```

We define the variable needed to solve the problem, in this case y is a binary variable that defines if an order is purchased or not, and x is the integer variable that defines the reorder quantity.

```
# Decision variable

y = mdl.binary_var_list(N, name='y')
x = mdl.integer_var_list(N,name='x')
```

The objective function makes use of the classic syntax used for the optimization in python (mdl.minimize for the minimization and for the summation inside the objective function mdl.sum).
We make use of a double summation: the exterior one is looped with a for cycle in the range of all the demands, the second summation is looped over the index of the first one plus a one.

```
# Objective function (Economic evaluation)
mdl.minimize(mdl.sum(hc*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N))
                + mdl.sum(kc*y[t] for t in range(N)))
```

The constraints used are as said in the mathematical model:

(1) Demand must be satisfied for every period
(2) Allows to link logically the variables yt and xt
(3) Reorder quantity equal or major than zero

```
# Constraints (Economic evaluation)

for t in range(N):
    mdl.add_constraint(mdl.sum(x[j]-D[j] for j in range(t+1)) >= 0)

for t in range(N):
    mdl.add_constraint(x[t]<=M*y[t])

for t in range(N):
    mdl.add_constraint(x[t]>=0)
```

Once the optimization is done, we want to evaluate the actual total cost in economic (Tc1 total cost) and total emission (Te1 total emission) terms.
We make use of the same function that we used in the minimization and we plug into it the results with the syntax sol.get_value

```
Tc1 = sum((hc * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))
        )+sum(kc * sol.get_value(y[t]) for t in range(N))

Te1 = sum((he * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))
        )+sum(ke * sol.get_value(y[t]) for t in range(N))
```

To evaluate our model with a focus on the environment performance, we use the same objective function used before with only two changes: the holding cost (hc) now is the holding emission (he) and the reorder cost (kc) now is the reorder emission (ke)

```
# Objective function (Emission evaluation)
mdl.minimize(mdl.sum(he*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N))
            + mdl.sum(ke*y[t] for t in range(N)))
```

Once the optimization is done, we want to evaluate the actual cost in economic (Tc2 total cost) and emission (Te2 total emission) terms.

```
Tc2 = sum((hc * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))
        ) + sum(kc * sol.get_value(y[t]) for t in range(N))

Te2 = sum((he * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))
        ) + sum(ke * sol.get_value(y[t]) for t in range(N))
```

We put the results funded in the two different optimizations in a vector COST_econ for the evaluation from the economic point of view and in COST_emission for environmental evaluation. With this data we can find the economic and environmental improvement if the company switches from an economic to environmental perspective.

```
ImprovECONOMIC = ((COST_emission[0]-COST_econ[0])/(COST_econ[0]))*100

ImprovEINV = ((COST_econ[1]-COST_emission[1])/(COST_econ[1]))*100

print('The costs have increased by :',ImprovECONOMIC,'% and the emission are reduced by',ImprovEINV,'%')
```

For the second part of the homework we can keep our constraints the same, we only need to switch the objective function, considering half part as economic optimization and the other half as environmental optimization.

```
# OBJECTIVE FUNCTION

mdl.minimize(0.5*(mdl.sum(he*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N)) + mdl.sum(ke*y[t] for t in range(N)))+
            (0.5*(mdl.sum(hc*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N)) + mdl.sum(kc*y[t] for t in range(N)))))
```

Like before we find the actual costs for this optimization.

```
Tc3 = sum((hc * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))
          ) + sum(kc * sol.get_value(y[t]) for t in range(N))

Te3 = sum((he * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))
          ) + sum(ke * sol.get_value(y[t]) for t in range(N))
```

Now we can plot the three point:
COST_econ: the anchor point obtained from the economic evaluation
COST_emission : the anchor point obtained from the environmental evaluation
COST_half : the point obtained using half economic and half environmental evaluation

```
# PLOT THE THREE MAIN POINT
plt.legend(title='Legend', bbox_to_anchor=(1.05, 1), loc='upper left')
plt.scatter(COST_half[0],COST_half[1])
plt.scatter(COST_emission[0],COST_emission[1])
plt.scatter(COST_econ[0],COST_econ[1])
plt.xlabel('Te [kg CO2]')
plt.ylabel('Tc [€]')
plt.title('Pareto front D')
plt.show()
```

Now we populate the non-dominated solutions by shifting from the original weight value with a value that varies from 0 (economic anchor point) to 1(environmental anchor point).For every iteration of the " for " cycle, with the value of the variables obtained from the optimization, we find a point with the value of the total (TCO) cost and the total emissions(TEN).

```
TEN = []
TCO = []
cmap_green = get_cmap('Greens')
cmap_red = get_cmap('Reds')
cmap_blue = get_cmap('Blues')


for percentage in range(11):

    mdl.minimize((percentage/10)*(mdl.sum(he*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N)) + mdl.sum(ke*y[t] for t in range(N)))+
                 (round(1-(percentage/10),2)*(mdl.sum(hc*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N)) + mdl.sum(kc*y[t] for t in range(N)))))

    sol = mdl.solve()

    TCO.append([sum((hc * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))) + sum(kc * sol.get_value(y[t]) for t in range(N))])

    TEN.append([sum((he * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))) + sum(ke * sol.get_value(y[t]) for t in range(N))])

    plt.scatter(TEN,TCO)


plt.plot([point[0] for point in TEN], [point[0] for point in TCO], linestyle='-', color=cmap_blue(0.7))
```

At the end we want to change the input factor and re-populate the non-dominated solutions, the idea is to change only one parameter each time we are re-populating the solution with a lower and higher value.

```
kc = 100
kc1 = 300
hc = 1  # Holding cost
ke = 100     # Reorder emission
he = 6  # Holding emission

for percentage in range(11):

    mdl.minimize((percentage/10)*(mdl.sum(he*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N)) + mdl.sum(ke*y[t] for t in range(N)))+
                  (round(1-(percentage/10),2)*(mdl.sum(hc*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N)) + mdl.sum(kc*y[t] for t in range(N)))))

    sol = mdl.solve()

    TC.append([sum((hc * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))) + sum(kc * sol.get_value(y[t]) for t in range(N))])

    TE.append([sum((he * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))) + sum(ke * sol.get_value(y[t]) for t in range(N))])

    plt.scatter(TE,TC)

    mdl.minimize((percentage/10)*(mdl.sum(he*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N)) + mdl.sum(ke*y[t] for t in range(N)))+
                 (round(1-(percentage/10),2)*(mdl.sum(hc*mdl.sum(x[j]-D[j] for j in range(G+1))for G in range(N)) + mdl.sum(kc1*y[t] for t in range(N)))))

    sol = mdl.solve()

    TC2.append([sum((hc * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))) + sum(kc1 * sol.get_value(y[t]) for t in range(N))])

    TE2.append([sum((he * sum(sol.get_value(x[j]) - D[j] for j in range(G + 1)) for G in range(N))) + sum(ke * sol.get_value(y[t]) for t in range(N))])

    plt.scatter(TE2,TC2)
```

4. Results and insights

In the following sections, we will present the results obtained from the different evaluations. At the beginning our focus was on the economic performance of the model, this result in the following combination of decision variable:

```
y_0 = 1
y_7 = 1
x_0 = 34
x_7 = 101
```

We have only two orders, one at the beginning with 34 products and the other in the seventh period of 101 units.

And this results in the minimum cost achievable of **658.0 €**

After the economic evaluation we proceed with the focus from the environmental point of view:

```
y_0 = 1
y_3 = 1
y_7 = 1
y_8 = 1
y_10 = 1
y_12 = 1
x_0 = 17
x_3 = 17
x_7 = 15
x_8 = 53
x_10 = 22
x_12 = 11
```

In this case we have six orders: at the beginning of 17 units, third period of 17 units, seventh period of 15 units, period eight of 53 units, period ten of 22 units and period twelve of 11 units.
The minimum emissions achievable are **798.0 kgCO2.**

If now we consider that the company switched from the economic perspective to the environment one we have the following result:

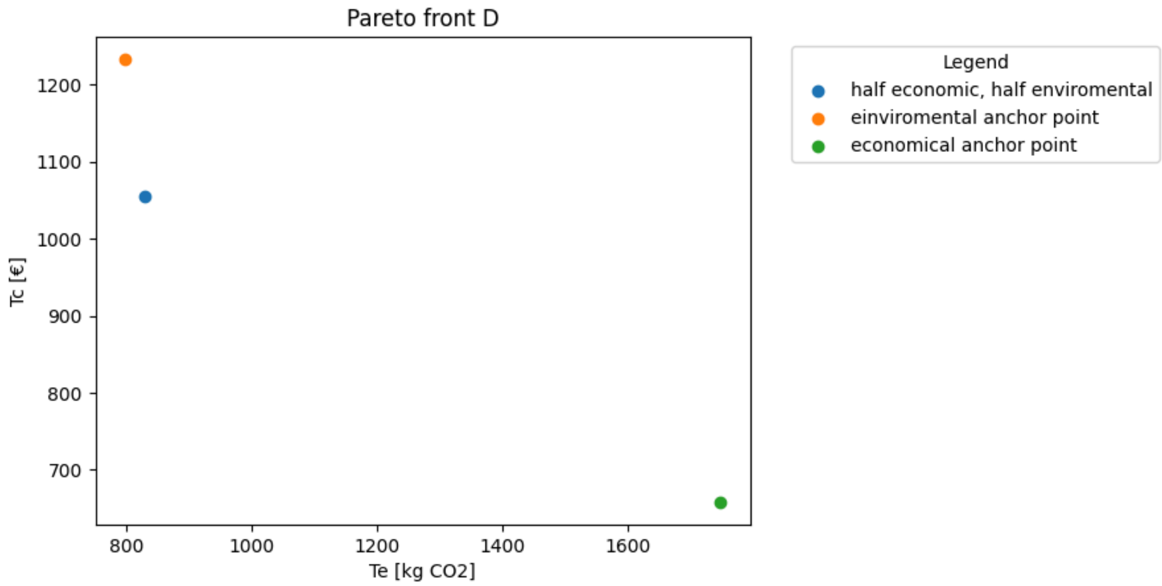
```
The costs have increased by : 87.38601823708207 % and the emission are reduced by 54.347826086956516 %
```

Now we are considering that the decision maker wants to base his decision on a total objective function that includes the environmental perspective as 50% of its total performance the resulting decision variables are:

```
y_0 = 1
y_3 = 1
y_7 = 1
y_8 = 1
y_10 = 1
x_0 = 17
x_3 = 17
x_7 = 15
x_8 = 53
x_10 = 33
```

In this case we have five orders:
at the beginning of 17 units, the second order at the third period of 17 units, at the seventh period an order of 15 units, period eight order of 53 units, the last order at the tenth period of 33 units.

If for each evaluation done at this point we calculate the total cost and the total emission we obtain three point that we can plot:



The three point are:

```
[658.0, 1748.0]
```

- ECONOMIC PERFORMANCE EVALUATION

As we might expect, in this case we have the lowest cost and the highest CO_2 emissions.

`[1233.0, 798.0]`

- ENVIRONMENTAL PERFORMANCE EVALUATION

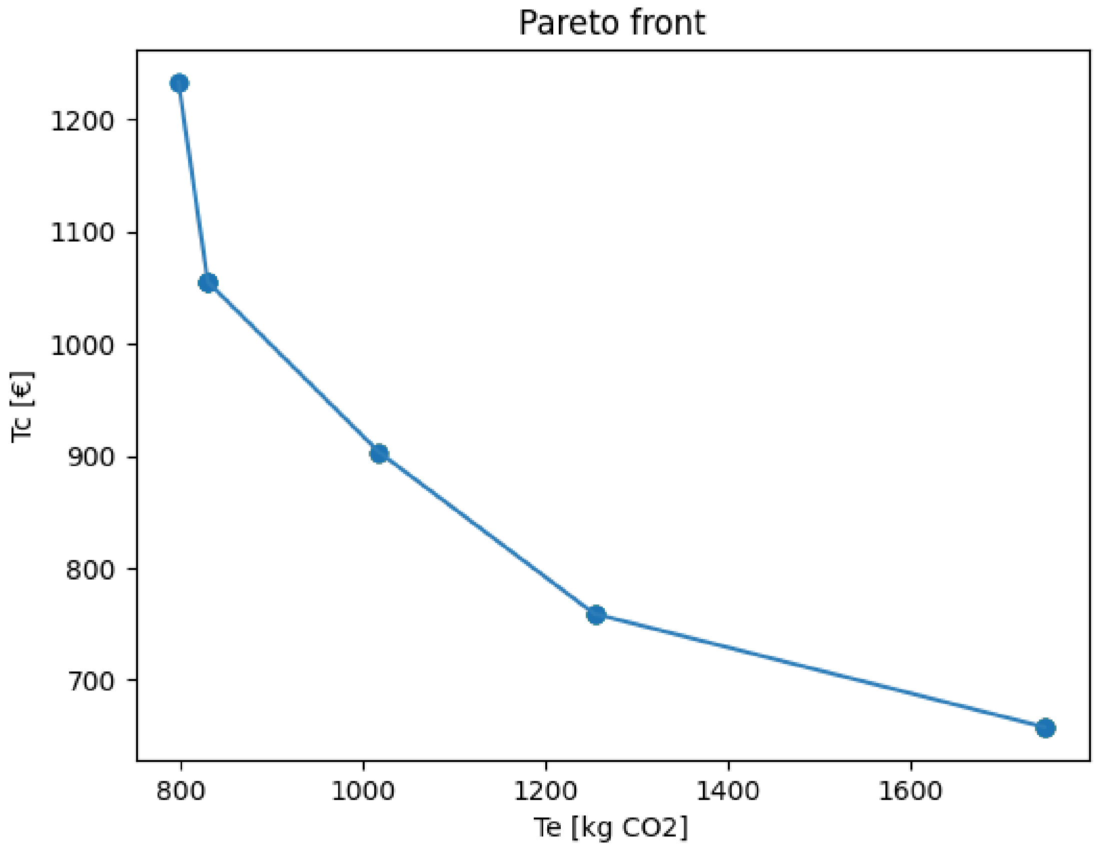
on the contrary in this case we have the lowest emissions and the highest cost

`[1055.0, 830.0]`

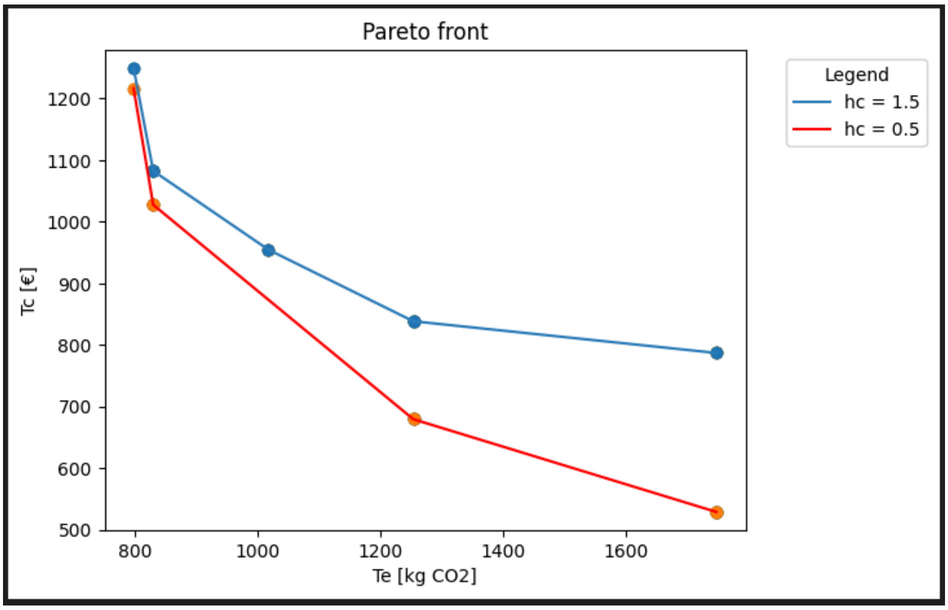
- ENVIRONMENTAL PROSPECTIVE AT 50 %

in this case it is important to notice that only considering the environmental aspects at the 50% we obtain only 32 KgCO2 more than the evaluation totally focused on the environmental performance.

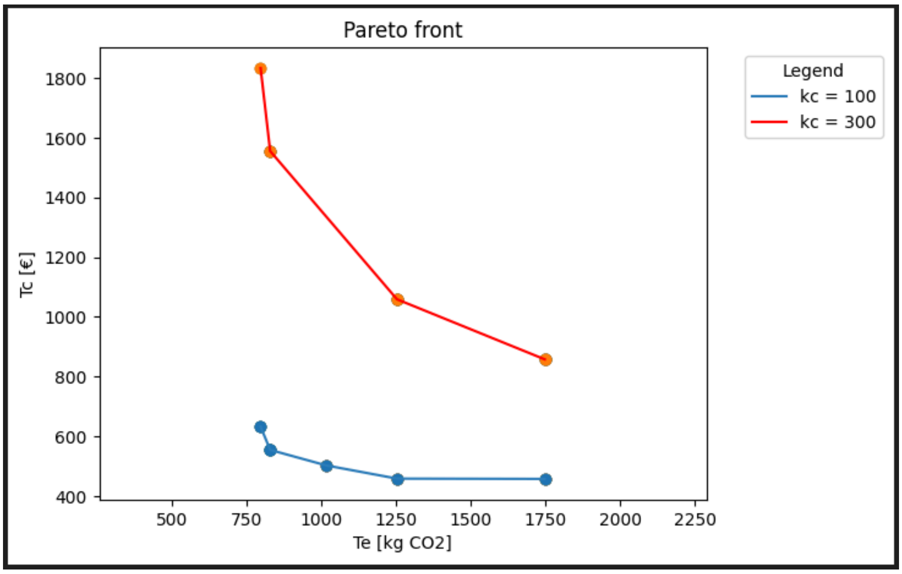
By shifting the weight value that decide the percentage of the aspects of our evaluation we obtain the Pareto front:



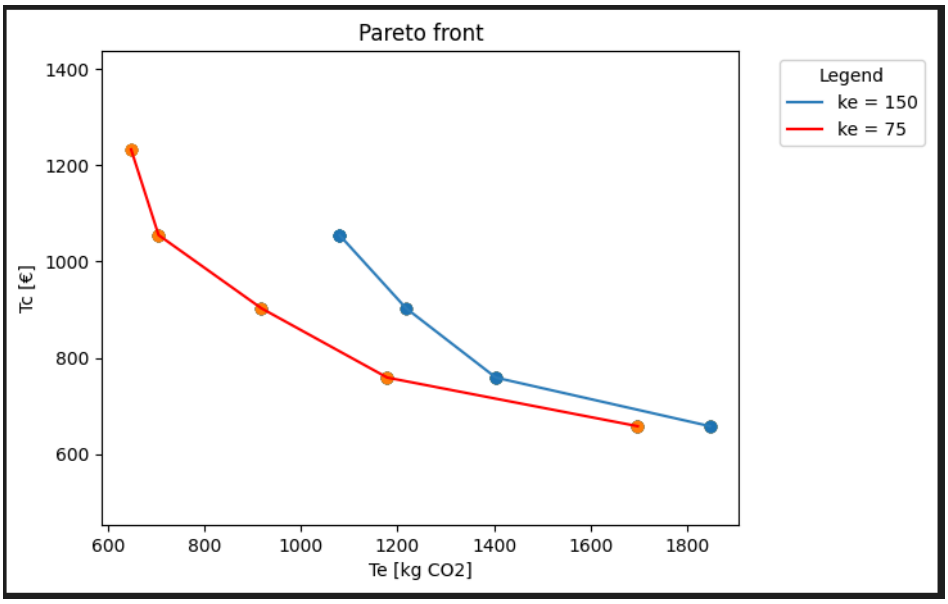
After obtaining a code capable of populating the non-dominated solutions for difference weight value, we decided to change the input factors. To obtain good results, we change the given input factors one at a time with a higher and a lower value.



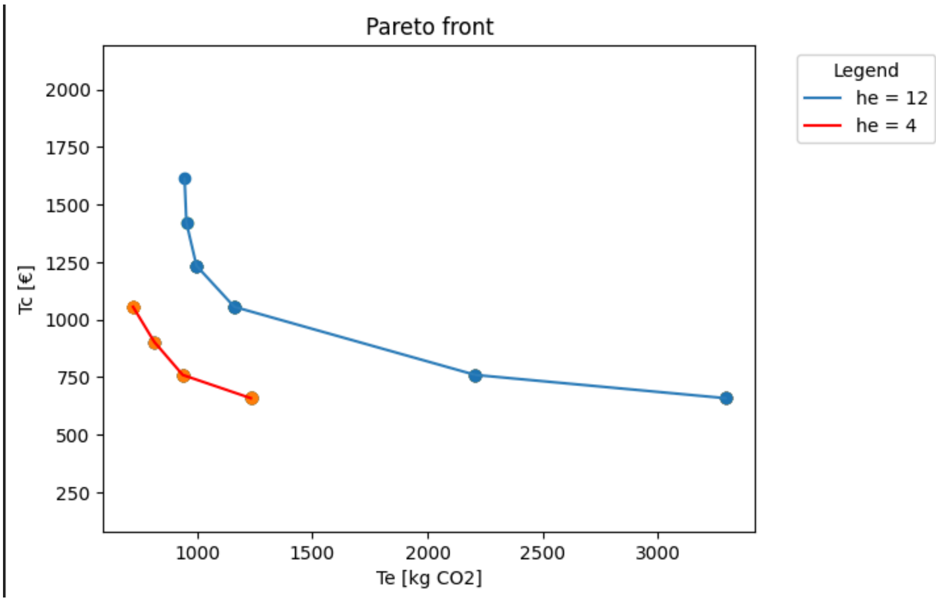
Changing the holding costs involves that in the case of a bigger holding cost we obtain the same amount of emission but, if we look at the full economic evaluation we obtain higher cost and in the case of lower holding cost we obtain lower cost, on the other hand if we look at the full environmental evaluation we obtain almost the same result, in terms of cost and emissions, as having the original holding cost. This change affects only the optimization with a weight value near the full economic optimization.



Changing the reorder cost consists in a considerable increase of cost, especially for what concerns the complete environmental evaluation. This result does not surprise us, for the fact that evaluating from the environmental point of view results in higher number of reorders, having higher cost for each reorder results in higher total cost. For this reason the difference between the two Pareto fronts is very high when we are considering only the environmental performance. If we are considering the economic performance we have an increase/decrease of the cost coherent with the new value of kc.



If now we change the reorder emission as we can expect, we can notice a big difference in what the emissions are when we are evaluating the environmental performance, this is a consequence of, as said before, having more order. The costs are almost not influenced by the increase or decrease of ke.



The last input factor that we change is the holding emission. Changing this value results in a considerable change of total emissions. A higher value of he, if we are considering the economic performance, consist in a total emission of almost 3500 KgCO2 and if we are also considering the environmental performance the total emissions is over 1000 KgCO2. On the other hand if we have very low holding emission the amount of emission evaluating the economic performance didn't go above the 1300Kg CO2. Total costs are also affected negatively, when we have higher holding emission, the total cost with the environmental evaluation is over 1500€ but when we are evaluating the economic performance, the total cost didn't vary from the one with original input value. When we are considering the lower holding emission the total cost is reduced drastically compared to the higher holding emissions.