Course Mechatronics System Modelling

Spring-loaded mechanical system-Homework 2

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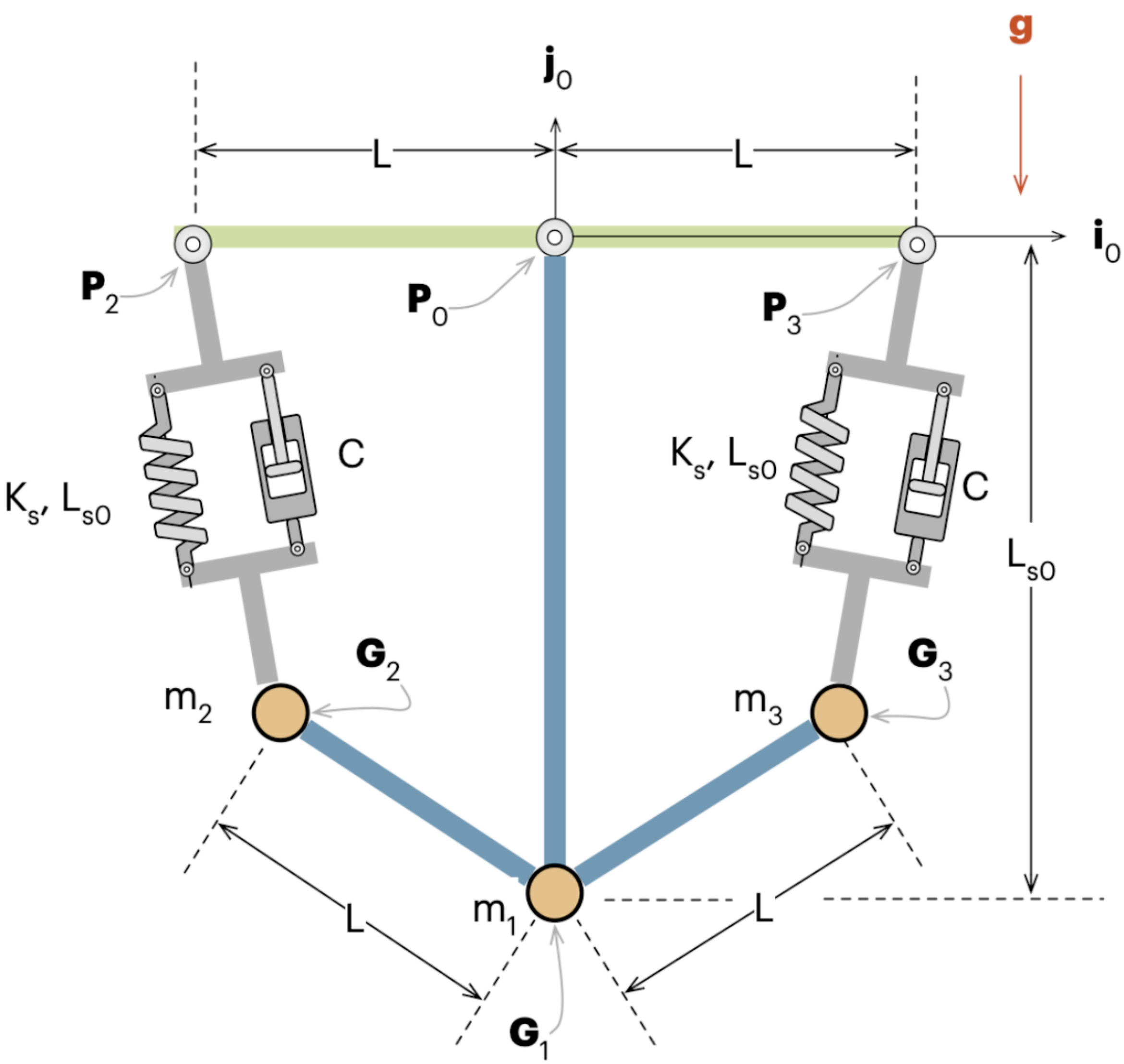


The mechanical system in the picture is made by three masses which are connected by two bars of the same lenght L.
The masses m2 and m3 are connected to ground by the spring-damper system with spring having natural length L__s0 and stiffeness and damping cooefficients respectivelly K__s2, K__s3

and C__s2 and C__s3.

Initialization

```
> restart:
  with(plots):
  with(LinearAlgebra):
  with(GraphTheory):

  interface(rtablesize=20):
  plots[setcolors]("spring"):
  c_set := ColorTools:-GetPalette("spring");

  FONT_TYPE     := "Helvetica":
  FONT_SIZE_LAB :=  14:
  FONT_SIZE_TIT :=  18:
  plots[setoptions](axes            = boxed,
                    size            = [800,200],
                    axis[2]         = [gridlines = [linestyle=dot]
  ],
                    legendstyle     = [location="top",font=
  [FONT_TYPE,FONT_SIZE_LAB]],
                    labelfont       = [FONT_TYPE,FONT_SIZE_LAB],
                    axesfont        = [FONT_TYPE,FONT_SIZE_LAB],
                    titlefont       = [FONT_TYPE,bold,
  FONT_SIZE_TIT],
                    captionfont     = [FONT_TYPE,italic,
  FONT_SIZE_TIT],
                    labeldirections = [horizontal , vertical]
  );
```

$c\_set :=$ ⟨Palette Spring: Blue Rose YellowGreen BlueGreen Violet Cobalt Yellow PurpleRed GreenBlue PaleGreen Orange Purple Green SeaBlue PaleYellow PaleBlueGreen⟩ **(1.1)**

```
> #alias(C=cos);
  #alias(S=sin);
```

List of MBD procedures

Primary rotation matrices and kinematic procedures

Primary axes rotation matrices

Rotation matrix around x axis

```
> Rx := theta -> <<1,0,0>|<0,cos(theta),sin(theta)>|<0,-sin
  (theta),cos(theta)>>:
  Rx(alpha);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{bmatrix} \quad \textbf{(2.1.1)}$$

Rotation matrix around y-axis

```
> Ry := theta -> <<cos(theta),0,-sin(theta)>|<0,1,0>|<sin
  (theta),0,cos(theta)>>:
  Ry(beta);
```

$$\begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \quad \textbf{(2.1.2)}$$

Rotation matrix around the z-axis

```
> Rz := theta -> <<cos(theta),sin(theta),0>|<-sin(theta),cos
  (theta),0>|<0,0,1>>:
  Rz(gamma);
```

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \textbf{(2.1.3)}$$

Transformation matrix

Create the transformation matrix given the rotation matrix and tre origin of the frame

```
> TM := proc(R::Matrix(3,3),P::{Vector(3),list},$)
    local tmp;
    if type(P,'list') then
      if nops(P) <> 3 then
        error "Error: expected dimension for %1 is 3", P;
      end ;
    end;
    tmp := <R|convert(P,Vector)>;
    <tmp,Transpose(<0,0,0,1>)>;

  end:
```

```
> TM(Rx(alpha),[x,y,z]),
  TM(Rx(alpha),<x,y,z>);
```

$$\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & \cos(\alpha) & -\sin(\alpha) & y \\ 0 & \sin(\alpha) & \cos(\alpha) & z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & \cos(\alpha) & -\sin(\alpha) & y \\ 0 & \sin(\alpha) & \cos(\alpha) & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(2.1.4)}$$

Compute the inverse of the Transformation Matrix

```
> invTM := proc(TM::Matrix(4,4),$)
    local R,P,tmp;

    R := TM[1..3,1..3];
    P := TM[1..3,4];
    tmp := <Transpose(R)|-R.P>;
    <tmp,Transpose(<0,0,0,1>)>;

  end proc:
```

Compute derivative of vector or of a list w.r.t. to a symbol or a function

```
> diffF := proc(v::{algebraic,list,Vector,Matrix},x::{symbol,
  function},$)
     local ss,xx, tmp;
     if type(x,function) then
       ss := x = xx ;
       tmp := subs( ss, v):
       subs( xx = x, map(e->diff(e,xx),tmp));
     else
       map(e->diff(e,x),v);
     end;

  end proc:
```

```
> diffF([x(t)^2+1],x(t))
```

$$[2\,x(t)] \tag{2.1.5}$$

```
> jacobianF := proc(L::{list,Vector},x::list,$)
    local i,j;
    convert([seq([seq(diffF(L[i],x[j]),j=1..nops(x))],i=1..nops
  (L))],Matrix);

  end proc:
> velocity := proc(v::{Vector},$)

    diffF(v,t);

  end:
```

Extract components of a vector

```
> comps_XYZ := proc(v::{Vector(3),list(3)})

    local i;

    seq(v[i],i=1..3);
```

```
  end proc:

  comps_XY := proc(v::{Vector(3),list(3)})

    local i;

    seq(v[i],i=1..2);


  end proc:
```

Angular velocity

```
> angularVelocity := proc(R::Matrix(3,3),$)
    local skew_ang,ang_vel;

    skew_ang := simplify(Transpose(R).map(diff,R,t)):
    ang_vel := <skew_ang[3,2],skew_ang[1,3],skew_ang[2,1]>;
    # skew_ang,ang_vel;
    ang_vel;

  end:
```

Skew matrix of rotation axis

```
> SkewMat := proc(v::{Vector,list},$)

  local vv,f;

  f := (x,y,z)-><<0,z,-y>|<-z,0,x>|<y,-x,0>>;

   f(v[1],v[2],v[3]);

  end:
> Skew := SkewMat(<n__x,n__y,n__z>);
```

$$Skew := \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix} \tag{2.1.6}$$

```
> angularVelocity(Rz(theta(t)).Rx(alpha(t)))
```

$$\begin{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t}\,\alpha(t) \\ \left(\frac{\mathrm{d}}{\mathrm{d}t}\,\theta(t)\right)\sin(\alpha(t)) \\ \left(\frac{\mathrm{d}}{\mathrm{d}t}\,\theta(t)\right)\cos(\alpha(t)) \end{bmatrix} \tag{2.1.7}$$

Cross product

```
> crossProduct := proc(v1::{Vector,list},v2::{Vector,list},$)

    SkewMat(v1).convert(v2,Vector);

  end;
```

crossProduct := **proc**(*v1*::{*Vector*, *list*}, *v2*::{*Vector*, *list*}, $) **(2.1.8)**

`.`(*SkewMat*(*v1*), *convert*(*v2*, *Vector*))

end proc

Dot product

```
> dotProduct := proc(v1::{Vector},v2::{Vector},$)

    Transpose(v1).v2;

  end;
```

dotProduct := **proc**(*v1*::{*Vector*}, *v2*::{*Vector*}, $) **(2.1.9)**

`.`(*LinearAlgebra*:-*Transpose*(*v1*), *v2*)

end proc

Procedures to visualise 3D objects

Procedure to display a frame as three cylindrical arrows

```
> draw_vec := proc(p::Vector(3),v::Vector(3),sf::scalar:=1,
  col::string:="DeepPink",$)

     #local ;
     #
     display(plots:-arrow(p,sf*v,color=col, shape=
  cylindrical_arrow),
               scaling=constrained);


  end:
```

Procedure to display a frame as three cylindrical arrows

```
> draw_frame := proc(R::Matrix(4,4),sf::scalar:=1,$)

     local p0,i1, j1, k1;
     # extract unit vectors from rotatin matrix

     p0 := convert(R[1..-2,4],list);
     i1 := R[1..-2,1];
     j1 := R[1..-2,2];
     k1 := R[1..-2,3];

     display(plots:-arrow(p0,sf*i1,color="Red",      shape=
```

```
cylindrical_arrow),
           plots:-arrow(p0,sf*j1,color="LimeGreen",shape=
cylindrical_arrow),
           plots:-arrow(p0,sf*k1,color="Blue",     shape=
cylindrical_arrow),
           plottools:-sphere(p0,0.05*sf,color="Goldenrod"),
           scaling=constrained);


end:
```

Procedure to rotate a body given a rotation matrix

```
> rotate_translate_body := proc(obj, R::Matrix(4,4),sf::scalar:=
1,$)

   local f, ff, FF,obj_new,FONT_TYPE,FONT_SIZE_LAB,
FONT_SIZE_TIT;

   FONT_TYPE := "Helvetica"; FONT_SIZE_LAB := 16;
FONT_SIZE_TIT := 18;
   # ccompute the projection of the coordinates of a point <x,
y,z>,
   # or rotate the coordinates of a point of a body
   FF := evalf(convert(R.<x,y,z,1>,list)):
   # create a function to apply rotation to a point <x,y,z>
   ff := unapply(FF[1..3],[x,y,z]):
   # create a transformation rule to apply to points of an
Maple graphical object
   f:= plottools[transform]((x,y,z)->ff(x,y,z)):

   obj_new := f(obj); #apply transformation to graphical
object

   # show the object
   display(obj_new,scaling=constrained,
           labelfont       = [FONT_TYPE,FONT_SIZE_LAB],
           axesfont        = [FONT_TYPE,FONT_SIZE_LAB],
           titlefont       = [FONT_TYPE,bold,FONT_SIZE_TIT]);



end:
```

Lagrange Equations

Kinetic energy

Input:

- G is the position of CoM in absolute reference frame

- T is the body rotation matrix of reference frame
- Ix, Iy, etc: are inertia tensor element in body fixed frame

```
> kineticEnergy := proc(m::scalar, G::Vector(3), T::Matrix(3,3),
  Ix::scalar,Iy::scalar,Iz::scalar, Ixy::scalar,Ixz::scalar,
  Iyz::scalar, $)
     local  VG,KET, gx,gy,gz,KER, Itensor, omega;
     Itensor := <<Ix,-Ixy,-Ixz>|<-Ixy,Iy,-Iyz>|<-Ixz,-Iyz,Iz>>;
  print(%);
     # kinetic energy due to translation motion - - - - -
     VG  := velocity(G);
     KET := 1/2*m*dotProduct(VG,VG): #when VG is linear, KET is
  quadratic
     #kinetic energy due to rotation motion - - - - - - -
     omega := angularVelocity(T);  #angular velocity with
  respect to the moving frame
     KER := 1/2 *Transpose(omega).Itensor.omega;
     if has(KER,{sin,cos}) then KER := simplify(KER,trig): end
  if:
     # total energy - - - - - - - - - - - - - - - - - - -
     KET + KER;
  end proc:
```

Potential energy

Compute the gravitational energy of a multibody system by evaluating the force field associated to the <_gravity> acceleration. "

```
> gravitationalEnergy := proc(m::scalar, G::Vector(3),
  gravityVec::Vector(3), $)
     local  VG,KET, gx,gy,gz,KER, Itensor, omega;
     -m*dotProduct(G,gravityVec);
     end proc:
```

Generalised forces and torques

```
> generalisedTorques := proc(R::Matrix(3,3),T::Vector(3),
  q::list,$)

     local i, Q_gen, dRdq,dOmega,  Tb;

      Q_gen := Vector(1..nops(q),[]);
      Tb := Transpose(R).T; # torque in moving frame

      for i from 1 to nops(q) do
        dRdq     := simplify(Transpose(R).diffF(R,q[i])):
```

```
        dOmega   := <dRdq[3,2],dRdq[1,3],dRdq[2,1]>:
        Q_gen[i] := dotProduct(Tb,dOmega);
      end do:

      Q_gen;

  end proc:
```

```
> (*
  RR := Rx(theta__x(t)).Ry(theta__y(t)).Rz(theta__z(t));

  # virtual rotation matrix in moving frame
  simplify(Transpose(RR).diffF(RR,theta__y(t)));[%[3,2],%[1,3],%
  [2,1]];

  # virtual rotation matrix
  simplify(Transpose(RR).diffF(RR,t),trig);
  AA,bb := GenerateMatrix([%[1,3],%[1,2],%[2,1]],diff([theta__x
  (t),theta__y(t),theta__z(t)],t));

  # Virtual displacement w.r.t. theta__y:


  AA.<0,1,0>;
  AA.<0,1,0>;
  *)
```

```
> generalisedForces := proc(P::Vector(3),F::Vector(3), q::list,
  $)

     local i, Q_gen, dPdq;

     Q_gen := Vector(1..nops(q),[]);

      for i from 1 to nops(q) do
        dPdq     := diffF(P,q[i]):
        Q_gen[i] := dotProduct(F,dPdq);
      end do:
      Q_gen;

  end proc:
```

```
> #generalisedForces(<x(t),y(t),0>+Rz(theta(t)).<Lx,Ly,0>,<Fx,
  Fy,0>,[x(t),y(t),theta(t)]);
```

```
> #generalisedTorques(Rz(theta(t)),<Tx,Ty,Tz>,[x(t),y(t),theta
  (t)]);
```

Lagrange equations

```
> lagrangeEquations := proc(L::scalar,Phi::list(algebraic),
  q::list(function))
     #description "Lagrange equations d/dt(dL/dqdot)- dL/dq":
```

```
    local qdot, Leqns,i, JqL:
    # - Qforce[i];

    if nops(Phi) <> 0 then
      JqL := Transpose(jacobianF(Phi,q)).<seq(lambda__||i(t),i=
 1..nops(Phi))>;
    else
     JqL := Vector(1..nops(q),fill=0);
     end;

    Leqns := Vector(1..nops(q),[]);

     for i from 1 to nops(q) do
       qdot := diff(q[i],t):
       Leqns[i]:= diff ( diffF(L,qdot), t) - diffF(L,q[i]);
     end do:
     Leqns-JqL;
 end:
```

```
> KE := kineticEnergy(m, <x(t),y(t),z(t)>,Rz(theta(t)),Ix,Iy,Iz,
  Ixy,Ixz,Iyz);
  PE := gravitationalEnergy(m, <x(t),y(t),z(t)>,<0,-g,0>);
```

$$\begin{bmatrix} Ix & -Ixy & -Ixz \\ -Ixy & Iy & -Iyz \\ -Ixz & -Iyz & Iz \end{bmatrix}$$

$$KE := \frac{m\left(\left(\frac{d}{dt}\,x(t)\right)^2+\left(\frac{d}{dt}\,y(t)\right)^2+\left(\frac{d}{dt}\,z(t)\right)^2\right)}{2}+\frac{\left(\frac{d}{dt}\,\theta(t)\right)^2 Iz}{2}$$

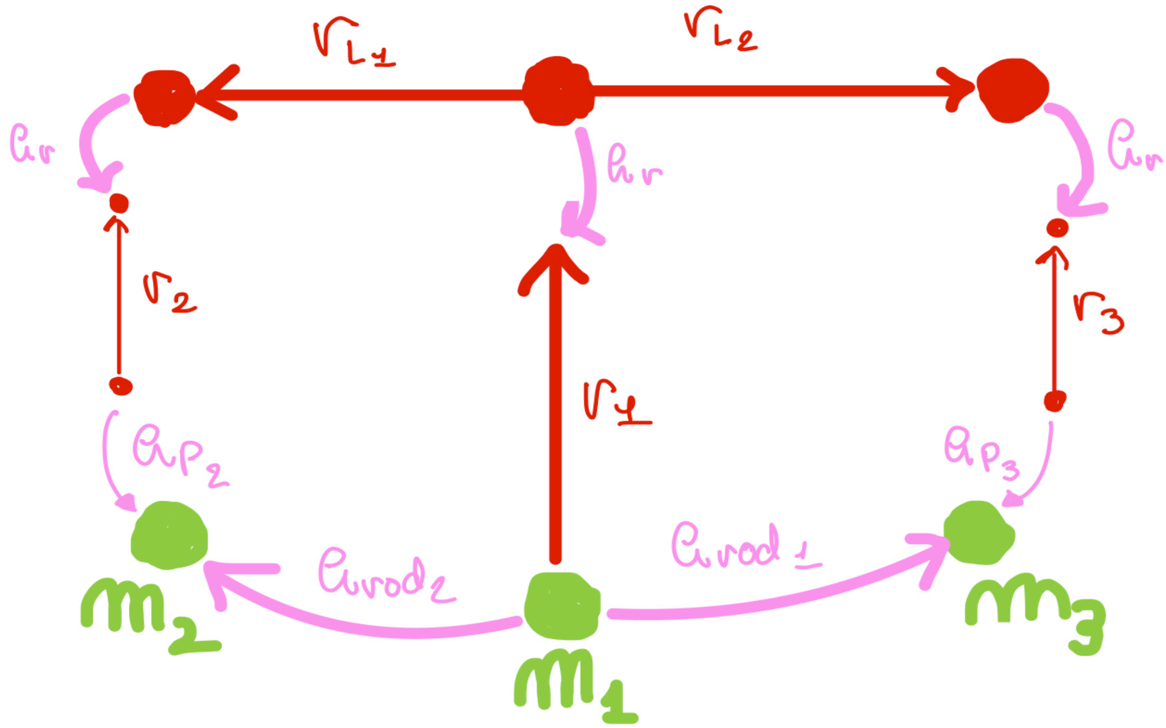
$$PE := m\,y(t)\,g \tag{2.3.1}$$

```
> lagrangeEquations(KE-PE,[],[x(t),y(t),z(t),theta(t)])
```

$$\begin{bmatrix} m\left(\frac{d^2}{dt^2}\,x(t)\right) \\ m\left(\frac{d^2}{dt^2}\,y(t)\right)+m\,g \\ m\left(\frac{d^2}{dt^2}\,z(t)\right) \\ \left(\frac{d^2}{dt^2}\,\theta(t)\right) Iz \end{bmatrix} \tag{2.3.2}$$

Kinematics

Linear Graph



Revolute joint

```
> hR := <0,0,0>;
```

$$hR := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.1}$$

Prismatic joint element

```
> hp := s(t)*<1,0,0>;
```

$$hp := \begin{bmatrix} s(t) \\ 0 \\ 0 \end{bmatrix} \tag{3.2}$$

Arm link elements fixed

```
> rL__1 := <-L,0,0>;
  rL__2 := <L,0,0>;
```

$$rL_1 := \begin{bmatrix} -L \\ 0 \\ 0 \end{bmatrix}$$

(3.3)

$$rL_2 := \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} \quad (3.3)$$

Arm link element

```
> r3 := Rz(theta__3(t)).<-L__s0,0,0>;
```

$$r3 := \begin{bmatrix} -\cos(\theta_3(t))\, L_{s0} \\ -\sin(\theta_3(t))\, L_{s0} \\ 0 \end{bmatrix} \quad (3.4)$$

Prismatic joint element

```
> hp3 := Rz(theta__3(t)).(s__3(t)*<1,0,0>);
```

$$hp3 := \begin{bmatrix} \cos(\theta_3(t))\, s_3(t) \\ \sin(\theta_3(t))\, s_3(t) \\ 0 \end{bmatrix} \quad (3.5)$$

Arm link element

```
> r1 := Rz(theta__1(t)).<-L__s0,0,0>;
```

$$r1 := \begin{bmatrix} -\cos(\theta_1(t))\, L_{s0} \\ -\sin(\theta_1(t))\, L_{s0} \\ 0 \end{bmatrix} \quad (3.6)$$

Arm link element

```
> r2 := Rz(theta__2(t)).<-L__s0,0,0>;
```

$$r2 := \begin{bmatrix} -\cos(\theta_2(t))\, L_{s0} \\ -\sin(\theta_2(t))\, L_{s0} \\ 0 \end{bmatrix} \quad (3.7)$$

Prismatic joint element

```
> hp2 := Rz(theta__2(t)).(s__2(t)*<1,0,0>);
```

$$hp2 := \begin{bmatrix} \cos(\theta_2(t))\, s_2(t) \\ \sin(\theta_2(t))\, s_2(t) \\ 0 \end{bmatrix} \quad (3.8)$$

Rotation Matrix 1

```
> Rs__1 := Rz(theta__1(t));
  is_uvec__1 := %[1..3,1]:
```

```
js_uvec__1 := %%[1..3,2]:

is_uvec__1,js_uvec__1;
```

$$Rs_1 := \begin{bmatrix} \cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0 \\ \sin(\theta_1(t)) & \cos(\theta_1(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta_1(t)) \\ \sin(\theta_1(t)) \\ 0 \end{bmatrix}, \begin{bmatrix} -\sin(\theta_1(t)) \\ \cos(\theta_1(t)) \\ 0 \end{bmatrix} \quad (3.9)$$

Rotation Matrix 2

```
> Rs__2 := Rz(theta__2(t));
  is_uvec__2 := %[1..3,1]:
  js_uvec__2 := %%[1..3,2]:

  is_uvec__2,js_uvec__2;
  rs__2 := Rs.<L__s+s__2(t),0,0>:
```

$$Rs_2 := \begin{bmatrix} \cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0 \\ \sin(\theta_2(t)) & \cos(\theta_2(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta_2(t)) \\ \sin(\theta_2(t)) \\ 0 \end{bmatrix}, \begin{bmatrix} -\sin(\theta_2(t)) \\ \cos(\theta_2(t)) \\ 0 \end{bmatrix} \quad (3.10)$$

Rotation Matrix 3

```
> Rs__3 := Rz(theta__3(t));
  is_uvec__3 := %[1..3,1]:
  js_uvec__3 := %%[1..3,2]:

  is_uvec__3,js_uvec__3;
  rs__3 := Rs.<L__s+s__3(t),0,0>:
```

$$Rs_3 := \begin{bmatrix} \cos(\theta_3(t)) & -\sin(\theta_3(t)) & 0 \\ \sin(\theta_3(t)) & \cos(\theta_3(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(3.11)

$$\left[\begin{array}{c}\cos(\theta_3(t))\\ \sin(\theta_3(t))\\ 0\end{array}\right], \left[\begin{array}{c}-\sin(\theta_3(t))\\ \cos(\theta_3(t))\\ 0\end{array}\right] \tag{3.11}$$

There are two circuits (2 rod joint and spring element) to derive the constraint equations

Rod element kinematic and constraint

```
> H__rod1 := -(rL__2-r3+hp3+r1);
  Phi1 := simplify( Transpose(H__rod1).H__rod1-L^2 );
```

$$H_{rod1} := \left[\begin{array}{c}-L - \cos(\theta_3(t))\, L_{s0} - \cos(\theta_3(t))\, s_3(t) + \cos(\theta_1(t))\, L_{s0}\\ -\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t) + \sin(\theta_1(t))\, L_{s0}\\ 0\end{array}\right]$$

$$\Phi 1 := \left(-L - \cos(\theta_3(t))\, L_{s0} - \cos(\theta_3(t))\, s_3(t) + \cos(\theta_1(t))\, L_{s0}\right)^2 + \left(-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t) + \sin(\theta_1(t))\, L_{s0}\right)^2 - L^2 \tag{3.12}$$

```
> H__rod2 := rL__1+r1+hp2-r2;
  Phi2 := simplify( Transpose(H__rod2).H__rod2-L^2 );
```

$$H_{rod2} := \left[\begin{array}{c}-L - \cos(\theta_1(t))\, L_{s0} + \cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0}\\ -\sin(\theta_1(t))\, L_{s0} + \sin(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, L_{s0}\\ 0\end{array}\right]$$

$$\Phi 2 := \left(-L - \cos(\theta_1(t))\, L_{s0} + \cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0}\right)^2 + \left(-\sin(\theta_1(t))\, L_{s0} + \sin(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, L_{s0}\right)^2 - L^2 \tag{3.13}$$

Overall kinematic elements

```
> rL__1,rL__2,r1,r2,r3,hp2,hp3;
```

$$\left[\begin{array}{c}-L\\ 0\\ 0\end{array}\right], \left[\begin{array}{c}L\\ 0\\ 0\end{array}\right], \left[\begin{array}{c}-\cos(\theta_1(t))\, L_{s0}\\ -\sin(\theta_1(t))\, L_{s0}\\ 0\end{array}\right], \left[\begin{array}{c}-\cos(\theta_2(t))\, L_{s0}\\ -\sin(\theta_2(t))\, L_{s0}\\ 0\end{array}\right], \left[\begin{array}{c}-\cos(\theta_3(t))\, L_{s0}\\ -\sin(\theta_3(t))\, L_{s0}\\ 0\end{array}\right], \left[\begin{array}{c}\cos(\theta_2(t))\, s_2(t)\\ \sin(\theta_2(t))\, s_2(t)\\ 0\end{array}\right], \left[\begin{array}{c}\cos(\theta_3(t))\, s_3(t)\\ \sin(\theta_3(t))\, s_3(t)\\ 0\end{array}\right] \tag{3.14}$$

Position and velocity analysis

The mechanical system there has 5 generalized coordinates and

3 DOF. In this analysis theta2 and theta3 are chosen as dipendent variables while s2,s3 and theta1 are the independent.

```
> Phi     := [Phi1,Phi2]: <%>;
  q_vars  := [s__2(t),theta__1(t),s__3(t),theta__2(t),theta__3
  (t)];
  qI_vars := [theta__1(t),s__2(t),s__3(t)];
  qD_vars := [theta__2(t),theta__3(t)];
```

$$\begin{bmatrix} \left(-L-\cos\left(\theta_3(t)\right) L_{s0}-\cos\left(\theta_3(t)\right) s_3(t)+\cos\left(\theta_1(t)\right) L_{s0}\right)^2+\left(\cdots\right. \\ \left(-L-\cos\left(\theta_1(t)\right) L_{s0}+\cos\left(\theta_2(t)\right) s_2(t)+\cos\left(\theta_2(t)\right) L_{s0}\right)^2+\left(\cdots\right. \end{bmatrix}$$

$$q\_vars := \left[s_2(t), \theta_1(t), s_3(t), \theta_2(t), \theta_3(t)\right]$$

$$qI\_vars := \left[\theta_1(t), s_2(t), s_3(t)\right]$$

$$qD\_vars := \left[\theta_2(t), \theta_3(t)\right] \qquad \textbf{(3.1.1)}$$

Position analysis: analytical

In this case the solution can be found analitically. The two solutions found correspond to the two mechanical system configurations.

```
> sol_kine_all := solve(Phi,qD_vars,explicit=true):
  nops(%);

  <sol_kine_all[1]>, <sol_kine_all[2]>;
```

$$4$$

$$\begin{bmatrix} \theta_2(t)=\arctan\left(\dfrac{4\,L\,L_{s0}\cos\left(\theta_1(t)\right)-\dfrac{2\,L_{s0}\left(2\,L\,L_{s0}^{\,2}\cos\left(\theta_1(t)\right)^2\right.}{}\cdots}{}\right. \\ \theta_3(t)=\arctan\left(-\dfrac{4\,L\,L_{s0}\cos\left(\theta_1(t)\right)+\dfrac{2\,L_{s0}\left(-2\,L\,L_{s0}^{\,2}\cos\left(\theta_1(t\right.\right.}{}\cdots}{}\right. \end{bmatrix}, \qquad \textbf{(3.1.2)}$$

$$\left[\begin{array}{l} \theta_2(t) = \arctan\left(\dfrac{4\,L\,L_{s0}\cos\left(\theta_1(t)\right) - \dfrac{2\,L_{s0}\left(2\,L\,L_{s0}^{\ 2}\cos\left(\theta_1(t)\right)^2\right.}{}}{}\right. \cdots \\ \theta_3(t) = \arctan\left(-\dfrac{4\,L\,L_{s0}\cos\left(\theta_1(t)\right) + \dfrac{2\,L_{s0}\left(-2\,L\,L_{s0}^{\ 2}\cos\left(\theta_1(t\right.\right.}{}}{}\right. \cdots \end{array}\right]$$

Compute velocity ratios:

Here we compute velocity ratios that are necessary for the virtual displacements

```
> JPhi_qD := jacobianF(Phi,qD_vars);
  JPhi_qI := jacobianF(Phi,qI_vars);

  taus := simplify( -MatrixInverse(JPhi_qD).JPhi_qI );
```

Velocities of dependent coordinates

```
  taus.<diff(qI_vars,t)>;
  vel_qD_vars := [seq(diff(qD_vars[i],t)=%[i],i=1..nops(qD_vars)
  )]:
```

$JPhi\_qD :=$

$$\left[\begin{array}{l} \cdots \\ 2\left(-L - \cos\left(\theta_1(t)\right)L_{s0} + \cos\left(\theta_2(t)\right)s_2(t) + \cos\left(\theta_2(t)\right)L_{s0}\right)\left(- \cdots\right. \end{array}\right]$$

$$JPhi\_qI := \left[\begin{array}{l} -2\left(-L - \cos\left(\theta_3(t)\right)L_{s0} - \cos\left(\theta_3(t)\right)s_3(t) + \cos\left(\theta_1(t)\right)L_{s0}\right) \cdots \\ 2\left(-L - \cos\left(\theta_1(t)\right)L_{s0} + \cos\left(\theta_2(t)\right)s_2(t) + \cos\left(\theta_2(t)\right)L_{s0}\right)\mathrm{si} \cdots \end{array}\right]$$

$$taus := \left[\begin{array}{l} \dfrac{\left(\left(\left(-L_{s0} - s_2(t)\right)\cos\left(\theta_2(t)\right) + L\right)\sin\left(\theta_1(t)\right) + \sin\left(\theta_2(t)\right)\cos\left(\right.\right.}{\left(L_{s0} + s_2(t)\right)\left(\left(\cos\left(\theta_1(t)\right)L_{s0} + L\right)\sin\left(\theta_2(t)\right) - L_{s0}\cos\left(\right.\right.} \cdots \\ \dfrac{\left(\left(\left(L_{s0} + s_3(t)\right)\cos\left(\theta_3(t)\right) + L\right)\sin\left(\theta_1(t)\right) - \sin\left(\theta_3(t)\right)\cos\left(\right.\right.}{\left(\left(-\cos\left(\theta_1(t)\right)L_{s0} + L\right)\sin\left(\theta_3(t)\right) + L_{s0}\cos\left(\theta_3(t)\right)\sin\left(\theta\right.\right.} \cdots \end{array}\right]$$

$$\left[\begin{array}{l} \dfrac{\left(\left(\left(-L_{s0} - s_2(t)\right)\cos\left(\theta_2(t)\right) + L\right)\sin\left(\theta_1(t)\right) + \sin\left(\theta_2(t)\right)\cos\right.}{\left(L_{s0} + s_2(t)\right)\left(\left(\cos\left(\theta_1(t)\right)L_{s0} + L\right)\sin\left(\theta_2(t)\right) - L\right.} \cdots \\ \dfrac{\left(\left(\left(L_{s0} + s_3(t)\right)\cos\left(\theta_3(t)\right) + L\right)\sin\left(\theta_1(t)\right) - \sin\left(\theta_3(t)\right)\cos\left(\theta\right.\right.}{\left(\left(-\cos\left(\theta_1(t)\right)L_{s0} + L\right)\sin\left(\theta_3(t)\right) + L_{s0}\cos\left(\theta_3(t)\right)\right.} \cdots \end{array}\right] \qquad \textbf{(3.1.3)}$$

Kinematics of points where weights, external forces are applied

```
> G1 := -r1:
  G2 := rL__1-r2+hp2:
  G3 := rL__2-r3+hp3:

  P0 := hR:
  P2 := rL__1:
  P3 := rL__2:
  G1,G2,G3,P0,P2,P3;
```

$$\begin{bmatrix} \cos(\theta_1(t))\,L_{s0} \\ \sin(\theta_1(t))\,L_{s0} \\ 0 \end{bmatrix}, \begin{bmatrix} -L+\cos(\theta_2(t))\,L_{s0}+\cos(\theta_2(t))\,s_2(t) \\ \sin(\theta_2(t))\,L_{s0}+\sin(\theta_2(t))\,s_2(t) \\ 0 \end{bmatrix}, \begin{bmatrix} L+\cos(\theta_3(t))\,L_{s0}+\cos(\theta_3(t))\,s_3(t) \\ \sin(\theta_3(t))\,L_{s0}+\sin(\theta_3(t))\,s_3(t) \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -L \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} \quad (3.1.4)$$

Virtual displacements occur at frozen time.

Formally the virtual displacements should be computed via the velocity ratios.
Here we compute the velocities which are then used as virtual velocities.

```
> VG1:= diffF(G1,t):
  VG2:= diffF(G2,t):
  VG3:= diffF(G3,t):
  VP0:= diffF(P0,t):
  VP2:= diffF(P2,t):
  VP3:= diffF(P3,t):
```

Principle of Virtual Work - equilibrium conditions

This section aims at finding the equilibrium conditions using the Principle of virtual work

Forces

Vector of forces in ground frame

```
> W1 := <0,-m__1*g,0>:
  W2 := <0,-m__2*g,0>:
  W3 := <0,-m__3*g,0>:
```

Spring and damper forces

```
> fs__2(t):= K__s2*s__2(t)+C__s2*diff(s__2(t),t); # force linear
```

```
model is defined with positive sign according to constitutive
equation of the element

fs__3(t):= K__s3*s__3(t)+C__s3*diff(s__3(t),t);

Fs__2 := -fs__2(t)*is_uvec__2; # force is defined with minus sign
since from the node balance it acts on the body
Fs__3 := -fs__3(t)*is_uvec__3;
```

$$fs_2(t) := K_{s2}\, s_2(t) + C_{s2} \left(\frac{d}{dt}\, s_2(t)\right)$$

$$fs_3(t) := K_{s3}\, s_3(t) + C_{s3} \left(\frac{d}{dt}\, s_3(t)\right)$$

$$Fs_2 := \begin{bmatrix} -\left(K_{s2}\, s_2(t) + C_{s2}\left(\frac{d}{dt}\, s_2(t)\right)\right)\cos\left(\theta_2(t)\right) \\ -\left(K_{s2}\, s_2(t) + C_{s2}\left(\frac{d}{dt}\, s_2(t)\right)\right)\sin\left(\theta_2(t)\right) \\ 0 \end{bmatrix}$$

$$Fs_3 := \begin{bmatrix} -\left(K_{s3}\, s_3(t) + C_{s3}\left(\frac{d}{dt}\, s_3(t)\right)\right)\cos\left(\theta_3(t)\right) \\ -\left(K_{s3}\, s_3(t) + C_{s3}\left(\frac{d}{dt}\, s_3(t)\right)\right)\sin\left(\theta_3(t)\right) \\ 0 \end{bmatrix} \quad \textbf{(4.1)}$$

Virtual work

Procedure to compute the dot (i.e. scalar product as vector products)

```
> dot_prod := proc(v1::Vector,v2::Vector)
    Transpose(v1).v2;
  end:
```

Spring and Damper work

```
> VW_spring_damper__2 := simplify(+dot_prod(Transpose(Rs__2).(+
  Fs__2),Transpose(Rs__2).VG2)
                      +dot_prod(Transpose(Rs__2).(-Fs__2),
  Transpose(Rs__2).VP2)
                      );
```

$$VW\_spring\_damper_2 := -\left(K_{s2}\, s_2(t) + C_{s2}\left(\frac{d}{dt}\, s_2(t)\right)\right)\left(\frac{d}{dt}\, s_2(t)\right) \quad \textbf{(4.2)}$$

```
> VW_spring_damper__3 := simplify(+dot_prod(Transpose(Rs__3).(+
  Fs__3),Transpose(Rs__3).VG3)
                      +dot_prod(Transpose(Rs__3).(-Fs__3),
  Transpose(Rs__3).VP3)
                      );
```

(4.3)

$$VW\_spring\_damper_3 := -\left(K_{s3}\, s_3(t) + C_{s3}\left(\frac{d}{dt}\, s_3(t)\right)\right)\left(\frac{d}{dt}\, s_3(t)\right) \qquad (4.3)$$

Overall Virtual Work

```
> PVW :=  dot_prod(W1,VG1)
         +dot_prod(W2,VG2)
         +dot_prod(W3,VG3)
         +VW_spring_damper__2
         +VW_spring_damper__3;
```

$$PVW := -m_1\, g\left(\frac{d}{dt}\, \theta_1(t)\right)\cos(\theta_1(t))\, L_{s0} - m_2\, g\left(\left(\frac{d}{dt}\, \theta_2(t)\right)\cos(\theta_2(t))\, L_{s0} + \left(\frac{d}{dt}\, \theta_2(t)\right)\cos(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\left(\frac{d}{dt}\, s_2(t)\right)\right) - m_3\, g\left(\left(\frac{d}{dt}\, \theta_3(t)\right)\cos(\theta_3(t))\, L_{s0} + \left(\frac{d}{dt}\, \theta_3(t)\right)\cos(\theta_3(t))\, s_3(t) + \sin(\theta_3(t))\left(\frac{d}{dt}\, s_3(t)\right)\right) - \left(K_{s2}\, s_2(t) + C_{s2}\left(\frac{d}{dt}\, s_2(t)\right)\right)\left(\frac{d}{dt}\, s_2(t)\right) - \left(K_{s3}\, s_3(t) + C_{s3}\left(\frac{d}{dt}\, s_3(t)\right)\right)\left(\frac{d}{dt}\, s_3(t)\right) \qquad (4.4)$$

Equation of motion

Substitute the virtual velocities of the dependent coordinates as a function of the independent

```
> simplify(subs( vel_qD_vars, PVW ));
  PVW_I := collect(%,diff(qI_vars,t));
```

$$-m_1\, g\left(\frac{d}{dt}\, \theta_1(t)\right)\cos(\theta_1(t))\, L_{s0} - \left(\left(\left(\left(\cos(\theta_1(t))\, L_{s0} + L\right)\cos(\theta_2(t))^2 + \left(-L_{s0} - s_2(t)\right)\cos(\theta_2(t)) + \sin(\theta_2(t))^2\left(\cos(\theta_1(t))\, L_{s0} + L\right)\right)\left(\frac{d}{dt}\, s_2(t)\right) + L_{s0}\cos(\theta_2(t))\left(-\sin(\theta_1(t))\left(L_{s0} + s_2(t)\right)\cos(\theta_2(t)) + \sin(\theta_2(t))\cos(\theta_1(t))\left(L_{s0} + s_2(t)\right) + L\sin(\theta_1(t))\right)\left(\frac{d}{dt}\, \theta_1(t)\right)\right) m_2\, g\right) \Big/ \left(\left(\cos(\theta_1(t))\, L_{s0} + L\right)\sin(\theta_2(t)) - L_{s0}\cos(\theta_2(t))\sin(\theta_1(t))\right) - \left(m_3\left(\left(\left(-\cos(\theta_1(t))\, L_{s0} + L\right)\cos(\theta_3(t))^2 + \left(L_{s0} + s_3(t)\right)\cos(\theta_3(t)) + \sin(\theta_3(t))^2\left(-\cos(\theta_1(t))\, L_{s0} + L\right)\right)\left(\frac{d}{dt}\, s_3(t)\right) + L_{s0}\cos(\theta_3(t))\left(\sin(\theta_1(t))\left(L_{s0} + s_3(t)\right)\cos(\theta_3(t)) - \sin(\theta_3(t))\cos(\theta_1(t))\left(L_{s0} + s_3(t)\right) + L\sin(\theta_1(t))\right)\left(\frac{d}{dt}\, \theta_1(t)\right)\right) g\right) \Big/ \left(\left(-\cos(\theta_1(t))\, L_{s0} + L\right)\sin(\theta_3(t)) + L_{s0}\cos(\theta_3(t))\sin(\theta_1(t))\right)$$

$$- \left(K_{s2}\, s_2(t) + C_{s2} \left(\frac{d}{dt}\, s_2(t)\right)\right) \left(\frac{d}{dt}\, s_2(t)\right) - \left(K_{s3}\, s_3(t) + C_{s3} \left(\frac{d}{dt}\, s_3(t)\right)\right) \left(\frac{d}{dt}\, s_3(t)\right)$$

$$PVW\_I := \Big(-m_1\, g \cos(\theta_1(t))\, L_{s0} - \big(L_{s0} \cos(\theta_2(t))\, (-\sin(\theta_1(t))\, (L_{s0} + s_2(t)) \cos(\theta_2(t)) + \sin(\theta_2(t)) \cos(\theta_1(t))\, (L_{s0} + s_2(t)) + L \sin(\theta_1(t)))\, m_2\, g\big) \Big/ \big((\cos(\theta_1(t))\, L_{s0} + L) \sin(\theta_2(t)) - L_{s0} \cos(\theta_2(t)) \sin(\theta_1(t))\big) \tag{4.5}$$

$$\cdot\, L) \sin(\theta_3(t)) + L_{s0} \cos(\theta_3(t)) \sin(\theta_1(t)))\Big) \left(\frac{d}{dt}\, \theta_1(t)\right) - C_{s2} \left(\frac{d}{dt}\, s_2(t)\right)^2 + \Big(- \big(\big((\cos(\theta_1(t))\, L_{s0} + L) \cos(\theta_2(t))^2 + (-L_{s0} - s_2(t)) \cos(\theta_2(t)) + \sin(\theta_2(t))^2 (\cos(\theta_1(t))\, L_{s0} + L)\big)\, m_2\, g\big) \Big/ \big((\cos(\theta_1(t))\, L_{s0} + L) \sin(\theta_2(t)) - L_{s0} \cos(\theta_2(t)) \sin(\theta_1(t))\big) - K_{s2}\, s_2(t)\Big) \left(\frac{d}{dt}\, s_2(t)\right) - C_{s3} \left(\frac{d}{dt}\, s_3(t)\right)^2 + \Big(- \big(m_3 \big((-\cos(\theta_1(t))\, L_{s0} + L) \cos(\theta_3(t))^2 + (L_{s0} + s_3(t)) \cos(\theta_3(t)) + \sin(\theta_3(t))^2 (-\cos(\theta_1(t))\, L_{s0} + L)\big)\, g\big) \Big/ \big((-\cos(\theta_1(t))\, L_{s0} + L) \sin(\theta_3(t)) + L_{s0} \cos(\theta_3(t)) \sin(\theta_1(t))\big) - K_{s3}\, s_3(t)\Big) \left(\frac{d}{dt}\, s_3(t)\right)$$

Collect for the independent coordinate virtual displacement

```
> AA,BB := GenerateMatrix([PVW_I],diff(qI_vars,t));
```

$$AA, BB := \left[-m_1\, g \cos(\theta_1(t))\, L_{s0} - \frac{L_{s0} \cos(\theta_2(t))\, (-\sin(\theta_1(t))\, (L_{s0} + s_2(t)) \cos \cdots}{(\cos(\theta_1(t))\, L_{s0} + \cdots} \cdots \right], \left[C_{s2} \left(\frac{d}{dt}\, s_2(t)\right)^2 + C_{s3} \left(\frac{d}{dt}\, s_3(t)\right)^2 \right] \tag{4.6}$$

The equations of motions are the element in the vector AA.

```
> eqns:=  AA[1,1..3];
  nops(%);
```

$$eqns := \left[\ -m_1\, g \cos\left(\theta_1(t)\right) L_{s0} - \frac{L_{s0} \cos\left(\theta_2(t)\right) \left(-\sin\left(\theta_1(t)\right) \left(L_{s0} + s_2(t)\right) \cos \cdots\right.}{\left(\cos\left(\theta_1(t)\right) L_{s0} + \cdots\right.} \cdots \ \right]$$

3 **(4.7)**

```
> PVW__eqns := subs(theta__1(t)=theta__1,theta__2(t)=theta__2,
  theta__3(t)=theta__3,s__2(t)=s__2,s__3(t)=s__3,(convert(eqns,set)
  union convert(Phi,set)));
  PVW__vars := convert(subs(theta__1(t)=theta__1,theta__2(t)=
  theta__2,theta__3(t)=theta__3,s__2(t)=s__2,s__3(t)=s__3,q_vars),
  set);
```

$$PVW_{eqns} := \Big\{ -\Big(m_3 \Big(\big(-\cos(\theta_1) L_{s0} + L\big) \cos(\theta_3)^2 + \big(L_{s0} + s_3\big) \cos(\theta_3) + \sin(\theta_3)^2 \big(-\cos(\theta_1) L_{s0} + L\big)\Big) g\Big) \Big/ \Big(\big(-\cos(\theta_1) L_{s0} + L\big) \sin(\theta_3) + L_{s0} \cos(\theta_3) \sin(\theta_1)\Big) - K_{s3}\, s_3,\ -\Big(\Big(\big(\cos(\theta_1) L_{s0} + L\big) \cos(\theta_2)^2 + \big(-L_{s0} - s_2\big) \cos(\theta_2) + \sin(\theta_2)^2 \big(\cos(\theta_1) L_{s0} + L\big)\Big) m_2\, g\Big) \Big/ \Big(\big(\cos(\theta_1) L_{s0} + L\big) \sin(\theta_2) - L_{s0} \cos(\theta_2) \sin(\theta_1)\Big) - K_{s2}\, s_2,\ \big(-L - \cos(\theta_1) L_{s0} + \cos(\theta_2) s_2 + \cos(\theta_2) L_{s0}\big)^2 + \big(-\sin(\theta_1) L_{s0} + \sin(\theta_2) s_2 + \sin(\theta_2) L_{s0}\big)^2 - L^2,\ \big(-L - \cos(\theta_3) L_{s0} - \cos(\theta_3) s_3 + \cos(\theta_1) L_{s0}\big)^2 + \big(-\sin(\theta_3) L_{s0} - \sin(\theta_3) s_3 + \sin(\theta_1) L_{s0}\big)^2 - L^2,\ -m_1\, g \cos(\theta_1) L_{s0} - \Big(L_{s0} \cos(\theta_2) \big(-\sin(\theta_1) \big(L_{s0} + s_2\big) \cos(\theta_2) + \sin(\theta_2) \cos(\theta_1) \big(L_{s0} + s_2\big) + L \sin(\theta_1)\big) m_2\, g\Big) \Big/ \Big(\big(\cos(\theta_1) L_{s0} + L\big) \sin(\theta_2) - L_{s0} \cos(\theta_2) \sin(\theta_1)\Big) - \Big(m_3 L_{s0} \cos(\theta_3) \big(\sin(\theta_1) \big(L_{s0} + s_3\big) \cos(\theta_3) - \sin(\theta_3) \cos(\theta_1) \big(L_{s0} + s_3\big) + L \sin(\theta_1)\big) g\Big) \Big/ \Big(\big(-\cos(\theta_1) L_{s0} + L\big) \sin(\theta_3) + L_{s0} \cos(\theta_3) \sin(\theta_1)\Big) \Big\}$$

$$PVW_{vars} := \{s_2, s_3, \theta_1, \theta_2, \theta_3\} \qquad \textbf{(4.8)}$$

We can substitute the data and solve the equations in order to find the equilibrium conditions of the mechanical system

```
> data := [L = 1, L__s0 = 1, m__1 = 10, m__2 = 2, m__3 = 30, g = 9.81,
  K__s2 = 5000, C__s2 = 10, K__s3 = 100000, C__s3 = 100];
```

$$data := [L = 1, L_{s0} = 1, m_1 = 10, m_2 = 2, m_3 = 30, g = 9.81, K_{s2} = 5000, C_{s2} = 10, K_{s3} = 100000, C_{s3} = 100] \quad (4.9)$$

```
> subs(data,PVW__eqns);
```

$$\Big\{-\big(294.30\,((-\cos(\theta_1)+1)\cos(\theta_3)^2 + (1+s_3)\cos(\theta_3) + \sin(\theta_3)^2(-\cos(\theta_1)+1))\big)\big/((-\cos(\theta_1)+1)\sin(\theta_3)+\cos(\theta_3)\sin(\theta_1)) - 100000\,s_3,$$
$$-\frac{1}{(\cos(\theta_1)+1)\sin(\theta_2)-\cos(\theta_2)\sin(\theta_1)}\big(19.62\,((\cos(\theta_1)+1)\cos(\theta_2)^2 + (-1-s_2)\cos(\theta_2) + \sin(\theta_2)^2(\cos(\theta_1)+1))\big) - 5000\,s_2, (-1-\cos(\theta_1) + \cos(\theta_2)\,s_2 + \cos(\theta_2))^2 + (-\sin(\theta_1) + \sin(\theta_2)\,s_2 + \sin(\theta_2))^2 - 1, (-1 - \cos(\theta_3) - \cos(\theta_3)\,s_3 + \cos(\theta_1))^2 + (-\sin(\theta_3) - \sin(\theta_3)\,s_3 + \sin(\theta_1))^2 - 1,$$
$$-98.10\cos(\theta_1) - \frac{1}{(\cos(\theta_1)+1)\sin(\theta_2)-\cos(\theta_2)\sin(\theta_1)}\big(19.62\cos(\theta_2)\,(-\sin(\theta_1)\,(1+s_2)\cos(\theta_2) + \sin(\theta_2)\cos(\theta_1)\,(1+s_2) + \sin(\theta_1))\big) - \big(294.30\cos(\theta_3)\,(\sin(\theta_1)\,(1+s_3)\cos(\theta_3) - \sin(\theta_3)\cos(\theta_1)\,(1+s_3) + \sin(\theta_1))\big)\big/((-\cos(\theta_1)+1)\sin(\theta_3)+\cos(\theta_3)\sin(\theta_1))\Big\} \quad (4.10)$$

Equilibrium conditions of the mechanical system

```
> sol_fs := fsolve(subs(data,PVW__eqns),PVW__vars,{s__2=-1..1,
  s__3=-1..1,theta__1=-Pi..0,theta__2=-Pi..0, theta__3=-Pi..0});
```

$$sol\_fs := \{s_2 = 0.003923999840, s_3 = 0.002942999986, \theta_1 = -1.570793602, \theta_2 = -1.570785943, \theta_3 = -1.570797927\} \quad (4.11)$$

System analysis

The following procedure is defined in order to display the configuration of the mechanical system

```
> vec_2D := proc(v::Vector(3))

    convert(v[1..2],list);

  end proc;
```

$vec\_2D := \mathbf{proc}(v::(Vector(3)))\ convert(v[1..2], list)\ \mathbf{end\ proc}$ **(4.1.1)**

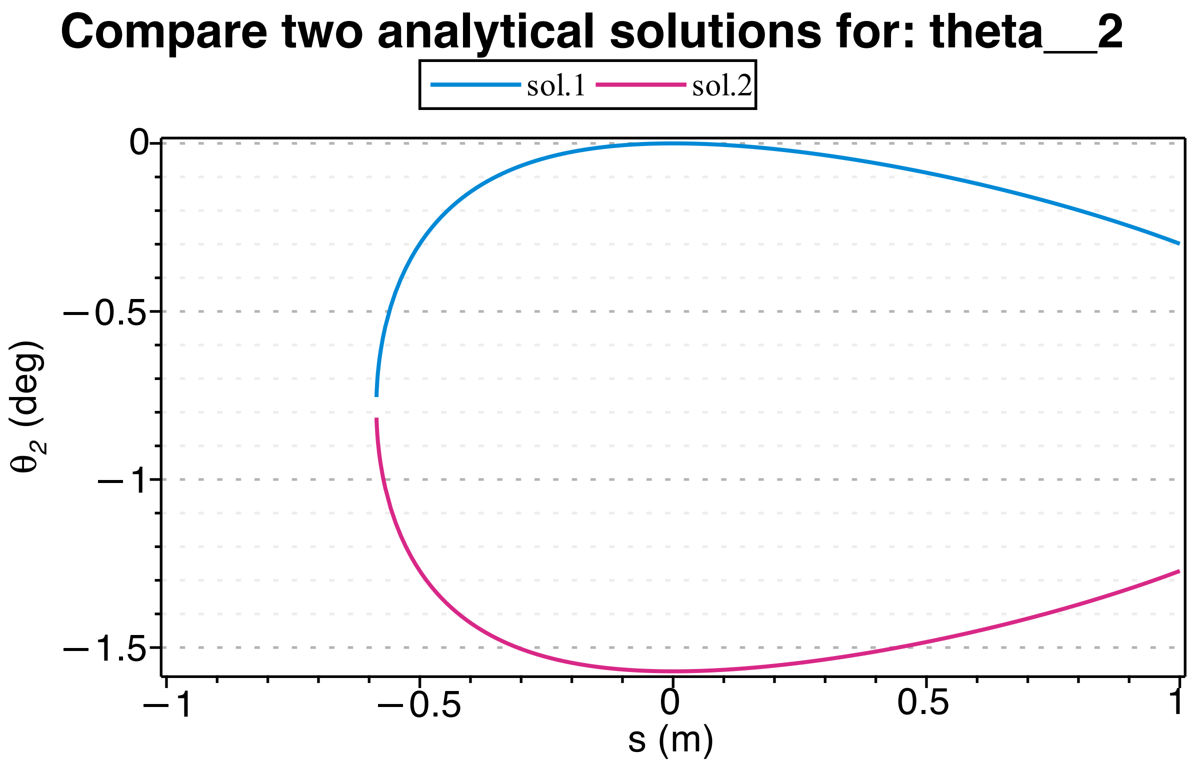
```
> draw_mech := proc(data, sol_kine, dof)

    local p0,p2,p3,pG1,pG2,pG3,LS1,LS2,LS3,SS2,SS3,PS1,PS2, PS3,
  VS;

    p0  := subs(sol_kine, dof,data, P0);
    p2  := subs(sol_kine, dof,data, P2);
    p3  := subs(sol_kine, dof,data, P3);
    pG1  := subs(sol_kine, dof,data, G1):
    pG2  := subs(sol_kine, dof,data, G2):
    pG3  := subs(sol_kine, dof,data, G3):
    LS1  := subs(sol_kine, dof,data, L__s0*is_uvec__1);
    LS2  := subs(sol_kine, dof,data, L__s0*is_uvec__2);
    LS3  := subs(sol_kine, dof,data, L__s0*is_uvec__3);
    SS2  := subs(sol_kine, dof,data, s__2(t)*is_uvec__2);
    SS3  := subs(sol_kine, dof,data, s__3(t)*is_uvec__3);
    PS1  := p0+LS1;
    PS2 := p2+LS2+SS2;
    PS3  := p3+LS3+SS3;

    display( plottools:-disk(vec_2D(p0),0.05,color="Gray"),
             plottools:-disk(vec_2D(p2),0.05,color="Gray"),
             plottools:-disk(vec_2D(p3),0.05,color="Gray"),
             plottools:-disk(vec_2D(PS1),0.05,color=
  "Goldenrod"),
             plottools:-disk(vec_2D(PS2),0.05,color=
  "Goldenrod"),
             plottools:-disk(vec_2D(PS3),0.05,color=
  "Goldenrod"),
             plottools:-line(vec_2D(p0),vec_2D(p2),color=
  "Green",thickness=4),
             plottools:-line(vec_2D(p0),vec_2D(p3),color=
  "Green",thickness=4),
             plottools:-line(vec_2D(p0),vec_2D(PS1),color=
  "DodgerBlue",thickness=4),
             plottools:-line(vec_2D(p2),vec_2D(PS2),color=
  "DodgerBlue",thickness=4),
```

```
            plottools:-line(vec_2D(p3),vec_2D(PS3),color=
"DodgerBlue",thickness=4),
            plottools:-line(vec_2D(PS1),vec_2D(PS2),color=
"DodgerBlue",thickness=4),
            plottools:-line(vec_2D(PS1),vec_2D(PS3),color=
"DodgerBlue",thickness=4),
            #plots:-arrow(vec_2D(p0),vec_2D(LS1),color="red",
thickness=3,shape=arrow),
            #plots:-arrow(vec_2D(p2),vec_2D(SS2),color=
"DarkOrange",thickness=2,shape=arrow),
            axes = boxed,
            scaling = constrained

   );

end proc:

anim_draw_mech := proc(data, sol_kine, dof::list, k)
  draw_mech(data, sol_kine, dof[k]);
end proc:
```

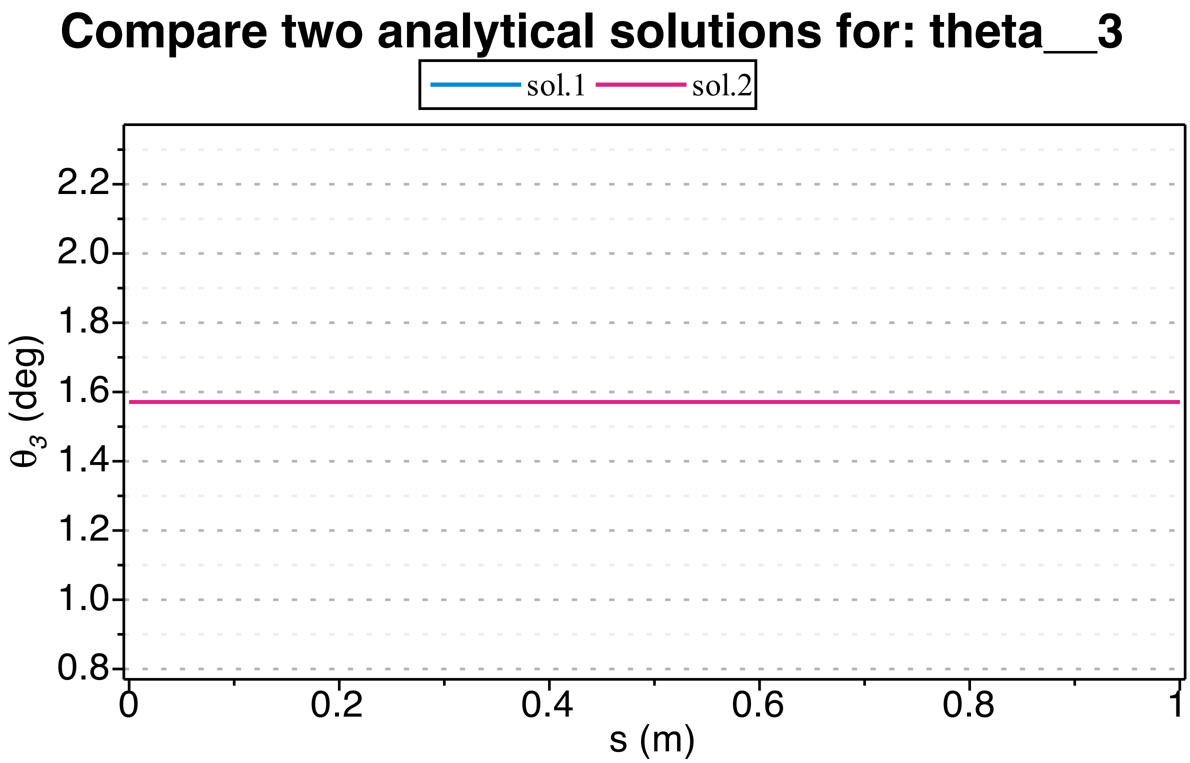
Kinematic Analytical Solution

In this section the two analytical solutions are plotted and compared both for theta2 and theta3.
Then the equilibrium configuration is plotted using the procedure defined previously

```
> display(plot(subs( sol_kine_all[1],data, s__2(t) =S, s__3(t)
  = 0.002942999986,theta__3(t) = -1.570797927,theta__1(t)=
  -1.570793602,theta__2(t)),  S=-1..1, color=c_set[1], legend
  = "sol.1"),
          plot(subs( sol_kine_all[2],data, s__2(t) = S, s__3
  (t) = 0.002942999986,theta__3(t) = -1.570797927,theta__1(t)=
  -1.570793602,theta__2(t)), S=-1..1, color=c_set[2], legend =
  "sol.2"),
          labels = ["s (m)", typeset(theta__2, " (deg)") ],
          size   = [800,300],
          title  = "Compare two analytical solutions for:
  theta__2");
```




```
> display(plot(subs( sol_kine_all[1],data, s__2(t) = S,s__3(t)
  = 0.003923999840,theta__1(t)=0.003923999840,theta__2(t) =
  -1.570785943, theta__3(t)), S=0..1,  color=c_set[1], legend=
  "sol.1"),
          plot(subs( sol_kine_all[2],data, s__2(t) = S,s__3(t)
  = 0.003923999840,theta__1(t)=0.003923999840,theta__2(t) =
  -1.570785943, theta__3(t)), S=0..1, color=c_set[2], legend=
  "sol.2"),
          labels = ["s (m)", typeset(theta__3, " (deg)") ],
         size   = [800,300] ,
          title  = "Compare two analytical solutions for:
  theta__3" );
```

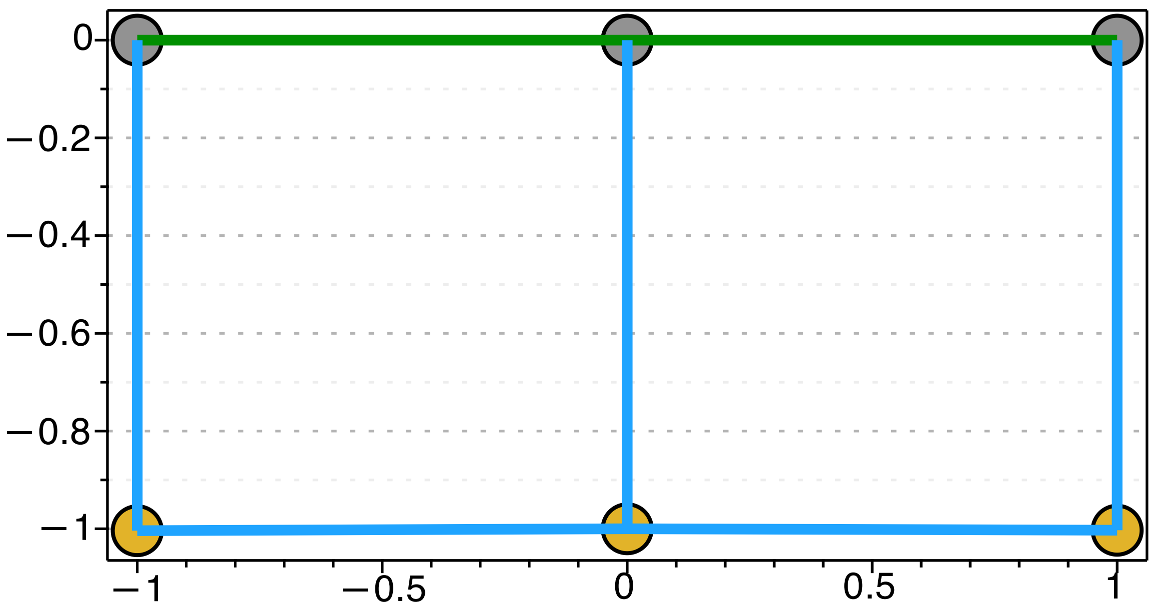



```
> solve(Phi,qD_vars,explicit=true,allsolutions=true,maxsols=4)
  :
```

Display the equilibrium configuration found in the previous section using the Principal of Virtual Work

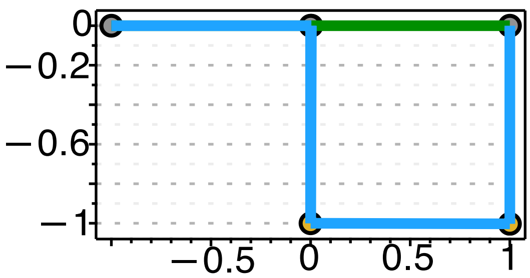
```
> display(draw_mech(data,sol_kine_all[2],[s__2(t) = op(2,
  sol_fs[1]), s__3(t) = op(2,sol_fs[2]), theta__1(t) =op(2,
  sol_fs[3]), theta__2(t) = op(2,sol_fs[4]), theta__3(t) = op
  (2,sol_fs[5])]),size=[800,400],title = "Equilibrium
  Confguration");
```

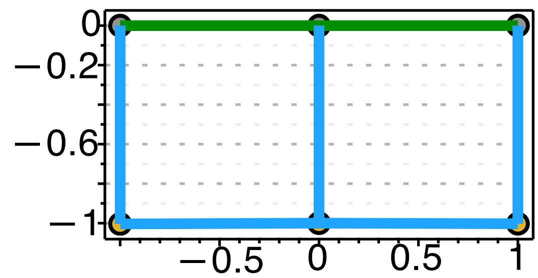
Equilibrium Confguration



Display the equilibrium found using both analytical solutions

```
> display(draw_mech(data,sol_kine_all[1],[s__2(t) = op(2,
  sol_fs[1]), s__3(t) = op(2,sol_fs[2]), theta__1(t) = op(2,
  sol_fs[3])]),size=[800,400]):
  display(draw_mech(data,sol_kine_all[2],[ s__2(t) = op(2,
  sol_fs[1]), s__3(t) = op(2,sol_fs[2]), theta__1(t) = op(2,
  sol_fs[3])]),size=[800,400]):
  DocumentTools:-Tabulate(Array(1..2,[%%,%]),interior=none,
  exterior=none,widthmode=percentage,width=100):
```





Kinematic Numerical Solution

This section aims at finding the numerical solutions of the mechanical system for given values of the independent variables.
Then the movement of the mechanical system is displayed and also the values of the dipendent variables (theta2,theta3) are plotted.

```
> x2y := {seq(qD_vars[i]=op(0,qD_vars[i]),i=1..nops(qD_vars))}
  ;
  y2x := map(x->rhs(x)=lhs(x),x2y);
  yy  := map(lhs,y2x);
```

$$x2y := \{\theta_2(t) = \theta_2, \theta_3(t) = \theta_3\}$$

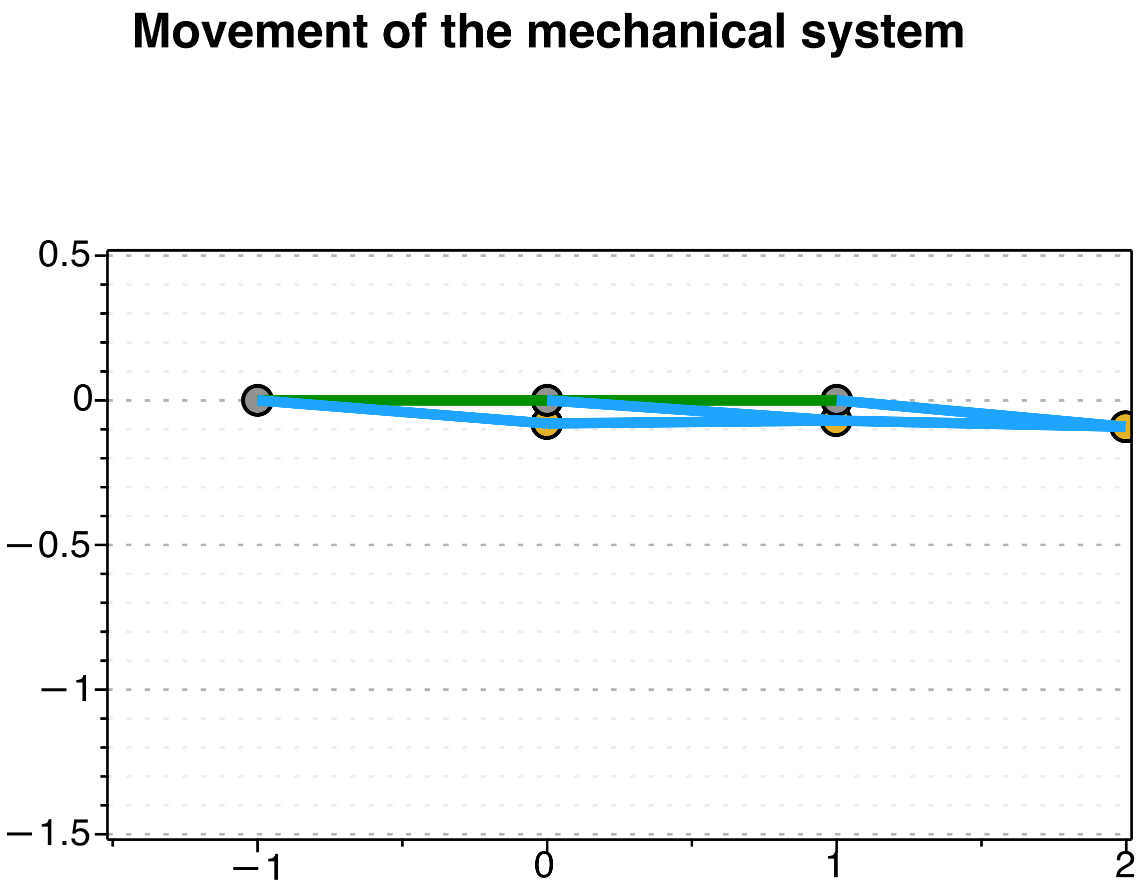
$$y2x := \{\theta_2 = \theta_2(t), \theta_3 = \theta_3(t)\}$$

$$yy := \{\theta_2, \theta_3\} \tag{4.1.2.1}$$

```
> NPTS   := 100:
  qI_vec := [seq(s__2(t) = 0.002+0.08/NPTS*i,i=0..NPTS),seq
  (s__3(t) = 0.002+0.28/NPTS*i,i=0..NPTS),seq(theta__1(t) =
  -0.04-1.5/NPTS*i,i=0..NPTS)]: # vector of independent
  variable's values
```

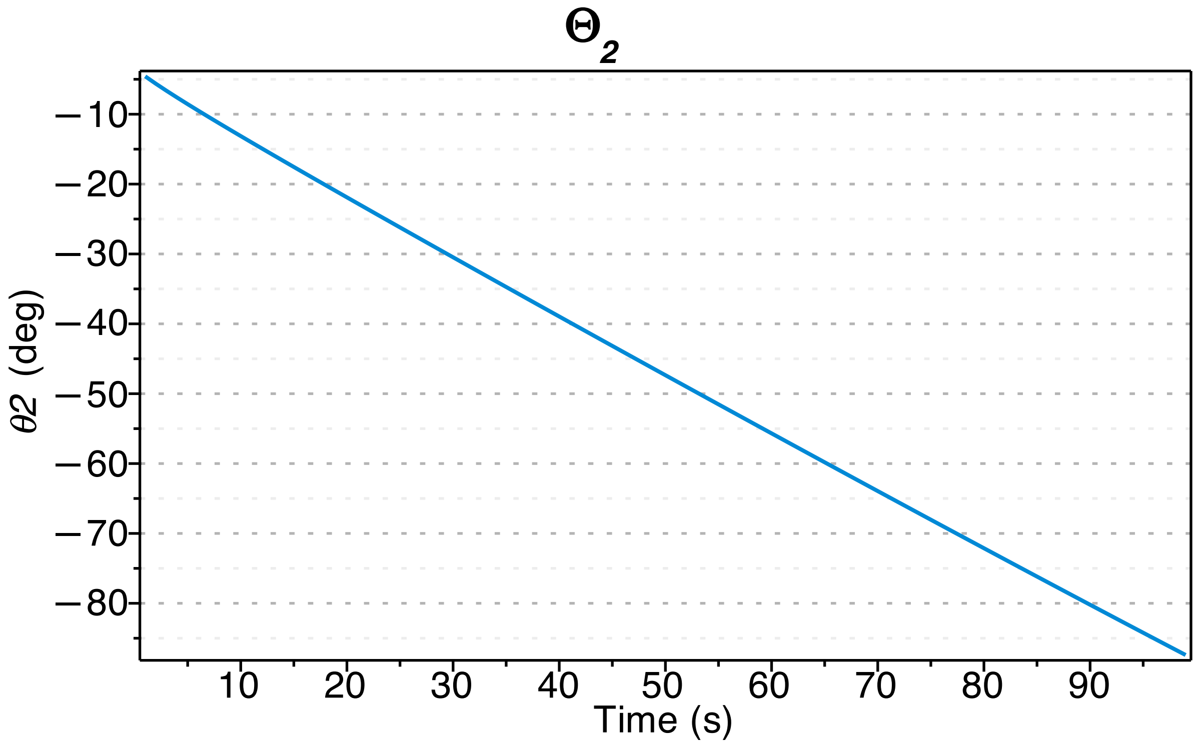
```
> sol_kine_num := Array(1..NPTS+1,[]): # vector to store the
  solution

  #first solution
  sol_kine_num[1] := subs( y2x, fsolve(convert(subs(data,
  qI_vec[1],qI_vec[102],qI_vec[204],theta__2(t)=theta__2,
```

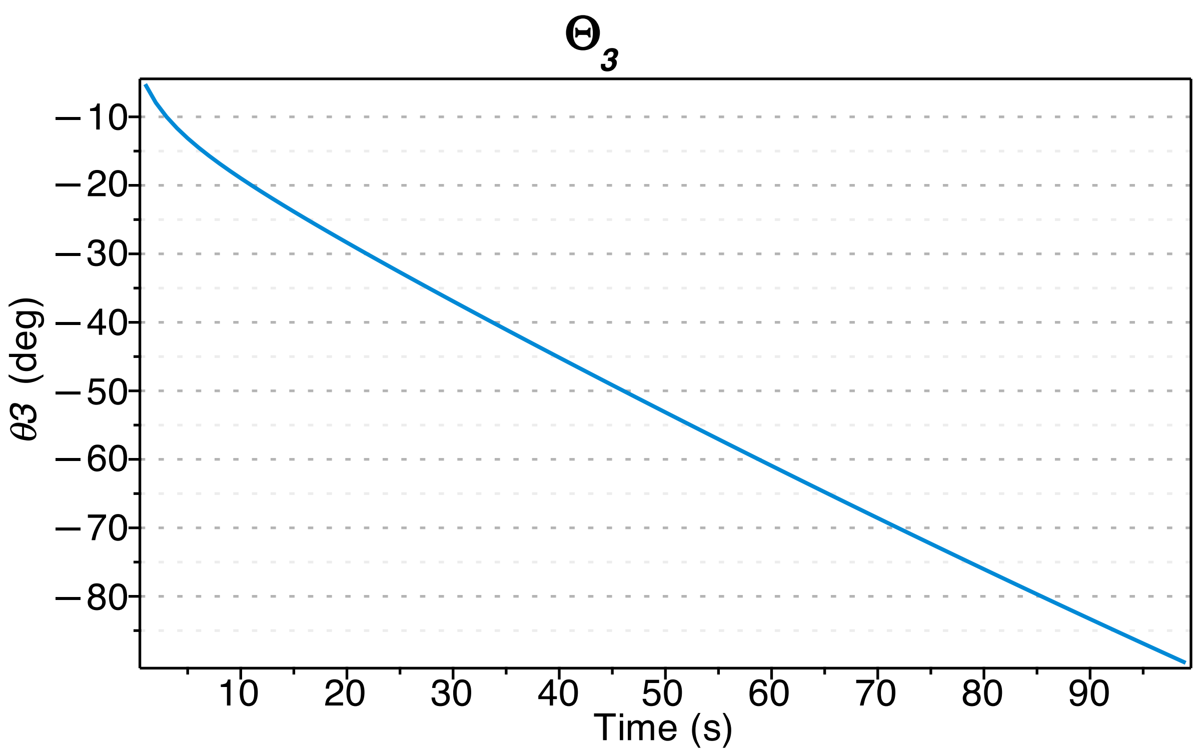
```
theta__3(t)=theta__3,x2y, Phi),set),yy, {theta__2 = -Pi/2..
Pi/2,theta__3 = -Pi/2..Pi/2}) ):

#cycle over the vector of positions of independent variables
and store di solution
for k from 2 to NPTS do
  sol_kine_num[k] := subs( y2x, fsolve(convert(subs(data,
qI_vec[k],qI_vec[101+k],qI_vec[203+k],theta__2(t)=theta__2,
theta__3(t)=theta__3,x2y, Phi),set),
                                       subs(x2y,sol_kine_num
[k-1]),
                                       {theta__2 =  -Pi/2..
Pi/2,theta__3 =  -Pi/2..Pi/2}  # add the ranges to search
the solution
                                       )
                          );

end do:
> anim_draw_mech_k := proc(data, sol_kine, dof::list, k)
  draw_mech(data, sol_kine[k], [dof[k],dof[k+101],dof[k+204]
]);
end proc:

animate(anim_draw_mech_k,[data,sol_kine_num, qI_vec,K],
        K=[seq(k,k=1..(NPTS-1))],
        size=[800,400],
        view=[-1.5..2,0.5..-1.5],
        title = "Movement of the mechanical system")
```



Plot the values found for the dipendent variables

```
> pointplot([seq(k,k=1..99)],[seq(subs(sol_kine_num[k],
  theta__2(t)*180/Pi),k=1..99)],
            connect = true,
            color   = c_set[1],
            labels  = ["Time (s)", typeset(theta2, " (deg)")
  ],
            title = Theta__2,
            size    = [800,300] );
  pointplot([seq(k,k=1..99)],[seq(subs(sol_kine_num[k],
  theta__3(t)*180/Pi),k=1..99)],
            connect = true,
            color   = c_set[1],
            labels  = ["Time (s)", typeset(theta3, " (deg)")
  ],
            title = Theta__3,
            size    = [800,300] );
```





Lagrange Equations

In this section the Lagrange equations of the mechanical system are derived

Lagrangian Equations

```
> gravity := <0,-g,0>;
```

$$gravity := \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} \quad \textbf{(5.1.1)}$$

Kinetic and potential energy for mass m1

```
> KE1 := kineticEnergy(m__1,G1,Rs__1, 0,0,0,0,0,0);
  PE1 := gravitationalEnergy(m__1,G1,gravity);
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$KE1 := \frac{m_1 \left(\left(\frac{d}{dt}\, \theta_1(t) \right)^2 \sin\left(\theta_1(t)\right)^2 L_{s0}{}^2 + \left(\frac{d}{dt}\, \theta_1(t) \right)^2 \cos\left(\theta_1(t)\right)^2 L_{s0}{}^2 \right)}{2}$$

$$PE1 := m_1 \sin\left(\theta_1(t)\right) L_{s0}\, g \quad \textbf{(5.1.2)}$$

Kinetic and potential energy for mass m2

```
> KE2 := simplify( kineticEnergy(m__2,G2,Rs__2, 0,0,0,0,0,0) );
  PE2 := gravitationalEnergy(m__2,G2,gravity);
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$KE2 := \frac{m_2 \left(\left(L_{s0} + s_2(t)\right)^2 \left(\frac{d}{dt}\, \theta_2(t) \right)^2 + \left(\frac{d}{dt}\, s_2(t) \right)^2 \right)}{2}$$

$$PE2 := m_2 \left(\sin\left(\theta_2(t)\right) L_{s0} + \sin\left(\theta_2(t)\right) s_2(t) \right) g \quad \textbf{(5.1.3)}$$

Kinetic and potential energy for mass m3

```
> KE3 := simplify( kineticEnergy(m__3,G3,Rs__3, 0,0,0,0,0,0) );
  PE3 := gravitationalEnergy(m__3,G3,gravity);
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$KE3 := \frac{m_3\left(\left(L_{s0}+s_3(t)\right)^2\left(\frac{d}{dt}\,\theta_3(t)\right)^2+\left(\frac{d}{dt}\,s_3(t)\right)^2\right)}{2}$$

$$PE3 := m_3\left(\sin\left(\theta_3(t)\right)L_{s0}+\sin\left(\theta_3(t)\right)s_3(t)\right)g \qquad (5.1.4)$$

Lagrangian function

```
> LagrF :=KE1+KE2+KE3-PE1-PE2-PE3;
```

$$LagrF := \frac{m_1\left(\left(\frac{d}{dt}\,\theta_1(t)\right)^2\sin\left(\theta_1(t)\right)^2L_{s0}^{\,2}+\left(\frac{d}{dt}\,\theta_1(t)\right)^2\cos\left(\theta_1(t)\right)^2L_{s0}^{\,2}\right)}{2} + \frac{m_2\left(\left(L_{s0}+s_2(t)\right)^2\left(\frac{d}{dt}\,\theta_2(t)\right)^2+\left(\frac{d}{dt}\,s_2(t)\right)^2\right)}{2} + \frac{m_3\left(\left(L_{s0}+s_3(t)\right)^2\left(\frac{d}{dt}\,\theta_3(t)\right)^2+\left(\frac{d}{dt}\,s_3(t)\right)^2\right)}{2} - m_1\sin\left(\theta_1(t)\right)L_{s0}\,g - m_2\left(\sin\left(\theta_2(t)\right)L_{s0}+\sin\left(\theta_2(t)\right)s_2(t)\right)g - m_3\left(\sin\left(\theta_3(t)\right)L_{s0} + \sin\left(\theta_3(t)\right)s_3(t)\right)g \qquad (5.1.5)$$

Generalized coordinates

```
> q_vars
```

$$\left[s_2(t), \theta_1(t), s_3(t), \theta_2(t), \theta_3(t)\right] \qquad (5.1.6)$$

```
> leqns := lagrangeEquations(LagrF,Phi,q_vars)
```

$$leqns := \left[\begin{array}{c} \cdots\; s_2(t)+\cos\left(\theta_2(t)\right)L_{s0}\big)\cos\left(\theta_2(t)\right)+2\left(-\sin\left(\theta_1(t)\right)L_{s0}+\mathrm{s}\cdots \\ \cdots_{3}(t)\big)\,s_3(t)+\sin\left(\theta_1(t)\right)L_{s0}\big)\cos\left(\theta_1(t)\right)L_{s0}\big)\,\lambda_1(t)-\big(2\left(-L\cdots \\ \cdots\big)\,s_3(t)+\cos\left(\theta_1(t)\right)L_{s0}\big)\cos\left(\theta_3(t)\right)-2\left(-\sin\left(\theta_3(t)\right)L_{s0}-\cdots \\ \cdots\big)\,s_2(t)+\cos\left(\theta_2(t)\right)L_{s0}\big)\left(-\sin\left(\theta_2(t)\right)s_2(t)-\sin\left(\theta_2(t)\right)L_{s0}\cdots \\ \cdots\big)\,s_3(t)+\cos\left(\theta_1(t)\right)L_{s0}\big)\left(\sin\left(\theta_3(t)\right)L_{s0}+\sin\left(\theta_3(t)\right)s_3(t)\right)\cdot\cdots \end{array}\right] \qquad (5.1.7)$$

```
> z_vars := [ op(q_vars), lambda__1(t)];
```

$$z\_vars := \left[s_2(t), \theta_1(t), s_3(t), \theta_2(t), \theta_3(t), \lambda_1(t)\right] \qquad (5.1.8)$$

```
> MM, ff := GenerateMatrix(convert(leqns,list),diff(q_vars,t,t))
```

$$MM, ff := \begin{bmatrix} m_2 & 0 & 0 & 0 \cdots \\ 0 & \frac{m_1 \left(2 L_{s0}^2 \cos(\theta_1(t))^2 + 2 L_{s0}^2 \sin(\theta_1(t))^2\right)}{2} & 0 & 0 \cdots \\ 0 & 0 & m_3 & 0 \cdots \\ 0 & 0 & 0 & m_2 \left(L_{s0} + \cdots\right. \\ 0 & 0 & 0 & 0 \cdots \end{bmatrix}, \quad \textbf{(5.1.9)}$$

$$\begin{bmatrix} \cdots \\ -m_1 g \cos(\theta_1(t)) L_{s0} + \left(-2 \left(-L - \cos(\theta_3(t)) L_{s0} - \cos(\theta_3(t) \cdots\right.\right. \\ \cdots \\ -2 m_2 \left(L_{s0} + s_2(t)\right) \left(\frac{d}{dt} \theta_2(t)\right) \left(\frac{d}{dt} s \cdots\right. \\ -2 m_3 \left(L_{s0} + s_3(t)\right) \left(\frac{d}{dt} \theta_3(t)\right) \left(\frac{d}{dt} s \cdots\right. \end{bmatrix}$$

Spring Ks2 and Damper Cs2 Forces

```
> fe__2(t) := -K__s2*s__2(t)-C__s2*diff(s__2(t),t); # spring
  force model
  FE__2 := <fe__2(t),0,0>;
```

$$fe_2(t) := -K_{s2}\, s_2(t) - C_{s2} \left(\frac{d}{dt} s_2(t)\right)$$

$$FE_2 := \begin{bmatrix} -K_{s2}\, s_2(t) - C_{s2} \left(\frac{d}{dt} s_2(t)\right) \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(5.1.10)}$$

Spring Ks3 and Damper Cs3 Forces

```
> fe__3(t) := -K__s3*s__3(t)-C__s3*diff(s__3(t),t); # spring
  force model
  FE__3 := <fe__3(t),0,0>;
```

$$fe_3(t) := -K_{s3}\, s_3(t) - C_{s3} \left(\frac{d}{dt} s_3(t)\right)$$

$$FE_3 := \begin{bmatrix} -K_{s3}\, s_3(t) - C_{s3} \left(\frac{d}{dt} s_3(t)\right) \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(5.1.11)}$$

```
> FE__2:=Rs__2.FE__2;
  FE__3:=Rs__3.FE__3;
```

$$FE_2 := \begin{bmatrix} \cos\left(\theta_2(t)\right)\left(-K_{s2}\, s_2(t) - C_{s2}\left(\frac{d}{dt}\, s_2(t)\right)\right) \\ \sin\left(\theta_2(t)\right)\left(-K_{s2}\, s_2(t) - C_{s2}\left(\frac{d}{dt}\, s_2(t)\right)\right) \\ 0 \end{bmatrix}$$

$$FE_3 := \begin{bmatrix} \cos\left(\theta_3(t)\right)\left(-K_{s3}\, s_3(t) - C_{s3}\left(\frac{d}{dt}\, s_3(t)\right)\right) \\ \sin\left(\theta_3(t)\right)\left(-K_{s3}\, s_3(t) - C_{s3}\left(\frac{d}{dt}\, s_3(t)\right)\right) \\ 0 \end{bmatrix} \quad \textbf{(5.1.12)}$$

```
> Q_genF := generalisedForces(G2,FE__2,q_vars)-generalisedForces
  (P2,FE__2,q_vars)+generalisedForces(G3,FE__3,q_vars)-
  generalisedForces(P3,FE__3,q_vars);
```

$$Q\_genF := \begin{bmatrix} \cos\left(\theta_2(t)\right)^2\left(-K_{s2}\, s_2(t) - \cdots \right. \\ \cdots \\ \cos\left(\theta_3(t)\right)^2\left(-K_{s3}\, s_3(t) - \cdots \right. \\ \cos\left(\theta_2(t)\right)\left(-K_{s2}\, s_2(t) - C_{s2}\left(\frac{d}{dt}\, s_2(t)\right)\right)\left(-\sin\left(\theta_2(t)\right) s_2(t \cdots \right. \\ \cos\left(\theta_3(t)\right)\left(-K_{s3}\, s_3(t) - C_{s3}\left(\frac{d}{dt}\, s_3(t)\right)\right)\left(-\sin\left(\theta_3(t)\right) L_{s0} \cdot \cdots \right. \end{bmatrix} \quad \textbf{(5.1.13)}$$

Full set of equations

```
> ode_sys := leqns-Q_genF;
```

(5.1.14)

$$ode\_sys := \left[\begin{array}{cc} \ldots & \ldots \\ \ldots & \frac{m_1 \left(2 \left(\frac{d^2}{dt^2} \theta_1(t)\right) \sin(\ldots}{} \ldots \\ \ldots & \ldots \\ \ldots _2 \left(L_{s0} + s_2(t)\right) \left(\frac{d}{dt} \theta_2(t)\right) \left(\frac{d}{dt} s_2(t)\right) + m_2 \left(L_{s0} + s_2(t)\right) \ldots \\ \ldots _3 \left(L_{s0} + s_3(t)\right) \left(\frac{d}{dt} \theta_3(t)\right) \left(\frac{d}{dt} s_3(t)\right) + m_3 \left(L_{s0} + s_3(t)\right) \ldots \end{array}\right] \quad \textbf{(5.1.14)}$$

Numerical solution and plots

In this section the consistent initial conditions for the dae dynamical system are computed, assuming that initially the two bars G1-G2 and G3-G1 are horizontal and bar P0-G1 is vertical.

Numerical solution Index Reduction

```
> sol_kine := solve(Phi,{theta__2(t),theta__3(t)}):
  s__2(t) = 0,s__3(t) = 0,theta__1(t)=-Pi/2;
  ics_p := {%} union subs( %, data, sol_kine );
```

$$s_2(t) = 0, s_3(t) = 0, \theta_1(t) = -\frac{\pi}{2}$$

$$ics\_p := \left\{ s_2(t) = 0, s_3(t) = 0, \theta_1(t) = -\frac{\pi}{2}, \theta_2(t) = \arctan\left(\frac{1}{\sin\left(-\frac{\pi}{2}\right)} \left(4 \cos\left(-\frac{\pi}{2}\right) - RootOf\left(\left(2 \cos\left(-\frac{\pi}{2}\right) + 2\right) \_Z^2 + \left(-8 \cos\left(-\frac{\pi}{2}\right)^2 - 16 \cos\left(-\frac{\pi}{2}\right) - 8\right) \_Z + 32 \cos\left(-\frac{\pi}{2}\right)^2 + 32 \cos\left(-\frac{\pi}{2}\right)\right) \cos\left(-\frac{\pi}{2}\right)\right.\right.\right. \quad \textbf{(6.1.1)}$$

$$- RootOf\left(\left(2\cos\left(-\frac{\pi}{2}\right)+2\right)\_Z^2+\left(-8\cos\left(-\frac{\pi}{2}\right)^2-16\cos\left(-\frac{\pi}{2}\right)-8\right)\_Z+32\cos\left(-\frac{\pi}{2}\right)^2+32\cos\left(-\frac{\pi}{2}\right)\right)+4\Bigg), RootOf\left(\left(2\cos\left(-\frac{\pi}{2}\right)+2\right)\_Z^2+\left(-8\cos\left(-\frac{\pi}{2}\right)^2-16\cos\left(-\frac{\pi}{2}\right)-8\right)\_Z+32\cos\left(-\frac{\pi}{2}\right)^2+32\cos\left(-\frac{\pi}{2}\right)\right)\Bigg), \theta_3(t)=\arctan\Bigg(-\frac{1}{4\sin\left(-\frac{\pi}{2}\right)}\Bigg(4\cos\left(-\frac{\pi}{2}\right)+RootOf\left(\left(-2\cos\left(-\frac{\pi}{2}\right)+2\right)\_Z^2+\left(8\cos\left(-\frac{\pi}{2}\right)^2-16\cos\left(-\frac{\pi}{2}\right)+8\right)\_Z+32\cos\left(-\frac{\pi}{2}\right)^2-32\cos\left(-\frac{\pi}{2}\right)\right)\cos\left(-\frac{\pi}{2}\right)-RootOf\left(\left(-2\cos\left(-\frac{\pi}{2}\right)+2\right)\_Z^2+\left(8\cos\left(-\frac{\pi}{2}\right)^2-16\cos\left(-\frac{\pi}{2}\right)+8\right)\_Z+32\cos\left(-\frac{\pi}{2}\right)^2-32\cos\left(-\frac{\pi}{2}\right)\right)-4\Bigg), \frac{1}{4}\Bigg(RootOf\left(\left(-2\cos\left(-\frac{\pi}{2}\right)+2\right)\_Z^2+\left(8\cos\left(-\frac{\pi}{2}\right)^2-16\cos\left(-\frac{\pi}{2}\right)+8\right)\_Z+32\cos\left(-\frac{\pi}{2}\right)^2-32\cos\left(-\frac{\pi}{2}\right)\right)\Bigg)\Bigg)\Bigg\}$$

```
> [s__2(t) = 0, diff(s__2(t),t) = 0,s__3(t) = 0,diff(s__3(t),t)
  = 0,theta__1(t)=-Pi/2,diff(theta__1(t),t) = 0];
```

$$\left[s_2(t)=0, \frac{d}{dt}\, s_2(t)=0, s_3(t)=0, \frac{d}{dt}\, s_3(t)=0, \theta_1(t)=-\frac{\pi}{2}, \frac{d}{dt}\, \theta_1(t)=0\right] \quad \textbf{(6.1.2)}$$

Initial conditions for the dae dynamical system

```
> ics_v := {%[2],%[4],%[6]} union subs( %, data, diff(sol_kine,
```

```
t) );

ics := ics_p union ics_v;
```

$$ics\_v := \left\{\frac{d}{dt}\, s_2(t) = 0, \frac{d}{dt}\, s_3(t) = 0, \frac{d}{dt}\, \theta_1(t) = 0, \frac{d}{dt}\, \theta_2(t) = 0, \frac{d}{dt}\, \theta_3(t) = 0\right\}$$

$$ics := \left\{\frac{d}{dt}\, s_2(t) = 0, \frac{d}{dt}\, s_3(t) = 0, \frac{d}{dt}\, \theta_1(t) = 0, \frac{d}{dt}\, \theta_2(t) = 0, \frac{d}{dt}\, \theta_3(t) = 0, s_2(t) = 0, s_3(t) = 0, \theta_1(t) = -\frac{\pi}{2}, \theta_2(t) = \arctan(-4 + RootOf(\_Z^2 - 4\,\_Z), RootOf(\_Z^2 - 4\,\_Z)), \theta_3(t) = \arctan\left(-1 - \frac{RootOf(\_Z^2 + 4\,\_Z)}{4}, \frac{RootOf(\_Z^2 + 4\,\_Z)}{4}\right)\right\} \quad \textbf{(6.1.3)}$$

```
> Phi_tt := diff(Phi,t,t);
```

$$Phi\_tt := \left[2\left(\left(\frac{d}{dt}\,\theta_3(t)\right)\sin(\theta_3(t))\,L_{s0} + \left(\frac{d}{dt}\,\theta_3(t)\right)\sin(\theta_3(t))\,s_3(t) - \cos(\theta_3(t))\left(\frac{d}{dt}\,s_3(t)\right) - \left(\frac{d}{dt}\,\theta_1(t)\right)\sin(\theta_1(t))\,L_{s0}\right)^2 + 2\,(-L - \cos(\theta_3(t))\,L_{s0} - \cos(\theta_3(t))\,s_3(t) + \cos(\theta_1(t))\,L_{s0})\left(\left(\frac{d^2}{dt^2}\,\theta_3(t)\right)\sin(\theta_3(t))\,L_{s0} + \left(\frac{d}{dt}\,\theta_3(t)\right)^2\cos(\theta_3(t))\,L_{s0} + \left(\frac{d^2}{dt^2}\,\theta_3(t)\right)\sin(\theta_3(t))\,s_3(t) + \left(\frac{d}{dt}\,\theta_3(t)\right)^2\cos(\theta_3(t))\,s_3(t) + 2\left(\frac{d}{dt}\,\theta_3(t)\right)\sin(\theta_3(t))\left(\frac{d}{dt}\,s_3(t)\right) - \cos(\theta_3(t))\left(\frac{d^2}{dt^2}\,s_3(t)\right) - \left(\frac{d^2}{dt^2}\,\theta_1(t)\right)\sin(\theta_1(t))\,L_{s0} - \left(\frac{d}{dt}\,\theta_1(t)\right)^2\cos(\theta_1(t))\,L_{s0}\right) + 2\left(-\left(\frac{d}{dt}\,\theta_3(t)\right)\cos(\theta_3(t))\,L_{s0} - \left(\frac{d}{dt}\,\theta_3(t)\right)\cos(\theta_3(t))\,s_3(t) - \sin(\theta_3(t))\left(\frac{d}{dt}\,s_3(t)\right) + \left(\frac{d}{dt}\,\theta_1(t)\right)\cos(\theta_1(t))\,L_{s0}\right)^2 + 2\,(-\sin(\theta_3(t))\,L_{s0} - \sin(\theta_3(t))\,s_3(t) + \sin(\theta_1(t))\,L_{s0})\left(-\left(\frac{d^2}{dt^2}\,\theta_3(t)\right)\cos(\theta_3(t))\,L_{s0} + \left(\frac{d}{dt}\,\theta_3(t)\right)^2\sin(\theta_3(t))\,L_{s0} - \left(\frac{d^2}{dt^2}\,\theta_3(t)\right)\cos(\theta_3(t))\,s_3(t) + \left(\frac{d}{dt}\,\theta_3(t)\right)^2\sin(\theta_3(t))\,s_3(t) - 2\left(\frac{d}{dt}\right.\right.\right. \quad \textbf{(6.1.4)}$$

$$\theta_3(t)\Big)\cos(\theta_3(t))\left(\frac{d}{dt}s_3(t)\right)-\sin(\theta_3(t))\left(\frac{d^2}{dt^2}s_3(t)\right)+\Big(\frac{d^2}{dt^2}$$

$$\theta_1(t)\Big)\cos(\theta_1(t))L_{s0}-\left(\frac{d}{dt}\theta_1(t)\right)^2\sin(\theta_1(t))L_{s0}\Big],2\Big(\Big(\frac{d}{dt}$$

$$\theta_1(t)\Big)\sin(\theta_1(t))L_{s0}-\left(\frac{d}{dt}\theta_2(t)\right)\sin(\theta_2(t))s_2(t)+\cos(\theta_2(t))\left(\frac{d}{dt}s_2(t)\right)$$

$$-\left(\frac{d}{dt}\theta_2(t)\right)\sin(\theta_2(t))L_{s0}\Big)^2+2\big(-L-\cos(\theta_1(t))L_{s0}+\cos(\theta_2(t))s_2(t)$$

$$+\cos(\theta_2(t))L_{s0}\big)\left(\left(\frac{d^2}{dt^2}\theta_1(t)\right)\sin(\theta_1(t))L_{s0}+\left(\frac{d}{dt}\theta_1(t)\right)^2\cos(\theta_1(t))L_{s0}\right.$$

$$-\left(\frac{d^2}{dt^2}\theta_2(t)\right)\sin(\theta_2(t))s_2(t)-\left(\frac{d}{dt}\theta_2(t)\right)^2\cos(\theta_2(t))s_2(t)-2\Big(\frac{d}{dt}$$

$$\theta_2(t)\Big)\sin(\theta_2(t))\left(\frac{d}{dt}s_2(t)\right)+\cos(\theta_2(t))\left(\frac{d^2}{dt^2}s_2(t)\right)-\Big(\frac{d^2}{dt^2}$$

$$\theta_2(t)\Big)\sin(\theta_2(t))L_{s0}-\left(\frac{d}{dt}\theta_2(t)\right)^2\cos(\theta_2(t))L_{s0}\Big)+2\Big(-\Big(\frac{d}{dt}$$

$$\theta_1(t)\Big)\cos(\theta_1(t))L_{s0}+\left(\frac{d}{dt}\theta_2(t)\right)\cos(\theta_2(t))s_2(t)+\sin(\theta_2(t))\left(\frac{d}{dt}s_2(t)\right)$$

$$+\left(\frac{d}{dt}\theta_2(t)\right)\cos(\theta_2(t))L_{s0}\Big)^2+2\big(-\sin(\theta_1(t))L_{s0}+\sin(\theta_2(t))s_2(t)$$

$$+\sin(\theta_2(t))L_{s0}\big)\left(-\left(\frac{d^2}{dt^2}\theta_1(t)\right)\cos(\theta_1(t))L_{s0}+\left(\frac{d}{dt}\theta_1(t)\right)^2\sin(\theta_1(t))L_{s0}\right.$$

$$+\left(\frac{d^2}{dt^2}\theta_2(t)\right)\cos(\theta_2(t))s_2(t)-\left(\frac{d}{dt}\theta_2(t)\right)^2\sin(\theta_2(t))s_2(t)+2\Big(\frac{d}{dt}$$

$$\theta_2(t)\Big)\cos(\theta_2(t))\left(\frac{d}{dt}s_2(t)\right)+\sin(\theta_2(t))\left(\frac{d^2}{dt^2}s_2(t)\right)+\Big(\frac{d^2}{dt^2}$$

$$\theta_2(t)\Big)\cos(\theta_2(t))L_{s0}-\left(\frac{d}{dt}\theta_2(t)\right)^2\sin(\theta_2(t))L_{s0}\Big)\Big]$$

```
> sys_indexred := [op(convert(ode_sys,list)),op(Phi_tt)]: <%>
```

$$\left[\begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 2\left(\left(\frac{d}{dt}\,\theta_3(t)\right)\sin\left(\theta_3(t)\right)L_{s0}+\left(\frac{d}{dt}\,\theta_3(t)\right)\sin\left(\theta_3(t)\right)s_3(t)-\cdots\right. \\ 2\left(\left(\frac{d}{dt}\,\theta_1(t)\right)\sin\left(\theta_1(t)\right)L_{s0}-\left(\frac{d}{dt}\,\theta_2(t)\right)\sin\left(\theta_2(t)\right)s_2(t)+\cdots\right. \end{array}\right] \quad (6.1.5)$$

```
> MM, bb := GenerateMatrix( sys_indexred,[op(diff(q_vars,t,t)),
  lambda__1(t),lambda__2(t)]);
```

$MM, bb :=$ **(6.1.6)**

$$\left[\begin{array}{cc} \cdots & -2\,(-\cdots \\ \cdots\ L_{s0} & -2\,(-L\ \cdots \\ \cdots & \cdots \\ \cdots & -2\,\left(-L-\cos\left(\theta_1(t)\right)L_{s0}+\cos\cdots\right. \\ \cdots\ t)\big)\,L_{s0}-\cos\left(\theta_3(t)\right)s_3(t)\big) & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \end{array}\right],$$

$$\begin{bmatrix} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots & \cdots \\ \cdots \left(\frac{d}{dt}\,\theta_3(t)\right)\sin\big(\theta_3(t)\big)\,L_{s0} + \left(\frac{d}{dt}\,\theta_3(t)\right)\sin\big(\theta_3(t)\big)\,s_3(t) - c & \cdots \\ \cdots \left(\frac{d}{dt}\,\theta_1(t)\right)\sin\big(\theta_1(t)\big)\,L_{s0} - \left(\frac{d}{dt}\,\theta_2(t)\right)\sin\big(\theta_2(t)\big)\,s_2(t) + c & \cdots \end{bmatrix}$$

```
> evalf(subs(ics, data, t=0, MM)):
  evalf(subs(ics, data, t=0, bb)):
  tmp:= LinearSolve(%%,%);
```

$$tmp := \begin{bmatrix} 9.81000000000000 \\ 4.79062896672952 \times 10^{-10} \\ 9.81000000000000 \\ 2.49112706232095 \times 10^{-9} \\ 2.49112706232095 \times 10^{-9} \\ 7.18594344981429 \times 10^{-9} \\ -4.79062896820952 \times 10^{-10} \end{bmatrix} \qquad \textbf{(6.1.7)}$$

```
> <[op(diff(q_vars,t,t)),lambda__1(t),lambda__1(t)]> =  tmp;

  ics_lambda := [lambda__1(t) = tmp[6],lambda__2(t) = tmp[7]];
```

$$\left[\begin{array}{c} \frac{d^2}{dt^2}\, s_2(t) \\ \frac{d^2}{dt^2}\, \theta_1(t) \\ \frac{d^2}{dt^2}\, s_3(t) \\ \frac{d^2}{dt^2}\, \theta_2(t) \\ \frac{d^2}{dt^2}\, \theta_3(t) \\ \lambda_1(t) \\ \lambda_1(t) \end{array}\right] = \left[\begin{array}{c} 9.81000000000000 \\ 4.79062896672952 \times 10^{-10} \\ 9.81000000000000 \\ 2.49112706232095 \times 10^{-9} \\ 2.49112706232095 \times 10^{-9} \\ 7.18594344981429 \times 10^{-9} \\ -4.79062896820952 \times 10^{-10} \end{array}\right]$$

$$ics\_lambda := \left[\lambda_1(t) = 7.18594344981429 \times 10^{-9}, \lambda_2(t) = -4.79062896820952 \times 10^{-10}\right] \tag{6.1.8}$$

System of DAE equations

```
> dae_sys  := convert(subs(data,[op(convert(ode_sys,list)),op
  (Phi)]),set): <op(%)>;
  full_ics := subs( t = 0, convert(convert(ics,set) union
  {ics_lambda[1]} union {ics_lambda[2]}, D));
```

$$\left[\begin{array}{cc} \cdots & \cdots \\ \cdots & \cdots \\ \cdots & 10 \left(\frac{d^2}{dt^2}\, \theta_1(t)\right) \sin(\theta_1(t))^2 + 10 \left(\frac{d^2}{dt^2}\, \theta_1(t)\right) \cos(\theta_1(t))^2 + \cdots \\ \cdots & 2\, \frac{d^2}{dt^2}\, s_2(t) - 2\, (1 + s_2(t \cdots \\ \cdots & 30\, \frac{d^2}{dt^2}\, s_3(t) - 30\, (1 + s_3(t))\, (\cdots \\ \cdots & + s_2(t))^2 \left(\frac{d^2}{dt^2}\, \theta_2(t)\right) + 19.62 \cos(\theta_2(t))\, s_2(t) + 19.62 \cos(\cdots \\ \cdots & {}_3(t))^2 \left(\frac{d^2}{dt^2}\, \theta_3(t)\right) + 294.30 \cos(\theta_3(t)) + 294.30 \cos(\theta_3(t)) \cdots \end{array}\right]$$

$$full\_ics := \{s_2(0) = 0, s_3(0) = 0, \lambda_1(0) = 7.18594344981429 \times 10^{-9}, \lambda_2(0) \tag{6.1.9}$$

$$= -4.79062896820952 \times 10^{-10}, \theta_1(0) = -\frac{\pi}{2}, \theta_2(0) = \arctan\left(-4 + RootOf(\_Z^2 - 4\_Z), RootOf(\_Z^2 - 4\_Z)\right), \theta_3(0) = \arctan\left(-1 - \frac{RootOf(\_Z^2 + 4\_Z)}{4}, \frac{RootOf(\_Z^2 + 4\_Z)}{4}\right), D(s_2)(0) = 0, D(s_3)(0) = 0, D(\theta_1)(0) = 0, D(\theta_2)(0) = 0, D(\theta_3)(0) = 0\}$$

```
> sol_dae := dsolve(convert(dae_sys,set) union full_ics,numeric,
  implicit=true,maxfun=300000);
```

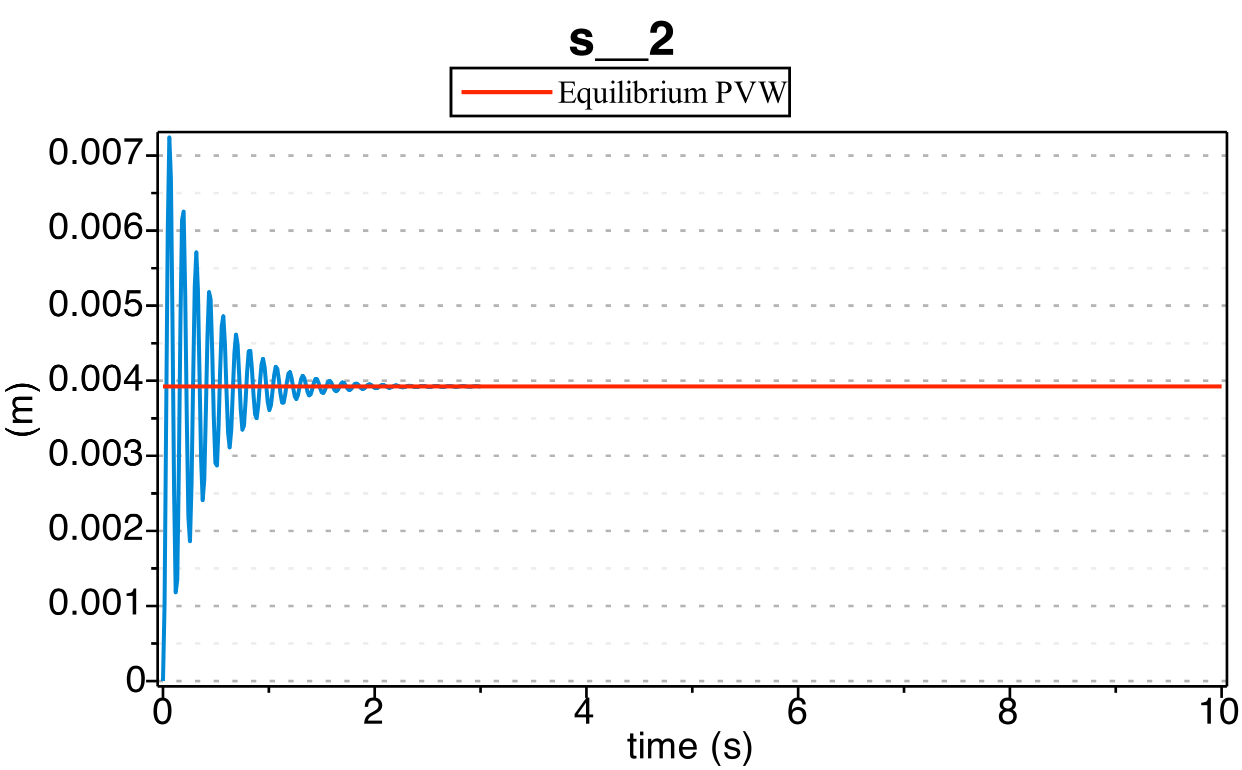
$$sol\_dae := \mathbf{proc}(x\_rkf45\_dae) \ \ldots \ \mathbf{end\ proc} \qquad \mathbf{(6.1.10)}$$

Plot the results, motion of masses and reaction forces

```
> TF := 10:
```

Plot of s2 with the equilibrium value found using PVW

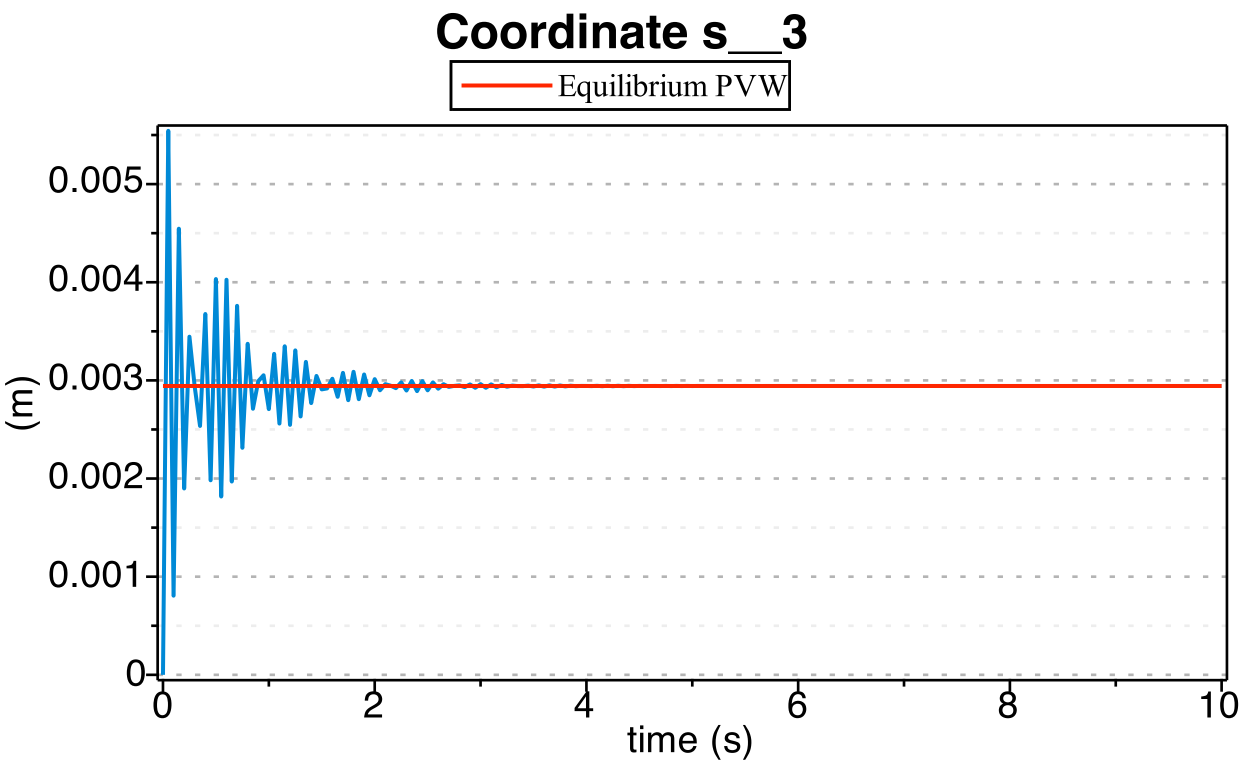
```
> display(odeplot(sol_dae,[t,s__2(t)],t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = " s__2 "),plot(op(2,sol_fs[1]),x=0..10,color=
  "red",legend="Equilibrium PVW"),size=[800,300]);
```



Plot of s3 with the equilibrium value found using PVW

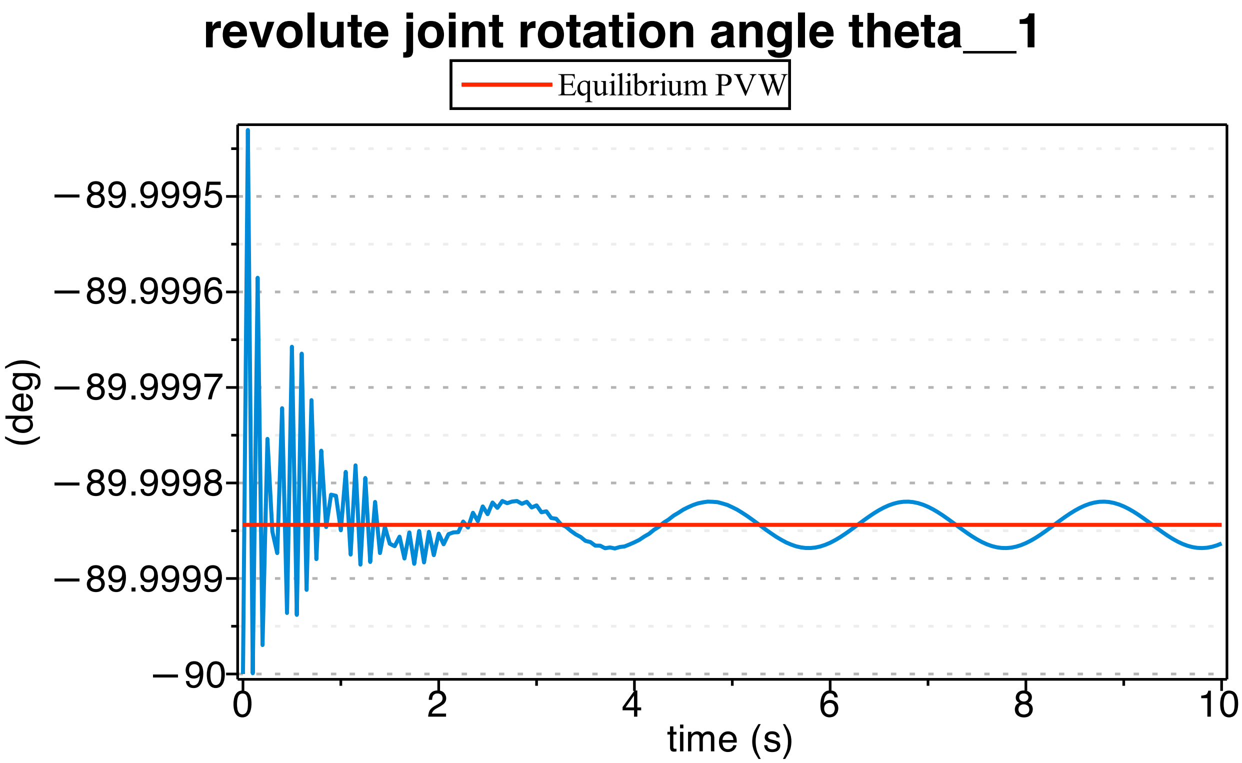
```
> display(odeplot(sol_dae,[t,s__3(t)],t=0..TF,
```

```
        labels = ["time (s)", "(m)"],
        title  = "Coordinate s__3"),plot(op(2,sol_fs[2]),x=0.
.10,color="red",legend="Equilibrium PVW"),size=[800,300]);
```



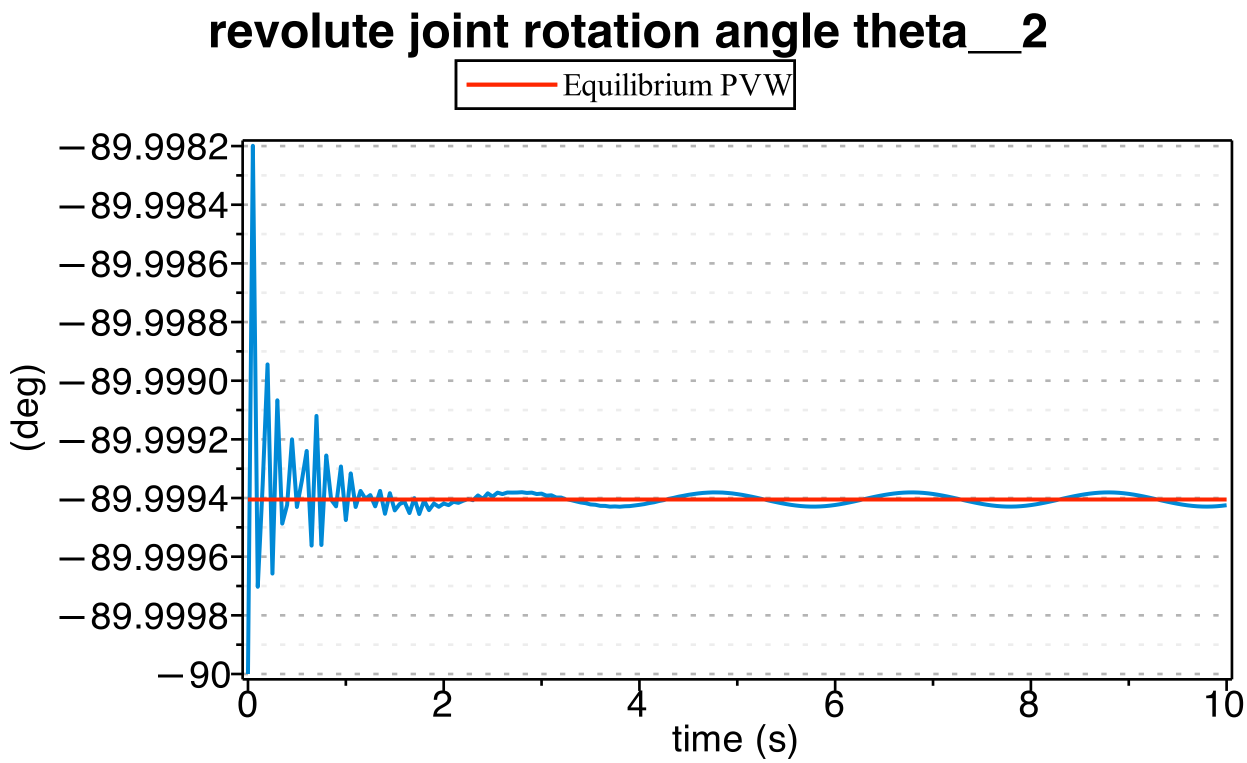
Plot of theta1 with the equilibrium value found using PVW

```
> display(odeplot(sol_dae,[t,theta__1(t)*180/Pi],t=0..TF,
        labels = ["time (s)", "(deg)"],
        title  = "revolute joint rotation angle theta__1"),
  plot(op(2,sol_fs[3]*180/Pi),x=0..10,color="red",legend=
  "Equilibrium PVW") ,size=[800,300]);
```



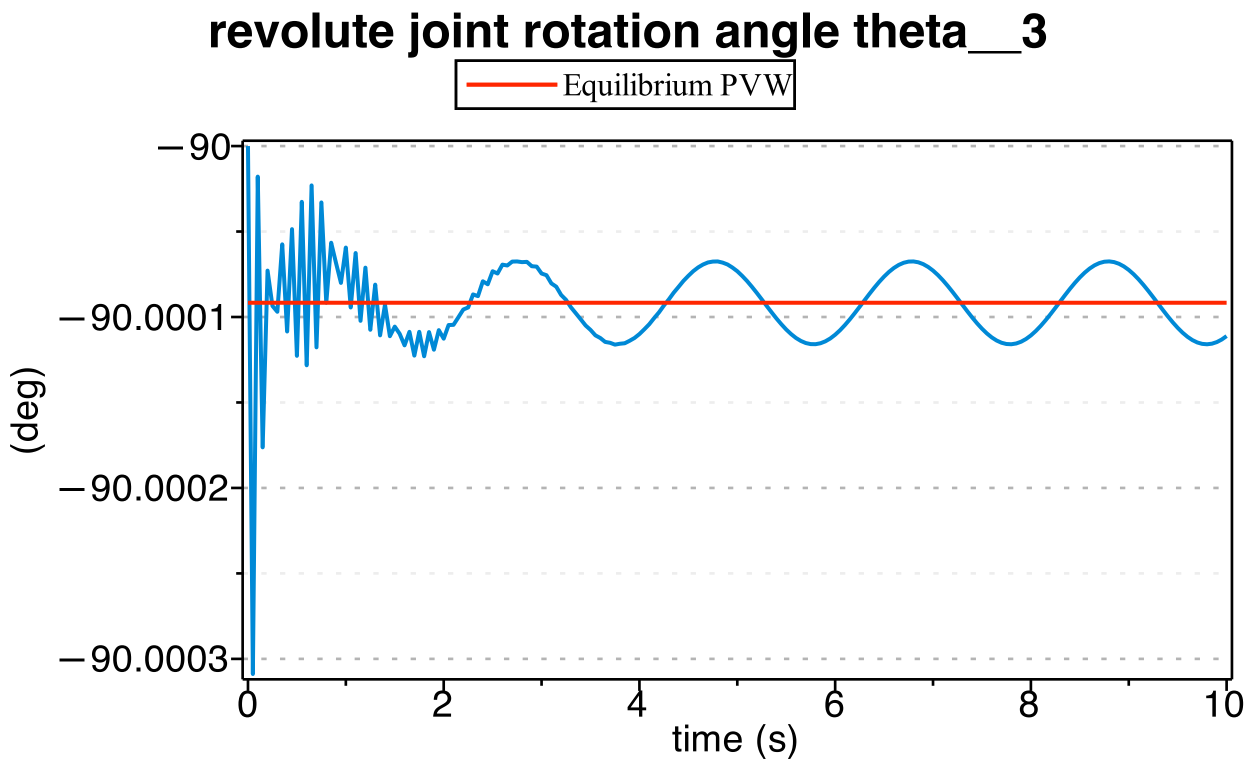
Plot of theta2 with the equilibrium value found using PVW

```
> display(odeplot(sol_dae,[t,theta__2(t)*180/Pi],t=0..TF,
          labels = ["time (s)", "(deg)"],
          title  = "revolute joint rotation angle theta__2" ),
  plot(op(2,sol_fs[4]*180/Pi),x=0..10,color="red",legend=
  "Equilibrium PVW") ,size=[800,300]);
```



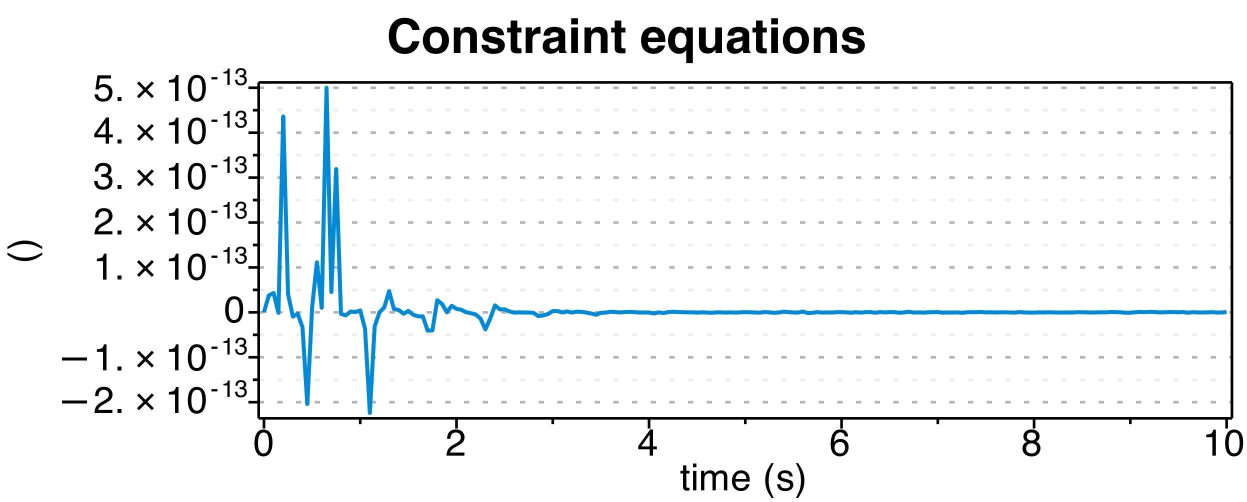
Plot of theta3 with the equilibrium value found using PVW

```
> display(odeplot(sol_dae,[t,theta__3(t)*180/Pi],t=0..TF,
          labels = ["time (s)", "(deg)"],
          title  = "revolute joint rotation angle theta__3" ),
  plot(op(2,sol_fs[5]*180/Pi),x=0..10,color="red",legend=
  "Equilibrium PVW") ,size=[800,300]);
```



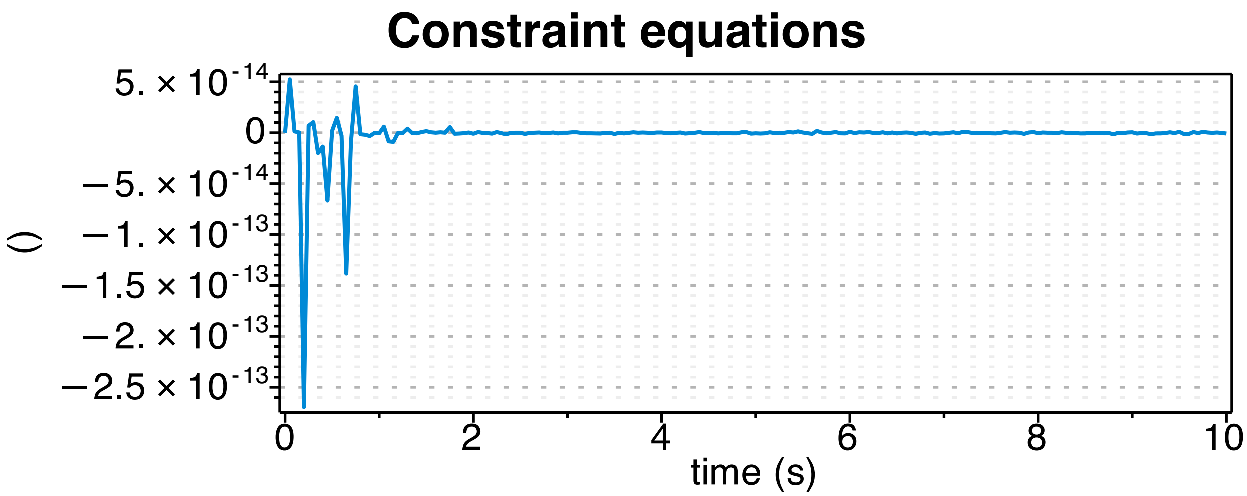
Plot of the constraint equation Phi1

```
> odeplot(sol_dae,subs(data,[t,Phi[1]]),t=0..TF,
          labels = ["time (s)", "()"],
          title  = "Constraint equations" );
```



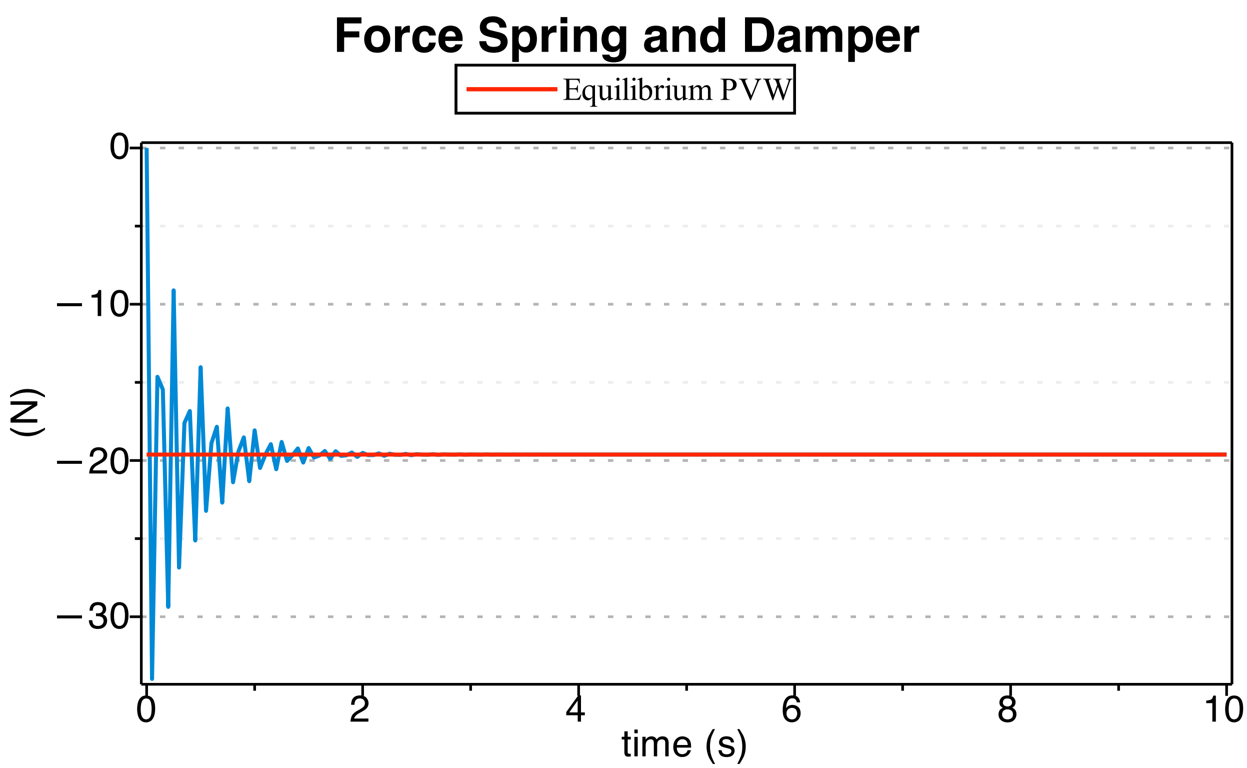
Plot of the constraint equation Phi2

```
> odeplot(sol_dae,subs(data,[t,Phi[2]]),t=0..TF,
          labels = ["time (s)", "()"],
          title  = "Constraint equations" );
```



Force of Spring K__s2 and Damper C__s2

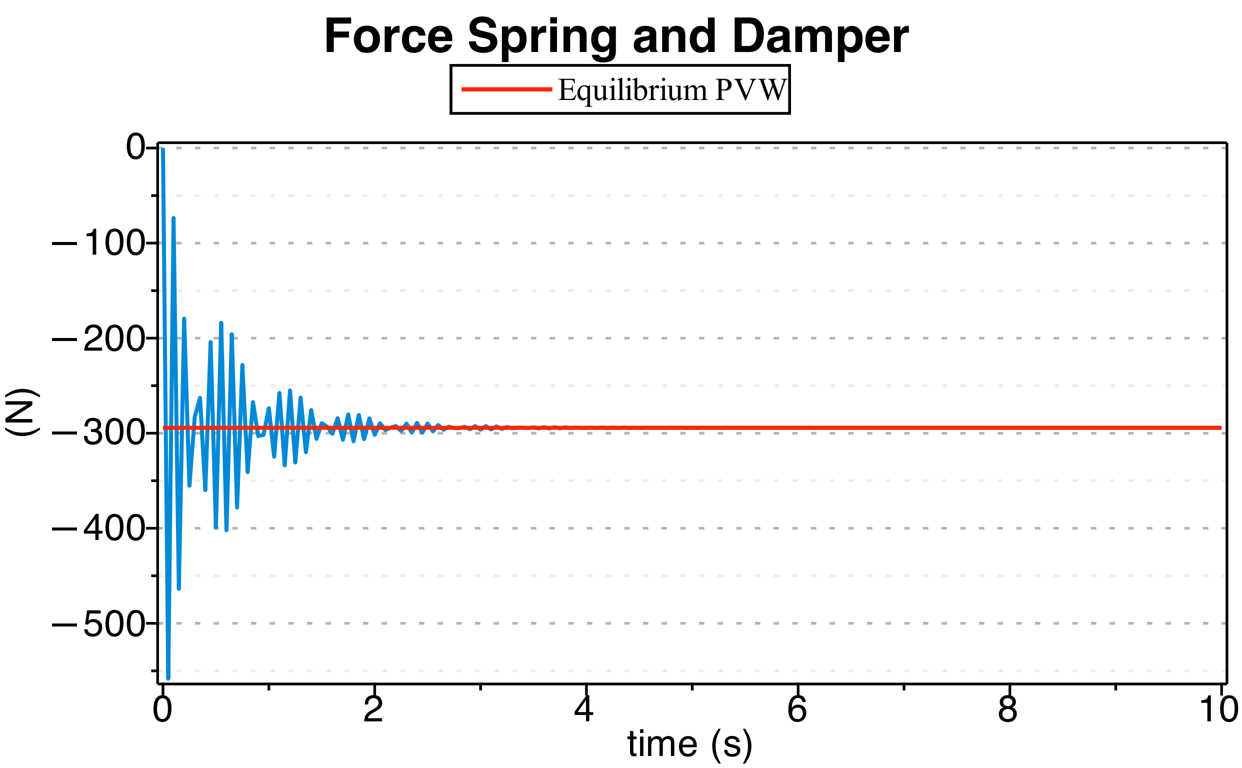
```
> display(odeplot(sol_dae,subs(data,[t,-K__s2*s__2(t)-C__s2*diff
  (s__2(t),t)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Force Spring and Damper"),plot( subs(data,-
  K__s2*op(2,sol_fs[1] )),x=0..10,color="red",legend=
  "Equilibrium PVW"),size=[800,300]);
```



Force of Spring K__s3 and Damper C__s3

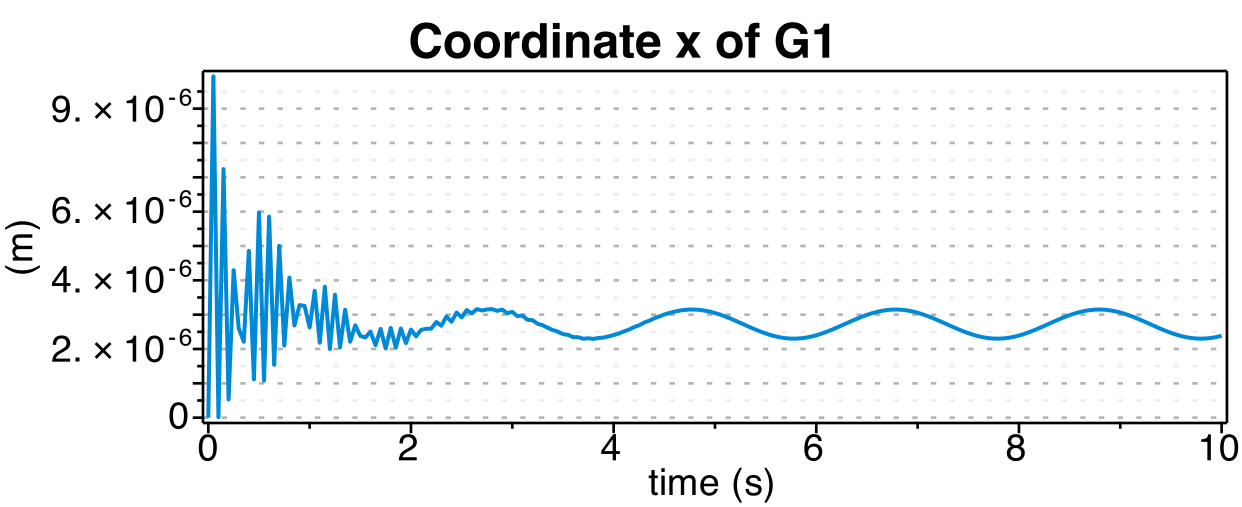
```
> display(odeplot(sol_dae,subs(data,[t,-K__s3*s__3(t)-C__s3*diff
  (s__3(t),t)]),t=0..TF,
          labels = ["time (s)", "(N)"],
```

```
        title  = "Force Spring and Damper "),plot( subs(data,-
K__s3*op(2,sol_fs[2] )),x=0..10,color="red",legend=
"Equilibrium PVW"),size=[800,300]);
```



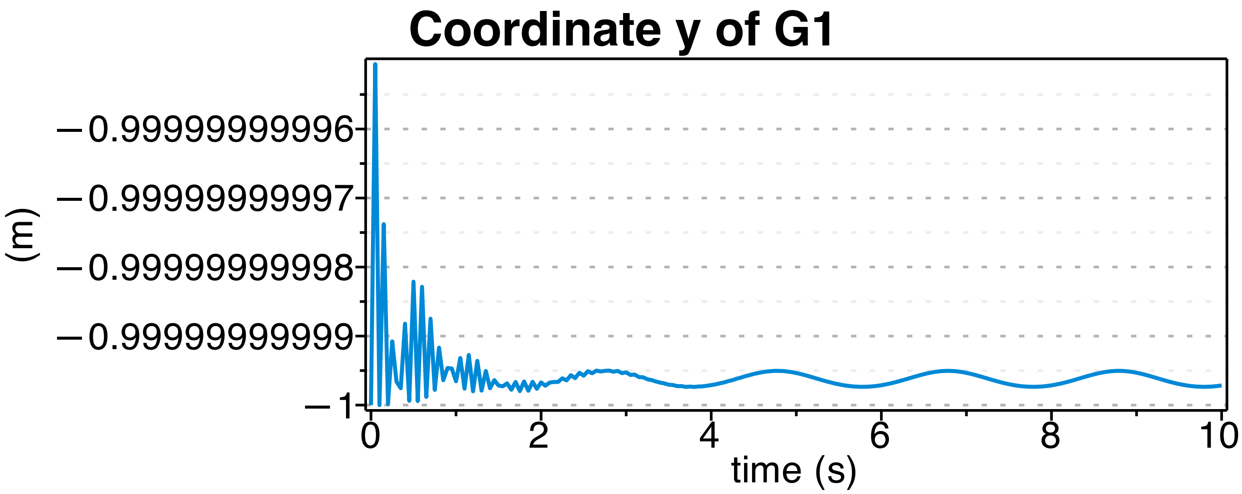
Plot the Motion of the different masses

```
> odeplot(sol_dae,subs(data,[t,G1[1]]),t=0..TF,
        labels = ["time (s)", "(m)"],
        title  = "Coordinate x of G1" );
```



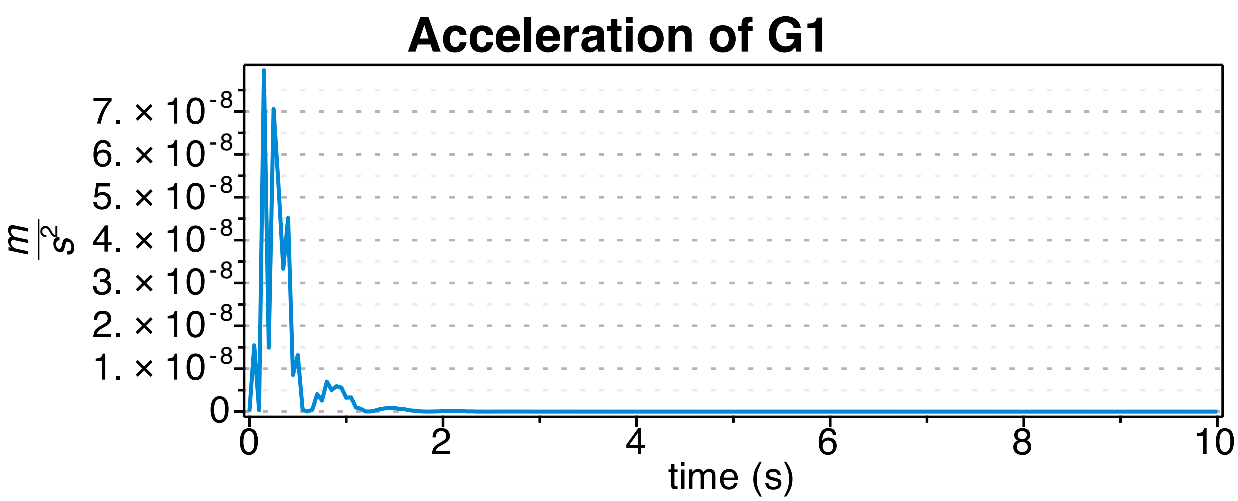

```
> odeplot(sol_dae,subs(data,[t,G1[2]]),t=0..TF,
        labels = ["time (s)", "(m)"],
```

```
        title  = "Coordinate y of G1");
```

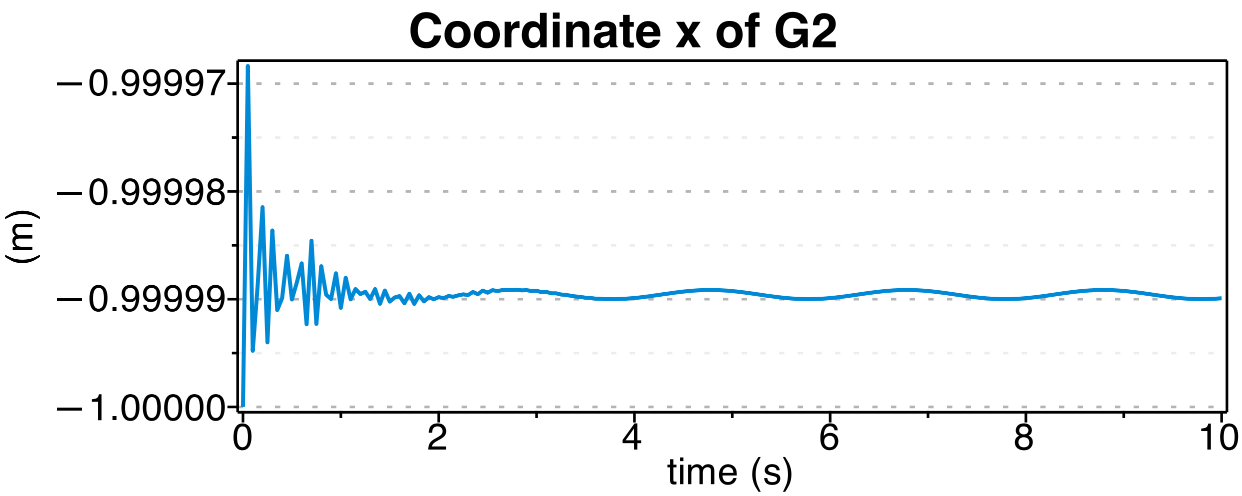


Plot the modulus of the acceleration of mass G1

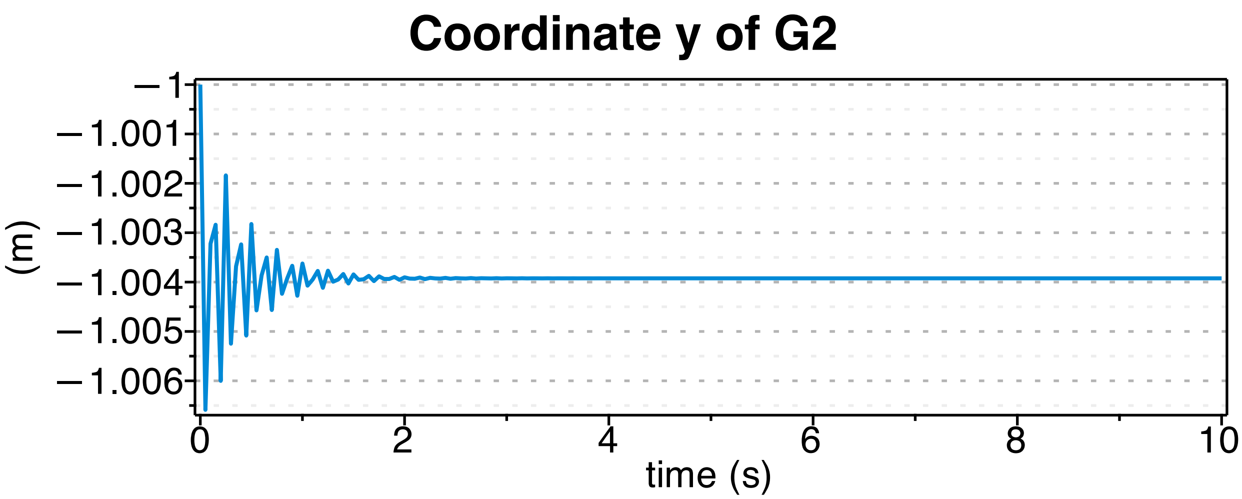
```
> odeplot(sol_dae,subs(data,[t,sqrt(diff(G1[1],t,t)^2+diff(G1
  [2],t,t)^2)]),t=0..TF,
        labels = ["time (s)", m/s^2],
        title  = "Acceleration of G1");
```




```
> odeplot(sol_dae,subs(data,[t,G2[1]]),t=0..TF,
        labels = ["time (s)", "(m)"],
        title  = "Coordinate x of G2" );
```

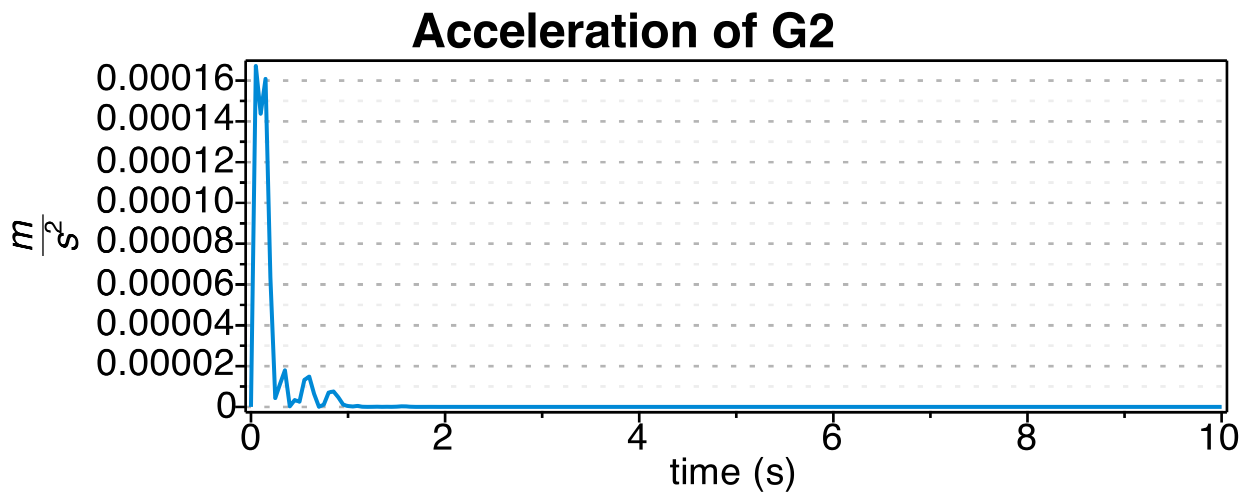



```
> odeplot(sol_dae,subs(data,[t,G2[2]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate y of G2");
```

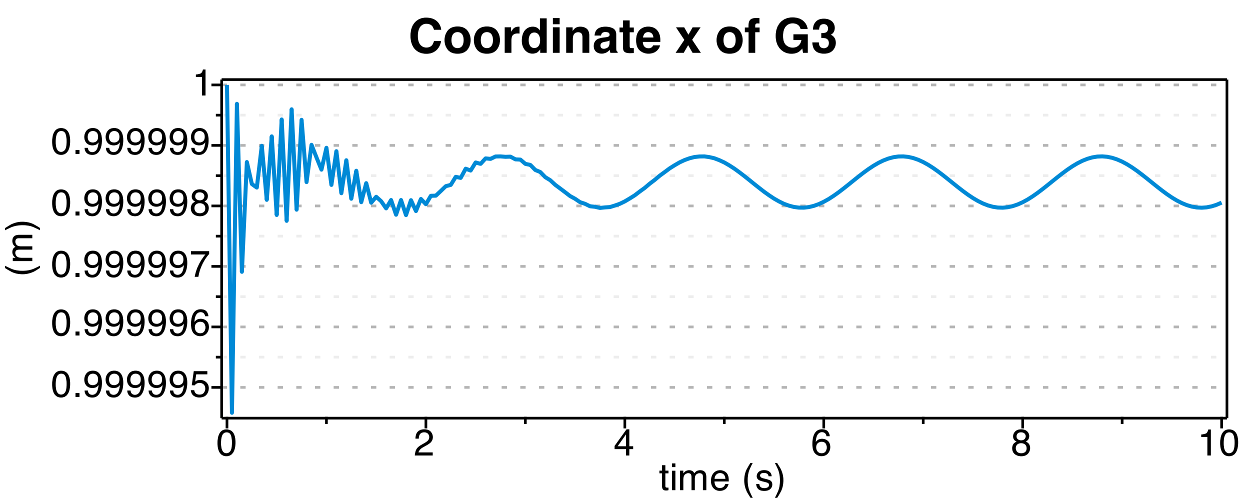


Plot the modulus of the acceleration of mass G2

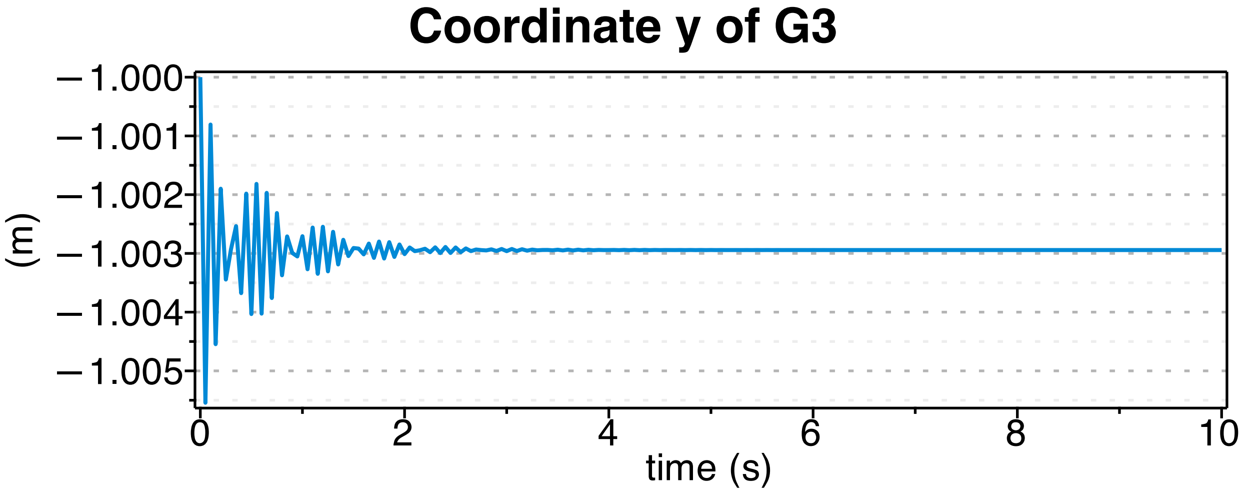
```
> odeplot(sol_dae,subs(data,[t,sqrt(diff(G2[1],t,t)^2+diff(G2
  [2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],
          title  = "Acceleration of G2");
```




```
> odeplot(sol_dae,subs(data,[t,G3[1]]),t=0..TF,
         labels = ["time (s)", "(m)"],
         title  = "Coordinate x of G3");
```

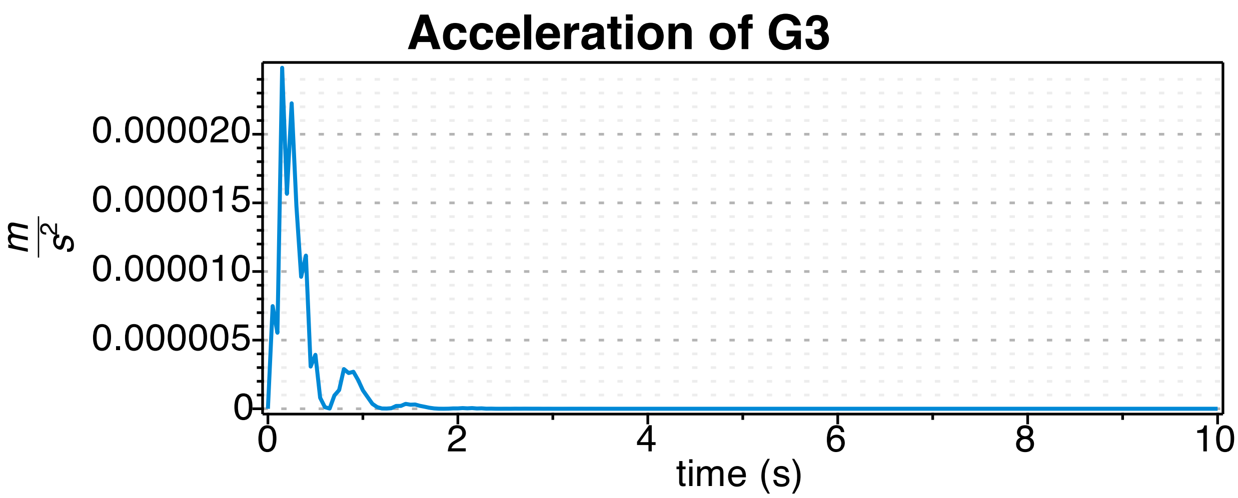



```
> odeplot(sol_dae,subs(data,[t,G3[2]]),t=0..TF,
         labels = ["time (s)", "(m)"],
         title  = "Coordinate y of G3");
```



Plot the modulus of the acceleration of mass G3

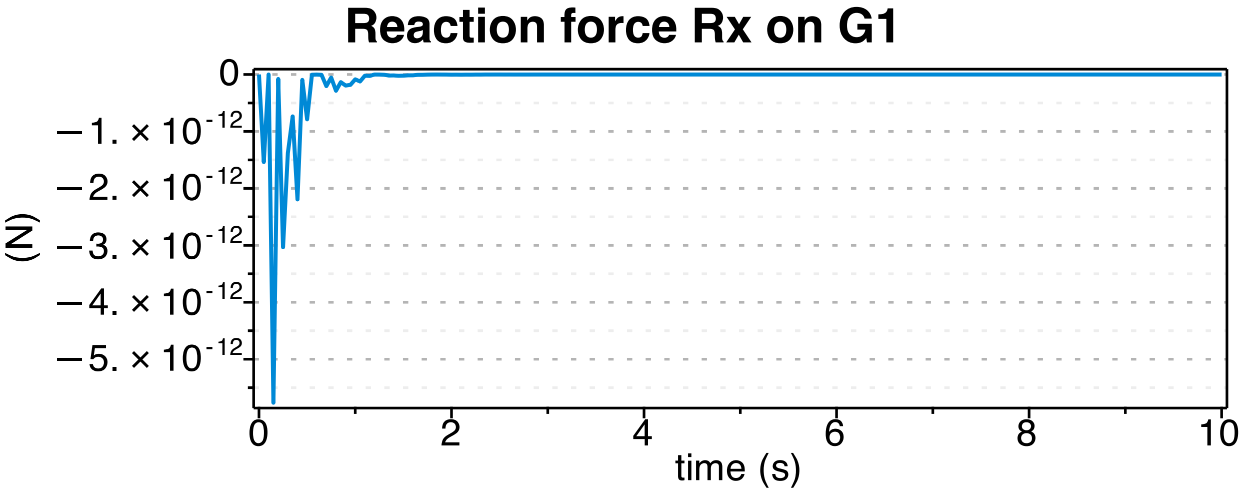
```
> odeplot(sol_dae,subs(data,[t,sqrt(diff(G3[1],t,t)^2+diff(G3
  [2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],
          title  = "Acceleration of G3");
```



Plot the Reaction forces on the different masses

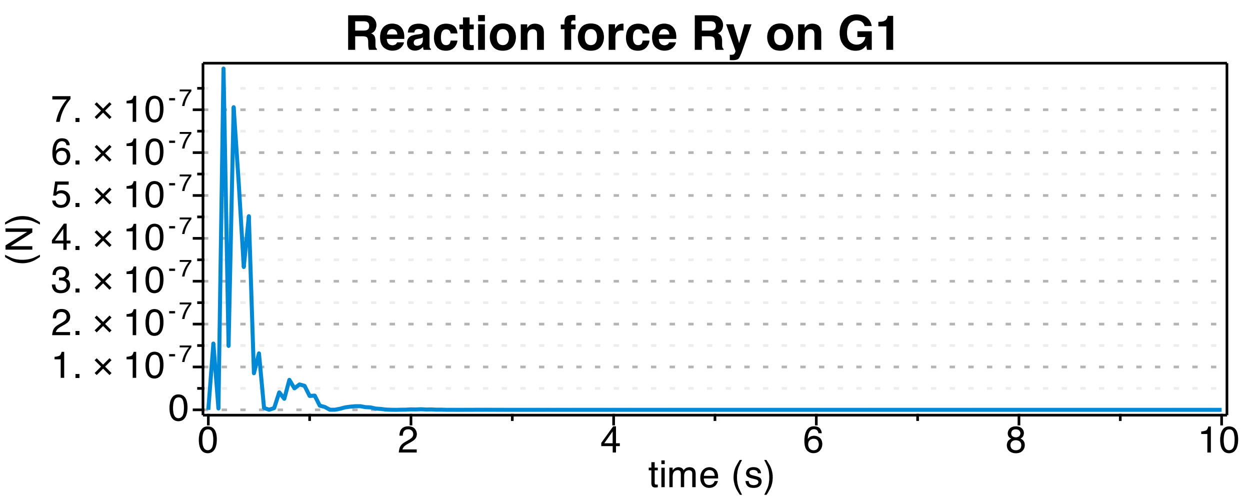
Plot the reaction force Rx acting on mass G1

```
> odeplot(sol_dae,subs(data,[t,m__1*diff(G1[1],t,t)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Reaction force Rx on G1" );
```



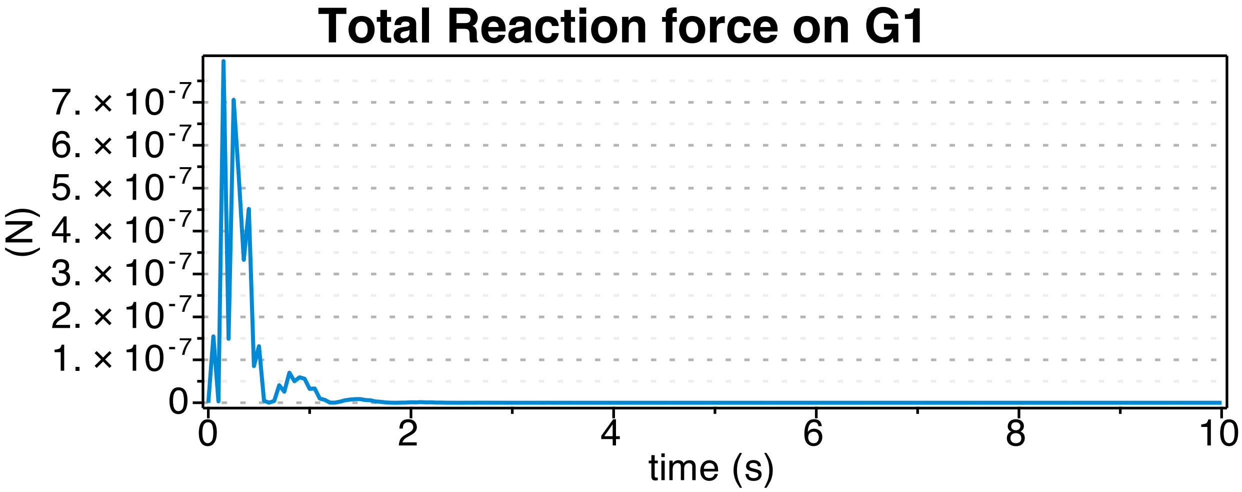
Plot the reaction force Ry acting on mass G1

```
> odeplot(sol_dae,subs(data,[t,m__1*diff(G1[2],t,t)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Reaction force Ry on G1" );
```



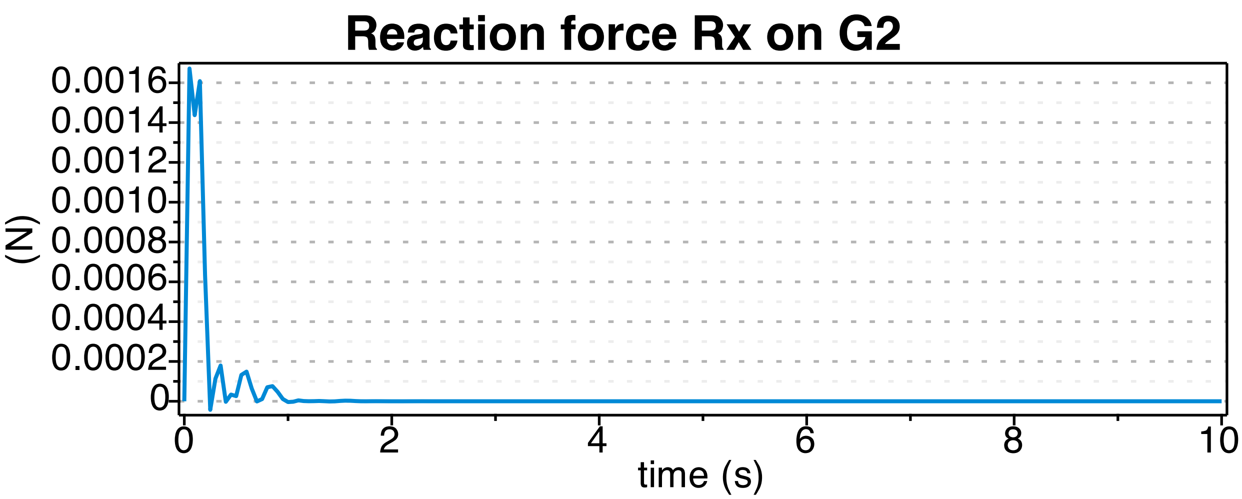
Plot the modulus of the reaction forces acting on mass G1

```
> odeplot(sol_dae,subs(data,[t,sqrt((m__1*diff(G1[1],t,t))^2+
  (m__1*diff(G1[2],t,t))^2)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Total Reaction force on G1" );
```



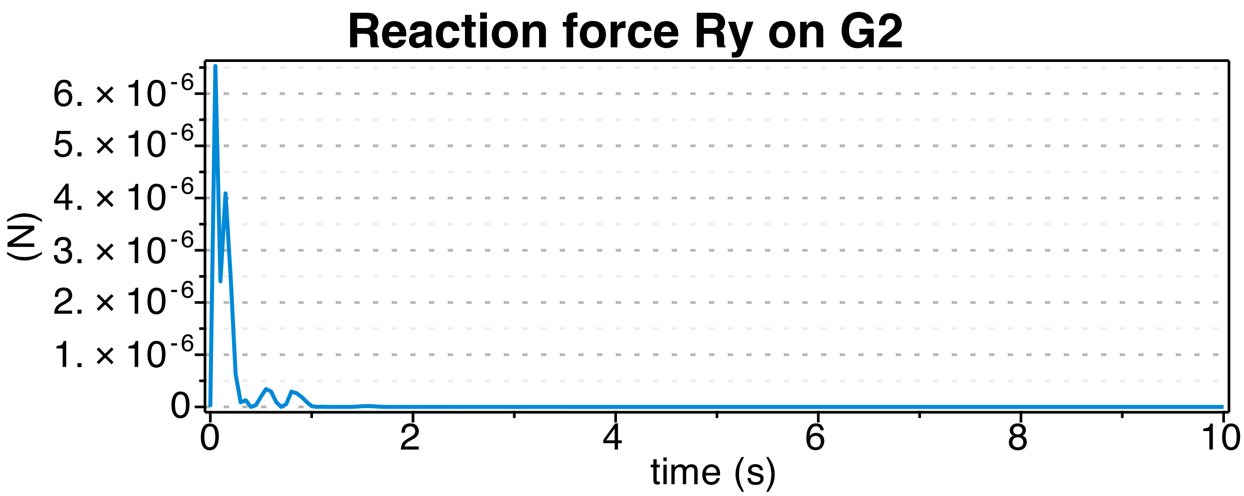
Plot the reaction force Rx acting on mass G2

```
> odeplot(sol_dae,subs(data,[t,m__1*diff(G2[1],t,t)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Reaction force Rx on G2" );
```



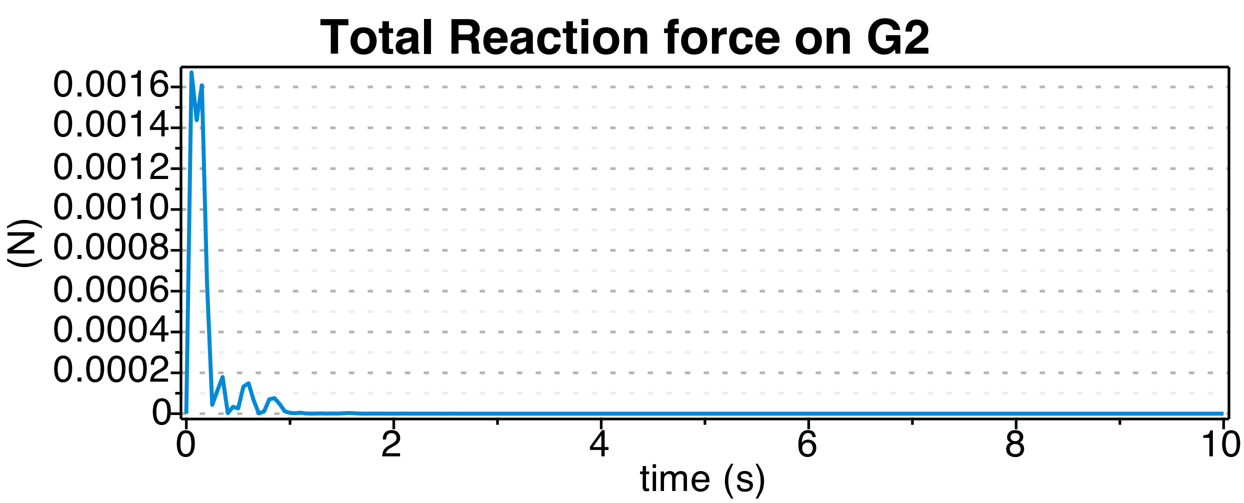
Plot the reaction force Ry acting on mass G2

```
> odeplot(sol_dae,subs(data,[t,m__1*diff(G2[2],t,t)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Reaction force Ry on G2" );
```



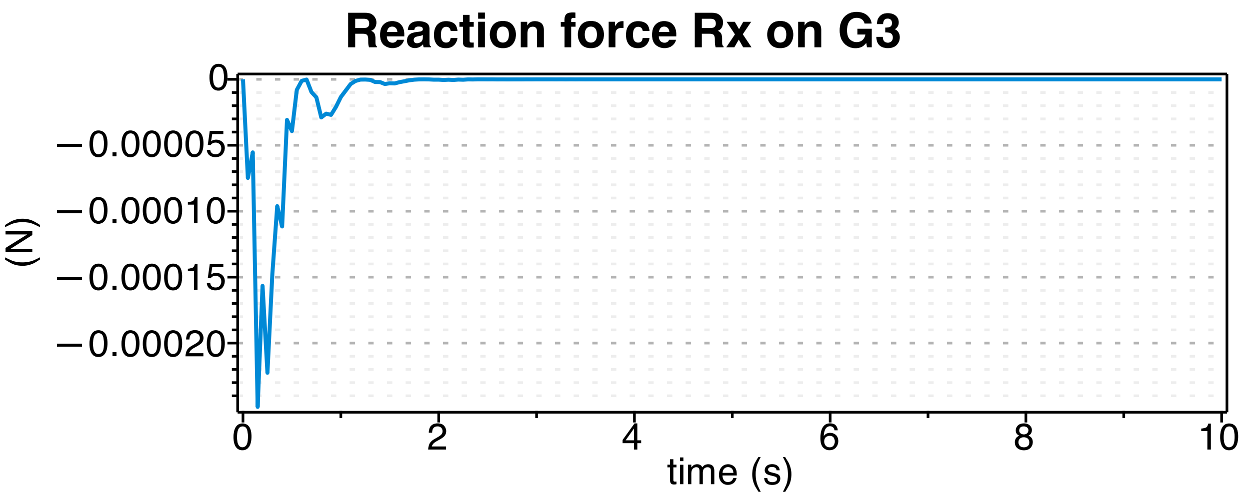
Plot the modulus of the reaction forces acting on mass G2

```
> odeplot(sol_dae,subs(data,[t,sqrt((m__1*diff(G2[1],t,t))^2+
  (m__1*diff(G2[2],t,t))^2)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Total Reaction force on G2" );
```



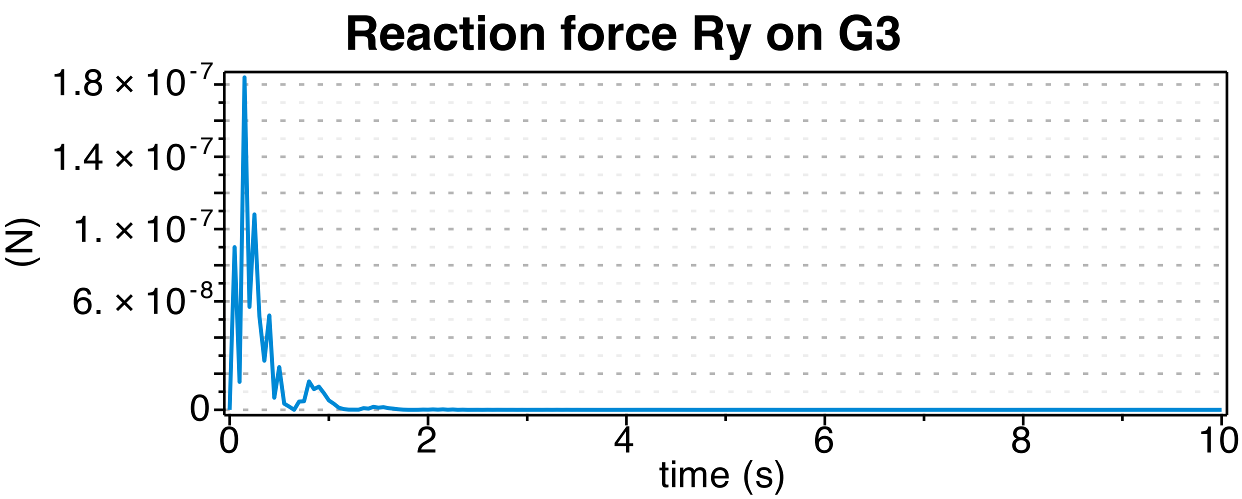
Plot the reaction force Rx acting on mass G3

```
> odeplot(sol_dae,subs(data,[t,m__1*diff(G3[1],t,t)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Reaction force Rx on G3" );
```



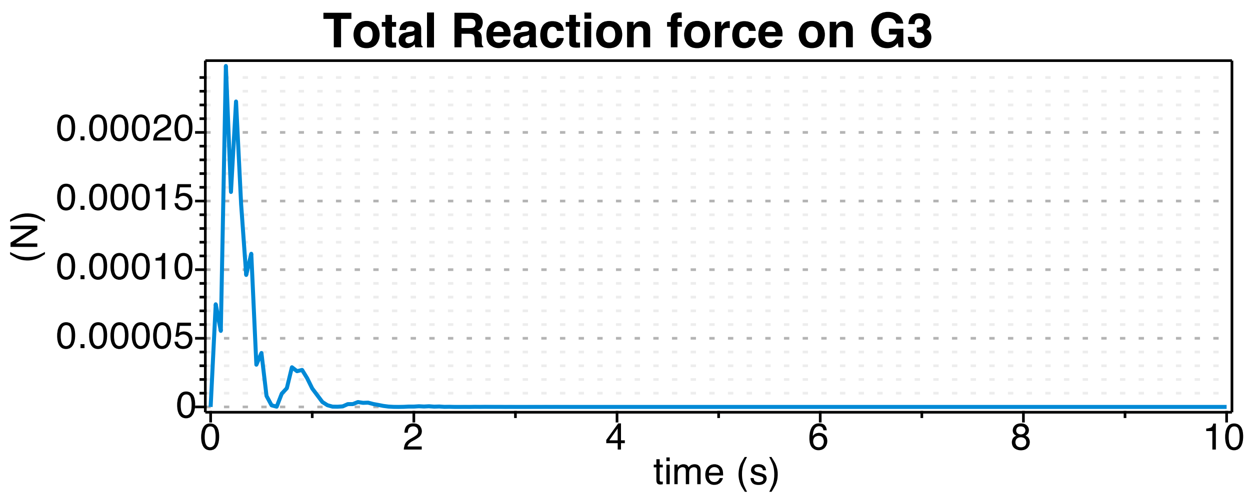
Plot the reaction force Ry acting on mass G3

```
> odeplot(sol_dae,subs(data,[t,m__1*diff(G3[2],t,t)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Reaction force Ry on G3" );
```



Plot the modulus of the reaction forces acting on mass G3

```
> odeplot(sol_dae,subs(data,[t,sqrt((m__1*diff(G3[1],t,t))^2+
  (m__1*diff(G3[2],t,t))^2)]),t=0..TF,
          labels = ["time (s)", "(N)"],
          title  = "Total Reaction force on G3" );
```



Index-reduction and projection method (prof. Bertolazzi)

In this section the dynamic system is solved using the index-reduction and projection method explained by prof. Bertolazzi.

```
> dae_sys := convert([op(convert(ode_sys,list)), op(Phi)],set):
  <op(%)>;
```

$$\left[\begin{array}{c} \cdots \quad \cdots \\ \cdots \quad \cdots \\ \cdots {}_{1}(t)\big)^{2} L_{s0}^{2}\Big) \\ \hline \end{array} + m_{1}\, g \cos\big(\theta_{1}(t)\big) L_{s0} - \big(-2\,\big(-L - \cos\big(\theta_{3}(t)\big) \cdots \\ \cdots\ s_{2}(t)\Big) - m_{2}\,\big(L_{s0} + s_{2}(t)\big)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,\theta_{2}(t)\right)^{2} + m_{2} \sin\big(\theta_{2}(t)\big)\, g - \cdots \\ \cdots\ s_{3}(t)\Big) - m_{3}\,\big(L_{s0} + s_{3}(t)\big)\left(\frac{\mathrm{d}}{\mathrm{d}t}\,\theta_{3}(t)\right)^{2} + m_{3} \sin\big(\theta_{3}(t)\big)\, g - \big(\cdots \\ \cdots (t) + \cos\big(\theta_{2}(t)\big) L_{s0}\big)\, g - \big(2\,\big(-L - \cos\big(\theta_{1}(t)\big) L_{s0} + \cos\big(\theta_{2}(\cdots \\ \cdots {}_{0} + \cos\big(\theta_{3}(t)\big)\, s_{3}(t)\big)\, g - \big(2\,\big(-L - \cos\big(\theta_{3}(t)\big) L_{s0} - \cos\big(\theta_{3}(\cdots \end{array}\right] \qquad \textbf{(6.3.1)}$$

```
> ALG := <
    dae_sys[1],
    dae_sys[2]
  >;
```

$$ALG := \left[\begin{array}{c} \left(-L - \cos\left(\theta_1(t)\right) L_{s0} + \cos\left(\theta_2(t)\right) s_2(t) + \cos\left(\theta_2(t)\right) L_{s0}\right)^2 + \left(\cdots \\ \left(-L - \cos\left(\theta_3(t)\right) L_{s0} - \cos\left(\theta_3(t)\right) s_3(t) + \cos\left(\theta_1(t)\right) L_{s0}\right)^2 + \left(\cdots \end{array}\right] \quad \textbf{(6.3.2)}$$

```
> ODE := <
    dae_sys[3],
    dae_sys[4],
    dae_sys[5],
    dae_sys[6],
    dae_sys[7]
  >;
```

$$ODE := \left[\begin{array}{c} \cdots \dfrac{\left(\theta_1(t)\right)^2 L_{s0}^2 + 2\left(\frac{d^2}{dt^2}\theta_1(t)\right)\cos\left(\theta_1(t)\right)^2 L_{s0}^2\right)}{2} + m_1 g \cos\left(\theta \cdots \\ \cdots \quad m_2\left(\frac{d^2}{dt^2}s_2(t)\right) - m_2\left(L_{s0} + s_2(t)\right. \cdots \\ \cdots \quad m_3\left(\frac{d^2}{dt^2}s_3(t)\right) - m_3\left(L_{s0} + s_3(t)\right) \cdots \\ \cdots)^2\left(\frac{d^2}{dt^2}\theta_2(t)\right) + m_2\left(\cos\left(\theta_2(t)\right) s_2(t) + \cos\left(\theta_2(t)\right) L_{s0}\right) g - \cdots \\ \cdots)^2\left(\frac{d^2}{dt^2}\theta_3(t)\right) + m_3\left(\cos\left(\theta_3(t)\right) L_{s0} + \cos\left(\theta_3(t)\right) s_3(t)\right) g - \cdots \end{array}\right] \quad \textbf{(6.3.3)}$$

```
> SS_DIFF_TO_DOT := diff(s__2(t),t)=x1(t),
                    diff(s__3(t),t)=x2(t),
                    diff(theta__1(t),t)=x3(t),
                    diff(theta__2(t),t)=x4(t),
                    diff(theta__3(t),t)=x5(t),
                    diff(x1(t),t)=x1dot(t),
                    diff(x2(t),t)=x2dot(t),
                    diff(x3(t),t)=x3dot(t),
                    diff(x4(t),t)=x4dot(t),
                    diff(x5(t),t)=x5dot(t);
```

$$SS\_DIFF\_TO\_DOT := \frac{d}{dt} s_2(t) = x1(t), \frac{d}{dt} s_3(t) = x2(t), \frac{d}{dt}\theta_1(t) = x3(t), \frac{d}{dt}\theta_2(t) = x4(t), \frac{d}{dt}\theta_3(t) = x5(t), \frac{d}{dt} x1(t) = x1dot(t), \frac{d}{dt} x2(t) = x2dot(t), \frac{d}{dt} x3(t) = x3dot(t), \frac{d}{dt} x4(t) = x4dot(t), \frac{d}{dt} x5(t) = x5dot(t) \quad \textbf{(6.3.4)}$$

```
> RM1:=x3dot(t)=solve(dae_sys[3], diff(theta__1(t),t,t));
```

$$RM1 := x3dot(t) \quad \textbf{(6.3.5)}$$

$$= \frac{1}{L_{s0}\, m_1 \left(\cos(\theta_1(t))^2 + \sin(\theta_1(t))^2\right)} \left(2 L_{s0} \sin(\theta_1(t)) \cos(\theta_3(t)) \lambda_1(t) + 2 L_{s0} \sin(\theta_1(t)) \cos(\theta_2(t)) \lambda_2(t) - 2 L_{s0} \cos(\theta_1(t)) \sin(\theta_3(t)) \lambda_1(t) - 2 L_{s0} \cos(\theta_1(t)) \sin(\theta_2(t)) \lambda_2(t) + 2 \sin(\theta_1(t)) \cos(\theta_3(t)) s_3(t) \lambda_1(t) + 2 \sin(\theta_1(t)) \cos(\theta_2(t)) s_2(t) \lambda_2(t) - 2 \cos(\theta_1(t)) s_3(t) \sin(\theta_3(t)) \lambda_1(t) - 2 \cos(\theta_1(t)) s_2(t) \sin(\theta_2(t)) \lambda_2(t) + 2 L \sin(\theta_1(t)) \lambda_1(t) - 2 L \sin(\theta_1(t)) \lambda_2(t) - \cos(\theta_1(t))\, g\, m_1\right)$$

```
> RM2:=x4dot(t)=solve(dae_sys[6], diff(theta__2(t),t,t));
```

$$RM2 := x4dot(t) = \frac{1}{m_2 \left(L_{s0} + s_2(t)\right)} \left(-2 L_{s0} \sin(\theta_1(t)) \cos(\theta_2(t)) \lambda_2(t) + 2 L_{s0} \cos(\theta_1(t)) \sin(\theta_2(t)) \lambda_2(t) + 2 L \sin(\theta_2(t)) \lambda_2(t) - \cos(\theta_2(t))\, g\, m_2 - 2 \left(\frac{d}{dt} s_2(t)\right) \left(\frac{d}{dt} \theta_2(t)\right) m_2\right) \quad \textbf{(6.3.6)}$$

```
> RM3:=x5dot(t)=solve(dae_sys[7], diff(theta__3(t),t,t));
```

$$RM3 := x5dot(t) = -\frac{1}{m_3 \left(L_{s0} + s_3(t)\right)} \left(-2 L_{s0} \cos(\theta_1(t)) \sin(\theta_3(t)) \lambda_1(t) + 2 L_{s0} \sin(\theta_1(t)) \cos(\theta_3(t)) \lambda_1(t) + 2 L \sin(\theta_3(t)) \lambda_1(t) + \cos(\theta_3(t))\, g\, m_3 + 2 \left(\frac{d}{dt} s_3(t)\right) \left(\frac{d}{dt} \theta_3(t)\right) m_3\right) \quad \textbf{(6.3.7)}$$

```
> RM4:=x1dot(t)=solve(dae_sys[4], diff(s__2(t),t,t));
```

$$RM4 := x1dot(t) = -\frac{1}{m_2} \left(C_{s2} \cos(\theta_2(t))^2 \left(\frac{d}{dt} s_2(t)\right) + C_{s2} \sin(\theta_2(t))^2 \left(\frac{d}{dt} s_2(t)\right) + K_{s2} \cos(\theta_2(t))^2 s_2(t) + K_{s2} \sin(\theta_2(t))^2 s_2(t) + 2 L_{s0} \sin(\theta_1(t)) \lambda_2(t) \sin(\theta_2(t)) + 2 L_{s0} \cos(\theta_1(t)) \cos(\theta_2(t)) \lambda_2(t) - 2 L_{s0} \cos(\theta_2(t))^2 \lambda_2(t) - 2 L_{s0} \lambda_2(t) \sin(\theta_2(t))^2 - m_2 \left(\frac{d}{dt} \theta_2(t)\right)^2 L_{s0} - 2 \cos(\theta_2(t))^2 \lambda_2(t) s_2(t) - 2 \lambda_2(t) \sin(\theta_2(t))^2 s_2(t) - m_2 \left(\frac{d}{dt} \theta_2(t)\right)^2 s_2(t) + 2 L \cos(\theta_2(t)) \lambda_2(t) + m_2 \sin(\theta_2(t))\, g\right) \quad \textbf{(6.3.8)}$$

```
> RM5:=x2dot(t)=solve(dae_sys[5], diff(s__3(t),t,t));
```

$$RM5 := x2dot(t) = -\frac{1}{m_3} \left(C_{s3} \cos(\theta_3(t))^2 \left(\frac{d}{dt} s_3(t)\right) + C_{s3} \sin(\theta_3(t))^2 \left(\frac{d}{dt} \right.\right. \quad \textbf{(6.3.9)}$$

$$s_3(t)\Big) + K_{s3}\cos(\theta_3(t))^2 s_3(t) + K_{s3}\sin(\theta_3(t))^2 s_3(t)$$

$$+ 2 L_{s0}\sin(\theta_1(t))\,\lambda_1(t)\sin(\theta_3(t)) + 2 L_{s0}\cos(\theta_1(t))\cos(\theta_3(t))\,\lambda_1(t)$$

$$- 2 L_{s0}\cos(\theta_3(t))^2\lambda_1(t) - 2 L_{s0}\lambda_1(t)\sin(\theta_3(t))^2 - m_3\left(\frac{d}{dt}\theta_3(t)\right)^2 L_{s0}$$

$$- 2\cos(\theta_3(t))^2\lambda_1(t)\,s_3(t) - 2\lambda_1(t)\sin(\theta_3(t))^2 s_3(t) - m_3\left(\frac{d}{dt}\theta_3(t)\right)^2 s_3(t)$$

$$- 2 L\cos(\theta_3(t))\,\lambda_1(t) + m_3\sin(\theta_3(t))\,g\Big)$$

```
> REMOVE_DOT:=subs(SS_DIFF_TO_DOT,[RM1,RM2,RM3,RM4,RM5]);
```

$$REMOVE\_DOT := \Big[x3dot(t) \tag{6.3.10}$$

$$= \frac{1}{L_{s0}\,m_1\left(\cos(\theta_1(t))^2 + \sin(\theta_1(t))^2\right)}\Big(2 L_{s0}\sin(\theta_1(t))\cos(\theta_3(t))\,\lambda_1(t)$$

$$+ 2 L_{s0}\sin(\theta_1(t))\cos(\theta_2(t))\,\lambda_2(t) - 2 L_{s0}\cos(\theta_1(t))\sin(\theta_3(t))\,\lambda_1(t)$$

$$- 2 L_{s0}\cos(\theta_1(t))\sin(\theta_2(t))\,\lambda_2(t) + 2\sin(\theta_1(t))\cos(\theta_3(t))\,s_3(t)\,\lambda_1(t)$$

$$+ 2\sin(\theta_1(t))\cos(\theta_2(t))\,s_2(t)\,\lambda_2(t) - 2\cos(\theta_1(t))\,s_3(t)\sin(\theta_3(t))\,\lambda_1(t)$$

$$- 2\cos(\theta_1(t))\,s_2(t)\sin(\theta_2(t))\,\lambda_2(t) + 2 L\sin(\theta_1(t))\,\lambda_1(t)$$

$$- 2 L\sin(\theta_1(t))\,\lambda_2(t) - \cos(\theta_1(t))\,g\,m_1\Big), x4dot(t) = \frac{1}{m_2\,(L_{s0} + s_2(t))}\Big($$

$$-2 L_{s0}\sin(\theta_1(t))\cos(\theta_2(t))\,\lambda_2(t) + 2 L_{s0}\cos(\theta_1(t))\sin(\theta_2(t))\,\lambda_2(t)$$

$$+ 2 L\sin(\theta_2(t))\,\lambda_2(t) - \cos(\theta_2(t))\,g\,m_2 - 2\,x1(t)\,x4(t)\,m_2\Big), x5dot(t) =$$

$$- \frac{1}{m_3\,(L_{s0} + s_3(t))}\Big(-2 L_{s0}\cos(\theta_1(t))\sin(\theta_3(t))\,\lambda_1(t)$$

$$+ 2 L_{s0}\sin(\theta_1(t))\cos(\theta_3(t))\,\lambda_1(t) + 2 L\sin(\theta_3(t))\,\lambda_1(t) + \cos(\theta_3(t))\,g\,m_3$$

$$+ 2\, x2(t)\, x5(t)\, m_3\Big), x1dot(t) = -\frac{1}{m_2}\Big(C_{s2} \cos\big(\theta_2(t)\big)^2\, x1(t)$$

$$+ C_{s2} \sin\big(\theta_2(t)\big)^2\, x1(t) + K_{s2} \cos\big(\theta_2(t)\big)^2\, s_2(t) + K_{s2} \sin\big(\theta_2(t)\big)^2\, s_2(t)$$

$$+ 2\, L_{s0} \sin\big(\theta_1(t)\big)\, \lambda_2(t) \sin\big(\theta_2(t)\big) + 2\, L_{s0} \cos\big(\theta_1(t)\big) \cos\big(\theta_2(t)\big)\, \lambda_2(t)$$

$$- 2\, L_{s0} \cos\big(\theta_2(t)\big)^2\, \lambda_2(t) - 2\, L_{s0}\, \lambda_2(t) \sin\big(\theta_2(t)\big)^2 - m_2\, x4(t)^2\, L_{s0}$$

$$- 2 \cos\big(\theta_2(t)\big)^2\, \lambda_2(t)\, s_2(t) - 2\, \lambda_2(t) \sin\big(\theta_2(t)\big)^2\, s_2(t) - m_2\, x4(t)^2\, s_2(t)$$

$$+ 2\, L \cos\big(\theta_2(t)\big)\, \lambda_2(t) + m_2 \sin\big(\theta_2(t)\big)\, g\Big), x2dot(t) =$$

$$-\frac{1}{m_3}\Big(C_{s3} \cos\big(\theta_3(t)\big)^2\, x2(t) + C_{s3} \sin\big(\theta_3(t)\big)^2\, x2(t) + K_{s3} \cos\big(\theta_3(t)\big)^2\, s_3(t)$$

$$+ K_{s3} \sin\big(\theta_3(t)\big)^2\, s_3(t) + 2\, L_{s0} \sin\big(\theta_1(t)\big)\, \lambda_1(t) \sin\big(\theta_3(t)\big)$$

$$+ 2\, L_{s0} \cos\big(\theta_1(t)\big) \cos\big(\theta_3(t)\big)\, \lambda_1(t) - 2\, L_{s0} \cos\big(\theta_3(t)\big)^2\, \lambda_1(t)$$

$$- 2\, L_{s0}\, \lambda_1(t) \sin\big(\theta_3(t)\big)^2 - m_3\, x5(t)^2\, L_{s0} - 2 \cos\big(\theta_3(t)\big)^2\, \lambda_1(t)\, s_3(t)$$

$$- 2\, \lambda_1(t) \sin\big(\theta_3(t)\big)^2\, s_3(t) - m_3\, x5(t)^2\, s_3(t) - 2\, L \cos\big(\theta_3(t)\big)\, \lambda_1(t)$$

$$+ m_3 \sin\big(\theta_3(t)\big)\, g\Big)\Big]$$

```
> ALG := subs(SS_DIFF_TO_DOT, convert(dae_sys, list));
```

$$ALG := \Big[\big(-L - \cos\big(\theta_1(t)\big)\, L_{s0} + \cos\big(\theta_2(t)\big)\, s_2(t) + \cos\big(\theta_2(t)\big)\, L_{s0}\big)^2 + \big(\qquad \textbf{(6.3.11)}$$

$$-\sin\big(\theta_1(t)\big)\, L_{s0} + \sin\big(\theta_2(t)\big)\, s_2(t) + \sin\big(\theta_2(t)\big)\, L_{s0}\big)^2 - L^2, \big(-L$$

$$- \cos\big(\theta_3(t)\big)\, L_{s0} - \cos\big(\theta_3(t)\big)\, s_3(t) + \cos\big(\theta_1(t)\big)\, L_{s0}\big)^2 + \big($$

$$-\sin\big(\theta_3(t)\big)\, L_{s0} - \sin\big(\theta_3(t)\big)\, s_3(t) + \sin\big(\theta_1(t)\big)\, L_{s0}\big)^2 - L^2,$$

$$\frac{m_1\left(2\, x3dot(t) \sin\big(\theta_1(t)\big)^2\, L_{s0}^{\,2} + 2\, x3dot(t) \cos\big(\theta_1(t)\big)^2\, L_{s0}^{\,2}\right)}{2}$$

$$+ m_1\, g \cos\big(\theta_1(t)\big)\, L_{s0} - \big(-2\, \big(-L - \cos\big(\theta_3(t)\big)\, L_{s0} - \cos\big(\theta_3(t)\big)\, s_3(t)$$

$$
\begin{aligned}
&+ \cos(\theta_1(t))\, L_{s0})\, \sin(\theta_1(t))\, L_{s0} + 2\, (-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t) \\
&+ \sin(\theta_1(t))\, L_{s0})\, \cos(\theta_1(t))\, L_{s0})\, \lambda_1(t) - (2\, (-L - \cos(\theta_1(t))\, L_{s0} \\
&+ \cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0})\, \sin(\theta_1(t))\, L_{s0} - 2\, (-\sin(\theta_1(t))\, L_{s0} \\
&+ \sin(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, L_{s0})\, \cos(\theta_1(t))\, L_{s0})\, \lambda_2(t), m_2\, \mathit{x1dot}(t) \\
&- m_2\, (L_{s0} + s_2(t))\, \mathit{x4}(t)^2 + m_2 \sin(\theta_2(t))\, g - (2\, (-L - \cos(\theta_1(t))\, L_{s0} \\
&+ \cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0})\, \cos(\theta_2(t)) + 2\, (-\sin(\theta_1(t))\, L_{s0} \\
&+ \sin(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, L_{s0})\, \sin(\theta_2(t)))\, \lambda_2(t) - \cos(\theta_2(t))^2\, (\\
&-K_{s2}\, s_2(t) - C_{s2}\, \mathit{x1}(t)) - \sin(\theta_2(t))^2\, (-K_{s2}\, s_2(t) - C_{s2}\, \mathit{x1}(t)), m_3\, \mathit{x2dot}(t) \\
&- m_3\, (L_{s0} + s_3(t))\, \mathit{x5}(t)^2 + m_3 \sin(\theta_3(t))\, g - (-2\, (-L - \cos(\theta_3(t))\, L_{s0} \\
&- \cos(\theta_3(t))\, s_3(t) + \cos(\theta_1(t))\, L_{s0})\, \cos(\theta_3(t)) - 2\, (-\sin(\theta_3(t))\, L_{s0} \\
&- \sin(\theta_3(t))\, s_3(t) + \sin(\theta_1(t))\, L_{s0})\, \sin(\theta_3(t)))\, \lambda_1(t) - \cos(\theta_3(t))^2\, (\\
&-K_{s3}\, s_3(t) - C_{s3}\, \mathit{x2}(t)) - \sin(\theta_3(t))^2\, (-K_{s3}\, s_3(t) - C_{s3}\, \mathit{x2}(t)), 2\, m_2\, (L_{s0} \\
&+ s_2(t))\, \mathit{x4}(t)\, \mathit{x1}(t) + m_2\, (L_{s0} + s_2(t))^2\, \mathit{x4dot}(t) + m_2\, (\cos(\theta_2(t))\, s_2(t) \\
&+ \cos(\theta_2(t))\, L_{s0})\, g - (2\, (-L - \cos(\theta_1(t))\, L_{s0} + \cos(\theta_2(t))\, s_2(t) \\
&+ \cos(\theta_2(t))\, L_{s0})\, (-\sin(\theta_2(t))\, s_2(t) - \sin(\theta_2(t))\, L_{s0}) + 2\, (-\sin(\theta_1(t))\, L_{s0} \\
&+ \sin(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, L_{s0})\, (\cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0})) \\
&\lambda_2(t) - \cos(\theta_2(t))\, (-K_{s2}\, s_2(t) - C_{s2}\, \mathit{x1}(t))\, (-\sin(\theta_2(t))\, s_2(t) \\
&- \sin(\theta_2(t))\, L_{s0}) - \sin(\theta_2(t))\, (-K_{s2}\, s_2(t) - C_{s2}\, \mathit{x1}(t))\, (\cos(\theta_2(t))\, s_2(t) \\
&+ \cos(\theta_2(t))\, L_{s0}), 2\, m_3\, (L_{s0} + s_3(t))\, \mathit{x5}(t)\, \mathit{x2}(t) + m_3\, (L_{s0} + s_3(t))^2\, \mathit{x5dot}(t) \\
&+ m_3\, (\cos(\theta_3(t))\, L_{s0} + \cos(\theta_3(t))\, s_3(t))\, g - (2\, (-L - \cos(\theta_3(t))\, L_{s0} \\
&- \cos(\theta_3(t))\, s_3(t) + \cos(\theta_1(t))\, L_{s0})\, (\sin(\theta_3(t))\, L_{s0} + \sin(\theta_3(t))\, s_3(t)) \\
&+ 2\, (-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t) + \sin(\theta_1(t))\, L_{s0})\, (-\cos(\theta_3(t))\, L_{s0} \\
&- \cos(\theta_3(t))\, s_3(t)))\, \lambda_1(t) - \cos(\theta_3(t))\, (-K_{s3}\, s_3(t) - C_{s3}\, \mathit{x2}(t))\, (\\
&-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t)) - \sin(\theta_3(t))\, (-K_{s3}\, s_3(t) \\
&- C_{s3}\, \mathit{x2}(t))\, (\cos(\theta_3(t))\, L_{s0} + \cos(\theta_3(t))\, s_3(t))]
\end{aligned}
$$

Reduce index by one [1]

Differentiate constraint not containing dot variables and save algebraic constraint for projection

```
> HIDDEN1 := [ALG[1],ALG[1]]:<%>;
```

$$\left[\begin{array}{l}\left(-L-\cos\left(\theta_1(t)\right) L_{s0}+\cos\left(\theta_2(t)\right) s_2(t)+\cos\left(\theta_2(t)\right) L_{s0} \cdots\right. \\ \left(-L-\cos\left(\theta_1(t)\right) L_{s0}+\cos\left(\theta_2(t)\right) s_2(t)+\cos\left(\theta_2(t)\right) L_{s0} \cdots\right.\end{array}\right] \tag{6.3.1.1}$$

```
> diff(ALG[1],t);
  diff(ALG[2],t);
```

$$2\left(-L-\cos\left(\theta_1(t)\right) L_{s0}+\cos\left(\theta_2(t)\right) s_2(t)+\cos\left(\theta_2(t)\right) L_{s0}\right)\left(\left(\frac{d}{dt}\theta_1(t)\right)\sin\left(\theta_1(t)\right) L_{s0}-\left(\frac{d}{dt}\theta_2(t)\right)\sin\left(\theta_2(t)\right) s_2(t)+\cos\left(\theta_2(t)\right)\left(\frac{d}{dt}s_2(t)\right)-\left(\frac{d}{dt}\theta_2(t)\right)\sin\left(\theta_2(t)\right) L_{s0}\right)+2\left(-\sin\left(\theta_1(t)\right) L_{s0}+\sin\left(\theta_2(t)\right) s_2(t)+\sin\left(\theta_2(t)\right) L_{s0}\right)\left(-\left(\frac{d}{dt}\theta_1(t)\right)\cos\left(\theta_1(t)\right) L_{s0}+\left(\frac{d}{dt}\theta_2(t)\right)\cos\left(\theta_2(t)\right) s_2(t)+\sin\left(\theta_2(t)\right)\left(\frac{d}{dt}s_2(t)\right)+\left(\frac{d}{dt}\theta_2(t)\right)\cos\left(\theta_2(t)\right) L_{s0}\right)$$

$$2\left(-L-\cos\left(\theta_3(t)\right) L_{s0}-\cos\left(\theta_3(t)\right) s_3(t)+\cos\left(\theta_1(t)\right) L_{s0}\right)\left(\left(\frac{d}{dt}\theta_3(t)\right)\sin\left(\theta_3(t)\right) L_{s0}+\left(\frac{d}{dt}\theta_3(t)\right)\sin\left(\theta_3(t)\right) s_3(t)-\cos\left(\theta_3(t)\right)\left(\frac{d}{dt}s_3(t)\right)-\left(\frac{d}{dt}\theta_1(t)\right)\sin\left(\theta_1(t)\right) L_{s0}\right)+2\left(-\sin\left(\theta_3(t)\right) L_{s0}-\sin\left(\theta_3(t)\right) s_3(t)+\sin\left(\theta_1(t)\right) L_{s0}\right)\left(-\left(\frac{d}{dt}\theta_3(t)\right)\cos\left(\theta_3(t)\right) L_{s0}-\left(\frac{d}{dt}\theta_3(t)\right)\cos\left(\theta_3(t)\right) s_3(t)-\sin\left(\theta_3(t)\right)\left(\frac{d}{dt}s_3(t)\right)+\left(\frac{d}{dt}\theta_1(t)\right)\cos\left(\theta_1(t)\right) L_{s0}\right) \tag{6.3.1.2}$$

```
> ALG[1] := subs(SS_DIFF_TO_DOT,diff(ALG[1],t));
  ALG[2] := subs(SS_DIFF_TO_DOT,diff(ALG[2],t));
```

$$ALG := \left[2\left(-L-\cos\left(\theta_1(t)\right) L_{s0}+\cos\left(\theta_2(t)\right) s_2(t)+\cos\left(\theta_2(t)\right) L_{s0}\right)\left(x3(t)\sin\left(\theta_1(t)\right) L_{s0}-x4(t)\sin\left(\theta_2(t)\right) s_2(t)+\cos\left(\theta_2(t)\right) x1(t)-x4(t)\sin\left(\theta_2(t)\right) L_{s0}\right)+2\left(-\sin\left(\theta_1(t)\right) L_{s0}\right.\right.$$

$$+\sin\left(\theta_2(t)\right) s_2(t)+\sin\left(\theta_2(t)\right) L_{s0}\right) \left(-x3(t) \cos\left(\theta_1(t)\right) L_{s0}$$

$$+x4(t) \cos\left(\theta_2(t)\right) s_2(t)+\sin\left(\theta_2(t)\right) x1(t)+x4(t) \cos\left(\theta_2(t)\right) L_{s0}\right), \left(\right.$$

$$-L-\cos\left(\theta_3(t)\right) L_{s0}-\cos\left(\theta_3(t)\right) s_3(t)+\cos\left(\theta_1(t)\right) L_{s0}\right)^2+\left(\right.$$

$$-\sin\left(\theta_3(t)\right) L_{s0}-\sin\left(\theta_3(t)\right) s_3(t)+\sin\left(\theta_1(t)\right) L_{s0}\right)^2-L^2,$$

$$\frac{m_1 \left(2\, x3dot(t) \sin\left(\theta_1(t)\right)^2 L_{s0}^{\;2}+2\, x3dot(t) \cos\left(\theta_1(t)\right)^2 L_{s0}^{\;2}\right)}{2}$$

$$+m_1\, g \cos\left(\theta_1(t)\right) L_{s0}-\left(-2 \left(-L-\cos\left(\theta_3(t)\right) L_{s0}-\cos\left(\theta_3(t)\right) s_3(t)\right.\right.$$

$$+\cos\left(\theta_1(t)\right) L_{s0}\right) \sin\left(\theta_1(t)\right) L_{s0}+2 \left(-\sin\left(\theta_3(t)\right) L_{s0}-\sin\left(\theta_3(t)\right) s_3(t)\right.$$

$$+\sin\left(\theta_1(t)\right) L_{s0}\right) \cos\left(\theta_1(t)\right) L_{s0}\right) \lambda_1(t)-\left(2 \left(-L-\cos\left(\theta_1(t)\right) L_{s0}\right.\right.$$

$$+\cos\left(\theta_2(t)\right) s_2(t)+\cos\left(\theta_2(t)\right) L_{s0}\right) \sin\left(\theta_1(t)\right) L_{s0}-2 \left(-\sin\left(\theta_1(t)\right) L_{s0}\right.$$

$$+\sin\left(\theta_2(t)\right) s_2(t)+\sin\left(\theta_2(t)\right) L_{s0}\right) \cos\left(\theta_1(t)\right) L_{s0}\right) \lambda_2(t), m_2\, x1dot(t)$$

$$-m_2 \left(L_{s0}+s_2(t)\right) x4(t)^2+m_2 \sin\left(\theta_2(t)\right) g-\left(2 \left(-L-\cos\left(\theta_1(t)\right) L_{s0}\right.\right.$$

$$+\cos\left(\theta_2(t)\right) s_2(t)+\cos\left(\theta_2(t)\right) L_{s0}\right) \cos\left(\theta_2(t)\right)+2 \left(-\sin\left(\theta_1(t)\right) L_{s0}\right.$$

$$+\sin\left(\theta_2(t)\right) s_2(t)+\sin\left(\theta_2(t)\right) L_{s0}\right) \sin\left(\theta_2(t)\right)\right) \lambda_2(t)-\cos\left(\theta_2(t)\right)^2 \left(\right.$$

$$-K_{s2}\, s_2(t)-C_{s2}\, x1(t)\right)-\sin\left(\theta_2(t)\right)^2 \left(-K_{s2}\, s_2(t)-C_{s2}\, x1(t)\right), m_3\, x2dot(t)$$

$$-m_3 \left(L_{s0}+s_3(t)\right) x5(t)^2+m_3 \sin\left(\theta_3(t)\right) g-\left(-2 \left(-L-\cos\left(\theta_3(t)\right) L_{s0}\right.\right.$$

$$-\cos\left(\theta_3(t)\right) s_3(t)+\cos\left(\theta_1(t)\right) L_{s0}\right) \cos\left(\theta_3(t)\right)-2 \left(-\sin\left(\theta_3(t)\right) L_{s0}\right.$$

$$-\sin\left(\theta_3(t)\right) s_3(t)+\sin\left(\theta_1(t)\right) L_{s0}\right) \sin\left(\theta_3(t)\right)\right) \lambda_1(t)-\cos\left(\theta_3(t)\right)^2 \left(\right.$$

$$-K_{s3}\, s_3(t)-C_{s3}\, x2(t)\right)-\sin\left(\theta_3(t)\right)^2 \left(-K_{s3}\, s_3(t)-C_{s3}\, x2(t)\right), 2\, m_2 \left(L_{s0}\right.$$

$$+s_2(t)\right) x4(t)\, x1(t)+m_2 \left(L_{s0}+s_2(t)\right)^2 x4dot(t)+m_2 \left(\cos\left(\theta_2(t)\right) s_2(t)\right.$$

$$+\cos\left(\theta_2(t)\right) L_{s0}\right) g-\left(2 \left(-L-\cos\left(\theta_1(t)\right) L_{s0}+\cos\left(\theta_2(t)\right) s_2(t)\right.\right.$$

$$+\cos\left(\theta_2(t)\right) L_{s0}\right) \left(-\sin\left(\theta_2(t)\right) s_2(t)-\sin\left(\theta_2(t)\right) L_{s0}\right)+2 \left(\right.$$

$$-\sin\left(\theta_1(t)\right) L_{s0}+\sin\left(\theta_2(t)\right) s_2(t)+\sin\left(\theta_2(t)\right) L_{s0}\right) \left(\cos\left(\theta_2(t)\right) s_2(t)\right.$$

$$+\cos\left(\theta_2(t)\right) L_{s0}\right)\right) \lambda_2(t)-\cos\left(\theta_2(t)\right) \left(-K_{s2}\, s_2(t)-C_{s2}\, x1(t)\right) \left(\right.$$

$$-\sin\left(\theta_2(t)\right) s_2(t)-\sin\left(\theta_2(t)\right) L_{s0}\right)-\sin\left(\theta_2(t)\right) \left(-K_{s2}\, s_2(t)\right.$$

$$- C_{s2}\, x1(t)\big)\,\big(\cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0}\big), 2\, m_3\, \big(L_{s0} + s_3(t)\big)\, x5(t)\, x2(t) + m_3\, \big(L_{s0} + s_3(t)\big)^2\, x5dot(t) + m_3\, \big(\cos(\theta_3(t))\, L_{s0} + \cos(\theta_3(t))\, s_3(t)\big)\, g - \big(2\, \big(-L - \cos(\theta_3(t))\, L_{s0} - \cos(\theta_3(t))\, s_3(t) + \cos(\theta_1(t))\, L_{s0}\big)\, \big(\sin(\theta_3(t))\, L_{s0} + \sin(\theta_3(t))\, s_3(t)\big) + 2\, \big(-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t) + \sin(\theta_1(t))\, L_{s0}\big)\, \big(-\cos(\theta_3(t))\, L_{s0} - \cos(\theta_3(t))\, s_3(t)\big)\big)\, \lambda_1(t) - \cos(\theta_3(t))\, \big(-K_{s3}\, s_3(t) - C_{s3}\, x2(t)\big)\, \big(-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t)\big) - \sin(\theta_3(t))\, \big(-K_{s3}\, s_3(t) - C_{s3}\, x2(t)\big)\, \big(\cos(\theta_3(t))\, L_{s0} + \cos(\theta_3(t))\, s_3(t)\big)\Big]$$

$$ALG := \Big[2\, \big(-L - \cos(\theta_1(t))\, L_{s0} + \cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0}\big)\, \big(x3(t)\, \sin(\theta_1(t))\, L_{s0} - x4(t)\, \sin(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, x1(t) - x4(t)\, \sin(\theta_2(t))\, L_{s0}\big) + 2\, \big(-\sin(\theta_1(t))\, L_{s0} + \sin(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, L_{s0}\big)\, \big(-x3(t)\, \cos(\theta_1(t))\, L_{s0} + x4(t)\, \cos(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, x1(t) + x4(t)\, \cos(\theta_2(t))\, L_{s0}\big), 2\, \big(-L - \cos(\theta_3(t))\, L_{s0} - \cos(\theta_3(t))\, s_3(t) + \cos(\theta_1(t))\, L_{s0}\big)\, \big(x5(t)\, \sin(\theta_3(t))\, L_{s0} + x5(t)\, \sin(\theta_3(t))\, s_3(t) - \cos(\theta_3(t))\, x2(t) - x3(t)\, \sin(\theta_1(t))\, L_{s0}\big) + 2\, \big(-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t) + \sin(\theta_1(t))\, L_{s0}\big)\, \big(-x5(t)\, \cos(\theta_3(t))\, L_{s0} - x5(t)\, \cos(\theta_3(t))\, s_3(t) - \sin(\theta_3(t))\, x2(t) + x3(t)\, \cos(\theta_1(t))\, L_{s0}\big), \frac{m_1\, \big(2\, x3dot(t)\, \sin(\theta_1(t))^2\, L_{s0}^2 + 2\, x3dot(t)\, \cos(\theta_1(t))^2\, L_{s0}^2\big)}{2} + m_1\, g\, \cos(\theta_1(t))\, L_{s0} - \big(-2\, \big(-L - \cos(\theta_3(t))\, L_{s0} - \cos(\theta_3(t))\, s_3(t) + \cos(\theta_1(t))\, L_{s0}\big)\, \sin(\theta_1(t))\, L_{s0} + 2\, \big(-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t) \tag{6.3.1.3}$$

$$
\begin{aligned}
&+ \sin(\theta_1(t))\, L_{s0})\cos(\theta_1(t))\, L_{s0})\, \lambda_1(t) - (2\,(-L - \cos(\theta_1(t))\, L_{s0} \\
&+ \cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0})\sin(\theta_1(t))\, L_{s0} - 2\,(-\sin(\theta_1(t))\, L_{s0} \\
&+ \sin(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, L_{s0})\cos(\theta_1(t))\, L_{s0})\, \lambda_2(t),\, m_2\, x1dot(t) \\
&- m_2\,(L_{s0} + s_2(t))\, x4(t)^2 + m_2 \sin(\theta_2(t))\, g - (2\,(-L - \cos(\theta_1(t))\, L_{s0} \\
&+ \cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0})\cos(\theta_2(t)) + 2\,(-\sin(\theta_1(t))\, L_{s0} \\
&+ \sin(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, L_{s0})\sin(\theta_2(t)))\, \lambda_2(t) - \cos(\theta_2(t))^2\,(\\
&-K_{s2}\, s_2(t) - C_{s2}\, x1(t)) - \sin(\theta_2(t))^2\,(-K_{s2}\, s_2(t) - C_{s2}\, x1(t)),\, m_3\, x2dot(t) \\
&- m_3\,(L_{s0} + s_3(t))\, x5(t)^2 + m_3 \sin(\theta_3(t))\, g - (-2\,(-L - \cos(\theta_3(t))\, L_{s0} \\
&- \cos(\theta_3(t))\, s_3(t) + \cos(\theta_1(t))\, L_{s0})\cos(\theta_3(t)) - 2\,(-\sin(\theta_3(t))\, L_{s0} \\
&- \sin(\theta_3(t))\, s_3(t) + \sin(\theta_1(t))\, L_{s0})\sin(\theta_3(t)))\, \lambda_1(t) - \cos(\theta_3(t))^2\,(\\
&-K_{s3}\, s_3(t) - C_{s3}\, x2(t)) - \sin(\theta_3(t))^2\,(-K_{s3}\, s_3(t) - C_{s3}\, x2(t)),\, 2\, m_2\,(L_{s0} \\
&+ s_2(t))\, x4(t)\, x1(t) + m_2\,(L_{s0} + s_2(t))^2\, x4dot(t) + m_2\,(\cos(\theta_2(t))\, s_2(t) \\
&+ \cos(\theta_2(t))\, L_{s0})\, g - (2\,(-L - \cos(\theta_1(t))\, L_{s0} + \cos(\theta_2(t))\, s_2(t) \\
&+ \cos(\theta_2(t))\, L_{s0})\,(-\sin(\theta_2(t))\, s_2(t) - \sin(\theta_2(t))\, L_{s0}) + 2\,(\\
&-\sin(\theta_1(t))\, L_{s0} + \sin(\theta_2(t))\, s_2(t) + \sin(\theta_2(t))\, L_{s0})\,(\cos(\theta_2(t))\, s_2(t) \\
&+ \cos(\theta_2(t))\, L_{s0}))\, \lambda_2(t) - \cos(\theta_2(t))\,(-K_{s2}\, s_2(t) - C_{s2}\, x1(t))\,(\\
&-\sin(\theta_2(t))\, s_2(t) - \sin(\theta_2(t))\, L_{s0}) - \sin(\theta_2(t))\,(-K_{s2}\, s_2(t) \\
&- C_{s2}\, x1(t))\,(\cos(\theta_2(t))\, s_2(t) + \cos(\theta_2(t))\, L_{s0}),\, 2\, m_3\,(L_{s0} \\
&+ s_3(t))\, x5(t)\, x2(t) + m_3\,(L_{s0} + s_3(t))^2\, x5dot(t) + m_3\,(\cos(\theta_3(t))\, L_{s0} \\
&+ \cos(\theta_3(t))\, s_3(t))\, g - (2\,(-L - \cos(\theta_3(t))\, L_{s0} - \cos(\theta_3(t))\, s_3(t) \\
&+ \cos(\theta_1(t))\, L_{s0})\,(\sin(\theta_3(t))\, L_{s0} + \sin(\theta_3(t))\, s_3(t)) + 2\,(\\
&-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t) + \sin(\theta_1(t))\, L_{s0})\,(-\cos(\theta_3(t))\, L_{s0} \\
&- \cos(\theta_3(t))\, s_3(t)))\, \lambda_1(t) - \cos(\theta_3(t))\,(-K_{s3}\, s_3(t) - C_{s3}\, x2(t))\,(\\
&-\sin(\theta_3(t))\, L_{s0} - \sin(\theta_3(t))\, s_3(t)) - \sin(\theta_3(t))\,(-K_{s3}\, s_3(t) \\
&- C_{s3}\, x2(t))\,(\cos(\theta_3(t))\, L_{s0} + \cos(\theta_3(t))\, s_3(t))]
\end{aligned}
$$

Reduce index by one [2]

Differentiate constraint not containing dot variables and save algebraic constraint for projection

```
> HIDDEN2 := [op(HIDDEN1), ALG[1],ALG[2]]:<%>;
```

$$\left[\begin{array}{c} \cdots \\ \cdots \\ 2\left(-L-\cos\left(\theta_1(t)\right)L_{s0}+\cos\left(\theta_2(t)\right)s_2(t)+\cos\left(\theta_2(t)\right)L\cdots\right. \\ 2\left(-L-\cos\left(\theta_3(t)\right)L_{s0}-\cos\left(\theta_3(t)\right)s_3(t)+\cos\left(\theta_1(t)\right)L\cdots\right. \end{array}\right] \quad \textbf{(6.3.2.1)}$$

```
> diff(ALG[1],t);
  diff(ALG[2],t);
```

$$2\left(\left(\frac{d}{dt}\theta_1(t)\right)\sin\left(\theta_1(t)\right)L_{s0}-\left(\frac{d}{dt}\theta_2(t)\right)\sin\left(\theta_2(t)\right)s_2(t)\right.$$
$$\left.+\cos\left(\theta_2(t)\right)\left(\frac{d}{dt}s_2(t)\right)-\left(\frac{d}{dt}\theta_2(t)\right)\sin\left(\theta_2(t)\right)L_{s0}\right)$$
$$\left(x3(t)\sin\left(\theta_1(t)\right)L_{s0}-x4(t)\sin\left(\theta_2(t)\right)s_2(t)+\cos\left(\theta_2(t)\right)x1(t)\right.$$
$$\left.-x4(t)\sin\left(\theta_2(t)\right)L_{s0}\right)+2\left(-L-\cos\left(\theta_1(t)\right)L_{s0}+\cos\left(\theta_2(t)\right)s_2(t)\right.$$
$$\left.+\cos\left(\theta_2(t)\right)L_{s0}\right)\left(\left(\frac{d}{dt}x3(t)\right)\sin\left(\theta_1(t)\right)L_{s0}+x3(t)\left(\frac{d}{dt}\right.\right.$$
$$\left.\theta_1(t)\right)\cos\left(\theta_1(t)\right)L_{s0}-\left(\frac{d}{dt}x4(t)\right)\sin\left(\theta_2(t)\right)s_2(t)-x4(t)\left(\frac{d}{dt}\right.$$
$$\left.\theta_2(t)\right)\cos\left(\theta_2(t)\right)s_2(t)-x4(t)\sin\left(\theta_2(t)\right)\left(\frac{d}{dt}s_2(t)\right)-\left(\frac{d}{dt}\right.$$
$$\left.\theta_2(t)\right)\sin\left(\theta_2(t)\right)x1(t)+\cos\left(\theta_2(t)\right)\left(\frac{d}{dt}x1(t)\right)-\left(\frac{d}{dt}\right.$$
$$\left.\left.x4(t)\right)\sin\left(\theta_2(t)\right)L_{s0}-x4(t)\left(\frac{d}{dt}\theta_2(t)\right)\cos\left(\theta_2(t)\right)L_{s0}\right)+2\left(-\left(\frac{d}{dt}\right.\right.$$
$$\left.\theta_1(t)\right)\cos\left(\theta_1(t)\right)L_{s0}+\left(\frac{d}{dt}\theta_2(t)\right)\cos\left(\theta_2(t)\right)s_2(t)+\sin\left(\theta_2(t)\right)\left(\frac{d}{dt}\right.$$
$$\left.\left.s_2(t)\right)+\left(\frac{d}{dt}\theta_2(t)\right)\cos\left(\theta_2(t)\right)L_{s0}\right)\left(-x3(t)\cos\left(\theta_1(t)\right)L_{s0}\right.$$
$$\left.+x4(t)\cos\left(\theta_2(t)\right)s_2(t)+\sin\left(\theta_2(t)\right)x1(t)+x4(t)\cos\left(\theta_2(t)\right)L_{s0}\right)+2\left(\right.$$
$$\left.-\sin\left(\theta_1(t)\right)L_{s0}+\sin\left(\theta_2(t)\right)s_2(t)+\sin\left(\theta_2(t)\right)L_{s0}\right)\left(-\left(\frac{d}{dt}\right.\right.$$
$$\left.x3(t)\right)\cos\left(\theta_1(t)\right)L_{s0}+x3(t)\left(\frac{d}{dt}\theta_1(t)\right)\sin\left(\theta_1(t)\right)L_{s0}+\left(\frac{d}{dt}\right.$$
$$\left.x4(t)\right)\cos\left(\theta_2(t)\right)s_2(t)-x4(t)\left(\frac{d}{dt}\theta_2(t)\right)\sin\left(\theta_2(t)\right)s_2(t)$$
$$+x4(t)\cos\left(\theta_2(t)\right)\left(\frac{d}{dt}s_2(t)\right)+\left(\frac{d}{dt}\theta_2(t)\right)\cos\left(\theta_2(t)\right)x1(t)$$
$$+\sin\left(\theta_2(t)\right)\left(\frac{d}{dt}x1(t)\right)+\left(\frac{d}{dt}x4(t)\right)\cos\left(\theta_2(t)\right)L_{s0}-x4(t)\left(\frac{d}{dt}\right.$$

$$\theta_2(t)\Big)\sin\big(\theta_2(t)\big)\,L_{s0}\Big)$$

$$2\left(\left(\frac{d}{dt}\theta_3(t)\right)\sin\big(\theta_3(t)\big)\,L_{s0}+\left(\frac{d}{dt}\theta_3(t)\right)\sin\big(\theta_3(t)\big)\,s_3(t)\right. \tag{6.3.2.2}$$

$$-\cos\big(\theta_3(t)\big)\left(\frac{d}{dt}s_3(t)\right)-\left(\frac{d}{dt}\theta_1(t)\right)\sin\big(\theta_1(t)\big)\,L_{s0}\Big)$$

$$\big(x5(t)\sin\big(\theta_3(t)\big)\,L_{s0}+x5(t)\sin\big(\theta_3(t)\big)\,s_3(t)-\cos\big(\theta_3(t)\big)\,x2(t)$$

$$-x3(t)\sin\big(\theta_1(t)\big)\,L_{s0}\big)+2\,\big(-L-\cos\big(\theta_3(t)\big)\,L_{s0}-\cos\big(\theta_3(t)\big)\,s_3(t)$$

$$+\cos\big(\theta_1(t)\big)\,L_{s0}\big)\left(\left(\frac{d}{dt}x5(t)\right)\sin\big(\theta_3(t)\big)\,L_{s0}+x5(t)\left(\frac{d}{dt}\right.\right.$$

$$\theta_3(t)\Big)\cos\big(\theta_3(t)\big)\,L_{s0}+\left(\frac{d}{dt}x5(t)\right)\sin\big(\theta_3(t)\big)\,s_3(t)+x5(t)\left(\frac{d}{dt}\right.$$

$$\theta_3(t)\Big)\cos\big(\theta_3(t)\big)\,s_3(t)+x5(t)\sin\big(\theta_3(t)\big)\left(\frac{d}{dt}s_3(t)\right)+\left(\frac{d}{dt}\right.$$

$$\theta_3(t)\Big)\sin\big(\theta_3(t)\big)\,x2(t)-\cos\big(\theta_3(t)\big)\left(\frac{d}{dt}x2(t)\right)-\left(\frac{d}{dt}\right.$$

$$x3(t)\Big)\sin\big(\theta_1(t)\big)\,L_{s0}-x3(t)\left(\frac{d}{dt}\theta_1(t)\right)\cos\big(\theta_1(t)\big)\,L_{s0}\Big)+2\left(-\left(\frac{d}{dt}\right.\right.$$

$$\theta_3(t)\Big)\cos\big(\theta_3(t)\big)\,L_{s0}-\left(\frac{d}{dt}\theta_3(t)\right)\cos\big(\theta_3(t)\big)\,s_3(t)-\sin\big(\theta_3(t)\big)\left(\frac{d}{dt}\right.$$

$$s_3(t)\Big)+\left(\frac{d}{dt}\theta_1(t)\right)\cos\big(\theta_1(t)\big)\,L_{s0}\Big)\,\big(-x5(t)\cos\big(\theta_3(t)\big)\,L_{s0}$$

$$-x5(t)\cos\big(\theta_3(t)\big)\,s_3(t)-\sin\big(\theta_3(t)\big)\,x2(t)+x3(t)\cos\big(\theta_1(t)\big)\,L_{s0}\big)+2\,\big($$

$$-\sin\big(\theta_3(t)\big)\,L_{s0}-\sin\big(\theta_3(t)\big)\,s_3(t)+\sin\big(\theta_1(t)\big)\,L_{s0}\big)\left(-\left(\frac{d}{dt}\right.\right.$$

$$x5(t)\Big)\cos\big(\theta_3(t)\big)\,L_{s0}+x5(t)\left(\frac{d}{dt}\theta_3(t)\right)\sin\big(\theta_3(t)\big)\,L_{s0}-\left(\frac{d}{dt}\right.$$

$$x5(t)\Big)\cos\big(\theta_3(t)\big)\,s_3(t)+x5(t)\left(\frac{d}{dt}\theta_3(t)\right)\sin\big(\theta_3(t)\big)\,s_3(t)$$

$$-x5(t)\cos\big(\theta_3(t)\big)\left(\frac{d}{dt}s_3(t)\right)-\left(\frac{d}{dt}\theta_3(t)\right)\cos\big(\theta_3(t)\big)\,x2(t)$$

$$-\sin\big(\theta_3(t)\big)\left(\frac{d}{dt}x2(t)\right)+\left(\frac{d}{dt}x3(t)\right)\cos\big(\theta_1(t)\big)\,L_{s0}-x3(t)\left(\frac{d}{dt}\right.$$

$$\theta_1(t)\Big)\sin\big(\theta_1(t)\big)\,L_{s0}\Big)$$

```
> ALG[1] := subs(REMOVE_DOT,subs(SS_DIFF_TO_DOT,diff(ALG[1],t)
  )):
  ALG[2] := subs(REMOVE_DOT,subs(SS_DIFF_TO_DOT,diff(ALG[2],t)
  )):
```

Multiplier appears, try to solve for dot variables and lambda.
Use GenerateMatrix for the generation of the linear system.

```
> VARS := [x1dot(t),x2dot(t),x3dot(t),x4dot(t),x5dot(t),
  lambda__1(t),lambda__2(t)];
```

$$VARS := \left[x1dot(t), x2dot(t), x3dot(t), x4dot(t), x5dot(t), \lambda_1(t), \lambda_2(t)\right] \quad (6.3.2.3)$$

```
> M, R := GenerateMatrix( convert(ALG,list), VARS );
```

$$M, R := \begin{bmatrix} 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & \frac{m_1\left(2 L_{s0}^2 \cos\left(\theta_1(t)\right)^2 + 2 L_{s0}^2 \sin\left(\theta_1(t)\right)^2\right)}{2} & \cdots \\ m_2 & 0 & 0 & \cdots \\ 0 & m_3 & 0 & \cdots \\ 0 & 0 & 0 & m_2 \cdots \\ 0 & 0 & 0 & \cdots \end{bmatrix}, \quad (6.3.2.4)$$

$$\begin{bmatrix} -2\left(x3(t)\sin\left(\theta_1(t)\right) L_{s0} - x4(t)\sin\left(\theta_2(t)\right) s_2(t) + \cos\left(\theta_2 \cdots\right.\right. \\ -2\left(x5(t)\sin\left(\theta_3(t)\right) L_{s0} + x5(t)\sin\left(\theta_3(t)\right) s_3(t) - \mathrm{co} \cdots\right. \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

```
> REMOVET := s__2(t) = s__2,
             s__3(t) = s__3,
             theta__1(t) = theta__1,
             theta__2(t) = theta__2,
             theta__3(t) = theta__3,
             x1(t) = x1,
```

```
             x2(t) = x2,
             x3(t) = x3,
             x4(t) = x4,
             x5(t) = x5:
> Mf := subs(data,REMOVET,M);
```

$$Mf := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 10\cos(\theta_1)^2 + 10\sin(\theta_1)^2 & 0 & 0 & \cdots \\ 2 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 30 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 2\left(1+s_2\right)^2 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 30\left(1+s_3\right. & \cdots \end{bmatrix} \quad \textbf{(6.3.2.5)}$$

```
> Rf:=subs(data,REMOVET,R);
```

$$Rf := \begin{bmatrix} -2\left(x3\sin(\theta_1) - x4\sin(\theta_2)\,s_2 + \cos(\theta_2)\,x1 - \right. & \cdots \\ -2\left(x5\sin(\theta_3) + x5\sin(\theta_3)\,s_3 - \cos(\theta_3)\,x2 - x3\sin(\theta_1)\right) & \cdots \\ & \cdots \\ & \cdots \\ & \cdots \\ & \cdots \\ & \cdots \end{bmatrix} \quad \textbf{(6.3.2.6)}$$

```
> RHS_ODE := proc( Z, t,Mf,Rf )
    local g    := 9.81,var,


          MM, RR, SOL;

  var := [  s__2 = Z[1],
          s__3 = Z[2],
```

```
        theta__1 = Z[3],
        theta__2 = Z[4],
        theta__3 = Z[5],
        x1= Z[6],
        x2= Z[7],
        x3= Z[8],
        x4= Z[9],
        x5= Z[10]];



  SOL := LinearAlgebra[LinearSolve](subs(var, Mf),subs(var,
 Rf));

 Vector[column](10, [ Z[6],Z[7],Z[8], Z[9],Z[10], SOL[1], SOL
 [2],SOL[3], SOL[4],SOL[5]  ]);

 end proc;
```

$RHS\_ODE := \mathbf{proc}(Z, t, Mf, Rf)$ **(6.3.2.7)**

$\mathbf{local}\ g, var, MM, RR, SOL;$

$g := 9.81;$

$var := [s[2] = Z[1], s[3] = Z[2], \theta[1] = Z[3], \theta[2] = Z[4], \theta[3] = Z[5], x1 = Z[6], x2 = Z[7], x3 = Z[8], x4 = Z[9], x5 = Z[10]];$

$SOL := LinearAlgebra[LinearAlgebra{:-}LinearSolve](subs(var, Mf), subs(var, Rf));$

$Vector[column](10, [Z[6], Z[7], Z[8], Z[9], Z[10], SOL[1], SOL[2], SOL[3], SOL[4], SOL[5]])$

end proc

```
> subs(REMOVET,HIDDEN2);
```

$$\Big[\big(-L-\cos(\theta_1)\,L_{s0}+\cos(\theta_2)\,s_2+\cos(\theta_2)\,L_{s0}\big)^2+\big(-\sin(\theta_1)\,L_{s0}+\sin(\theta_2)\,s_2+\sin(\theta_2)\,L_{s0}\big)^2-L^2, \big(-L-\cos(\theta_1)\,L_{s0}+\cos(\theta_2)\,s_2+\cos(\theta_2)\,L_{s0}\big)^2+\big(-\sin(\theta_1)\,L_{s0}+\sin(\theta_2)\,s_2+\sin(\theta_2)\,L_{s0}\big)^2-L^2, 2\,\big(-L-\cos(\theta_1)\,L_{s0}+\cos(\theta_2)\,s_2+\cos(\theta_2)\,L_{s0}\big)\,\big(x3\sin(\theta_1)\,L_{s0}-x4\sin(\theta_2)\,s_2+\cos(\theta_2)\,x1-x4\sin(\theta_2)\,L_{s0}\big)+2\,\big(-\sin(\theta_1)\,L_{s0}+\sin(\theta_2)\,s_2+\sin(\theta_2)\,L_{s0}\big)\,\big(-x3\cos(\theta_1)\,L_{s0}+x4\cos(\theta_2)\,s_2+\sin(\theta_2)\,x1+x4\cos(\theta_2)\,L_{s0}\big), 2\,\big(-L-\cos(\theta_3)\,L_{s0}-\cos(\theta_3)\,s_3+\cos(\theta_1)\,L_{s0}\big)\,\big(x5\sin(\theta_3)\,L_{s0}+x5\sin(\theta_3)\,s_3-\cos(\theta_3)\,x2-x3\sin(\theta_1)\,L_{s0}\big)+2\,\big(-\sin(\theta_3)\,L_{s0}-\sin(\theta_3)\,s_3+\sin(\theta_1)\,L_{s0}\big)\,\big(-x5\cos(\theta_3)\,L_{s0}-x5\cos(\theta_3)\,s_3-\sin(\theta_3)\,x2+x3\cos(\theta_1)\,L_{s0}\big)\Big] \quad \textbf{(6.3.2.8)}$$

```
> PROJECT := proc( Z, t ) # in general DAE coeffs depend on
  time (not in this case)
    local g := 9.81,
          s__2,s__3,theta__1,theta__2, theta__3, x1,x2,x3,x4,
  x5,
          fun, constraints, SOL;


    fun := (s__2-Z[1])^2+
           (s__3-Z[2])^2+
           (theta__1-Z[3])^2+
           (theta__2-Z[4])^2+
           (theta__3-Z[5])^2+
           (x1-Z[6])^2+
           (x2-Z[7])^2+
           (x3-Z[8])^2+
           (x4-Z[9])^2+
           (x5-Z[10])^2;

    constraints := { (-L - cos(theta__1)*L__s0 + cos(theta__2)
  *s__2 + cos(theta__2)*L__s0)^2 + (-sin(theta__1)*L__s0 + sin
  (theta__2)*s__2 + sin(theta__2)*L__s0)^2 - L^2=0, (-L - cos
  (theta__1)*L__s0 + cos(theta__2)*s__2 + cos(theta__2)*L__s0)
  ^2 + (-sin(theta__1)*L__s0 + sin(theta__2)*s__2 + sin
  (theta__2)*L__s0)^2 - L^2=0, 2*(-1 - cos(theta__3) - cos
  (theta__3)*s__3 + cos(theta__1))*(x5*sin(theta__3) + x5*sin
  (theta__3)*s__3 - cos(theta__3)*x2 - x3*sin(theta__1)) + 2*
  (-sin(theta__3) - sin(theta__3)*s__3 + sin(theta__1))*(-x5*
  cos(theta__3) - x5*cos(theta__3)*s__3 - sin(theta__3)*x2 +
  x3*cos(theta__1))=0, 2*(-L - cos(theta__3)*L__s0 - cos
  (theta__3)*s__3 + cos(theta__1)*L__s0)*(x5*sin(theta__3)*
  L__s0 + x5*sin(theta__3)*s__3 - cos(theta__3)*x2 - x3*sin
  (theta__1)*L__s0) + 2*(-sin(theta__3)*L__s0 - sin(theta__3)*
  s__3 + sin(theta__1)*L__s0)*(-x5*cos(theta__3)*L__s0 - x5*
  cos(theta__3)*s__3 - sin(theta__3)*x2 + x3*cos(theta__1)*
  L__s0)=0
  };
    SOL := Optimization[Minimize](
      fun, constraints, initialpoint = {s__2=Z[1],s__3=Z[2],
  theta__1=Z[3],theta__2=Z[4],theta__3=Z[5],x1=Z[6],x2=Z[7],
  x3=Z[8],x4=Z[9],x5=Z[10]}
    );
    subs(SOL[2],<s__2,s__3,theta__1,theta__2,theta__3,x1,x2,
  x3,x4,x5>);
  end proc;
```

$PROJECT := \mathbf{proc}(Z, t)$ **(6.3.2.9)**

$\mathbf{local}\ g, s[2], s[3], \theta[1], \theta[2], \theta[3], x1, x2, x3, x4, x5, fun, constraints, SOL;$

$g := 9.81;$

```
fun := (s[2] − Z[1])^2 + (s[3] − Z[2])^2 + (θ[1] − Z[3])^2 + (θ[2
] − Z[4])^2 + (θ[3] − Z[5])^2 + (x1 − Z[6])^2 + (x2 − Z[7])^2
 + (x3 − Z[8])^2 + (x4 − Z[9])^2 + (x5 − Z[10])^2;
constraints := {2 * ( − 1 − cos(θ[3]) − cos(θ[3]) * s[3] + cos(θ[1]))
 * (x5 * sin(θ[3]) + x5 * sin(θ[3]) * s[3] − cos(θ[3]) * x2 − x3 * sin(θ[1
])) + 2 * ( − sin(θ[3]) − sin(θ[3]) * s[3] + sin(θ[1])) * ( − x5 * cos(θ
[3]) − x5 * cos(θ[3]) * s[3] − sin(θ[3]) * x2 + x3 * cos(θ[1])) = 0, 2 * (
 − L − cos(θ[3]) * L[s0] − cos(θ[3]) * s[3] + cos(θ[1]) * L[s0]) * (x5
 * sin(θ[3]) * L[s0] + x5 * sin(θ[3]) * s[3] − cos(θ[3]) * x2 − x3 * sin(θ
[1]) * L[s0]) + 2 * ( − sin(θ[3]) * L[s0] − sin(θ[3]) * s[3] + sin(θ[1])
 * L[s0]) * ( − x5 * cos(θ[3]) * L[s0] − x5 * cos(θ[3]) * s[3] − sin(θ[3])
 * x2 + x3 * cos(θ[1]) * L[s0]) = 0, ( − L − cos(θ[1]) * L[s0] + cos(θ[2
]) * s[2] + cos(θ[2]) * L[s0])^2 + ( − sin(θ[1]) * L[s0] + sin(θ[2]) * s
[2] + sin(θ[2]) * L[s0])^2 − L^2 = 0};
SOL := Optimization[Minimize](fun, constraints, initialpoint = {s[2] = Z[1],
s[3] = Z[2], θ[1] = Z[3], θ[2] = Z[4], θ[3] = Z[5], x1 = Z[6], x2 = Z[7], x3
 = Z[8], x4 = Z[9], x5 = Z[10]});
subs(SOL[2], < s[2], s[3], θ[1], θ[2], θ[3], x1, x2, x3, x4, x5 > )
end proc
```

```
> ADVANCE := proc( Z_in, t_begin, t_end, dt,Mf,Rf,
  do_projection := true )
    local Z0, Z1,
          t, pc, last_pc := -1,
          # output values
        T,s2,s3,theta1,theta2, theta3,x1,x2,x3,x4,x5;

    # project initial condition
    Z0 := PROJECT( Z_in );

    # initialize output list
    T  := [t_begin];
    s2 := [Z0[1]];
    s3 := [Z0[2]];
    theta1 := [Z0[3]];
    theta2 := [Z0[4]];
    theta3 := [Z0[5]];
    x1 := [Z0[6]];
    x2 := [Z0[7]];
    x3 := [Z0[8]];
    x4 := [Z0[9]];
```

```
  x5 := [Z0[10]];

  # advance
  t := t_begin;
  while t < t_end do
    pc := 100*(t-t_begin)/(t_end-t_begin);
    if pc > last_pc+0.5 then
      printf("[%g%%] t=%g\r", pc, t );
      last_pc := pc;
    end if;

    # advance using improved Euler (or Collatz)
    # half forward Euler step
    Z1 := Z0 + (dt/2)*RHS_ODE( Z0, t,Mf,Rf );
    t  := t+dt/2;
    # full iproved step
    Z1 := Z0 + dt*RHS_ODE( Z1, t ,Mf,Rf);
    t  := t+dt/2;

    # project step to Z0
    if do_projection then
      Z0 := PROJECT( Z1, t );
    else
      Z0 := Z1;
    end if;
    # save solution to listì
    T  := [op(T),t];
    s2 := [op(s2),Z0[1]];
    s3 := [op(s3),Z0[2]];
    theta1 := [op(theta1),Z0[3]];
    theta2 := [op(theta2),Z0[4]];
    theta3 := [op(theta3),Z0[5]];
    x1 := [op(x1),Z0[6]];
    x2 := [op(x2),Z0[7]];
    x3 := [op(x3),Z0[8]];
    x4 := [op(x4),Z0[9]];
    x5 := [op(x5),Z0[10]];
  end;
  table( [ "t" = T,"s__2" = s2, "s__3" = s3, "theta__1" =
theta1,"theta__2" = theta2, "theta__3" = theta3,"x1" = x1,
"x2" = x2,"x3" = x3, "x4" = x4,"x5" = x5 ]);
end proc;
```

$ADVANCE := \mathbf{proc}(Z\_in, t\_begin, t\_end, dt, Mf, Rf, do\_projection := true)$ **(6.3.2.10)**

$\mathbf{local}\ Z0, Z1, t, pc, last\_pc, T, s2, s3, \theta 1, \theta 2, \theta 3, x1, x2, x3, x4, x5;$

$last\_pc := -1;$

$Z0 := PROJECT(Z\_in);$

$T := [t\_begin];$

```
s2 := [Z0[1]];
s3 := [Z0[2]];
θ1 := [Z0[3]];
θ2 := [Z0[4]];
θ3 := [Z0[5]];
x1 := [Z0[6]];
x2 := [Z0[7]];
x3 := [Z0[8]];
x4 := [Z0[9]];
x5 := [Z0[10]];
t := t_begin;
while t < t_end do
    pc := 100 * (t - t_begin) / (t_end - t_begin);
    if last_pc + 0.5 < pc then printf("[%g%%] t=%g
", pc, t); last_pc := pc end if;
    Z1 := Z0 + 1/2 * dt * RHS_ODE(Z0, t, Mf, Rf);
    t := t + 1/2 * dt;
    Z1 := Z0 + dt * RHS_ODE(Z1, t, Mf, Rf);
    t := t + 1/2 * dt;
    if do_projection then Z0 := PROJECT(Z1, t) else Z0 := Z1 end if;
    T := [op(T), t];
    s2 := [op(s2), Z0[1]];
    s3 := [op(s3), Z0[2]];
    θ1 := [op(θ1), Z0[3]];
    θ2 := [op(θ2), Z0[4]];
    θ3 := [op(θ3), Z0[5]];
    x1 := [op(x1), Z0[6]];
    x2 := [op(x2), Z0[7]];
    x3 := [op(x3), Z0[8]];
    x4 := [op(x4), Z0[9]];
    x5 := [op(x5), Z0[10]]
end do;
    table(["t" = T, "s__2" = s2, "s__3" = s3, "theta__1" = θ1, "theta__2" = θ2,
    "theta__3" = θ3, "x1" = x1, "x2" = x2, "x3" = x3, "x4" = x4, "x5" = x5])
end proc
```

Test forward integration and plots

Define the initial conditions

```
> INI := <0,0,-Pi/2,-Pi/2,-Pi/2,0,0,0,0,0>
```

$$INI := \begin{bmatrix} 0 \\ 0 \\ -\frac{\pi}{2} \\ -\frac{\pi}{2} \\ -\frac{\pi}{2} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6.3.3.1}$$

```
> RES := PROJECT(INI,0);
```

Warning, no iterations performed as initial point satisfies first-order conditions

$$RES := \begin{bmatrix} 0. \\ 0. \\ -1.57079632679490 \\ -1.57079632679490 \\ -1.57079632679490 \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \end{bmatrix} \tag{6.3.3.2}$$

```
> RHS_ODE(INI, 0, Mf, Rf);
```

(6.3.3.3)

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 9.810000000 \\ 9.810000000 \\ 4.790628970 \times 10^{-10} \\ 2.491127062 \times 10^{-9} \\ 2.491127062 \times 10^{-9} \end{bmatrix} \qquad \textbf{(6.3.3.3)}$$

```
> RES := ADVANCE( evalf(INI), 0, 10, 0.01,Mf,Rf, true ):
Warning, no iterations performed as initial point satisfies first-order conditions
[0%] t=0
[0.6%] t=0.06
[1.2%] t=0.12
[1.8%] t=0.18
[2.4%] t=0.24
[3%] t=0.3
[3.6%] t=0.36
[4.2%] t=0.42
[4.8%] t=0.48
[5.4%] t=0.54
[6%] t=0.6
[6.6%] t=0.66
[7.2%] t=0.72
[7.8%] t=0.78
[8.4%] t=0.84
[9%] t=0.9
[9.6%] t=0.96
[10.2%] t=1.02
[10.8%] t=1.08
[11.4%] t=1.14
[12%] t=1.2
[12.6%] t=1.26
[13.2%] t=1.32
[13.8%] t=1.38
[14.4%] t=1.44
[15%] t=1.5
[15.6%] t=1.56
[16.2%] t=1.62
[16.8%] t=1.68
[17.4%] t=1.74
```

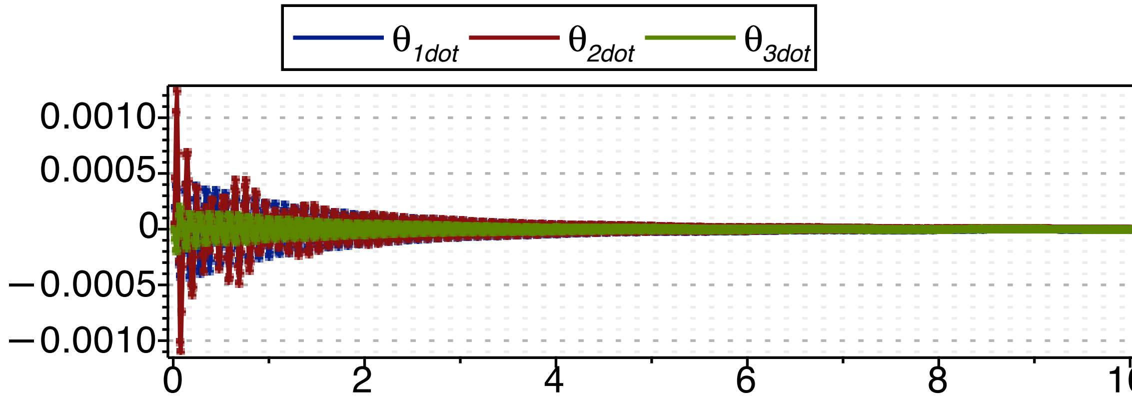
```
[18%] t=1.8
[18.6%] t=1.86
[19.2%] t=1.92
[19.8%] t=1.98
[20.4%] t=2.04
[21%] t=2.1
[21.6%] t=2.16
[22.2%] t=2.22
[22.8%] t=2.28
[23.4%] t=2.34
[24%] t=2.4
[24.6%] t=2.46
[25.2%] t=2.52
[25.8%] t=2.58
[26.4%] t=2.64
[27%] t=2.7
[27.6%] t=2.76
[28.2%] t=2.82
[28.8%] t=2.88
[29.4%] t=2.94
[30%] t=3
[30.6%] t=3.06
[31.2%] t=3.12
[31.8%] t=3.18
[32.4%] t=3.24
[33%] t=3.3
[33.6%] t=3.36
[34.2%] t=3.42
[34.8%] t=3.48
[35.4%] t=3.54
[36%] t=3.6
[36.6%] t=3.66
[37.2%] t=3.72
[37.8%] t=3.78
[38.4%] t=3.84
[39%] t=3.9
[39.6%] t=3.96
[40.2%] t=4.02
[40.8%] t=4.08
[41.4%] t=4.14
[42%] t=4.2
[42.6%] t=4.26
[43.2%] t=4.32
[43.8%] t=4.38
[44.4%] t=4.44
[45%] t=4.5
[45.6%] t=4.56
[46.2%] t=4.62
```

```
[46.8%] t=4.68
[47.4%] t=4.74
[48%] t=4.8
[48.6%] t=4.86
[49.2%] t=4.92
[49.8%] t=4.98
[50.4%] t=5.04
[51%] t=5.1
[51.6%] t=5.16
[52.2%] t=5.22
[52.8%] t=5.28
[53.4%] t=5.34
[54%] t=5.4
[54.6%] t=5.46
[55.2%] t=5.52
[55.8%] t=5.58
[56.4%] t=5.64
[57%] t=5.7
[57.6%] t=5.76
[58.2%] t=5.82
[58.8%] t=5.88
[59.4%] t=5.94
[60%] t=6
[60.6%] t=6.06
[61.2%] t=6.12
[61.8%] t=6.18
[62.4%] t=6.24
[63%] t=6.3
[63.6%] t=6.36
[64.2%] t=6.42
[64.8%] t=6.48
[65.4%] t=6.54
[66%] t=6.6
[66.6%] t=6.66
[67.2%] t=6.72
[67.8%] t=6.78
[68.4%] t=6.84
[69%] t=6.9
[69.6%] t=6.96
[70.2%] t=7.02
[70.8%] t=7.08
[71.4%] t=7.14
[72%] t=7.2
[72.6%] t=7.26
[73.2%] t=7.32
[73.8%] t=7.38
[74.4%] t=7.44
[75%] t=7.5
```

```
[75.6%] t=7.56
[76.2%] t=7.62
[76.8%] t=7.68
[77.4%] t=7.74
[78%] t=7.8
[78.6%] t=7.86
[79.2%] t=7.92
[79.8%] t=7.98
[80.4%] t=8.04
[81%] t=8.1
[81.6%] t=8.16
[82.2%] t=8.22
[82.8%] t=8.28
[83.4%] t=8.34
[84%] t=8.4
[84.6%] t=8.46
[85.2%] t=8.52
[85.8%] t=8.58
[86.4%] t=8.64
[87%] t=8.7
[87.6%] t=8.76
[88.2%] t=8.82
[88.8%] t=8.88
[89.4%] t=8.94
[90%] t=9
[90.6%] t=9.06
[91.2%] t=9.12
[91.8%] t=9.18
[92.4%] t=9.24
[93%] t=9.3
[93.6%] t=9.36
[94.2%] t=9.42
[94.8%] t=9.48
[95.4%] t=9.54
[96%] t=9.6
[96.6%] t=9.66
[97.2%] t=9.72
[97.8%] t=9.78
[98.4%] t=9.84
[99%] t=9.9
[99.6%] t=9.96
```

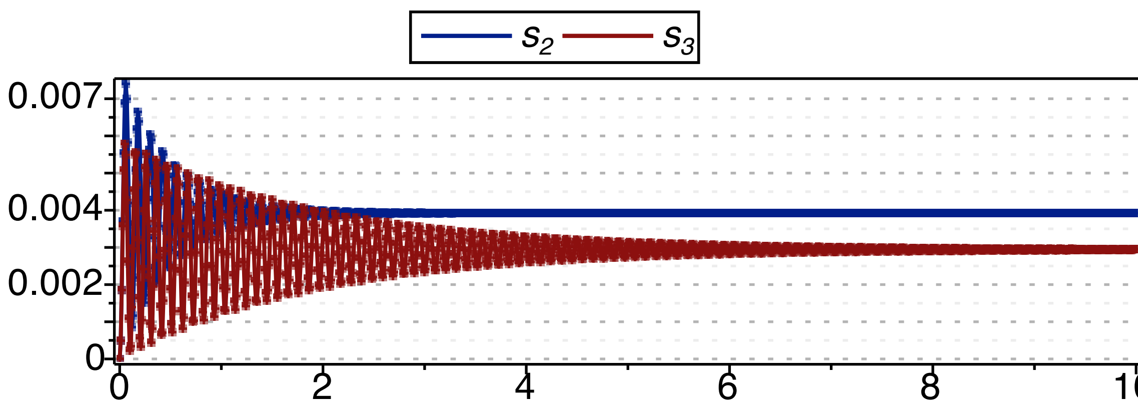
Plot of theta1dot, theta2dot and theta3dot

```
> dataplot( RES["t"], [ RES["x3"], RES["x4"], RES["x5"] ],
  symbolsize=1,legend=[theta__1dot,theta__2dot,theta__3dot] );
```



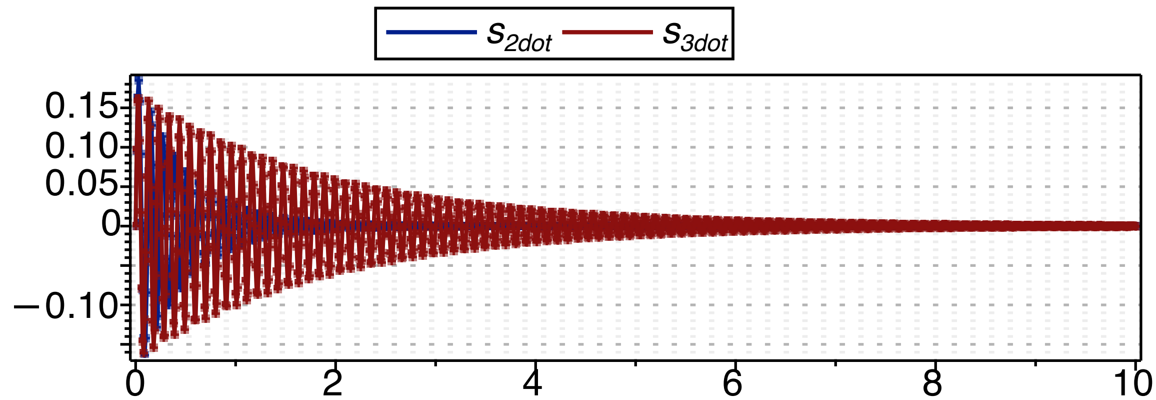
Plot of s2 and s3

```
> dataplot( RES["t"], [ RES["s__2"], RES["s__3"] ],symbolsize=
  1,legend=[s__2,s__3]  );
```



Plot of s2dot and s3dot

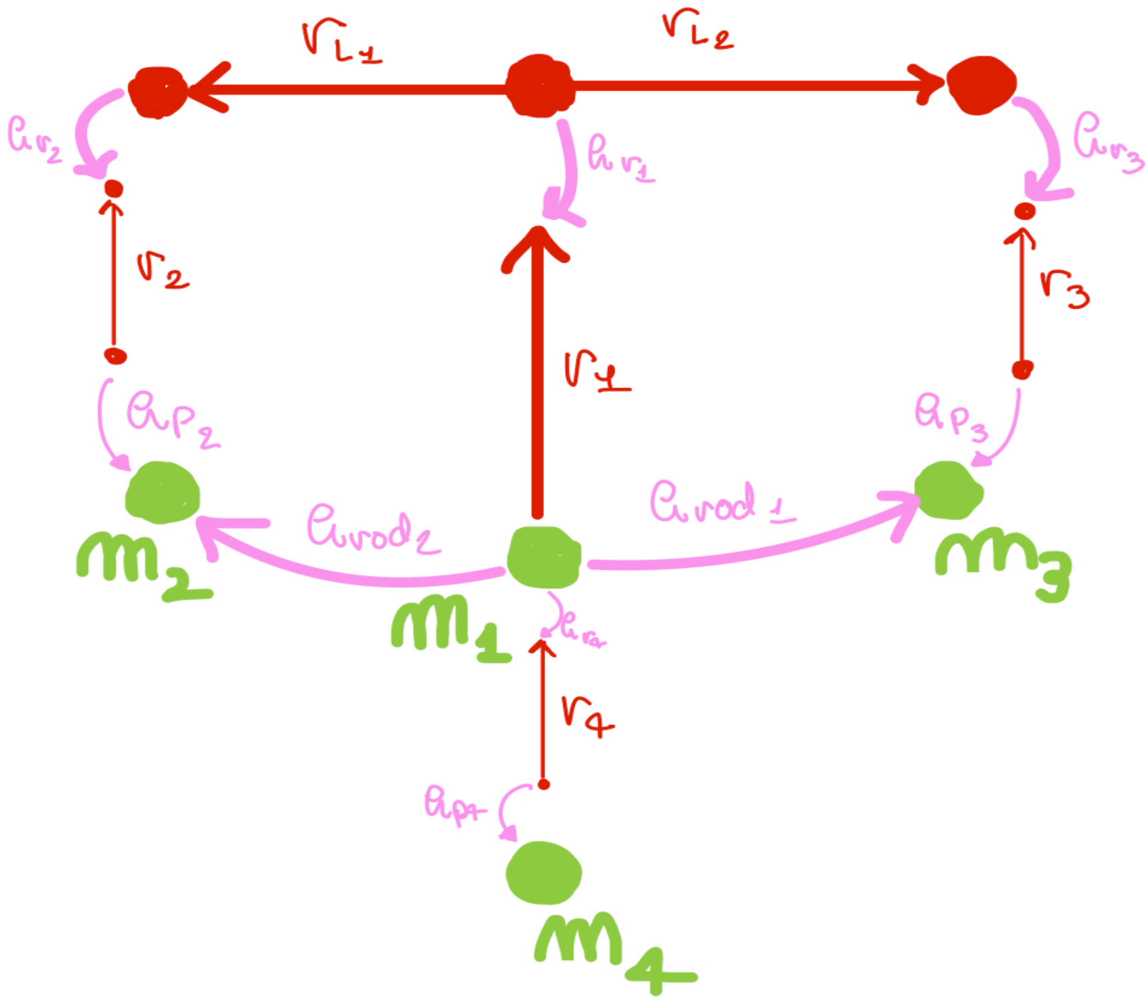
```
> dataplot( RES["t"], [ RES["x1"], RES["x2"] ],symbolsize=1,
  legend=[s__2dot,s__3dot]  );
```



Optional: optimisation

In this section a variable mass m4 is connected to G1 with a starting distance of Ls0 and with a damper with variable damping.
The kinematic of the new mechanical system is modelled and then different values for the mass and damper are used and evaluated in order to minimise the settling time of the system.

Linear Graph



Kinematic with mass m4 and damper added

In this section the kinematic model of the new mechanical system is defined and the displayed

```
> r4 := Rz(theta__4(t)).<-L__s0,0,0>;
```

$$r4 := \begin{bmatrix} -\cos\left(\theta_4(t)\right) L_{s0} \\ -\sin\left(\theta_4(t)\right) L_{s0} \\ 0 \end{bmatrix} \tag{7.1.1}$$

```
> hp4 := Rz(theta__4(t)).(s__4(t)*<1,0,0>);
```

$$hp4 := \begin{bmatrix} \cos\left(\theta_4(t)\right) s_4(t) \\ \sin\left(\theta_4(t)\right) s_4(t) \\ 0 \end{bmatrix} \tag{7.1.2}$$

```
> Rs__4 := Rz(theta__4(t));
```

```
is_uvec__4 := %[1..3,1]:
js_uvec__4 := %%[1..3,2]:

is_uvec__4,js_uvec__4;
```

$$Rs_4 := \begin{bmatrix} \cos(\theta_4(t)) & -\sin(\theta_4(t)) & 0 \\ \sin(\theta_4(t)) & \cos(\theta_4(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta_4(t)) \\ \sin(\theta_4(t)) \\ 0 \end{bmatrix}, \begin{bmatrix} -\sin(\theta_4(t)) \\ \cos(\theta_4(t)) \\ 0 \end{bmatrix} \tag{7.1.3}$$

```
> G4 := -r1-r4+hp4;
```

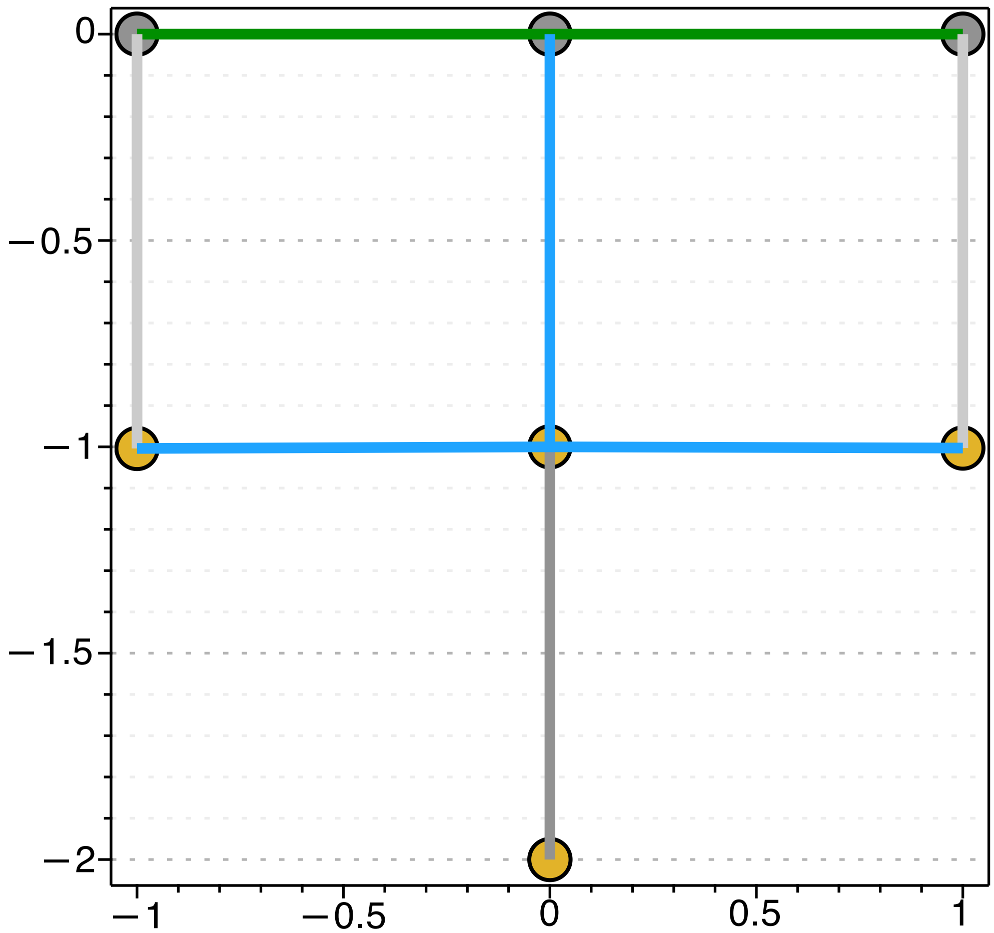
$$G4 := \begin{bmatrix} \cos(\theta_1(t))\, L_{s0} + \cos(\theta_4(t))\, L_{s0} + \cos(\theta_4(t))\, s_4(t) \\ \sin(\theta_1(t))\, L_{s0} + \sin(\theta_4(t))\, L_{s0} + \sin(\theta_4(t))\, s_4(t) \\ 0 \end{bmatrix} \tag{7.1.4}$$

The following procedure is defined in order to display the configuration of the mechanical system with the mass m4 and the dumper added

```
> draw_mech__2 := proc(data, sol_kine, dof)

    local p0,p2,p3,pG1,pG2,pG3,pG4,LS1,LS2,LS3,LS4,SS2,SS3,SS4,
  PS1,PS2, PS3, VS;


    p0  := subs(sol_kine, dof,data, P0);
    p2  := subs(sol_kine, dof,data, P2);
    p3  := subs(sol_kine, dof,data, P3);
    pG1  := subs(sol_kine, dof,data, G1):
    pG2  := subs(sol_kine, dof,data, G2):
    pG3  := subs(sol_kine, dof,data, G3):
    pG4  := subs(sol_kine, dof,data, G4):
    LS1  := subs(sol_kine, dof,data, L__s0*is_uvec__1);
    LS2  := subs(sol_kine, dof,data, L__s0*is_uvec__2);
    LS3  := subs(sol_kine, dof,data, L__s0*is_uvec__3);
    LS4  := subs(sol_kine, dof,data, L__s0*is_uvec__4);
    SS2  := subs(sol_kine, dof,data, s__2(t)*is_uvec__2);
    SS3  := subs(sol_kine, dof,data, s__3(t)*is_uvec__3);
    SS4  := subs(sol_kine, dof,data, s__4(t)*is_uvec__4);
    PS1  := p0+LS1;
    PS2 := p2+LS2+SS2;
    PS3  := p3+LS3+SS3;
```

```
  display( plottools:-disk(vec_2D(p0),0.05,color="Gray"),
           plottools:-disk(vec_2D(p2),0.05,color="Gray"),
           plottools:-disk(vec_2D(p3),0.05,color="Gray"),
           plottools:-disk(vec_2D(PS1),0.05,color=
"Goldenrod"),
           plottools:-disk(vec_2D(PS2),0.05,color=
"Goldenrod"),
           plottools:-disk(vec_2D(PS3),0.05,color=
"Goldenrod"),
           plottools:-disk(vec_2D(pG4),0.05,color=
"Goldenrod"),
           plottools:-line(vec_2D(p0),vec_2D(p2),color=
"Green",thickness=4),
           plottools:-line(vec_2D(p0),vec_2D(p3),color=
"Green",thickness=4),
           plottools:-line(vec_2D(p0),vec_2D(PS1),color=
"DodgerBlue",thickness=4),
           plottools:-line(vec_2D(p2),vec_2D(PS2),color=
"grey",thickness=4),
            plottools:-line(vec_2D(PS1),vec_2D(pG4),color=
"Gray",thickness=4),
           plottools:-line(vec_2D(p3),vec_2D(PS3),color=
"grey",thickness=4),
           plottools:-line(vec_2D(PS1),vec_2D(PS2),color=
"DodgerBlue",thickness=4),
           plottools:-line(vec_2D(PS1),vec_2D(PS3),color=
"DodgerBlue",thickness=4),
           axes = boxed,
           scaling = constrained
   );

end proc:

anim_draw_mech := proc(data, sol_kine, dof::list, k)
  draw_mech(data, sol_kine, dof[k]);
end proc:
```

```
> display(draw_mech__2(data,sol_kine_all[2],[s__2(t) = op(2,
sol_fs[1]), s__3(t) = op(2,sol_fs[2]), theta__1(t) =op(2,
sol_fs[3]), theta__2(t) = op(2,sol_fs[4]), theta__3(t) = op(2,
sol_fs[5]),theta__4(t)=3*Pi/2,s__4(t)=0]),size=[800,400]);
```



Lagrangian Equations

Kinetic and potential energy for mass m4

```
> KE4 := simplify( kineticEnergy(m__4,G4,Rs__4, 0,0,0,0,0,0) );
  PE4 := gravitationalEnergy(m__4,G4,gravity);
```

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$KE4 := \frac{1}{2}\left(m_4\left(\left(-\left(\frac{\mathrm{d}}{\mathrm{d}t}\theta_1(t)\right)\sin(\theta_1(t))\,L_{s0} - \left(\frac{\mathrm{d}}{\mathrm{d}t}\theta_4(t)\right)\sin(\theta_4(t))\,L_{s0} - \left(\frac{\mathrm{d}}{\mathrm{d}t}\theta_4(t)\right)\sin(\theta_4(t))\,s_4(t) + \cos(\theta_4(t))\left(\frac{\mathrm{d}}{\mathrm{d}t}s_4(t)\right)\right)^2 + \left(\left(\frac{\mathrm{d}}{\mathrm{d}t}\theta_1(t)\right)\cos(\theta_1(t))\,L_{s0} + \left(\frac{\mathrm{d}}{\mathrm{d}t}\theta_4(t)\right)\cos(\theta_4(t))\,L_{s0} + \left(\frac{\mathrm{d}}{\mathrm{d}t}\right.\right.\right.\right.$$

$$\theta_4(t)\Big)\cos\big(\theta_4(t)\big)\, s_4(t) + \sin\big(\theta_4(t)\big)\left(\frac{d}{dt}\, s_4(t)\right)\Big)^2\Big)\Big)$$

$$PE4 := m_4\left(\sin\big(\theta_1(t)\big)\, L_{s0} + \sin\big(\theta_4(t)\big)\, L_{s0} + \sin\big(\theta_4(t)\big)\, s_4(t)\right) g \qquad \textbf{(7.2.1)}$$

Lagrangian function

```
> LagrF :=KE1+KE2+KE3+KE4-PE1-PE2-PE3-PE4;
```

$$LagrF := \frac{m_1\left(\left(\frac{d}{dt}\,\theta_1(t)\right)^2 \sin\big(\theta_1(t)\big)^2 L_{s0}^{\ 2} + \left(\frac{d}{dt}\,\theta_1(t)\right)^2 \cos\big(\theta_1(t)\big)^2 L_{s0}^{\ 2}\right)}{2} \qquad \textbf{(7.2.2)}$$

$$+ \frac{m_2\left(\big(L_{s0} + s_2(t)\big)^2 \left(\frac{d}{dt}\,\theta_2(t)\right)^2 + \left(\frac{d}{dt}\, s_2(t)\right)^2\right)}{2}$$

$$+ \frac{m_3\left(\big(L_{s0} + s_3(t)\big)^2 \left(\frac{d}{dt}\,\theta_3(t)\right)^2 + \left(\frac{d}{dt}\, s_3(t)\right)^2\right)}{2} + \frac{1}{2}\Bigg(m_4\Bigg(\Bigg(\Bigg($$

$$-\left(\frac{d}{dt}\,\theta_1(t)\right)\sin\big(\theta_1(t)\big)\, L_{s0} - \left(\frac{d}{dt}\,\theta_4(t)\right)\sin\big(\theta_4(t)\big)\, L_{s0} - \Bigg(\frac{d}{dt}$$

$$\theta_4(t)\Bigg)\sin\big(\theta_4(t)\big)\, s_4(t) + \cos\big(\theta_4(t)\big)\left(\frac{d}{dt}\, s_4(t)\right)\Bigg)^2 + \Bigg(\Bigg(\frac{d}{dt}$$

$$\theta_1(t)\Bigg)\cos\big(\theta_1(t)\big)\, L_{s0} + \left(\frac{d}{dt}\,\theta_4(t)\right)\cos\big(\theta_4(t)\big)\, L_{s0} + \Bigg(\frac{d}{dt}$$

$$\theta_4(t)\Bigg)\cos\big(\theta_4(t)\big)\, s_4(t) + \sin\big(\theta_4(t)\big)\left(\frac{d}{dt}\, s_4(t)\right)\Bigg)^2\Bigg)\Bigg) - m_1 \sin\big(\theta_1(t)\big)\, L_{s0}\, g$$

$$- m_2\left(\sin\big(\theta_2(t)\big)\, L_{s0} + \sin\big(\theta_2(t)\big)\, s_2(t)\right) g - m_3\Big(\sin\big(\theta_3(t)\big)\, L_{s0}$$

$$+ \sin\big(\theta_3(t)\big)\, s_3(t)\Big) g - m_4\Big(\sin\big(\theta_1(t)\big)\, L_{s0} + \sin\big(\theta_4(t)\big)\, L_{s0}$$

$$+ \sin\big(\theta_4(t)\big)\, s_4(t)\Big) g$$

```
> q_vars__opt := [s__2(t),theta__1(t),s__3(t),s__4(t),theta__2
  (t),theta__3(t),theta__4(t)];
```

$$q\_vars_{opt} := \left[s_2(t), \theta_1(t), s_3(t), s_4(t), \theta_2(t), \theta_3(t), \theta_4(t)\right] \qquad \textbf{(7.2.3)}$$

```
> leqns__opt := lagrangeEquations(LagrF,Phi,q_vars__opt)
```

$$leqns_{opt} := \left[\begin{array}{r} \cdots \\ \dfrac{m_1 \left(2 \left(\frac{d^2}{dt^2} \theta_1(t)\right) \sin(\theta_1(t))^2 L_{s0}^2 + 2 \left(\frac{d^2}{dt^2} \theta_1(t)\right) \cos(\theta_1(t))\right.}{2} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \dfrac{m_4 \left(2 \left(- \right.\right.}{} \cdots \end{array}\right] \quad (7.2.4)$$

```
> z_vars := [ op(q_vars__opt), lambda__1(t)];
```

$$z\_vars := [s_2(t), \theta_1(t), s_3(t), s_4(t), \theta_2(t), \theta_3(t), \theta_4(t), \lambda_1(t)] \quad (7.2.5)$$

```
> MM, ff := GenerateMatrix(convert(leqns__opt,list),diff
  (q_vars__opt,t,t))
```

$$MM, ff := \left[\begin{array}{llr} m_2 & 0 & \cdots \\ 0 & \dfrac{m_1 \left(2 L_{s0}^2 \cos(\theta_1(t))^2 + 2 L_{s0}^2 \sin(\theta_1(t))^2\right)}{2} + & \cdots \\ 0 & 0 & \cdots \\ 0 & \dfrac{m_4 \left(2 \sin(\theta_4(t)) \cos(\theta_1(t)) L_{s0} \right.}{2} & \cdots \\ 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 0 & \dfrac{m_4 \left(-2 \left(-\sin(\theta_4(t)) L_{s0} - \sin(\theta_4(t)) s_4(t)\right) \sin(\theta_1(t)) L_{s0}\right.}{2} & \cdots \end{array}\right], \quad (7.2.6)$$

$$\left[\begin{array}{c} \ldots \\ -\frac{m_4\left(-2\left(-\left(\frac{d}{dt}\theta_1(t)\right)^2\cos(\theta_1(t))\,L_{s0}-\left(\frac{d}{dt}\theta_4(t)\right)^2\cos(\theta\right.\right.}{} \ldots \\ \ldots \\ \ldots \\ \ldots \\ \ldots \\ -\frac{m_4\left(2\left(-\left(\frac{d}{dt}\theta_1(t)\right)^2\cos(\theta_1(t))\,L_{s0}-\left(\frac{d}{dt}\right.\right.\right.}{} \ldots \end{array}\right]$$

Damper Cs4 Force

```
> fe__4(t) := -C__s4*diff(s__4(t),t); # damper force model
  FE__4 := <fe__4(t),0,0>;
```

$$fe_4(t) := -C_{s4}\left(\frac{d}{dt}s_4(t)\right)$$

$$FE_4 := \left[\begin{array}{c} -C_{s4}\left(\frac{d}{dt}s_4(t)\right) \\ 0 \\ 0 \end{array}\right] \qquad (7.2.7)$$

```
> FE__4:=Rs__4.FE__4;
```

$$FE_4 := \left[\begin{array}{c} -\cos(\theta_4(t))\,C_{s4}\left(\frac{d}{dt}s_4(t)\right) \\ -\sin(\theta_4(t))\,C_{s4}\left(\frac{d}{dt}s_4(t)\right) \\ 0 \end{array}\right] \qquad (7.2.8)$$

```
> Q_genF__opt := generalisedForces(G2,FE__2,q_vars__opt)-
  generalisedForces(P2,FE__2,q_vars__opt)+generalisedForces(G3,
  FE__3,q_vars__opt)-generalisedForces(P3,FE__3,q_vars__opt)+
  generalisedForces(G4,FE__4,q_vars__opt)-generalisedForces(G3,
  FE__4,q_vars__opt);
```

(7.2.9)

$Q\_genF_{opt} :=$ (7.2.9)

$$\left[\begin{array}{c} \cdots \\ \cdots \\ \text{co}\cdots \\ \cdots \\ \cdots \\ \cos(\theta_3(t))\left(-K_{s3}\,s_3(t) - C_{s3}\left(\frac{d}{dt}\,s_3(t)\right)\right)\left(-\sin(\theta_3(t))\,L_{s0} - \cdots\right. \\ \cdots \end{array}\right]$$

Full set of equations

```
> ode_sys__opt := leqns__opt-Q_genF__opt;
```

$ode\_sys_{opt} :=$ (7.2.10)

$$\left[\begin{array}{c} \cdots \\ \frac{m_1\left(2\left(\frac{d^2}{dt^2}\,\theta_1(t)\right)\sin(\theta_1(t))^2 L_{s0}^2 + 2\left(\frac{d^2}{dt^2}\,\theta_1(t)\right)\cos(\theta_1(t\cdots\right.\right.}{2} \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \frac{m_4\left(2\left(-\left(\frac{d^2}{dt^2}\,\theta_1(t)\right)\sin(\theta_1(t\cdots\right.\right.}{} \end{array}\right]$$

Tune the mass and damping coefficient to minimise the settling time of the system

In this section the mechanical system is solved assuming a mass m4=1 and a damper coefficient Cs4=10.
In the following sections the values of mass and damper are changed in order to find the best combination.

```
> data__opt := [L = 1, L__s0 = 1, m__1 = 10, m__2 = 2, m__3 =
  30, g = 9.81,
  K__s2 = 5000, C__s2 = 10, K__s3 = 100000, C__s3 = 100,C__s4=
  10,m__4=1];
```

$$data_{opt} := \left[L = 1, L_{s0} = 1, m_1 = 10, m_2 = 2, m_3 = 30, g = 9.81, K_{s2} = 5000, C_{s2} = 10, K_{s3} = 100000, C_{s3} = 100, C_{s4} = 10, m_4 = 1\right] \quad (7.3.1)$$

```
> sys_indexred__opt := [op(convert(ode_sys__opt,list)),op
  (Phi_tt)]: <%>
```

$$\left[\begin{array}{c} \ldots \\ \frac{m_1\left(2\left(\frac{d^2}{dt^2}\theta_1(t)\right)\sin\left(\theta_1(t)\right)^2 L_{s0}^2 + 2\left(\frac{d^2}{dt^2}\theta_1(t)\right)\cos\left(\theta_1(t)\right)\ldots}{2} \ldots \\ \ldots \\ \ldots \\ \ldots \\ \ldots \\ \frac{m_4\left(2\left(-\left(\frac{d^2}{dt^2}\theta_1(t)\right)\sin\left(\theta_1(t)\right)\ldots\right.\right.}{} \ldots \\ \ldots \\ \ldots \end{array}\right] \quad (7.3.2)$$

```
> MM, bb := GenerateMatrix( sys_indexred__opt,[op(diff
  (q_vars__opt,t,t)),lambda__1(t),lambda__2(t)]);
```

(7.3.3)

$MM, bb :=$ **(7.3.3)**

$$\left[\begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 2\left(-L - \cos\left(\theta_1(t)\right) L_{s0} + \cos\left(\theta_2(t)\right) s_2(t) + \cos\left(\theta_2(t)\right) L_{s0}\right) \cos\cdots \end{array}\right],$$

$$\left[\begin{array}{l} \ldots \\ \ldots \\ \ldots \\ \ldots \\ \ldots \\ \ldots \\ -\frac{m_4\left(2\left(-\left(\frac{\mathrm{d}}{\mathrm{d}t}\theta_1(t)\right)^2\cos\left(\theta_1(t)\right)L_{s0}-\left(\frac{\mathrm{d}}{\mathrm{d}t}\theta_4(t)\right)^2\cos\left(\theta_4(\right.\right.\right.}{} \ldots \\ \ldots \\ \ldots \end{array}\right]$$

```
> [s__2(t) = 0, diff(s__2(t),t) = 0,s__3(t) = 0, diff(s__3(t),t)
  = 0,s__4(t) = 0, diff(s__4(t),t) = 0,theta__1(t) = -Pi/2,diff
  (theta__1(t),t)=0,theta__2(t) = -Pi/2,diff(theta__2(t),t) = 0,
  theta__3(t) = -Pi/2,diff(theta__3(t),t)=0,theta__4(t) = -Pi/2,
  diff(theta__4(t),t)=0];
```

$$\left[s_2(t)=0,\ \frac{\mathrm{d}}{\mathrm{d}t}s_2(t)=0,\ s_3(t)=0,\ \frac{\mathrm{d}}{\mathrm{d}t}s_3(t)=0,\ s_4(t)=0,\ \frac{\mathrm{d}}{\mathrm{d}t}s_4(t)=0,\ \theta_1(t)=-\frac{\pi}{2},\ \frac{\mathrm{d}}{\mathrm{d}t}\theta_1(t)=0,\ \theta_2(t)=-\frac{\pi}{2},\ \frac{\mathrm{d}}{\mathrm{d}t}\theta_2(t)=0,\ \theta_3(t)=-\frac{\pi}{2},\ \frac{\mathrm{d}}{\mathrm{d}t}\theta_3(t)=0,\ \theta_4(t)=-\frac{\pi}{2},\ \frac{\mathrm{d}}{\mathrm{d}t}\theta_4(t)=0\right] \tag{7.3.4}$$

```
> ics_v__opt := {%} union subs( %, data, diff(sol_kine,t) );
```

$$ics\_v_{opt} := \left\{0=0,\ \left[s_2(t)=0,\ \frac{\mathrm{d}}{\mathrm{d}t}s_2(t)=0,\ s_3(t)=0,\ \frac{\mathrm{d}}{\mathrm{d}t}s_3(t)=0,\ s_4(t)=0,\ \frac{\mathrm{d}}{\mathrm{d}t}s_4(t)=0,\ \theta_1(t)=-\frac{\pi}{2},\ \frac{\mathrm{d}}{\mathrm{d}t}\theta_1(t)=0,\ \theta_2(t)=-\frac{\pi}{2},\ \frac{\mathrm{d}}{\mathrm{d}t}\theta_2(t)=0,\ \theta_3(t)=-\frac{\pi}{2},\ \frac{\mathrm{d}}{\mathrm{d}t}\right.\right. \tag{7.3.5}$$

$$\theta_3(t) = 0, \theta_4(t) = -\frac{\pi}{2}, \frac{d}{dt}\,\theta_4(t) = 0\Big]\Big]\Big\}$$

```
> ics__opt := ics_v__opt[2];
```

$$ics_{opt} := \Big[s_2(t) = 0, \frac{d}{dt}\,s_2(t) = 0, s_3(t) = 0, \frac{d}{dt}\,s_3(t) = 0, s_4(t) = 0, \frac{d}{dt}\,s_4(t) = 0, \theta_1(t) = -\frac{\pi}{2}, \frac{d}{dt}\,\theta_1(t) = 0, \theta_2(t) = -\frac{\pi}{2}, \frac{d}{dt}\,\theta_2(t) = 0, \theta_3(t) = -\frac{\pi}{2}, \frac{d}{dt}\,\theta_3(t) = 0, \theta_4(t) = -\frac{\pi}{2}, \frac{d}{dt}\,\theta_4(t) = 0\Big] \quad \textbf{(7.3.6)}$$

```
> evalf(subs(ics__opt, data__opt, t=0, MM)):
  evalf(subs(ics__opt, data__opt, t=0, bb)):
  tmp:= LinearSolve(%%,%);
```

$$tmp := \begin{bmatrix} 9.81000000000000 \\ 4.79062896530095 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706217810 \times 10^{-9} \\ 2.49112706217810 \times 10^{-9} \\ 1.53300126946991 \times 10^{-9} \\ 7.18594344767143 \times 10^{-9} \\ -4.79062896678096 \times 10^{-10} \end{bmatrix} \quad \textbf{(7.3.7)}$$

```
> <[op(diff(q_vars__opt,t,t)),lambda__1(t),lambda__1(t)]> =
  tmp;

  ics_lambda := [lambda__1(t) = tmp[7],lambda__2(t) = tmp[8]];
```

$$\begin{bmatrix} \frac{d^2}{dt^2} s_2(t) \\ \frac{d^2}{dt^2} \theta_1(t) \\ \frac{d^2}{dt^2} s_3(t) \\ \frac{d^2}{dt^2} s_4(t) \\ \frac{d^2}{dt^2} \theta_2(t) \\ \frac{d^2}{dt^2} \theta_3(t) \\ \frac{d^2}{dt^2} \theta_4(t) \\ \lambda_1(t) \\ \lambda_1(t) \end{bmatrix} = \begin{bmatrix} 9.81000000000000 \\ 4.79062896530095 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706217810 \times 10^{-9} \\ 2.49112706217810 \times 10^{-9} \\ 1.53300126946991 \times 10^{-9} \\ 7.18594344767143 \times 10^{-9} \\ -4.79062896678096 \times 10^{-10} \end{bmatrix}$$

$$ics\_lambda := \left[\lambda_1(t) = 1.53300126946991 \times 10^{-9}, \lambda_2(t) = 7.18594344767143 \times 10^{-9}\right] \quad \textbf{(7.3.8)}$$

```
> dae_sys  := convert(subs(data__opt,[op(convert(ode_sys__opt,
  list)),op(Phi)]),set): <op(%)>;
  full_ics := subs( t = 0, convert(convert(ics__opt,set) union
  {ics_lambda[1]} union {ics_lambda[2]}, D));
```

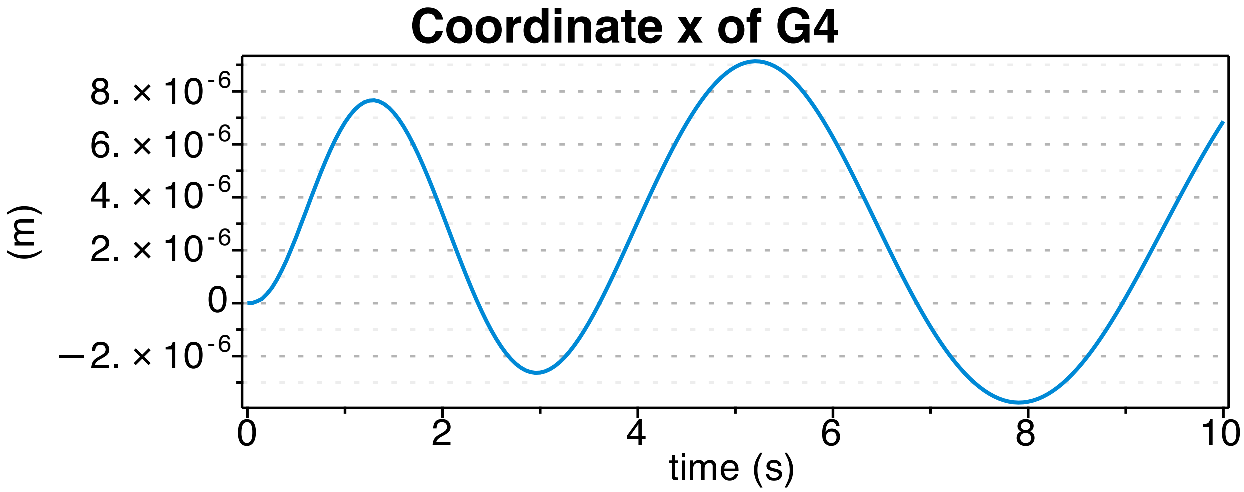
$$\left[\begin{array}{r} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 10\left(\frac{d^2}{dt^2}\theta_1(t)\right)\sin\left(\theta_1(t)\right)^2+10\left(\frac{d^2}{dt^2}\theta_1(t)\right)\cos\left(\theta_1(t)\right)^2-\left(-\cdots\right. \\ \cdots \end{array}\right]$$

$$full\_ics := \left\{s_2(0)=0, s_3(0)=0, s_4(0)=0, \lambda_1(0)=1.53300126946991\times 10^{-9}, \lambda_2(0) = 7.18594344767143\times 10^{-9}, \theta_1(0)=-\frac{\pi}{2}, \theta_2(0)=-\frac{\pi}{2}, \theta_3(0)=-\frac{\pi}{2}, \theta_4(0) = -\frac{\pi}{2}, D(s_2)(0)=0, D(s_3)(0)=0, D(s_4)(0)=0, D(\theta_1)(0)=0, D(\theta_2)(0) = 0, D(\theta_3)(0)=0, D(\theta_4)(0)=0\right\} \quad \textbf{(7.3.9)}$$

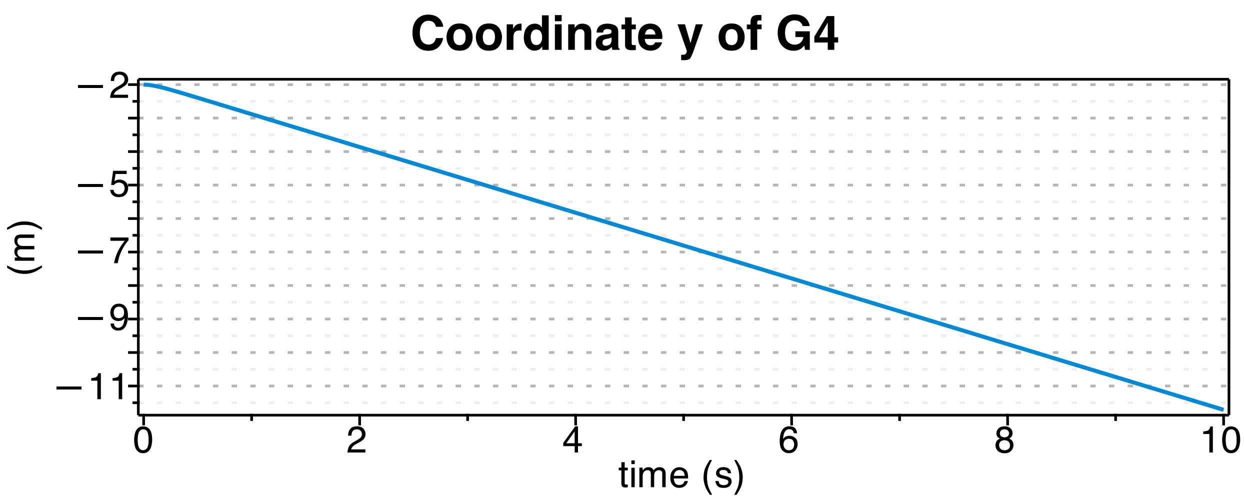
```
> sol_dae__opt := dsolve(convert(dae_sys,set) union full_ics,
  numeric,implicit=true,maxfun=300000);
```

$$sol\_dae_{opt} := \textbf{proc}(x\_rkf45\_dae)\ \ldots\ \textbf{end proc} \quad \textbf{(7.3.10)}$$

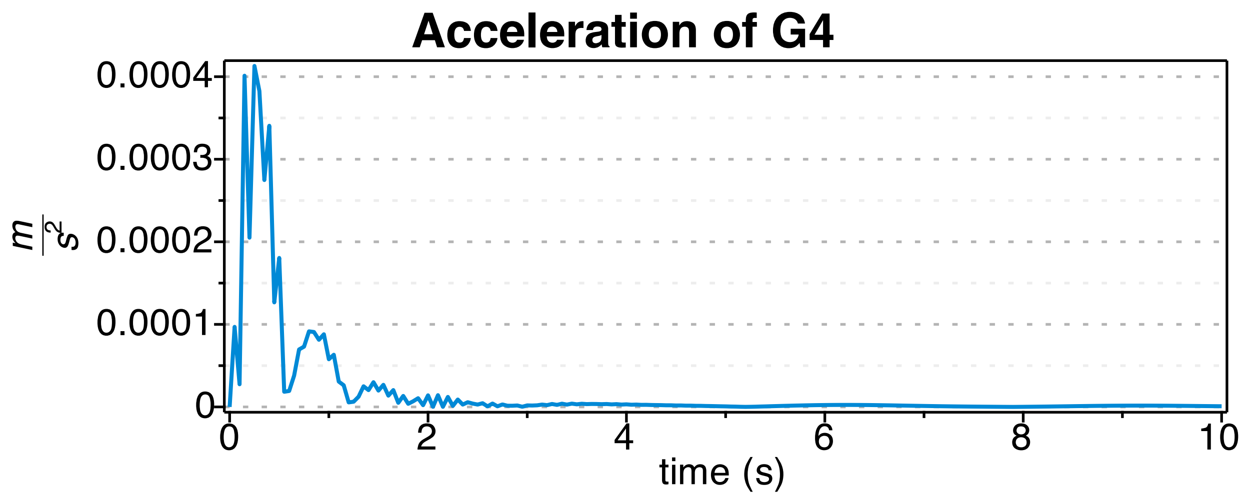
```
> TF := 10:
> odeplot(sol_dae__opt,subs(data,[t,G4[1]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate x of G4");
```



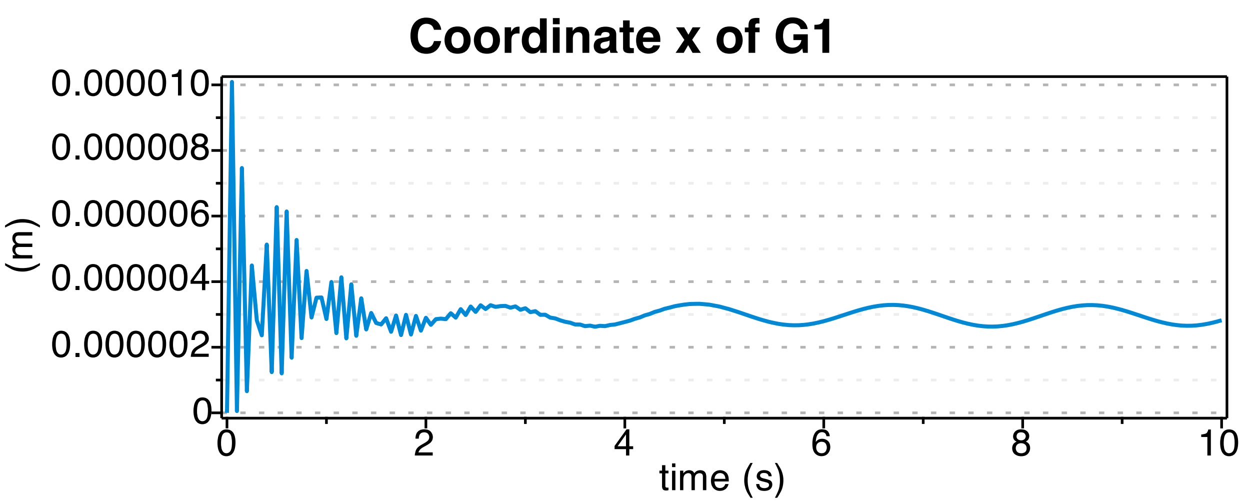

```
> odeplot(sol_dae__opt,subs(data,[t,G4[2]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate y of G4");
```



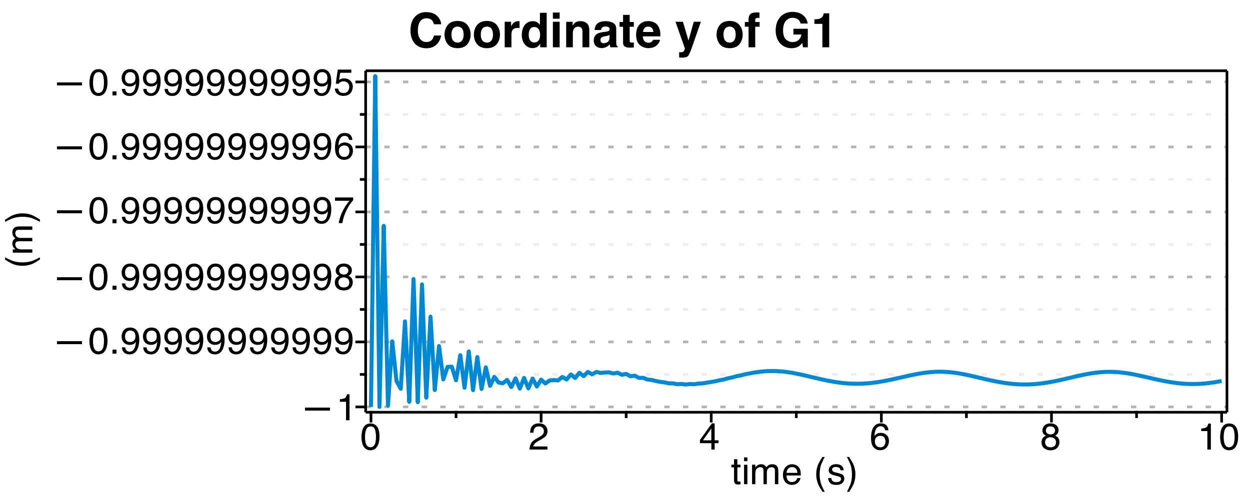

```
> odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G4[1],t,t)^2+diff
  (G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],
          title  = "Acceleration of G4");
```



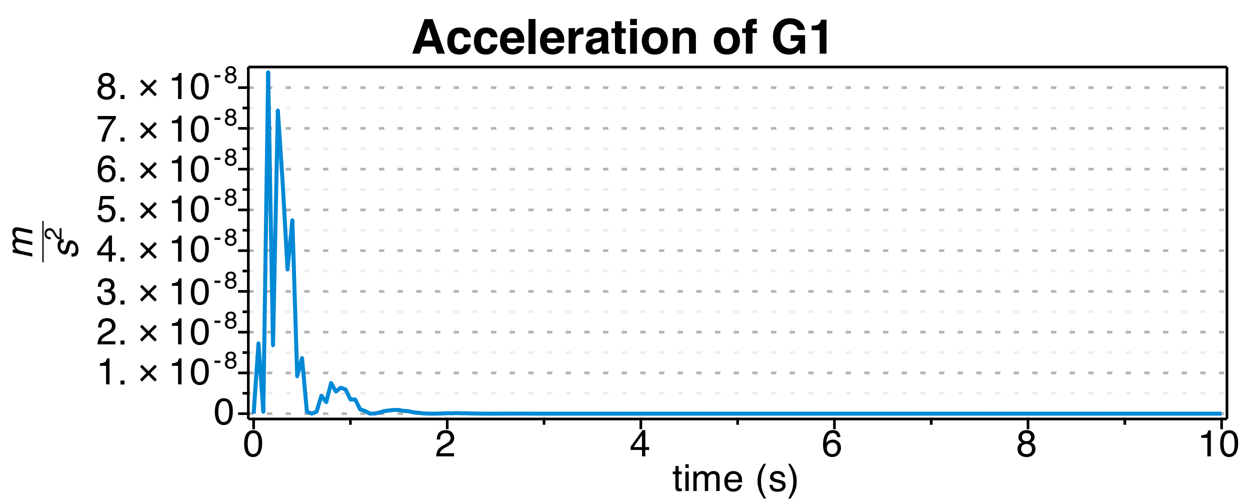

```
> odeplot(sol_dae__opt,subs(data,[t,G1[1]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate x of G1");
```




```
> odeplot(sol_dae__opt,subs(data,[t,G1[2]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate y of G1");
```




```
> odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G1[1],t,t)^2+diff
  (G1[2],t,t)^2)]),t=0..TF,
         labels = ["time (s)", m/s^2],
         title  = "Acceleration of G1");
```



Simulation with m4 = 10 and Cs4 = 10

In this section the mechanical system is solved assuming a mass m4=10 and a damper coefficient Cs4=10

```
> data__opt := [L = 1, L__s0 = 1, m__1 = 10, m__2 = 2, m__3 =
  30, g = 9.81,
  K__s2 = 5000, C__s2 = 10, K__s3 = 100000, C__s3 = 100,C__s4=
  10,m__4=10];
```

$$data_{opt} := [L = 1, L_{s0} = 1, m_1 = 10, m_2 = 2, m_3 = 30, g = 9.81, K_{s2} = 5000, C_{s2} = 10, K_{s3} = 100000, C_{s3} = 100, C_{s4} = 10, m_4 = 10] \quad \textbf{(7.4.1)}$$

```
> evalf(subs(ics__opt, data__opt, t=0, MM)):
  evalf(subs(ics__opt, data__opt, t=0, bb)):
  tmp:= LinearSolve(%%,%);
```

$$tmp := \begin{bmatrix} 9.81000000000000 \\ 4.79062896434858 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706208286 \times 10^{-9} \\ 2.49112706208286 \times 10^{-9} \\ 1.53300126956514 \times 10^{-9} \\ 7.18594344624286 \times 10^{-9} \\ -4.79062896582857 \times 10^{-10} \end{bmatrix} \tag{7.4.2}$$

```
> <[op(diff(q_vars__opt,t,t)),lambda__1(t),lambda__1(t)]> =
  tmp;

  ics_lambda := [lambda__1(t) = tmp[7],lambda__2(t) = tmp[8]];
```

$$\begin{bmatrix} \frac{d^2}{dt^2}\, s_2(t) \\ \frac{d^2}{dt^2}\, \theta_1(t) \\ \frac{d^2}{dt^2}\, s_3(t) \\ \frac{d^2}{dt^2}\, s_4(t) \\ \frac{d^2}{dt^2}\, \theta_2(t) \\ \frac{d^2}{dt^2}\, \theta_3(t) \\ \frac{d^2}{dt^2}\, \theta_4(t) \\ \lambda_1(t) \\ \lambda_1(t) \end{bmatrix} = \begin{bmatrix} 9.81000000000000 \\ 4.79062896434858 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706208286 \times 10^{-9} \\ 2.49112706208286 \times 10^{-9} \\ 1.53300126956514 \times 10^{-9} \\ 7.18594344624286 \times 10^{-9} \\ -4.79062896582857 \times 10^{-10} \end{bmatrix}$$

$$ics\_lambda := \left[\lambda_1(t) = 1.53300126956514 \times 10^{-9}, \lambda_2(t) = 7.18594344624286 \times 10^{-9}\right] \tag{7.4.3}$$

```
> dae_sys  := convert(subs(data__opt,[op(convert(ode_sys__opt,
  list)),op(Phi)]),set): <op(%)>;
  full_ics := subs( t = 0, convert(convert(ics__opt,set) union
```

```
{ics_lambda[1]} union {ics_lambda[2]}, D));
```

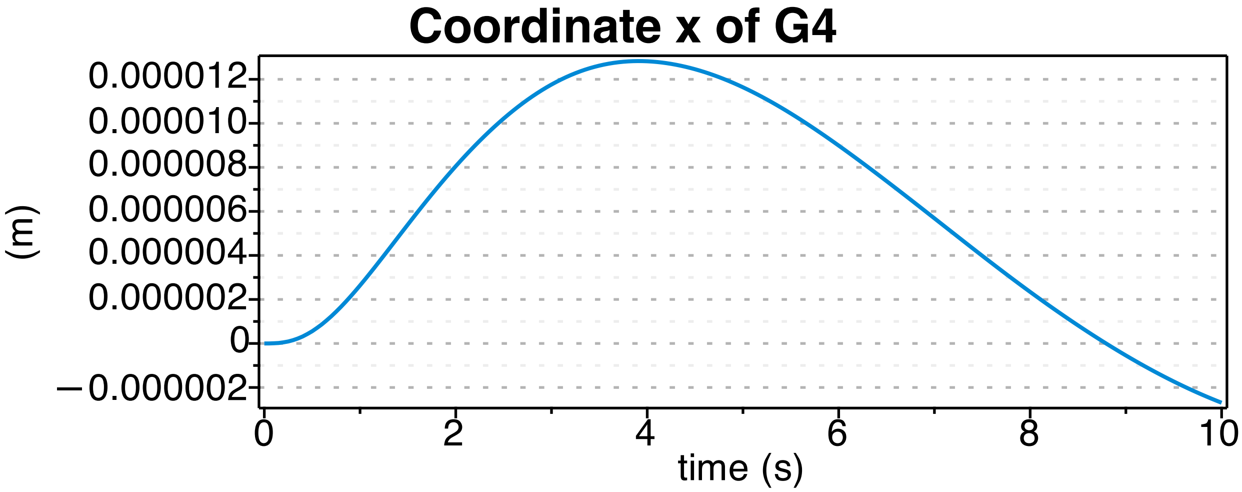
$$\left[\begin{array}{c} \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ 10\left(\frac{d^2}{dt^2}\theta_1(t)\right)\sin\left(\theta_1(t)\right)^2+10\left(\frac{d^2}{dt^2}\theta_1(t)\right)\cos\left(\theta_1(t)\right)^2-10 \dots \\ \dots \end{array}\right]$$

$$full\_ics := \left\{s_2(0)=0, s_3(0)=0, s_4(0)=0, \lambda_1(0)=1.53300126956514\times 10^{-9}, \lambda_2(0) = 7.18594344624286\times 10^{-9}, \theta_1(0)=-\frac{\pi}{2}, \theta_2(0)=-\frac{\pi}{2}, \theta_3(0)=-\frac{\pi}{2}, \theta_4(0) = -\frac{\pi}{2}, D(s_2)(0)=0, D(s_3)(0)=0, D(s_4)(0)=0, D(\theta_1)(0)=0, D(\theta_2)(0) = 0, D(\theta_3)(0)=0, D(\theta_4)(0)=0\right\} \quad \textbf{(7.4.4)}$$

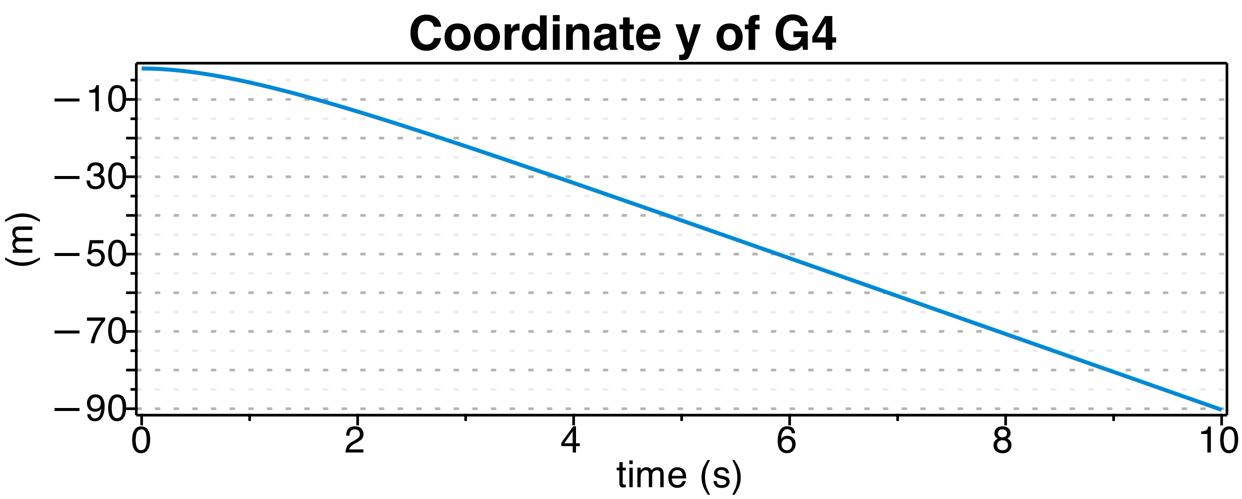
```
> sol_dae__opt2 := dsolve(convert(dae_sys,set) union full_ics,
  numeric,implicit=true,maxfun=300000);
```

$$sol\_dae_{opt2} := \textbf{proc}(x\_rkf45\_dae)\ \dots\ \textbf{end proc} \quad \textbf{(7.4.5)}$$

```
> odeplot(sol_dae__opt2,subs(data,[t,G4[1]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate x of G4");
```

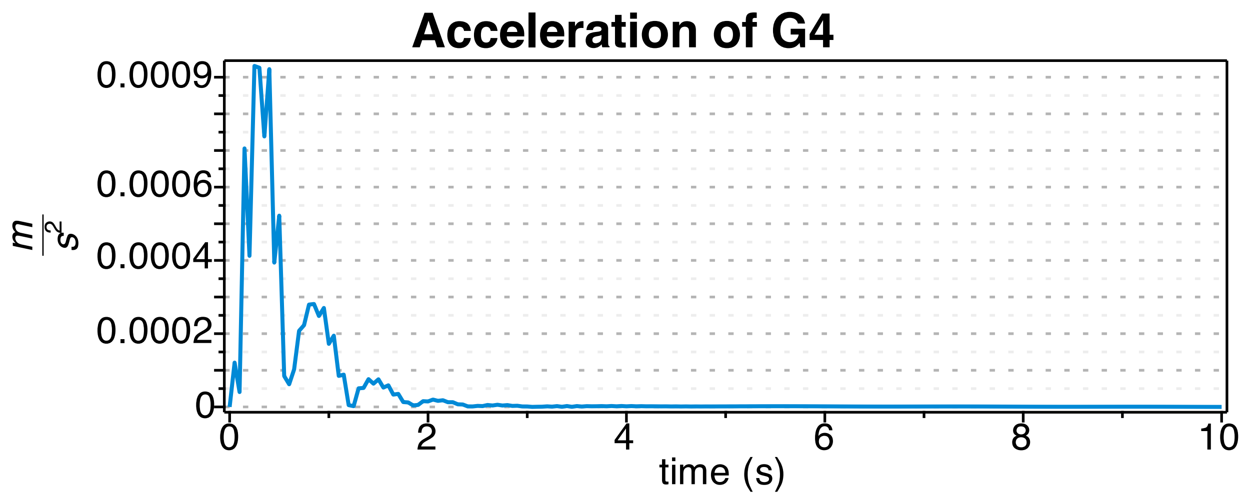



```
> odeplot(sol_dae__opt2,subs(data,[t,G4[2]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate y of G4");
```

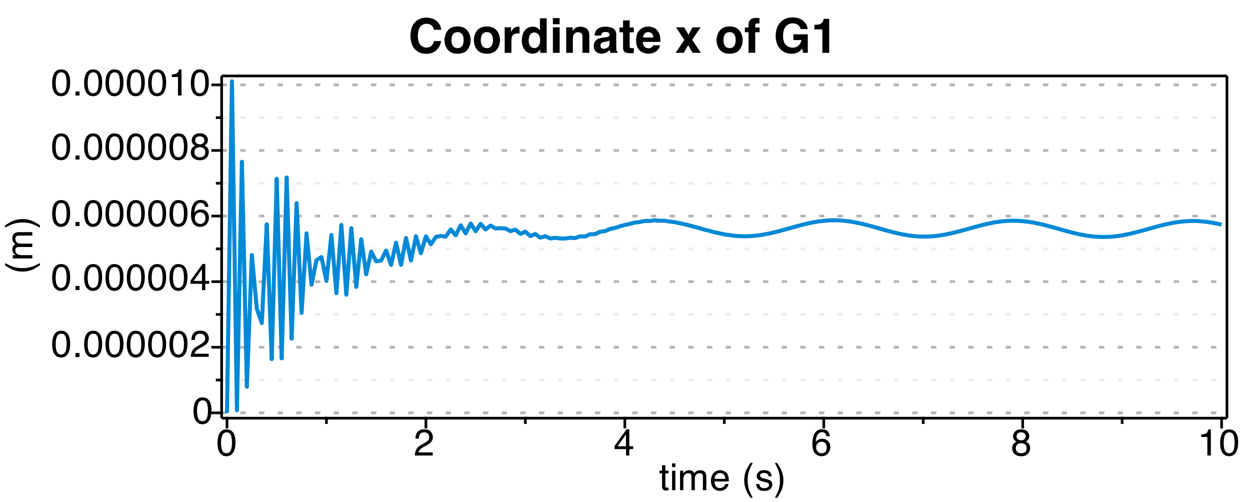


Plot the modulus of the acceleration of mass G4

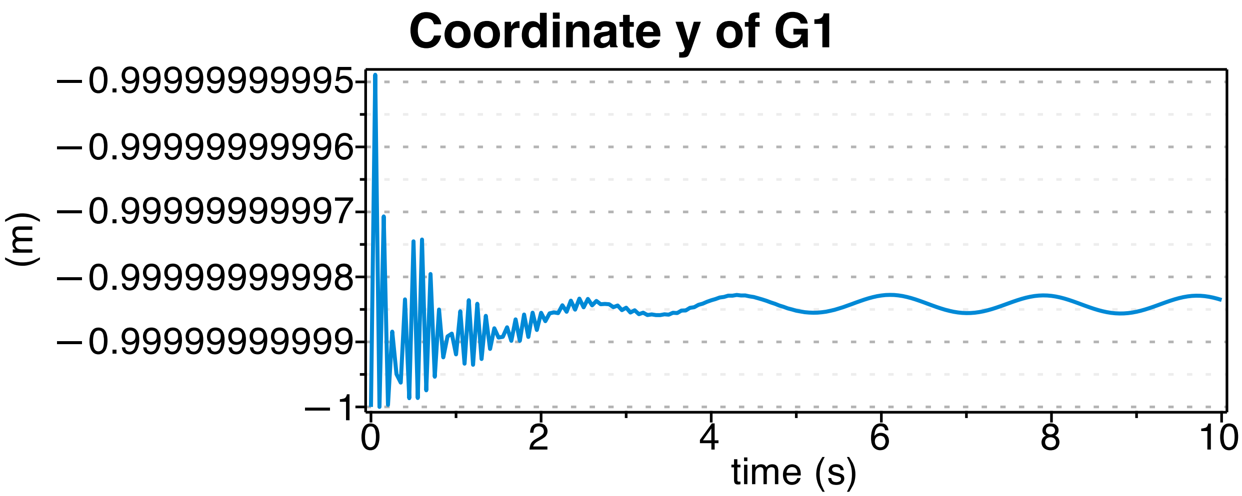
```
> odeplot(sol_dae__opt2,subs(data,[t,sqrt(diff(G4[1],t,t)^2+diff
  (G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],
          title  = "Acceleration of G4");
```




```
> odeplot(sol_dae__opt2,subs(data,[t,G1[1]]),t=0..TF,
         labels = ["time (s)", "(m)"],
         title  = "Coordinate x of G1");
```




```
> odeplot(sol_dae__opt2,subs(data,[t,G1[2]]),t=0..TF,
         labels = ["time (s)", "(m)"],
         title  = "Coordinate y of G1");
```



Simulation with m4 = 10 and Cs4 = 100

In this section the mechanical system is solved assuming a mass m4=10 and a damper coefficient Cs4=100

```
> data__opt := [L = 1, L__s0 = 1, m__1 = 10, m__2 = 2, m__3 =
  30, g = 9.81,
  K__s2 = 5000, C__s2 = 10, K__s3 = 100000, C__s3 = 100,C__s4=
  100,m__4=10];
```

$$data_{opt} := [L = 1, L_{s0} = 1, m_1 = 10, m_2 = 2, m_3 = 30, g = 9.81, K_{s2} = 5000, C_{s2} = 10, K_{s3} = 100000, C_{s3} = 100, C_{s4} = 100, m_4 = 10] \quad \textbf{(7.5.1)}$$

```
> evalf(subs(ics__opt, data__opt, t=0, MM)):
  evalf(subs(ics__opt, data__opt, t=0, bb)):
  tmp:= LinearSolve(%%,%);
```

$$tmp := \begin{bmatrix} 9.81000000000000 \\ 4.79062896434858 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706208286 \times 10^{-9} \\ 2.49112706208286 \times 10^{-9} \\ 1.53300126956514 \times 10^{-9} \\ 7.18594344624286 \times 10^{-9} \\ -4.79062896582857 \times 10^{-10} \end{bmatrix} \quad \textbf{(7.5.2)}$$

```
> <[op(diff(q_vars__opt,t,t)),lambda__1(t),lambda__1(t)]> =
  tmp;

  ics_lambda := [lambda__1(t) = tmp[7],lambda__2(t) = tmp[8]];
```

$$\begin{bmatrix} \frac{d^2}{dt^2}\, s_2(t) \\ \frac{d^2}{dt^2}\, \theta_1(t) \\ \frac{d^2}{dt^2}\, s_3(t) \\ \frac{d^2}{dt^2}\, s_4(t) \\ \frac{d^2}{dt^2}\, \theta_2(t) \\ \frac{d^2}{dt^2}\, \theta_3(t) \\ \frac{d^2}{dt^2}\, \theta_4(t) \\ \lambda_1(t) \\ \lambda_1(t) \end{bmatrix} = \begin{bmatrix} 9.81000000000000 \\ 4.79062896434858 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706208286 \times 10^{-9} \\ 2.49112706208286 \times 10^{-9} \\ 1.53300126956514 \times 10^{-9} \\ 7.18594344624286 \times 10^{-9} \\ -4.79062896582857 \times 10^{-10} \end{bmatrix}$$

$$ics\_lambda := \left[\lambda_1(t) = 1.53300126956514 \times 10^{-9}, \lambda_2(t) = 7.18594344624286 \times 10^{-9}\right] \quad \textbf{(7.5.3)}$$

```
> dae_sys  := convert(subs(data__opt,[op(convert(ode_sys__opt,
  list)),op(Phi)]),set): <op(%)>;
  full_ics := subs( t = 0, convert(convert(ics__opt,set) union
  {ics_lambda[1]} union {ics_lambda[2]}, D));
```

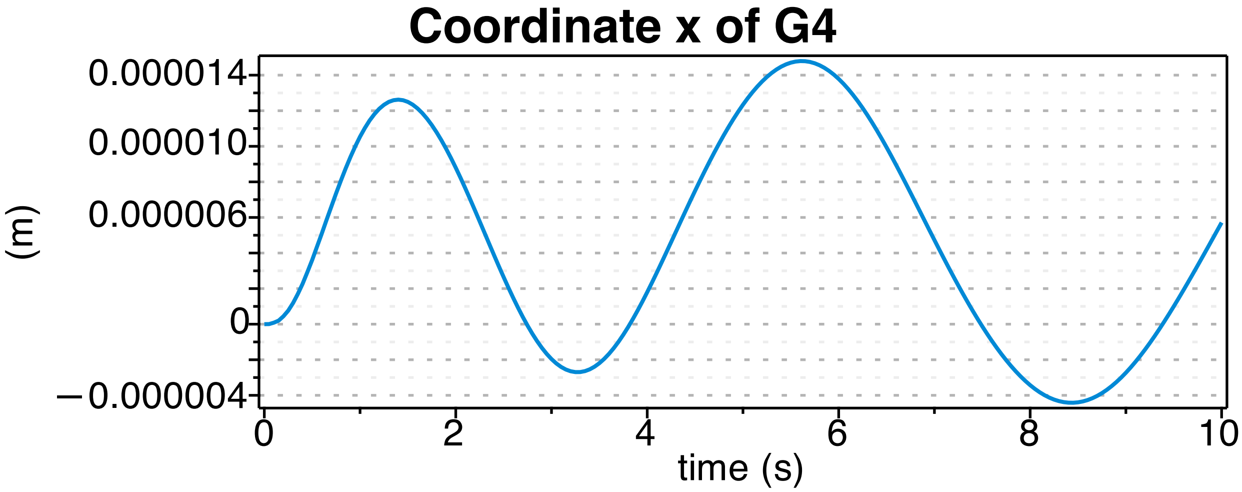
$$\left[\begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 10\left(\frac{d^2}{dt^2}\theta_1(t)\right)\sin(\theta_1(t))^2 + 10\left(\frac{d^2}{dt^2}\theta_1(t)\right)\cos(\theta_1(t))^2 - 10 \cdots \\ \cdots \end{array}\right]$$

$$full\_ics := \{s_2(0) = 0, s_3(0) = 0, s_4(0) = 0, \lambda_1(0) = 1.53300126956514 \times 10^{-9}, \lambda_2(0) = 7.18594344624286 \times 10^{-9}, \theta_1(0) = -\frac{\pi}{2}, \theta_2(0) = -\frac{\pi}{2}, \theta_3(0) = -\frac{\pi}{2}, \theta_4(0) = -\frac{\pi}{2}, D(s_2)(0) = 0, D(s_3)(0) = 0, D(s_4)(0) = 0, D(\theta_1)(0) = 0, D(\theta_2)(0) = 0, D(\theta_3)(0) = 0, D(\theta_4)(0) = 0\} \quad \textbf{(7.5.4)}$$

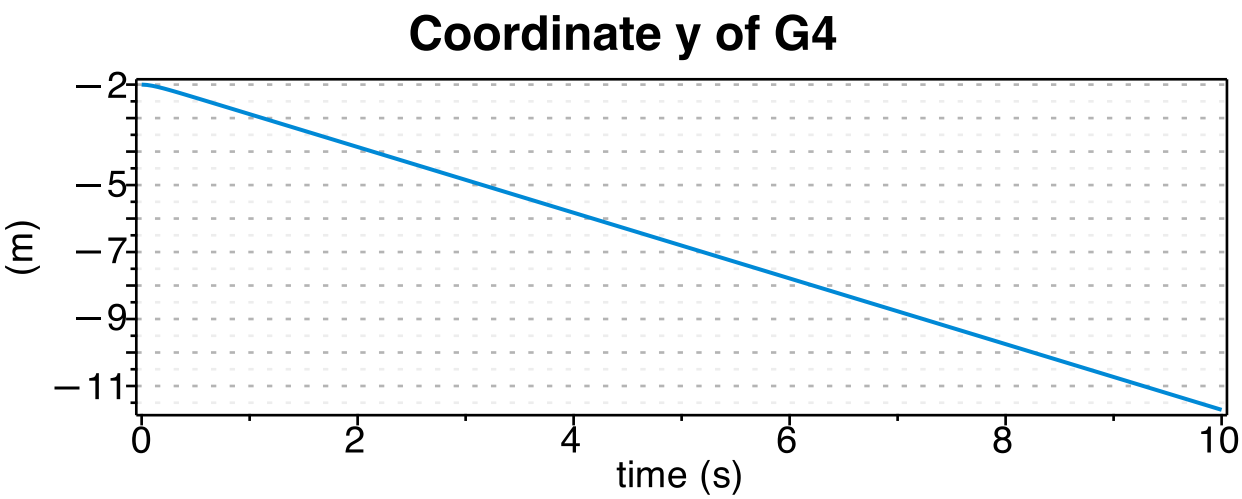
```
> sol_dae__opt3 := dsolve(convert(dae_sys,set) union full_ics,
  numeric,implicit=true,maxfun=300000);
```

$$sol\_dae_{opt3} := \textbf{proc}(x\_rkf45\_dae) \ \ldots \ \textbf{end proc} \quad \textbf{(7.5.5)}$$

```
> odeplot(sol_dae__opt3,subs(data,[t,G4[1]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate x of G4");
```

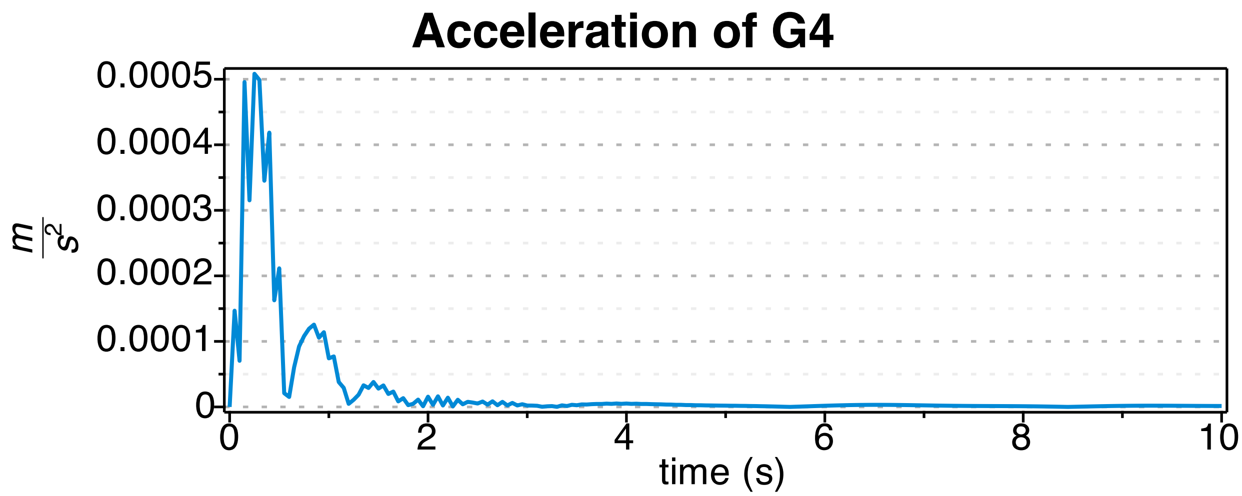



```
> odeplot(sol_dae__opt3,subs(data,[t,G4[2]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate y of G4");
```

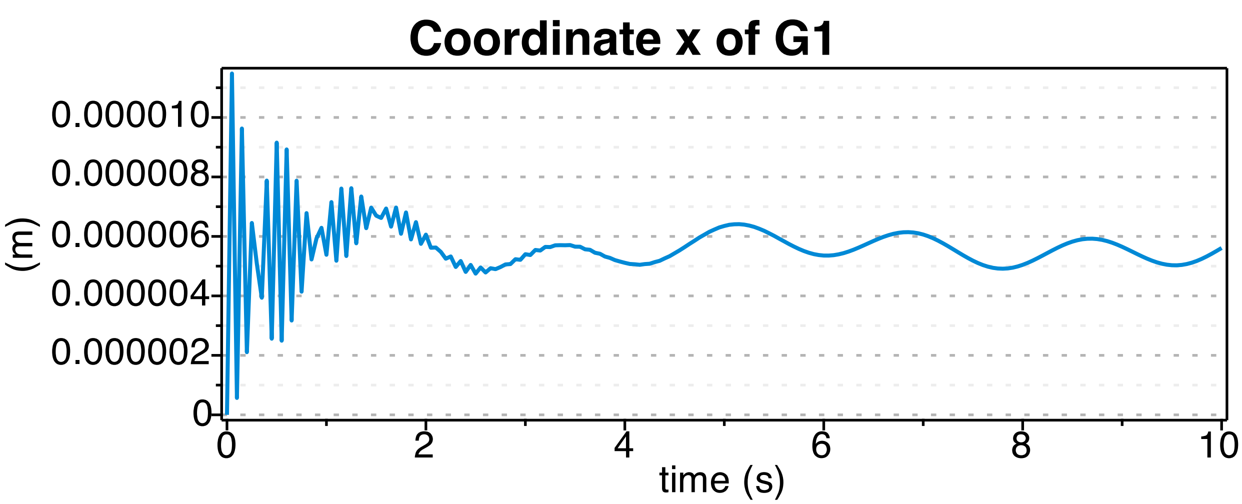


Plot the modulus of the acceleration of mass G4

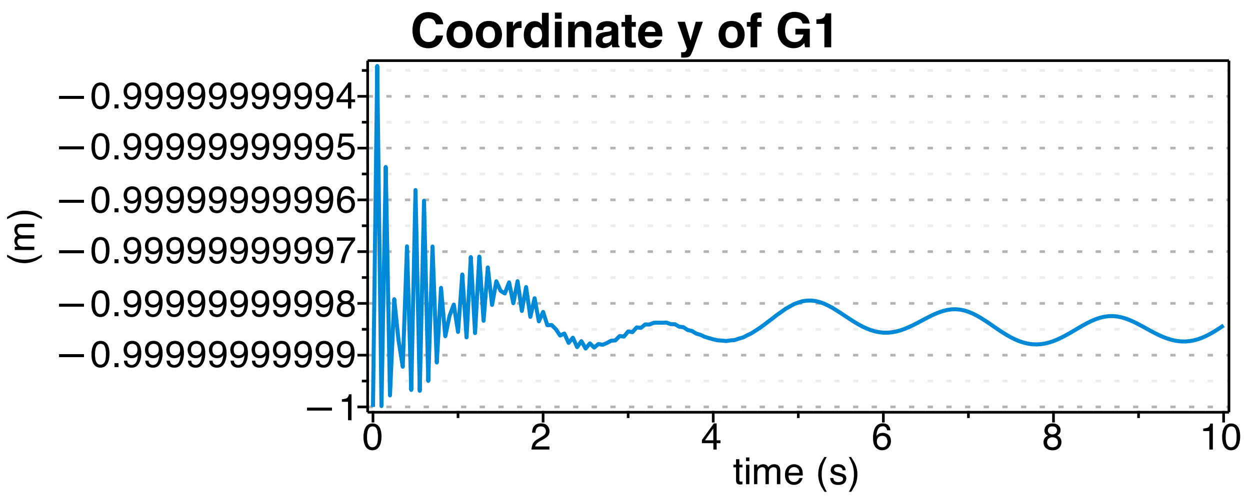
```
> odeplot(sol_dae__opt3,subs(data,[t,sqrt(diff(G4[1],t,t)^2+diff
  (G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],
          title  = "Acceleration of G4");
```




```
> odeplot(sol_dae__opt3,subs(data,[t,G1[1]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate x of G1");
```




```
> odeplot(sol_dae__opt3,subs(data,[t,G1[2]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate y of G1");
```



Simulation with m4 = 100 and Cs4 = 100

In this section the mechanical system is solved assuming a mass m4=100 and a damper coefficient Cs4=100

```
> data__opt := [L = 1, L__s0 = 1, m__1 = 10, m__2 = 2, m__3 = 
  30, g = 9.81,
  K__s2 = 5000, C__s2 = 10, K__s3 = 100000, C__s3 = 100,C__s4=
  100,m__4=100];
```

$$data_{opt} := [L = 1, L_{s0} = 1, m_1 = 10, m_2 = 2, m_3 = 30, g = 9.81, K_{s2} = 5000, C_{s2} = 10, K_{s3} = 100000, C_{s3} = 100, C_{s4} = 100, m_4 = 100] \quad (7.6.1)$$

```
> evalf(subs(ics__opt, data__opt, t=0, MM)):
  evalf(subs(ics__opt, data__opt, t=0, bb)):
  tmp:= LinearSolve(%%,%);
```

$$tmp := \begin{bmatrix} 9.81000000000000 \\ 4.79062895244380 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706089238 \times 10^{-9} \\ 2.49112706089238 \times 10^{-9} \\ 1.53300127075562 \times 10^{-9} \\ 7.18594342838571 \times 10^{-9} \\ -4.79062895392381 \times 10^{-10} \end{bmatrix} \quad (7.6.2)$$

```
> <[op(diff(q_vars__opt,t,t)),lambda__1(t),lambda__1(t)]> = 
  tmp;

  ics_lambda := [lambda__1(t) = tmp[7],lambda__2(t) = tmp[8]];
```

$$\begin{bmatrix} \frac{d^2}{dt^2}\, s_2(t) \\ \frac{d^2}{dt^2}\, \theta_1(t) \\ \frac{d^2}{dt^2}\, s_3(t) \\ \frac{d^2}{dt^2}\, s_4(t) \\ \frac{d^2}{dt^2}\, \theta_2(t) \\ \frac{d^2}{dt^2}\, \theta_3(t) \\ \frac{d^2}{dt^2}\, \theta_4(t) \\ \lambda_1(t) \\ \lambda_1(t) \end{bmatrix} = \begin{bmatrix} 9.81000000000000 \\ 4.79062895244380 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706089238 \times 10^{-9} \\ 2.49112706089238 \times 10^{-9} \\ 1.53300127075562 \times 10^{-9} \\ 7.18594342838571 \times 10^{-9} \\ -4.79062895392381 \times 10^{-10} \end{bmatrix}$$

$$ics\_lambda := \left[\lambda_1(t) = 1.53300127075562 \times 10^{-9}, \lambda_2(t) = 7.18594342838571 \times 10^{-9}\right] \tag{7.6.3}$$

```
> dae_sys  := convert(subs(data__opt,[op(convert(ode_sys__opt,
  list)),op(Phi)]),set): <op(%)>;
  full_ics := subs( t = 0, convert(convert(ics__opt,set) union
  {ics_lambda[1]} union {ics_lambda[2]}, D));
```

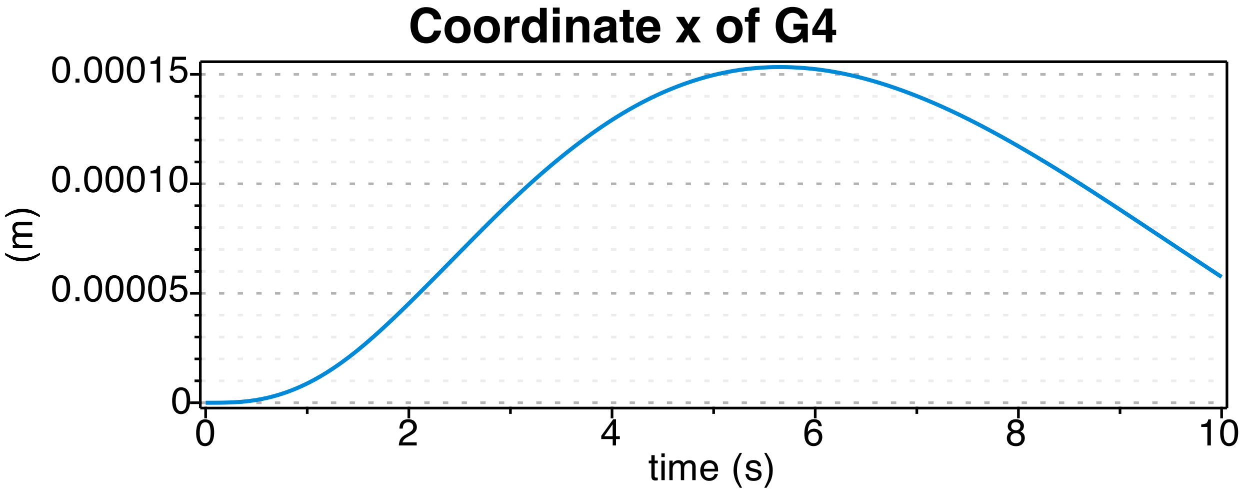
$$\left[\begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 10 \left(\frac{d^2}{dt^2}\,\theta_1(t)\right)\sin\left(\theta_1(t)\right)^2 + 10\left(\frac{d^2}{dt^2}\,\theta_1(t)\right)\cos\left(\theta_1(t)\right)^2 - 100 \cdots \\ \cdots \end{array}\right]$$

$$full\_ics := \left\{s_2(0) = 0, s_3(0) = 0, s_4(0) = 0, \lambda_1(0) = 1.53300127075562 \times 10^{-9}, \lambda_2(0) = 7.18594342838571 \times 10^{-9}, \theta_1(0) = -\frac{\pi}{2}, \theta_2(0) = -\frac{\pi}{2}, \theta_3(0) = -\frac{\pi}{2}, \theta_4(0) = -\frac{\pi}{2}, D(s_2)(0) = 0, D(s_3)(0) = 0, D(s_4)(0) = 0, D(\theta_1)(0) = 0, D(\theta_2)(0) = 0, D(\theta_3)(0) = 0, D(\theta_4)(0) = 0\right\} \quad \textbf{(7.6.4)}$$

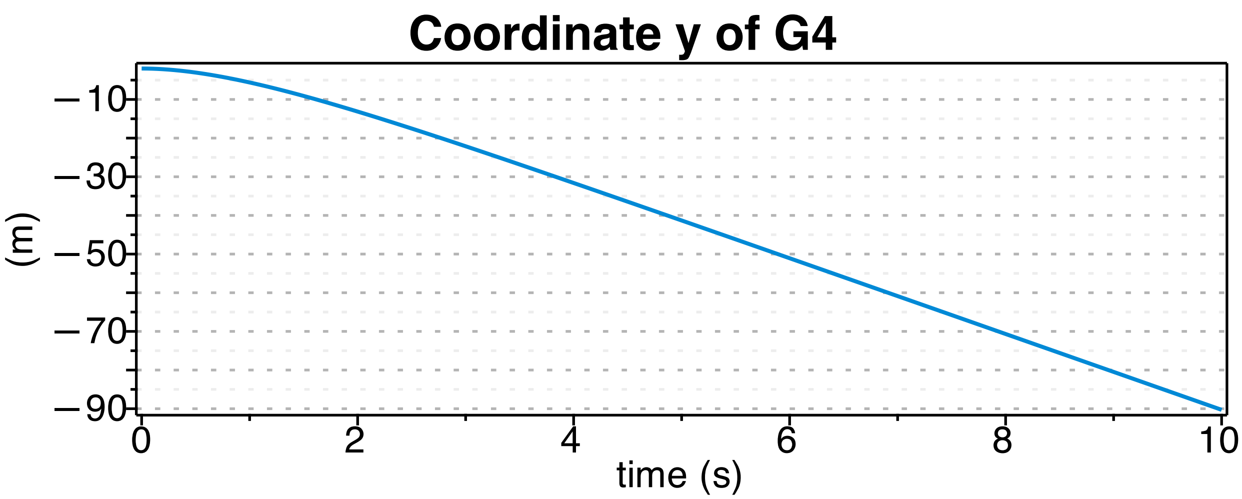
```
> sol_dae__opt4 := dsolve(convert(dae_sys,set) union full_ics,
  numeric,implicit=true,maxfun=300000);
```

sol_dae_{opt4} := **proc**(x_rkf45_dae) ... **end proc** **(7.6.5)**

```
> odeplot(sol_dae__opt4,subs(data,[t,G4[1]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate x of G4");
```

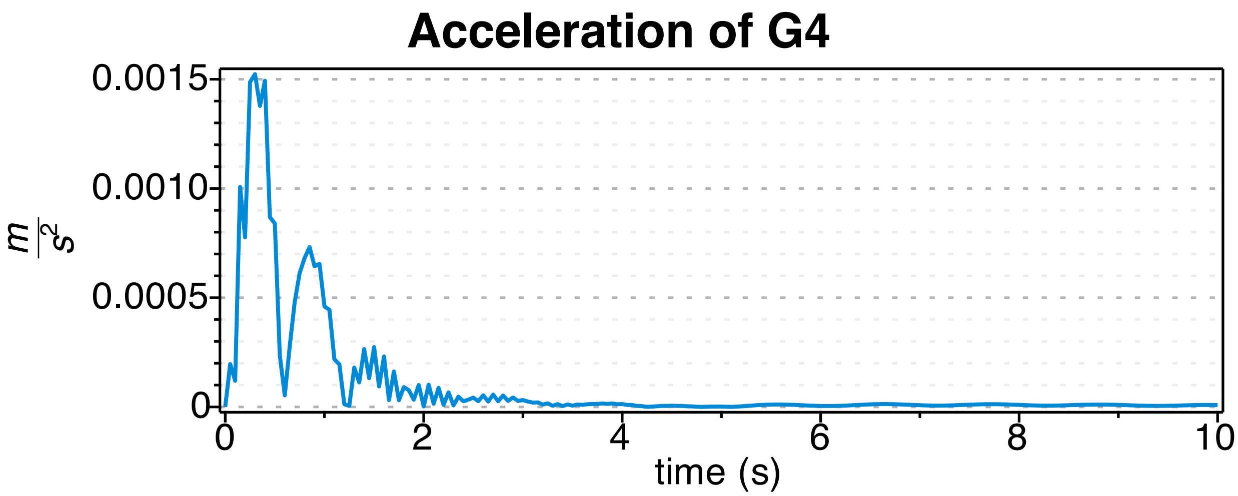



```
> odeplot(sol_dae__opt4,subs(data,[t,G4[2]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate y of G4");
```

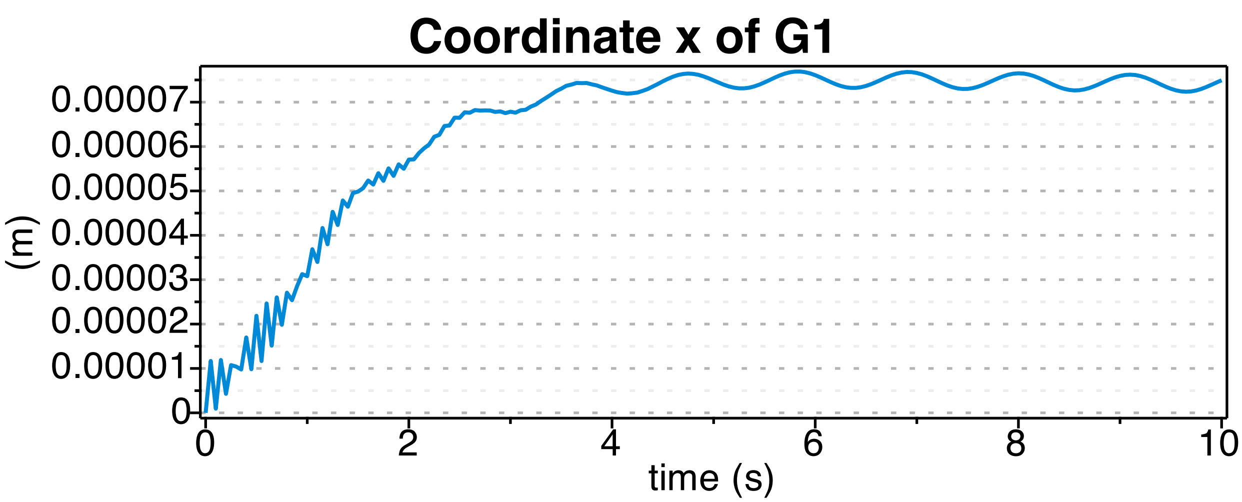


Plot the modulus of the acceleration of mass G4

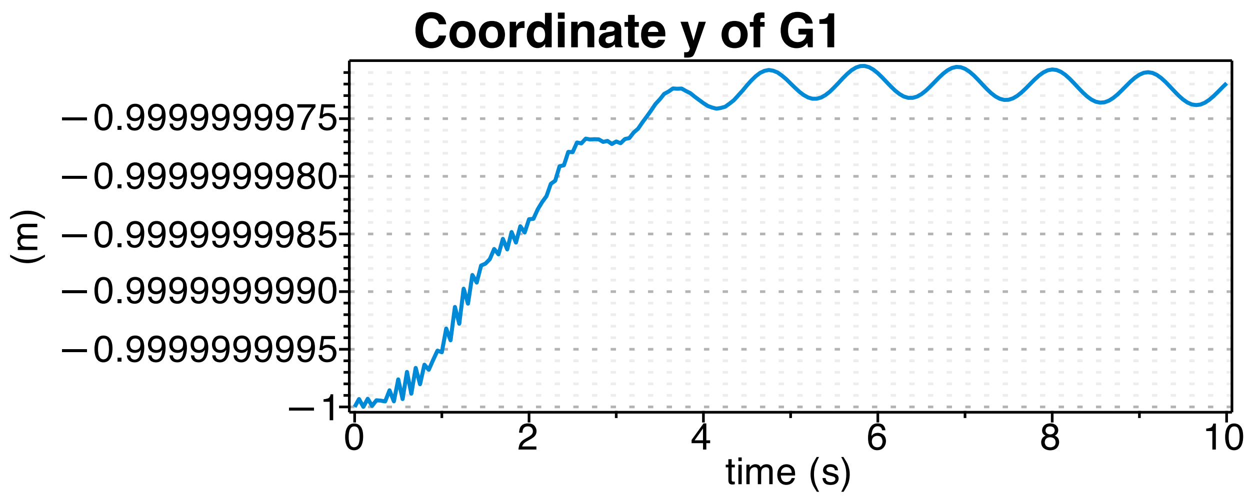
```
> odeplot(sol_dae__opt4,subs(data,[t,sqrt(diff(G4[1],t,t)^2+diff
  (G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],
          title  = "Acceleration of G4");
```




```
> odeplot(sol_dae__opt4,subs(data,[t,G1[1]]),t=0..TF,
         labels = ["time (s)", "(m)"],
         title  = "Coordinate x of G1");
```




```
> odeplot(sol_dae__opt4,subs(data,[t,G1[2]]),t=0..TF,
         labels = ["time (s)", "(m)"],
         title  = "Coordinate y of G1");
```



Simulation with m4 = 1 and Cs4 = 1

In this section the mechanical system is solved assuming a mass m4=1 and a damper coefficient Cs4=1

```
> data__opt := [L = 1, L__s0 = 1, m__1 = 10, m__2 = 2, m__3 =
  30, g = 9.81,
  K__s2 = 5000, C__s2 = 10, K__s3 = 100000, C__s3 = 100,C__s4=1,
  m__4=1];
```

$$data_{opt} := [L = 1, L_{s0} = 1, m_1 = 10, m_2 = 2, m_3 = 30, g = 9.81, K_{s2} = 5000, C_{s2} = 10, K_{s3} = 100000, C_{s3} = 100, C_{s4} = 1, m_4 = 1] \quad \textbf{(7.7.1)}$$

```
> evalf(subs(ics__opt, data__opt, t=0, MM)):
  evalf(subs(ics__opt, data__opt, t=0, bb)):
  tmp:= LinearSolve(%%,%);
```

$$tmp := \begin{bmatrix} 9.81000000000000 \\ 4.79062896530095 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706217810 \times 10^{-9} \\ 2.49112706217810 \times 10^{-9} \\ 1.53300126946991 \times 10^{-9} \\ 7.18594344767143 \times 10^{-9} \\ -4.79062896678096 \times 10^{-10} \end{bmatrix} \quad \textbf{(7.7.2)}$$

```
> <[op(diff(q_vars__opt,t,t)),lambda__1(t),lambda__1(t)]> =
  tmp;

  ics_lambda := [lambda__1(t) = tmp[7],lambda__2(t) = tmp[8]];
```

$$\begin{bmatrix} \frac{d^2}{dt^2} s_2(t) \\ \frac{d^2}{dt^2} \theta_1(t) \\ \frac{d^2}{dt^2} s_3(t) \\ \frac{d^2}{dt^2} s_4(t) \\ \frac{d^2}{dt^2} \theta_2(t) \\ \frac{d^2}{dt^2} \theta_3(t) \\ \frac{d^2}{dt^2} \theta_4(t) \\ \lambda_1(t) \\ \lambda_1(t) \end{bmatrix} = \begin{bmatrix} 9.81000000000000 \\ 4.79062896530095 \times 10^{-10} \\ 9.81000000000000 \\ 9.81000000000000 \\ 2.49112706217810 \times 10^{-9} \\ 2.49112706217810 \times 10^{-9} \\ 1.53300126946991 \times 10^{-9} \\ 7.18594344767143 \times 10^{-9} \\ -4.79062896678096 \times 10^{-10} \end{bmatrix}$$

$$ics\_lambda := \left[\lambda_1(t) = 1.53300126946991 \times 10^{-9}, \lambda_2(t) = 7.18594344767143 \times 10^{-9}\right] \tag{7.7.3}$$

```
> dae_sys  := convert(subs(data__opt,[op(convert(ode_sys__opt,
  list)),op(Phi)]),set): <op(%)>;
  full_ics := subs( t = 0, convert(convert(ics__opt,set) union
  {ics_lambda[1]} union {ics_lambda[2]}, D));
```

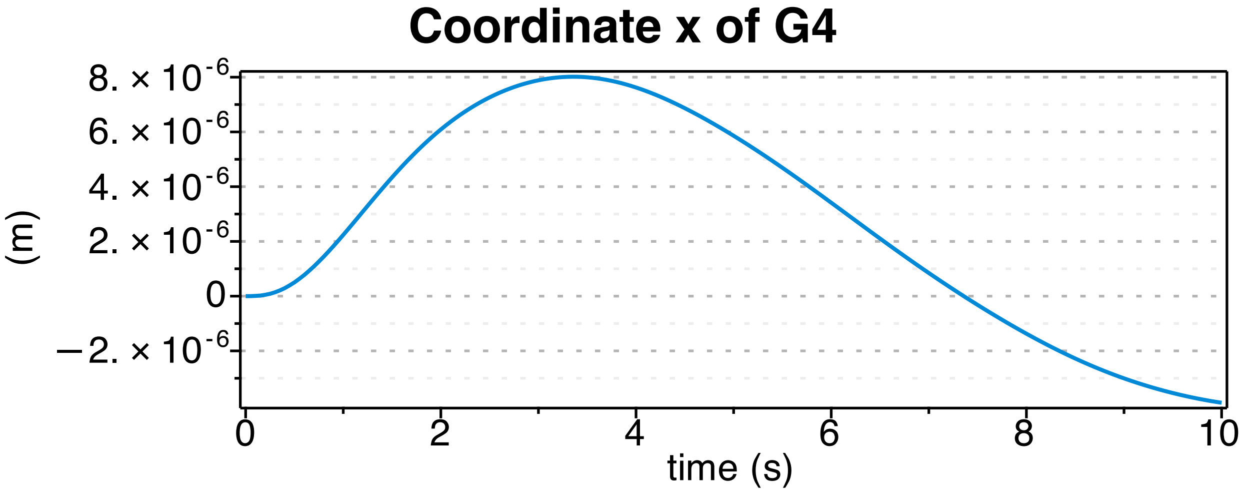
$$\left[\begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ 10\left(\frac{d^2}{dt^2}\,\theta_1(t)\right)\sin\left(\theta_1(t)\right)^2+10\left(\frac{d^2}{dt^2}\,\theta_1(t)\right)\cos\left(\theta_1(t)\right)^2-\left(-\cdots\right. \\ \cdots \end{array}\right]$$

$$full\_ics := \left\{s_2(0)=0, s_3(0)=0, s_4(0)=0, \lambda_1(0)=1.53300126946991\times 10^{-9}, \lambda_2(0) = 7.18594344767143\times 10^{-9}, \theta_1(0)=-\frac{\pi}{2}, \theta_2(0)=-\frac{\pi}{2}, \theta_3(0)=-\frac{\pi}{2}, \theta_4(0) = -\frac{\pi}{2}, D(s_2)(0)=0, D(s_3)(0)=0, D(s_4)(0)=0, D(\theta_1)(0)=0, D(\theta_2)(0) = 0, D(\theta_3)(0)=0, D(\theta_4)(0)=0\right\} \quad \textbf{(7.7.4)}$$

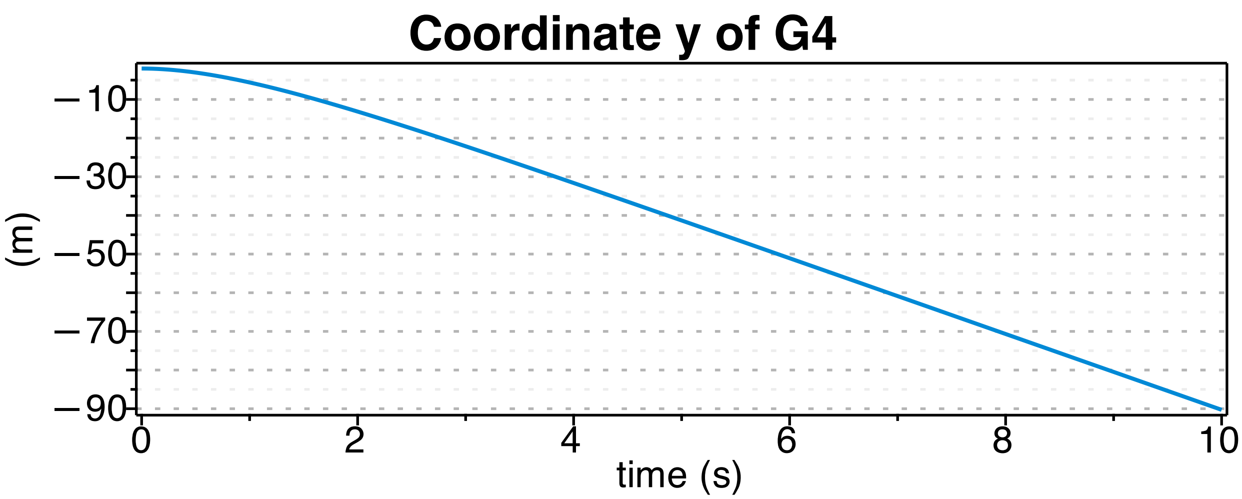
```
> sol_dae__opt5 := dsolve(convert(dae_sys,set) union full_ics,
  numeric,implicit=true,maxfun=300000);
```

$$sol\_dae_{opt5} := \textbf{proc}(x\_rkf45\_dae)\ \ldots\ \textbf{end proc} \quad \textbf{(7.7.5)}$$

```
> odeplot(sol_dae__opt5,subs(data,[t,G4[1]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate x of G4");
```

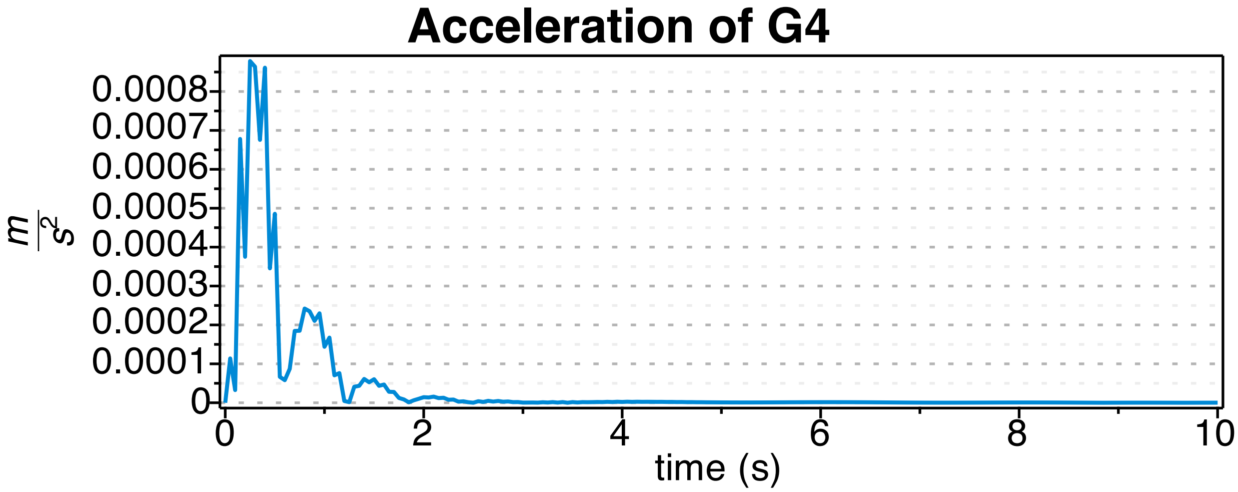



```
> odeplot(sol_dae__opt5,subs(data,[t,G4[2]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate y of G4");
```

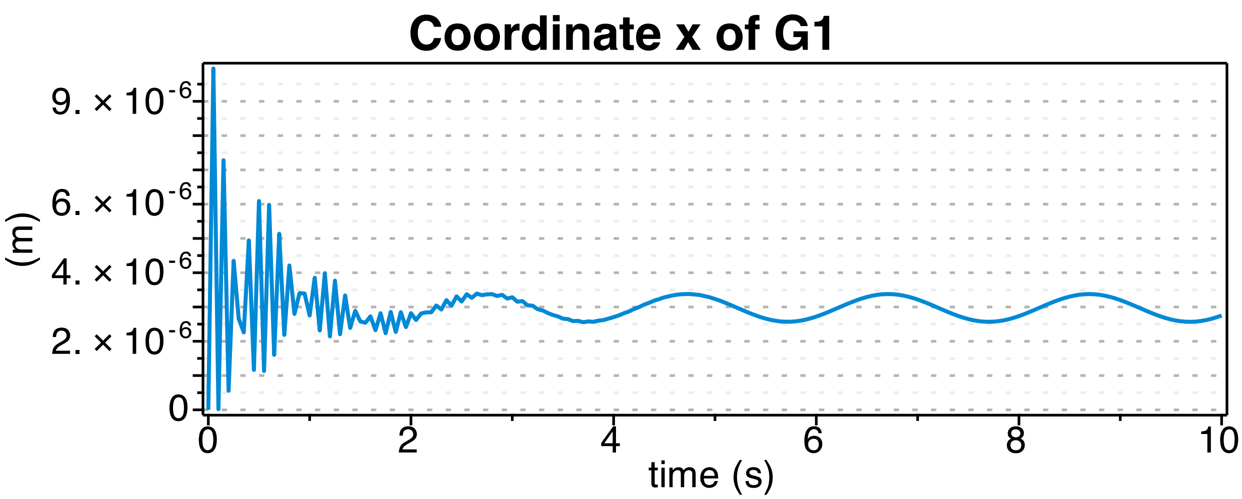


Plot the modulus of the acceleration of mass G4

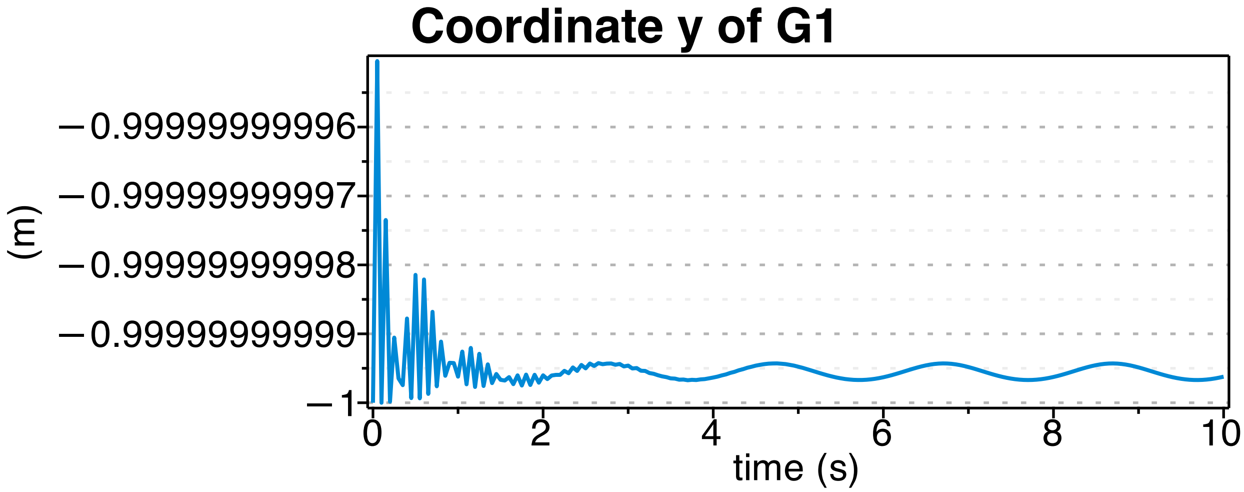
```
> odeplot(sol_dae__opt5,subs(data,[t,sqrt(diff(G4[1],t,t)^2+diff
  (G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],
          title  = "Acceleration of G4");
```




```
> odeplot(sol_dae__opt5,subs(data,[t,G1[1]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate x of G1");
```




```
> odeplot(sol_dae__opt5,subs(data,[t,G1[2]]),t=0..TF,
          labels = ["time (s)", "(m)"],
          title  = "Coordinate y of G1");
```



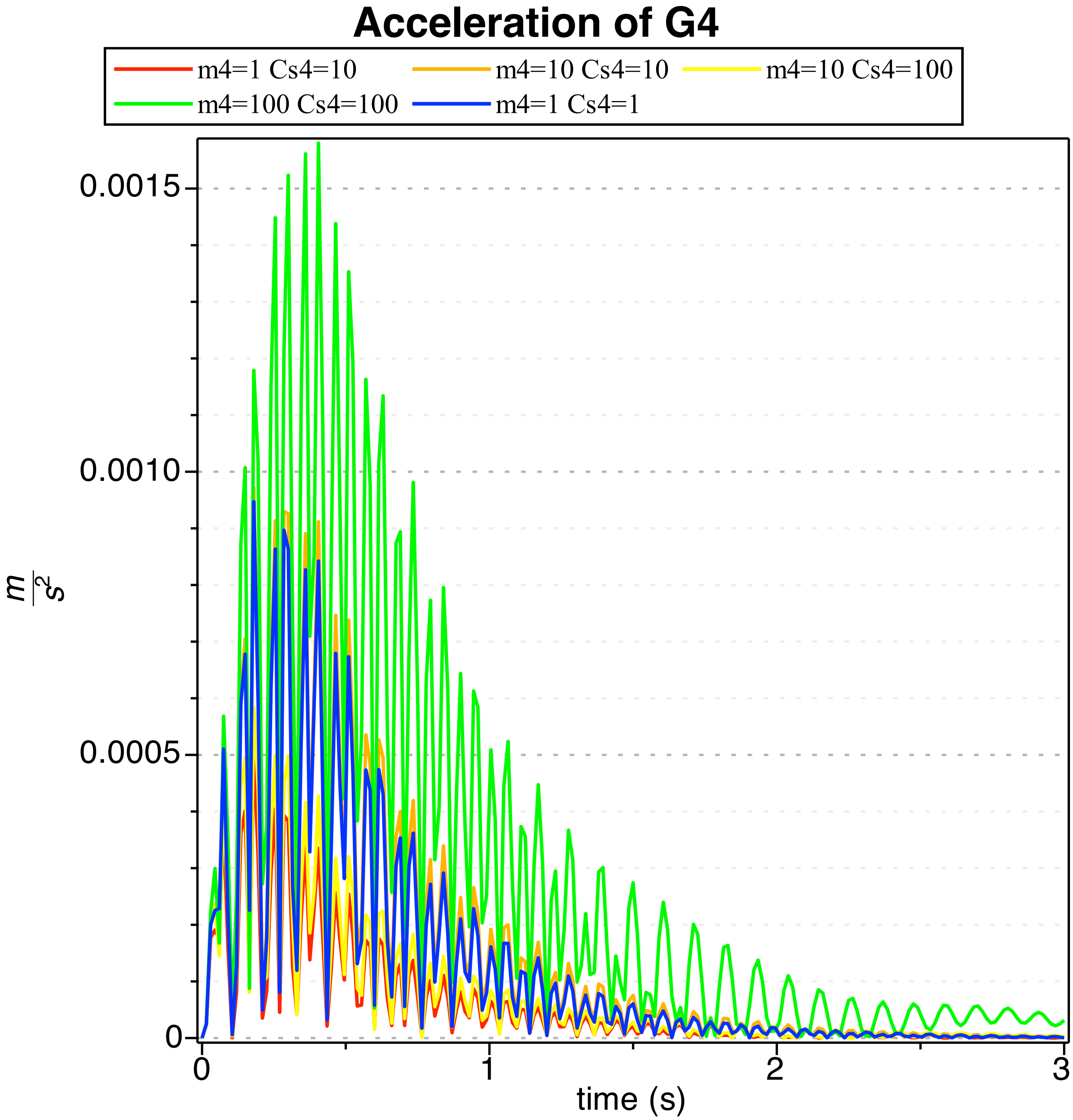
Result comparison

This section aims at finding the best values of mass and damping by evaluating the plot in the different scenario

```
> TF:=3:
```

Plot the modulus of the acceleration of mass G4 in the 5 different scenarios

```
> display(odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G4[1],t,t)
  ^2+diff(G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],title  = "Acceleration of
  G4"),
  odeplot(sol_dae__opt2,subs(data,[t,sqrt(diff(G4[1],t,t)^2+diff
  (G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot(sol_dae__opt3,subs(data,[t,sqrt(diff(G4[1],t,t)^2+diff
  (G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],title  = "Acceleration of
  G4"),
  odeplot(sol_dae__opt4,subs(data,[t,sqrt(diff(G4[1],t,t)^2+diff
  (G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot(sol_dae__opt5,subs(data,[t,sqrt(diff(G4[1],t,t)^2+diff
  (G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  color=["Red","Orange","Yellow","green","Blue"],legend=["m4=1
  Cs4=10 ","m4=10 Cs4=10 ","m4=10 Cs4=100 ","m4=100 Cs4=100 ",
  "m4=1 Cs4=1 "],size=[800,500]);
```

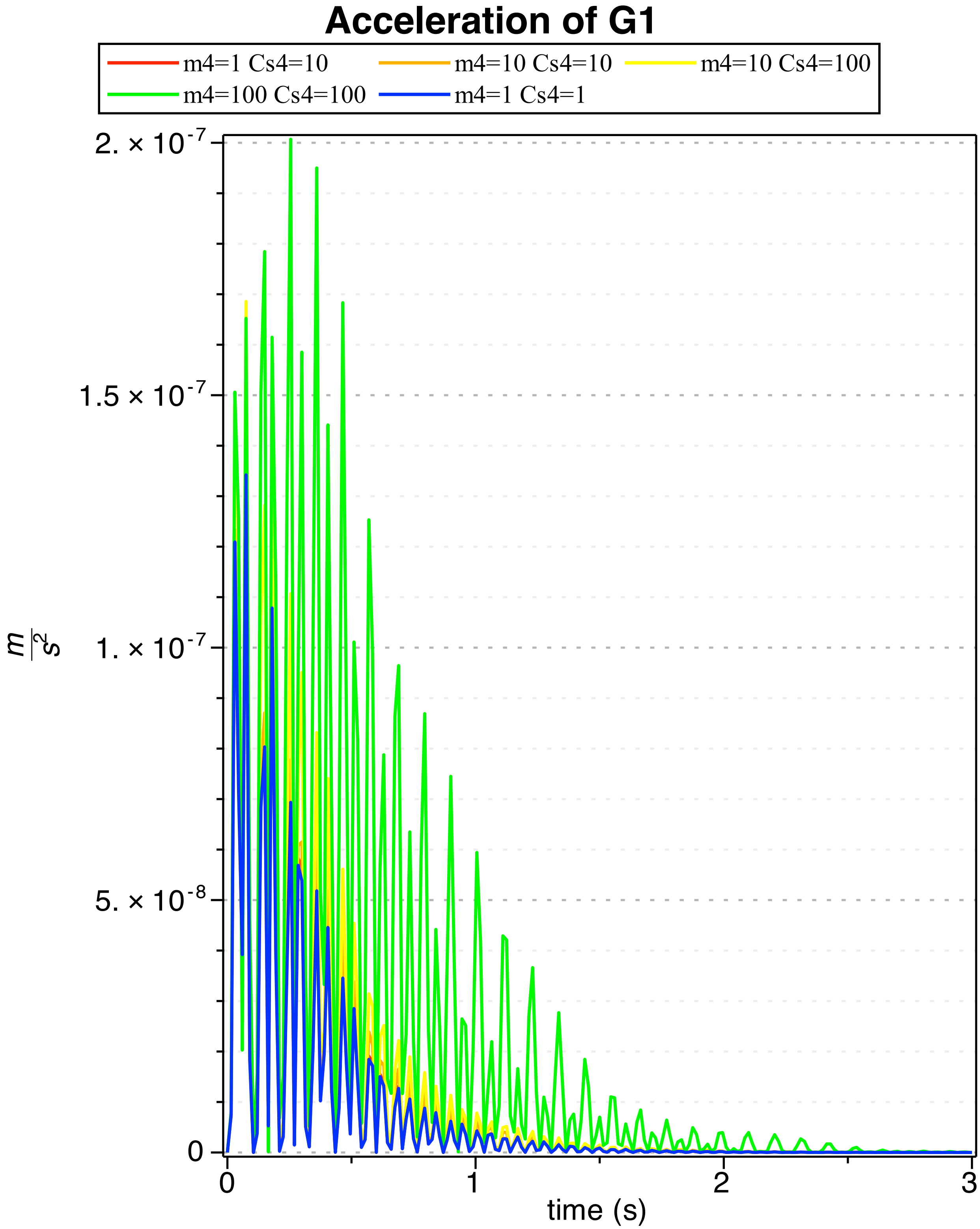


From the previous plot it appears clearly that the configuration with m4=100 and Cs4=100 is the most unstable one.
Also the blue one (m4=1 and Cs4=1) and the orange one (m4= 10 and Cs4=10) appear not be optimal.
The red configuration and the yellow configuration look the most stable ones.

Plot the modulus of the acceleration of mass G1 in the 5 different scenarios

```
> display(odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G1[1],t,t)
  ^2+diff(G1[2],t,t)^2)]),t=0..TF,
```

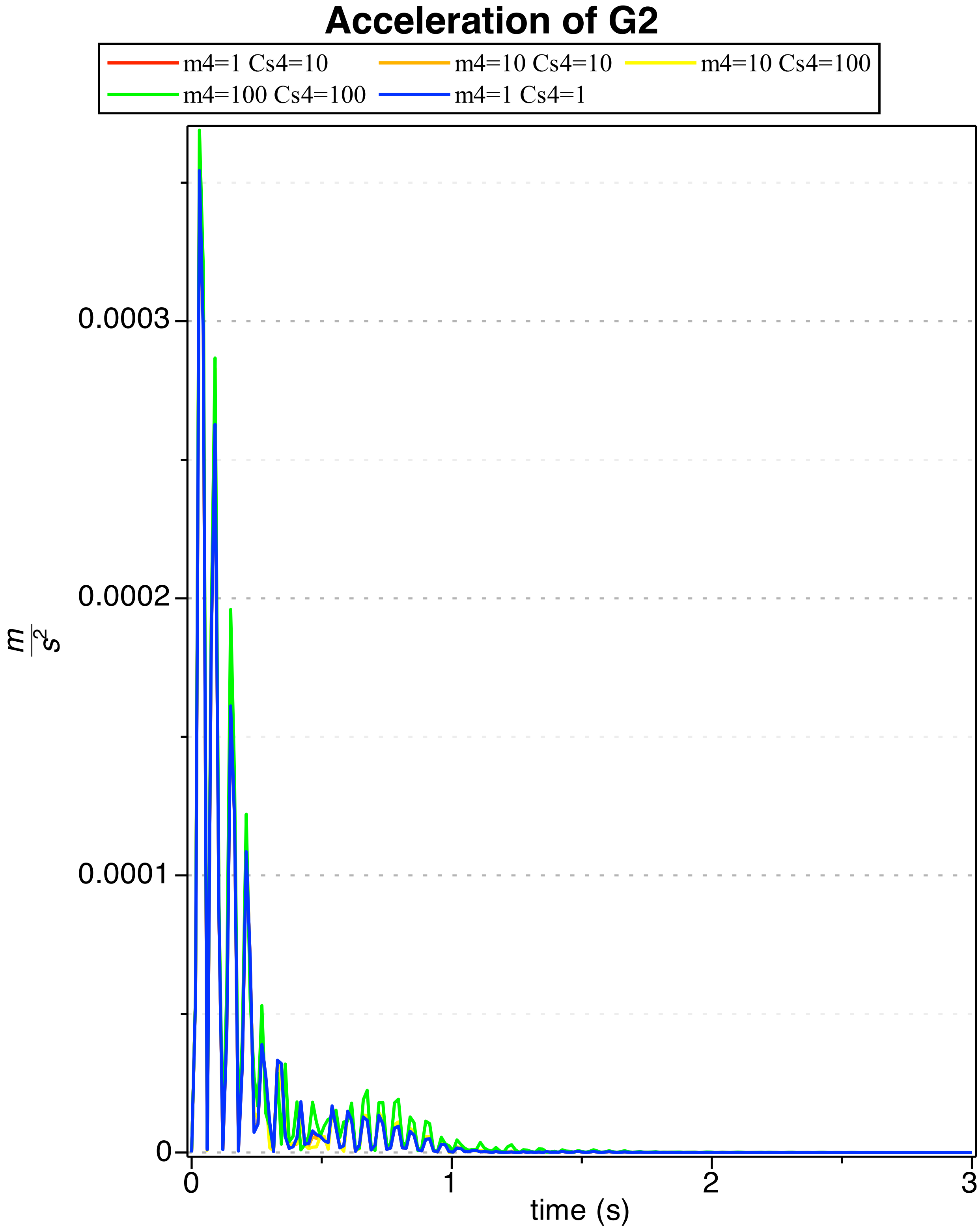
```
        labels = ["time (s)", m/s^2]),
odeplot(sol_dae__opt2,subs(data,[t,sqrt(diff(G1[1],t,t)^2+diff
(G1[2],t,t)^2)]),t=0..TF,
        labels = ["time (s)", m/s^2]),
odeplot(sol_dae__opt3,subs(data,[t,sqrt(diff(G1[1],t,t)^2+diff
(G1[2],t,t)^2)]),t=0..TF,
        labels = ["time (s)", m/s^2]),
odeplot( sol_dae__opt4,subs(data,[t,sqrt(diff(G1[1],t,t)^2+
diff(G1[2],t,t)^2)]),t=0..TF,
        labels = ["time (s)", m/s^2]),
odeplot(sol_dae__opt5,subs(data,[t,sqrt(diff(G1[1],t,t)^2+diff
(G1[2],t,t)^2)]),t=0..TF,
        labels = ["time (s)", m/s^2],title  = "Acceleration of
G1"),
color=["Red","Orange","Yellow","green","Blue"],legend=["m4=1
Cs4=10 ","m4=10 Cs4=10 ","m4=10 Cs4=100 ","m4=100 Cs4=100 ",
"m4=1 Cs4=1 "],labels = ["time (s)", m/s^2],size=[800,600]);
```



From the previous plot the red configuration (m4=1 and Cs4=10) looks the most stable ones.

Plot the modulus of the acceleration of mass G2 in the 5 different scenarios

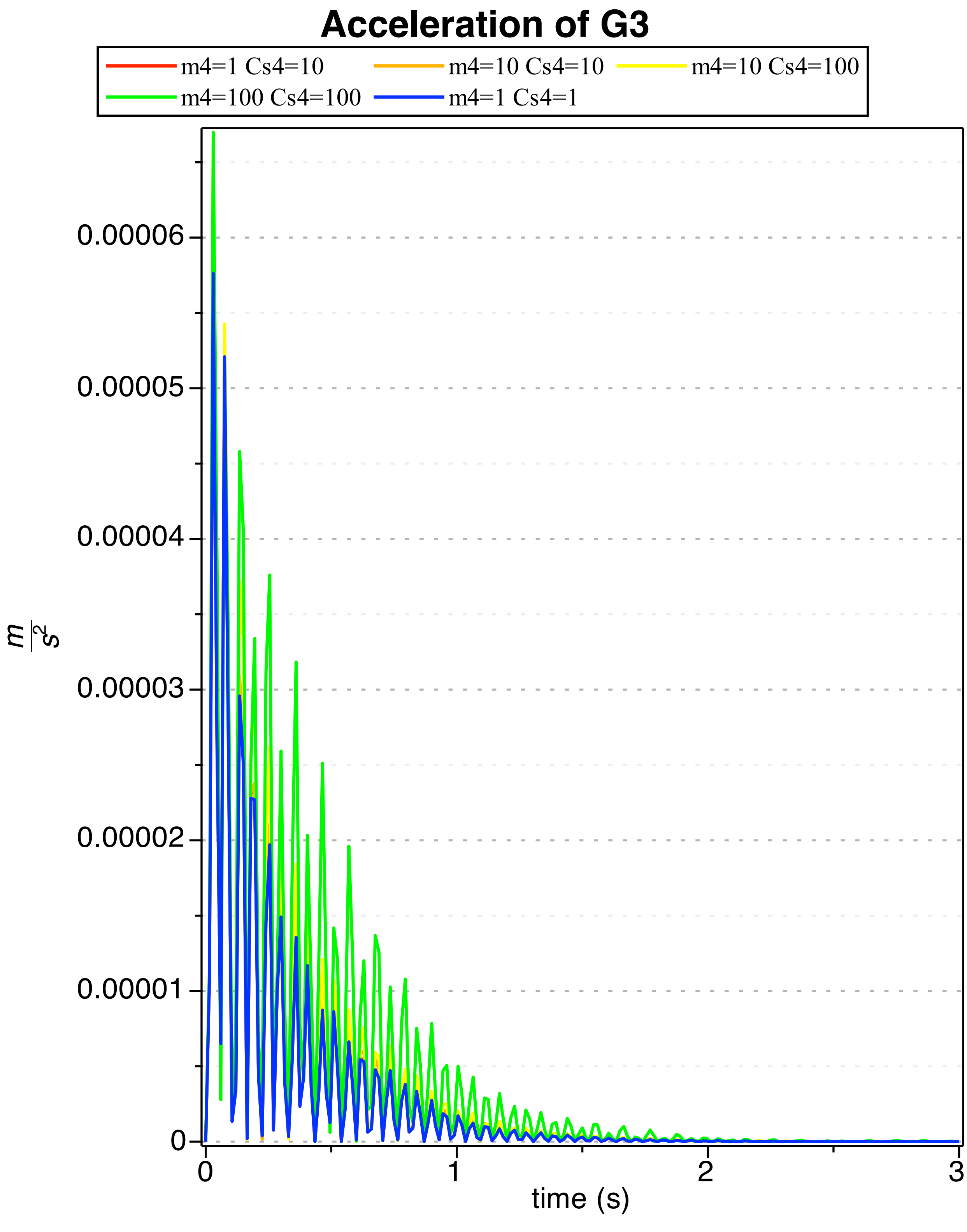
```
> display(odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G2[1],t,t)
  ^2+diff(G2[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot(sol_dae__opt2,subs(data,[t,sqrt(diff(G2[1],t,t)^2+diff
  (G2[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot(sol_dae__opt3,subs(data,[t,sqrt(diff(G2[1],t,t)^2+diff
  (G2[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot( sol_dae__opt4,subs(data,[t,sqrt(diff(G2[1],t,t)^2+
  diff(G2[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot(sol_dae__opt5,subs(data,[t,sqrt(diff(G2[1],t,t)^2+diff
  (G2[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],title  = "Acceleration of
  G2"),
  color=["Red","Orange","Yellow","green","Blue"],legend=["m4=1
  Cs4=10 ","m4=10 Cs4=10 ","m4=10 Cs4=100 ","m4=100 Cs4=100 ",
  "m4=1 Cs4=1 "],labels = ["time (s)", m/s^2],size=[800,600]);
```



From the previous plot the red configuration (m4=1 and Cs4=10) looks the most stable ones.

Plot the modulus of the acceleration of mass G3 in the 5 different scenarios

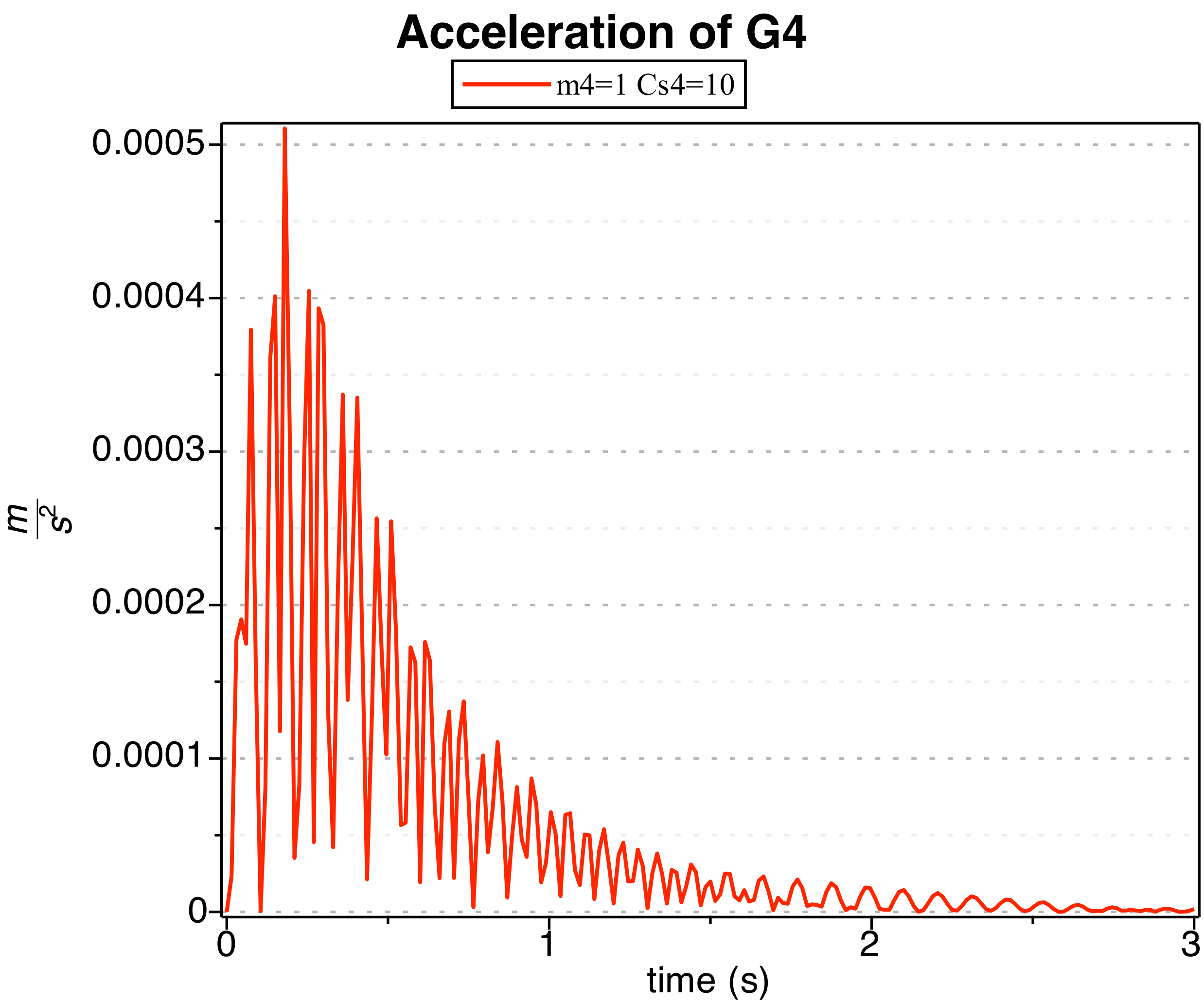
```
> display(odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G3[1],t,t)
  ^2+diff(G3[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot(sol_dae__opt2,subs(data,[t,sqrt(diff(G3[1],t,t)^2+diff
  (G3[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot(sol_dae__opt3,subs(data,[t,sqrt(diff(G3[1],t,t)^2+diff
  (G3[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot( sol_dae__opt4,subs(data,[t,sqrt(diff(G3[1],t,t)^2+
  diff(G3[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),
  odeplot(sol_dae__opt5,subs(data,[t,sqrt(diff(G3[1],t,t)^2+diff
  (G3[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2],title  = "Acceleration of
  G3"),
  color=["Red","Orange","Yellow","green","Blue"],legend=["m4=1
  Cs4=10 ","m4=10 Cs4=10 ","m4=10 Cs4=100 ","m4=100 Cs4=100 ",
  "m4=1 Cs4=1 "],labels = ["time (s)", m/s^2],size=[800,600]);
```



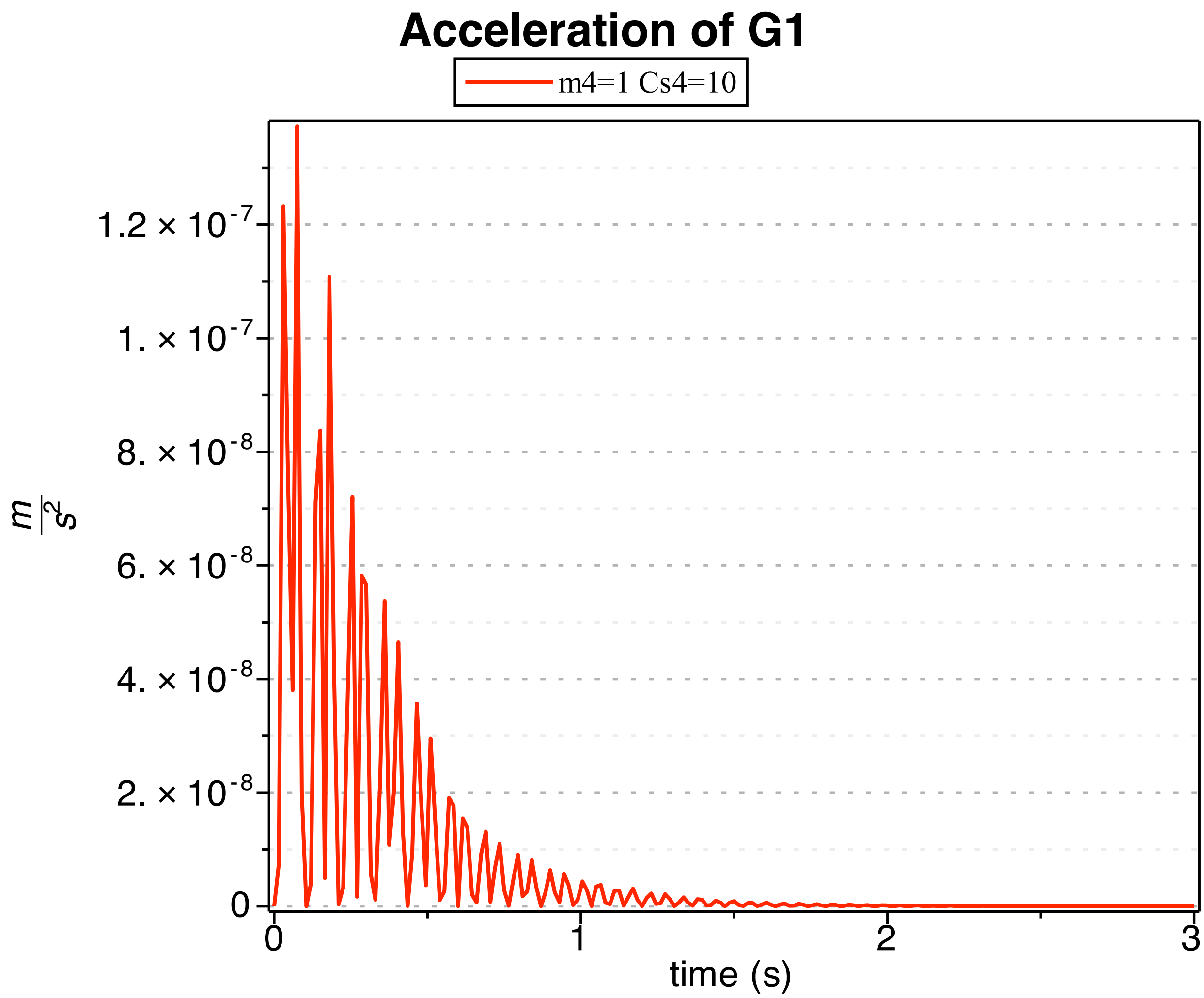
Optimized Configuration with m4=1 and Cs4=10

```
> display(odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G4[1],t,t)
  ^2+diff(G4[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),title  = "Acceleration
  of G4",
```

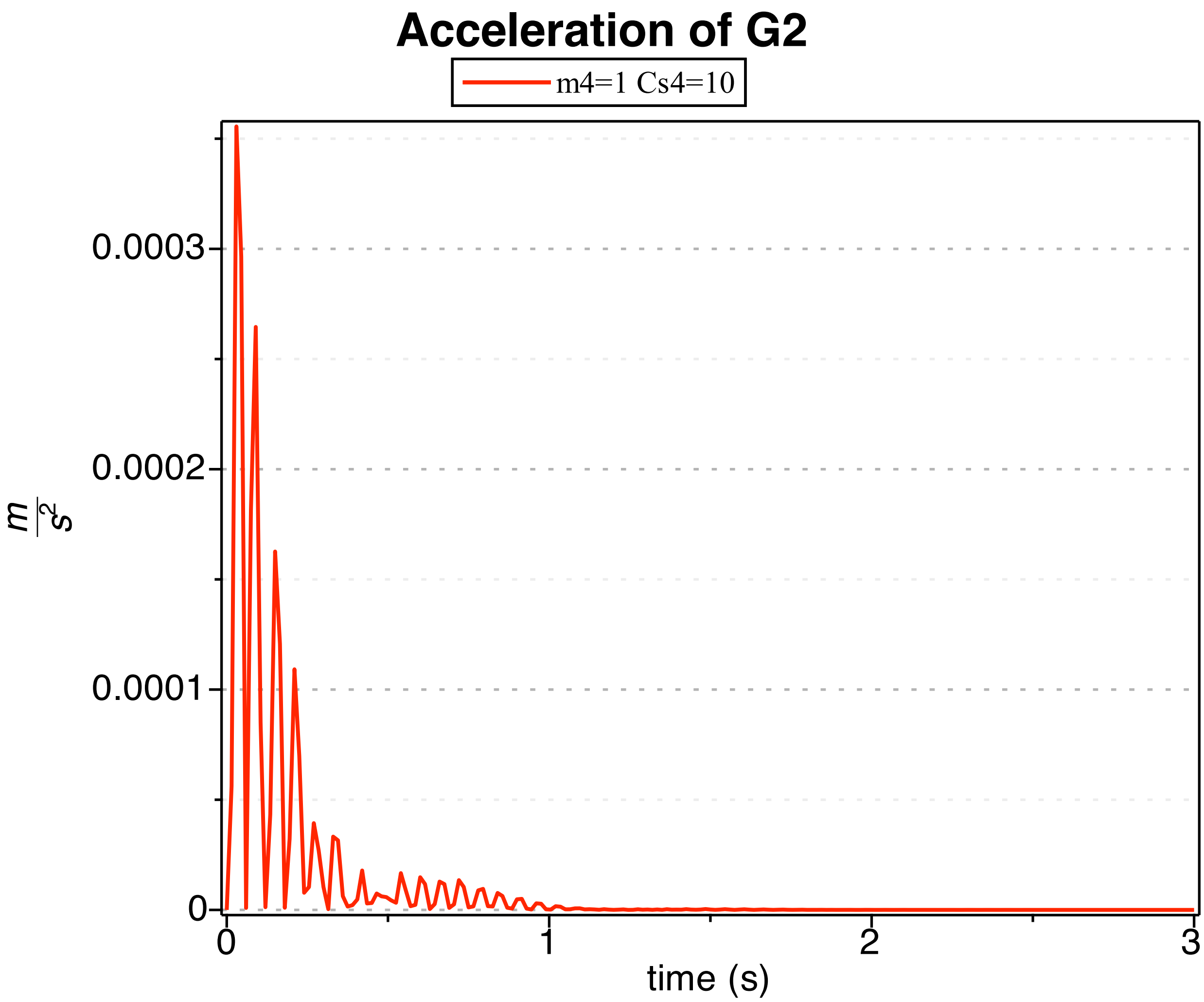
```
color="Red",legend="m4=1 Cs4=10 ",labels = ["time (s)",
m/s^2],size=[800,400]);
```



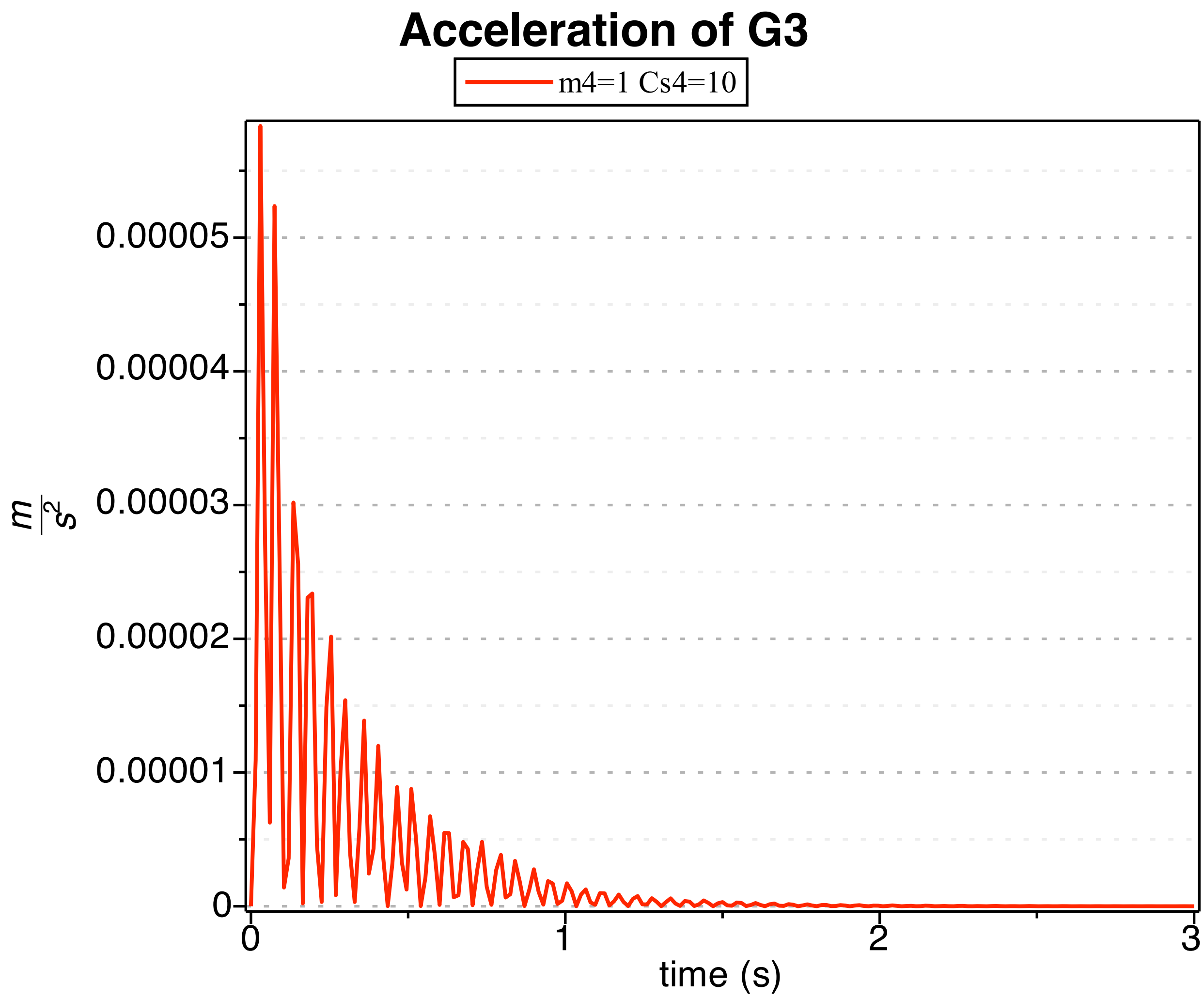

```
> display(odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G1[1],t,t)
^2+diff(G1[2],t,t)^2)]),t=0..TF,
        labels = ["time (s)", m/s^2]),title  = "Acceleration
of G1",
color="Red",legend="m4=1 Cs4=10 ",labels = ["time (s)",
m/s^2],size=[800,400]);
```




```
> display(odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G2[1],t,t)
  ^2+diff(G2[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),title  = "Acceleration
  of G2",
  color="Red",legend="m4=1 Cs4=10 ",labels = ["time (s)",
  m/s^2],size=[800,400]);
```




```
> display(odeplot(sol_dae__opt,subs(data,[t,sqrt(diff(G3[1],t,t)
  ^2+diff(G3[2],t,t)^2)]),t=0..TF,
          labels = ["time (s)", m/s^2]),title  = "Acceleration
  of G3",
  color="Red",legend="m4=1 Cs4=10 ",labels = ["time (s)",
  m/s^2],size=[800,400]);
```



Acceleration of G3
m4=1 Cs4=10
0.00005
0.00004
0.00003
0.00002
0.00001
0
$\frac{m}{s^2}$
0
1
2
3
time (s)