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Research Statement

I am broadly interested in the mathematical foundations of computer science [Wig19]. More specifically, my research focuses on the still largely mysterious boundary between efficient and intractable computation. I am particularly attracted to mathematical structures and properties enabling efficient algorithms, and conversely, also to those underpinning hardness or obstructions for such algorithms. So far, my work has involved convex optimization, coding theory and expansion including their intersections. In this statement, I will first briefly describe the objects and tools most relevant to my research until now indicating where my research fits in. Then, I will provide more details in a summary of my completed projects all done in various collaborations. Finally, I will point some future directions.

Sum-of-Squares: Convex optimization has played an important role in the design of efficient algorithms for combinatorial optimization problems [Vaz01, WS11, GW95]. Linear programming and semi-definite programming reshaped our understanding of efficient computation. A natural generalization of these powerful tools is embodied in the Sum-of-Square (SOS) semi-definite hierarchy [Las15, FKP19, Dar20c], which captures the state-of-the-art polynomial time guarantees for many combinatorial optimization problems [Rag08, ARV04, BRS11, GS11]. Given this success, SOS has a double role in the study of computation. On the positive side, it can serve as a powerful tool to help advance the frontiers of efficient algorithms, and alternatively, a hardness result against this hierarchy can serve as a proxy for hardness, particularly useful for average case problems [BHK+16, KMOW17, GJJ+20] for which the sophisticated PCP machinery, e.g., [ALM+98, Din06, Aro98], for NP-hardness is not readily available. As I will discuss in more detail shortly, my research and interests span both algorithms [AJT19, AJQ+20, JQST20] and lower bounds [GJJ+20] involving this hierarchy. I believe my most important contribution so far is a decoding algorithm based on the SOS hierarchy [JQST20].

Codes: Roughly speaking, coding theory is the study of properties and algorithms related to sets of strings (*codes*) over finite alphabets [GRS19, vL99, Dar20a]. Typically, to really "unlock" interesting properties of codes, one brings to bear way more structure such as algebraic properties of low degree polynomials or structural properties of expanders graphs (the latter playing an important role in my research [AJQ+20, JQST20, JST20]). The quest towards optimal parameter trade-offs for codes is particularly important. For a concrete example, consider all the information communicated and stored in binary form. To protect it against corruptions, we might want to encode it using binary codes which are sufficiently robust to allow recovery from possible corruptions. Being able to encode with the least amount of redundancy while providing the desired robustness guarantees may have several benefits (e.g., reduced energy consumption, faster communication time, reduced storage requirement and less waste). Since coding theory is a reasonably mature field dating back to seminal work of Shannon [Sha49] and Hamming [Ham50], one might guess that, by now, binary codes are thoroughly understood. Surprisingly, this is not at all the case and in several aspects binary codes are the elusive case compared to their larger alphabet counterparts [Gur09, Gur10]. For instance, in the case of general adversarial corruptions, we know since the 1950's [Gil52, Var57] that random binary codes achieve near optimal redundancy (rate) versus robustness (distance) trade-off, the so-called Gilbert–Varshamov (GV) bound. However, it was only in a recent breakthrough work that the first **explicit** construction of very robust (i.e., large distance) **binary** codes with parameters near the GV bound were discovered by Ta-Shama [TS17]. Ta-Shma's construction relies on the pseudorandom properties of expander graphs and it was **not** known to be efficiently decodable. In [JQST20], we used the SOS hierarchy to design polynomial time unique decoding algorithms for Ta-Shma's codes.

Expanders: Expansion is a phenomenon at the core of a myriad of results in theoretical computer science. To give a few examples, there are several expander based code constructions [ABN+92, SS96, GKO+17, HRW17, TS17, DHK+19], the combinatorial proof of the PCP Theorem [Din06] and agreement testers [DK17, DD19]. Roughly speaking, an expander graph is an explicit sparse well-connected graph admitting a variety of pseudorandom properties [HLW06, Chu97]. Expander graphs and coding theory have a synergetic two way multi-faceted relationship. As a concrete example (and the connection appearing in my research [AJQ+20, JQST20, JST20]), expander graphs can be used to resource efficiently amplify the distance of a base code. Now, we give a gist of how the expander properties may be useful in this process: (i) an expander being pseudorandom can imply large distance, (ii) an expander being sparse can imply lower redundancy, and (iii) an expander being explicit can imply explicit code. More recently, a systematic study of **hypergraph** expanders, the so-called high-dimensional expanders (see [Lub18, Dar20b]), has emerged. This brought a new set of questions ranging from coding theory to design of very efficient PCPs.

Completed Research Projects

In the following, I give brief descriptions of my completed projects in reverse chronological order, all done in various collaborations (please see my website for further details).

- **Unique decoding near optimal binary codes:** In [JQST20], we give polynomial time *unique decoding* algorithms for nearly optimal, in terms of redundancy (rate) versus robustness (distance) trade-off (i.e., near the so-called Gilbert–Varshamov bound [Gil52, Var57]), explicit **binary** codes of large distance. These codes are (essentially) those explicit binary codes of distance $1/2 - \varepsilon$ and rate $\Omega(\varepsilon^{2+o(1)})$ arising from the breakthrough construction of Ta-Shma [TS17]. Our algorithms are based on Sum-of-Squares hierarchy and use as a starting point a decoding framework from our earlier work [AJQ+20]. Our main contribution consists in overcoming the far from optimal rates from [AJQ+20] to operate in this near optimal regime for unique decoding. This result can be seen as a step towards a better understanding of the elusive case of binary codes in the general adversarial error model of Hamming [Ham50]. These algorithms are just a proof of concept showing that polynomial time algorithms exist in this previously unattained regime. This result opened avenues for our recent near-linear time unique algorithms [JST20] using a novel algorithmic weak regularity decomposition in the style of Frieze and Kannan [FK96] but for sparse tensors supported on expending hypergraphs. We hope that these techniques can also open avenues to list decoding algorithms with near optimal parameters, this being a major open problem in the field [Gur09, Gur10].

- **SOS lower bounds for the Sherrington–Kirkpatrick model:** In [GJJ+20], we show that even as many as n^δ levels[1] of the SOS hierarchy (subexponential running time) still fail to provide a tighter energy upper bounds on the Sherrington–Kirkpatrick (SK) Hamiltonian [SK75] than the trivial, and non-tight, spectral bound. The SK Hamiltonian is a widely studied fundamental model of spin-glass in statistical physics admitting surprising connections [Tal06, Pan14, MS16, KB19, Mon19, MRX20] (e.g., the max-cut value of a random d-regular graphs is "determined" by this model [DMS17]). Despite this mouthful description, the SK model simply consists in maximizing a familiar[2] quadratic form $x^t Mx$, with x ranging in $\{\pm 1\}^n$ and the twist that M is an $n \times n$ random symmetric matrix with independent Gaussian entries[3]. With high probability this quadratic form has value $\approx 1.5264 \cdot n^{3/2}$ whereas SOS thinks its value is $(2 + o(1)) \cdot n^{3/2}$ (the spectral bound). We actually obtain lower bounds for another natural problem by doing *Fourier analysis of random matrices* [AMP20], and then via known connections [MRX20] derive the SK lower bound.

- **Tighter bounds for the Birkhoff graph:** In [CJ20], we provide tighter bounds for the independence number of the Birkhoff graph family. This is a family of Cayley graphs on the symmetric group S_n encoding the skeleton of the so-called Birkhoff polytope of doubly stochastic matrices [Bir46, Bar02]. Our results are obtained by making the beautiful representation theoretic techniques of Kane, Lovett and Rao [KLR17] "higher-order" (by analyzing more irreducible representations) and using linear programming. In particular, this allows us to improve their upper bound from $O(n!/\sqrt{2}^n)$ to $O(n!/1.97^n)$. By known connections this readily implies stronger alphabet lower bounds for a family of codes for distributed storage [GHJY14, GHK$^+$17].

- **List decoding framework for binary codes:** In [AJQ$^+$20], we provide a *list decoding* framework for distance amplified codes based on expanding structures: high-dimensional expanders (as in Dinur and Kaufman definition [DK17]) and walks on expander graphs. Roughly, list decoding is a relaxed decoding model in which we double the unique decoding radius at the expense of possibly having a small list of codewords rather than at most one. We view the problem of *unique decoding* as solving a suitable Max k-CSP (Constraint Satisfaction Problem) instance, which can be solved using our earlier work [AJT19] based on the SOS hierarchy. To obtain the list decoding framework, we maximize an entropic proxy[4] while solving a k-CSP. This makes the SOS solution rich enough so that we can "recover" a list of all the desired codewords from it. A noteworthy novelty of this work is that this framework can decode distance amplified **binary** codes obtained from **binary** codes of smaller distance. Although this gives a new "genuinely binary" algorithmic technique, our rates were very far from the state-of-the-art results which typically rely on large alphabet techniques. Despite this parameter shortcoming, this framework served as our starting point for *unique decoding* results of nearly optimal codes [JQST20] mentioned above.

- **Approximating k-CSPs on expanding structures:** In [AJT19], we give polynomial time approximation algorithms for k-CSPs (Constraint Satisfaction Problems) on suitably [5] expanding hypergraphs, which is a class of structures containing high-dimensional expanders (as in Dinur and Kaufman definition [DK17]) as an important special case. Naturally, the quality of approximation crucially depends on quality of expansion of these hypergraphs.

[1] Here, $\delta > 0$ is an universal constant.

[2] Note that when M is a the Laplacian of graph this maximization problem becomes the familiar MaxCut problem.

[3] More precisely, M is from the Gaussian orthogonal ensemble GOE(n) (see [Tao12]).

[4] This entropic idea was independently used for list decoding in the context of machine learning by Karmalkar, Klivans and Kothari [KKK19] and Raghavendra and Yau [RY19]. Our results deal with finite fields/alphabets whereas theirs deal with $\mathbb{R}$. Among others, this leads to a variety of technical differences.

[5] More precisely, we generalize to hypergraphs the notion of threshold rank of a graph [BRS11] which is a robust version of expansion tolerating a few,i.e., $O(1)$, large eigenvalues (the rank) in the adjacency matrix of a graph.

Our algorithmic results are based on the SOS hierarchy and generalize the 2-CSPs results of [BRS11, GS11]. Our results can be seen as a step towards better understanding structural properties of hypergraphs making k-CSPs easy to approximate. On the flip side, this result can also better inform what particular conditions[6] to avoid when trying to design hard k-CSP instances using hypergraphs, say, when designing a PCP. Via known connections, our algorithms translate into approximation algorithms for quantum k-CSPs, the so-called k-local Quantum Hamiltonians. However, contrary to the classical case where PCP theorems are known, the Quantum PCP Hamiltonian Conjecture [AAV13] is widely open.

Future Directions

In this section, I will point some reasonably concrete near future directions, but I stress that I am not claiming any progress just expressing some of my interests[7]. Most of these projects are being pursued in various collaborations. Some of these problems might be quite challenging in which case any improved understanding might already constitute a nice outcome. I hope that during the postdoc serendipitous new research directions and techniques will emerge by being in a new research environment.

The following are some directions being actively pursued.

- **Explicit binary codes near *list decoding capacity*:** List decoding provides a candidate bridge to achieve the best parameters of the simpler error model of Shannon [Sha49] in the general adversarial error regime of Hamming [Ham50]. Finding explicit optimal binary codes admitting efficient list decoding algorithms is a major open problem in coding theory [Gur09, Gur10]. Prior to the work of Ta-Shma [TS17], we had no idea what **explicit** near optimal binary codes for unique decoding looked like. Now, we do and more recently obtained unique decoding algorithms for them [JQST20, JST20]. Before trying to extend our algorithms to list decoding achieving near optimal parameters[8], we can ask a weaker yet quite challenging information theoretic question of whether Ta-Shma's codes are *combinatorially list decodable*, i.e., all list sizes are small regardless of the computational cost to find them. Resolving this question seems to require a reasonably more sophisticated use of the pseudorandom properties of expander graphs than done in [TS17], and hopefully might deepen our understanding of the connection between expanders and codes. Currently, only random ensembles of binary codes are known to achieve (near) optimal list decoding radius versus rate trade-offs[9] [MRRZ$^+$19, GR08, GRS19], so only randomized analysis are available.

- **What exactly are optimal binary codes?:** Another longstanding mystery about binary codes is that we still do not have a fine-grained understanding of the rate versus distance trade-offs they achieve. We know that random binary codes achieve great parameters, but we do not know whether this is best possible. For codes of distance $1/2 - \varepsilon$, a random code achieves rate $\Omega(\varepsilon^2)$ whereas the best upper bound (impossibility result) is $O(\varepsilon^2 \log(1/\varepsilon))$ (see [Del75, MRRW77, Lau07, NS09, Alo09, Val19] for some upper bound related results). Improving any of these bounds (if possible) would be wonderful. For now, we are trying to tighten the upper bound. The state-of-the-art upper bound can be obtained via a theoretical analysis of linear programs [MRRW77]. We are trying to understand a natural hierarchy of

[6] Of course the connection of, say, high-dimensional expanders and CSPs can take many forms, see [DFHT20] for a variation that already yields SOS hard instances.

[7] Moreover, I will omit any partial approaches and results that we may have.

[8] With respect to list decoding radius and rate.

[9] These random ensembles actually achieve optimal list decoding radius versus rate trade-offs.

linear programs (for linear codes) and semi-definite programs (for non-linear codes) based on Fourier analysis and representation theory, respectively.

- **Efficient list decoding for random LDPC codes:** Recently, a random ensemble of low-density parity-check (LDPC), i.e., when the parity check matrix has a constant number of non-zero entries in each row, codes was shown to achieve list decoding capacity. However, the problem of efficient list decoding was left open while the problem of unique decoding was previously known to take only linear time [MRRZ+19]. In this project, we are trying to understand this algorithmic list problem for LDPC codes.

- **Are there *good* quantum LDPC codes?:** Classical codes having constant rate and constant relative distance (i.e., good codes) are known even if we require them to be low-density parity-check (LDPC) codes. However, no such quantum code is known [EKZ20, KT20, HHO20]. We are studying the best rate distance trade-offs of quantum LDPCs together with companion decoding algorithms (this project is in its initial stages). Expansion seems to play an important role in the construction of this kind of quantum code and convex optimization techniques may be helpful for decoding.

The following directions are being cautiously and less actively pursued for now.

- **SOS lower bounds for independence number of sparse random graphs:** It seems that we have a long way to go in terms of acquiring a systematic understanding of the limitations of the Sum-of-Squares hierarchy. Towards this direction, an elegant open problem is to show limitations on the SOS power of certifying good upper bounds on the independence number of sparse $\mathcal{G}_{n,d/n}$ Erdős–Rényi random graphs, for $d = O(1)$ or at most some slowly growing function of n. This problem has some structural similarities to the celebrated planted clique problem on the dense $\mathcal{G}_{n,1/2}$ Erdős–Rényi random graphs [BHK+16]. However, it challenges the current SOS lower bound machinery in several aspects.

- **Subexponential time guarantees for MaxCut:** What are the best approximation guarantees for the MaxCut problem in the $1 - \varepsilon$ regime that can obtained with subexponential time Sum-of-Squares (i.e., using $o(n)$ levels)?

- **PCP for MA:** Can we design a suitable PCP theorem for MA? Interestingly, this question seems to lie between the widely open Quantum PCP Hamiltonian Conjecture [AAV13] and the now well established PCP theorems for NP. Using a recent connection [AB19], a suitable PCP for MA could imply MA=NP.

With the emerging theory of high-dimensional expanders, the hope for a *good* (constant rate and constant relative distance) Locally Testable Codes (LTCs) was reinvigorated [DDHRZ20]. I keep an open eye for any opportunities in this direction.

Combinatorial ideas gave us a good control on the construction of expanders graphs, as in the zig-zag product of [RVW00], and the PCP Theorem by gap amplification of [Din06] (providing alternatives to the previous more algebraic approaches), I wonder whether the Unique Games Conjecture [Kho10] can be tackled using combinatorial ideas.

Long Term Views

As theoretical computer science experiences a revolution in terms of its increased diversity, depth and impact to other sciences, I hope to contribute to this multidisciplinary effort by establishing bridges among its various (and constantly emerging) ramifications as well as by deepening its connection to mathematics. I am certain that the Sum-of-Squares hierarchy, coding theory and expansion will be part of my future explorations, but I want to keep an evergrowing horizon.

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