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## Mathematical modelling and simulation of the behaviour of the steam turbine

Mircea Dulau<sup>a,\*</sup>, Dorin Bica<sup>b</sup>

<sup>a,b</sup>*Department of Electrical and Computer Engineering, "Petru Maior" University of Tîrgu-Mureş, 1 N.Iorga st., 540088*

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### Abstract

Model simulations are becoming very important in dynamic power system analyses. However, no mathematical system can exactly model a physical process. Based on mathematical models of the processes and design calculations, PC programs allow simulation and the determination of the control system performances. This paper presents the mathematical modelling of the steam turbine unit, developed based on the continuity equation. This model is used to determine the simulation diagram for the steam turbine with high, medium and low pressure sections. Using Matlab/Simulink software facilities, have been simulated the behaviour of the shaft torque, depending of the control valves opening, with uncertain parameters of the process, than the step response of the steam turbine, with load and proportional control algorithm.

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### 1. Introduction

The steam turbine convert stored energy of high pressure and high temperature steam into rotary energy, which is turn converted into electrical energy by the generator. Each turbine section consists of a set of a moving blades attached to rotor and a set of stationary vanes in which steam is accelerated to high velocity (Fig. 1).

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\* Corresponding author. Tel.: +4-075-224-9499.

E-mail address: [mircea.dulau@ing.upm.ro](mailto:mircea.dulau@ing.upm.ro)

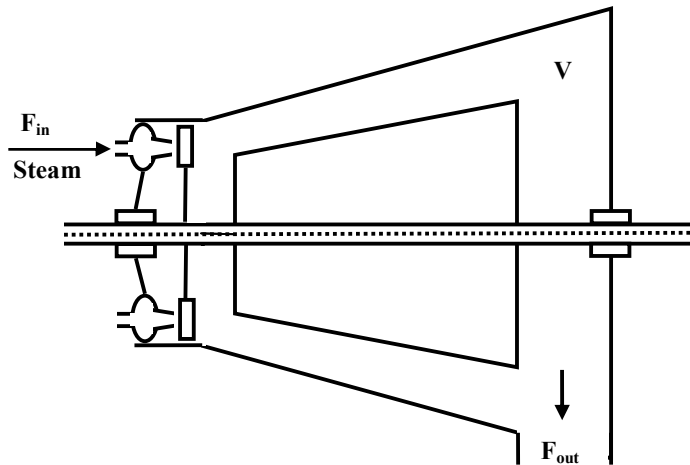


Fig. 1. Steam turbine unit.

The kinetic energy of the high velocity steam is converted into shaft torque [2,4,5,9]. A large variety of steam turbines have been built, with respect to the capacity, application and desired performance. To increase the thermal efficiency in applications, steam turbines consist of multistage steam expansion.

## 2. Mathematical modelling of steam turbine unit

In many cases, the steam turbine models are simplified, many intermediate variables are omitted and only map input variables to outputs as outlined in [2,3,9,10,12,13].

In these conditions, the input-output mathematical model (the transfer function) of a steam turbine from Fig. 1 and the expression for mechanical power developed by a turbine are based on the continuity equation:

$$\frac{dW}{dt} = V \frac{d\rho}{dt} = F_{in}(t) - F_{out}(t) \quad (1)$$

where:  $W$  is the weight of steam in turbine [kg];  $V$  – volume of turbine [m<sup>3</sup>];  $\rho$  – density of steam [kg/m<sup>3</sup>];  $F$  – steam mass flow rate [kg/s];  $t$  – time [sec.].

Assuming the flow out of the turbine to be proportional to pressure in the turbine [9]:

$$F_{out} = P \frac{F_0}{P_0} \quad (2)$$

where:  $P$  – pressure of steam in the turbine [kPa];  $P_0$  – rated pressure;  $F_0$  – rated flow out of turbine. With constant temperature in the turbine:

$$\frac{d\rho}{dt} = \frac{dP}{dt} \cdot \frac{\partial \rho}{\partial P} \quad (3)$$

From equations (1)...(3), result the mathematical model:

$$F_{in}(t) - F_{out}(t) = V \frac{dP}{dt} \cdot \frac{\partial \rho}{\partial P} = V \frac{\partial \rho}{\partial P} \cdot \frac{P_0}{F_0} \cdot \frac{dF_{out}}{dt} = T_T \frac{dF_{out}}{dt} \quad (4)$$

$$T_T \frac{dF_{out}}{dt} + F_{out}(t) = F_{in}(t) \quad (5)$$

and, after Laplace transform, the transfer function of a steam turbine unit:

$$H_T(s) = \frac{F_{out}(s)}{F_{in}(s)} = \frac{1}{T_T s + 1} \quad (6)$$

where:  $T_T = V \frac{P_0}{F_0} \cdot \frac{\partial \rho}{\partial P}$  is the time constant [sec.].

The turbine torque is proportional to the steam flow rate:

$$T_m(t) = k \cdot F(t) \quad (7)$$

where:  $k$  is a proportional constant.

The change in density of steam with respect to pressure ( $\partial \rho / \partial P$ ) at a given temperature may be determined from tables [14].

### 3. Block diagram of steam turbine configuration

Depending on the turbine configuration, gas units consist of high pressure (HP), medium pressure (MP) and low pressure (LP) turbine sections (Fig. 2). A steam turbine is equipped with sets of valves: high pressure valves (HPV), re-heater valves (RHV) [15,17].

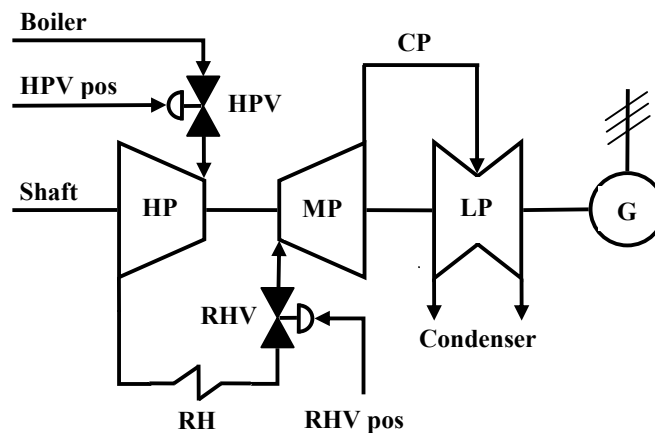


Fig. 2. Steam turbine configuration.

Steam enters to the HP section through the HPV and the inlet piping. The HP exhaust steam is passed through the re-heater (RH). Re-heater section is a large heat exchanger, which has significant thermal capacity and steam mass storage. In the reheat type turbine, the steam upon leaving the HP section returns to the boiler, where is passed through a RH before returning to the MP section. The reheat steam flows into the MP turbine section through the RHV and the inlet piping. The crossover piping (CP) provides a path for the steam from MP section exhaust to the LP inlet.

Based on Fig. 2 and equation (6), result the block diagram of the turbine configuration, used for simulation (Fig. 3), where [2,8,9,17]:

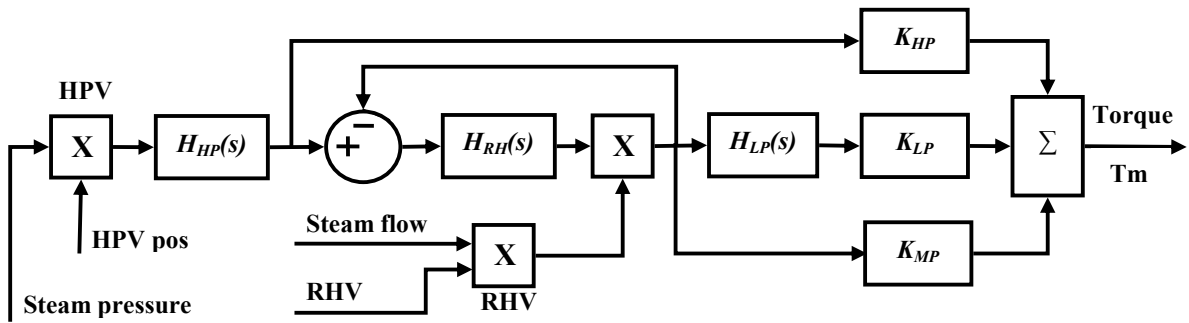


Fig. 3. Block diagram of steam turbine configuration.

$$H_{HP}(s) = \frac{1}{T_{HP}s + 1} \quad (8)$$

$$H_{RH}(s) = \frac{1}{T_{RH}s} \quad (9)$$

$$H_{LP}(s) = \frac{1}{T_{LP}s + 1} \quad (10)$$

The response of steam flow to a change in HPV opening exhibits a time constant  $T_{HP}$  due to the changing time to the HP section. Usually,  $T_{HP} = (0.2 \dots 0.3)$  sec. The steam flow in the MP and LP sections can change only with the build-up of pressure in the re-heater volume. The re-heater holds a substantial amount of steam and the time constant is  $T_{RH} = (5 \dots 10)$  sec. The steam flows into the LP sections, associated with the crossover piping, and express an additional time constant  $T_{LP} \cong 0.5$  sec.

The MP and LP sections generate about (60...80) % of the total turbine power. The sum of the power fractions of the various turbines sections is [9,15,17]:

$$K_{HP} + K_{LP} + K_{MP} = 1 \quad (11)$$

If  $T_{LP} \ll T_{RH}$ , then  $T_{LP}$  is negligible in comparison with  $T_{RH}$ , and a simplified transfer function of the turbine relating perturbed values of the torque and HPV position ( $\Delta T_m / \Delta Z_{HPV}$ ) may be written as follows:

$$\frac{\Delta T_m}{\Delta Z_{HPV}} = \frac{K_{HP}}{T_{HP}s + 1} + \frac{1 - K_{HP}}{(T_{HP}s + 1)(T_{RH}s + 1)} = \frac{T_{RH}K_{HP}s + 1}{(T_{HP}s + 1)(T_{RH}s + 1)} \quad (12)$$

If the steam turbine is of a single reheat type, the transfer function may be approximated by:

$$\frac{\Delta T_m}{\Delta Z_{HPV}} \cong \frac{T_{RH}K_{HP}s + 1}{T_{RH}s + 1} \quad (13)$$

No mathematical system can exactly model a physical process. Typically, the flow  $F$ , the pressure  $P$ , the density  $\rho$  etc. are experimentally measured and/or calculated, leading to the confidence intervals for time values and not just a single value:

$$\begin{aligned} T_{HP} &\in [T_{HPnom} \pm \Delta T_{HP}] \\ T_{RH} &\in [T_{RHnom} \pm \Delta T_{RH}] \end{aligned} \quad (14)$$

The values  $T_{HPnom}, T_{RHnom}$  are called nominal values and the  $\Delta T_{HP}, \Delta T_{RH}$  are called the maximum deviations from the nominal values [1,6,7,11,16]. As follow, in practice, results many admissible transfer functions for this process, one for each possible combination of  $T_{HP}, T_{RH}$  in the given intervals.

#### 4. Simulation of the steam turbine configuration

Based on block diagram of the steam turbine (Fig. 3) and used Matlab/Simulink facilities [1,6,16], result the shaft torque, as an output variable, depending on the control valves position, HPV and RHV, as input variables, where 100% means fully open (Fig. 4, Fig. 5).

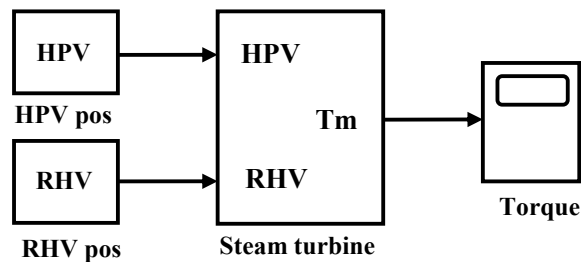


Fig. 4. Simulink block diagram of steam turbine configuration.

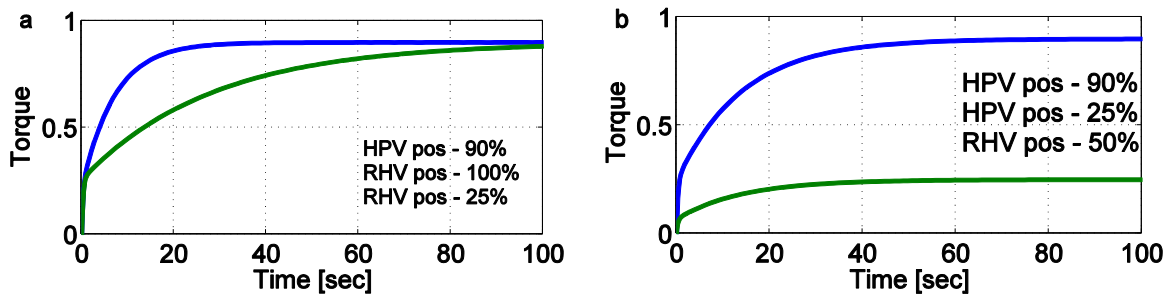


Fig. 5. Shaft torque to different opening positions of: (a) RHV; (b) HPV.

The HPV position (HPV pos) modulates the steam flow through the turbine for load control during normal operation. The RHV with valve position (RHV pos) is normally used only for rapid control of turbine mechanical power in the event of an overspeed.

Fig. 6 shows the step response of the steam turbine, described by the simplified transfer functions (12) and (13).

Some of parameters from these relationships have exact value (for example  $K_{RH} = 0.3$ ) and some of these varies within a specific range of values, defining the uncertain model of the process, as follow [9,15,17]:

- the nominal value of  $T_{RH} = 7.5$  sec. and a range between (5...10) sec.;
- the nominal value of  $T_{HP} = 0.25$  sec. and a range between (0.2...0.3) sec.

To find how variations of the parameters from the nominal model affect the process, with Matlab techniques are generated a number of random samples of the uncertain parameters and plot the corresponding step responses [1,16].

The graphics from Fig. 6 show the same type of damped step response, with similar transient performances, so, in order to simulate the behaviour of the steam turbine is adequate to use the transfer function from relation (13).

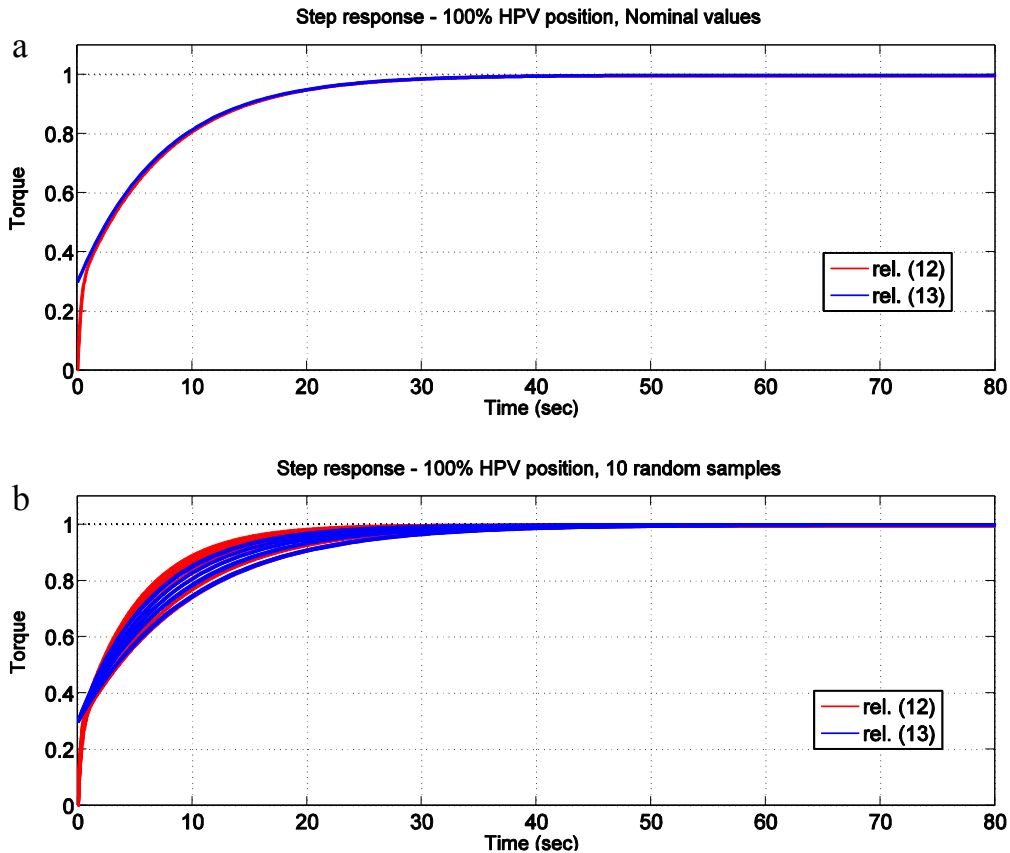


Fig. 6. Shaft torque: (a) nominal values; (b) random samples.

It is considered that the steam turbine works with an isolated load, described by the transfer function:  $H_L(s) = 1/(T_L s)$ , and a proportional algorithm (controller), in feedback connection (Fig. 7).

With simulation values:  $T_L = 12$  sec.,  $T_{RH} = 7.5$  sec. and  $K_{HP} = 0.3$  it is determined the values of tuning parameter  $K_C$ , for which the speed is stable and the step response is damped or critically damped.

Based on characteristic equation of the close-loop system and values for critical damping, any positive value,  $K_C > 0$ , will lead to the stable response, and  $0 < K_C < 0.67$ , will lead to the (critically) damped stable response (Fig. 8).

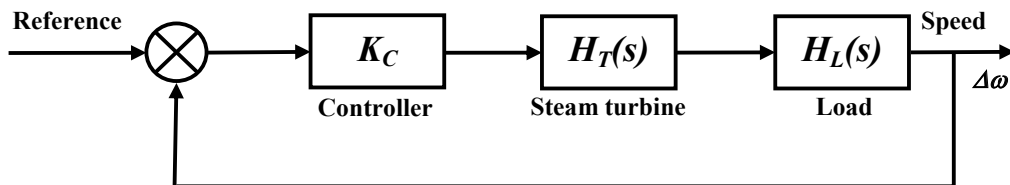


Fig. 7. Steam turbine closed-loop block diagram.

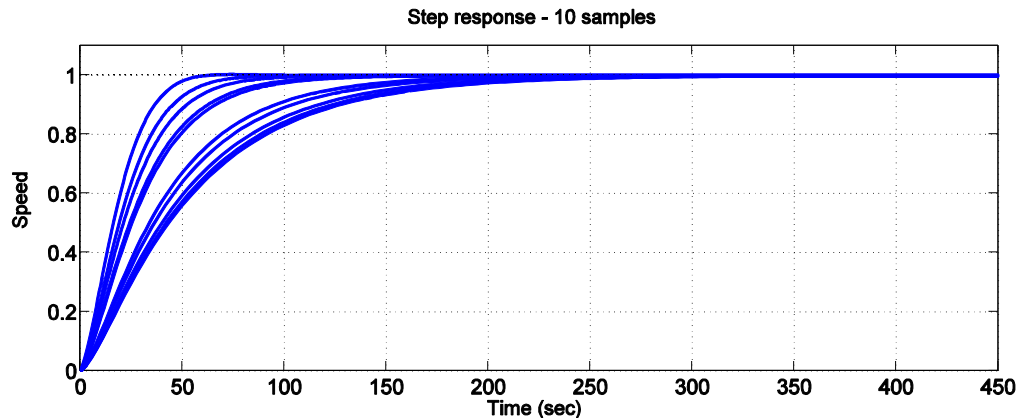


Fig. 8. Steam turbine closed-loop step response.

## 5. Conclusions

Usually, the steam turbine transfer function is characterized by two time constants. In order to analyze the behaviour, simulation results show that it can use a first order transfer function, despite the variation of the steam turbine parameters.

Largely, the dynamic response of a steam turbine is influenced by two factors: entrained steam into high pressure turbine section and the storage action in the re-heater. The dynamic response of a steam turbine can be related in terms of changes in steam valves opening (HPV position and RHV position).

In order to meet the active power demand, inputs to the generator (steam parameters) must be controlled, otherwise, the generator speed will vary.

One of the control key refers to establish the conditions of the stable response, with steam turbine, generator and different type of controllers, in close-loop connection.

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