Introduction to high energy physics

Grant

Update: May 16, 2022

1 Quarks and leptons

1.1 Preamble

Nothing to say.

1.1.1 Why high energies?

Particle physics deals with the study of the elementary constituents of matter. We need to talk about "pointlike". It is a conception depends on the spatial resolution of the probe used to investigate possible structure. The resolution is Δr if two points in an object can just be resolved as separate when they are a distance Δr apart.

Assuming the probing beam itself consists of pointlike particles, the resolution is limited by the de Broglie wavelength of these particles, which is $\lambda = h/p$ where p is the beam momentum and h is Planck's constant. Thus beams of high momentum have short wavelengths and can have high resolution.

In an optical micro scope, the resolution is given by

$$\Delta r \simeq \lambda / \sin\theta$$

where θ is the angular aperture of the light beam used to view the structure of an object.

Substituting the de Broglie relation, the resolution becomes

$$\Delta \simeq \frac{\lambda}{\sin\theta} = \frac{h}{p\sin\theta}$$

so that Δr is inversely proportional to the momentum transferred to the photons, or other particles in an incident beam, when these are scattered by the target. Therefore, the first reason is resolution.

The second reason is that many of the elementary particles are extremely massive and the energy required to create them is correspondingly large.

1.1.2 Units in high energy physics

Table 1: Units in high energy physics(1)

Quantity	High energy unit	Value in SI units
length	$1fm$	$10^{-15}m$
energy	$1GeV = 10^9 eV$	$1.602 \times 19^{-10}J$
mass, E/c^2	$1GeV/c^2$	$1.78 \times 10^{-27}kg$
$\hbar = h/(2\pi)$	$6.588 \times 10^{-25}GeV \cdot s$	$1.055 \times 10^{-34}J \cdot s$
c	$2.998 \times 10^{23}fm \cdot s^{-1}$	$2.998 \times 10^{8}m \cdot s^{-1}$
$\hbar c$	$0.1975GeV \cdot fm$	$3.162 \times 10^{-26}J \cdot m$

Table 2: Units in high energy physics(2)

natural units, $\hbar = c = 1$	
mass, Mc^2/c^2	$1GeV$
length, $\hbar c/(Mc^2)$	$1GeV^{-1} = 0.1975fm$
time, $\hbar c/(Mc^3)$	$1GeV^{-1} = 6.59 \times 10^{-25}s$
Heaviside-Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$	
fine structure constant	$\alpha = e^2/(4\pi) = 1/137.06$

In the SI system the unit electric charge, e, is measured in coulombs and the fine structure constant is given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \simeq \frac{1}{137}$$

Here ϵ_0 is the permittivity of free space, while its permeability is defined as μ_0, such that $\epsilon_0\mu_0 = 1/c^2$.

For interaction in general, such units are not useful and we can define e in Heaviside-Lorentz units, which require $\epsilon_0 = \mu_0 = \hbar = c = 1$, so that

$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

with similar definitions that relate charges and coupling constants analogous to α in the other interactions.

1.1.3 Relativistic transformations

The relativistic relation between total energy E, the vector 3-momentum **p**(with Cartesian components p_x, p_y, p_z) and the rest mass m for a free particle is

$$E^2 = \mathbf{p}^2c^2 + m^2c^4$$

or, in units with $c = 1$

$$E^2 = \mathbf{p}^2 + m^2$$

The components p_x, p_y, p_z, E can be written as components of an energy-momentum 4-vector p_μ, where $\mu = 1, 2, 3, 4$. In the Minkowski convention, the three momentum(or space) components are taken to be real and the energy (or time) component to be imaginary, as follows:

$$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = iE$$

so that

$$p^2 = \sum_\mu p_\mu^2 = \mathbf{p}^2 - E^2 = -m^2$$

Thus p^2 is a relativistic invariant. Its value is $-m^2$, where m is the rest mass, and clearly has the same value in all reference frames. If $E, \mathbf{p}$ refer to the values measured in the lab frame Σ then those in another frame, say Σ', moving along the x-axis with velocity βc are found from the Lorentz transformation, given in matrix form by

$$p'_\mu = \sum_{\nu=1}^{4} \alpha_{\mu\nu} p_\nu$$

where

$$\alpha_{\mu\nu} = \begin{vmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{vmatrix}$$

and $\gamma = 1/\sqrt{1-\beta^2}$. Thus

$$
\begin{aligned}
p_1' &= \gamma p_1 + i\beta\gamma p_4 \\
p_2' &= p_2 \\
p_3' &= p_3 \\
p_4' &= -i\beta\gamma p_1 + \gamma p_4
\end{aligned}
$$

In terms of energy and momentum

$$
\begin{aligned}
p_x' &= \gamma(p_x - \beta E) \\
p_y' &= p_y \\
p_z' &= p_z \\
E' &= \gamma(E - \beta p_x)
\end{aligned}
$$

with $\mathbf{p}'^2 - E'^2 = -m^2$.

The above transformations apply equally to the space-time coordinates, making the replacements $p_1 \to x_1(= x), p_2 \to x_2(= y), p_3 \to x_3(= z), p_4 \to x_4(= it)$.

The 4-momentum squared above is an example of a Lorentz scalar, i.e. the invariant scalar product of the two 4-vectors. Another example is the phase of a plane wave, which determines whether it is at a crest or a trough and which must be the same for all observers. With $\mathbf{k}$ and ω as the propagation vector and the angular frequency, and in units $\hbar = c = 1$

$$
phase = \mathbf{k} \cdot \mathbf{x} - \omega t = \mathbf{p} \cdot \mathbf{x} - Et = \sum p_\mu x_\mu
$$

The Minkowski notation used here for 4-vectors defines the metric, namely the square of the 4-vector momentum $p = (\mathbf{p}, iE)$ so that

$$
metric = (4 - momentum)^2 = (3 - momentum)^2 - (energy)^2
$$

In analogy with the space-time components, the components $p_{x,y,z}$ of 3-momentum are said to be spacelike and the energy component E, timelike. Thus, if q denotes the 4-momentum transfer in a reaction, i.e. is $q = p - p'$ where p, p' are the initial and final 4-momenta, then $q^2 > 0$ means spacelike, otherwise, timelike.

A different notation is used in texts on field theory. These avoid the use of the imaginary fourth component and introduce the negative sign via the metric tensor $g_{\mu\nu}$. The scalar product of 4-vectors A and B is then defined as

$$
AB = g_{\mu\nu} A_\mu B_\nu = A_0 B_0 - \mathbf{A} \cdot \mathbf{B}
$$

where all the components are real. Here $\mu, \nu = 0$ stand for the energy(time) component and $\mu, \nu = 1, 2, 3$ for the momentum(space) components, and

$$g_{00} = +1, \quad g_{11} = g_{22} = g_{33} = -1, \quad g_{\mu\nu} = 0 \; for \; \mu \neq \nu$$

This metric results in Lorentz scalars with sign opposite to those using the Minkowski convention, so that a spacelike(timelike) 4-momentum has $q^2 < 0 (q^2 > 0)$ respectively.

Sometimes, to avoid writing negative quantities, re-definitions have to be made. In deep inelastic electron scattering, q^2 is spacelike and negative, and in discussing such processes it has become common to define the positive quantity $Q^2 = -q^2$.

1.1.4 Fixed-target and colliding beam accelerators

Consider the energy available for particle creation in fixed-target and in colliding-beam accelerators.

Suppose an incident particle of mass m_A, total energy E_A and momentum $\mathbf{p}_A$ hits a target particle of mass m_B, energy E_B, momentum $\mathbf{p}_B$. The total 4-momentum, squared, of the system is

$$p^2 = (\mathbf{p}_A + \mathbf{p}_B)^2 - (E_A + E_B)^2 = -m_A^2 - m_B^2 + 2\mathbf{p}_A \cdot \mathbf{p}_B - 2E_A E_B$$

The centre-of-momentum system(cms) is defined as the reference frame in which the total 3-momentum is zero. If the total energy in the cms is denoted E^*, then we also have $p^2 = -E^{*2}$.

Suppose first of all that the target particle is at rest in the laboratory(lab) system, so that $\mathbf{p}_B = 0$ and $E_B = m_B$. Then

$$E^{*2} = -p^2 = m_A^2 + m_B^2 + 2m_B E_A$$

Secondly, suppose that the incident and target particles travel in opposite directions. Then, with p_A and p_B denoting the absolute values of the 3-momenta, the above equation gives

$$E^{*2} = 2(E_A E_B + p_A p_B) + (m_A^2 + m_B^2) \simeq 4E_A E_B$$

if $m_A, m_B \ll E_A, E_B$. This result is for a head-on collision. For two beams crossing at an angle θ, the result would be $E^{*2} = 2E_A E_B(1 + \cos\theta)$.

Note that the cms energy available for new particle creation in a collider with equal energies E in the two beams rises linearly with E, i.e. $E^* \simeq 2E$, while for a fixed-target machine the cms energy rises as the square root of the incident energy, $E^* \simeq \sqrt{2m_B E_A}$. Therefore, the highest possible energies for creating new particles are to be found at colliding-beam accelerators.

1.2 The Standard Model of particle physics

Practically all experimental data from high energy experiments can be accounted for by the so-called Standard Model of particles and their interactions.

1.2.1 The fundamental fermions

All matter is built from a small number of fundamental spin 1/2 particles, or fermions: six quarks and six leptons.

Table 3: The fundamental fermions

Particle	Flavour			$Q/\|e\|$
leptons	e	μ	τ	-1
	ν_e	ν_μ	ν_τ	0
quarks	u	c	t	+2/3
	d	s	b	-1/3

In the table, the quark masses increase from left to right, just as they do for the leptons. And, just as for the leptons, the quarks are grouped into pairs differing by on unit of electric charge.

The quark type or 'flavour' is denoted by a symbol:

- u→up
- d→down
- s→strange
- c→charmed
- b→bottom
- t→top

The 's for strange' quark terminology came about because theses quarks turned out to be constituents of the so-called 'strange particles' discovered in cosmic rays. Their behaviour was strange in the sense that they were produced prolifically in strong interactions, and therefore would be expected to decay on a strong interaction timescale(10^{-23}s); instead they decayed extremely slowly, by weak interactions.

The solution to this puzzle was that these particles carried a new quantum number, S for strangeness, conserved in strong interactions - so that they were always produced in pairs with $S = +1$ and $S = -1$ but they decayed singly and weakly, with a change in strangeness, $\Delta S = \pm 1$, into non-strange particles.

A proton consists of uud, and a neutron consists of ddu. The common material of the present universe is the stable particles, i.e. the electrons e and the u and d quarks. The heavier quarks s, c, b, t also combine to form particles akin to, but much heavier than, the proton and neutron, but these are unstable and decay rapidly(in typically 10^{-13}s) to u, d combinations, just as heavy leptons decay to electrons.

If we allow a new degree of freedom colour(three colours), we'll find that the total charge of all the fermions is zero.

1.2.2 The interactions

The Standard Model also comprises interactions of particles. There are four types of fundamental interaction or field, as follows.

1. **Strong interactions** are responsible for binding the quarks in the neutron and proton, and the neutrons and protons within nuclei. The interquark force is mediated by a massless particle, the gluon.
2. **Electromagnetic interactions** are responsible for virtually all the phenomena in extra-nuclear physics, in particular for the bound states of electrons with nuclei. These interactions are mediated by photon exchange.
3. **Weak interactions** are typified by the slow process of nuclear β-decay. The mediators of the weak interactions are the $W^{\pm}$ and Z^0 bosons, with masses of order 100 times the proton mass.
4. **Gravitational interactions** act between all types of particle. On the scale of experiments in particle physics, gravity is by far the weakest of the universe. It is supposedly mediated by exchange of a spin boson, the graviton.

Table 4: The boson mediators

Interaction	Mediator	Spin/parity
strong	gluon, G	1^-
electromagnetic	photon, γ	1^-
weak	$W^{\pm}$, Z^0	1^-, 1^+
gravity	gravition, g	2^+

Weak and electromagnetic interactions can indeed be unified, and would have the same strength at very high energies; only at lower energies is the symmetry broken so that their apparent strengths are very different.

To indicate the relative magnitudes of the four types of interaction, the comparative strengths of the force between two protons when just in contact are very roughly as follows.

- strong: 1
- electromagnetic: 10^{-2}
- weak: 10^{-7}
- gravity: 10^{-39}

As for timescales, the Uncertainty Principle relates the lifetime and the uncertainty in energy of a state. An unstable particle does not have a unique mass, but a distribution with 'width' $\Gamma = \hbar/\tau$. So, when τ is very short, its value can be inferred from the measured width Γ.

1.2.3 Limitations of the Standard Model

1. Gravitational interactions are not included.
2. Neutrinos are assumed to be massless, but there is growing evidence that neutrinos do have finite masses.
3. The model is somewhat inelegant, as it contains some 17 arbitrary parameters.
4. The origin of the parameters and the underlying reasons for the 'xerox copies' - six quark and six lepton flavours - is not at all understood.

We require new and presently unknown physics beyond it. But equally, it seems fairly certain that the model will form an integral and important part of a more complete theory of particles in the far future.

1.3 Particle classification: fermions and bosons

Fundamental particles are of two types.

- **Fermions**: particles with half-integral spin and obey Fermi-Dirac statistics.
- **Bosons**: particles with integral spin and obey Bose-Einstein statistics.

The statistics obeyed by a particle determines how the wavefunction ψ describing an ensemble of identical particles behaves under interchange of any pair of particles. Clearly $|\psi|^2$ cannot be altered by the interchange, since particles are indistinguishable. Thus, under interchange $\psi \to \pm\psi$. There is a fundamental theorem, which is a sacrosanct principle of quantum field theory.

- under exchange of identical bosons $\psi \to +\psi$; ψ is symmetric
- under exchange of identical fermions $\psi \to -\psi$; ψ is antisymmetric

Pauli principle: two or more indentical fermions cannot exist in the same quantum state.

One exciting possible extension beyond the Standard Model is the concept of supersymmetry, which predicts that, at a high energy scale there should be fermion-boson symmetry. Each fermion will have a boson partner and vice versa.

1.4 Particles and antiparticles

The total energy E can in principle assume negative as well as positive values,

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

Classically, negative energies for free particles appear to be meaningless. In quantum mechanics, however, we represent the amplitude of an infinite stream of particles travelling along the positive x-axis with 3-momentum p by the plane wavefunction

$$\psi = Ae^{-i(Et-px)/\hbar}$$

where the angular frequency is $\omega = E/\hbar$, the wavenumber is $k = p/\hbar$ and A is a normalisation constant.

As t increases, the phase advances in the direction of increasing x. Formally, however, it can also represent particles of energy $-E$ and momentum $-p$ travelling in the negative x-direction and backwards in time(i.e. replacing Et by $(-E)(-t)$ and px by $(-p)(-x)$).

Such a stream of negative electrons flowing backwards in time is equivalent to positive charges flowing forward, and thus having $E > 0$. Hence, the negative energy particle states are connected with the existence of positive energy antiparticles of exactly equal but opposite electrical charge and magnetic moment, and otherwise identical.

Dirac's original picture of antimatter was that the vacuum actually consisted of an infinitely deep sea of completely filled negative energy levels. A positive energy electron was prevented from falling into a negative energy state, with release of energy, by the Pauli principle. If one supplies energy $E > 2mc^2$, a negative energy electron could be lifted into a positive energy state. However, such a picture is not valid for the pair creation of bosons.

In the relativistic case, ψ should be treated as an operator that creates or destroys particles. Negative energies are simply associated with destruction operators acting on positive energy particles to reduce the energy within the system. The absorption or destruction of a negative energy particle is again interpreted as the creation of a positive energy antiparticle, with opposite charge, and vice versa.

For fermions only there is a conservation law: the difference in the number of fermions and antifermions is a constant. Thus fermions and antifermions can only be created or destroyed in pairs.

1.5 Free particle wave equations

1.6 Helicity states: helicity conservation

1.7 Lepton flavours

1.8 Quark flavours

1.9 The cosmic connection

1.9.1 Early work in cosmic rays

1.9.2 Particle physics in cosmology

2 Interactions and fields

3 Invariance principles and conservation laws

4 Quarks in hadrons

5 Lepton and quark scattering

6 Quark interactions and QCD

7 Weak interactions

8 Electroweak interactions and the Standard Model

9 Physics beyond the Standard Model

10 Particle physics and cosmology

11 Experimental methods