Introduction to high energy physics

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1 Quarks and leptons

1.1 Preamble

Nothing to say.

1.1.1 Why high energies?

Particle physics deals with the study of the elementary constituents of matter. We need to talk about "pointlike". It is a conception depends on the spatial resolution of the probe used to investigate possible structure. The resolution is Δr if two points in an object can just be resolved as separate when they are a distance Δr apart.

Assuming the probing beam itself consists of pointlike particles, the resolution is limited by the de Broglie wavelength of these particles, which is $\lambda = h/p$ where p is the beam momentum and h is Planck's constant. Thus beams of high momentum have short wavelengths and can have high resolution.

In an optical micro scope, the resolution is given by

$$\Delta r \simeq \lambda / \sin\theta$$

where θ is the angular aperture of the light beam used to view the structure of an object.

Substituting the de Broglie relation, the resolution becomes

$$\Delta \simeq \frac{\lambda}{\sin\theta} = \frac{h}{p\sin\theta}$$

so that Δr is inversely proportional to the momentum transferred to the photons, or other particles in an incident beam, when these are scattered by the target. Therefore, the first reason is resolution.

The second reason is that many of the elementary particles are extremely massive and the energy required to create them is correspondingly large.

1.1.2 Units in high energy physics

Table 1: Units in high energy physics(1)

Quantity	High energy unit	Value in SI units
length	$1fm$	$10^{-15}m$
energy	$1GeV = 10^9 eV$	$1.602 \times 19^{-10} J$
mass, E/c^2	$1GeV/c^2$	$1.78 \times 10^{-27} kg$
$\hbar = h/(2\pi)$	$6.588 \times 10^{-25} GeV \cdot s$	$1.055 \times 10^{-34} J \cdot s$
c	$2.998 \times 10^{23} fm \cdot s^{-1}$	$2.998 \times 10^8 m \cdot s^{-1}$
$\hbar c$	$0.1975 GeV \cdot fm$	$3.162 \times 10^{-26} J \cdot m$

Table 2: Units in high energy physics(2)

natural units, $\hbar = c = 1$	
mass, Mc^2/c^2	$1GeV$
length, $\hbar c/(Mc^2)$	$1GeV^{-1} = 0.1975 fm$
time, $\hbar c/(Mc^3)$	$1GeV^{-1} = 6.59 \times 10^{-25} s$
Heaviside-Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$	
fine structure constant	$\alpha = e^2/(4\pi) = 1/137.06$

In the SI system the unit electric charge, e, is measured in coulombs and the fine structure constant is given by

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \simeq \frac{1}{137}$$

Here ϵ_0 is the permittivity of free space, while its permeability is defined as μ_0, such that $\epsilon_0\mu_0 = 1/c^2$.

For interaction in general, such units are not useful and we can define e in Heaviside-Lorentz units, which require $\epsilon_0 = \mu_0 = \hbar = c = 1$, so that

$$\alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

with similar definitions that relate charges and coupling constants analogous to α in the other interactions.

1.1.3 Relativistic transformations

The relativistic relation between total energy E, the vector 3-momentum **p**(with Cartesian components p_x, p_y, p_z) and the rest mass m for a free particle is

$$E^2 = \mathbf{p}^2 c^2 + m^2 c^4$$

or, in units with $c = 1$

$$E^2 = \mathbf{p}^2 + m^2$$

The components p_x, p_y, p_z, E can be written as components of an energy-momentum 4-vector p_μ, where $\mu = 1, 2, 3, 4$. In the Minkowski convention, the three momentum(or space) components are taken to be real and the energy (or time) component to be imaginary, as follows:

$$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = iE$$

so that

$$p^2 = \sum_\mu p_\mu^2 = \mathbf{p}^2 - E^2 = -m^2$$

Thus p^2 is a relativistic invariant. Its value is $-m^2$, where m is the rest mass, and clearly has the same value in all reference frames. If $E, \mathbf{p}$ refer to the values measured in the lab frame Σ then those in another frame, say Σ', moving along the x-axis with velocity βc are found from the Lorentz transformation, given in matrix form by

$$p'_\mu = \sum_{\nu=1}^{4} \alpha_{\mu\nu} p_\nu$$

where

$$\alpha_{\mu\nu} = \begin{vmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{vmatrix}$$

and $\gamma = 1/\sqrt{1-\beta^2}$. Thus

$$
\begin{aligned}
p_1' &= \gamma p_1 + i\beta\gamma p_4 \\
p_2' &= p_2 \\
p_3' &= p_3 \\
p_4' &= -i\beta\gamma p_1 + \gamma p_4
\end{aligned}
$$

In terms of energy and momentum

$$
\begin{aligned}
p_x' &= \gamma(p_x - \beta E) \\
p_y' &= p_y \\
p_z' &= p_z \\
E' &= \gamma(E - \beta p_x)
\end{aligned}
$$

with $\mathbf{p}'^2 - E'^2 = -m^2$.

The above transformations apply equally to the space-time coordinates, making the replacements $p_1 \to x_1(= x), p_2 \to x_2(= y), p_3 \to x_3(= z), p_4 \to x_4(= it)$.

The 4-momentum squared above is an example of a Lorentz scalar, i.e. the invariant scalar product of the two 4-vectors. Another example is the phase of a plane wave, which determines whether it is at a crest or a trough and which must be the same for all observers. With $\mathbf{k}$ and ω as the propagation vector and the angular frequency, and in units $\hbar = c = 1$

$$
phase = \mathbf{k} \cdot \mathbf{x} - \omega t = \mathbf{p} \cdot \mathbf{x} - Et = \sum p_\mu x_\mu
$$

The Minkowski notation used here for 4-vectors defines the metric, namely the square of the 4-vector momentum $p = (\mathbf{p}, iE)$ so that

$$
metric = (4 - momentum)^2 = (3 - momentum)^2 - (energy)^2
$$

In analogy with the space-time components, the components $p_{x,y,z}$ of 3-momentum are said to be spacelike and the energy component E, timelike. Thus, if q denotes the 4-momentum transfer in a reaction, i.e. is $q = p - p'$ where p, p' are the initial and final 4-momenta, then $q^2 > 0$ means spacelike, otherwise, timelike.

A different notation is used in texts on field theory. These avoid the use of the imaginary fourth component and introduce the negative sign via the metric tensor $g_{\mu\nu}$. The scalar product of 4-vectors A and B is then defined as

$$
AB = g_{\mu\nu} A_\mu B_\nu = A_0 B_0 - \mathbf{A} \cdot \mathbf{B}
$$

where all the components are real. Here $\mu, \nu = 0$ stand for the energy(time) component and $\mu, \nu = 1, 2, 3$ for the momentum(space) components, and

$$g_{00} = +1, \quad g_{11} = g_{22} = g_{33} = -1, \quad g_{\mu\nu} = 0 \; for \; \mu \neq \nu$$

This metric results in Lorentz scalars with sign opposite to those using the Minkowski convention, so that a spacelike(timelike) 4-momentum has $q^2 < 0 (q^2 > 0)$ respectively.

Sometimes, to avoid writing negative quantities, re-definitions have to be made. In deep inelastic electron scattering, q^2 is spacelike and negative, and in discussing such processes it has become common to define the positive quantity $Q^2 = -q^2$.

1.1.4 Fixed-target and colliding beam accelerators

Consider the energy available for particle creation in fixed-target and in colliding-beam accelerators.

Suppose an incident particle of mass m_A, total energy E_A and momentum $\mathbf{p}_A$ hits a target particle of mass m_B, energy E_B, momentum $\mathbf{p}_B$. The total 4-momentum, squared, of the system is

$$p^2 = (\mathbf{p}_A + \mathbf{p}_B)^2 - (E_A + E_B)^2 = -m_A^2 - m_B^2 + 2\mathbf{p}_A \cdot \mathbf{p}_B - 2E_A E_B$$

The centre-of-momentum system(cms) is defined as the reference frame in which the total 3-momentum is zero. If the total energy in the cms is denoted E^*, then we also have $p^2 = -E^{*2}$.

Suppose first of all that the target particle is at rest in the laboratory(lab) system, so that $\mathbf{p}_B = 0$ and $E_B = m_B$. Then

$$E^{*2} = -p^2 = m_A^2 + m_B^2 + 2m_B E_A$$

Secondly, suppose that the incident and target particles travel in opposite directions. Then, with p_A and p_B denoting the absolute values of the 3-momenta, the above equation gives

$$E^{*2} = 2(E_A E_B + p_A p_B) + (m_A^2 + m_B^2) \simeq 4E_A E_B$$

if

$$m_A, m_B \ll E_A, E_B$$

. This result is for a head-on collision. For two beams crossing at an angle θ, the result would be $E^{*2} = 2E_A E_B(1 + \cos\theta)$.

Note that the cms energy available for new particle creation in a collider with equal energies E in the two beams rises linearly with E, i.e. $E^* \simeq 2E$, while for a fixed-target machine the cms energy rises as the square root of the incident energy, $E^* \simeq \sqrt{2m_B E_A}$. Therefore, the highest possible energies for creating new particles are to be found at colliding-beam accelerators.

1.2 The Standard Model of particle physics

1.2.1 The fundamental fermions

1.2.2 The interactions

1.2.3 Limitations of the Standard Model

1.3 Particle classification: fermions and bosons

1.4 Particles and antiparticles

1.5 Free particle wave equations

1.6 Helicity states: helicity conservation

1.7 Lepton flavours

1.8 Quark flavours

1.9 The cosmic connection

1.9.1 Early work in cosmic rays

1.9.2 Particle physics in cosmology

2 Interactions and fields

3 Invariance principles and conservation laws

4 Quarks in hadrons

5 Lepton and quark scattering

6 Quark interactions and QCD

7 Weak interactions

8 Electroweak interactions and the Standard Model

9 Physics beyond the Standard Model

10 Particle physics and cosmology

11 Experimental methods