GPS Based Inertial Navigation For Low-Thrust Spacecraft In Cislunar Space

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ABSTRACT

With renewed intrest in manned and robotic missions to Earth's moon with the advent of NASA's Artemis Program, robust tecniques for navigating spacecraft within cislunar space are necessary.

INTRODUCTION

TECHNICAL PLAN

A feasibility study will be performed to investigate the robustness of GPS based inertial navitgation for a spacecraft traversing from a Geostationary Transfer Orbit (GTO) to the target Near-Rectilinear Halo Orbit (NRHO) of the Artemis Lunar Gateway station. The true GTO to NRHO trajectory has be generated for a low-thrust spacecraft by employing Circular Restricted Three-Body Problem (CR3BP) dynamics, such that fuel use is minimized. An Extended Kalman Filter (EKF) has also been developed, which employs GPS pseudorange and accelerometer measurements, for estimating the inertial states (e.g., position and velocity) and mass of a spacecraft as it travels along the true trajectory. An Unscented Kalman Filter (UKF) will also be developed which estimates the inertial states and mass by employing the same measurements. The performance of both the EKF and UKF will be analyzed through Monte Carlo analysis.

True Trajectory Generation

The true low-thrust trajectory traversed by the spacecraft has been generated with CR3BP dynamics and a fully continuous, constant specific impluse thrust model by employing the *indirect* approach to optimizing a spacecraft trajectory. Although the details of indirect optimal control are outside of the scope of this project, this category of trajectory optimization methods employ variational calculus and Pontryagin's Maximum Principle to analytically derive the necesary conditions of optimality. This process results in the formation of a two-point boundary value problem (TPBVP) which is solved by fiding the set of time varying co-state (adjoint) variables at the initial epoch such that the boudary conditions are satisfied. Details of several tecniques used to generate the true trajectory can be found in Reference 1.

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Table 1. Low-Thrust Trajectory Parameters

Constant	Value	Units
Initial Mass (m_0)	1500	kg
Exhaust Velocity (c)	29.43	km/s
Specific Impulse (I_{sp})	3000	s
Max Thrust (T_{max})	10	N
Time of Flight ($t_f - t_0$)	33.0	days

By solving the TPBVP derived through indirect optimal control for a spacecraft with parameters shown in Table 1, the initial co-states were found to be

$$\boldsymbol{\lambda}_r(t_0) = \begin{bmatrix} 5.963713007227692 & 12.999481348176722 & 1.6584130970663207 \end{bmatrix}^T$$

$$\boldsymbol{\lambda}_v(t_0) = \begin{bmatrix} -0.04062357612068792 & 0.018558730104232632 & -0.0004993971529784666 \end{bmatrix}^T$$

$$\lambda_m(t_f) = 0.12200533102974148$$

which result in the trajectory shown in Figure 1, where coasting arcs are shown in blue, thrusting arcs in red, the

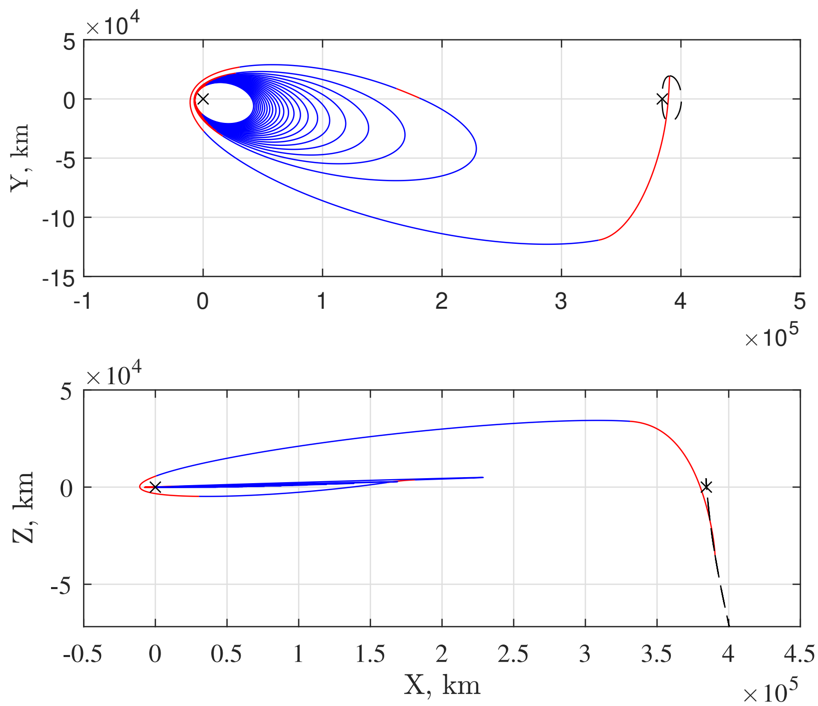

Figure 1. True Low-Thrust Trajectory in Inertial Reference Frame

target NRHO in dashed black lines, and the location of the Earth and Moon as black $\times$ symbols (please note the trajectory is shown in an inertial reference frame and not the CR3BP synodic reference frame). As can be seen, the trajectory consits of multiple sprirals about the Earth with gradually increasing apogee before finally reaching the NRHO. Therefore, this trajectory should provide a good indication of the developed filters performance in both near Earth and cislunar space.

Measurement Modeling

GPS Pseudoranges: A common GPS pseudorange model is given by[2,3]

$$p_{s,k} = ||\boldsymbol{\rho}_{s/rk,k}|| + c(\Delta t_k - \Delta t_k^s) + \phi_k + \nu_{GPS,k} \tag{1}$$

$$\boldsymbol{\rho}_{s/rk,k} = \mathbf{r}_k^i + \mathbf{T}_b^i \mathbf{r}_{rx}^b - \mathbf{r}_{s,k}^i + \mathbf{T}_{b,s}^i \mathbf{r}_{pc,s}^b \tag{2}$$

where $p_{s,k}$ is the pseudorange generated from GPS satelite s at time t_k, Δt is the GNSS receiver clock bias, Δt_k^s is the clock bias of GPS satellite s, c is the speed of light in a vacuum, $\phi_{s,k}$ is the ionospheric delay, and $\nu_{GPS,k}$ is the reciever white noise. The geometric range is the Euclidean norm of the vector from the phase center of the GPS transmitter and GNSS reciever phase center $\boldsymbol{\rho}_{s/rk,k}$, where $\mathbf{r}_k^i$ is the inertial position of the satelite, $\mathbf{T}_b^i(\bar{\mathbf{q}}_{m,k}) \in SO(3)$ is the rotation from the body frame of the spacecraft to the inertial frame, $\mathbf{r}_{rx}^b$ is the location of the GNSS reciever phase center in the spacecrafts body frame, $\mathbf{r}_{s,k}^i$ is the inertial position of the center of mass of GPS satellite s, $\mathbf{T}_{b,s}^i \in SO(3)$ is the rotation from the body frame of GPS satelite s to the inertial frame, and $\mathbf{r}_{pc,s}^b$ is the location of the phase center in the GPS body frame.

For the purposes of this work, we will assume the phase centers of both the GNSS reciever and all GPS transmitters are located at their respective spacecraft's center of mass (i.g., $\mathbf{r}_{rx}^b = \mathbf{r}_{tx}^b = \mathbf{0}_{3\times1}$), thereby decoupling attitde from the pseudorange measurements. We will also assume the GNSS reciever clock is perfect, such that $\Delta t = 0$, and ionospheric effects are negligable. Therefore, our GPS pseudorange model reduces to

$$p_{s,k} = ||\mathbf{r}_k^i - \mathbf{r}_{s,k}^i|| - c\Delta t_k^s + \nu_{GPS,k} \tag{3}$$

with relevent partials given by

$$\left.\frac{\partial p_s}{\partial \mathbf{r}}\right|_{\mathbf{x}=\mathbf{x}^*} \approx \frac{\mathbf{r}_k^{i^T} - \mathbf{r}_{s,k}^{i^T}}{||\mathbf{r}_k^i - \mathbf{r}_{s,k}^i||} \tag{4}$$

where $(\cdot)_k^*$ is employed to indicated a term associated with the reference trajectory at time t_k.

High precision ephemeris data projects provided by IGS[4] will be considered as truth and employed to compute the position of GPS satelites at signal transmission times when generating simulated GPS pseudorange measurements. Perturbed trajectory solutions from the same ephemeris will also be employed when computing expected GPS pseudorange measurements so simulate much less precise broadcast ephemeris. IGS ephemeris data products are defined with respect to the Earth fixed IGS14 reference frame, and therefore must be rotated to the Geocentric Celestial Reference Frame (e.g., J200 frame) before they can be employed for inertial navigation. Details of this rotation can be found in Refercne 5.

Inertial Measurement Unit A highly simplified model of IMU accelerometer measurements will be employed in which accelerometer scale factor error, bias, nonorthoganality, and misalighment are assumed negligable and is given by

$$\mathbf{a}_{IMU,k} = \mathbf{T}_b^{IMU} \mathbf{T}_b^{i^T} \mathbf{a}_k^i + \nu_{IMU,k} \tag{5}$$

where $\mathbf{a}_k^i$ is the inertial acceleration of the spacecraft at time t_k, $\mathbf{T}_b^{IMU} \in SO(3)$ is the rotation from the spacecraft body frame to the IMU frame, and $\nu_{IMU,k}$ is accelerometer white noise. We will also assume the spacecraft body frame and IMU frame are always perfectly aligned with the inertial frame (e.g., $\mathbf{T}_b^{IMU} = \mathbf{T}_b^i = \mathbf{I}_{3\times3}$), thereby allowing the attitde of the spacecraft to be ignored as attitude estimation is outside of the scope of this work. The relevent partials of the accelerometer measurements, assuming perfectly spherical gravitational effects from both the Earth and Moon, are then given by

$$\left.\frac{\partial \mathbf{a}_{IMU}}{\partial \mathbf{r}}\right|_{\mathbf{x}=\mathbf{x}^*} = \frac{\mu}{r^3}\left(\frac{3}{r^2}\mathbf{r}\mathbf{r}^T - \mathbf{I}_{3\times3}\right) + \frac{\mu_l}{r_{ls}^3}\left(\frac{3}{r_{ls}^2}\mathbf{r}_{ls}\mathbf{r}_{ls}^T - \mathbf{I}_{3\times3}\right) \tag{6}$$

$$\left.\frac{\partial \mathbf{a}_{IMU}}{\partial m}\right|_{\mathbf{x}=\mathbf{x}^*} = -\frac{uT_{max}}{m^2}\boldsymbol{\alpha} \tag{7}$$

where $\mathbf{r}_{ls}$ is the vector from the spacecraft to the Moon, r and r_{ls} are the Euclidian norm of $\mathbf{r}$ and $\mathbf{r_{ls}}$ respectively, μ and μ_l are the gravitational parameters of the Earth and Moon respectively, T_{max} is the spacecrafts maximum possible thrust, u is the thrust throttling factor, and $\boldsymbol{\alpha}$ is the direction of applied propulsive force.

PRELIMINARY AND EXPECTED RESULTS

Spacecraft used 127 kg of fuel. Small ammount makes mass hardly observable.

REFERENCES

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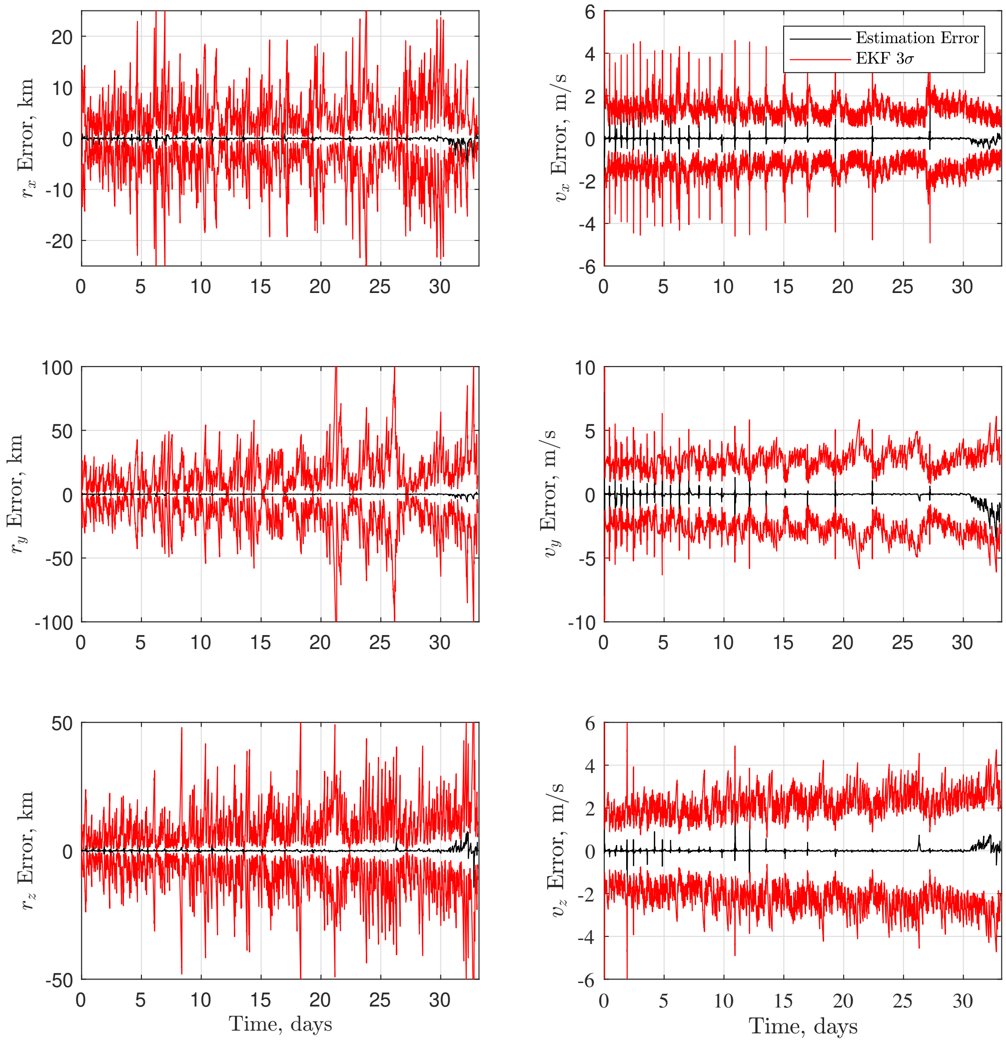

Figure 2. Position and Velocity Estimation Error and Uncertainty

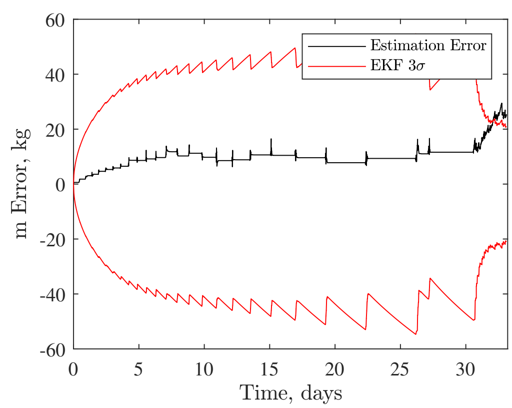

Figure 3. Mass Estimation Error and Uncertainty

[5] D. A. Vallado and W. D. McClain, *Fundamentals of Astrodynamics and Applications*. Microcosm Press, Hawthorne, CA, 2013.