# Figgie

Grant Stenger

May 21, 2019

## Goal

We want to know what the starting probability that a specified suit is the common suit given that we are dealt $n$ cards of that suit initially.

### Notation

$C =$ "Suit is common" (12 of 40 cards are of the common suit)

$n =$ "Initially dealt $n$ of this suit" (of the 10 cards dealt, $n$ are of this suit)

## Solution

#### Bayes' Theorem

$p(C \mid n) = \frac{p(n|C)p(C)}{p(n|C)p(C)+p(n|C')p(C')}$

**Task 1:** $p(n \mid C)$

Restated: Given that a suit is common, what's the probability that we would draw exactly $n$ of them initially?

$$p(n \mid C) = \frac{\binom{10}{n}\frac{12!}{(12-n)!}\frac{(40-12)!}{((40-12)-(10-n))!}}{\frac{40!}{10!}}$$

| $n$ | $p(\text{dealt } n \mid C \text{ "suit is common"})$ |
|---|---|
| 0 | 1.55% |
| 1 | 9.78% |
| 2 | 24.20% |
| 3 | 30.73% |
| 4 | 22.00% |
| 5 | 9.18% |
| 6 | 2.22% |
| 7 | 0.31% |
| 8 | 0.02% |
| 9 | 0.00% |
| 10 | 0.00% |

**Task 2:** $p(C) = 25\%$

**Task 3:** $p(n \mid C')$

Restated: Given that a suit is not the common suit, what's the probability that we would draw exactly $n$ of them initially?

$$p(n \mid C') = \frac{\binom{10}{n}\frac{10!}{(10-n)!}\frac{(40-10)!}{((40-10)-(10-n))!}}{\frac{40!}{10!}}$$

| $n$ | $p$(dealt $n$ \| $C$' "suit is not common") |
|---|---|
| 0 | 3.54% |
| 1 | 16.89% |
| 2 | 31.07% |
| 3 | 28.82% |
| 4 | 14.71% |
| 5 | 4.24% |
| 6 | 0.68% |
| 7 | 0.06% |
| 8 | 0.00% |
| 9 | 0.00% |
| 10 | 0.00% |

**Task 4:** $p(C') = 75\%$

**Putting It All Together**

$$p(C \mid n) = \frac{p(n|C)p(C)}{p(n|C)p(C)+p(n|C')p(C')} = \frac{\frac{\binom{10}{n}\frac{12!}{(12-n)!}\frac{(40-12)!}{((40-12)-(10-n))!}}{\frac{40!}{10!}}\frac{1}{4}}{\frac{\binom{10}{n}\frac{12!}{(12-n)!}\frac{(40-12)!}{((40-12)-(10-n))!}}{\frac{40!}{10!}}\frac{1}{4}+\frac{\binom{10}{n}\frac{10!}{(10-n)!}\frac{(40-10)!}{((40-10)-(10-n))!}}{\frac{40!}{10!}}\frac{3}{4}}$$

| $n$ | $p$($C$ is the common suit \| $n$ are dealt) |
|---|---|
| 0 | 12.5% |
| 1 | 16.1% |
| 2 | 20.6% |
| 3 | 26.2% |
| 4 | 33.3% |
| 5 | 41.9% |
| 6 | 52.3% |
| 7 | 63.9% |
| 8 | 76.1% |
| 9 | 87.3% |
| 10 | 95.7% |