General Physics 2

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CAS PY 212

Contents

Lecture 3: 34 ppl left

Tue 10 Sep 2024 17:00

- Discussion this week - counts
- Prelab - due sunday midnight
- homework - both sheet and mastering prob sheet, due friday
- start doing the reading first

If a neutrally charged object is placed in an electric field, a dipole can form (charges separate within the object), generating a force.

Positive charges feel force in direction of electric field, negative charges opposite.

Definition 0.1: Charge Density

Charge density is just charge per unit length.

$$\lambda \equiv \frac{q}{L}.$$

for charge q and length L.

Consider a rod of length l with a uniform charge Q. Take some differential length dl. It will have charge dq. At some point, it gives the electric field dE with direction $\hat{\mathbf{r}}$. In this case, we have

$$\mathbf{E} = \int k_e \frac{\mathrm{d}q}{r^2}\hat{\mathbf{r}}.$$

At the center, the y component of $\mathbf{E}$ is $\mathbf{0}$, and the horizontal component will be found with $dE_x = dE\cos\theta$ with $\lambda = \frac{Q}{L}$.

dE is the differential magnitude of $\mathbf{E}$ induced by some dq. And $\cos\theta = \frac{d}{r}$, where d is now the length from the center of the rod

Let r be the length from some differential charge dq to a charge, d the length from the center to the charge.

We get

$$dE_x = k_e \frac{dq}{x^2+y^2}\frac{x}{\sqrt{x^2+y^2}}.$$

EXPLANATION

$$dE = \frac{dq}{r^2}.$$

Ok so basically we are replacing q with λL which makes sense and then integrating on y where y spans the rod. r is going to represent all the vector magnitudes. Wow this is aids without calc 3.

so each differential, of the rod can be treated as a point charge. it exerts ad ifferential electic field dE_x, since y s cancel. Then this is just $dE\cos\theta$ obviously, and we know $E = k\frac{q}{r^2}$, so we just make q a dq. Then we have

$$dE_x = k_e\frac{dq}{r^2}\cos\theta.$$

To get $\cos\theta$, we just do horizontal over hypotenuse:

$$\frac{x}{\sqrt{x^2+y^2}}.$$

The hypotenuse in this case is the leg r going from dL (differential of rod) to our test charge. This is convenient, because then

$$r^2 = x^2 + y^2.$$

We then just integrate along this:

$$E_x = \int_{-\frac{L}{2}}^{\frac{L}{2}} k_e\frac{\mathrm{d}q}{x^2+y^2}\frac{x}{\sqrt{x^2+y^2}}.$$

HALF RING

Take a differential of length dL. Our test charge is in the middle of the half ring. The distance from the center point to the differential of length is $\sqrt{x^2+y^2}$. The x directions obv cancel (ring is top half), so we just do y. The y component is $dE_y = dE\sin\theta$. Then $dE = k_e\frac{dq}{x^2+y^2}$. It's the same god damn integral except in y, but we're integrating along a circle. We can use the arclength at some θ, which is the angle times the

Okay the distance to the length differential is a now and our angle is θ. We get

$$dE_y = -dE\sin\theta.$$

The minus is because pointing down in this example yknow fuck vectors right??? Take $\theta \in [0,\pi]$. We integrate along θ. We still know

$$dE_y = -dE\sin\theta = -k_e\frac{dq}{r^2}\sin\theta.$$

Take $s = a\theta$ (arclength). Then $dq = ad\theta$. The charge of our differential dq is just $\lambda a d\theta$. a is just the radius. Substitute:

$$dE_y = -k\frac{\lambda a\mathrm{d}\theta}{a^2}\sin\theta.$$

Note that $\lambda = \frac{Q}{\pi a}$. We then just have

$$E_y = \int -k_e\frac{\lambda a\mathrm{d}\theta}{a^2}.$$

RING OF CHARGE, SOME DISTANCE AWAY (x)

Integrate along the angle around the ring ϕ. The charge of each length differential will just be like

$$dE_x = k_e\frac{dq}{r^2}\cos\theta = k_e\frac{\lambda a\mathrm{d}\phi}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}}.$$

Since $\lambda = \frac{Q}{2a\pi}$, we finally have

$$E_x = \int_0^{2\pi} k_e\left(\frac{Q}{2a\pi}\right)\frac{a}{x^2+a^2}\frac{x}{\sqrt{x^2+a^2}}\,\mathrm{d}\phi.$$

We can always give the representation with λ as well.

Doing this integral for fun. Notice that fucking everything is constant so we have

$$E_x = 2\pi k_e \left(\frac{Q}{2\pi}\right) \frac{1}{x^2+a^2} \frac{x}{\sqrt{x^2+a^2}} = k_e \frac{Qx}{(x^2+a^2)^{\frac{3}{2}}}.$$

WHAT ABOUT A DISK, SAME PROBLEM BUT NOT A RING NOW?

Well, we just found the charge for a ring, so lets integrate along 0 to the radius of each ring charge. Except now since we are on a differential length, our Q becomes a dq, and we need to find this again. The **surface charge density** is called σ, so $dq = \sigma dA = \sigma(2\pi a da)$. So we write

$$E_x = \int_0^R \frac{x\sigma\,(2\pi r)}{(x^2+r^2)^{3/2}}\,\mathrm{d}r.$$

Replaced a with r. Gay naming scheme.

Lecture 4: Bro this prof went from attending a talk on BSM Higgs interactions to teaching undergrad E&M wtf

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Theorem 0.1 (Gauss' Law)

For a gaussian surface S with enclosed charge q_{enc},

$$\Phi = \iint_S \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{q_{\text{enc}}}{\epsilon_0}.$$

To use Gauss' law,

- Identify the symmetry of the situation
- Choose the gaussian surface over which Ebf is constant
- Calculate the charge encosed

For example, with an infinitely long rod and a finite cylinder around it, since $\mathbf{E}$ is parallel to $\hat{\mathbf{n}}$, we have the surface integral just being the integral of $\|\mathbf{E}\|$ over the surface. This is easy.

Calculate E for a hollow, spherical shell of radius a with charge Q evenly distributed over its surface. Take an exterior circle with radius $r > a$ as a gaussian surface. We expect $\mathbf{E}$ to be constant at all points, and parallel to $\hat{\mathbf{n}}$. We can then apply Gauss' law. The total interior charge is Q, so we have

$$\iint_S \mathbf{E} \cdot \mathrm{d}\mathbf{S} = \frac{Q}{\epsilon_0}.$$

It follows since $\mathbf{E}$ is constant in magnitude that

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0} \implies E = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q}{r^2}.$$

Recall here that SA for a sphere is $4\pi r^2$.

For inside, we give a gaussian surface $r < a$. However, the enclosed charge in this sphere is 0, meaning the electric field within the hollow sphere is 0. We can take r arbitrarily close to a.

HOLY SHIT THE FORMULA FOR ELECTRIC FORCE/FIELD DUE TO A POINT CHARGE IS DUE TO IT FALLING OFF AS THE FIELD SPREADS OUT ACROSST HE SURFACE AREA OF AN ARBITRARY SPHERE OF SPACE AT A DISTANCE OMG

For a solid sphere, we have for $r < a$ the charge will be ρV, where ρ is the charge density per unit volume and V is the volume within the gausian surface of radius r. Then we have $E(4\pi r^2) = \rho V = \rho\left(\frac{4}{3}\pi r^3\right)\frac{1}{\epsilon_0}$.

THICK INFINITE LINE OF CHARGE. Rod has radius R with charge density ρ. Shove a cylinder inside it with radius $r > R$. By Gauss' Law, $\Phi = \rho\pi R^2 \ell \frac{1}{\epsilon_0}$ for some length ℓ. We have

$$E(2\pi r\ell) = \rho\pi R^2 \ell \frac{1}{\epsilon_0} \implies E = \frac{\rho R^2}{2r\epsilon_0}.$$

Now take $r < R$. We have the enclosed charge $\rho V = \rho\pi r^2 \ell$. By gauss' law,

$$\Phi = \rho\pi r^2 \ell \frac{1}{\epsilon_0}.$$

It follows that

$$E(2\pi r\ell) = \frac{\rho\pi r^2 \ell}{\epsilon_0} \implies E = \frac{\rho r}{2\epsilon_0}.$$

NExt: Thick, infinite slab. Thickness of $2d$ and charge density ρ. For $x > d$, we have a cyllinder implanted into the plane, with curved side parallel to $\mathbf{E}$ and circular sides w/ rad r perpendicular, so we integrate on those sides. We have

$$\Phi = \frac{1}{\epsilon_0}\rho\pi r^2 2d.$$

By gauss' law,

$$E(2\pi r^2) = \frac{1}{\epsilon_0}\pi\rho r^2 2d \implies E = \frac{\rho d}{\epsilon_0}.$$

Now put the cylinder inside the slab. The enclosed charge is ρV, and the volume is $2x\pi r^2$, where x is distance from the center. We have

$$\Phi = \frac{\rho}{\epsilon_0} 2x\pi r^2.$$

By gauss' law,

$$E(2\pi r^2) = \frac{1}{\epsilon_0} 2\rho x\pi r^2 \implies E = \frac{x\rho}{\epsilon_0}.$$

0.1 Charges on Conductors

Within a solud conductor, $\mathbf{E} = \mathbf{0}$ always. The charges go to the surface of the solid, so it's like the charged surface we saw earlier. This is electrostatic, meaning "you wait a long time after you charge it".

Suppose now there is a hole within the conductor. We can put a Gaussian surface within the insulator, which musdt have 0 flux since it's inside the conductor, so the cavity has no charge.

If there is a point charge within the cavity, there is still 0 net charge and 0 flux, meaning the negative charges must accumulateon the inside of the cavity equal and opposite to q point charge.