6/4

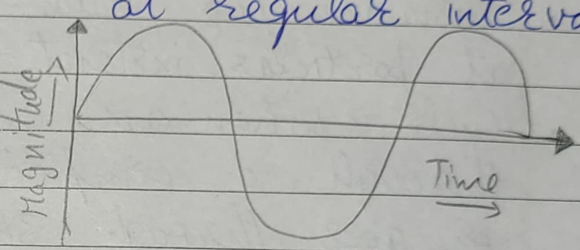
Unit → 2

AC Circuits

Alternating Voltage & Current

A voltage that varies in both magnitude and polarity with respect to time.

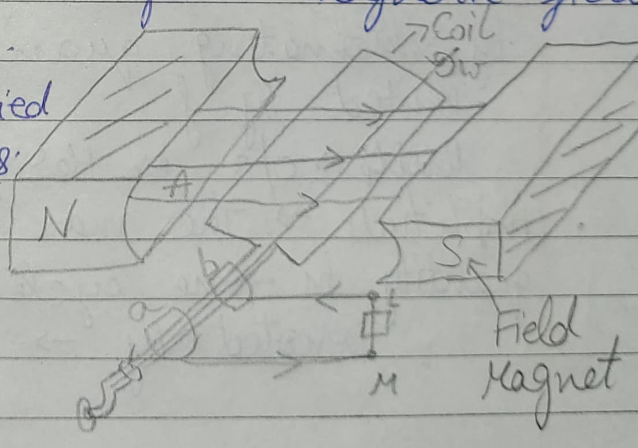
A type of electrical current; in which direction of flow of e switches back and forth at regular intervals.



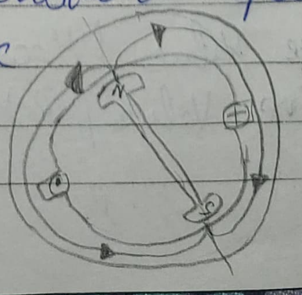

→ Generation of Alternating Voltage and Current.

An Alternating voltage can be generated either by;

(i) Rotating a coil in uniform magnetic field at constant speed.
It is generally applied in small a.c. generators.




(ii) by rotating a uniform magnetic field within a stationary coil at constant speed.
It is applied in large a.c generators.



Date / /
Page No.

In both the cases; magnetic field is cut by conductors and emf is induced in them.

Important Terms :-

- Wave form → The shape of curve obtained by plotting instantaneous values of alternating quantity (voltage or current) along y-axis and time or angle ($\theta = \omega t$) along x-axis.
- Instantaneous Value (e/i) ⇒ The value of an alternating at particular instant of time in the cycle.
- Cycle → When an alternating quantity goes through a complete set of +ve and -ve values or goes through 360 electrical degrees; it is said to have completed one cycle.
- Alternation → One half cycle
  180 Electrical Degrees.
- Time Period (T) → The time taken in sec to complete one cycle by alternating quantity.
- Frequency → Number of cycles made per sec by alternating quantity.
  Denoted by $f$
  Units → c/s or Hertz (Hz).
- Amplitude → The max value attained by alternating quantity in one cycle.
  Denoted by → $E_m$ ($V_m$) → Max value of voltage
  $I_m$ → Max value of current.

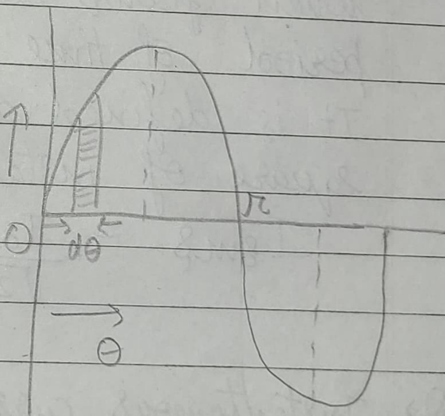
Peak value / Max value

⇒ Values Of Alternating Voltage and Current
1. Peak Value
2. Average Value / Mean Value
3. Effective Value / R.M.S Value.

1. **Peak Value** → The max value attained by alternating quantity during one cycle. This is also called max value / crest value / Amplitude.
A Sinusoidal Alternating quantity obtains max value at 90°.
Alternating voltage → $E_m$
Alternating Current → $I_m$
It is imp in case of testing dielectric strength of insulating materials.

2. **Average Value** → The arithmetic average of all instantaneous values considered of an alternating quantity over one cycle.
Average Value over a complete cycle is zero.
→ Average Value Of Sinusoidal current.
$i = I_m \sin \theta$

$\theta = \omega t \; ; \; I_m = I_0$




$I = I_0 \sin \omega t$

$I_{av} = \dfrac{I}{T} = \dfrac{q}{T}$

$dq = I \cdot dt$

$\int_0^{T/2} dq = \int_0^{T/2} I_0 \sin \omega t \, dt$

$q = I_0 \left[ \dfrac{-\cos \omega t}{\omega} \right]_0^{T/2}$

$q = \dfrac{-I_0}{2\pi/T} \left[ \cos \dfrac{2\pi t}{T} \right]_0^{T/2}$

$q = -\dfrac{I_0 T}{2\pi} \left[ \cos \pi - \cos 0 \right]$

$q = -\dfrac{I_0 T}{2\pi} \left[ -1 - 1 \right] = \dfrac{I_0 T}{\pi}$

Date / /
Page No.

$$I_{av} = \frac{q}{t}$$

$$= \frac{I_0 T}{\pi t} \qquad [t \rightarrow T/2]$$

$$\Rightarrow \frac{2 I_0 \cancel{T}}{\pi \cancel{T}}$$

$$I_{av} \Rightarrow \frac{2 I_0}{\pi}$$

$$\boxed{I_{av} = 0.637 I_0}$$

#

3. Effective / R.M.S Value → The steady current, which when flows through a resistor of known resistance for a given period of time than as a result the same quantity of heat is produced by alternating current when flows through same resistor for same period of time.
It is defined as square root of mean of square of instantaneous values.

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \boxed{0.707 I_0}$$

#

Q→ Instantaneous current is given by;
$i = 20 \sin 314t$. Find rms and average value of a.c.

Sol→ $I = I_0 \sin \omega t$

$I_0 = 20A$

$\omega = 314$

$$I_{rms} \Rightarrow I_0 (0.707)$$
$$= 20 (0.707)$$
$$= 14.14A$$

$$I_{av} = 0.637 I_0$$
$$= 0.637 (20)$$
$$= 12.74 A$$

Date / /
Page No.

Q→ Equation is $I = 42.42 \sin 628t$
Determine.
(i) Max value (ii) Frequency (iii) RMS value (iv) Average value (v) Form Factor.

Sol→ $I = I_0 \sin \omega t$

$I = 42.42 \sin 628t$

$I_0 = 42.42\ A$

$\omega = 628$ rad/sec

(i) Max value $= I_0 = \boxed{42.42\ A}$

(ii) Frequency (f)

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{628}{2 \times 3.14} \Rightarrow \boxed{314\ Hz}$$

(iii) RMS value $\Rightarrow I_0 (0.707)$

$\Rightarrow 42.42 \times 0.707$

$\Rightarrow \boxed{29.99\ A}$

(iv) Average value $\Rightarrow 0.637\ I_0$

$\Rightarrow 0.637\ (42.42)$

$\Rightarrow \boxed{27.02\ A}$

(v) Form factor $\Rightarrow \frac{I_{rms}}{I_{av}}$

$\Rightarrow \frac{29.99}{27.02}$

$\Rightarrow \boxed{1.10}$ #

⇒ Formulas

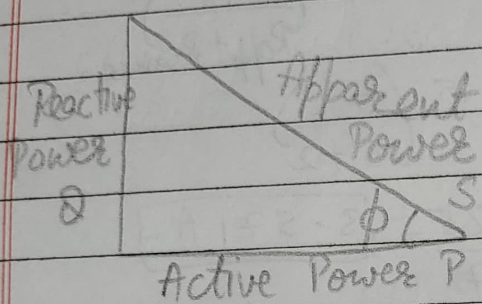
- $I_{rms} = \frac{I_{max}}{\sqrt{2}} \Rightarrow 0.707\ I_{max}$
- Form Factor $= \frac{I_{rms}}{I_{avg}}$ | $\frac{E_{rms}}{E_{avg}}$
- Peak Factor $= \frac{I_{max}}{I_{rms}}$
- Average value $= \frac{2 I_0}{\pi} \Rightarrow 0.637\ I_0$
- Frequency $\Rightarrow \omega = 2\pi f$

Unit → 2 [Continue...]

## AC Circuits

→ Introduction [Already Done]

### Power Triangle




① Active Power

$P = V_{rms} I_{rms} \cdot \cos\phi$ (watts)

② Apparent Power

$S = V_{rms} I_{rms}$ (volt - Amp) (VA)

③ Reactive Power

$Q = V_{rms} I_{rms} \sin\phi$ (Volt - Amp - Reactive) (VAR)

$\phi = 0^\circ$

$P = VI$

{ Using Ohm's law $V = I \cdot R$

$P = I^2 R$

Date / /
Page No.

→ Power Factor [Unit - less quantity].
- A factor on which entire true power of circuit depends.
- It is the ratio of active power to apparent power.
- It is defined as the cosine of the phase difference angle b/w supply voltage and current.

Methods to calculate Power Factor.

① $\cos \phi$

② $\cos \phi = \frac{R}{Z}$

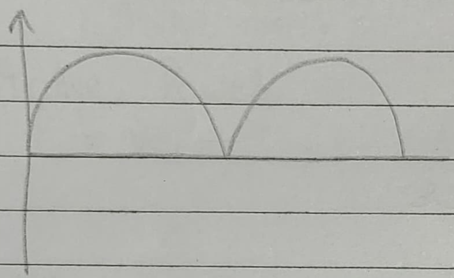
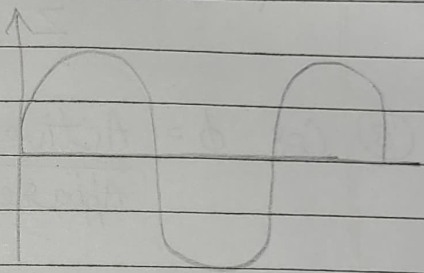
③ $\cos \phi = \frac{\text{Active Power}}{\text{Apparent Power}}$

→ Active Power
- It is always positive
- It does not changes its direction
- Power flow is always from source to load.
- Transfers real energy, does work.
- Measured in watts (w).
- Denoted by 'P'.

→ Apparent Power
- Power that is transferred by conductors
- Measured in volt - Amp (V-A).
- Denoted by 'S'.

## → Reactive Power

- It does not do any work.
- Result of current transferring, no energy.
- Measured in VARs.
- Imaginary part of apparent Power.

| | Active Power | Reactive Power |
|---|---|---|
| 1. | Real power which is actually utilised in the circuit. | The power which flows back and forth like a pendulum without doing any useful work in circuit. |
| 2. |  |  |
| 3. | Does not change it's direction and remains positive all the time. | Changes it's direction periodically and it is positive as well as negative. |
| 4. | Measured using wattmeter. | Measured using VAR meter. |
| 5. | It produces heat, mechanical power, light, etc. | It only represents power that oscillates back and forth. |
| 6. | It is max in purely ~~reactive~~ resistive circuit, and 0 in purely reactive circuit. | It is max in purely reactive circuit, and 0 in purely resistive circuit. |
| 7. | It's a function of circuits dissipative elements, usually resistance. | It's a function of circuits reactance. |

## Formula

1. Power Factor $(\cos\phi) = \dfrac{\text{Active Power}}{\text{Apparent Power}} = \dfrac{KW}{KVA}$

2. $P = V I \cos\phi$

   $I = \dfrac{P}{V\cos\phi}$ ; $\cos\phi$ = Power Factor

3. For $= X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

   For $= X_L = \omega L = 2\pi f L$

4. $\cos\phi = \dfrac{R}{Z} = \dfrac{\text{Resistance}}{\text{Impedance}}$

5. $Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (X_L - X_C)^2}$

6. $I = I_m \sin\omega t$
7. $P$ (Active Power) $= VI\cos\theta$
8. $Q$ (Reactive Power) $= VI\sin\theta$
9. (Apparent Power) $S = \sqrt{P^2 + Q^2}$
10. $R = \dfrac{V}{I}$

    $Z$ = Total Resistance
    $= R + X_L + X_C$

    $Z = \dfrac{V}{I}$ ; unit $(\Omega)$.

11. $\tan\phi = \dfrac{X_L - X_C}{R}$

    $\phi = \tan^{-1} \dfrac{X_L - X_C}{R}$

Date / /
Page No.

(12) $P = VI\cos\phi$

$VI = \frac{P}{\cos\phi}$

$\boxed{KVA = \frac{KW}{\cos\phi}}$

Numericals

Q→ A series circuit consists of a 250 Ω non-inductive resistor, a 3-H inductor of negligible resistance, a 8.5 μF capacitor of negligible resistance. If voltage is 240V at 50 Hz. Calculate (i) circuit current (ii) circuit P.F.

Sol→ $R = 250\ \Omega$ ; $L = 3H$ ; $C = 8.5\ \mu F$ ; $V = 240V$

$f = 50\ Hz$

(i). $I = \frac{V}{R}$

$= \frac{V}{Z}$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(250)^2 + (942 - 374.67)^2}$

$= 619.97\ \Omega$

$I = \frac{240}{619.97} = \boxed{0.387\ A}$ #

(ii) Power Factor => $\cos\phi$

$\phi = \tan^{-1}\frac{X_L - X_C}{R}$

$= \tan^{-1}\frac{942 - 374.67}{250}$

$= 66.22°$

P.F $= \cos\phi = \cos 66.22°$

$= \boxed{0.403}$ (lagging).

Date / /
Page No.

Q→ If an inductive coil is connected to a DC supply at 240 V, the current in coil is 15 A. When same coil is connected to an ac supply at 230 V, 50 Hz, the current is 14 A. (a). Calculate;
(i). resistance, (ii) Impedance, (iii) Reactance,
(iv). Inductance (v). P.F of coil
(b). If supply frequency is to be altered to 60 Hz at 240 V; find current in circuit.

Sol. (a) (i) $R = \frac{V}{I} = \frac{240}{15} = \boxed{16\,\Omega}$

(ii). $Z_L = \frac{V}{I} = \frac{240}{14} = \boxed{17.14\,\Omega}$

(iii). Reactance $X_L = \sqrt{Z_L^2 - R^2}$

$= \sqrt{(17.14)^2 - (16)^2}$
$= \boxed{6.14\,\Omega}$

(iv). Inductance

$L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = \frac{6.14}{2 \times 3.14 \times 50} = \boxed{19.57\text{ mH}}$

(v) P.F

$\cos\phi = \frac{R}{Z_L} = \frac{16}{17.14} = \boxed{0.933}$

(b). $X_L = 2\pi f L$
$= 2 \times 3.14 \times 60 \times 19.55 \times 10^{-3}$
$= 7.366\,\Omega$

$Z_L = \sqrt{R^2 + X_L^2}$
$= \sqrt{16^2 + (7.366)^2} = 17.614\,\Omega$

$I = \frac{V}{Z_L} = \frac{240}{17.614} = \boxed{13.62\text{ A}}$ #.