1. Free Electron Theory

why → To understand electrical, thermal, and magnetic properties of solids.
→ It is applicable for both metal & non metal.

Classical (Assumptions)

- Proposed by Drude and Lorentz in 1900.
- The valence e^- are free to move about whole volume of metal.
- Free e^- move in random direction and collide with either +ve ions fixed to lattice or the free e^-. All collissions are elastic in nature that means no loss of energy.
- Momentum of free e^- obeys the laws of classical kinetic Theory of gases.
- Electron velocities in metal obey Maxewell Boltzmann distribution.
- When electric feild is applied e^- move in direction opposite of the feild.
- Mutual repulsion among the e^- are ignored so that they move in all direction with all possible velocities
- In the absence of the field the energy associated with an e^- at temperature T is $\frac{3}{2}KT$.

$$\frac{3}{2}kT = \frac{1}{2}mV_{th}^2 \quad V_{th} \rightarrow \text{Thermal Velocity}$$

k = Boltzmann Constant

Success of free e^-

- Verifies ohm's law
- Explains electrical conductivity
- " Thermal conductivity
- Derives Wiedemann - Franz law.
- Explains optical property of metals.

Drawbacks

- Not explain photoelectric effect, compton Effect and black body radiation.
- Electrical conductivity of semiconductor and insulator not explains.
- Fails to explain specific heat & heat capacity
- Fails to explain superconductivity.
- " " " ferromagnetism.

Quantum free e^- theory:

- Sommerfield developed in 1928.
- e^- obey quantum laws.
- free e^- have constant potential, means even if you change the current the the voltage remains constant.

→

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2. Density of states:

Density: Per unit volume
State: Allowed solution which satisfy boundary condition.
The density of states can be represented in form of $\rho(k)$ and $\rho(E)$

$\rho(k)dk$ is the number of states between k and $k+dk$ per unit volume of metal
$\rho(E)dE$ is the number of states between E and $E+dE$ per unit volume.
$\Rightarrow$ For every value of k there is a value of E

The number of states between $k+dk$ is same as $E+dE$

So $\rho(E)dE = \rho(k)dk$

Now according to Bloch Theorem

$$\psi_k = \underbrace{U_k(r)}_{\text{Periodic}} \underbrace{e^{i\vec{k}\cdot\vec{r}}}_{\text{plane wave}}$$

we know probability of finding e^- is $|\psi_k|^2$

Therefore for $2\vec{k}\cdot\vec{r} = N.2\pi$

Here $k = ik_x + jk_y + kk_z$

Now in all 3 dimension

$2k_x l_x = m.2\pi \Rightarrow k_x = \pi/l_x$

$2k_y l_y = p.2\pi \Rightarrow k_y = \pi/l_y$

$2k_z l_z = q.2\pi \Rightarrow k_z = \pi/l_z$

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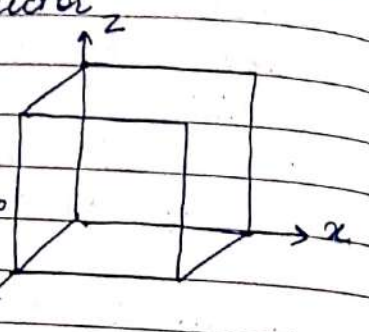
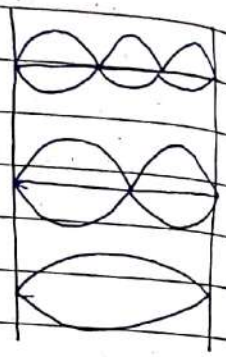
Now consider a semiconductor



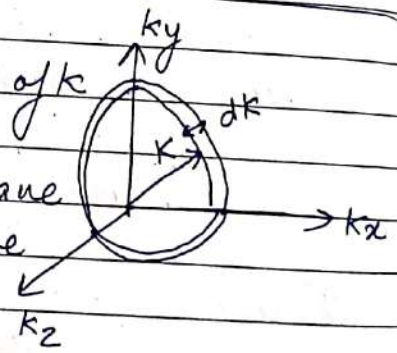
⇒ E^- is inside a semiconductor not outside

- Probability of finding e^- is zero at boundary.
- So ψ should be a standing wave means at corner value is zero



→ So standing boundary wave condition is applied.

→ Now in k space any value of k is possible



- But because of standing wave boundary condition only +ve value are possible

- Volume of one allowed state in k space

$$k_x k_y k_z = \frac{\pi}{l_x}\frac{\pi}{l_y}\frac{\pi}{l_z}$$

Volume of $\frac{1}{8}^{th}$ of Sphere ⇒ $\frac{1}{8}4\pi k^2 dk \times 2$

$\times 2$ → because it is momentum space So both spin.

Total no. of states = $\dfrac{\frac{1}{8}\times 4\pi k^2 dk \times 2}{\text{Volume of one state}}$

$\rho(k)\,dk$

$\rho(k$

We know $\rho(E)$

Now from $E = E_c$

$\frac{dE}{dk}$

Putting $\rho(E)$

from E

$(E -$

$\rho(E)$

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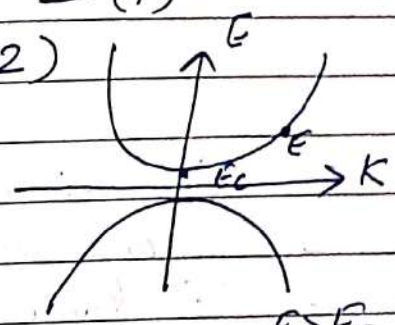
$$\rho(k)\{dk\} = \frac{1}{\frac{\pi}{l_x}\frac{\pi}{l_y}\frac{\pi}{l_z}}\left(4\pi k^2 dk\right)\times 2 \quad (l_x l_y l_z)$$

$$\rho(k)dk = \frac{k^2}{\pi^2}dk$$

We know $\rho(E)dE = \rho(k)\,dk$ —(1)

$\rho(E) = \rho(k)/dE/dk$ —(2)



$E > E_c$

Now from parabolic approx

$$E = E_c + \underbrace{\frac{\hbar^2 k^2}{2m_c}}_{\text{Kinetic Energy}} \quad -(3)$$

$$\frac{dE}{dk} = \frac{\hbar^2}{2m_c}(2k) \quad -(4)$$

Putting value in eq^n 2

$$\rho(E) = \frac{k^2}{\pi^2}\frac{2m_c}{\hbar^2(2k)} = \frac{k}{2\pi^2}\left(\frac{2m_c}{\hbar^2}\right) \quad -(5)$$

from eq (3)

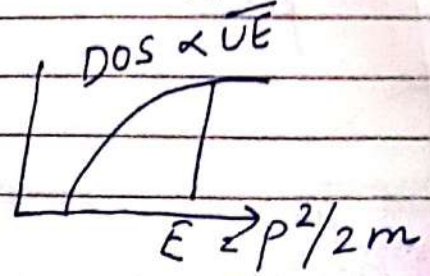
$$E = E_c + \frac{\hbar^2 k^2}{2m_c}$$

$$(E - E_c)\frac{2m_c}{\hbar^2} = k^2$$

$$k = \left(\frac{2m_c}{\hbar^2}\right)^{\frac{1}{2}}(E - E_c)^{\frac{1}{2}}$$

$$\rho(E) = \frac{1}{2\pi^2}\frac{2m_c}{\hbar^2}\times\left(\frac{2m_c}{\hbar^2}\right)^{\frac{1}{2}}(E - E_c)^{\frac{1}{2}}$$

$$\rho(E) = \frac{1}{2\pi^2}\left(\frac{2m_c}{\hbar^3}\right)^{3/2}(E - E_c)^{\frac{1}{2}}$$



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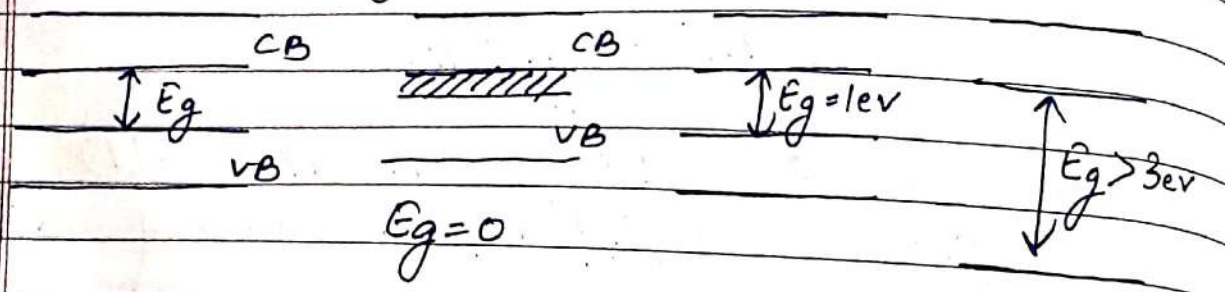
3. E-K Diagram and Direct Indirect



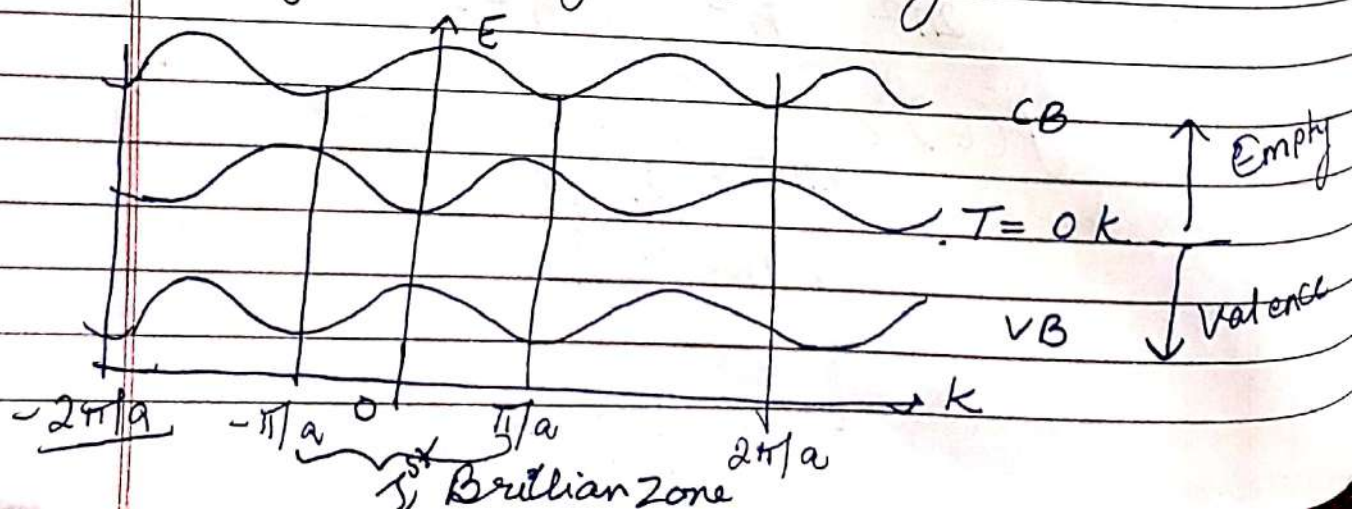
EK Diagrams

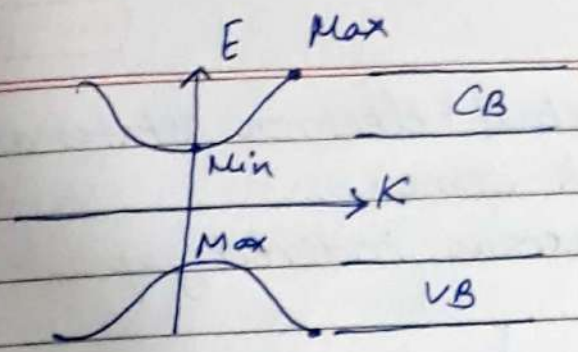
- Shows Band gap energy only
- To know more electrical and optical properties of semiconductor we have to study this
- Shows characteristics of particular semiconductor
- " relation b/w energy and momentum of available states for electron
- K → momentum, E = energy.
  K is a wave ~~no~~ vector which is a way to represent momentum
- It is obtained by solving Scrodinger Equation.

$$\psi = U_K e^{iKx} \quad \text{– (Bloch)}$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V) = 0$$

Value of ψ changes with k.
Also E & V change
Solving schrodinger eqn we get





K = Propagation constant / wave vector

$$K = \frac{2\pi}{\lambda}$$

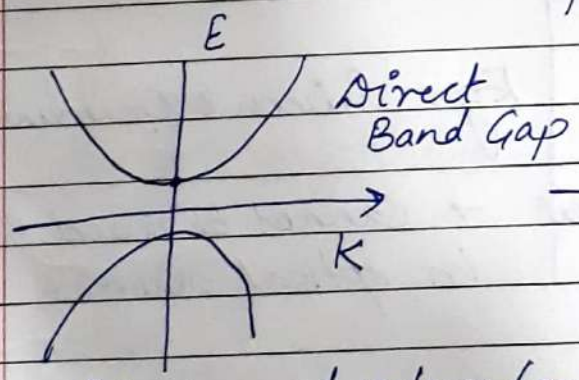
According to De Broglie

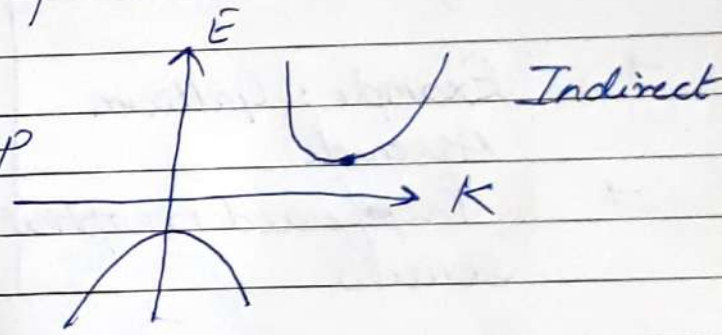
$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda} \times \frac{2\pi}{2\pi}$$

$$\frac{h}{2\pi} = \hbar \Rightarrow \boxed{p = \hbar k}$$

p = momentum.





Maxima of valence band align with minima of conduction band at given value of K.

Significance of E-k Diagram

- Helps in theoritical, experimental & technical application
- Indicates band gap (E_g)
- Shows Electron (hole) mobility
- Explains Electron hole effective mass.

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- Indicates how actual electron states are equally spaced in k-space.
- Shows direct versus indirect gap.

DBG	IBG
→ A direct recombination takes place with release of energy equal to energy difference between recombing particle.	- Due to relative difference in momentum, first momenta align then recombination occurs with release of energy.
→ Efficiency factor is higher.	Lower
→ Example : Gallium Arsenide	Eg → Silicon & Germanium
→ Preferred in optical sources	→ cannot be used in optical sources.

4. Kronig
- Why
- Free e
of solid
in cons
- It explai
- Failed
- Bloch p
in cons
periodic
- Kronig
square
- Explain

V=m

V=0

Solving

$V_0 = 0$

$V(r) \propto \frac{1}{r^2}$

I

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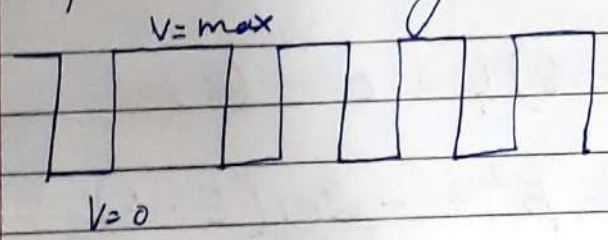
4. Kronig Penney Model:

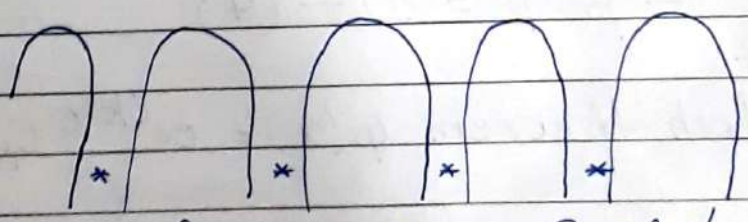
- Why this Model?
- Free e^- theory was able to explain properties of solid. The basic assumption was e^- moves in constant potential.
- It explained electrical & thermal emission of solid
- Failed to explain classification of Solids.
- Bloch proposed e^- inside the material are not in constant potential but they are moving in periodic potential well.
- Kronig and Penny define the periodic 1-d square potential well.
- Explained mainly Band in solids & classification



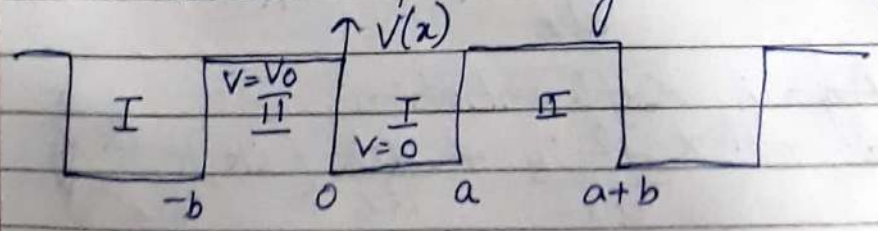
Solving Kronig Penny

$V_0 = 0$

$V(r) \propto \frac{1}{r^2}$



Potential function of one-D single crystal



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According to Bloch Theorem periodically varying potential energy function

$$\psi(x) = U_k(x)\, e^{ikx}$$

The schrodinger wave eqn

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

For I Region (From 0 to a) $V = 0$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \quad -(1)$$

For II - Region $(-b < x < 0)$ $V = V_0$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi = 0 \quad -(2)$$

$$\alpha^2 = \frac{2mE}{\hbar^2} \quad \& \quad \beta^2 = -\frac{2m}{\hbar^2}(E-V_0)$$

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \quad -(3)$$

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \quad -(4)$$

Bloch theorem $\psi(x) = e^{ikx}\, U_k(x) \quad -(5)$

$$\frac{d\psi_x}{dx} = e^{ikx}\frac{d}{dx}U_k + U_k\, ik\, e^{ikx}$$

Again Differentiation.

$$\frac{d^2\psi_x}{dx^2} = e^{ikx}\frac{d^2U_k}{dx^2} + \frac{dU_k}{dx}(ik)e^{ikx} + \frac{dU_k}{dx}(ik)e^{ikx} + U_k(ik)^2 e^{ikx}$$

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$$\frac{d^2\psi}{dx^2} = e^{ikx}\left(\frac{d^2u_k}{dx^2} + \frac{2ik\,du_k}{dx} - u_k k^2\right) \quad -(6)$$

on putting 5 and 6 in (3)

$$e^{ikx}\left(\frac{d^2u_k}{dx^2} + 2ik\frac{du_k}{dx} - u_k k^2\right) + \alpha^2 e^{ikx} u_k = 0$$

$$\frac{d^2u_k}{dx^2} + 2ik\frac{du}{dx} + (\alpha^2 - k^2)u_k = 0 \quad -(7)$$

$$\frac{d^2u_k}{dx^2} + 2ik\frac{du_k}{dx} - (\beta^2 + k^2)u_k = 0 \quad -(8)$$

From 7 & 8 the general solution will be

$$u_1(x) = Ae^{i(\alpha-k)x} + Be^{-i(\alpha+k)x} \quad -(9)$$

$$u_2(x) = Ce^{i(\beta-k)x} + De^{-i(\beta+k)x} \quad -(10)$$

Considering boundary condition $x = 0$

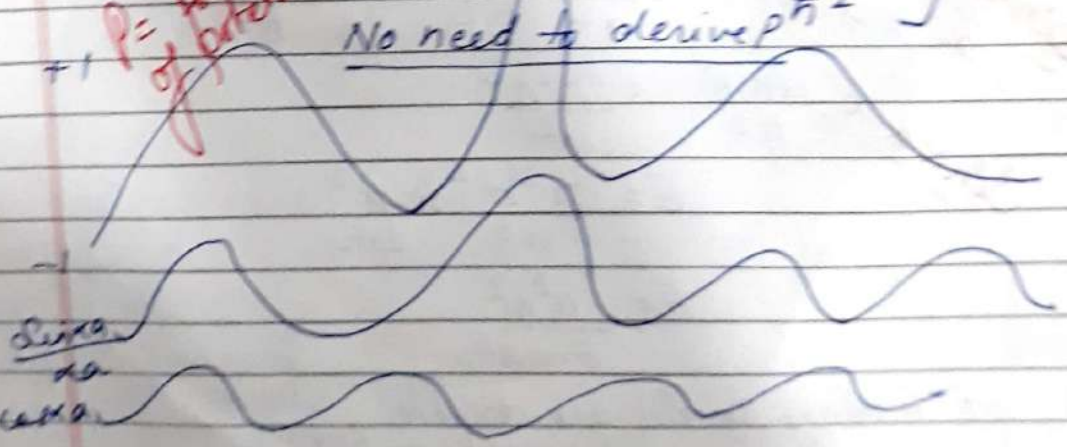
$$u_1(0) = u_2(0)$$

$$A + B = C + D$$

By solving $P\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka$

$$\left[P = \frac{mV_0 ba}{\hbar^2}\right]$$

No need to derive P



Divide

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Case I

$V_0 = 0$ and $P = 0$

Means no potential Barrier

$\cos \alpha a = \cos ka$

$\alpha = k$

$\alpha^2 = k^2$

$$\frac{2mE}{\hbar^2} = \frac{4\pi^2}{\lambda^2}$$

$$E = \frac{4\pi^2}{\lambda^2} \times \frac{\hbar^2}{4\pi^2 2m} \quad \left[\because \hbar = \frac{h}{2\pi}\right]$$

We know

$\lambda = h/mv$ (De Broglie)

$$E = \frac{h^2}{2m} \times \frac{m^2v^2}{h^2}$$

$$E = \frac{1}{2}mv^2 \quad E = \frac{p^2}{2m}$$

So here e^- is free and moving, it is condition of conductor.

Case II $P \to \infty$ means $V_0 = \infty$

$\sin \alpha a = 0$

Divide

$$\frac{P\sin\alpha a + \cos\alpha a}{\alpha a} = \frac{P\cos ka}{P}$$

$\alpha a = n\pi \Rightarrow \alpha = n\pi/a$

$$\alpha^2 = \frac{n^2\pi^2}{a^2} \quad \left[\alpha^2 = \frac{2mE}{\hbar^2}\right]$$

$\sin \alpha a = 0$

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{n^2\pi^2}{a^2} \times \frac{\hbar^2}{2m}$$

$$E = \frac{n^2h^2}{8ma^2}$$

→ Energy levels are discrete

Similar to Potential box, NO KG

5. Probability of Occupation & Fermi level.

we know density of states in conduction Band

$$\rho(E) = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^3}\right)^{3/2}(E-E_C)^{\frac{1}{2}} \quad \text{where } E > E_C$$

we know that carrier concentration of e^-

$$n = \int_{E_C}^{\infty} n(E)\,dE$$ where $n(E)$ is carrier density

$$n(E) = \int \rho(E)\,F(E)\,dE$$

$k = 1.38 \times 10^{-23}\,J/K$

where $\rho(E)$ = density of states

$F(E)$ = Probability of occupation.

⇒ The probability of occupation is given by Fermi Function $F(E)$:

$$F(E) = \frac{1}{e^{(E-E_f)/KT}+1}$$

$k \rightarrow$ Boltzmann constant

It is also called Fermi dirac distribution fn.

$F(E)$ denotes distribution of e^- in energy bands that is C.B and VB.

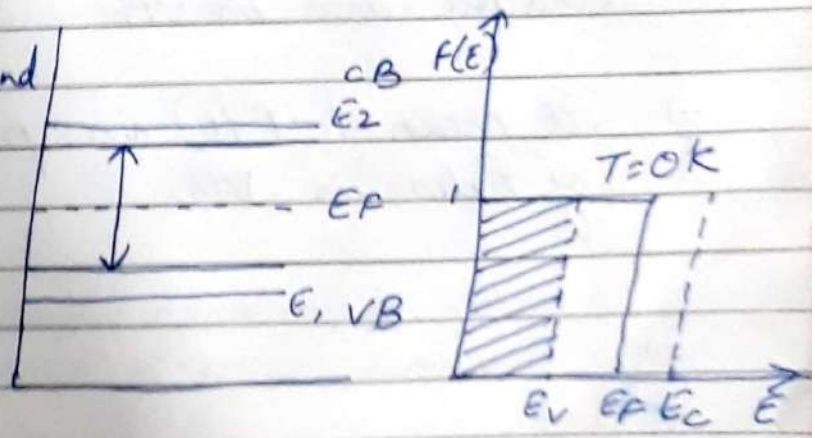
E_f = fermi Energy is a constant

E_f = Fermi energy which is maximum energy occupied by e^- at 0K.

Case I In valence Band

$T = 0K$ & $E_1 < E_f$

on putting the value in $F(E)$



$$F(E) = \frac{1}{e^{-\infty}+1} = 1$$

This indicates probability of finding e^- in valence band at 0K is 1

Case II : $T = 0K$, $E = E_g > E_f$

$$F(E) = \frac{1}{e^{\infty}+1} = \frac{1}{\infty} = 0$$

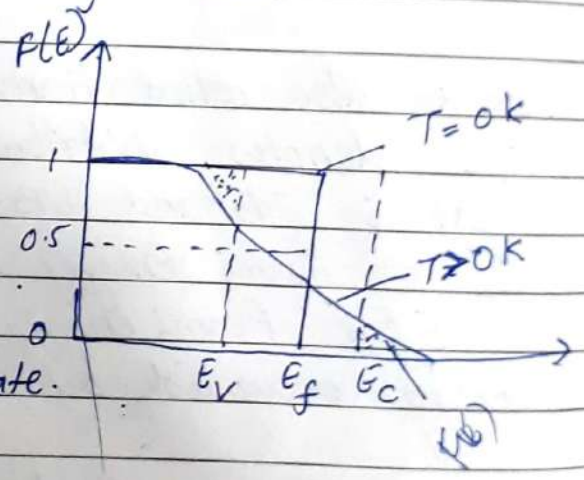
This shows probability of finding e^- in conduction band is zero at $T = 0K$.

Case III $T > 0K$; $E = E_f$

$$F(E) = \frac{1}{e^{0/kT}+1} \Rightarrow \frac{1}{2} = 0.5$$

It shows probability of finding an e^- at any temp $T > 0K$ at $E = E_f = 0.5$

F(E) is probability of find e^- in V.B



⇒ 1 − F(E) is probability of finding the vacant state. Shaded area in VB.

⇒ It mean 1 − F(E) is probability of occupation of holes in VB.

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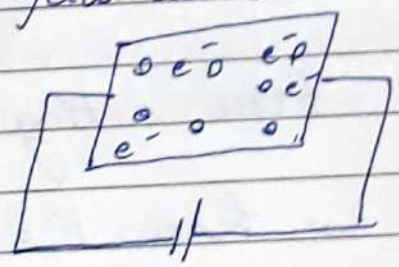
Ch.6 Effective Mass.

It is the mass exhibited by an e^- when inside the semiconductor

⇒ e^- moving in the semiconductor is moving under the influence of atoms and other e^-. According to ohm's law.

⇒ Due to that there is an electrostatic feild acting on e^- known as crystal feild. This is called internal electric feild.

⇒ On applying an external feild as shown



$F = F_{ext} + F_{internal}$
which is total force on e^-

⇒ If we define mass of an e^- as effective mass than e^- can be treated as free e^-. It is represented by m^*.

⇒ So you need not consider internal electric field.

⇒ When effective mass is small(e^-) the mobility of e^- is higher. When effective mass is higher the mobility is smaller.

$$m^*_{e^-} < m^*_H \Rightarrow \mu_e > \mu_{h^+}$$

We know that De Broglie equation is when e^- moves than a group of wave associate with it.

Group Velocity $V_g = \frac{d\omega}{dk}$ —(1)

$\omega \rightarrow$ angular velocity

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now we know $E = h\nu = \frac{h\omega}{2\pi} = \hbar\omega$ — (2)

From eq (2) $\omega = E/\hbar$, putting in (1)

$$v_g = \frac{1}{\hbar}\frac{dE}{dk}$$

on differentiating with respect to t

$$a = \frac{dv_g}{dt} = \frac{1}{\hbar}\frac{d^2E}{dk^2}\frac{dk}{dt} \quad — (3)$$

we know that By debroglie $p = \frac{h}{\lambda} \times \frac{2\pi}{2\pi}$

$$p \Rightarrow \hbar k$$

Force acting on e⁻ $F = \frac{dp}{dt} = \hbar\frac{dk}{dt}$ (4)

we know $F = m^* a$

$$m^* = \frac{F}{a} \qquad m^* = \frac{\hbar\frac{dk}{dt}}{\frac{1}{\hbar}\frac{d^2E}{dk^2}\cdot\frac{dk}{dt}}$$

$$m^* = \frac{1}{\hbar^2}\left(\frac{d^2E}{dk^2}\right)^{-1}$$

Fermi level: defined as reference level that gives probability of occupancy of states

Maximum energy that an electron in metal has at absolute zero temperature is called fermi level.

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5 Optical Loss and Gain

General physics Terms.

① Planck's Theory:

→ Energy is not radiated or emitted continuosly. It is emitted in small portions called quanta.

→ Radiation when in form of light, each particle is called photon

→ 1 quantum of energy is called photon.

→ $E = h\nu$

② Electromagnetic Radiation: Phenomenon by which electromagnetic waves travel through space.

- Does not need any material medium.
- Both components are perpendicular planes.

③ De Broglie Equation:

- Used to define wave properties of matter.
- Describes wave nature of e^-

EMR has dual nature

- Having momentum
- wave nature

Very low mass particles moving at speed less than light behave like particle and wave.

$$\lambda = \frac{h}{mv}$$

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4. Bohr's Hypothesis:
Explained that electrons move in fixed orbitals (shells) and not anywhere in between.
and also explained each shell has fixed energy.

Angular momentum of an e^- revolving around nucleus as quantized
Angular mom. of $e^- = mvr$

$$mvr = \frac{nh}{2\pi}$$

$$\lambda = \frac{h}{mv}$$

$$mv = \frac{h}{\lambda}$$

$$\frac{hr}{\lambda} = \frac{nh}{2\pi} \quad = 2\pi r = n\lambda$$

$$\text{or } \frac{1}{\lambda} = \frac{n}{2\pi r}$$

$$mv = \frac{nh}{2\pi r} \Rightarrow mvr = \frac{nh}{2\pi}$$

5. Dielectric constant:

Dielectric is insulating material or very poor conductor of current.
Dielectric constant is defined as insulating material that can store some charge when placed b/w 2 metallic plates.

6. Coulomb's
F

7. photoel
EMR

8. Schro
d

9. Electric
q_1 q_2
from
F = G

E

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in fixed orbitals
between.
has fixed

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6. Coulomb's Law:

$$F \propto \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_r\epsilon_0}\frac{q_1 q_2}{d^2}$$

$$F = k\frac{q_1 q_2}{r^2}$$

7. photoelectric effect: Emission of e^- when EMR such as light hits a material.

8. Schrodinger Eqn

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

9. Electric Field: If we have several point charges $q_1\ q_2 \dots q_n$ at distances $r_1\ r_2 \dots r_n$ from Q, the total force on Q =

$$F = F_1 + F_2 + \dots = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 Q}{r_1^2}\hat{r}_1 + \frac{q_2 Q}{r_2^2}\hat{r}_2 + \dots\right)$$

$$F = QE$$

$$E(r) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

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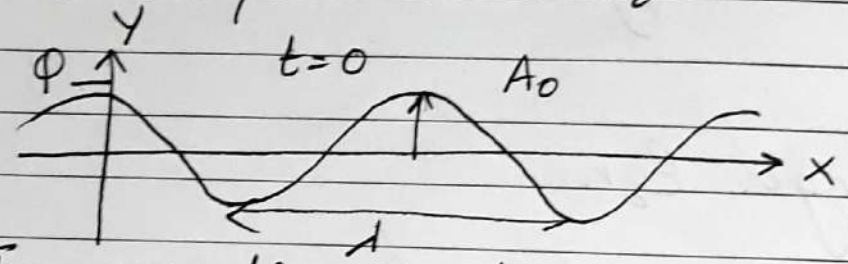
Schrodinger wave Eqn.

It is a mathematical expression describing the energy and position of e^- in space and time, taking into account the matter wave nature of e^- inside an atom

1. Classical plane wave eqn.



The one dimensional wave eqn

$$\nabla^2 \psi = \frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2}$$

The amplitude(y) of progressive sinusoidal wave

$$y = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi t}{T} + \phi\right),$$

$\phi \rightarrow$ phase difference of wave

$t =$ time in seconds.

$$y = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi T}{t}\right) \text{ (no phase diff)}$$

$$= A\cos\left(\frac{2\pi x}{\lambda} - 2\pi\nu t\right), \quad \nu = \frac{1}{T}$$

in general $y = e^{i\left(\frac{2\pi x}{\lambda} - 2\pi\nu t\right)}$

$$= e^{i\left(2\pi\nu t - \frac{2\pi x}{\lambda}\right)}$$

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2. Broglie Hypothesis of Matter waves:

$$E = h\nu = 2\pi\hbar\nu$$

$$\hbar = \frac{h}{2\pi}$$

$$\nu = \frac{E}{2\pi\hbar}$$

According to de Broglie $\lambda = \frac{h}{mv}$

$$\frac{2\pi\hbar}{mv} = \frac{2\pi\hbar}{p}$$

Amplitude = wavefunction = ψ

$$= e^{-i\left(2\pi\nu t - \frac{2\pi x}{\lambda}\right)}$$

Ohm's law: Current through a conductor between 2 points is directly proportional to voltage across 2 points

$$V = IR$$

Wiedemann Franz law:
States that ratio of electronic contribution of thermal conductivity K to electrical conductivity σ of metal is proportional to temperature based on semi classical treatment of electron gas.

$$K\sigma = LT \quad (L = \text{Lorentz number})$$

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Classical Kinetic Theory of Gases:

PN Junction

When P type semiconductor is suitably joined to n-type semiconductor, contact surface is called pn junction.

Properties of PN Junction:

- The free e^- near the junction in n region begin to diffuse across the junction into p region where they combine with holes near the junction. The result is that n region loses free e^- as they diffuse into junction. This creates a layer of pentavalent charges (positive) near junction. As e^- move across the junction, the p region loses holes as e^- and holes combine. The result is that there is a region of negative charges (trivalent) near the junction. These 2 layers of +ve and negative charges creates depletion region. The depletion layer is formed very quickly and is very thin.

Once pn junction is formed and depletion layer is created, the diffussion of free e^- stops. Depletion region acts as a barrier to further movement of e^-. The positive and negative charges set up electric field.

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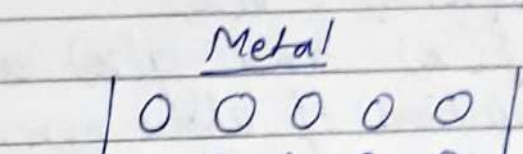
Bloch Theorem:

In a metals e^- have variable potential which is periodic in nature.
Bloch helped to find wave function of e^- in periodic potential.

Metal

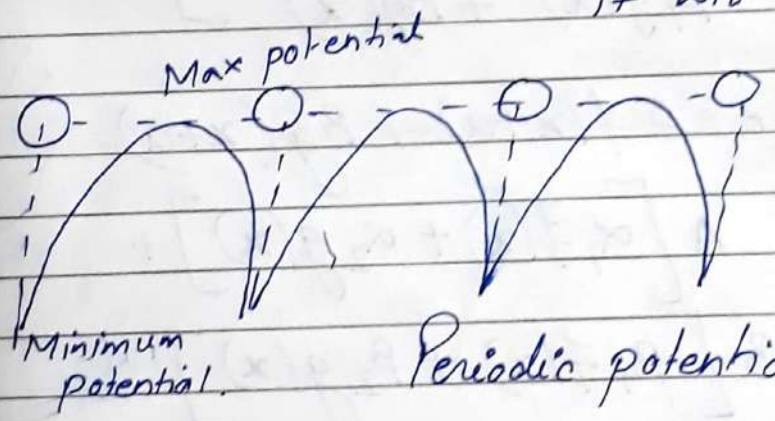


Periodic potential

$$U = \frac{1}{4\pi\epsilon_0}\frac{Ze(-e)}{r}$$

$$U = -\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}$$

$U \propto \frac{1}{r}$ when r will increa U decrease but because of negative sign it will increase.



Periodic potential.

Statement of Bloch Theorem:

If an e^- is in periodic potential then potential wavefunction $= \psi(x) = \underbrace{e^{\pm ikx}}_{\text{plane wave eqn}} \underbrace{U_k(x)}_{\text{Bloch fn}}$

$$U_k(x+a) = U_k(x)$$

$$\psi(x+a) = e^{\pm ikx} e^{\pm ika} U_k(x+a)$$

$$\psi(x+a) = e^{\pm ika} \psi(x)$$

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Proof

Schrodinger Eqn of e^- moving in 1-D periodic potential

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left[E - V(x)\right]\psi = 0 \quad -(1)$$

$$V(x+a) = V(x)$$

Let $f(x)$ and $g(x)$ are 2 independent Solution of above eqn.

$$\psi(x) = Af(x) + Bg(x) \quad -(2)$$

$f(x+a)$ and $g(x+a)$ are independent solution

$$\left.\begin{aligned} f(x+a) &= \alpha_1 f(x) + \alpha_2 g(x) \\ g(x+a) &= \beta_1 f(x) + \beta_2 g(x) \end{aligned}\right] (3)$$

$$\psi(x+a) = Af(x+a) + Bg(x+a)$$

$$\psi(x+a) = A\left[\alpha_1 f(x) + \alpha_2 g(x)\right] + B\left[\beta_1 f(x) + \beta_2 g(x)\right]$$

$$\psi(x+a) = (A\alpha_1 + B\beta_1) f(x) + (A\alpha_2 + B\beta_2) g(x)$$

Choose

$$\left.\begin{aligned} A\alpha_1 + B\beta_1 &= QA \\ A\alpha_2 + B\beta_2 &= QB \end{aligned}\right] -(4)$$

$$\psi(x+a) = Q\left[Af(x) + Bg(x)\right]$$

$$\psi(x+a) = Q\,\psi(x) \quad -(5)$$

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From eq^n 4

$$(\alpha_1 - Q)A + (\beta_1)B = 0$$

$$(\alpha_2)A + (\beta_2 - Q)B = 0$$

If we choose A & B

$$\begin{vmatrix} \alpha_1 - Q & \beta_1 \\ \alpha_2 & \beta_2 - Q \end{vmatrix} = 0$$

$$Q^2 - (\alpha_1 + \beta_2)Q + \alpha_1\beta_2 - \alpha_2\beta_1 = 0 \quad (6)$$

$$\therefore \alpha_1\beta_2 - \alpha_2\beta_1 = 1$$

$f(x)$ and $g(x)$ are solutions

$$\frac{d^2 f(x)}{dx^2} + \frac{2m}{\hbar^2}\left[E - V(x)\right]f(x) = 0 \quad (7)$$

$$\frac{d^2 g(x)}{dx^2} + \frac{2m}{\hbar^2}\left[(E - V(x)\right]g(x) = 0 \quad (8)$$

Multiply eq (7) by $g(x)$ and eq(8) by $f(x)$, then subtract.

$$g(x)\frac{d^2 f(x)}{dx} - f(x)\frac{d^2 g(x)}{dx^2} = 0$$

Integrate.

$$\underbrace{g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}}_{\text{wranskian } w(x)} = \text{Constant}$$

$$\underline{w(x+a) = w(x)}$$

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$$w(x+a) = g(x+a)\frac{df(x+a)}{dx} - f(x+a)\frac{dg(x+a)}{dx}$$

$$w(x+a) = \left[\beta_1 f(x) + \beta_2 g(x)\right] d\left[\frac{\alpha_1 f(x) + \alpha_2 g(x)}{dx}\right]$$

$$- \left[\alpha_1 f(x) + \alpha_2 g(x)\right] d\left[\frac{\beta_1 f(x) + \beta_2 g(x)}{dx}\right]$$

$$w(x+a) = \alpha_1\beta_2 - \alpha_2\beta_1\left[g(x)\frac{df(x)}{dx} - f(x)\frac{dg(x)}{dx}\right]$$

$$w(x+a) = \alpha_1\beta_2 - \alpha_2\beta_1 w(x)$$

$$\alpha_1\beta_2 - \alpha_2\beta_1 = 1$$

So eqn (6) becomes

$$Q^2 - (\alpha_1 + \beta_2)Q + 1 = 0$$

$$Q^2 - \mu Q + 1 = 0 \quad \text{— (10)}$$

$$Q_1 = \frac{\mu + \sqrt{\mu^2 - 4}}{2}$$

$$Q_2 = \frac{\mu - \sqrt{\mu^2 - 4}}{2}$$

Case 1 $\mu^2 < 4$

Q_1 & Q_2 are complex & conjugate roots

$Q_1 = e^{ika}$, $Q = e^{-ika}$

$$\psi(x+a) = e^{\pm ika}\psi(x)$$

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$x+a)\frac{dg(x+a)}{dx}$

$g(x)]$

$f(x)+\beta_2 f(x)]$

$f(x)\frac{dg(x)}{dx})$

ate roots

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Case 2 $\mu^2 > 4$

θ_1 and θ_2 are real roots

Here wave function is absent

Means e^- are absent.

Places where e^- are absent is Forbidden gap

" " e^- " present " bands.

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Free e^- theory of metal

The free e^- theory classical was proposed by drude in 1900. He suggested that metal crystal consist of +ve ion cores with valence e^-. These valance e^- were also known as conduction electrons because these are responsible for conduction of electricity. These conduction e^- were supposed to move freely among the positive metal ion cores.

2 mark
Q1 What were the assumption of drude model?

Ans 1. The Columb's attraction b/w positive ion core and valence electron is neglected but on the other hand it is also considered that this Columb attraction does not allow to leave the surface of metal.

2. The potential field due to these metal ions is supposed to be uniform and hence the potential energy of e^- may be taken as constant and hence zero for mathematical convience.

Hence, for studying the behaviour of e^- only Kinetic energy of e^- was consider.

3. The mutual repulsion b/w e^- is also neglected.

e^- are fermions

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Hence, it was suggested that the free e^- in metal behave like atoms ~~and~~ or molecules of a perfect gas. Hence these e^- are sometime known as free e^- gas or gas of free e^- in a metal or free e^- cloud or fermi gas.

2 mark

This fermi gas is much different from ordinary gas.

i) Ordinary gas has neutral particle but fermi gas has -vely charged particles.

ii) The no of atoms or molecules per unit volume for ordinary gas is 10^{25} per m^3 where as for fermi gas the no of particles per unit volume is about 10^{29} per m^3.

Therefore, the ordinary gas theory could not be applied ~~for~~ to the system. To overcome this drawback ~~lauer~~ lorentz in 1909 proposed that maxwell-boltzmann statistics must be applied to such kind of system.

Hence, classical theory is also known as drude lorentz theory.

On the basis of free e^- theory following properties of solid have been explained successfully

i) →

(1) Electrical Conductivity :- It is defined as the quantity of electricity that flows unit time per unit area of cross section of the conductor per unit potential gradient.

Acc. to free e^- theory in a solid the e^- move freely if E is the applied electric field.

$$F = ma \quad -(1) \quad \text{(classical)}$$
$$F = eE \quad -(2) \quad \text{(universal)}$$

equating (1) and (2)

$$ma = eE$$
$$a = \frac{eE}{m} \qquad \left[\because a = \frac{d^2x}{dt^2}\right]$$

$$\boxed{\frac{d^2x}{dt^2} = \frac{eE}{m}} \quad -(3)$$

$\lambda \rightarrow$ mean free path (dist.)
$\tau \rightarrow$ Relaxation time

$$\tau = \frac{\lambda}{v} \quad -(4)$$

integrating eq. (3) we get

$$\frac{dx}{dt} = \frac{eEt}{m} + c$$

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At $t = 0$, $\frac{dx}{dt} = 0$

$$\frac{eE(0)}{m} + c = 0$$

$$\therefore c = 0$$

$$\therefore \frac{dx}{dt} = \frac{eEt}{m}$$

So the average velocity b/w two successive collision is given as

$$\bar{v} = \frac{1}{\tau}\int_0^{\tau} \frac{eEt}{m}\,dt$$

MB

$$\bar{v} = \frac{eE\tau}{2m}$$

$$\left[\ \vec{v} = \frac{1}{\tau}\left[\frac{eEt^2}{2m}\right]_0^{\tau}\right.$$

$$= \frac{1}{\tau}\left[\frac{eE(\tau)^2}{2m} - \frac{eE(0)^2}{2m}\right]$$

$$= \frac{1}{\tau}\left[\frac{eE\tau^2}{2m}\right]$$

$$= \frac{eE\tau}{2m}$$

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$$\bar{v} = \frac{eE\lambda}{2mv} \quad \text{— (5)} \qquad [\text{putting value of } \tau]$$

$\vec{v}$ = average velocity

v = instantaneous velocity.

Also we know that from free gas theory (drude)

$$\frac{1}{2}mv^2 = \frac{3}{2}KT$$

where K is boltzmann constant.
T is temperature

$$mv = \frac{3KT}{v}$$

Substitute in (5)

$$\vec{v} = \frac{eE\lambda \cdot v}{2 \cdot 3KT} = \frac{eE\lambda v}{6KT} \quad \text{— (6)}$$

If n is the no. density of e^- or e^- per unit volume, then the current density is given by

$$i = ne\bar{v}$$

Substituting (6) here

$$i = ne\left(\frac{eE\lambda v}{6KT}\right)$$

$$i = \frac{ne^2E\lambda v}{6KT} \quad \text{— (7)}$$

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If q is the charge flowing through the conductor of cross section A in time t, then the charge is, ~~equal to~~

$$q = \sigma A E t$$

where σ is electrical conductivity.

$$\frac{q}{t} = i = \sigma A E \quad — (8)$$

Comparing (7) and (8)

$$\frac{ne^2 E v \lambda}{6KT} = \sigma A E$$

$$\boxed{\sigma = \frac{ne^2 \lambda v}{6AKT}}$$

For a unit cross section area, $\boxed{\sigma = \frac{ne^2 \lambda v}{6KT}}$

This shows that electrical conductivity of a material depends upon no. density of free e^-. And this result was in strong agreement with experimental result.

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(2) Ohm's law :-

from eq. (7)

$$i = \frac{ne^2 E \lambda v}{6KT}$$

ie. for a specific material and at a given temp.,

$$i \propto E$$

This shows that current is proportional to applied field which is ohm's law. and is in strong agreement with experimental results

current density

$J = i$ [$J = \sigma E$ (microscopic Ohm's law)]
[$V = IR$ (macroscopic Ohm's law)]

conductivity, electric field

(3) Thermal Conductivity :-

We know that if there is no temp. diff. b/w two pts. in a material than there is no transfer of energy.

So to discuss about thermal conductivity we consider a specimen having a temp. gradient. Let us say there is a metallic rod with ends A and B such that A is at higher temp. as compared to B. Therefore, conduction of heat takes place from A to B through collision of e^-.

During the collision e^- near pt. A loose the K.E. where as e^- near the pt. B gain the K.E.

The amt. of heat passing through cross-section of the rod per unit area per second is given by

$$Q = \frac{1}{3} m v \lambda \frac{dE}{dt} \quad \text{—— (9)} \quad \text{(free)}$$

From Kinetic theory of gases we know that,

$$E = \frac{3}{2} KT \rightarrow \text{temp}$$

$$\frac{dE}{dt} = \frac{3}{2} K \frac{dT}{dt} \rightarrow \text{time}$$

Substitute $\frac{dE}{dt} = \frac{3}{2} K \frac{dT}{dt}$ in eq. (9)

$$Q = \frac{1}{3} m v \lambda \left[\frac{3}{2} K \frac{dT}{dt} \right]$$

$$= \frac{1}{2} m v \lambda \underset{\text{boltzmann}}{K} \frac{dT}{dt} \quad \text{—— (10)}$$

Also from thermodynamics, it is known that

$$Q = \underset{\text{thermal conductivity}}{k} \frac{dT}{dt} \quad \text{—— (11)} \quad \text{(maxwell boltzmann)}$$

Comparing (10) and (11)

$$\frac{1}{2} m v \lambda K \frac{dT}{dt} = k \frac{dT}{dt}$$

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$$\boxed{k = \frac{1}{2} m v \lambda K}$$

This value of thermal conductivity is verified experimentally and hence free e^- theory successfully explains the thermal conductivity.

(4) Wiedemann - Franz law :-

Wiedemann and Franz in 1853 discovered that all good electrical conductor are also good thermal conductors and the ratio of thermal conductivity to electrical conductivity (k/σ) is constant for all metals except at very very low temp.

$$k = \frac{1}{2} m v \lambda K \qquad \sigma = \frac{n e^2 \lambda v}{3KT}$$

$$\frac{k}{\sigma} = \frac{\frac{1}{2} m v \lambda K}{\frac{n e^2 \lambda v}{3KT}}$$

$$= \frac{K}{\frac{e^2}{3KT}}$$

$$\boxed{\frac{K}{\sigma} = \frac{3K^2 T}{e^2}} \Rightarrow \boxed{\frac{K}{\sigma} \propto T}$$

Therefor at given temp K/σ is constant

(3) Lustre and opacity of metals :-

When a E.M. radiations falls on metals it produced forced oscillation in free e^- having same velocity and frequency as ~~that~~ of E.M. radiations.

Thus energy of incident radiations is observed by free e^- and metals appear to be opaque.

The excited e^- while returning to its initial state emits a photon having same energy as is observed initially. This energy is given out in the form of visible light in all direction but only light rays directed towards the metal surface can get through and hence the metal appears to reflect virtually all the light i.e. incident on it giving it the characteristic of metallic lustre.

There were few failures of drude lorentz theory which are as follows :-

i. It does not explain the heat capacity of metals.

ii. It fails to explain the ~~paragem~~ paramagnetic susceptibility of conduction e^-.

iii) It fails to explain that why metals prefer certain structure.
iv) This theory does not explain the long mean free path at low temp.
v) This theory does not explain the temp. variation of electrical resistivity.
vi) This theory does not explain why some crystals are metal.

Sommerfeld's Quantum theory :-

As it was observed acc. to classical theory, few of the experimental fact were not explained. The major reason was the application of maxwell boltzmann statistics on the motion of e^- / free e^- gas. However, in 1928 sommerfeld suggested that e^- gas should be treated quantum mechanically. That is e^- are to be consider indistinguishable in the light of Pauli's exclusion principle i.e. no two e^- can be in the same quantum state.

Hence, He suggested that fermi-Dirac statistic lee applied to e^- gas instead of maxwell boltzmann statistics. And Hence he was successful in explaning various experimentale facts.

Subpart 5M

* Free e^- gas in one dimensional box :-

The assumptions of sommerfeld's model were

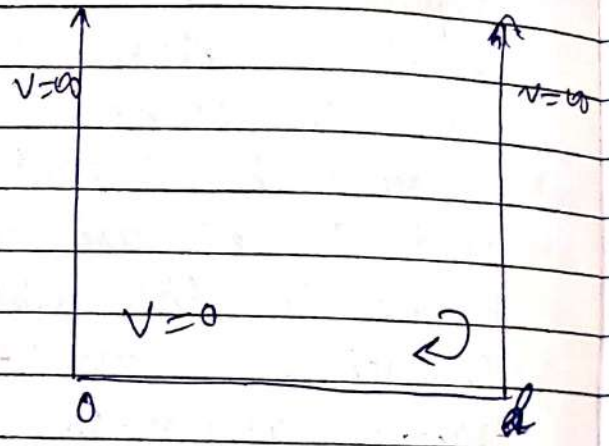
same as drude lorentz classical theory i.e. e^- is consider to be moving in constant potential. But since e^- here has to be treated quantum mechanically and so we consider this e^- in 1D infinite potential well.

The concept of e^- in 1D infinite potential well evolves from the fact that e^- is moving in the metal under constant potential but e^- cannot leave the metal. So, mathematically the potential just on the surface of metal is consider to be infinite.

The particle in 1D IPW (Infinite potential well) is represented as

$V(x) = 0 \quad 0 < x < l$

$V(x) = \infty \quad x \leq 0$ and $x \geq l$



Since e^- has to be consider Quantum mechanically therefore, its motion or wave function is studied through / analysis / discussion through SWT.

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar}(E - V(x))\psi(x) = 0 \quad \text{— (1)}$$

for $V = 0$
$0 < x < l$

where 0 to l is atomic dimension

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2} E\,\psi(x) = 0$$

$$\frac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0 \quad \text{— (2)}$$

where $k^2 = \frac{2mE}{\hbar^2}$ — (3)

$$[D^2 + K^2]\,\psi(x) = 0$$

where $D^2 = \frac{d^2}{dx^2}$

$$D^2 + k^2 = 0$$
$$D^2 = -k^2$$
$$D = \pm i k$$

$$\psi(x) = A\cos kx + B\sin kx \quad \text{— (4)}$$

Applying boundary condition on (4)

Ist B.C.-

$[\cos 0 = 1,\ \sin 0 = 1]$

At $x = 0$, $\psi(x) = 0$

$$\psi(0) = A\cos k(0) + B\sin k(0)$$
$$0 = A(1) + B(0)$$
$$\therefore \boxed{A = 0}$$

$\therefore$ from (4)

$$\psi(x) = B\sin K(x) \quad \text{— (5)}$$

~~IInd~~ Applying B.C. on (5)

IInd B.C.

At $x = l$, $\psi(x) = 0$

$[\sin^{-1} = n\pi]$

$$\psi(l) = B\sin kl$$
$$0 = B\sin kl$$

either $B = 0$ or $\sin kl = 0$

B can't be zero

$\because \sin kl = 0$

$$kl = \sin^{-1} 0$$
$$kl = n\pi$$
$$k = \frac{n\pi}{l}$$

Squaring b.s.;

$$k^2 = \frac{n^2\pi^2}{l^2}$$

from (3)

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{l^2}$$

$$\boxed{E = \frac{n^2\pi^2\hbar^2}{2l^2m}}$$

So $E_1 = \dfrac{n^2\pi^2\hbar^2}{2ml^2}$

$E_2 = 4E_1$

$E_3 = 9E_1$

$E_4 = 16 E_1$

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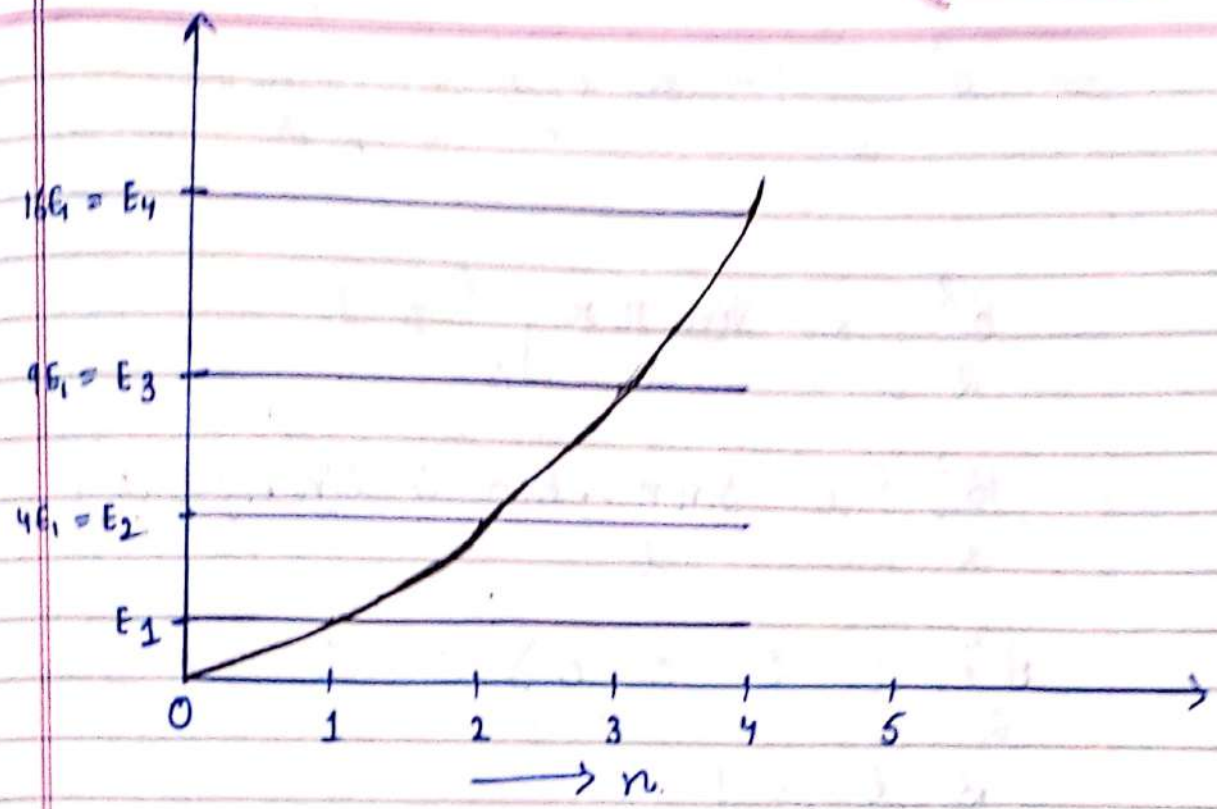


It is seen that energy spacing b/w two levels depend upon the length l as well as n. This spacing b/w energy level is important only when l is of the order of atomic dimensions.

$$\psi = B \sin \frac{n\pi}{l} x \qquad \text{(putting K in eq. (3))}$$

Applying concept of probability

$$\int_0^l |\psi|^2 \, dx = 1$$

$$\int_0^l B^2 \sin\left(\frac{n\pi}{l}\right)x \; \sin\left(\frac{n\pi}{l}\right)x \, dx = 1$$

$$B^2 \int_0^l \sin^2\left(\frac{n\pi}{l}\right)x \, dx = 1$$

$$B^2 = \frac{\int_0^l 1 - \cos 2\left(\frac{n\pi}{l}\right) x \, dx}{2} = 1$$

$$\frac{B^2}{2}\left[x - \frac{\sin n\pi x}{l}\right]_0^l = 1$$

$$\frac{B^2}{2}\left[l - \frac{\sin n\pi l}{l} - 0 \; \frac{\sin n\pi}{l}(0)\right]$$

$$\frac{B^2}{2}(l - 0 - 0 - 0) = 1$$

$$\frac{B^2 l}{2} = 1$$

$$B^2 = \frac{2}{l}$$

$$B = \sqrt{\frac{2}{l}}$$

Thus $\psi(x) = \sqrt{\frac{2}{l}} \sin\left(\frac{n\pi x}{l}\right)$

Fermi Energy :- It is know important to observe that how $N e^-$ are distributed in this 1D lattice bind.

Electrons are indistinguishable particle and follow pauli exclusion principle that no two e^- can be in same quantum state i.e. each quantum state can be

occupy by at the most one electron 'N'.
In the case of atoms, these quantum state are referred as orbitals. But acc. to quantum mechanics there can be more than one quantum state having the same energy. These states are called Degenerate states.

If we consider a linear solid (1D). then there are two quantum no. of associated to conduction e^- orbital. These are n and s.

where $n = 1, 2, 3 - - -$ so on

$$s = \mp \frac{1}{2}$$

n represent the energy level / orbital level.
s represent the spins.

It means that for every end s can have two values Hence at single level of n. two e^- are accomodated. Therefore each energy level is doubly degenerate.

If we consider an example of sodium which has an atomic no. 11. Then n is filled upto 6th level. $n=6$ such that there are two e^- from $n = 1$ to 5 and one e^- in $n = 6$.

levels for n less than 6 are completely filled and for n greater than 6 are completely empty. For a general discussion, In the case of even no of e^- we can write that $\boxed{n = \frac{N}{2}}$

n = Quantum (energy levels)
N = no. of e^-

Here n_f is the quantum no representing the uppermost filled energy level and this level is called fermi level. And the energy associated with this level is called fermi energy.

$$E = \frac{n_f^2 \pi^2 h^2}{2ml^2}$$

(putting n_f)

$$\boxed{E = \frac{\hbar^2}{2m}\left(\frac{N\pi}{2L}\right)^2}$$

We can see from this value of fermi energy that it is dependent upon no. of e^- N and size of box L.

For $\frac{N}{L} = 1\, e^-/A^\circ = 10^8\, e^-/cm$

$$\boxed{E_f = 9.6\, eV}$$

Physically it means, that to accomodate $10^8 e^-$ in one cm ($10^8 e^-/cm$) energy of uppermost e^- should be 9.6 eV.

$$E_0 = 2\sum_{n=1}^{N/2} E_n$$

energy of levels $\boxed{E_0 = \sum_{n=1}^{N/2} E_n}$

energy of e^- ∴ That each level can accomodate 2 e^-

$$\boxed{E_0 = 2\sum_{n=1}^{N/2} E_n}$$

$$E_0 = 2\sum_{n=1}^{N/2} E_n$$

$$= 2\sum_{n=1}^{N/2} \frac{n^2\pi^2\hbar^2}{2l^2m}$$

$$= \frac{\cancel{2}\pi^2\hbar^2}{\cancel{2}mL^2}\sum_{n=1}^{N/2} n^2 \qquad \left[\sum n^2 = \frac{1}{3}n^3\right]$$

$$= \frac{\pi^2\hbar^2}{mL^2}\times\frac{1}{3}n^3 \qquad \left[= \frac{1}{3}\left(\frac{N}{2}\right)^3\right]$$

$$= \cancel{\frac{n^3\pi^2\hbar^2}{8mL^2}} \qquad n = \frac{N}{2}$$

$$= \frac{\pi^2\hbar^2}{mL^2}\cdot\frac{1}{3}\left(\frac{N}{2}\right)^3$$

$$= \cancel{\frac{\pi^2\hbar^2}{\left(\frac{N}{2}\right)^3}} \quad \frac{\pi^2\hbar^2}{mL^2}\frac{1}{3}\times\frac{N^3}{8}$$

$$= \underbrace{\frac{\hbar^2}{2m}\left(\frac{N\pi}{2L}\right)^2}_{\Rightarrow E_F}\cdot\frac{1}{3}N$$

$$\boxed{E = \frac{1}{3}NE_F}$$

It is clear from this expression that for 1-D problem the average K.E. in the ground state is 1/3 of that of fermi energy.

for 2M

imp

Density of States :- It is defined as no. of electronic or quantum state per unit energy range.

Mathematically, it is given as

$$\boxed{D(E) = \frac{dn}{dE}}$$

where dn = no. of quantum state present in the Energy range b/w E and $E+dE$

Since each level can accomodate two e⁻

$$\therefore \boxed{D(E) = \frac{2\,dn}{dE}}$$

$$E = \frac{n^2\pi^2\hbar^2}{2mL}$$

$$\frac{dE}{dn} = \frac{d}{dn}\left(\frac{n^2\pi^2\hbar^2}{2mL^2}\right)$$

$$= \frac{\pi^2\hbar^2}{2mL^2}\frac{d}{dn}n^2$$

$$\frac{dE}{dn} = \frac{n\pi^2\hbar^2}{mL^2}$$

$$\frac{dn}{dE} = \frac{mL}{n\pi^2\hbar^2} = \frac{mL}{\pi^2\hbar^2}\left(\frac{1}{n}\right).$$

$$\rightarrow D(E) = 2\frac{mL^2}{\pi^2\hbar^2}\left(\frac{1}{n}\right)$$

from $E = \frac{n^2\pi^2\hbar^2}{2mL}$

$$\frac{2Eml}{\pi^2\hbar^2} = n^2$$

$$\frac{1}{n^2} = \frac{\pi^2\hbar^2}{2Eml}$$

$$\frac{1}{n} = \left(\frac{\pi^2\hbar^2}{2mL^2}\frac{1}{E}\right)^{1/2}$$

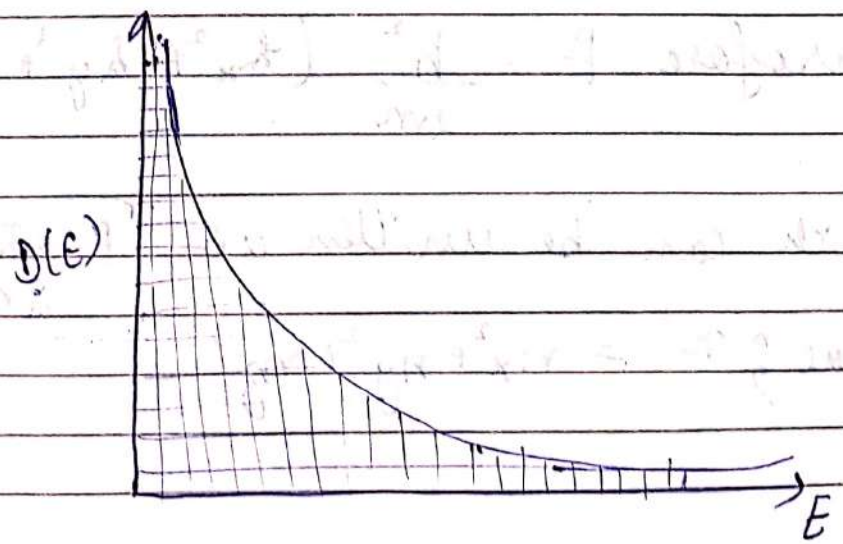
$$D(E) = \frac{2mL^2}{\pi^2\hbar^2} \times \left(\frac{\pi^2\hbar^2}{2mL^2}\frac{1}{E}\right)^{1/2}$$

$$= \frac{2mL^2\pi\hbar}{\pi^2\hbar^2L^2}\left(\frac{1}{2mE}\right)^{1/2}$$

$$= \frac{2mL}{\pi\hbar}\left(\frac{1}{2mE}\right)^{1/2}$$

$$= \frac{L}{\pi\hbar}\left(\frac{2m}{E}\right)^{1/2}$$

$$\boxed{D(E) \propto \frac{L}{\pi\hbar}\left(\frac{2m}{E}\right)^{1/2}}$$



* Free e^- gas in 3-D:

The SWE for 3-D is given as:

$$\nabla^2 \psi = \frac{2mE}{\hbar^2}\psi$$

for constant and zero potential.

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

laplacian operator

The solution of 3D equation is given as

$$\psi = \left(\frac{8}{L^3}\right)^{1/2} \sin k_x x \sin k_y y \sin k_z z$$

where $k_x = \frac{n_x \pi}{L}$, $k_y = \frac{n_y \pi}{L}$, $k_z = \frac{n_z \pi}{L}$

In this case also

$$k^2 = \frac{2mE}{\hbar^2}, \quad k^2 = k_x^2 + k_y^2 + k_z^2$$

$$\therefore k_x^2 + k_y^2 + k_z^2 = \frac{2mE}{\hbar^2}$$

Therefore $E = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2 + k_z^2\right)$

which can be written as $\boxed{E = \frac{\hbar^2}{2mL^2} n^2}$

where; $n^2 = n_x^2 + n_y^2 + n_z^2$

applying the periodic boundary condition

$$= \psi(x+L, y, z) = \psi(x, y+L, z) \quad (1)$$

$$= \psi(x, y, z+L)$$

$$\Rightarrow \psi(x, y, z)$$

The wave function satisfied the three Dimension SWE for constant potential and periodic b.c. is given as

$$\psi(\vec{r}) = A e^{i(\vec{k}\cdot\vec{r})}$$

which can be written as $\Rightarrow A e^{i(k_x x + k_y y + k_z z)}$

Applying the periodic b.c. to this wave function we get

$$A e^{i k_x x L} = 1$$

$$\therefore k_x = 0, \pm\frac{2\pi}{L}, \pm\frac{4\pi}{L}, \text{———} \frac{2p\pi}{L}$$

|||ly k_y, k_z have same values if we use the periodic B.c.

The value of normalisation constant is given by the condition

$$\int_0\int_0\int_0 A e^{i(\vec{k}\cdot\vec{r})} A e^{-i(\vec{k}\cdot\vec{r})} dr = 1$$

$$\therefore A = \left(\frac{1}{V}\right)^{1/2} \quad \text{where } V = L^3$$

$\therefore$ The total wave function is

$$\boxed{\psi(\vec{r}) = \left(\frac{1}{V}\right)^{1/2} e^{(\vec{k}\cdot\vec{r})}}$$

* * * *

$$\nabla^2\psi + \frac{2}{\hbar^2} mE\psi = 0 \rightarrow 3d$$

$$\text{as } \frac{d^2}{dx^2}\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0 \rightarrow 1D$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\psi(x,y,z,t) = \psi(x,y,z)\,T(t)$$

different

$$\psi(x,y,z) = \psi(x)\,\psi(y)\,\psi(z)$$

$$\frac{\partial^2}{\partial x^2}\psi(x) + \frac{2mE}{\hbar^2}\psi(x) = 0 \longrightarrow \left(\frac{2}{L}\right)^{1/3} \sin k_x x$$

$$\frac{\partial^2}{\partial x^2}\psi(y) + \frac{2mE}{\hbar^2}\psi(y) = 0 \longrightarrow \left(\frac{2}{L}\right)^{1/3} \sin_y k_y$$

$$\frac{\partial^2}{\partial y^2}\psi(z) + \frac{2mE}{\hbar^2}\psi(z) = 0 \longrightarrow \left(\frac{2}{L}\right)^{1/3} \sin_z k_z$$

$$\psi = \left(\frac{8}{L}\right)^{1/3} \sin k_x x \, \sin k_y y \, \sin k_z z$$

$k = \frac{n\pi}{L}$ and $k^2 = k_x^2 + k_y^2 + k_z^2$

$$n^2 = n_x^2 + n_y^2 + n_z^2$$

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[k-space = n space]

The Fermi Energy [E_{F_0}] :-

It is the energy which corresponds to energy level below which all quantum states are filled and above it all the states are completely empty at 0K. Each quantum level in 3-D is determined by three quantum numbers : k_x, k_y, k_z and each level is occupied by two e^-, one will spin up and other will spin down.

In the ground state of system of n free e^- the occupied levels may be represented as pts. inside a sphere in k-space. The energy at the surface of sphere is the fermi energy. Fermi energy is given as

$$\boxed{E_{F_0} = \frac{n_f^2 \pi^2 \hbar^2}{2mL^2}}$$

We know find the no of pts. in k-space represented by n_f. Each pt. occupies on an average a unit volume in the integer space and vol. of +ve octant is given by

$$\frac{1}{8}\left(\frac{4}{3}\pi n_f^3\right)$$ → vol. of sphere

n_f = radius of sphere

and hence the no of pts. in an octant is also $\frac{1}{8}\left(\frac{4}{3}\pi n_f^3\right)$.

When we consider the spin of an e^- the no. of pts. become twice this no ie. $2\left[\frac{1}{8}\left(\frac{4}{3}\pi n_f^3\right)\right]$

If N is no density ie. no of e^- per unit volume

$$N = \frac{\text{no of } e}{\text{Vol.}} = \frac{\text{no of } e^-}{L^3}$$

then the total no. of e^- is NL^3.

$$\therefore NL^3 = 2 \times \frac{1}{8}\left(\frac{4}{3}\pi n_F^3\right)$$

$$n_F^3 = \frac{\cancel{8}}{\cancel{2}} NL^3 \times \frac{3}{\cancel{4}} \times \frac{1}{\pi}$$

$$\frac{n_F^3}{L^3} = \frac{3N}{\pi}$$

$$\left(\frac{n_F}{L}\right)^3 = \frac{3N}{\pi}$$

$$\boxed{\frac{n_F}{L} = \left(\frac{3N}{\pi}\right)^{1/3}}$$

Substituting $\frac{n_F}{L}$ in $E_{F_0} = \frac{n_F^2 \pi^2 \hbar^2}{2mL^2}$

$$E_{F_0} = \frac{n_F^2}{L^2} \cdot \frac{\pi^2 \hbar^2}{2m}$$

$$= \left[\left(\frac{3N}{\pi}\right)^{1/3}\right]^2 \cdot \frac{\pi^2 \hbar^2}{2m}$$

$$= \left(\frac{3N}{\pi}\right)^{2/3} \cdot \frac{\pi^2 \hbar^2}{2m}$$

$$\boxed{E_{F_0} = \frac{\hbar^2}{2m}\left(3\pi^2 N\right)^{2/3}}$$

The fermi energy is function of density of e^-. It is also concluded that at absolute 0 e^- have finite energy

Imp

Band theory of Solids :-

The free e^- theory (→ sommerfeld) of metals successfully explained various properties of metal like heat capacity, electrical & Thermal conductivity, magnetic susceptibility and many more.

Failure of Sommerfeld

There were still many properties which were unexplained on the basis of this theory :-

i) Diff. b/w inductors, conductors and semiconductor

ii) It has found that divalent (2 valency) metals like Be, and Cd and trivalent (3 valency) metals like Al and In are not good conductors. Even though the

theory says that conductivity is directly proportional to e^- concentration. Rather monovalent atom like Cu, Ag, Au are good conductor even though they have less conc. of free e^- as compared to divalent and trivalent metal.

iii) The shape of fermi surface is found to be non-spherical where as according to free e^- theory it should be spherical.

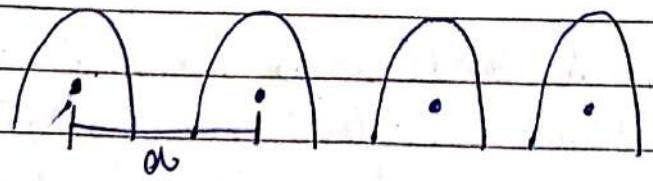
iv) Some of metals exhibit +ve hall coefficient while free e^- theory of metals predict that hall coeff. should be -ve for all metals.

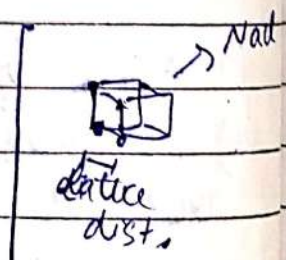
↳ Conductor ⇒ electric field or magnetic field apply ⟹ Potential diff. occurs → called as hall voltage

↳ effect measure is called hall coefficient.

Assumption led to failure of free e^- theory:

The failure of free e^- theory is due to over simplified assumption that e^- moves in the region of constant potential or zero potential. However, this is not the actual case because pot. experienced by e^- is very complicated and to regionable approximation we can assume that e^- are moving in a region of periodic potential of +ve ion core having periodicity equal to the length of lattice. The periodic potential can be depected as follows :-



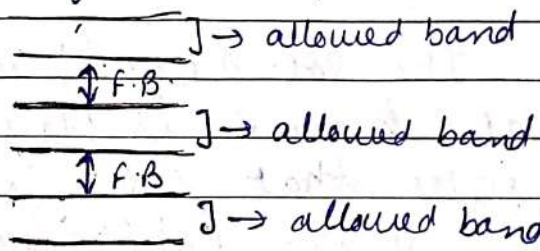


Wrong assumption of free e⁻ model.

i) This periodic potential is assumed to be extending upto infinity in all the directions but it is interrupted at the surface of the metal.
ii) The regular arrangement of atom in the crystal was not taken into account in free e⁻ theory. All these facts let to the development of band theory of solid.

When we consider the e⁻ in periodic potential we get following results:-

① There exist allowed energy bands seperated by forbidden band gaps or forbidden energy gaps.



② Acc. to free e⁻ theory the energy of e⁻ is given as $\frac{n^2\pi^2\hbar^2}{2ma^2}$.

where as when we consider periodic potential the energy is a periodic function of k.

$k^2 = \frac{2mE^2}{\hbar^2}$	But here
$E^2 = \frac{\hbar^2 k^2}{2m}$ → free e⁻ theory no periodically	E depends upon K but periodically
$k = \frac{n\pi}{L}$	↳ Same repeat after specific interval

i.e. graph will same after some specific value of k.

Bloch's Theorem :-

The periodic potential in 1-D can be written as :-

$$V(x+a) = V(x) = V(x-a) = V(x+2a) \quad \text{—— (1)}$$

[after a interval potential is repeated]

Also the SWE for 1D is given as :-

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V(x))\psi = 0 \quad \text{—— (2)}$$

where V is given by eq. (1)

The sol. of eq. (2) is given by bloch theorem also known as <u>flouquet's theorem</u> which states that sol. is of type

$$\psi(x) = \exp(\pm ikx)\, U_k(x) \;/\; e^{\pm ikx}\, U_k(x) \quad \text{—— (3)}$$

where $U_k = U_k(x+a)$ —— (4)

↳ due to periodicity depends upon a !

It is observed from eq. (3) that solution is same as of constant potential modulated by a function $U_k(x)$ which is again periodic with the periodicity of lattice.

~~Functions~~ equations (3) and (4) are called <u>bloch's function</u> and have the following property.

$$\psi(x+a) = e^{\pm ik(x+a)}\, U_k(x+a)$$

f^n = function.

$$\boxed{\psi(x+a) = \gamma\,\psi(x)}$$

where $\gamma = e^{\pm ika}$

So, we see that wave function after the period a is equal to initial wave f^n. which is modulated by factor γ (gamma) which is further dependent upon the periodicity of lattice.

SWE is a second order differential eq^n. Therefore, it has only <u>two real independent</u> solutions.

Let us say <u>$f(x)$ and $g(x)$</u> and rest all other solutions are linear combination of the independent solution.

$\therefore$ If $f(x+a)$ and $g(x+a)$ are to be the sol^n of given SWE then they can be expressed as

$$f(x+a) = \alpha_1 f(x) + \alpha_2 g(x)$$
$$g(x+a) = \beta_1 f(x) + \beta_2 g(x)$$

where α and β are functions of E.

Let $\psi(x)$ be the another sol^n of eq^n (2) therefore, it must be represented as

$$\psi(x) = A f(x) + B g(x)$$

and

$$\psi(x+a) = A f(x+a) + B g(x+a)$$

Substitution values of $f(x+a)$ and $g(x+a)$ here

$$\psi(x+a) = A\left[\alpha_1 f(x) + \alpha_2 g(x)\right] + B\left[\beta_1 f(x) + \beta_2 g(x)\right]$$

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$$= A\alpha_1 f(x) + A\alpha_2 g(x) + B\beta_1 f(x) + B\beta_2 g(x)$$

$$\psi(x+a) = [A\alpha_1 + B\beta_1] f(x) + [A\alpha_2 + B\beta_2] g(x)$$

~~Also $\psi(x+a)$ further can be written as~~

~~$\psi(x+a)$ can~~

$$\psi(x+a) = \gamma\,\psi(x)$$

$$= \gamma[A f(x) + B g(x)]$$

equating both equation

$$A\alpha_1 + B\beta_1 = \gamma A$$

$$(\alpha_1 - \gamma)A + \beta_1 B = 0$$

$$A\alpha_2 + B\beta_2 = \gamma B$$

$$(\alpha_2 - \gamma)A + B\beta_2 = 0 \quad \text{— (5)}$$

Set of eq. (5) have non-vanishing sol^n for A and B if and only if

$$\begin{vmatrix} \alpha_1 - \gamma & \beta_1 \\ \alpha_2 & \beta_2 - \gamma \end{vmatrix} = 0$$

$$(\alpha_1 - \gamma)(\beta_2 - \gamma) - \beta_1\alpha_2 = 0$$

$$\alpha_1\beta_2 - \underline{\alpha_1\gamma} - \underline{\gamma\beta_2} + \gamma^2 - \beta_1\alpha_2 = 0$$

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$$\gamma^2 - (\alpha_1 + \beta_2)\gamma + (\alpha_1 \beta_2 - \alpha_2 \beta_1) = 0$$

Since $\alpha_1 \beta_2 - \alpha_2 \beta_1 = 1$

$$\therefore \gamma^2 - (\alpha_1 + \beta_2)\gamma + 1 = 0 \quad \text{—} \quad (6)$$

Hence two cases exist for quadratic eq^n as sol^n of eq^n are is

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case I

For the energy ranges $(\alpha_1 + \beta_2)^2 < 4$

In this case will be complex conjugate of each other, can be written as

$$\gamma_1 = e^{ika} \quad \text{and} \quad \gamma_2 = e^{-ika}$$

where k is real and corresponding wave f^n $\psi_1(x)$ and $\psi_2(x)$ can be written as

$$\psi_1(x+a) = e^{ika}\psi(x)$$

and

$$\psi_2(x+a) = e^{-ika}\psi(x)$$

And in general,

$$\psi(x+a) = e^{\pm ika}\psi(x)$$

These f^n are called bloch's functions.

Case II for the energy ranges $(\alpha_1 + \beta_2)^2 < 4$.

In this case roots are real and will be reciprocal to each other which correspond to following wave f^n.

$$\psi_1(x) = e^{\mu x} U_K(x)$$

$$\psi_2(x) = e^{-\mu x} U_K(x)$$

where μ is real.

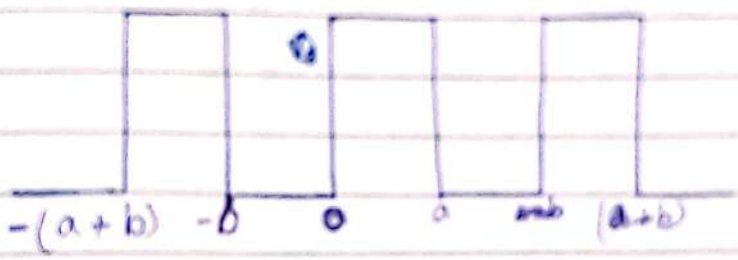
These are not acceptable wave f^n. Since these are not bounded and approaches to $\pm\infty$ when $x \to \infty$.

Thus, we find that energy spectrum of an e^- in a periodic potential consists of allowed and forbidden energy regions and bands.

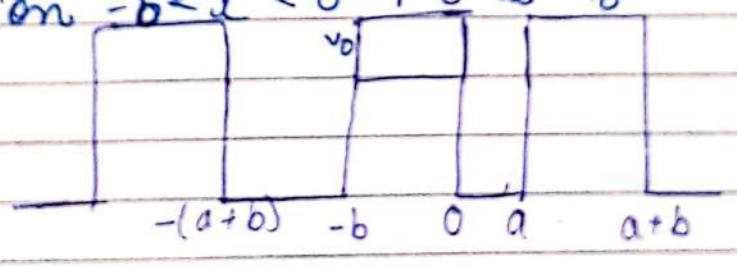
Imp The Kronig - Penny

Kronig-Penny Model :-

Kronig and Penny studied the behaviour of e^- in the periodic potential by considering the simplest 1-D model. It was assumed that pot. energy of e^- has a shape of square wells as shown in fig.



The period of the pot.^{n} is $a+b$ for the region ~~less than~~ $0 < x < a$. the potential energy is assumed to be zero. and for the region $-b < x < 0$. P.E. is V_0.



mathematically,

for $0 < x < a$ $\quad V(x) = 0$

for $-b < x < 0$ $\quad V(x) = V_0$

This type of periodic pot.^{n} is an approximation of an actual region. The SWE for both regions

can be written as

$$\frac{d^2\psi_1}{dx^2} + \alpha^2\psi_1 = 0 \quad — (1)$$

also for region $-b<x<0$ at considering $E<V_0$ the SWE is

$$\frac{d^2\psi_2}{dx^2} - \beta^2\psi_2 = 0 \quad — (2)$$

[S.W.E. Solve] Blue

Since the potential is periodic hence, the Sol^n must be of the form of Bloch's f^n

$$\therefore \psi_1(x) = e^{ikx} U_1(x)$$

where $U_1(x)$ is periodic f^n of x with period $a+b$.

Also $\psi_2(x) = e^{ikx} U_2(x)$

Finding the values of $\frac{d^2\psi_1}{dx^2}$ and $\frac{d^2\psi_2}{dx^2}$ and substituting in eq. ① and ② The sol^n are of the form

$$U_1 = Ae^{i(\alpha+k)x} + Be^{-i(\alpha+k)x} \quad \text{— ③}$$

for $0<x<a$

$$U_2 = Ce^{(\beta-ik)x} + De^{-(\beta+ik)x} \quad \text{— ④}$$

for $-b<x<a$

Applying the B.C.

① $U_1(0) = U_2(0)$

② $\left.\frac{dU_1}{dx}\right|_{x=0} = \left.\frac{dU_2}{dx}\right|_{x=0}$

③ $U_1(a) = U_2(-b)$

④ $\left.\frac{dU_1}{dx}\right|_{x=a} = \left.\frac{dU_2}{dx}\right|_{x=-b}$

Applying these 4 Boundary conditions to eq^n ③ and ④ and eliminating A, B, C, D.

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and by determinant method we see that solutions are possible iff

$$\begin{vmatrix} 1 & 1 \\ i(\alpha-k) & -i(\alpha+k) \\ e^{i(\alpha-k)a} & e^{-i(\alpha+k)a} \\ i(\alpha-k)e^{i(\alpha-k)a} & i(\alpha+k)e^{-i} \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ i(\alpha-k) & -i(\alpha+k) & \beta-ik & -(\beta-ik) \\ e^{i(\alpha-k)a} & e^{-i(\alpha+k)a} & e^{-(\beta-ik)b} & e^{(\beta-ik)b} \\ i(\alpha-k)e^{i(\alpha-k)a} & -i(\alpha+k)e^{-i(\alpha+k)a} & (\beta-ik)e^{-(\beta-ik)b} & -(\beta+ik)e^{(\beta}\end{vmatrix}$$

$$= 0$$

By solving the matric we get following equation,

$$\frac{\beta^2+\alpha^2}{2\alpha\beta}\sinh\beta b\sin\alpha a + \cosh\beta b\cos\alpha a = \cos k(a+b) \quad \text{— (5)}$$

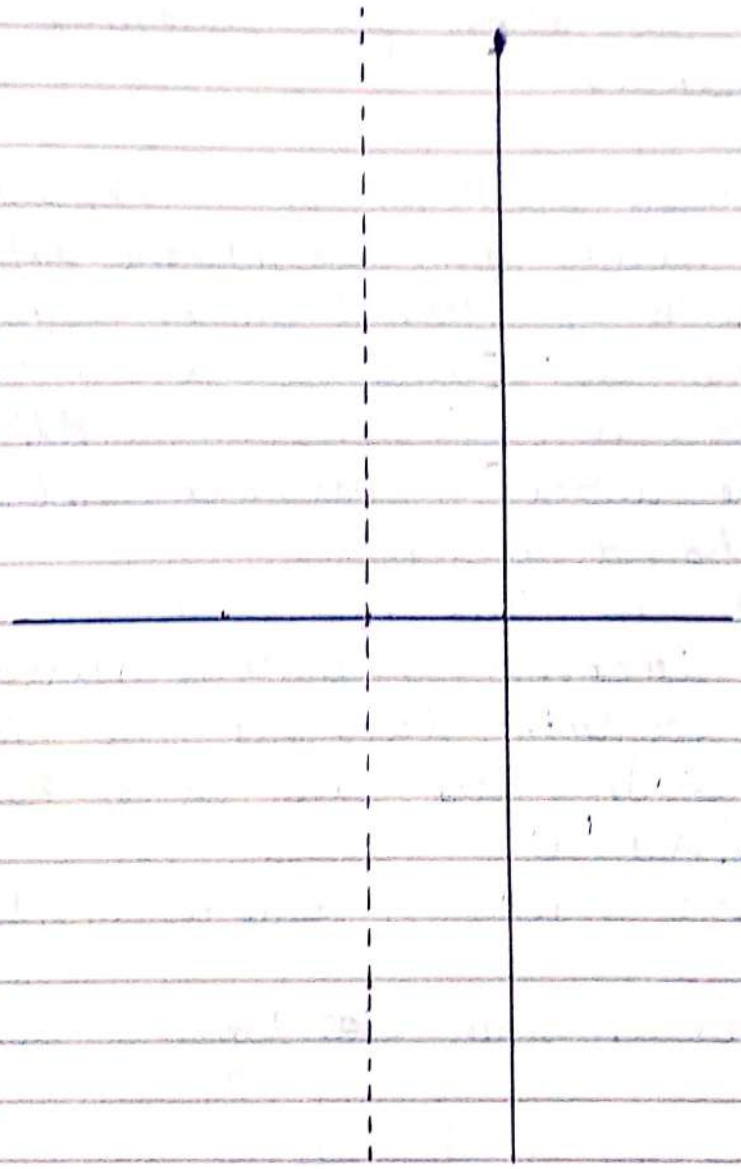
To consider the more similar case it was assumed that $V_0 \to \infty$ and $b \to 0$ but the product of $V_0 b$ remains finite and is known as <u>barrier strength</u>.
Under these conditions, $\sinh\beta b \to \beta b$
$\cosh\beta b \to \beta b$ and
$\beta^2+\alpha^2 \to \beta^2$

Thus from eq. (5) we get,

$$\frac{P\sin\alpha a}{\alpha a} + \cos\alpha a = \cos ka \quad \text{— (6)}$$

where $\beta = \frac{ma}{\hbar^2} V_0 b$

β is the measure of pot^n barrier. larger the value of β means that the given e^- is more strongly bound to a particular pot^n well.



The rt. hand side of eq.^{n} (6) can have the values b/w +1 and -1 and hence only those values of αa are allowed which make L.H.S. of this eq.^{n} lie b/w +1 and -1. Since α^2 proportional to energy $[\alpha^2 \propto E]$ Hence, the x-axis gives the measure of energy.

From the above fig. following pts. can be concluded.

1. The energy spectrum of e^- consist of allowed energy bands and forbidden gaps.
2. The width of allowed energy region increases with inc. in value of αa i.e. energy (E)
3. With inc. in P i.e. inc. in binding energy of e^- the width of particular allowed energy band decreases.

i) $P \to \infty$, in this case, the allowed region becomes infinitely narrow and energy spectrum becomes the ~~light~~ line spectrum. In this case eq. (6) has solution iff $\sin \alpha a = 0$

$$\therefore \alpha a = \pm n\pi \qquad \text{and } n = 0, 1, 2, 3 \ldots$$

And hence, since $\alpha^2 = \frac{2mE}{\hbar^2}$

$$\therefore E = \frac{\hbar^2 \alpha^2}{2m} \qquad \left[\alpha = \frac{n\pi}{a}\right]$$

$$= \frac{\hbar^2 n^2 \pi^2}{2ma}$$

This is the result of for a particle in a box having

infinite potential.

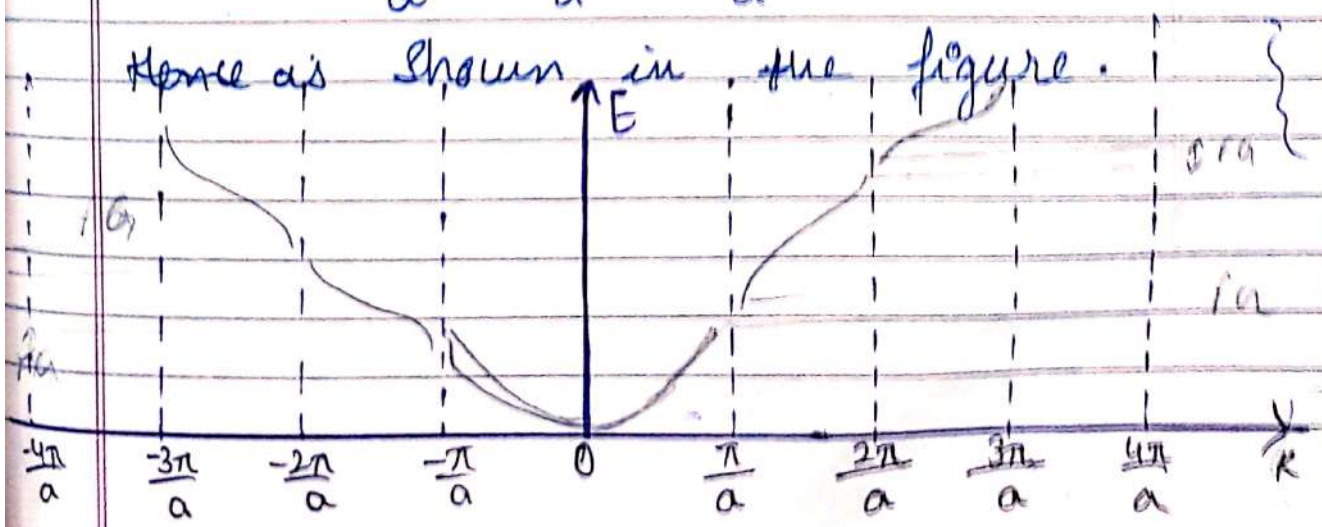
ii) $P \to 0$, in this case, eq. ⑥ implies that
$\cos \alpha a = \cos ka$
$\therefore \alpha a = ka$
$\alpha = k$
can be $\alpha^2 = k^2$
Substitute α^2 here
$$\frac{2mE}{\hbar^2} = k^2$$
$$E = \frac{k^2\hbar^2}{2m}$$

This is the energy of free e^- for which any energy value is possible.

iii) At the boundary of allowed band, ~~cos ka~~
$\cos ka = \pm 1$
$ka = \pm n\pi$
$$k = \frac{\pm n\pi}{a}$$

$\therefore$ The discontinuity in the energy occur at $k = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}$ - - -

Hence as shown in the figure.



If we consider in 1-D mono atomic lattice representing a value dividing into a value discontinuity into segment $\pm \frac{\pi}{a}$

Then these lines segment are called ~~Brollouing zones~~ Brillouin zones.

First zone extends from $-\frac{\pi}{a}$ to $+\frac{\pi}{a}$.

Second zone consist of two part ie $\frac{\pi}{a}$ to $\frac{2\pi}{a}$

~~an~~ and $-\frac{\pi}{a}$ to $-\frac{2\pi}{a}$

Thus first Brillouin zone is represented ~~the~~ by segment $-\frac{\pi}{a} < K < \frac{\pi}{a}$ and

Second Brillouin zone is represented by the segment $-\frac{2\pi}{a} < k < \frac{\pi}{a}$ and $\frac{\pi}{a} < K < \frac{2\pi}{a}$

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Effective mass of an e^- :-

* Velocity of e^- :- Acc. to de-broglie theory. The velocity of e^- is given as,

$$v = \frac{d\omega}{dk}$$ — omega

Also we know that,

$$E = \hbar\omega$$

$$\omega = \frac{E}{\hbar}$$

$$\frac{d\omega}{dt} = \frac{1}{\hbar} dE$$

$$v = \frac{1}{\hbar}\frac{dE}{dk} \quad \text{— (1)}$$

This shows that velocity is function of k. also for free e^-,

$$E = \frac{\hbar^2 k^2}{2m}$$

$$v = \frac{1}{\hbar}\frac{d(\hbar^2 k^2/2m)}{dk}$$

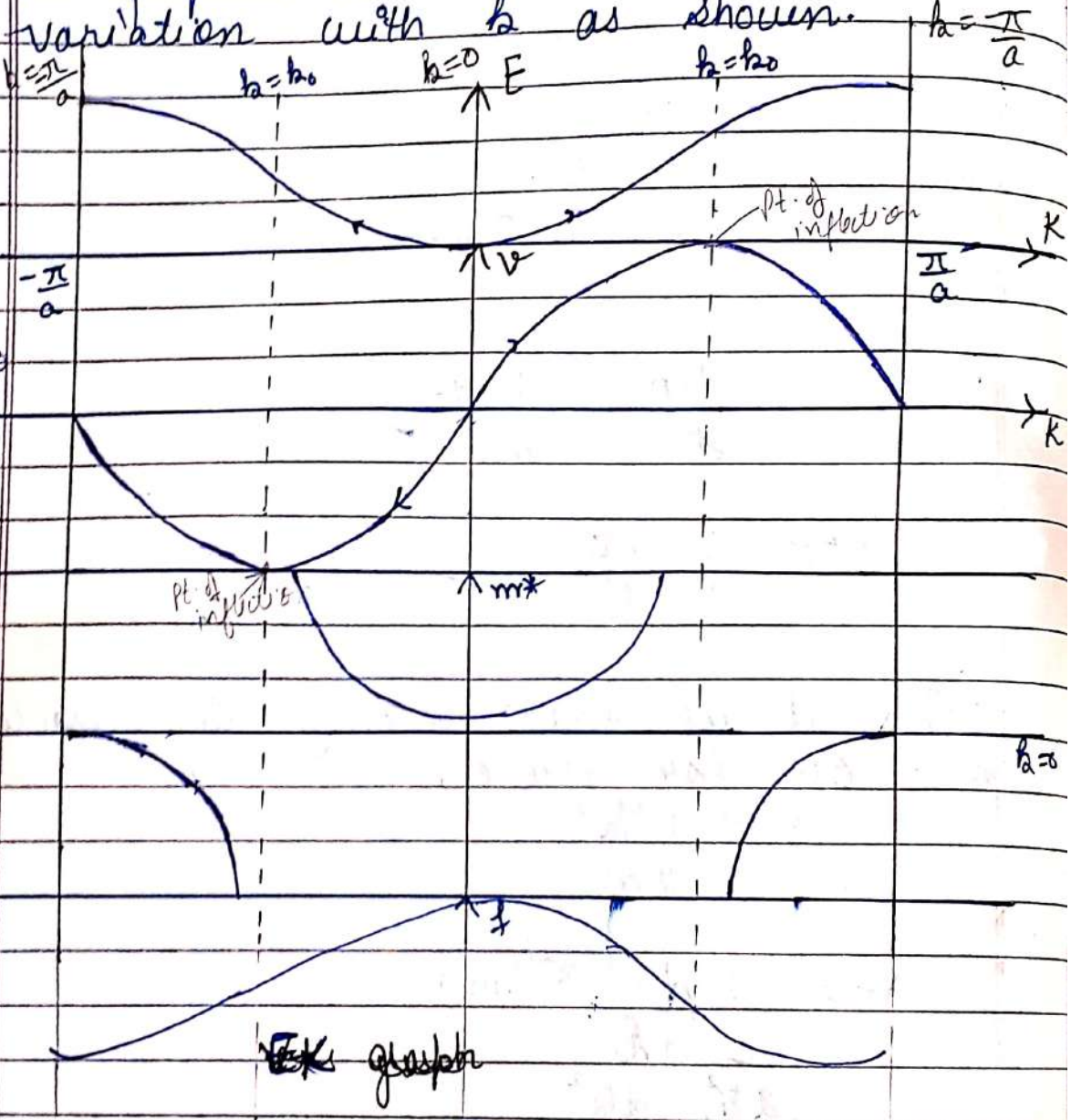
$$= \frac{2\hbar^2}{\hbar\, 2m}\frac{dk^2}{dk} \qquad \hbar k = p$$

$$= \frac{2\hbar k}{m}$$

$$v = \frac{p}{m}$$

This shows that even the energy of free e^- also satisfies the eq-(1)

In the band theory E in general is not proportional to k^2. rather it has variation with k as shown.



EK graph

from the expression (1) we can calculate velocity and vk graph is shown in figure.

It is observed that at the bottom and top of the band velocity is zero. We also see

that velocity increases with k and is maximum at a pt. known as pt. of inflection. Beyond this pt. velocity dec. with inc. in Energy.

As it is seen that vel. changes its behaviour at pt. of inflection and this is one of the most significant feature of band theory/model because this is quite diff. from free e^- theory which says that $E \propto k^2$.

The mass of an e^- in a crystal is in general diff from its free mass. This mass is called effective mass. In the E.F. & due to this field e^- start moving & gain energy. The gain in energy is given by

$$dE = efv\,dt \qquad \left[v = \frac{1}{\hbar}\frac{dE}{dk}\right]$$

$$dE = ef\frac{1}{\hbar}\frac{dE}{dk}dt \qquad [\text{from } (1)]$$

$$\frac{dK}{dt} = \frac{eF}{\hbar}$$

also $a = \dfrac{dv}{dt}$

$$a = \frac{1}{\hbar}\frac{d}{dk}\left(\frac{dE}{dt}\right)$$

$$= \frac{1}{\hbar}\frac{d}{dt}\left(\frac{dE}{dk}\right)\frac{dk}{dk}$$

$$= \frac{1}{\hbar}\frac{dk}{dt}\left(\frac{d^2E}{dk^2}\right)$$

HR

$$a = \frac{eF}{\hbar^2}\frac{d^2E}{dk^2} \quad \text{— (1)}$$

F = electric field

$$a = \frac{eF}{m} \quad \text{— (2)}$$

Comparing (1) (2)

$$\frac{eF}{m} = \frac{eF}{\hbar^2}\left(\frac{d^2E}{dk^2}\right)$$

$$m^* = \hbar^2\left(\frac{d^2E}{dk^2}\right)^{-1}$$

effective mass

It is seen that e^- behaves as if it has an effective mass (m^*) given by $\hbar^2\left(\frac{d^2E}{dk^2}\right)^{-1}$

The behaviour of effective mass along with k is represented in the given graph.

It is observed that m^* is +ve in lower half and -ve in upper half of the energy band and also effective mass become infinite at inflection pt. i.e. $k = k_0$.

This means that in upper half the e^- behaves as +vely charged particle which is usually reffered as hole. This same thing can be observed from the graph of velocity.

Let us suppose that e^- starts at $k=0$ when an E.F. is applied. The wave function/factor increase linearly with time until the vel. reaches its max. value. Beyond the max. value the same field produces decrease in velocity. So the mass becomes -ve in upper half.

It is important to define a quantity called degree of freedom.

Degree of freedom :- It is quantity which is defined as mass of free e^- upon the effective mass

$$f_k = \frac{m}{m^*} = \frac{m}{\hbar^2}\left(\frac{d^2E}{dk^2}\right)$$

shows particle depends on how many parameter.

f_k = max value = 1

f_k versus k is shown in fig. It is observed that when m^* is large then f_k is small i.e particle is heavy and for free e^- i.e. $m = m^*$ then $f_k = 1$

The concept of effective mass arises due to interaction of e^- wave packet with periodic lattice.

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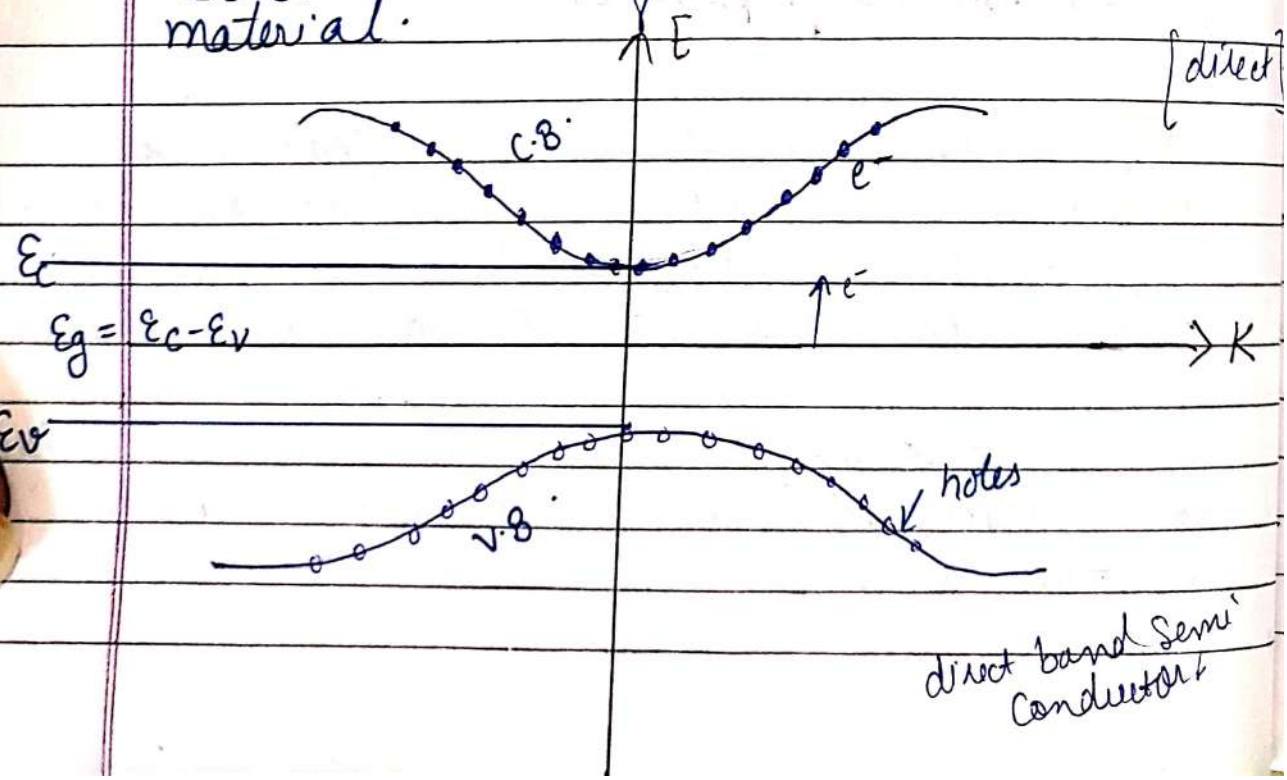
Note:- In most of the conductors $m^* = m$ because the band is partially filled but in semiconductor and insulator which have almost filled valence band the effective mass place an imp. role.

In Conduction process, The concept of effective mass is able to explain many experimental observations like high electronic specific heat and high paramagnetic susceptabilities.

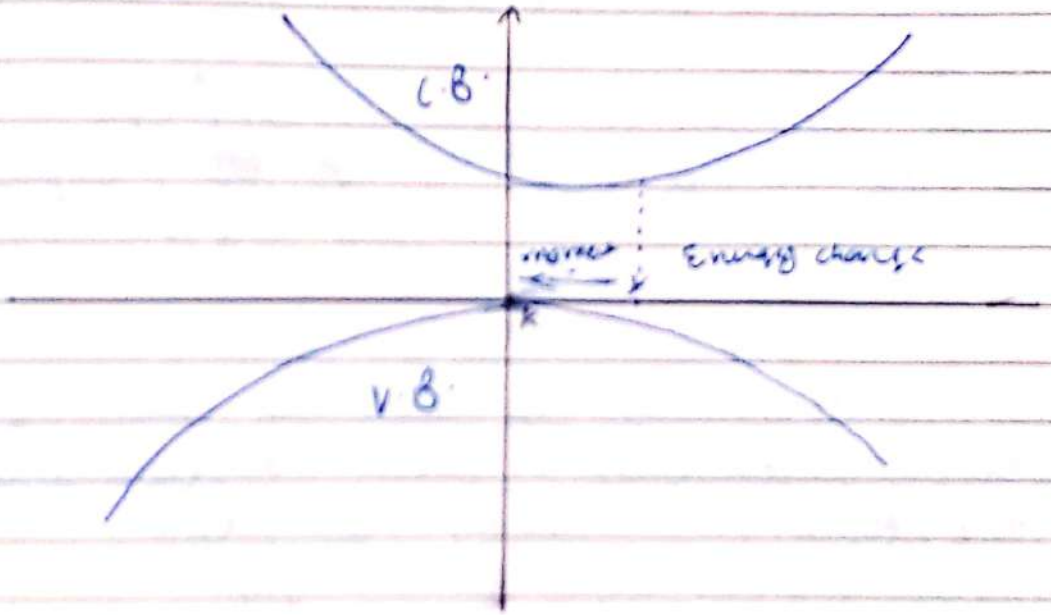
E-K diagram (2 mark)

The E-K diagram shows characteristics of particular semiconductor material. Its shows the relationship b/w energy and momentum [$p = \hbar k$] of available quantum mechanical state for the e^- in the material.



direct band semi conductor

This diagram indicates how the actual e^- states are equally spaced in K-space which means that densities of states depends upon the slope of E-K curves. It also shows that whether the material is direct band gap semiconductor or indirect band gap semiconductor.



direct – CB lowest Pt, VB highest Pt } → have same value of momentum
→ change in energy evolved.

Indirect – minimum pt. of C.B, Top of VB } → don't have same value of momentum
→ change of energy and momentum is evolved

Electrical Conductivity in terms of Band Gap Energy :-

As we know that electrical conductivity of semiconductor is given as

$$\sigma = e n_i (\mu_e + \mu_h)$$

Also we know that

$$n_i = 2\left(\frac{2\pi K}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} \exp\left(\frac{-E_g}{2KT}\right)$$

$\left(\frac{V}{E}\right)$

The mobility μ_e and μ_h have a temp. dependence which largely cancels the term $T^{3/2}$. Hence, the term

$$n_i = 2\left(\frac{2\pi K}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} \exp\left(\frac{-E_g}{2KT}\right)$$

Cancel out

Hence, the conductivity can also be written as

$$\sigma = \text{Const.} \times \exp\left(\frac{-E_g}{2KT}\right)$$

$\therefore$ Resistance can also be written as

$$\rho = \frac{1}{\sigma} = \text{Const} + \exp\left(\frac{E_g}{2KT}\right)$$

$$\log \rho = \log \text{Const} + \frac{E_g}{2KT}$$