UNIT-3

LIGHT AMPLIFICATION by STIMULATED EMISSION OF RADIATION

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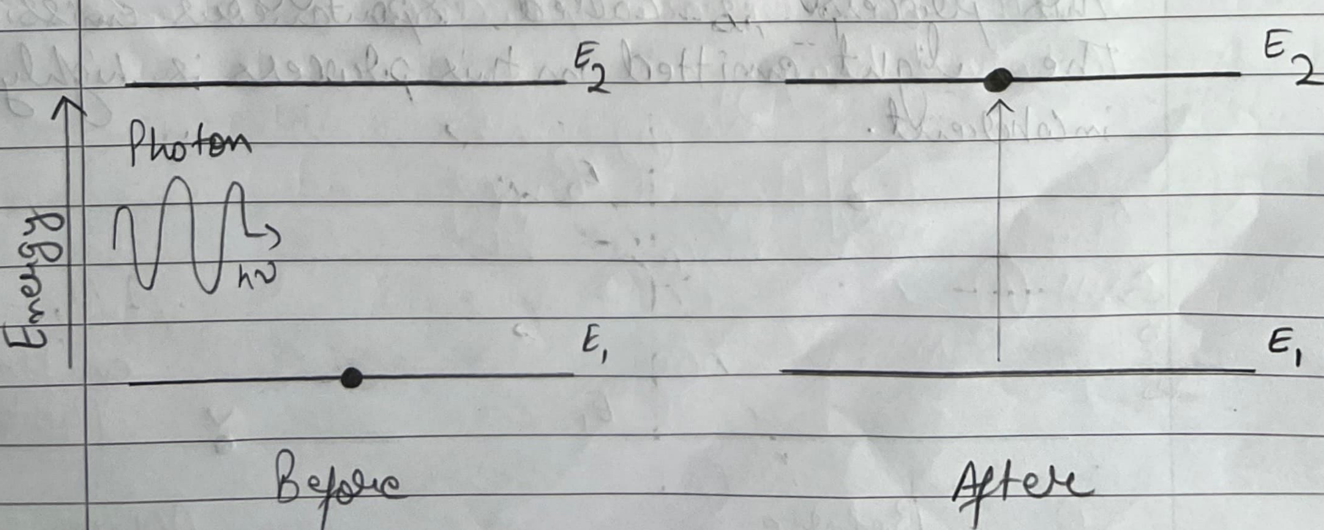
Laser emits light through a process called stimulated emission of radiation which amplifies or increases the intensity of light and produces highly directional light.

Absorption :- When an electron in the ground state E_1 absorb an incident photon, its energy increases by an amount of $h\nu$ and it goes to the excited state E_2. This process is called absorption.



$$A + h\nu \longrightarrow A^*$$

ground state energy — excited state energy

The number of absorption transitions occuring in the material at any istant will be proportional to number of electron present in state E_1 and incident radiation density Q.

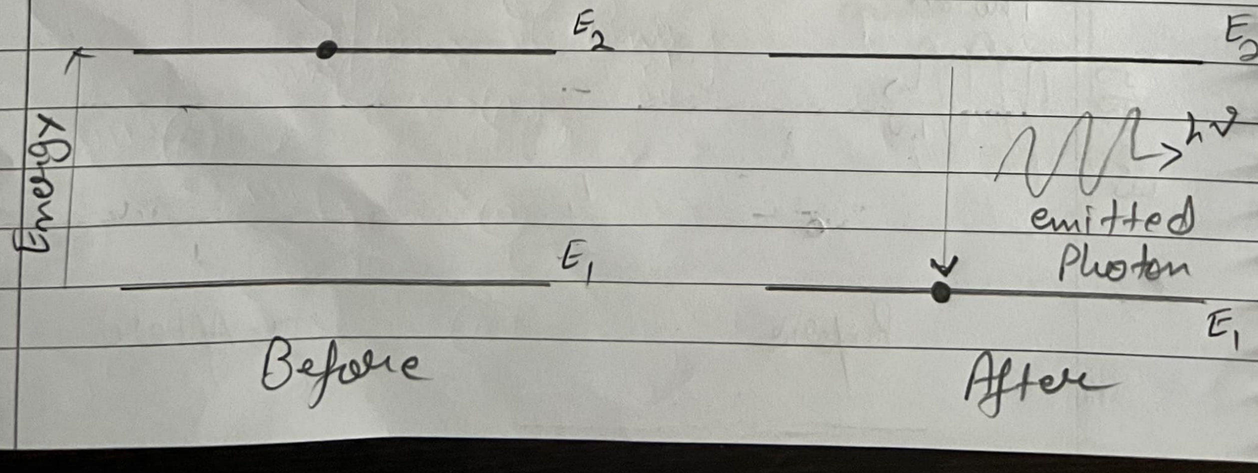
$$N_{ab} \propto N_1 Q \Delta t$$

$$\boxed{N_{ab} = B_{12} N_1 Q \Delta t}$$

B_{12} → Probability of absorption transition

Spontaneous Emission :-

Normally the electron in excited state is an unstable state where the life time of an electron is very short, around 10^{-8} sec. Hence the electron in the excited state E_2 returns to the ground state spontaneously by releasing one photon of energy $h\nu$. This process is called spontaneous emission. The light emitted in this process is highly incoherent.



$$A^* \longrightarrow A + h\nu$$

A^* → excited state energy

A → ground state energy

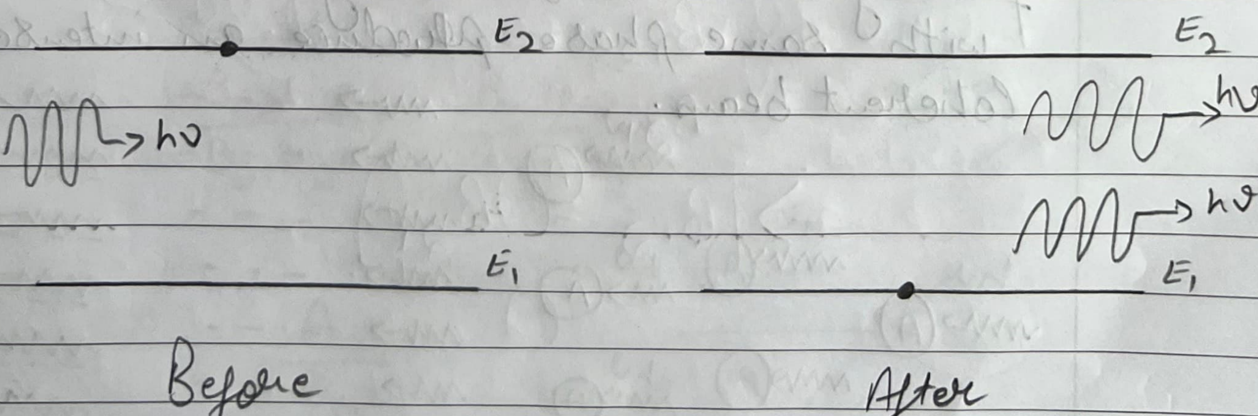
The number of spontaneous emissions at any instant will be proportional to the number of electrons in excited state.

$$N_{sp} \propto N_2 \Delta t$$

$$\boxed{N_{sp} = A_{21} N_2 \Delta t}$$

A_{21} → Probability of spontaneous transitions

Stimulated Emission :- In this process, an incident photon is absorbed by an excited electron, as a result of which the electron becomes unstable in E_2 and makes a transition to the ground state releasing two photons. This process is called stimulated emission.



$$A^* + h\nu \longrightarrow A + 2h\nu$$

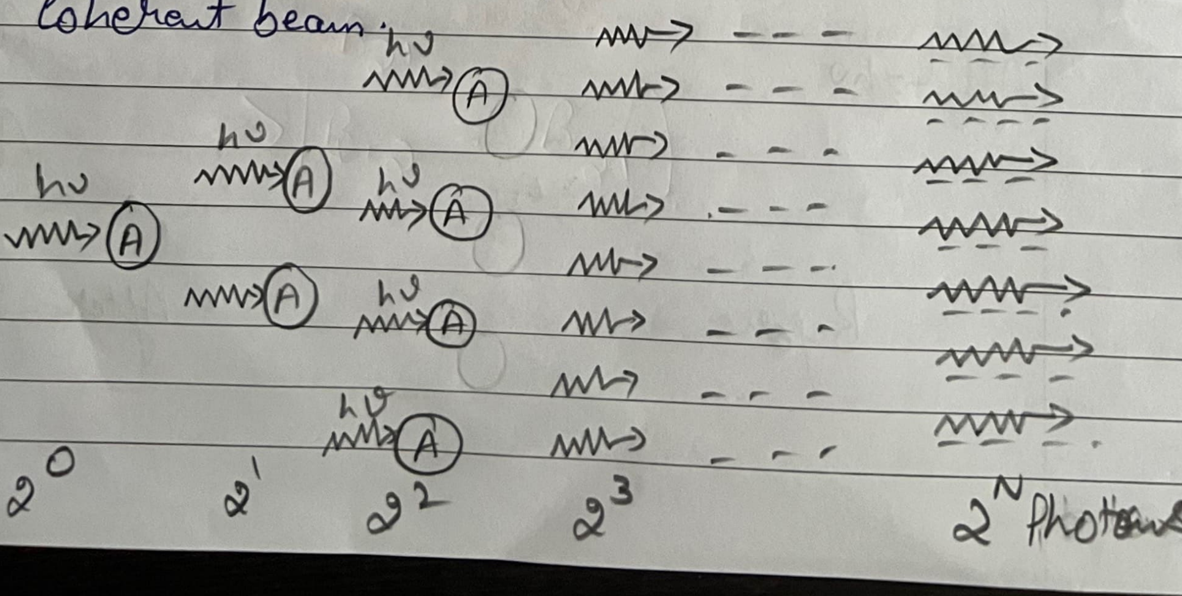
The number of Stimulated transitions will be proportional to the number of electrons in the excited state and radiation density Q.

$$N_{st} \propto N_2 \, Q \Delta t$$

$$\boxed{N_{st} = B_{21} N_2 \, Q \Delta t}$$

B_{21} → Probability of Stimulated transitions

One photon induces an electron to emit two photons, these two photons deexcite two more electrons thereby emitting four photons and so on. The combined effect of all emission from a large sample of atoms leads to amplification of light. The constructive interference of many waves traveling in same direction with same phase produce an intense coherent beam.



Einstein Coefficients :-

Under the thermal equilibrium the number of upward transition is equal to number of downward transition per unit volume per second.

$$\therefore \quad N_{sp} + N_{st} = N_{ab}$$

$$A_{21} N_2 \Delta t + B_{21} N_{21} Q \Delta t = B_{12} N_1 Q \Delta t$$

$$A_{21} N_2 + B_{21} N_2 Q = B_{12} N_1 Q \quad \text{——— } (1)$$

$$A_{21} N_2 = B_{12} N_1 Q - B_{21} N_2 Q$$

Therefore $$Q = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

divide by $B_{21} N_2$

$$Q = \frac{\frac{A_{21} \cancel{N_2}}{B_{21} \cancel{N_2}}}{\frac{B_{12} N_1}{B_{21} N_2} - \cancel{\frac{B_{21} N_2}{B_{21} N_2}}}$$

$$Q = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \quad \text{——— } (2)$$

but $\frac{N_1}{N_2} = e^{\frac{E_2 - E_1}{kT}}$ $\left[E_2 - E_1 = h\nu\right]$

$$Q = \frac{\frac{A_{21}}{B_{21}}}{\frac{B_{12}}{B_{21}} e^{\frac{h\nu}{kT}} - 1} \quad \text{--- } (3)$$

eq (3) must agree with Plank's energy distribution formula.

$$Q = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right] \quad \text{--- } (4)$$

by Comparing eq (3) and (4)

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}}$$

and

$$\frac{B_{12}}{B_{21}} = 1$$

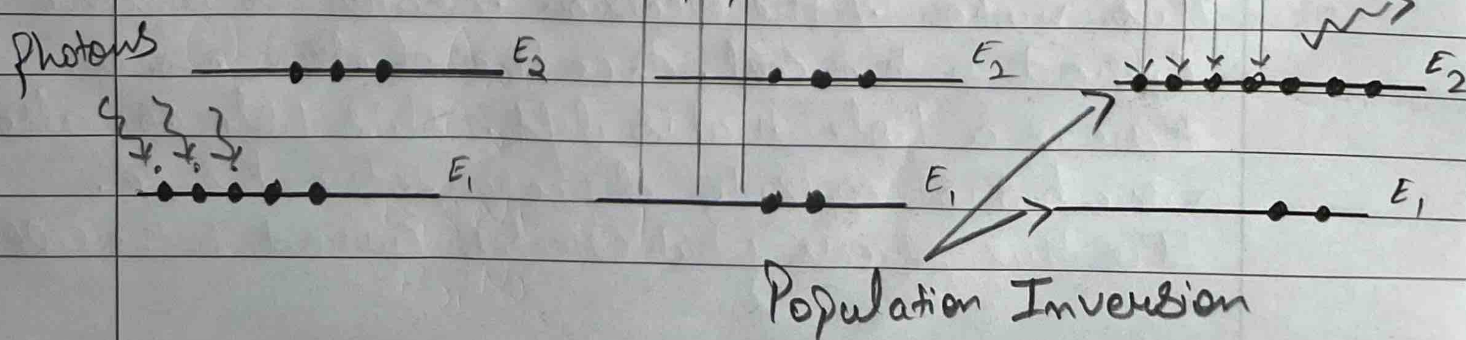
$$\Rightarrow \boxed{B_{12} = B_{21}}$$

Population Inversion :- The non-equilibrium state in which the population N_2 of the upper energy level exceeds to a large extent the population N_1 of the lower energy level is known as the state of population inversion.

→ How to achieve Population Inversion?

• **Pumping :-** In order to achieve population inversion, energy should be supplied to the laser medium. The process of supplying energy to the laser medium is called pumping. By this process electrons are raised from the lower level to the upper level.

1) **Optical Pumping :-** In this method, light is used to supply energy to the laser medium. When light source provides enough energy to the lower energy state electrons in the laser medium jump into the higher energy state E_3. The electrons in the higher energy state do not stay for long time. After a very short time, they fall back to the next lower state E_2 by releasing energy. This method is used in Solid State lasers.



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2) Electric Discharge or Excitation by Electrons :-

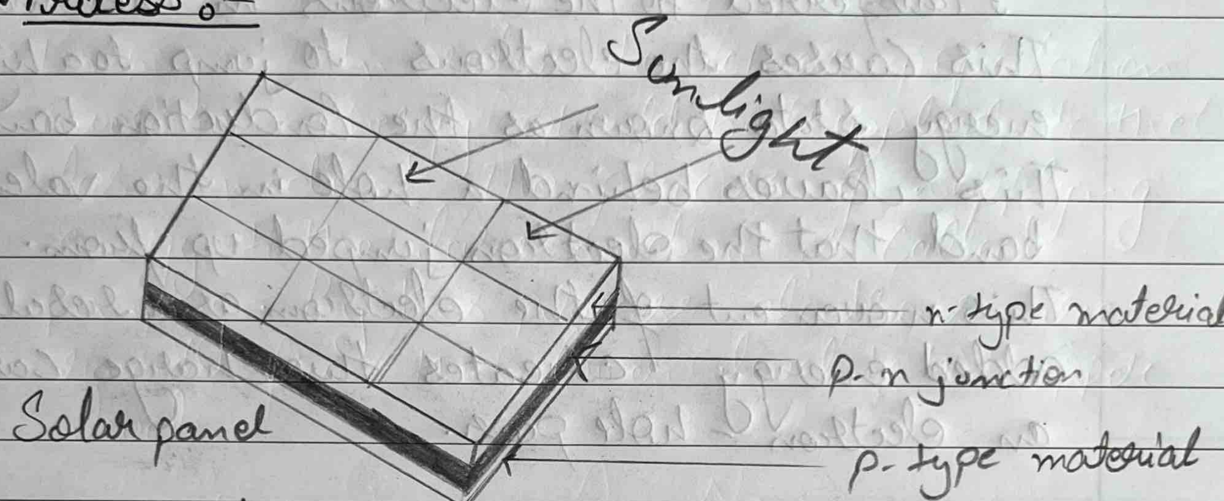
Electric Discharge refers to flow of electrons or electric current through gas, liquid or solid.

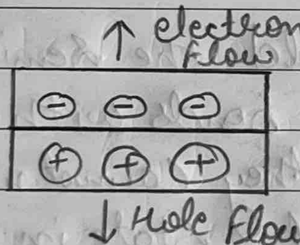
In this method of pumping, electric discharge acts as a energy source. A high voltage electric discharge is passed through the laser medium. The intense electric field accelerates the electrons to high speeds and they collide with neutral neutral atoms. As a result, the electrons in the lower energy state gains sufficient energy from external electrons and jumps into the higher energy state. This method of pumping is used in gas lasers.

3) **Thermal Pumping :-** In this method, population inversion is achieved by supplying heat into the laser medium. When heat is supplied to the laser medium, the lower energy state electrons gains sufficient energy and jumps into the higher energy level.

Photovoltaic Effect :- The photovoltaic effect is a process that generates voltage or electric current in a photovoltaic cell when it is exposed to sunlight. It is this effect that makes solar panels useful, as it is how the cells within the panel convert sunlight to electrical energy.

• **Process :-**





The Photovoltaic effect occurs in solar cells. These solar cells are composed of two different types of semiconductors, P type and N type, that are joined together to create a P-N junction. By joining these two types of semiconductors, an electric field is formed in the region of the junction as electrons move to +ve P side and holes move to -ve N side.

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Light is composed of photons, which are simply small bundles of electromagnetic radiation or energy. When light of a suitable wavelength is incident on these cells, energy from the photon is transferred to an atom of the semiconducting material in the P-N junction. Specifically the energy is transferred to the electrons in the material. This causes the electrons to jump to a higher energy state known as the conduction band. This leaves behind a hole in the valence band that the electron jumped up from. This movement of the electron as a result of added energy creates two charge carriers, an electron - hole pair.

When unexcited, electrons hold the semiconducting material together by forming bonds with the surrounding atoms and thus they cannot move. However in their excited state in conduction band, these electrons are free to move through the material. Because of the electric field that exists as a result of the P-N junction, electrons and holes move in the opposite direction as expected. Insted of being attracted to the P side, the freed electron tends to move to the n side. This motion of the electron creates an electric current in the cell. Once the electron moves, there is a hole that is left. This hole can also move but is opposite direction to the P side. It is this process which creates current in the cell.

Applications of Semiconductor LASER :-

1) They are used as a laser pointer.

2) They are used for storing data on CD or DVD.

3) They are used in optical fiber Communication to provide high-frequency waves for modulating the low frequency signal.

4) They are used as a pumping source in a Solid-State laser.

5) Lasers are used in Laser printing, Bar Code Scanner, Holograms.

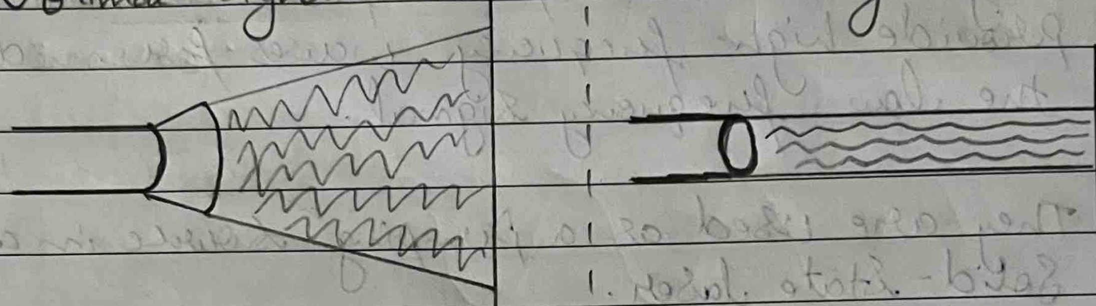
6) Lasers are also used in Military equipments for target designation, Laser weapons, Laser range-finder.

7) Laser is used in Medical Science for surgery of eyes, dentistry, dermatology.

Characteristics of LASER :-

- **Directionality :-** The property of the laser that it travels in one direction only without separating is known as directionality.

Normal Light | Laser light



- **Intensity :-** Since the laser gives out light into a narraw beam and the light energy is Concentrated in a region of a small area, therefore the intensity of the laser beam is very high.

Intensity of a wave is defined as the energy per unit time per unit area.

$$\therefore \quad I = \frac{E/t}{A}$$

$$= \frac{P}{A} \quad (\text{watt } m^{-2})$$

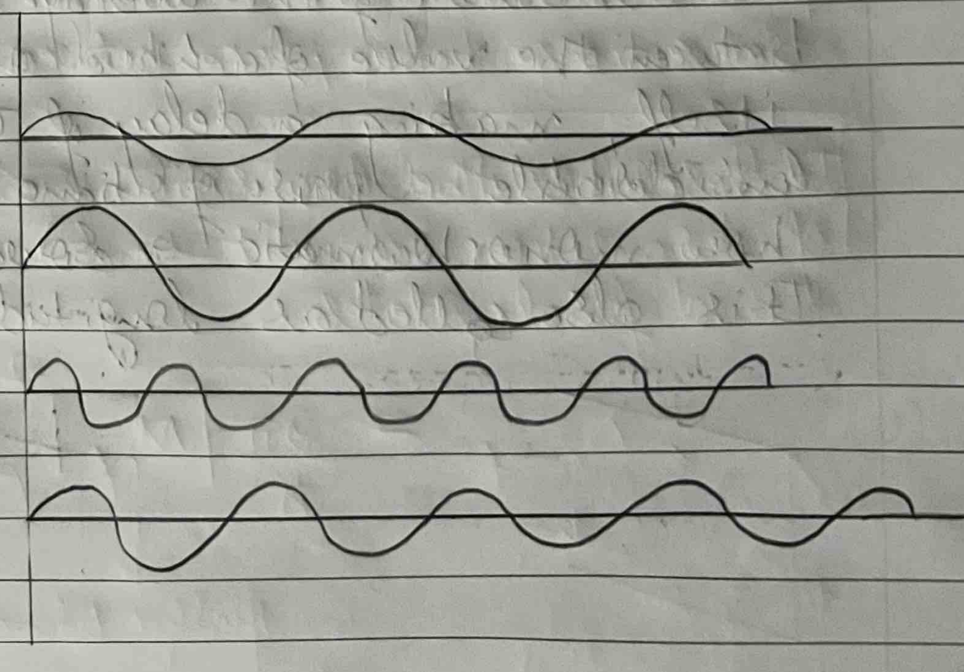
For a typical laser, the $\boxed{I = 10^{13} \ W m^{-2}}$

order of intensity

• <u>Monochromaticity :-</u> Monochromaticity is a property of light containing only one wavelength.
The photons emitted from ordinary light source have different energies, frequencies, wavelengths or colours. Whereas in laser, all the emitted photons have the same energy, frequency or wavelength. Therefore laser light covers a very narrow range of frequencies or wavelengths.

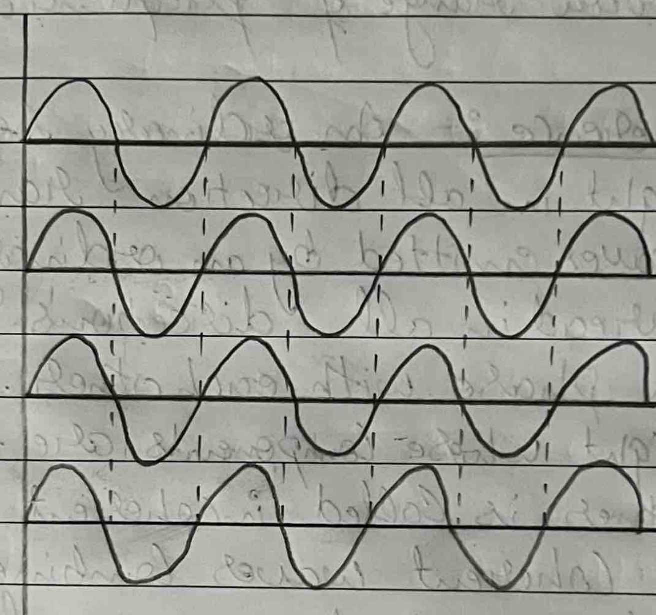
• <u>Coherence :-</u> An ordinary source of light emits light in all direction randomly. The light waves emitted by an ordinary source of light spread in all directions and are not in phase with each other. Such a beam of light whose components are not in phase with each other is called incoherent beam of light.
If coherent waves combine, then the amplitude of resultant wave is always less than the amplitude of any of the combining waves.

Incoherent Waves

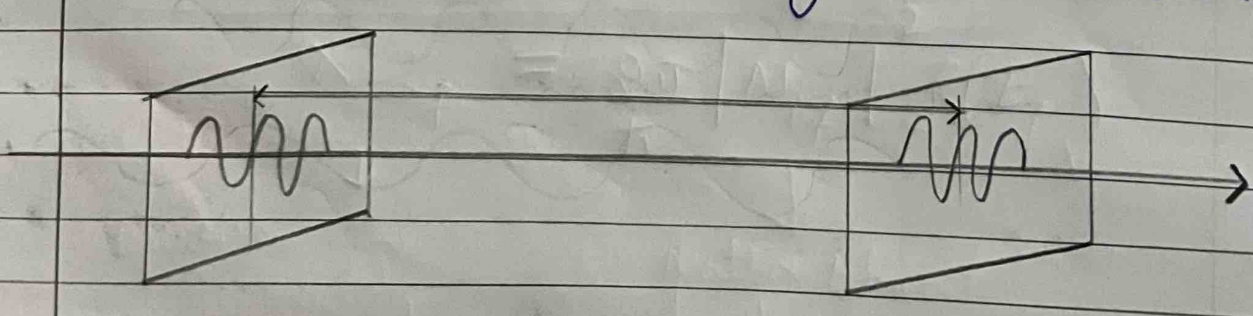


Light waves emitted by laser are always in phase with each other at every point in space. Hence, a laser light beam is known as Coherent beam of light. If Coherent waves combine, then the amplitude of the resultant wave is always greater than the amplitude of any of the combining waves.

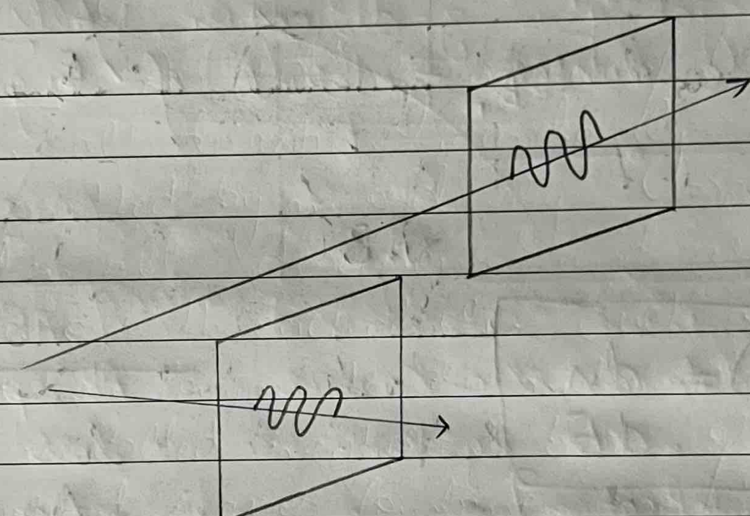
Coherent Waves



-> <u>Temporal Coherence</u> :- Temporal Coherence is a measure of the average relationship between the value of a wave and the coherence itself, making a delay of τ, at any considerable pair of time. It measures how monochromatic a source of light is. It is also called as longitudinal coherence.



→ Spatial Coherence :- Spatial Coherence describes Correlation in a plane perpendicular to the direction of propagation. It is defined as the predictable Correlation of amplitude and phase at one point on the wave with respect to another point on a second wave, then the waves are said to be spatial coherence. Spatial Coherence is also called as transverse Coherence.



Joint Density of States :- The Joint Density of states gives the number of pairs of states (One in Valence Band and one in Conduction Band) per unit volume per unit photon energy interval whose energy difference equals to $h\nu$.

For a given energy $(h\nu)$, there are several pairs of $(E_2 - E_1)$. So, therefore, there are no. of pairs of states available for a photon of energy $(h\nu)$ to interact with semiconductor. This is given by Joint Density of States.

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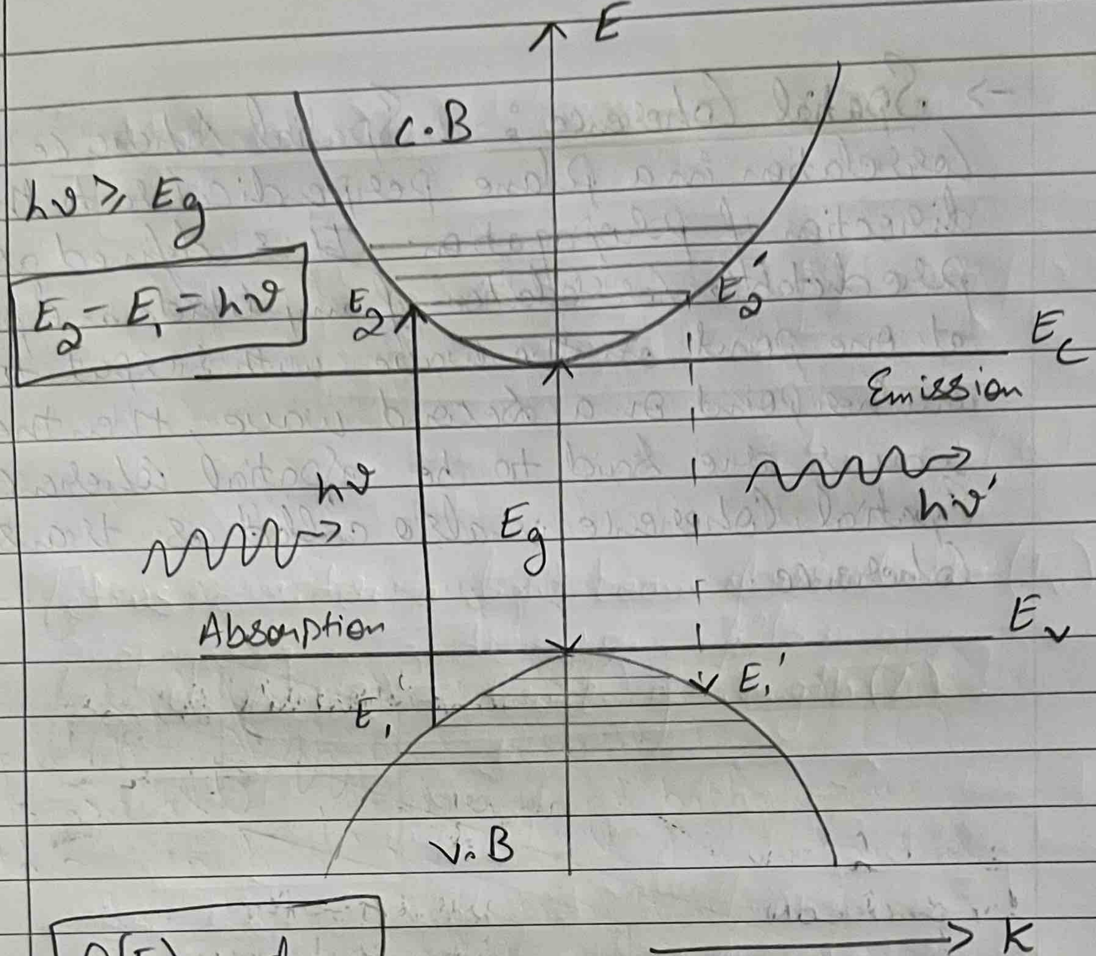

$$\boxed{D(E) = \frac{dn}{dE}}$$

Fermi's - Golden Rule :- Fermi-Golden rule provides the rate at which atomic or electronic transitions take place between two states.

It describes the transition rate (i.e. Probability of transition per unit time) between initial and final states.

$$\lambda_{i \to f} = \frac{2\pi}{\hbar} |M_{if}|^2 \rho(E_f)$$

$\lambda_{i \to f}$ → transition Probability

M_{if} → Matrix Element → indicates Coupling b/w initial and final states

$\rho(E_f)$ → density of final states

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Thus, the transition rate depend upon the strength of the coupling b/w the initial and final states of a system and upon the number of ways the transition can happen (i.e the density of final states.

A transition will take ~~plame~~ place more rapidly if the coupling is stronger.

$$M_{if} = \int \psi_f^{*} \, V \, \psi_i \, d\tau$$

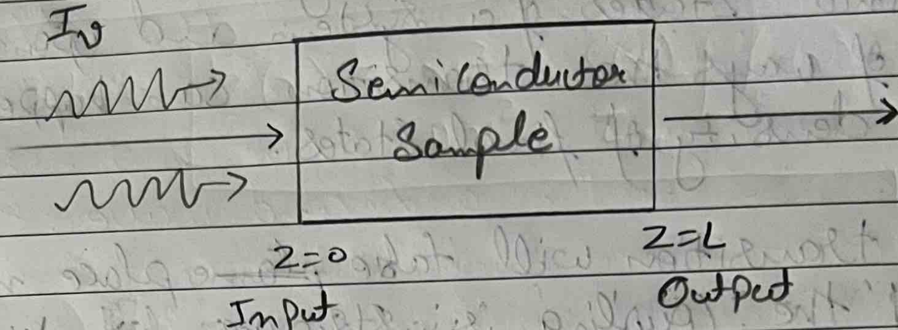
V → Potential Operator that causes the transition to occur

ψ_f^{*} → Wave function for final wave

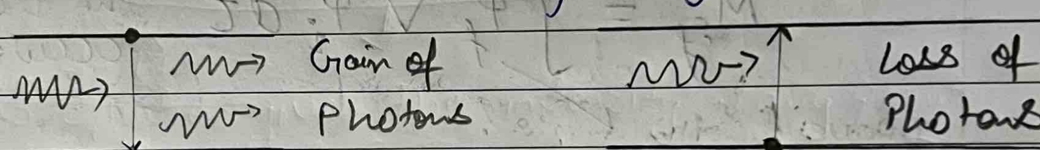
ψ_i → wave function for initial wave

In semiconductors, the Fermi-Golden rule can be used for calculating the transition probability rate for an electron that is excited by a photon from the valence band to conduction band in a direct bandgap semiconductor, and also for when the electron recombines with the hole and emits a photon.

Optical Gain and Loss:-



Let us Consider a light beam of irradiance (I_ν), propagating in z-direction, falls on a Semiconductor material of length (L).



Stimulated Emission	Absorption
Stimulate emission Causes 'gain' in the System. [Amplification]	Absorption Causes 'loss' in the System [Attenuation]

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The rate of stimulated emission is given by

$$\boxed{r_{st}(\nu) = B N_2 \, u(\nu) \, g(\nu)} \quad \text{--- (1)}$$

B -> Einstein's Coefficient

N_2 -> No. of e^- per unit volume in upper Level

$u(\nu)$ -> Energy density of Photons

$$\left[\frac{\text{Energy}}{\text{Volume}} = \frac{n h \nu}{V} \right]$$

$g(\nu)$ -> Line shape function strengthing interaction at frequency (ν)

In Semiconductors, we deal with electrons and holes (insted of N_2 and N_1) in the conduction and valence bands.

For Semiconductors :-

$$\boxed{r_{st}(\nu) = B\,\rho(\nu)\,f_e(\nu)\,u(\nu)} \quad \text{--- (2)}$$

$\rho(\nu) \rightarrow$ Optical Joint density of States
$f_e(\nu) \rightarrow$ Probability of emission
$u(\nu) \rightarrow$ Energy density at frequency (ν)

number of emissions = Joint density of States X Probability of emission
↓
no. of Photons emitted

The rate of Stimulated absorption is given by:-

$$\boxed{r_{ab}(\nu) = B\,\rho(\nu)\,f_a(\nu)\,u(\nu)} \quad \text{--- (3)}$$

$f_a(\nu) \rightarrow$ Probability of Absorption

Now, the variation of intensity (I_ν) with distance (z) within the Sample is given by:-

$$\boxed{\frac{dI_\nu}{dz} = \frac{v^2}{8\pi\nu^2}\,\frac{\rho(\nu)}{\tau_r}\left[f_e(\nu) - f_a(\nu)\right] I_\nu(z)} \quad \text{--- (4)}$$

$\tau_r \rightarrow$ Recombination lifetime $= \frac{1}{A}$

$$\boxed{\frac{dI_\nu}{dz} = \gamma(\nu)\, I_\nu(z)} \quad \text{--- (5)}$$

where $$\boxed{\gamma(\nu) = \frac{\lambda^2}{8\pi \nu^2} \frac{g(\nu)}{\tau_r} \left[f_e(\nu) - f_a(\nu) \right]} \quad \text{--- (6)}$$

$\gamma(\nu) \rightarrow$ Gain Coefficient

The Solution of eq (5) can be written as

$$\boxed{I_\nu(z) = I_\nu(0)\, e^{\gamma z}} \quad \text{--- (7)}$$

This equation gives the intensity at any point within the Sample [any value of distance z]

$I_\nu(0) \rightarrow$ Intensity at input [z=0]

$I_\nu(z) \rightarrow$ Intensity at any point within the Sample

using eq (7), intensity at output (z=L) is

$$\boxed{I_\nu(L) = I_\nu(0)\, e^{\gamma L}} \quad \text{--- (8)}$$

Now 'Gain' is defined as:-

$$\boxed{\frac{\text{Output}}{\text{Input}} = \frac{I_\nu(L)}{I_\nu(0)} = e^{\gamma L}} \quad \text{--- (9)}$$

If, $I_\nu(L) > I_\nu(0)$ → γ is positive, there is a gain in the system [Amplification]

from eq (6)

$$f_e(\nu) - f_a(\nu) > 0$$

$$\boxed{f_e(\nu) > f_a(\nu)} \quad — (10)$$

Probability of Emission > Probability of Absorption

[$\gamma > 0$ → Gain or Amplification
$\gamma < 0$ → Loss or Attenuation]

Density of States for Phonons :-

• **Phonon** :- The quanta of lattice vibrational energy (i.e. Sound) is known as Phonon.

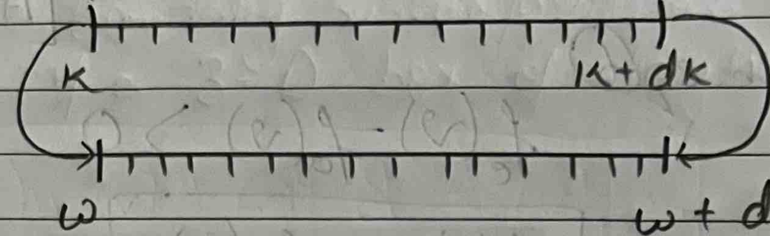
Atoms of the crystal vibrate which create normal modes.
→ Normal modes are the vibrations of a particular frequency.
[i.e longitudinal or transversal]

Let $D(\omega)\, d\omega$ = no. of modes b/w frequency ω and $\omega + d\omega$

$D(\omega)$ = density of states in ω-range

$D(K)\, dK$ = no. of modes b/w wave vector K and $K + dK$

$D(k)$ = density of states in k-range



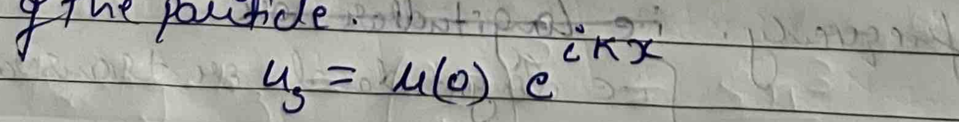
$$D(k)\,dk = D(\omega)\,d\omega$$

Total no. of modes are equal

$$D(\omega) = D(k)\frac{dk}{d\omega}$$

→ **In 1-D :-**

Consider the boundary value problem for vibrations of a 1-D line of length L carrying $N+1$ particles at separation a:



Each normal vibrational mode has the form of a standing wave, where u_s is the displacement of the particle

$$u_s = u(0)\,e^{ikx}$$

On applying the periodic boundary conditions over primitive cells within a cube of side L

$$e^{ikx} = e^{ik(x+L)}$$

$$e^{ikx} = e^{ikx}\cdot e^{ikL}$$

$$e^{ikL} = 1$$

$$\cos KL + i \sin KL = 1 + i0$$

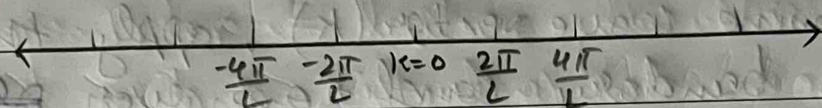
$$\Rightarrow \cos KL = 1 \quad , \quad i \sin KL = 0$$

$$\sin KL = 0$$

$$KL = \pm n\pi \quad , \text{ where } n = 0, 2, 4 ---$$

$$K = \pm \frac{n\pi}{L}$$

$$K = 0, \pm \frac{2\pi}{L}, \pm \frac{4\pi}{L}, \pm \frac{6\pi}{L} ----$$



$\therefore$ length corresponding to one state $= \frac{2\pi}{L}$

no. of states in $\frac{2\pi}{L}$ length $= 1$

no. of states in 1 length $= \dfrac{1}{\frac{2\pi}{L}} = \dfrac{L}{2\pi}$

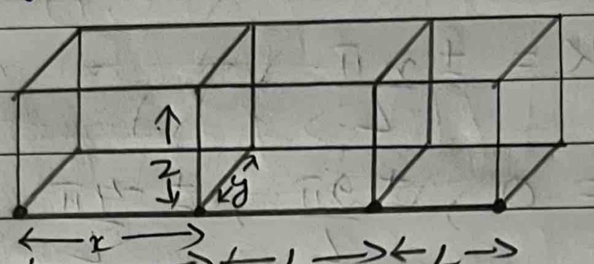
$$\frac{\text{no. of states}}{\text{length}} = \frac{L}{2\pi}$$

$$\therefore D(\omega) = \frac{L}{2\pi}$$

$$\Rightarrow D(\omega) \propto \omega^0$$

→ **In 3-D :-**

Consider the bondary value problem for vibrations of a 3-D box of length L.



To determine wave vector (K), apply the periodic bondary conditions over primitive cell within a cube of length L.

$$e^{i(K_x x + K_y y + K_z z)} = e^{iK_x(x+L) + iK_y(y+L) + iK_z(z+L)}$$

$$e^{iK_x x} + e^{iK_y y} + e^{iK_z z} = \left(e^{iK_x x} + e^{iK_y y} + e^{iK_z z}\right) e^{iK_x K_y K_z L}$$

$$e^{iK_x K_y K_z L} = 1$$

$$\therefore \; K_x, K_y, K_z = 0, \pm\frac{2\pi}{L}, \pm\frac{4\pi}{L}, \pm\frac{6\pi}{L} \ldots$$

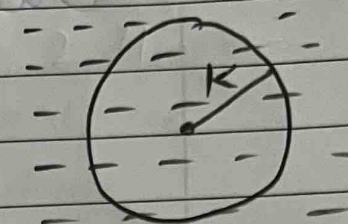
$$\text{Volume} = K_x \cdot K_y \cdot K_z = \left(\frac{2\pi}{L}\right)^3 = \frac{8\pi^3}{L^3}$$

no. of states in $\left(\frac{2\pi}{L}\right)^3$ volume = 1

no. of states in 1 volume = $\dfrac{1}{\left(\frac{2\pi}{L}\right)^3}$

$$\frac{\text{no. of states}}{\text{volume}} = \left(\frac{L}{2\pi}\right)^3$$

K sphere



no. of states in volume $\frac{4}{3}\pi K^3 =$

$$N = \left(\frac{L}{2\pi}\right)^3 \cdot \frac{4}{3}\pi K^3$$

$$\boxed{k = \frac{2\pi}{\lambda} \qquad k = \frac{\overset{\omega}{2\pi\nu}}{v}}$$

$$N = \left(\frac{L}{2\pi}\right)^3 \cdot \frac{4}{3}\pi\left(\frac{\omega}{v}\right)^3$$

L^3 = volume of cube $\qquad v = \lambda\nu \qquad \lambda = \frac{v}{\nu}$

$$N = \frac{L^3}{\underset{2}{\cancel{8}}\pi^{\cancel{3}2}} \cdot \frac{\cancel{4}}{3}\cancel{\pi}\frac{\omega^3}{v^3}$$

$$N = \frac{V\cdot\omega^3}{6\pi^2 v^3}$$

$V \to$ volume of cube

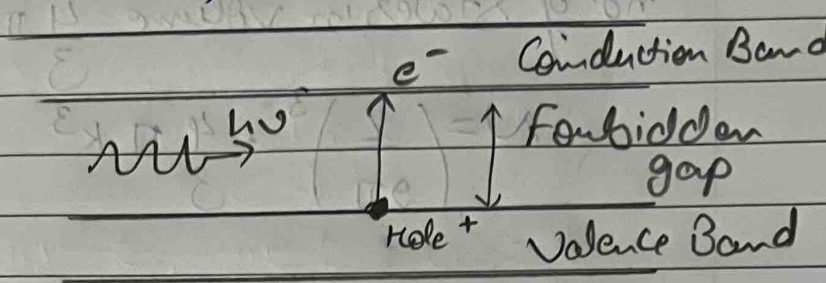
$$D(\omega) = \frac{dN}{d\omega} = \frac{d\left(\frac{V\cdot\omega^3}{6\pi^2 v^3}\right)}{d\omega}$$

$$D(\omega) = \frac{\cancel{3}\, V\cdot\omega^2}{{}_2\cancel{6}\pi^2 v^3}$$

$$D(\omega) = \frac{V\cdot\omega^2}{2\pi^2 v^3}$$

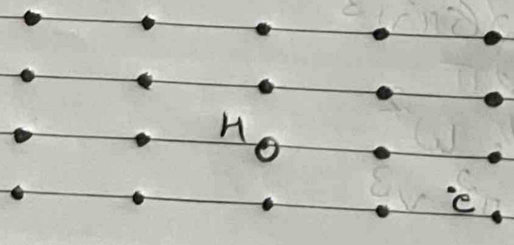
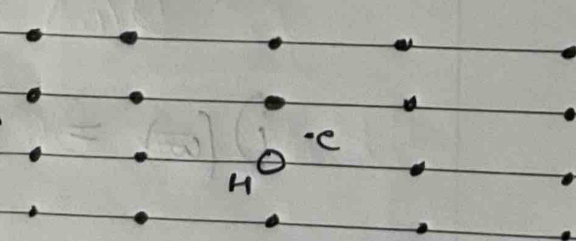
$$D(\omega) \propto \omega^2$$

Exciton :- An exciton is a bound state of electron and hole which is attracted by an electrostatic Coulomb force.



- It is electrically neutral => net charge is zero.
- It can be in semiconductors and in insulators
- They transport energy without charge transfer.
- They have slightly less energy than unbounded electron.
- They have a time period, after that it will dissociate or recombine.

→ Types of Exciton :-

Wannier Mott Exciton	Frenkel Exciton
	
→ It is free to move within the crystal lattice.	→ It is tightly bounded within the crystal lattice.
→ Found in organic semiconductor Si, Ge etc.	→ Found in inorganic material, insulators.

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-> distance b/w e^- and hole is large.	-> distance b/w e^- and hole is small.
-> Large exciton.	-> Small exciton.
-> Force b/w e^- and hole is 0.01 eV i.e binding energy.	-> Force b/w e^- and hole is 0.1 - 1eV i.e Binding energy.
-> Found in low bandgap semiconductors.	-> Found in materials with large bandgap.
-> Formed in materials with large dielectric constant.	-> Formed in materials with low dielectric constant.