2/Sep

Maths

Matrices

Additive Identity [0] zero
Remains Same

Multiplicative Identity [1] one
Number multiplied by any number gives same number.

Zero Matrix
or
Null Matrix
↓
A matrix where all the elements are zero.

Identity Matrix or unitary Matrix
↓
A square matrix ~~of~~ where all the non diagonal elements are zero and diagonal elements are all one.

* **Diagonal Matrix**
A square Matrix is said to be diagonal matrix iff [if and only if];
i) All the non diagonal elements are zero.
(ii). Diagonal elements can have any value.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -7 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

But we cannot have all three to be zero in diagonal entries.

* **Scalar Matrix**
A diagonal matrix where all the diagonal elements are same.

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$$

* **Transpose** of a Matrix.
obtained by inter changing rows & columns.

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 4 & 7 \end{bmatrix} \quad A^t / A' / A^T = \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 5 & 7 \end{bmatrix}$$

* Symmetric Matrix

A square matrix A will be called a symmetric matrix if; $A^T = A$.

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 4 & 9 & 8 \\ 2 & 8 & -3 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 4 & 2 \\ 4 & 9 & 8 \\ 2 & 8 & -3 \end{bmatrix}$$

$$A = A^T$$

* Skew - Symmetric Matrix

A square matrix A will be called a skew symmetric matrix if $A^T = -A$

$$A = \begin{bmatrix} 0 & 7 & 5 \\ -7 & 0 & 9 \\ -5 & -9 & 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 0 & -7 & -5 \\ 7 & 0 & -9 \\ 5 & 9 & 0 \end{bmatrix}$$

$$A^T = -A \qquad A^T = -\begin{bmatrix} 0 & 7 & 5 \\ -7 & 0 & 9 \\ -5 & -9 & 0 \end{bmatrix}$$

Q→ $A = \begin{bmatrix} 3 & -2 & -4 \\ 1 & 5 & 6 \\ 2 & -3 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -4 & 2 \\ 1 & -5 & 0 \\ 2 & 1 & -2 \end{bmatrix}$

verify;

(i) $(A+B)^T = A^T + B^T$

(ii) $(AB)^T = B^T A^T$

(iii) $(A-B)^T = A^T - B^T$

(iv) $(BA)^T = A^T B^T$

Reversal law $\rightarrow (AB)^T = B^T A^T$

$(AB)^T \neq A^T B^T$,

(i) $A+B = \begin{bmatrix} 3+1 & -2-4 & -4+2 \\ 1+1 & 5-5 & 6+0 \\ 2+2 & -3+1 & 9-2 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & -6 & -2 \\ 2 & 0 & 6 \\ 4 & -2 & 7 \end{bmatrix}$

$(A+B)^T = \begin{bmatrix} 4 & 2 & 4 \\ -6 & 0 & -2 \\ -2 & 6 & 7 \end{bmatrix}$

$A^T = \begin{bmatrix} 3 & 1 & 2 \\ -2 & 5 & -3 \\ -4 & 6 & 9 \end{bmatrix}$ $B^T = \begin{bmatrix} 1 & 1 & 2 \\ -4 & -5 & 1 \\ 2 & 0 & -2 \end{bmatrix}$

$A^T + B^T \Rightarrow \begin{bmatrix} 3+1 & 1+1 & 2+2 \\ -2-4 & 5-5 & -3+1 \\ -4+2 & 6+0 & 9-2 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 2 & 4 \\ -6 & 0 & -2 \\ -2 & 6 & 7 \end{bmatrix}$

$(A+B)^T = A^T + B^T$

Hence verified.

(ii). $(AB)^T \Rightarrow B^T A^T$

$AB = \begin{bmatrix} 3-2-8 & -12+10-4 & 6+8 \\ 1+5+12 & -4-25+6 & 2-12 \\ 2-3+18 & -8+15+9 & 4-18 \end{bmatrix} = \begin{bmatrix} -7 & -6 & 14 \\ 18 & -23 & -10 \\ 17 & 16 & -14 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} -7 & 18 & 17 \\ -6 & -23 & 16 \\ 14 & -10 & -14 \end{bmatrix}$

$B^T \Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ -4 & -5 & 1 \\ 2 & 0 & -2 \end{bmatrix}$ $A^T \begin{bmatrix} 3 & 1 & 2 \\ -2 & 5 & -3 \\ -4 & 6 & 9 \end{bmatrix}$

$B^T A^T \Rightarrow \begin{bmatrix} 3-2-8 & 1+5+12 & 2-3+18 \\ -12+10-4 & -4-25+6 & -8+15+9 \\ 6+8 & 2-12 & 4-18 \end{bmatrix} = \begin{bmatrix} -7 & 18 & 17 \\ -6 & -23 & 16 \\ 14 & -10 & -14 \end{bmatrix}$

$(AB)^T = B^T A^T$

Hence verified.

(iii) $A-B = \begin{bmatrix} 3-1 & -2+4 & -4-2 \\ 1-1 & 5+5 & 6-0 \\ 2-2 & -3-1 & 9+2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -6 \\ 0 & 10 & 6 \\ 0 & -4 & 11 \end{bmatrix}$

$(A-B)^T = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 10 & -4 \\ -6 & 6 & 11 \end{bmatrix}$

$A^T = \begin{bmatrix} 3 & 1 & 2 \\ -2 & 5 & -3 \\ -4 & 6 & 9 \end{bmatrix}$ $B^T = \begin{bmatrix} 1 & 1 & 2 \\ -4 & -5 & 1 \\ 2 & 0 & -2 \end{bmatrix}$

$A^T - B^T = \begin{bmatrix} 3-1 & 1-1 & 2-2 \\ -2+4 & 5+5 & -3-1 \\ -4-2 & 6-0 & 9+2 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 2 & 10 & -4 \\ -6 & 6 & 11 \end{bmatrix}$

$(A-B)^T = A^T - B^T$; Hence verified

(iv) $(BA)^T = A^T B^T$

$BA \Rightarrow \begin{bmatrix} 3-4+4 & -2-20-6 & -4-24+18 \\ 3-5 & -2-25 & -4-30 \\ 6+1-4 & -4+5+6 & -8+6-18 \end{bmatrix}$

$BA \Rightarrow \begin{bmatrix} 3 & -28 & -10 \\ -2 & -27 & -34 \\ 3 & 7 & -20 \end{bmatrix}$ $(BA)^T \Rightarrow \begin{bmatrix} 3 & -2 & 3 \\ -28 & -27 & 7 \\ -10 & -34 & -20 \end{bmatrix}$

$A^T B^T \Rightarrow \begin{bmatrix} 3-4+4 & 3-5 & 6+1-4 \\ -2-20-6 & -2-25 & -4+5+6 \\ -4-24+18 & -4-30 & -8+6-18 \end{bmatrix}$

$A^T B^T \Rightarrow \begin{bmatrix} 3 & -2 & 3 \\ -28 & -27 & 7 \\ -10 & -34 & -20 \end{bmatrix}$

So; $(BA)^T = A^T B^T$

Hence verified.

3/Sep

Mathematics

Determinants -> Algebric solution of square matrix

2 X 2

I ↖ − ↗ II

| | -> Parallel lines for (determinant).

→ $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = 3 - 8 = -5$

→ $B = \begin{bmatrix} -3 & 7 \\ -2 & 1 \end{bmatrix}$

$|B| = \begin{vmatrix} -3 & 7 \\ -2 & 1 \end{vmatrix} = -3 + 14 = 11$

→ $D = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 4 & 5 \\ 2 & 3 & 1 \end{bmatrix}$

$|D| = \begin{vmatrix} 3^{+} & 1^{-} & 2^{+} \\ 1 & 4 & 5 \\ 2 & 3 & 1 \end{vmatrix}$

$= 3(4-15) - 1(1-10) + 2(3-8)$

$= 3(-11) - 1(-9) + 2(-5)$

$= -33 + 9 - 10 = -33 - 1$

$= \boxed{-34}$

→ $E = \begin{bmatrix} -1 & -3 & -5 \\ 8 & -2 & -4 \\ 7 & 2 & 1 \end{bmatrix}$

$|E|$ Find

$$|E| = \begin{vmatrix} -1 & -3 & -5 \\ 8 & -2 & -4 \\ 7 & 2 & 1 \end{vmatrix}$$

$\Rightarrow -1(-2+8) + 3(8+28) - 5(16+14)$

$\Rightarrow -1(6) + 3(36) - 5(30)$

$\Rightarrow -6 + 108 - 150$

$\Rightarrow -48$

$\rightarrow A = \begin{bmatrix} 3 & 5 & 2 \\ 1 & 6 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

$|A^2|$ Find?

$A^2 \Rightarrow \begin{bmatrix} 3 & 5 & 2 \\ 1 & 6 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 5 & 2 \\ 1 & 6 & 1 \\ 2 & 1 & 2 \end{bmatrix} \Rightarrow$

$\Rightarrow \begin{bmatrix} 9+5+4 & 15+30+2 & 6+5+4 \\ 3+6+2 & 5+36+1 & 2+6+2 \\ 6+1+4 & 10+6+2 & 4+1+4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 18 & 47 & 15 \\ 11 & 42 & 10 \\ 11 & 18 & 9 \end{bmatrix}$

$|A^2| \Rightarrow 18(378-180) - 47(99-110) + 15(198-462)$

$\Rightarrow 18(198) - 47(-11) + 15(-264)$

$\Rightarrow 3564 + 517 - 3960$

$\Rightarrow$ ~~4897 8041~~ $\boxed{121}$ #

$\Rightarrow B = \begin{bmatrix} -3 & 4 & 5 \\ 1 & 2 & 7 \\ 9 & 1 & 0 \end{bmatrix}$

$B^2 \Rightarrow \begin{bmatrix} -3 & 4 & 5 \\ 1 & 2 & 7 \\ 9 & 1 & 0 \end{bmatrix} \begin{bmatrix} -3 & 4 & 5 \\ 1 & 2 & 7 \\ 9 & 1 & 0 \end{bmatrix}$

$B^2 \Rightarrow \begin{bmatrix} +9+4+45 & -12+8+5 & -15+28 \\ -3+2+63 & 4+4+7 & 5+14 \\ -27+1 & 36+2 & 45+7 \end{bmatrix}$

$B^2 \Rightarrow \begin{bmatrix} 58 & 1 & 13 \\ 62 & 15 & 19 \\ -26 & 38 & 52 \end{bmatrix}$

$|B^2| \Rightarrow 58(780-722) - 1(3224+494) + 13(2356+390)$

$\Rightarrow 58(58) - 1(3718) + 13(2746)$

$\Rightarrow 3364 - 3718 + 35698$

$\Rightarrow 3364 + 32170$

$\Rightarrow \boxed{35534}$

$39062 - 3718$

$\boxed{35344}$

6/Sep Maths.

→ If a set of eq has as many variables as eq, it is called as simultaneous equations.

Eg; $3x - 4z = 4$
$2y - 3x = 7$
$5y + 2z = -1$

$x - 2y + 3z = -4$
$2x + y - 7z = 3$
$2x + 5y - z = 2$

$\updownarrow$

$3x + 0y - 4z = 4$
$-3x + 2y + 0z = 7$
$0x + 5y + 2z = -1$

Methods to solve simultaneous eq's.
① Crammar's Rule
② Matrix Inversion Method

I → Crammar's Rule;

$a_1x + b_1y + c_1z = p$
$a_2x + b_2y + c_2z = q$
$a_3x + b_3y + c_3z = r$

I Convert into matrix

Δ = Delta

$$\underbrace{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} p \\ q \\ r \end{bmatrix}}_{B}$$

II Find $|A| = \Delta$ find $\Delta_1, \Delta_2, \Delta_3$

[To find $\Delta_1, \Delta_2, \Delta_3$
$\Delta_1 = \begin{bmatrix} p & b_1 & c_1 \\ q & b_2 & c_2 \\ r & b_3 & c_3 \end{bmatrix}$ $\Delta_2 = \begin{bmatrix} a_1 & p & c_1 \\ a_2 & q & c_2 \\ a_3 & r & c_3 \end{bmatrix}$
$\Delta_3 = \begin{bmatrix} a_1 & b_1 & p \\ a_2 & b_2 & q \\ a_3 & b_3 & r \end{bmatrix}$]

III Check the value of Δ

$\boxed{\text{If } \Delta \neq 0}$ (solvable)
System is consistent & has unique solution
$x = \frac{\Delta_1}{\Delta}$ $y = \frac{\Delta_2}{\Delta}$ $z = \frac{\Delta_3}{\Delta}$

$\boxed{\text{If } \Delta = 0}$

If $\Delta_1, \Delta_2, \Delta_3$ are also all zero
System is consistent, & has infinite solutions

If $\Delta_1, \Delta_2, \Delta_3$ are ~~all~~ not zero
System is inconsistent, so there will be no-sol.

8/Sep Maths

Q → Solve by Cramer's Rule.

(1)

$$2x - y + 2z = 3$$
$$x + y - 3z = 1$$
$$3x + 2y + z = 4$$

$$\underset{A}{\begin{bmatrix} 2 & -1 & 2 \\ 1 & 1 & -3 \\ 3 & 2 & 1 \end{bmatrix}} \times \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}}$$

$$|A| = 2(1+6) + 1(1+9) + 2(2-3)$$
$$= 2(7) + 1(10) + 2(-1)$$
$$= 14 + 10 - 2 \Rightarrow 22$$

$$|A| = \Delta$$

$$\Delta_1 \Rightarrow \begin{vmatrix} 3 & -1 & 2 \\ 1 & 1 & -3 \\ 4 & 2 & 1 \end{vmatrix}$$

$$\Rightarrow 3(1+6) + 1(1+12) + 2(2-4)$$
$$\Rightarrow 3(7) + 1(13) + 2(-2)$$
$$\Rightarrow 21 + 13 - 4 \Rightarrow 34 - 4 \Rightarrow 30$$

$$\Delta_2 \Rightarrow \begin{vmatrix} 2 & 3 & 2 \\ 1 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix}$$

$$\Rightarrow 2(1+12) - 3(1+9) + 2(4-3)$$
$$\Rightarrow 2(13) - 3(10) + 2(1)$$
$$\Rightarrow 26 - 30 + 2 \Rightarrow -2$$

$$\Delta_3 \Rightarrow \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 3 & 2 & 4 \end{vmatrix}$$

$$\Rightarrow 2(4-2) + 1(4-3) + 3(2-3)$$
$$\Rightarrow 2(+2) + 1(1) + 3(-1)$$
$$\Rightarrow +4 + 1 - 3 \Rightarrow +2$$

$\Delta \neq 0$

System is consistent and has unique sol.

$$x = \frac{\Delta_1}{\Delta} = \frac{\cancel{30}^{15}}{\cancel{22}_{11}} = \boxed{\frac{15}{11}}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-\cancel{2}}{\cancel{22}} = \boxed{\frac{-1}{11}}$$

$$z = \frac{\Delta_3}{\Delta} = \frac{+\cancel{2}\,1}{\cancel{22}\,11} \Rightarrow \boxed{\frac{+1}{11}}$$

→ To check the value of $x\ y\ z$.

→ $2x - y + 2z = 3$

$$2\left(\frac{15}{11}\right) + \frac{1}{11} + 2\left(\frac{1}{11}\right) = 3$$

$$\frac{30}{11} + \frac{1}{11} + \frac{2}{11} = 3$$

$$\frac{33}{11} = 3$$

$$\boxed{3 = 3}$$

→ $3x + 2y + z = 4$

$$3\left(\frac{15}{11}\right) + 2\left(\frac{-1}{11}\right) + \frac{1}{11} = 4$$

$$\frac{45}{11} - \frac{2}{11} + \frac{1}{11} = 4$$

$$\frac{44}{11} = 4$$

$$\boxed{4 = 4}$$

→ $x + y - 3z = 1$

$$\frac{15}{11} + \left(\frac{-1}{11}\right) - 3\left(\frac{1}{11}\right) = 1$$

$$\frac{15}{11} - \frac{1}{11} - \frac{3}{11} = 1$$

$$\frac{15 - 4}{11} = 1$$

$$\frac{11}{11} = 1$$

$$\boxed{1 = 1}$$

Q→ $2x - y + 3z = 1$

② $4x - 2y + 6z = 2$

$x + 5y - z = 3$

$$\underset{A}{\begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ 1 & 5 & -1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}$$

$|A| = 2(2-30) + 1(-4-6) + 3(20+2)$

$\Rightarrow 2(-28) + 1(-10) + 3(22)$

$\Rightarrow -56 - 10 + 66$

$\Rightarrow -66 + 66$

$\boxed{|A| \Rightarrow 0} \Rightarrow \Delta$

$$\Delta_1 \Rightarrow \begin{vmatrix} 1 & -1 & 3 \\ 2 & -2 & 6 \\ 3 & 5 & -1 \end{vmatrix}$$

$\Rightarrow 1(2-30) + 1(-2-18) + 3(10+6)$

$\Rightarrow 1(-28) + 1(-20) + 3(16)$

$\Rightarrow -28 - 20 + 48$

$\Rightarrow -48 + 48 \Rightarrow \boxed{0}$

$$\Delta_2 \Rightarrow \begin{vmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & -1 \end{vmatrix}$$

$\Rightarrow 2(-2-18) - 1(-4-6) + 3(12-2)$

$\Rightarrow 2(-20) - 1(-10) + 3(10)$

$\Rightarrow -40 + 10 + 30$

$\Rightarrow -40 + 40 \Rightarrow 0$

$$\Delta_3 \Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ 4 & -2 & 2 \\ 1 & 5 & 3 \end{vmatrix}$$

$\Rightarrow 2(-6-10)+1(12-2)+1(20+2)$

$\Rightarrow 2(-16)+1(10)+1(22)$

$\Rightarrow -32+10+22$

$\Rightarrow -32+32$

$\Delta_3 \Rightarrow 0$

$\Delta = 0$

$\Delta_1 = \Delta_2 = \Delta_3 = 0$

System is consistent and has infinite sol.

Q → $3x - 2y + 4z = 1$

(3) $6x - 4y + 3z = 2$

$9x - 6y + 8z = 1$

$$\begin{bmatrix} 3 & -2 & 4 \\ 6 & -4 & 3 \\ 9 & -6 & +8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$|A| \Rightarrow 3(-32+18)+2(+48-27)+4(-36+36)$

$\Rightarrow 3(-14)+2(21)+0$

$\Rightarrow -42+42$

$|A| \Rightarrow 0$

$$\Delta_1 \Rightarrow \begin{vmatrix} 1 & -2 & 4 \\ 2 & -4 & 3 \\ 1 & -6 & 8 \end{vmatrix}$$

$\Rightarrow 1(-32+18) + 2(16-3) + 4(-12+4)$

$\Rightarrow 1(-14) + 2(13) + 4(-8)$

$\Rightarrow -14 + 26 - 32$

$\Rightarrow -46 + 26 \Rightarrow -20$

$$\Delta_2 \Rightarrow \begin{vmatrix} 3 & 1 & 4 \\ 6 & 2 & 3 \\ 9 & 1 & 8 \end{vmatrix}$$

$\Rightarrow 3(16-3) - 1(48-27) + 4(6-18)$

$\Rightarrow 3(13) - 1(21) + 4(-12)$

$\Rightarrow 39 - 21 - 48$

$\Rightarrow 39 - 69$

$\Rightarrow -30$

$$\Delta_3 \Rightarrow \begin{vmatrix} 3 & -2 & 1 \\ 6 & -4 & 2 \\ 9 & -6 & 1 \end{vmatrix}$$

$\Rightarrow 3(-4+12) + 2(6-18) + 1(-36+36)$

$\Rightarrow 3(8) + 2(-12) + 0$

$\Rightarrow 24 - 24 + 0$

$\Delta_3 \Rightarrow 0$

$\Delta. = 0$

$\Delta_1 = -20 \quad \Delta_2 = -30 \quad \Delta_3 = 0$

System is inconsistent and has no sol.

Q→ $2x - 3y = 1$
$x + 3z = 11$
$x + 2y + z = 7$

$$\underset{A}{\begin{bmatrix} 2 & -3 & 0 \\ 1 & 0 & 3 \\ 1 & 2 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 1 \\ 11 \\ 7 \end{bmatrix}}$$

$|A| = \Delta = \boxed{-18}$

$\Delta_1 = -36 \qquad \Delta_2 = -18 \qquad \Delta_3 = -54$

Since $\Delta \neq 0$

System is consistent & will have unique sol.

$x = \frac{\Delta_1}{\Delta} = \frac{-36}{-18} = 2$

$y = \frac{\Delta_2}{\Delta} = \frac{-18}{-18} = 1$

$z = \frac{\Delta_3}{\Delta} = \frac{-54}{-18} = 3$

Q→ $x - y + z = 3$
$2x + y - z = 2$
$-x - 2y + 2z = 1$

$$\underset{A}{\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}}$$

$$\Delta = |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{vmatrix} = 1(2-2) + 1(4-1) + 1(-4+1)$$

$\Rightarrow 0 + 1(3) + 1(-3)$

$\Rightarrow 3 - 3 \Rightarrow \boxed{0}$

$$\Delta_1 = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & 2 \end{vmatrix} = 3(2-2) + 1(4+1) + 1(-4-1)$$

$$= 0 + 5 + 1(-5)$$

$$\Rightarrow 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 3 & +1 \\ 2 & 2 & -1 \\ -1 & 1 & 2 \end{vmatrix} = 1(4+1) - 3(4-1) + 1(2+2)$$

$$\Rightarrow 5 - 3(3) + 1(4)$$

$$\Rightarrow 5 - 9 + 4 \Rightarrow 0$$

$$\Delta_3 = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -1 & -2 & 1 \end{vmatrix} = 1(1+4) + 1(2+2) + 3(-4+1)$$

$$\Rightarrow 5 + 1(4) + 3(-3)$$

$$\Rightarrow 0$$

Since $\Delta = 0$

$\Delta_1 = \Delta_2 = \Delta_3 = 0$

System is ~~in~~consistent & has ~~no~~ infinite sol.

Q→ $2x + 5y = 7$
$6x + 15y = 13$

$$\underset{A}{\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 7 \\ 13 \end{bmatrix}}$$

$$\Delta = |A| = \begin{vmatrix} 2 & 5 \\ 6 & 15 \end{vmatrix} = 30 - 30$$
$$= 0$$

$$\Delta_1 = \begin{vmatrix} 7 & 5 \\ 13 & 15 \end{vmatrix} = 105 - 65$$
$$\Rightarrow 40$$

$$\Delta_2 = \begin{vmatrix} 2 & 7 \\ 6 & 13 \end{vmatrix} = 26 - 42$$
$$\Rightarrow -16$$

$\cancel{\Delta_3 =}$

$\Delta = 0$

$\Delta_1, \Delta_2 \neq 0$

System is inconsistent & has no sol.

(1) $x + 2y + 3z = 14$
$3x + y + 2z = 11$
$2x + 3y + z = 11$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 1(1-6) - 2(3-4) + 3(9-2)$$
$$= -5 - 2(-1) + 3(7)$$
$$= -5 + 2 + 21$$
$$\Rightarrow \boxed{18}$$

$$\Delta_1 = \begin{vmatrix} 14 & 2 & 3 \\ 11 & 1 & 2 \\ 11 & 3 & 1 \end{vmatrix}$$

$$\Rightarrow 14(1-6) - 2(11-22) + 3(33-11)$$
$$\Rightarrow 14(-5) - 2(-11) + 3(22)$$
$$\Rightarrow -70 + 22 + 66$$
$$\Rightarrow -70 + 88 \Rightarrow \boxed{18}$$

$$\Delta_2 = \begin{vmatrix} 1 & 14 & 3 \\ 3 & 11 & 2 \\ 2 & 11 & 1 \end{vmatrix}$$

$$\Rightarrow 1(11-22) - 14(3-4) + 3(33-22)$$
$$\Rightarrow -11 - 14(-1) + 3(11)$$
$$\Rightarrow -11 + 14 + 33 \Rightarrow -11 + 47$$
$$\Rightarrow \boxed{36}$$

$$\Delta_3 \Rightarrow \begin{vmatrix} 1 & 2 & 14 \\ 3 & 1 & 11 \\ 2 & 3 & 11 \end{vmatrix}$$

$$\Rightarrow 1(11-33) - 2(33-22) + 14(9-2)$$
$$\Rightarrow -22 - 2(11) + 14(7)$$
$$\Rightarrow -22 - 22 + 98 \Rightarrow -44 + 98 \Rightarrow \boxed{54}$$

$\Delta \neq 0$

system is consistent & has unique sol.

$$x = \frac{\Delta_1}{\Delta} = \frac{18}{18} = 1$$

$$y = \frac{\Delta_2}{\Delta} = \frac{36}{18} = 2$$

$$z = \frac{\Delta_3}{\Delta} = \frac{54}{18} = 3$$

(2) $3x - 5z = -1$

$2x + 7y = 6$

$x + y + z = 5$

$$|A| = \begin{vmatrix} 3 & 0 & -5 \\ 2 & 7 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 3(7) - 0 - 5(2-7)$$

$\Rightarrow 21 - 5(-5)$

$\Rightarrow 21 + 25 \Rightarrow \boxed{46}$

$$\Delta_1 \Rightarrow \begin{vmatrix} -1 & 0 & -5 \\ 6 & 7 & 0 \\ 5 & 1 & 1 \end{vmatrix} \Rightarrow -1(7) - 5(6-35)$$

$\Rightarrow -7 - 5(-29)$

$\Rightarrow -7 + 145 \Rightarrow \boxed{138}$

$$\Delta_2 \Rightarrow \begin{vmatrix} 3 & -1 & -5 \\ 2 & 6 & 0 \\ 1 & 5 & 1 \end{vmatrix} \Rightarrow 3(6-0) + 1(2) - 5(10-6)$$

$\Rightarrow 18 + 2 - 20 \Rightarrow \boxed{0}$

$$\Delta_3 \Rightarrow \begin{vmatrix} 3 & 0 & -1 \\ 2 & 7 & 6 \\ 1 & 1 & 5 \end{vmatrix} \Rightarrow 3(35-6) - 1(2-7)$$

$\Rightarrow 3(29) - 1(-5) \Rightarrow 87 + 5$

$\Rightarrow \boxed{92}$

~~x~~ System is consistent & has unique sol.

$x \Rightarrow \frac{138}{46} \Rightarrow \boxed{3}$ $\quad y \Rightarrow \boxed{0}$ $\quad z \Rightarrow \frac{92}{46} \Rightarrow \boxed{2}$

(13) $6x + y - 3z = 5$
$x + 3y - 2z = 5$
$2x + y + 4z = 8$

$$|A| = \begin{vmatrix} 6 & 1 & -3 \\ 1 & 3 & -2 \\ 2 & 1 & 4 \end{vmatrix} = 6(12+2) - 1(4+4) - 3(1-6)$$
$$= 6(14) - 1(8) - 3(-5)$$
$$\Rightarrow 84 - 8 + 15$$
$$\Rightarrow 99 - 8 \Rightarrow \boxed{91}$$

$$\Delta_1 \Rightarrow \begin{vmatrix} 5 & 1 & -3 \\ 5 & 3 & -2 \\ 8 & 1 & 4 \end{vmatrix} \Rightarrow 5(12+2) - 1(20+16) - 3(5-24)$$
$$\Rightarrow 70 - 36 + 57$$
$$\Rightarrow 70 + 21 \Rightarrow \boxed{91}$$

$$\Delta_2 = \begin{vmatrix} 6 & 5 & -3 \\ 1 & 5 & -2 \\ 2 & 8 & 4 \end{vmatrix} \Rightarrow 6(20+16) - 5(4+4) - 3(8-10)$$
$$\Rightarrow 6(36) - 5(8) - 3(-2)$$
$$\Rightarrow 216 - 40 + 6$$
$$\Rightarrow 222 - 40 \Rightarrow \boxed{182}$$

$$\Delta_3 = \begin{vmatrix} 6 & 1 & 5 \\ 1 & 3 & 5 \\ 2 & 1 & 8 \end{vmatrix} \Rightarrow 6(24-5) - 1(8-10) + 5(1-6)$$
$$\Rightarrow 6(19) + 2 + 5(-5)$$
$$\Rightarrow 114 + 2 - 25 \Rightarrow 116 - 25$$
$$\Rightarrow \boxed{91}$$

$$x = \frac{\Delta_1}{\Delta} = \frac{91}{91} = 1$$
$$y = \frac{\Delta_2}{\Delta} = \frac{182}{91} = 2$$
$$z = \frac{\Delta_3}{\Delta} = \frac{91}{91} = 1$$

4. $x + y + z = 1$
$x + 2y + 3z = 4$
$x + 3y + 5z = 7$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} \Rightarrow 1(10-9) - 1(5-3) + 1(3-2)$$
$$\Rightarrow 1(1) - 1(2) + 1(1)$$
$$\Rightarrow 1 - 2 + 1 \Rightarrow \boxed{0}$$

$$\Delta_1 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 3 \\ 7 & 3 & 5 \end{vmatrix} \Rightarrow 1(10-9) - 1(20-21) + 1(12-14)$$
$$\Rightarrow 1 + 1 + 1(-2)$$
$$\Rightarrow 2 - 2 \Rightarrow \boxed{0}$$

$$\Delta_2 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 7 & 5 \end{vmatrix} \Rightarrow 1(20-21) - 1(5-3) + 1(7-4)$$
$$\Rightarrow -1 - 2 + 3$$
$$\Rightarrow -3 + 3 \Rightarrow \boxed{0}$$

$$\Delta_3 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \end{vmatrix} \Rightarrow 1(14-12) - 1(7-4) + 1(3-2)$$
$$\Rightarrow 2 - 3 + 1 \Rightarrow \boxed{0}$$

System is consistent & has infinite sol.

14/Sep

Maths

Adjoint of a Matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} \text{cofactor} \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} -1 & -5 & 4 \\ 0 & -3 & 2 \\ 1 & 7 & -5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ -5 & -3 & 7 \\ 4 & 2 & -5 \end{bmatrix}$$

	Minor		Cofactor
R_1	1	-1	$+ -1 = -1$
	2	5	$- 5 = -5$
	3	4	$+ 4 = 4$
R_2	3	0	$- 0 = 0$
	1	-3	$+ -3 = -3$
	2	-2	$- -2 = 2$
R_3	2	1	$+ 1 = 1$
	2	-7	$- -7 = 7$
	3	-5	$+ -5 = -5$

Q→ $A \Rightarrow \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

$$\text{Adj } A = \begin{bmatrix} 2 & 3 & -9 \\ -2 & -1 & 7 \\ 2 & -1 & -1 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2 & -2 & 2 \\ 3 & -1 & -1 \\ -9 & 7 & -1 \end{bmatrix}$$

	Minor		Cofactor
R_1	2	2	$+ 2 = 2$
	3	-3	$- -3 = 3$
	1	-9	$+ -9 = -9$
R_2	3	2	$- 2 = -2$
	4	-1	$+ -1 = -1$
	2	-7	$- -7 = 7$
R_3	3	2	$+ 2 = 2$
	1	1	$- 1 = -1$
	1	-1	$+ -1 = -1$

Q → A ⇒ $\begin{bmatrix} 7 & -3 & 2 \\ -1 & 4 & 3 \\ -4 & 2 & 5 \end{bmatrix}$

Adj A ⇒ $\begin{bmatrix} 14 & -7 & 14 \\ 19 & 43 & -26 \\ -17 & -23 & 25 \end{bmatrix}^T$

⇒ $\begin{bmatrix} 14 & 19 & -17 \\ -7 & 43 & -23 \\ 14 & -26 & 25 \end{bmatrix}$

Minor	Cofactor
$R_1 = 7 \quad 14$	$+14 = 14$
$= -3 \quad 7$	$-7 = -7$
$= 2 \quad 14$	$+14 = 14$
$R_2 = -1 \quad -19$	$- -19 = 19$
$= 4 \quad 43$	$+43 = 43$
$= 3 \quad 26$	$-26 = -26$
$R_3 = -4 \quad -17$	$+ -17 = -17$
$= 2 \quad 23$	$-23 = -23$
$= 5 \quad 25$	$+25 = 25$

15/Sep. Maths

Inverse of a Matrix

→ Additive Inverse.
$3 + ? = 0$ Additive Identity

→ Multiplicative Inverse
$3 \times ? = 1$ multiplicative Identity

Let A be any matrix then Inverse of A will be denoted by A^{-1}.

$$A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

Note A^{-1} exists if & only if $|A| \neq 0$

If $|A| = 0$; A^{-1} is not possible.

Q→ $A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 1 & 0 \\ 3 & 2 & -3 \end{bmatrix}$ find A^{-1}.

$$|A| = \begin{vmatrix} 3 & 1 & 4 \\ 2 & 1 & 0 \\ 3 & 2 & -3 \end{vmatrix} = 3(-3-0) - 1(-6-0) + 4(4-3)$$
$$= 1$$

$|A| = 1 \neq 0$
A^{-1} exists.

Find adj A.

adj $A \Rightarrow \begin{bmatrix} -3 & 11 & -4 \\ 6 & -21 & 8 \\ 1 & -3 & 1 \end{bmatrix}$

$A^{-1} \Rightarrow \frac{1}{1}$ adj A

$\boxed{A^{-1} \Rightarrow \text{adj } A}$

Q → $A \Rightarrow \begin{bmatrix} 3 & 2 & 4 \\ 1 & 5 & 3 \\ 2 & 6 & -2 \end{bmatrix}$

$A \Rightarrow \begin{vmatrix} 3 & 2 & 4 \\ 1 & 5 & 3 \\ 2 & 6 & -2 \end{vmatrix}$ $\Rightarrow 3(-10-18) - 2(-2-6) + 4(6-10)$

$\Rightarrow 3(-28) - 2(-8) + 4(-4)$

$\Rightarrow -84 + 16 - 16$

$\Rightarrow \boxed{-84} + 2 \Rightarrow \boxed{-82}$

$\boxed{|A| = -87}$

Adj $A \Rightarrow \begin{vmatrix} -28 & -8 & -4 \\ 28 & -14 & +14 \\ -14 & +5 & 13 \end{vmatrix}^T$

$\Rightarrow \begin{bmatrix} -28 & 28 & -14 \\ -8 & -14 & -5 \\ -4 & +14 & 13 \end{bmatrix}$

$A^{-1} \Rightarrow \frac{1}{-84} \begin{bmatrix} -28 & 28 & -14 \\ -8 & -14 & -5 \\ -4 & -14 & 13 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} +\frac{1}{3} & -\frac{1}{3} & +\frac{1}{6} \\ +\frac{2}{21} & \frac{1}{6} & +\frac{5}{84} \\ \frac{1}{21} & -\frac{1}{6} & -\frac{13}{84} \end{bmatrix}$

To check $AA^{-1} = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q → $A = \begin{bmatrix} -3 & -4 & 5 \\ 2 & -6 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

$$\begin{array}{ccc} + & - & + \end{array}$$

$|A| = \begin{vmatrix} -3 & -4 & 5 \\ 2 & -6 & 1 \\ 2 & -3 & 4 \end{vmatrix}$ → $-3(-24+3)+4(8-2)+5(-6+12)$

→ $-3(-21)+4(6)+5(6)$

→ $63+24+30$

→ $\boxed{117}$

Adj A → $\begin{bmatrix} -21 & -6 & 6 \\ 1 & -22 & -17 \\ 26 & 13 & -10 \end{bmatrix}^T$

→ $\begin{vmatrix} -21 & 1 & 26 \\ -6 & -22 & 13 \\ 6 & -17 & +26 \end{vmatrix}$

A^{-1} → $\frac{1}{117}\begin{bmatrix} -21 & 1 & 26 \\ -6 & -22 & 13 \\ 6 & -17 & +26 \end{bmatrix}$

0/9

Maths.

Rank will be zero if null matrix is given.

Rank of a Matrix → No of non-zero eqns of a matrix.

Rank can never be zero or (-)ve.

(ρ) Rho → Symbol of Rank

Q→ $A = \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 3 & 5 & 2 & 1 \\ 1 & 4 & 3 & 4 \\ 6 & 10 & 4 & 2 \end{bmatrix}$

Elementary Row Transformation

$R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 3 & 5 & 2 & 1 \\ 1 & 4 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\rho(A) = 2}$

→ Shortcut Method [determinant method] (Square matrix)

Q→ $A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 5 & 10 & 15 \end{bmatrix}_{3\times3}$

$B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 6 \\ 3 & 4 & -1 \end{bmatrix}_{3\times3}$

$|A| \Rightarrow 2(30-30) - 4(15-15) + 6(0)$

$|A| \Rightarrow 0$

If $|A| = 0$

order ≠ Rank ≠ 3

then find $|A| = 2\times2$ order

$|A_1| = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} = 0$ $|A_2| = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix} = 0$

$|A_3| = \begin{bmatrix} 1 & 2 \\ 5 & 10 \end{bmatrix} = 0$ $|A_4| = \begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix} = 0$

$\rho(A) \neq 2$

So, by default Ans = $\boxed{\rho = 1}$

$|B| \Rightarrow 1(-5-24) - 3(-2-18) + 2(8-15)$

$\Rightarrow 1(-29) - 3(-20) + 2(-7)$

$\Rightarrow -29 + 60 - 14$

$\Rightarrow -43 + 60 \Rightarrow 17$

If $|B| \neq 0$

order = Rank = $\rho = 3$

Q→ $A = \begin{bmatrix} 7 & 3 & 2 \\ 1 & 5 & 3 \\ 28 & 12 & 8 \end{bmatrix}$

$|A| \Rightarrow 7(40-36) - 3(8-84) + 2(12-140)$

$\Rightarrow 7(4) - 3(-76) + 2(-128)$

$\Rightarrow 28 + 228 - 256$

$\Rightarrow 256 - 256 \Rightarrow \boxed{0}$

$|A_1| \Rightarrow \begin{bmatrix} 7 & 3 \\ 1 & 5 \end{bmatrix} \Rightarrow 35 - 3 \Rightarrow 32$

$\rho(A) = 2$

[Rectangular Matrix]

Q→ $A \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 5 & 3 \\ 2 & 4 & 6 & 8 \end{bmatrix}_{3\times 4}$

firstly check the order.

$\rho(A)$ can not be more than 3 [smallest element in order]

Possible ranks = 3, 2, 1

Now find 3×3

$A_1 \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \\ 2 & 4 & 6 \end{bmatrix} = 0$ $\quad A_2 \Rightarrow \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 3 \\ 4 & 6 & 8 \end{bmatrix} = 0$

$A_3 \Rightarrow \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 3 \\ 2 & 4 & 6 \end{bmatrix} = 0$ $\quad A_4 \Rightarrow \begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 3 \\ 2 & 6 & 8 \end{bmatrix} = 0$

$\rho \neq 3$

Now find 2 x 2

$A_1 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $\Rightarrow 1 - 4$ $\Rightarrow -3$

$A_2 = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$ $\Rightarrow 10 - 3$ $\Rightarrow 7$

$A_3 \Rightarrow \begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}$ $A_3 \Rightarrow 9 - 20$ $\Rightarrow -11$

$\boxed{\rho = 2}$ #

4/oct

Eigen values

Let A be any matrix then λ will be called Eigen value of A iff $AX = \lambda X$.
where X is any random matrix.

$$AX = \lambda X$$
$$AX - \lambda X = 0$$
$$AX - \lambda I X = 0$$
$$(A - \lambda I) X = 0$$

characteristic eq is given by;

$$|A - \lambda I| = 0$$

$$A - \lambda I = \begin{bmatrix} & \\ & \end{bmatrix} - \lambda \begin{bmatrix} & \\ & \end{bmatrix}$$

Quadratic eq.
$ax^2 + bx + c = 0$
Cubic eq
$ax^3 + bx^2 + cx + d = 0$
Biquadratic eq
$ax^4 + bx^3 + cx^2 + dx + e = 0$

* Cubic eq:

$$x^3 - 6x^2 + 11x - 6 = 0$$

Hit & trial method;

Put $x = 1 \quad -1$

$2 \quad -2$

$3 \quad -3$

$x = 1$ $\quad 1 - 6 + 11 - 6 = 0$

$12 - 12 = 0$

$0 = 0$

So; $(x-1)$ is one factor.

$$
\begin{array}{l}
x-1 \,\big)\, x^3 - 6x^2 + 11x - 6 \,\big(\, x^2 - 5x + 6 \\
\quad x^3 - x^2 \\
\quad\quad - \quad + \\
\hline
\quad\quad -5x^2 + 11x - 6 \\
\quad\quad -5x^2 + 5x \\
\quad\quad + \quad\quad - \\
\hline
\quad\quad\quad 6x - 6 \\
\quad\quad\quad 6x - 6 \\
\quad\quad\quad - \quad + \\
\hline
\quad\quad\quad 0
\end{array}
$$

$$x^3 - 6x^2 + 11x - 6 = 0$$

$$(x-1).(x^2 - 5x + 6) = 0$$

$$(x-1)\,(x^2 - 3x - 2x + 6) = 0$$

$$(x-1)\,(x-3)\,(x-2) = 0$$

$x \Rightarrow \boxed{1, 2, 3}$ #

Quadratic eq.

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x^2 - 5x + 6 = 0$$

$$x = \frac{-(-5) \pm \sqrt{25 - 4(6)}}{2}$$

$$\Rightarrow \frac{5 \pm \sqrt{25-24}}{2} \Rightarrow \frac{5 \pm 1}{2}$$

$$x \Rightarrow \frac{5+1}{2} \Rightarrow \frac{6}{2} \Rightarrow \boxed{3} \qquad x \Rightarrow \frac{5-1}{2} \Rightarrow \frac{4}{2} \Rightarrow \boxed{2}$$

5/oct

II. Matrix Inversion Method

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

I
$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$
$$A \quad X \quad = B$$

II Find $|A|$

If $|A| \neq 0$

System is consistent & has unique sol.

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

If $|A| = 0$

Find (adj A)

Find (adj A) B

$$\begin{bmatrix} = \\ = \\ = \end{bmatrix}$$

If (adj A) B = all zero
System is consistent & has infinite sol

If (adj A) B ≠ all zero
System is inconsistent & has no sol.

Q→ $x+y+z=2$
$x+2y+3z=5$
$2x+3y+4z=11$

$$\begin{bmatrix}1&1&1\\1&2&3\\2&3&4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\5\\11\end{bmatrix}$$

$|A| = 1(8-9)-1(4-6)+1(3-4)$
$= 1(-1)-1(-2)+1(-1)$
$= -1+2-1 \Rightarrow -2+2 = \boxed{0}$

$|A| = 0$

$\text{adj } A = [\text{Cofactor}]^T$

$$\begin{bmatrix}-1&2&-1\\-1&2&-1\\1&-2&1\end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix}-1&-1&1\\2&2&-2\\-1&-1&1\end{bmatrix}$$

	Minor	Cofactor
R_1		
	1 -1	+ -1 = -1
	1 -2	− -2 = 2
	1 -1	+ -1 = -1
R_2		
	1 1	− 1 = -1
	2 2	+ 2 = 2
	3 1	− 1 = -1
R_3		
	2 1	+ 1 = 1
	3 2	− 2 = -2
	4 1	+ 1 = 1

$(\text{adj } A)\ B$

$$\begin{bmatrix}-1&-1&1\\2&2&-2\\-1&-1&1\end{bmatrix}\begin{bmatrix}2\\5\\11\end{bmatrix}$$

$$\begin{bmatrix}-2-5+11\\4+10-22\\-2-5+11\end{bmatrix}=\begin{bmatrix}4\\-8\\4\end{bmatrix}\neq 0$$

System is inconsistent & has no sol.

Q→ ~~x+y+~~ $2x - y + 3z = 9$

$x + y + z = 6$

$x - y + z = 2$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$|A| = 2(1+1) + 1(1-1) + 3(-1-1)$

$= 4 + 0 + 3(-2)$

$\Rightarrow 4 - 6 \Rightarrow -2$

$|A| \neq 0$

System is consistent & has unique sol.

$\text{adj}A = [\text{Cofactor}]^T$

$$\begin{bmatrix} 2 & 0 & -2 \\ -2 & -1 & 1 \\ -4 & 1 & 3 \end{bmatrix}^T$$

$$\begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

	Minor	Cofactor
R_1		
	$2 \quad 2$	$+2 = 2$
	$-1 \quad 0$	$-0 = 0$
	$3 \quad -2$	$+ -2 = -2$
R_2		
	$1 \quad 2$	$-2 = -2$
	$1 \quad -1$	$+ -1 = -1$
	$1 \quad -1$	$- -1 = 1$
R_3		
	$1 \quad -4$	$+ -4 = -4$
	$-1 \quad -1$	$- -1 = 1$
	$1 \quad 3$	$+ 3 = 3$

$A^{-1} = \frac{1}{|A|}(\text{adj}A)$

$\Rightarrow \frac{1}{-2}(\text{adj}A)$

$$A^{-1} \Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & -\frac{3}{2} \end{bmatrix}$$

$$X = A^{-1} B$$

$$= \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1/2 & -1/2 \\ 1 & -1/2 & -3/2 \end{bmatrix} \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} -9+6+4 \\ 0+3-3 \\ 9-3-3 \end{bmatrix} = \begin{bmatrix} -9+10 \\ 0 \\ 9-6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$x = 1$; $y = 0$; $z = 3$

12/oct

Gauss Elimination Method

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

After applying conditions.

$$A = \begin{bmatrix} a & b & c \\ d & e & 0 \\ g & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad \left(\begin{array}{l} \text{Min } 1(0) \text{ in } R_2 \\ \text{Min } 2(0) \text{ in } R_3 \end{array} \right)$$

$$ax + by + cz = p$$
$$dx + ey = q$$
$$gx = r$$

$$\boxed{x = \frac{r}{g}} \quad \boxed{y \Rightarrow -} \quad \boxed{z = -}$$

Q→ $x + y + z = 2$
$x + 2y + 3z = 5$
$2x + 3y + 4z = 11$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 11 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 11 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$x + y + z = 2$$
$$y + 2z = 3$$
$$0 = 4$$

No Solution

Q→ $y + z = 2$
$2x + 3z = 5$
$x + y + z = 3$

$$\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 0 & 1 & 1 \\ 2 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$$

$y + z = 2$
$2x + 3z = 5$
$x = 1$

$x = 1$
$2 + 3z = 5$
$3z = 3$
$z = 1$

$y = 2 - 1$
$y = 1$

8 →

$x + y + z = 3$

$x + 2y + 2z = 5$

$3x + 4y + 4z = 11$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 3 & 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$x + y + z = 3$

$y + z = 2$

$x = 1$

$\cancel{x = x}$

$\cancel{y = x}$

Infinite sol.

Q→ $x - 2y + z = 0$
$2x + y - 3z = 5$
$4x - 7y + z = -1$

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -3 \\ 4 & -7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 2 \\ 2 & 1 & -3 \\ 4 & -7 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 5 & -5 \\ 4 & -7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix}$$

$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 5 & -5 \\ 0 & +1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix}$$

$R_3 \to 5R_3 - R_2$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 5 & -5 \\ 0 & 0 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -10 \end{bmatrix}$$

$x - 2y + z = 0$
$5y - 5z = 5$
$-10z = -10$
$z = 1$

$5y = 5 + 5$
$y = 2$

$x - 4 + 1 = 0$
$x = 3$

$$\begin{bmatrix} - & - & - \\ 0 & - & - \\ 0 & 0 & - \end{bmatrix}$$ Echlon matrix

$$\left[\begin{array}{ccc|c} 1 & -2 & 2 & 0 \\ 2 & 1 & -3 & 5 \\ 4 & -7 & 1 & -1 \end{array}\right]$$ → Augmented matrix

Q→ $2x - 2y = -6$

$x - y + z = 1$

$3y - 2z = -5$

$$\begin{bmatrix} 2 & -2 & 0 \\ 1 & -1 & 1 \\ 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \\ -5 \end{bmatrix}$$

$R_2 \to 2R_2 - R_1$

$-2+2$

$-2+1$

$2+6$

$$\begin{bmatrix} 2 & -2 & 0 \\ 0 & 0 & 2 \\ 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 8 \\ -5 \end{bmatrix}$$

$R_2 \longleftrightarrow R_3$

$$\begin{bmatrix} 2 & -2 & 0 \\ 0 & 3 & -2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ -5 \\ 8 \end{bmatrix}$$

$2x - 2y = -6$

$3y - 2z = -5$

$2z = 8$

$z = 4$

$3y = -5 + 8$

$y = 1$

$2x = -6 + 2$

$x = -2$

18/oct.

Gauss Jordan Method

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$A \quad X = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$x = p$

$y = q$

$z = r$

Q→ $2x + y + 2z = 10$

$x + 2y + z = 8$

$3x + y - z = 2$

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 2 \end{bmatrix}$$

$R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 2 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 2R_1$

$R_3 \longrightarrow R_3 - 3R_1$

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$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & -5 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -6 \\ -22 \end{bmatrix}$$

$R_2 \to R_2 / -3$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & -5 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ -22 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$
$R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ -12 \end{bmatrix}$$

$R_3 \to R_3 / -4$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & +1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$$

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1$; $y = 2$; $z = 3$

Q→ ① If $A^2 - 8.A + KI = 0$

Find K. Given $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

Sol→ $A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1-0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} = 0$

$\begin{bmatrix} -7+K & 0 \\ 0 & -7+K \end{bmatrix} = 0$

$-7 + K = 0$

$\boxed{K = 7}$ #

Q→ $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Sol→ $A - \lambda I \Rightarrow \begin{bmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{bmatrix}$

$|A - \lambda I| \Rightarrow 1-\lambda\left[(5-\lambda)(1-\lambda)-1\right] - 1\left[1-\lambda-3\right] + 3\left[1-15+3\lambda\right]$

$\Rightarrow 1-\lambda\left[5-5\lambda-\lambda+\lambda^2-1\right] - 1\left[-\lambda-2\right] + 3\left[-14+3\lambda\right]$

$\Rightarrow 1-\lambda\left[\lambda^2-6\lambda+4\right] + \left[\lambda+2\right] + 3\left[-14+3\lambda\right]$

$\Rightarrow \lambda^2 - \lambda^3 - \cancel{6\lambda} + 6\lambda^2 + 4 - \cancel{4\lambda} + \cancel{\lambda} + 2 - 42 + \cancel{9\lambda}$

$\Rightarrow -\lambda^3 + 7\lambda^2 - 36$

-10
$+10$
42
-6
36

Put $\lambda \Rightarrow 3$

$-27 + 63 - 36 \Rightarrow 63 - 63 \Rightarrow 0$

$\boxed{\lambda - 3}$ is the factor.

$$
\begin{array}{l}
\lambda - 3 \;\big)\; -\lambda^3 + 7\lambda^2 - 36 \;\big(\; -\lambda^2 + 4\lambda + 12 \\
\quad\quad -\lambda^3 + 3\lambda^2 \\
\quad\quad + \quad\quad - \\
\hline
\quad\quad\quad 4\lambda^2 - 36 \\
\quad\quad\quad -4\lambda^2 \quad -12\lambda \\
\quad\quad\quad\quad\quad + \\
\hline
\quad\quad\quad\quad -36 + 12\lambda \\
\quad\quad\quad\quad -36 + 12\lambda \\
\quad\quad\quad\quad + \quad\quad - \\
\hline
\quad\quad\quad\quad 0
\end{array}
$

$$\Rightarrow (\lambda-3)\left[-\lambda^2+4\lambda+12\right]$$

$$\Rightarrow (\lambda-3)\left[-\lambda^2+6\lambda-2\lambda+12\right]$$

$$\Rightarrow (\lambda-3)\left(\lambda(-\lambda+6)+2(-\lambda+6)\right)$$

$$\Rightarrow (\lambda-3)(-\lambda+6)(\lambda+2)$$

$$\Rightarrow \boxed{\lambda \Rightarrow 3, 6, -2}$$

Eigen vectors.

For -2 $\quad (A+2I)X=0$

$$\left[\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}+2\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}\right]X=0$$

$$\begin{bmatrix}3&1&3\\1&7&1\\3&1&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$

$$\begin{cases}3x+y+3z=0\\ x+7y+z=0 \longrightarrow x=-z-7y\\ 3x+y+3z=0 \longrightarrow 3(-z-7y)+y+3z=0\end{cases}$$

$$-\cancel{3z}-21y+y+\cancel{3z}=0$$

$$-20y=0$$

$$\boxed{y=0}$$

$$\frac{x}{1}=\frac{-z}{1}$$

$$\begin{bmatrix}1\\0\\-1\end{bmatrix}$$

For (3)

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-2x + y + 3z = 0 \rightarrow (1)$

$x + 2y + z = 0 \rightarrow (2)$

$3x + y - 2z = 0 \rightarrow (3)$

subtract (1), (3)

$$\begin{array}{r} -2x + y + 3z = 0 \\ 3x + y - 2z = 0 \\ - \quad - \quad + \\ \hline -5x + 5z = 0 \\ \hline -x + z = 0 \end{array}$$

$\boxed{z = x}$

In eq (2)

$z + 2y + z = 0$

$y + z = 0$

$-y = +z$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

For (8)

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-5x + y + 3z = 0 \rightarrow (1)$

$x - y + z = 0 \rightarrow (2)$

$3x + y - 5z = 0 \rightarrow (3)$

Subtract (1), (3)

$$\begin{array}{r} -5x + y + 3z = 0 \\ 3x + y - 5z = 0 \\ - \quad - \quad + \\ \hline -8x + 8z = 0 \\ \hline z = x \end{array}$$

In eq (2)

$x - y + x = 0$

$2x - y = 0$

$2x = y$

$\boxed{\frac{x}{1} = \frac{y}{2}}$

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

17/11/21

Normal Form of a matrix.

Q→ Reduce the matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 6 & 5 \\ 3 & 5 & 4 \end{bmatrix}$ into normal form & hence find rank of A.

Sol→ $A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 6 & 5 \\ 3 & 5 & 4 \end{bmatrix}$

$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -6 & -3 \\ 0 & -4 & -2 \end{bmatrix}$$

~~5-8~~

~~5-4~~

~~4-6~~

$-6-0$

$C_2 \to C_2 - 3C_1$

$C_3 \to C_3 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -6 & -3 \\ 0 & -4 & -2 \end{bmatrix}$$

$R_2 \to R_2 / -6$

~~$R_3 \to R_3 / -2$~~

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/2 \\ 0 & -4 & -2 \end{bmatrix}$$

$R_3 \to R_3 + 4R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to 2C_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

which is in normal form;

So $\rho(A) = 2$