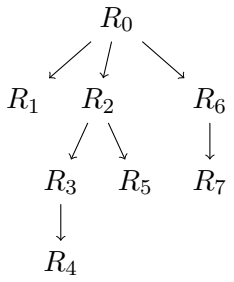
6 Exercise Sheet 6

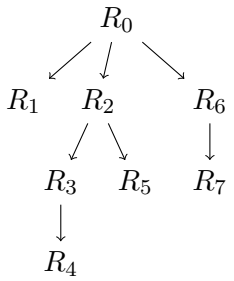
6.1 Exercise 1

$|R_0| = 20, |R_1| = 10, |R_2| = 50, |R_3| = 40, |R_4| = 50, |R_5| = 55, |R_6| = 50, |R_7| = 60$
$f_{0,1} = 0.1, f_{0,2} = 0.2, f_{0,6} = 0.2, f_{6,7} = 0.2, f_{2,5} = 0.2, f_{2,3} = 0.2, f_{3,4} = 0.3$

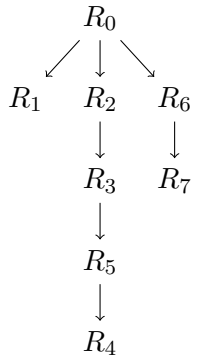
6.1.1 Give the precedence graph rooted in R_0

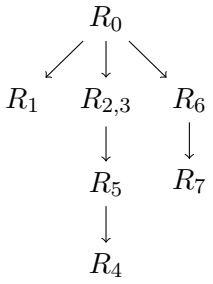


6.1.2 Perform the IKKBZ algorithm Root at R_0



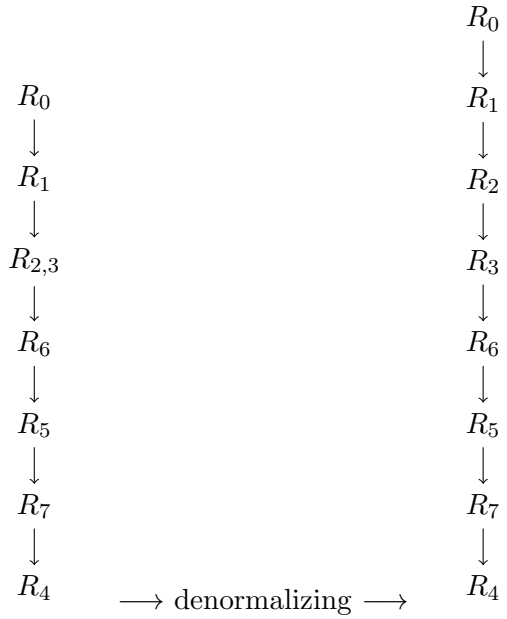
Relation	n	s	C	T	rank
R_1	10	0.1	1	1	0
R_2	50	0.2	10	10	$\frac{9}{10} = 0.9$
R_3	40	0.2	8	8	$\frac{7}{8} = 0.875$
R_4	50	0.3	15	15	$\frac{14}{15} = 0.933$
R_5	55	0.2	11	11	$\frac{10}{11} = 0.909$
R_6	50	0.2	10	10	$\frac{9}{10} = 0.9$
R_7	60	0.2	12	12	$\frac{11}{12} = 0.917$





$\longrightarrow$ Merge R_2 and R_3 $\longrightarrow$

Relation	n	s	C	T	rank
R_1	10	0.1	1	1	0
R_4	50	0.3	15	15	$\frac{14}{15} = 0.933$
R_5	55	0.2	11	11	$\frac{10}{11} = 0.909$
R_6	50	0.2	10	10	$\frac{9}{10} = 0.9$
R_7	60	0.2	12	12	$\frac{11}{12} = 0.917$
$R_{2,3}$			90	80	$\frac{79}{90} = 0.878$
R_2	50	0.2	10	10	$\frac{9}{10} = 0.9$
R_3	40	0.2	8	8	$\frac{7}{8} = 0.875$



Resulting Join Tree =
$((((((R_0 \bowtie R_1) \bowtie R_2) \bowtie R_3) \bowtie R_6) \bowtie R_5) \bowtie R_7) \bowtie R_4 = RJT$

$M = R_0 \bowtie R_1$
$C_{out}(M) = |R_0| * |R_1| * s_1 = 20$

$P = (R_0 \bowtie R_1) \bowtie R_2$
$C_{out}(P) = |M| * |R_2| * s_2 + C_{out}(M) = 220$

$Q = ((R_0 \bowtie R_1) \bowtie R_2) \bowtie R_3$
$C_{out}(Q) = |P| * |R_3| * s_3 + C_{out}(P) = 1'820$

$S = (((R_0 \bowtie R_1) \bowtie R_2) \bowtie R_3) \bowtie R_6$
$C_{out}(S) = |Q| * |R_6| * s_6 + C_{out}(Q) = 17'820$

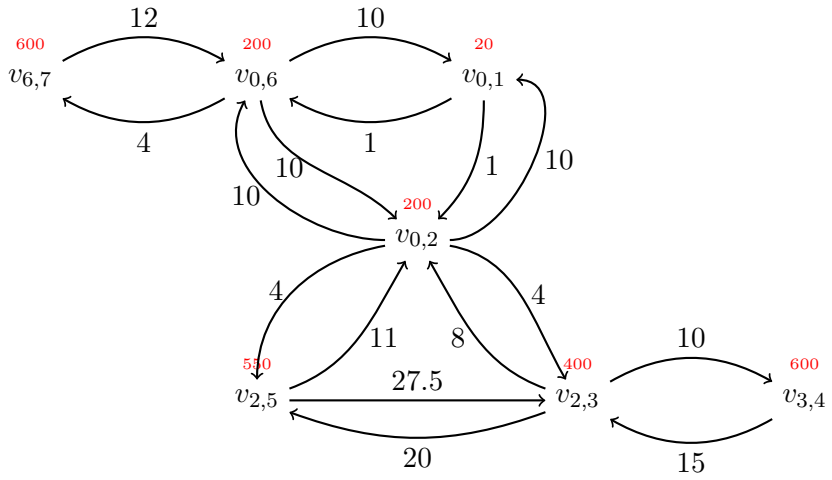
$T = ((((R_0 \bowtie R_1) \bowtie R_2) \bowtie R_3) \bowtie R_6) \bowtie R_5$
$C_{out}(T) = |S| * |R_5| * s_5 + C_{out}(S) = 193'820$

$U = (((((R_0 \bowtie R_1) \bowtie R_2) \bowtie R_3) \bowtie R_6) \bowtie R_5) \bowtie R_7$
$C_{out}(U) = |T| * |R_7| * s_7 + C_{out}(T) = 2'305'820$

$C_{out}(RJT) = |U| * |R_4| * s_4 + C_{out}(U) =$ **$33'985'820$**

6.1.3 Perform the MVP algorithm

There are no edges with weights < 1, so we start with phase 2:
Weighted-Join-Graph G:



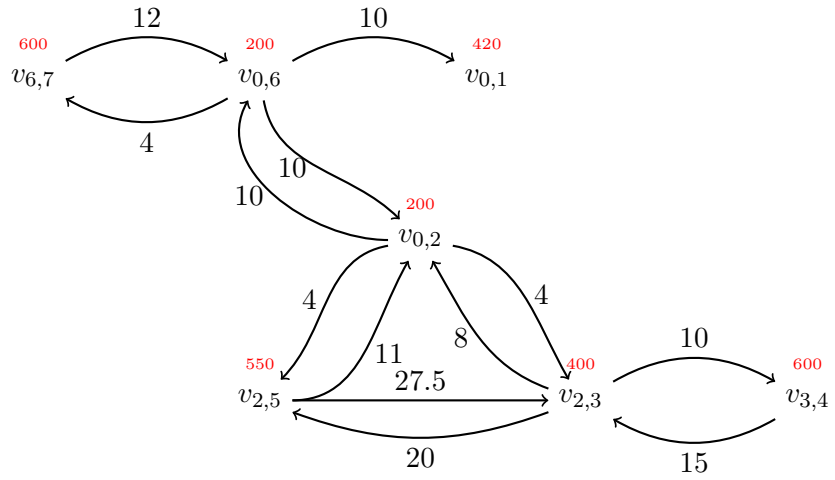
Spanning Tree S:

$v_{6,7}$ $v_{0,6}$ $v_{0,1}$

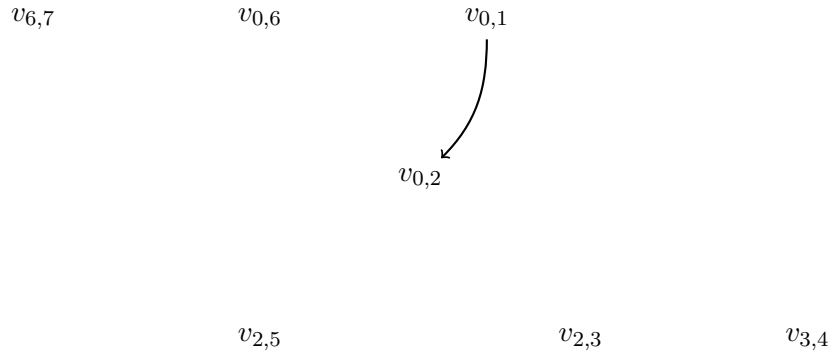
$v_{0,2}$

$v_{2,5}$ $v_{2,3}$ $v_{3,4}$

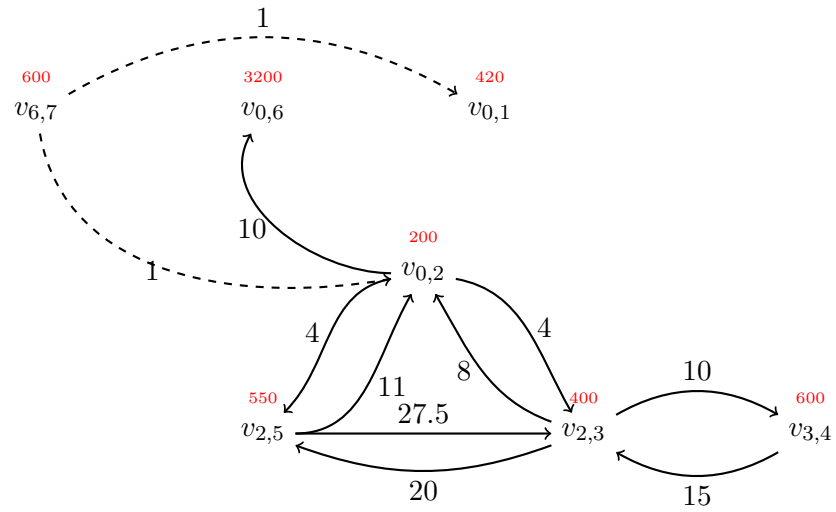
$Q_1 = \emptyset$
$Q_2 = \{v_{0,1}, v_{0,6}, v_{0,2}, v_{2,3}, v_{2,5}, v_{3,4}, v_{6,7}\}$
Consider edge $v_{0,1} \to v_{0,2}$.
New cost: $\text{cost}(v'_{0,1}) = 10 \cdot \frac{1}{10} \cdot 20 \cdot \frac{1}{5} \cdot 50 + 20 + 200 = 420$



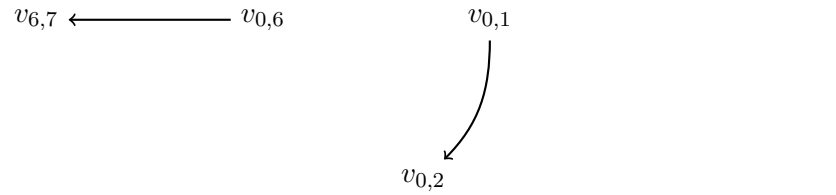
Spanning Tree S:



$Q_1 = \emptyset$
$Q_2 = \{v_{0,6}, v_{0,2}, v_{2,3}, v_{2,5}, v_{3,4}, v_{6,7}\}$
Consider edge $v_{0,6} \to v_{6,7}$.
New cost: $\text{cost}(v'_{0,6}) = 60 \cdot \frac{1}{5} \cdot 50 \cdot \frac{1}{5} \cdot 20 + 600 + 200 = 3200$

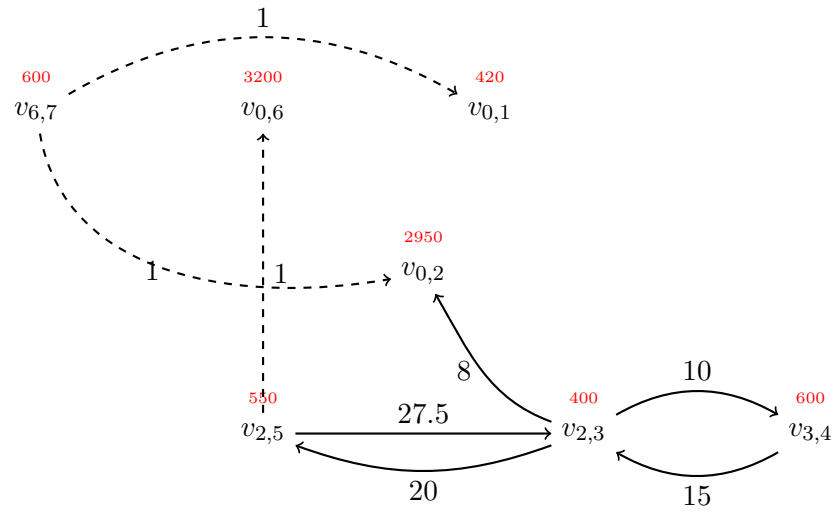


Spanning Tree S:

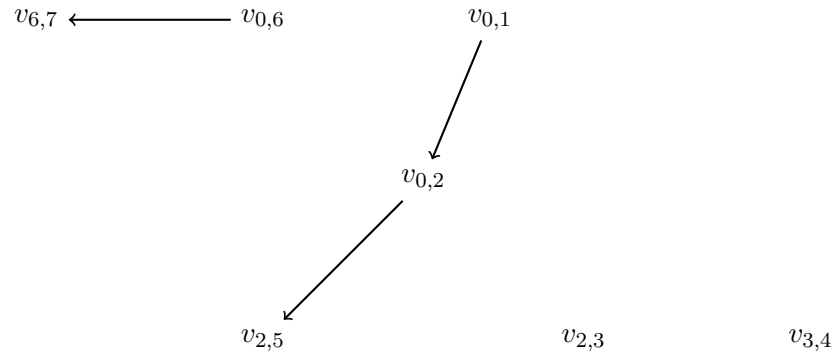


$v_{2,5}$ $v_{2,3}$ $v_{3,4}$

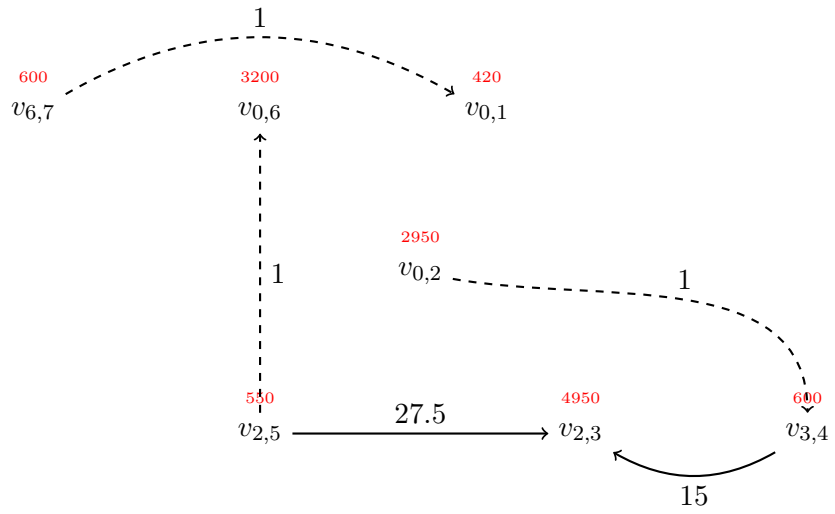
$Q_1 = \emptyset$
$Q_2 = \{v_{0,2}, v_{2,3}, v_{2,5}, v_{3,4}, v_{6,7}\}$
Consider edge $v_{0,2} \to v_{2,5}$.
New cost: $\text{cost}(v'_{0,2}) = 20 \cdot \frac{1}{5} \cdot 50 \cdot \frac{1}{5} \cdot 55 + 200 + 550 = 2950$



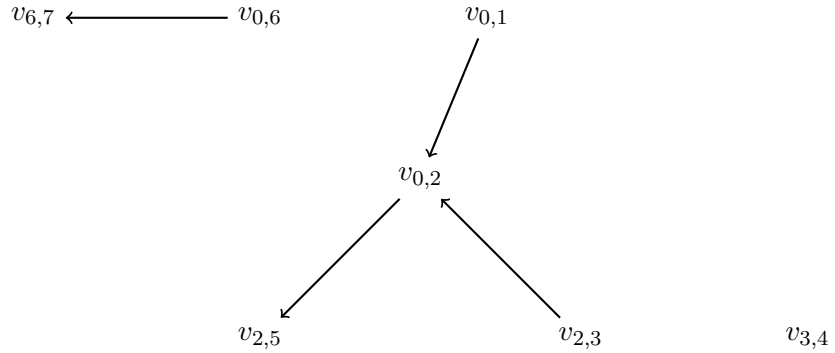
Spanning Tree S:



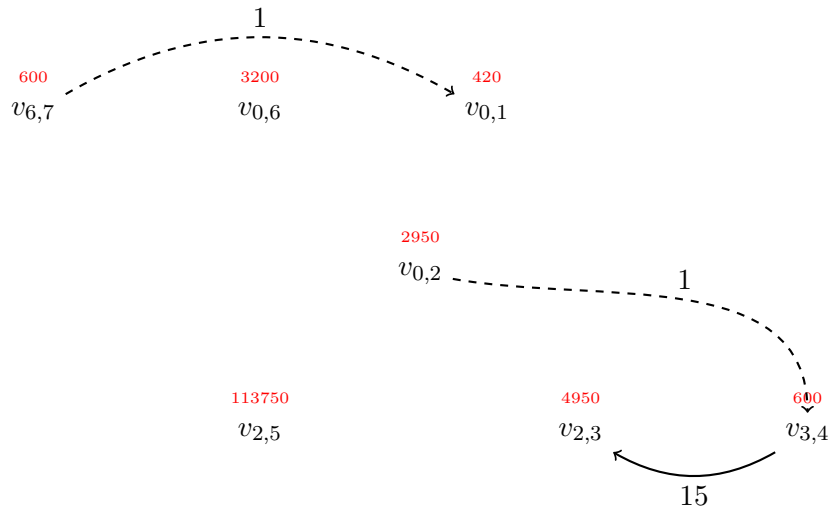
$Q_1 = \emptyset$
$Q_2 = \{v_{2,3}, v_{2,5}, v_{3,4}, v_{6,7}\}$
Consider edge $v_{2,3} \to v_{0,2}$.
New cost: $\text{cost}(v'_{2,3}) = 20 \cdot \frac{1}{5} \cdot 50 \cdot \frac{1}{5} \cdot 40 + 2950 + 400 = 4950$



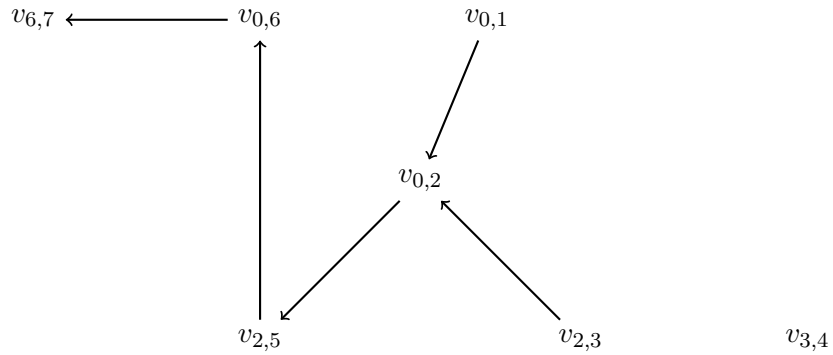
Spanning Tree S:



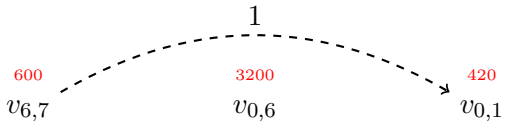
$Q_1 = \emptyset$
$Q_2 = \{v_{2,5}, v_{3,4}, v_{6,7}\}$
Consider edge $v_{2,5} \to v_{0,6}$.
New cost: $\text{cost}(v'_{2,5}) = 50 \cdot \frac{1}{5} \cdot 55 \cdot 20 \cdot \frac{1}{5} \cdot 50 + 550 + 3200 = 113750$

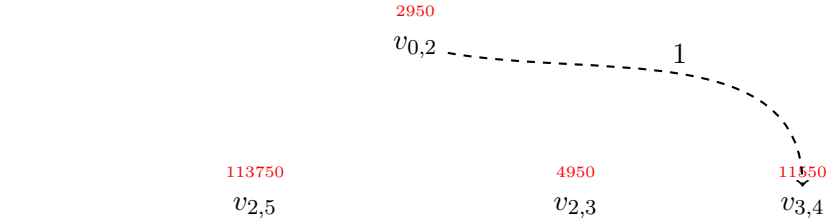


Spanning Tree S:

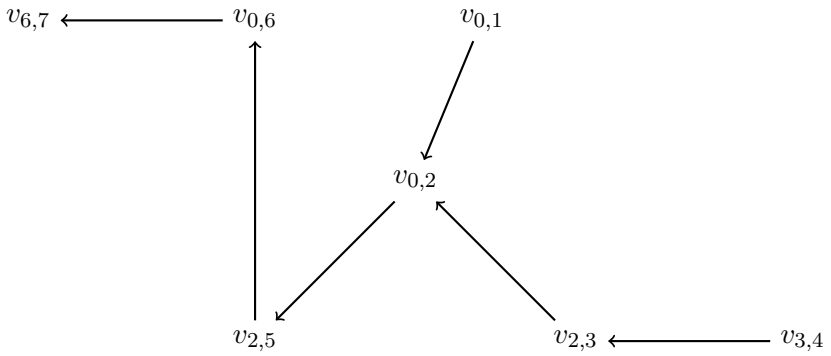


$Q_1 = \emptyset$
$Q_2 = \{v_{3,4}, v_{6,7}\}$
Consider edge $v_{3,4} \to v_{2,3}$.
New cost: $\text{cost}(v'_{3,4}) = 50 \cdot \frac{3}{10} \cdot 40 \cdot \frac{1}{5} \cdot 50 + 600 + 4950 = 11550$





Spanning Tree S:



spanning tree is complete $\Rightarrow$ stop.
Resulting join tree: