Training Graph Neural Networks

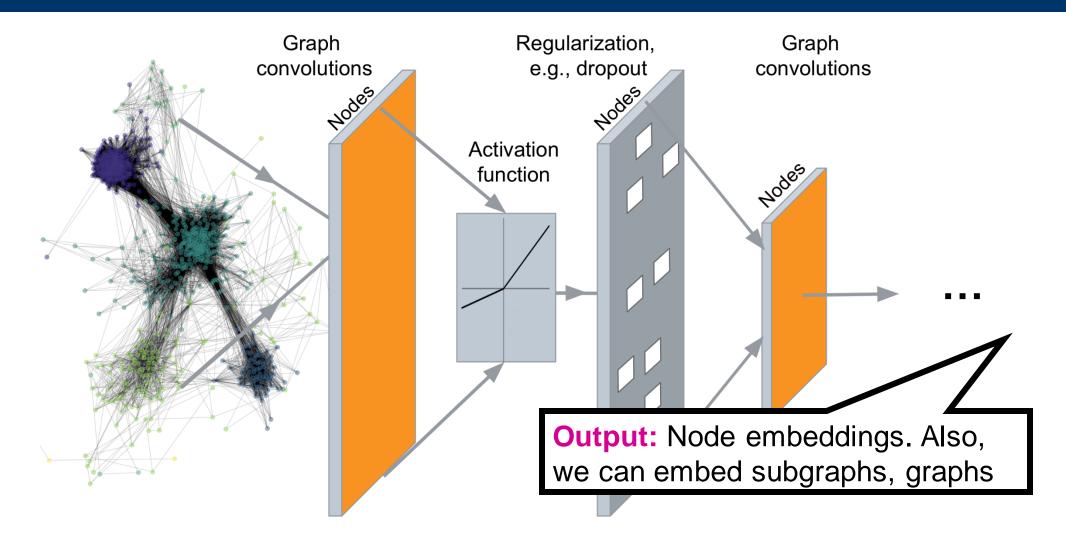
CPSC483: Deep Learning on Graph-Structured Data

Rex Ying

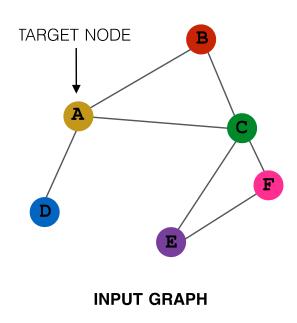
Readings

- Readings are updated on the website (syllabus page)
- Lecture 4 readings:
 - Semi-Supervised Classification with Graph Convolutional Networks
 - Principled Neighborhood Aggregation on Graph Nets
- Lecture 5 readings:
 - Design Space of Graph Neural Networks
 - OGB Datasets

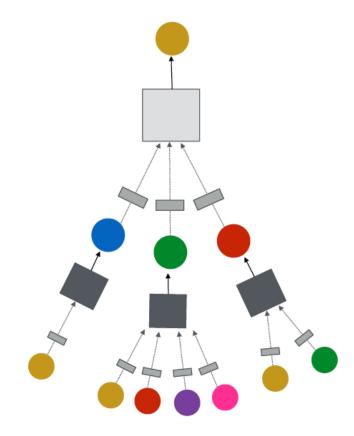
Recap: Deep Graph Encoders



Recap: A General GNN Framework

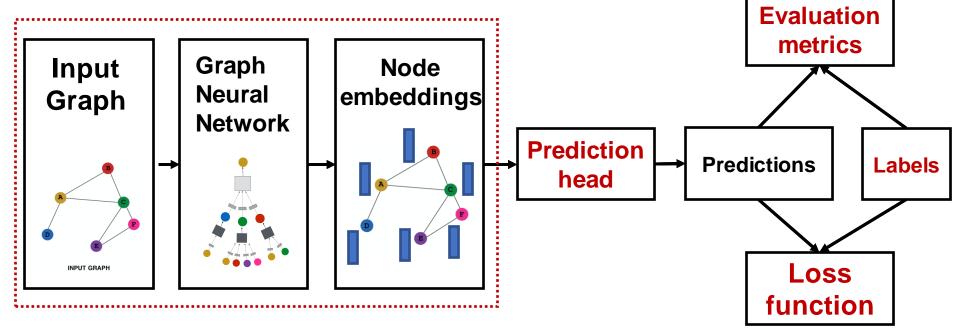


Next: How do we train / test a GNN?



GNN Training Pipeline (1)

So far what we have covered



Output of a GNN: set of node embeddings

$$\{\mathbf{h}_{v}^{(L)}, \forall v \in G\}$$

Outline of Today's Lecture

GNN Prediction Heads

GNN Predictions & Labels

GNN Loss Functions

GNN Evaluation Metrics

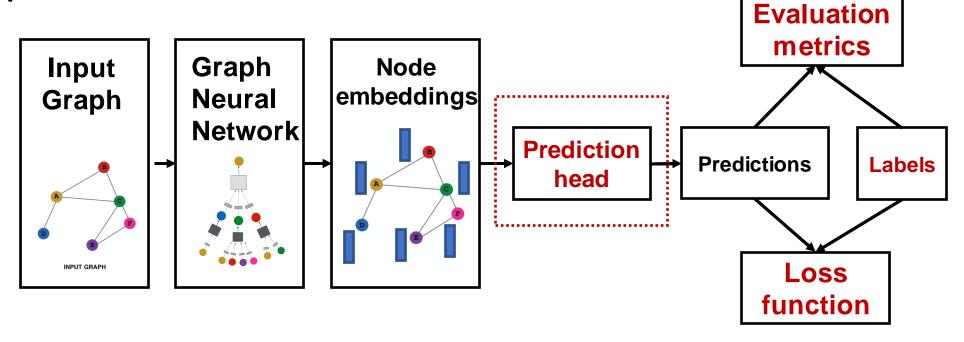
GNN Dataset Split

Part 1

GNN Prediction Heads

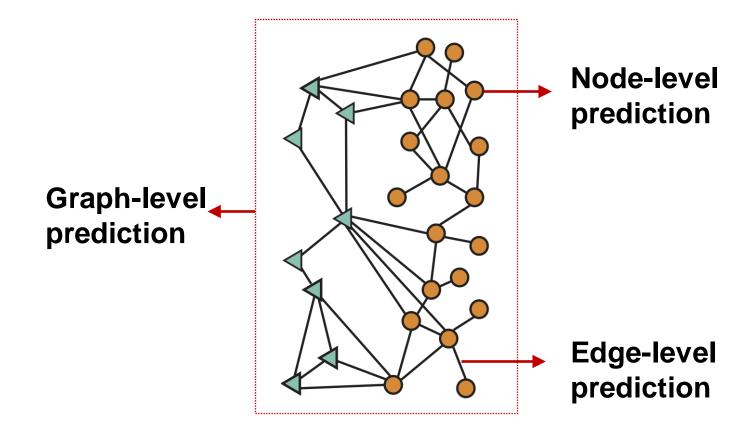
GNN Training Pipeline (2)

- (1) Different prediction heads:
 - Node-level tasks
 - Edge-level tasks
 - Graph-level tasks



GNN Predictions Heads

• Idea: Different task levels require different prediction heads



Prediction Heads: Node-Level

- Node-level prediction: We can directly make prediction using node embeddings!
- After GNN computation, we have d-dim node embeddings: $\{\mathbf{h}_v^{(L)} \in \mathbb{R}^d, \forall v \in G\}$
- Suppose we want to make k-way prediction
 - Classification: classify among k categories
 - Regression: regress on k targets
- $\widehat{\mathbf{y}}_{v} = \text{Head}_{\text{node}}(\mathbf{h}_{v}^{(L)}) = \mathbf{W}^{(H)}\mathbf{h}_{v}^{(L)}$
 - $\mathbf{W}^{(H)} \in \mathbb{R}^{k \times d}$: We map node embeddings from $\mathbf{h}_v^{(L)} \in \mathbb{R}^d$ to $\widehat{\mathbf{y}}_v \in \mathbb{R}^k$ so that we can compute the loss

Prediction Heads: Edge-Level (1)

- Edge-level prediction: Make prediction using pairs of node embeddings
- Suppose we want to make a k-way prediction

•
$$\widehat{\mathbf{y}}_{uv} = \operatorname{Head}_{\operatorname{edg}e}(\mathbf{h}_{u}^{(L)}, \mathbf{h}_{v}^{(L)})$$

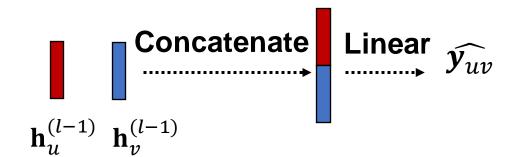
$$\mathbf{h}_{u}^{(L)}$$
• $\widehat{\mathbf{y}}_{uv} = \operatorname{Head}_{\operatorname{edg}e}(\mathbf{h}_{u}^{(L)}, \mathbf{h}_{v}^{(L)})$

• What are the options for $\operatorname{Head}_{\operatorname{edg}e}(\mathbf{h}_{u}^{(L)}, \mathbf{h}_{v}^{(L)})$?

Prediction Heads: Edge-Level (2)

• Options for $\operatorname{Head}_{\operatorname{edg}e}(\mathbf{h}_{u}^{(L)}, \mathbf{h}_{v}^{(L)})$:

- (1) Concatenation + Linear
 - $\hat{y}_{uv} = \text{Linear}(\text{Concat}(\mathbf{h}_u^{(L)}, \mathbf{h}_v^{(L)}))$



We will see it in Graph Attention (lecture 7)

• Here Linear(·) will map 2-dimensional embeddings (since we concatenated embeddings) to k-dimentional embeddings (k-way prediction)

Prediction Heads: Edge-Level (3)

• Options for $Head_{edge}(\mathbf{h}_{u}^{(L)}, \mathbf{h}_{v}^{(L)})$:

- (2) Dot product
 - $\widehat{\mathbf{y}}_{uv} = (\mathbf{h}_u^{(L)})^T \mathbf{h}_v^{(L)}$
 - This approach only applies to 1-way prediction (e.g., link prediction: predict the existence of an edge)
 - Applying to k-way prediction:
 - Let $\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(k)}$ be trainable weights:

$$\widehat{\mathbf{y}}_{uv}^{(1)} = (\mathbf{h}_{u}^{(L)})^{T} \mathbf{W}^{(1)} \mathbf{h}_{v}^{(L)}$$

$$\vdots$$

$$\widehat{\mathbf{y}}_{uv}^{(k)} = (\mathbf{h}_{u}^{(L)})^{T} \mathbf{W}^{(k)} \mathbf{h}_{v}^{(L)}$$

$$\widehat{\mathbf{y}}_{uv} = \text{Concat}(\widehat{\mathbf{y}}_{uv}^{(1)}, ..., \widehat{\mathbf{y}}_{uv}^{(k)}) \in \mathbb{R}^{k}$$

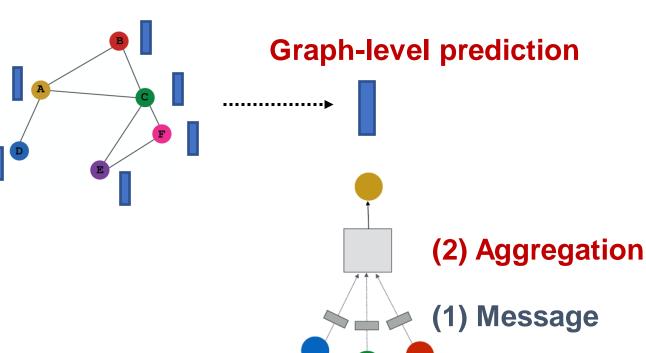
We will see it in Graph Attention (lecture 7)

Also called bilinear form

Prediction Heads: Graph-Level (1)

- Graph-level prediction: Make prediction using all the node embeddings in our graph
- Suppose we want to make k-way prediction
- $\widehat{\mathbf{y}}_G = \operatorname{Head}_{\operatorname{graph}}(\{\mathbf{h}_v^{(L)} \in \mathbb{R}^d, \forall v \in G\})$

 Head_{graph}(·) is similar to AGG(·) in a GNN layer!



Prediction Heads: Graph-Level (2)

- Options for $\operatorname{Head}_{\operatorname{graph}}(\{\mathbf{h}_v^{(L)} \in \mathbb{R}^d, \forall v \in G\})$
- (1) Global mean pooling

$$\widehat{\mathbf{y}}_G = \operatorname{Mean}(\{\mathbf{h}_v^{(L)} \in \mathbb{R}^d, \forall v \in G\})$$

• (2) Global max pooling

$$\widehat{\mathbf{y}}_G = \operatorname{Max}(\{\mathbf{h}_v^{(L)} \in \mathbb{R}^d, \forall v \in G\})$$

• (3) Global sum pooling

$$\widehat{\mathbf{y}}_G = \operatorname{Sum}(\{\mathbf{h}_v^{(L)} \in \mathbb{R}^d, \forall v \in G\})$$

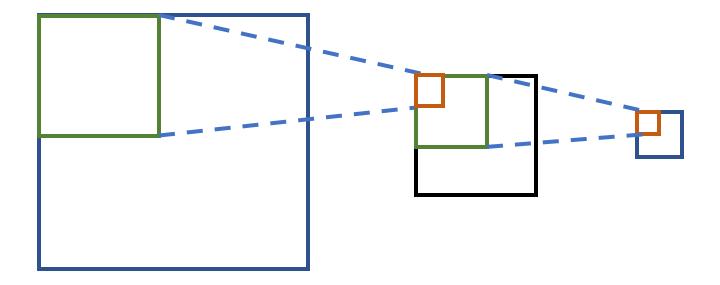
- These options work great for small graphs
- Can we do better for large graphs?

Issue of Global Pooling (1)

- Issue: Global pooling over a (large) graph will lose information
- Toy example: we use 1-dim node embeddings
 - Node embeddings for G_1 : $\{-1, -2, 0, 1, 2\}$
 - Node embeddings for G_2 : {-10, -20, 0, 10, 20}
 - Clearly G_1 and G_2 have very different node embeddings \rightarrow Their graph structures should be different
- If we do global sum pooling:
 - Prediction for $G_1: \hat{y}_{G_1} = \text{Sum}(\{-1, -2, 0, 1, 2\}) = 0$
 - Prediction for G_2 : $\hat{y}_{G_2} = \text{Sum}(\{-10, -20, 0, 10, 20\}) = 0$
 - We cannot differentiate G_1 and G_2 !

Issue of Global Pooling (2)

- Aggregate node embeddings
 - Naive approach: global mean/max/sum
 - Better pooling strategies that respects structure?
 - Here we consider hierarchical pooling analogous to CNNs

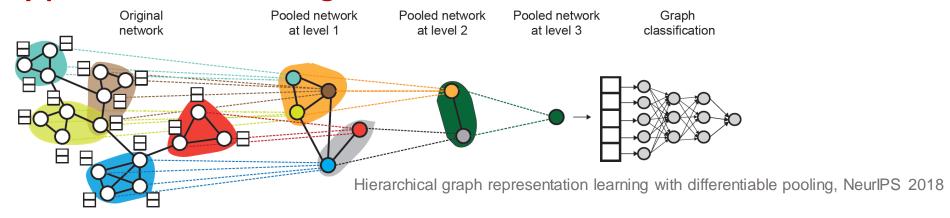


Hierarchical Global Pooling

- A solution: Let's aggregate all the node embeddings hierarchically
 - Toy example: We will aggregate via $ReLU(Sum(\cdot))$
 - We first separately aggregate the first 2 nodes and last 3 nodes
 - Then we aggregate again to make the final prediction
 - G_1 node embeddings: {-1, -2, 0, 1, 2}
 - Round 1: $\hat{y}_a = \text{ReLU}(\text{Sum}(\{-1, -2\})) = 0$, $\hat{y}_b = \text{ReLU}(\text{Sum}(\{0, 1, 2\})) = 3$
 - Round 2: $\hat{y}_G = \text{ReLU}(\text{Sum}(\{y_a, y_b\})) = 3$
 - G_2 node embeddings: $\{-10, -20, 0, 10, 20\}$
 - Round 1: $\hat{y}_a = \text{ReLU}(\text{Sum}(\{-10, -20\})) = 0$, $\hat{y}_b = \text{ReLU}(\text{Sum}(\{0, 10, 20\})) = 30$
 - Round 2: $\hat{y}_G = \text{ReLU}(\text{Sum}(\{y_a, y_b\})) = 30$
- Now we can differentiate G_1 and G_2 !

Hierarchical Pooling in Practice

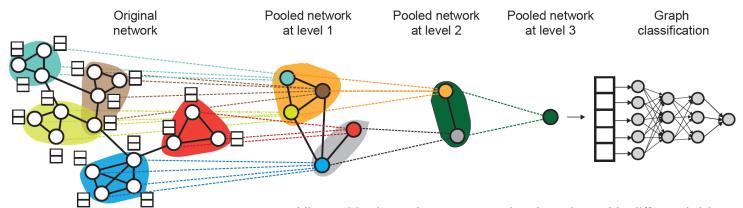
- DiffPool idea:
 - Hierarchically pool node embeddings



- Leverage 2 independent GNNs at each level
 - GNN A: Compute node embeddings
 - GNN B: Compute the cluster that a node belongs to
- GNNs A and B at each level can be executed in parallel

Hierarchical Pooling in Practice: DiffPool

DiffPool idea:



Hierarchical graph representation learning with differentiable pooling, NeurlPS 2018

- For each Pooling layer
 - Use clustering assignments from GNN B to aggregate node embeddings generated by GNN A
 - Create a single new node for each cluster, maintaining edges between clusters to generated a new pooled network
- Jointly train GNN A and GNN B

DiffPool Architecture (1)

Assuming general GNN model:

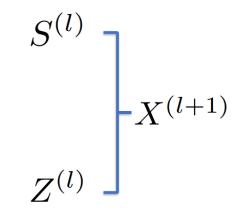
$$H^{(k)} = M(A, H^{(k-1)}; \theta^{(k)})$$

- For example: $\operatorname{ReLU}\left(\tilde{D}^{-\frac{1}{2}}\tilde{A}\tilde{D}^{-\frac{1}{2}}H^{(k-1)}W^{(k-1)}\right)$
- Two-tower architecture

$$Z^{(l)} = \text{GNN}_{l, \text{ embed}} \left(A^{(l)}, X^{(l)} \right) \text{ Embedding}$$

$$S^{(l)} = \text{softmax} \left(\text{GNN}_{l, \text{pool}} \left(A^{(l)}, X^{(l)} \right) \right) \text{ Assignment}$$

 Combine them to generate next-level representations and adjacency



DiffPool Architecture (2)

- Let n_l be the number of nodes at layer l
- Assignment matrix S dimension: $n_l \times n_{l+1}$
 - $S_{i,j}=1$ if i-th node for layer l belongs to j-th node/cluster in layer l+1
- Having computed Z and S:
- Compute **embedding** for layer l + 1:

$$X^{(l+1)} = S^{(l)^T} Z^{(l)} \in \mathbb{R}^{n_{l+1} \times d}$$

• Compute **adjacency matrix** for layer l + 1:

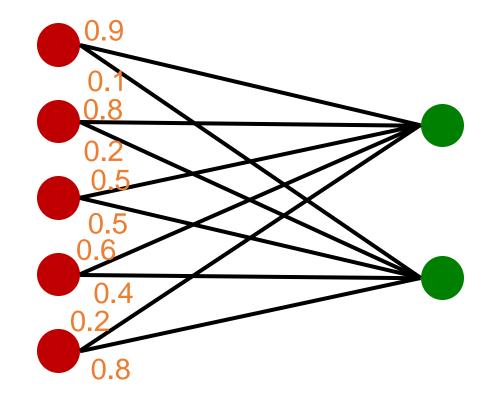
$$A^{(l+1)} = S^{(l)^T} A^{(l)} S^{(l)} \in \mathbb{R}^{n_{l+1} \times n_{l+1}}$$

DiffPool Architecture: Example

Computing node embeddings:

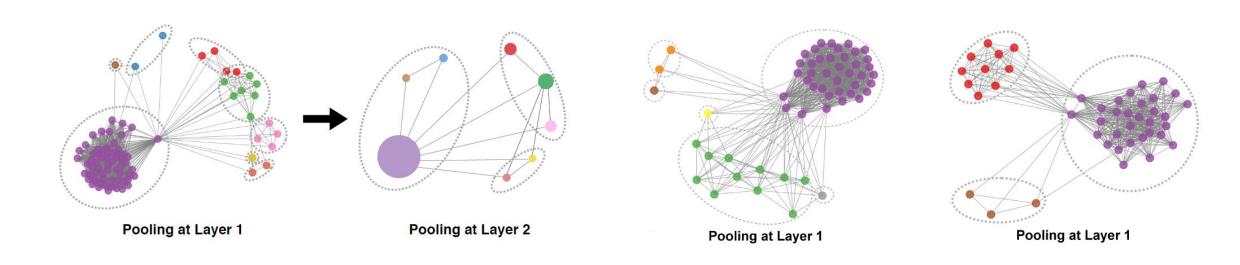
$$S^{(l)} = \begin{pmatrix} 0.9 & 0.1 \\ 0.8 & 0.2 \\ 0.5 & 0.5 \\ 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$$

 Computing adjacency matrix is analogous to this!



DiffPool Results

- Learns meaning clusters through downstream tasks
- An average of 6.27% improvement in accuracy for standard benchmarks



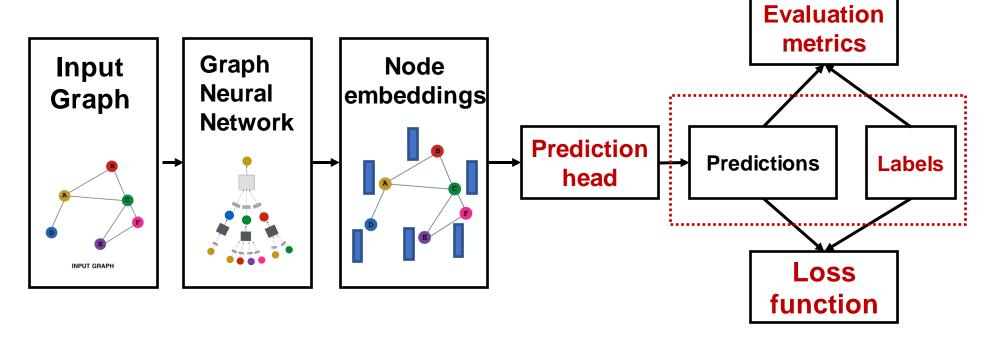
Hierarchical graph representation learning with differentiable pooling, NeurlPS 2018

Part 2

GNN Predictions & labels

GNN Training Pipeline (3)

- Where does ground-truth come from?
 - Supervised labels
 - Unsupervised signals



Supervised vs Unsupervised

- Supervised learning on graphs
 - Labels come from external sources
 - E.g., predict drug likeness of a molecular graph
- Unsupervised learning on graphs
 - Signals come from graphs themselves
 - E.g., link prediction: predict if two nodes are connected
- Sometimes the differences are blurry
 - We still have "supervision" in unsupervised learning
 - E.g., train a GNN to predict node clustering coefficient
 - An alternative name for "unsupervised" is "self-supervised"

Supervised Labels on Graphs

- Supervised labels come from the specific use cases. For example:
 - Node labels y_v : in a citation network, which subject area does a node belong to
 - Edge labels y_{uv} : in a transaction network, whether an edge is fraudulent
 - Graph labels y_G : among molecular graphs, the drug likeness of graphs
- Advice: Reduce your task to node / edge / graph labels, since they are easy to work with
 - E.g., we knew some nodes form a cluster. We can treat the cluster that a node belongs to as a node label

Unsupervised Signals on Graphs

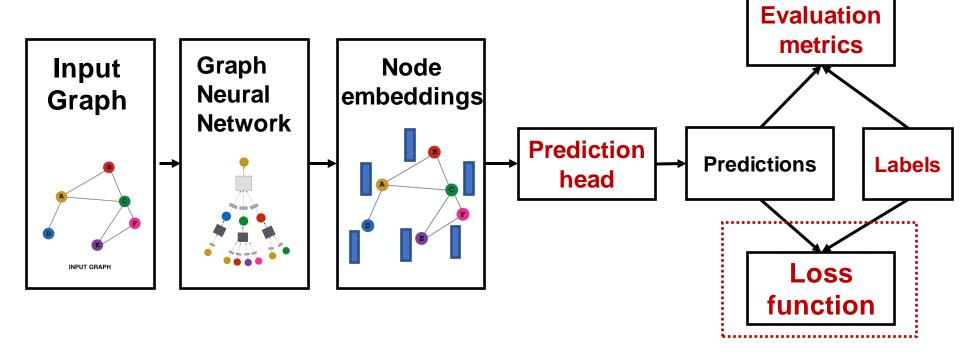
- The problem: sometimes we only have a graph, without any external labels
- The solution: "self-supervised learning", we can find supervision signals within the graph.
 - For example, we can let GNN predict the following:
 - Node-level y_v . Node statistics: such as clustering coefficient, PageRank, ...
 - Edge-level y_{uv} . Link prediction: hide the edge between two nodes, predict if there should be a link
 - Graph-level y_G . Graph statistics: for example, predict if two graphs are isomorphic
 - These tasks do not require any external labels!

Part 3

GNN Loss Functions

GNN Training Pipeline (4)

- How do we compute the final loss?
 - Classification loss
 - Regression loss



Settings for GNN Training

- The setting: We have N data points
 - Each data point can be a node/edge/graph
 - Node-level tasks: prediction $\widehat{m{y}}_v^{(i)}$, label $m{y}_v^{(i)}$
 - Edge-level tasks: prediction $\widehat{m{y}}_{uv}^{(i)}$, label $m{y}_{uv}^{(i)}$
 - Graph-level tasks: prediction $\widehat{m{y}}_G^{(i)}$, label $m{y}_G^{(i)}$
 - We will use prediction $\widehat{m{y}}^{(i)}$, label $m{y}^{(i)}$ to refer to **predictions at all tasks**

Classification or Regression

- Classification: labels $oldsymbol{y}^{(i)}$ with discrete value
 - E.g., Node classification: which category does a node belong to
- Regression: labels $y^{(i)}$ with continuous value
 - E.g., predict the drug likeness of a molecular graph

- GNNs can be applied to both settings
- Differences: loss function & evaluation metrics

Classification Loss

- As discussed in lecture 3, cross entropy (CE) is a very common loss function in classification
- *K*-way prediction for *i*-th data point:

$$CE(y^{(i)}, \widehat{y}^{(i)}) = -\sum_{j=1}^{K} y_j^{(i)} \log(\widehat{y}_j^{(i)})$$
Label Prediction
i-th data point
j-th class

where:

$$\mathbf{y}^{(i)} \in \mathbb{R}^K = \text{one-hot label encoding (e.g. } [0, 0, 1, 0, 0])$$

 $\widehat{\mathbf{y}}^{(i)} \in \mathbb{R}^K = \text{prediction after Softmax}(\cdot) \text{ (e.g. } [0.1, 0.3, 0.4, 0.1, 0.1])$

• Total loss over all *N* training examples

$$\mathcal{L} = \sum_{i=1}^{N} CE(\mathbf{y}^{(i)}, \widehat{\mathbf{y}}^{(i)})$$

Regression Loss

- For regression tasks we often use Mean Squared Error (MSE) a.k.a. L2 loss
- K-way regression for data point (i):

MSE $(y^{(i)}, \widehat{y}^{(i)}) = \sum_{i=1}^{K} (y_j^{(i)} - \widehat{y}_j^{(i)})^2$ _{j-th dimension}

where:

Label Prediction

 $y^{(i)} \in \mathbb{R}^k$ = Real valued vector of targets (e.g. [1.4, 2.3, 1.0, 0.5, 0.6]) $\hat{y}^{(i)} \in \mathbb{R}^k$ = Real valued vector of predictions(e.g. [0.9, 2.8, 2.0, 0.3, 0.8])

Total loss over all N training examples

$$\mathcal{L} = \sum_{i=1}^{N} MSE(\mathbf{y}^{(i)}, \widehat{\mathbf{y}}^{(i)})$$

Part 4

GNN Evaluation Metrics

GNN Training Pipeline (5)

- How do we measure the success of a GNN?
 - Regression
 - Classification **Evaluation** metrics Input Graph Node Neural embeddings Graph **Network Prediction Predictions** Labels head Loss function

Evaluation Metrics: Regression

- Standard evaluation metrics can be used for GNNs
 - (Content below can be found in any ML course)
 - In practice we often use sklearn for implementation
 - Suppose we make predictions for N data points
- Evaluate regression tasks on graphs:
 - Root mean square error (RMSE)

$$\sqrt{\sum_{i=1}^{N} \frac{(\boldsymbol{y}^{(i)} - \widehat{\boldsymbol{y}}^{(i)})^2}{N}}$$

Mean absolute error (MAE)

$$\frac{\sum_{i=1}^{N} \left| \boldsymbol{y}^{(i)} - \widehat{\boldsymbol{y}}^{(i)} \right|}{N}$$

Evaluation Metrics: Classification

- Evaluate classification tasks on graphs:
- (1) Multi-class classification
 - We simply report the accuracy (confusion matrix can also be used)

$$\frac{\mathbf{1}\left[\operatorname{argmax}(\widehat{\mathbf{y}}^{(i)}) = \mathbf{y}^{(i)}\right]}{N}$$

- (2) Binary classification
 - Metrics sensitive to classification threshold
 - Accuracy
 - Precision / Recall
 - If the range of prediction is [0,1], we will use 0.5 as threshold
 - Metric Agnostic to classification threshold
 - ROC AUC

Classification Metrics: Precision / Recall

Accuracy:

$$\frac{TP + TN}{TP + TN + FP + FN} = \frac{TP + TN}{|Dataset|}$$

Precision (P):

• Recall (R):

Micro-F1 Score:

$$\frac{TP}{TP + FP}$$

$$\frac{TP}{TP + FN}$$

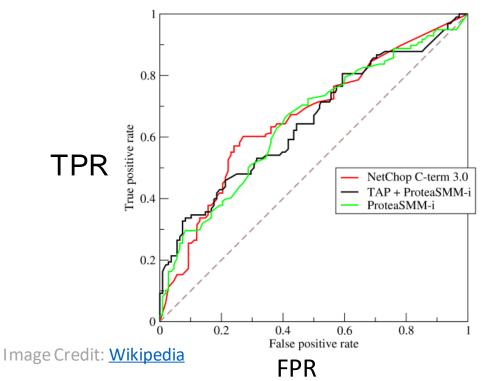
$$\frac{2PR}{P+R}$$

Confusion matrix

| | Actually Positive (1) | Actually Negative (0) |
|---------------------------|-----------------------------|-----------------------------|
| Predicted Positive (1) | True Positives (TPs) | False Positives (FPs) |
| Predicted Negative (0) | False Negatives (FNs) | True Negatives (TNs) |

Classification Metrics: ROC (1)

 ROC Curve: Captures the tradeoff in TPR and FPR as the classification threshold is varied for a binary classifier.



$$TPR = Recall = \frac{TP}{TP + FN}$$

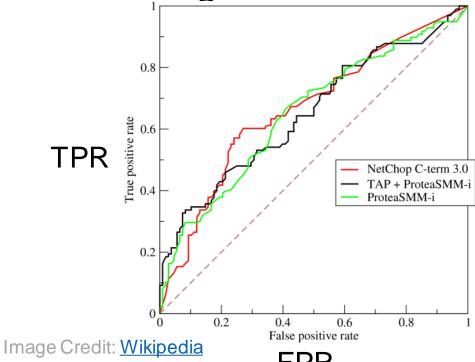
$$FPR = \frac{FP}{FP + TN}$$

Note: the dashed line represents performance of a random classifier

Classification Metrics: ROC (2)

ROC AUC: Area under the ROC Curve.

• Intuition: The probability that a classifier will rank a randomly chosen positive instance higher than a randomly chosen negative one



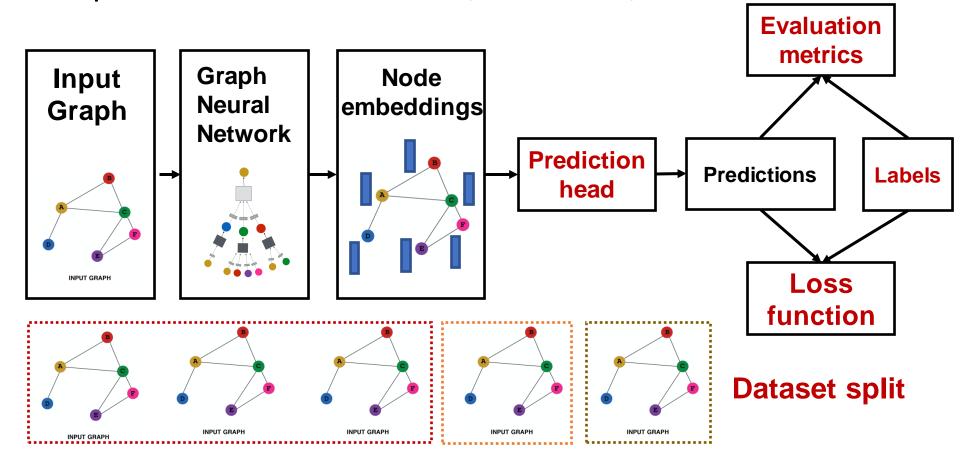
Note: the dashed line represents performance of a random classifier

Part 5

GNN Dataset Split

GNN Training Pipeline (6)

How do we split our dataset into train/validation/test set?

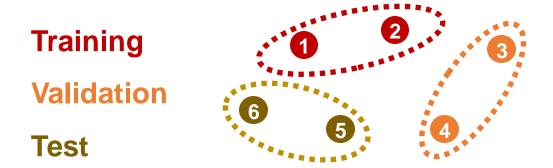


Dataset Split: Fixed / Random Split

- Fixed split: We will split our dataset once
 - Training set: used for optimizing GNN parameters
 - Validation set: develop model/hyperparameters
 - Test set: held out until we report final performance
- A concern: sometimes we cannot guarantee that the test set will really be held out
- Random split: we will randomly split our dataset into training / validation / test
 - We report average performance over different random seeds

Why Splitting Graphs is Special (1)

- Suppose we want to split an image dataset
 - Image classification: Each data point is an image
 - Here data points are independent
 - Image 5 will not affect our prediction on image 1



Why Splitting Graphs is Special (2)

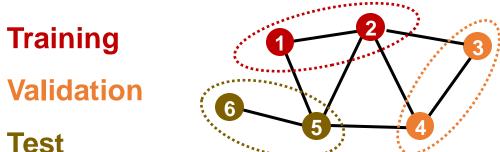
- Splitting a graph dataset is different!
 - Node classification: Each data point is a node
 - Here data points are NOT independent
 - **Node 5 will affect our prediction on node 1,** because it will participate in message passing → affect node 1's embedding

Training
Validation
Test

What are our options?

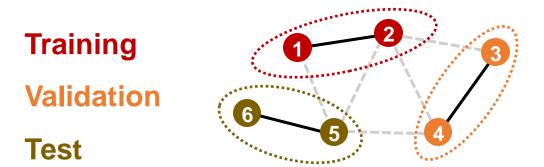
Why Splitting Graphs is Special (3)

- Solution 1 (Transductive setting): The input graph can be observed in all the dataset splits (training, validation and test set).
- We will only split the (node) labels
 - At training time, we compute embeddings using the entire graph, and train using node 1&2's labels
 - At validation time, we compute embeddings using the entire graph, and evaluate on node 3&4's labels



Why Splitting Graphs is Special (4)

- Solution 2 (Inductive setting): We break the edges between splits to get multiple graphs
 - Now we have 3 graphs that are independent. Node 5 will not affect our prediction on node 1 any more
 - At training time, we compute embeddings using the graph over node 1&2, and train using node 1&2's labels
 - At validation time, we compute embeddings using the graph over node 3&4, and evaluate on node 3&4's labels

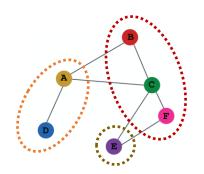


Transductive / Inductive Settings

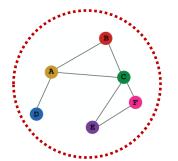
- Transductive setting: training / validation / test sets are on the same graph
 - The dataset consists of one graph
 - The entire graph can be observed in all dataset splits, we only split the labels
 - Only applicable to node / edge prediction tasks
- Inductive setting: training / validation / test sets are on different graphs
 - The dataset consists of multiple graphs
 - Each split can only observe the graph(s) within the split. A successful model should generalize to unseen graphs
 - Applicable to node / edge / graph tasks

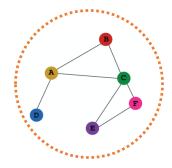
Example: Node Classification

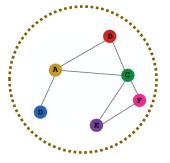
- Transductive node classification
 - All the splits can observe the entire graph structure, but can only observe the labels of their respective nodes
- Inductive node classification
 - Suppose we have a dataset of 3 graphs
 - Each split contains an independent graph



Training
Validation
Test



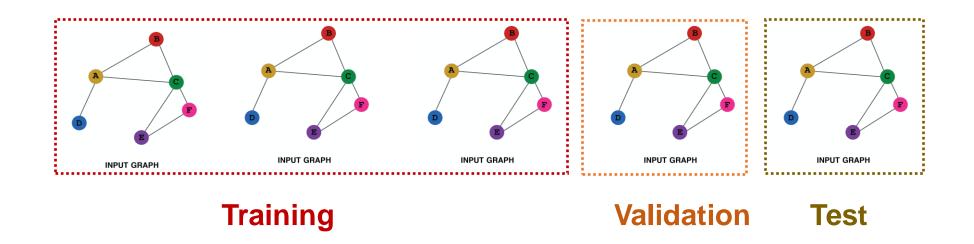




Training
Validation
Test

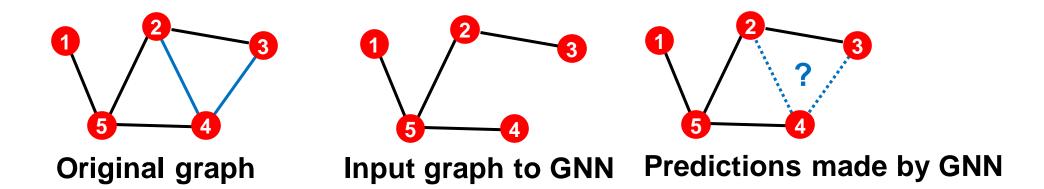
Example: Graph Classification

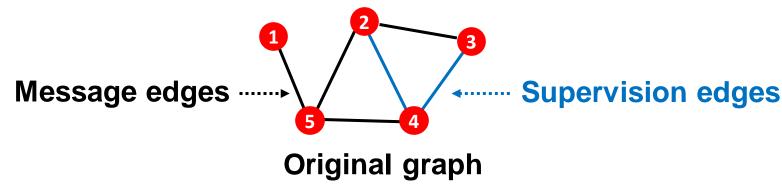
- Only the inductive setting is well defined for graph classification
 - Because we have to test on unseen graphs
 - Suppose we have a dataset of 5 graphs. Each split will contain independent graph(s).



Example: Link Prediction

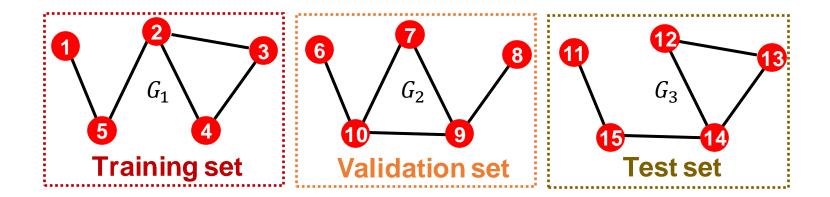
- Goal of link prediction: predict missing edges
- Setting up link prediction is tricky:
 - Link prediction is an unsupervised / self-supervised task. We need to create the labels and dataset splits on our own
 - Concretely, we need to hide some edges from the GNN and the let the GNN predict
 if the edges exist



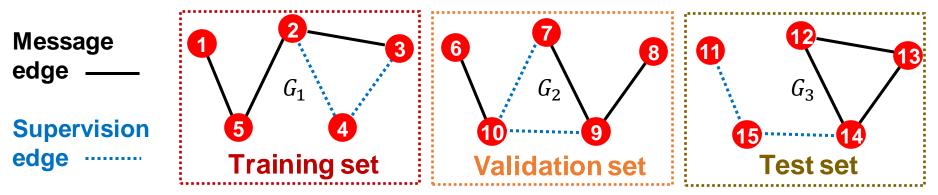


- For link prediction, we will split edges twice
- Step 1: Assign 2 types of edges in the original graph
 - Message edges: Used for GNN message passing
 - Supervision edges: Use for computing objectives
 - After step 1:
 - Only message edges will remain in the graph
 - Supervision edges are used as supervision for edge predictions made by the model, will not be fed into GNN!

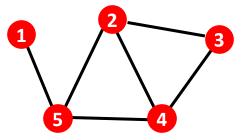
- Step 2: Split edges into train / validation / test
- Option 1: Inductive link prediction split
 - Suppose we have a dataset of 3 graphs. Each inductive split will contain an independent graph



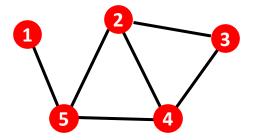
- Step 2: Split edges into train / validation / test
- Option 1: Inductive link prediction split
 - Suppose we have a dataset of 3 graphs. Each inductive split will contain an independent graph
 - In train or val or test set, each graph will have 2 types of edges: message edges + supervision edges
 - Supervision edges are not the input to GNN



- Option 2: Transductive link prediction split:
 - This is often the default setting when people talk about link prediction
 - Suppose we have a dataset of 1 (potentially very large) graph



- Option 2: Transductive link prediction split:
 - By definition of "transductive", the entire graph can be observed in all dataset splits
 - But since edges are both part of graph structure and the supervision, we need to hold out validation / test edges
 - To train the training set, we further need to hold out supervision edges for the training set

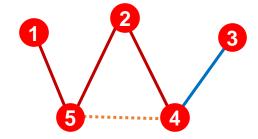


Next: we will show the exact settings

Option 2: Transductive link prediction split:

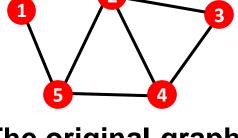
Why do we use growing number of edges?

- After training, supervision edges are known to GNN. Therefore, an ideal model should use supervision edges in message passing at validation time.
- The same applies to the test time.

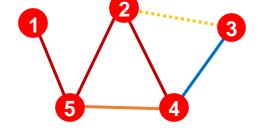






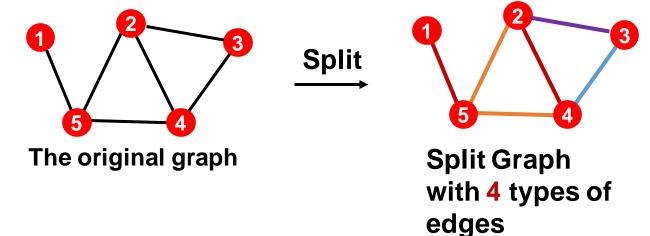


The original graph



(3) At test time:
Use training message edges
& training supervision edges
& validation edges to predict
test edges

• Summary: Transductive link prediction split:

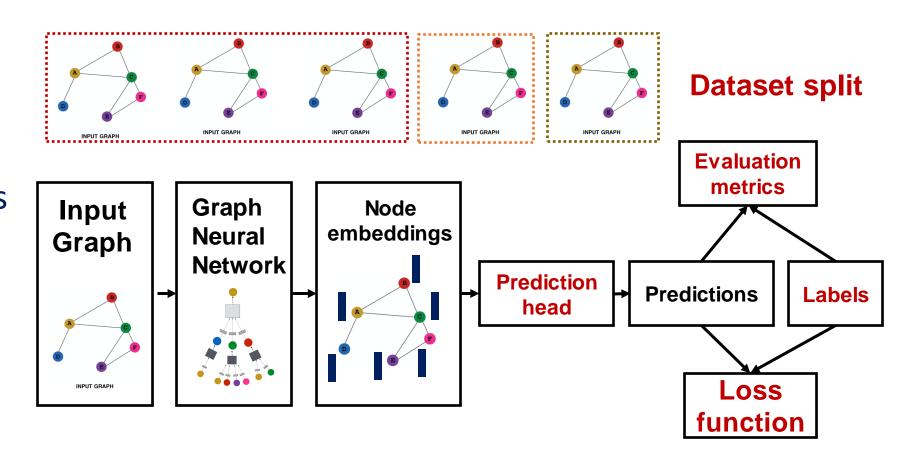


Training message edges
Training supervision edges
Validation edges
Test edges

- Note: Link prediction settings are tricky and complex. You may find papers
 do link prediction differently. But if you follow our reasoning steps, this
 should be the right way to implement link prediction
- Luckily, we have full support in <u>DeepSNAP</u> and <u>GraphGym</u>

GNN Training Pipeline (7)

- Implementation resources:
- <u>DeepSNAP</u> provides core modules for this pipeline
- GraphGym further implements the full pipeline to facilitate GNN design



Summary of the Lecture

We introduce the pipeline of GNN training

- Prediction Heads:
 - Node-level / Edge-level / Graph-level
- Predictions & Labels
 - Supervised / unsupervised
- Loss Functions:
 - Regression / Classification
- Evaluation Metrics:
 - Regression / Classification
- Dataset Split
 - Transductive / inductive
 - Node / edge / graph