

The LAGraph C API Specification Version 0.5

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others as they take on writing tasks

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Chapter 1

Introduction

The LAGraph C API defines a library of graph algorithms based on a representation of graphs in terms of linear algebra [2]. We assume an implementation of the LAGraph library is layered on top of a GraphBLAS library [1] and the objects manipulated by LAGraph are constructed from GraphBLAS objects.

LAGraph is built on a collection of objects exposed to the C programmer as opaque data types. Functions that manipulate these objects are referred to as *methods*. These methods fully define the interface to LAGraph objects to create or destroy them, modify their contents, and copy the contents of opaque objects into non-opaque objects; the contents of which are under direct control of the programmer.

The LAGraph C API is designed to work with C99 (ISO/IEC 9899:199) extended with *static type-based* and *number of parameters-based* function polymorphism, and language extensions on par with the `_Generic` construct from C11 (ISO/IEC 9899:2011).

The remainder of this document is organized as follows:

- Chapter 2: Basic Concepts
- Chapter 3: Objects
- Chapter 4: Functions
- Appendix A: Revision History
- Appendix B: Examples

Chapter 2

Basic Concepts

The GraphBLAS C API is used to construct graph algorithms expressed “in the language of linear algebra.” Graphs are expressed as matrices, and the operations over these matrices are generalized through the use of a semiring algebraic structure.

In this chapter, we will define the basic concepts used to define the GraphBLAS C API. We provide the following elements:

- Glossary of terms used in this document.
- Notation
- Execution model
- Error model

2.1 Glossary

2.1.1 Basic definitions

- *application*: A program that calls methods from the GraphBLAS C API to solve a problem.
- *GraphBLAS C API*: The application programming interface that fully defines the types, objects, literals, and other elements of the C binding to the GraphBLAS.

2.1.2 Objects and their structure

- *handle*: A variable that uses one of the GraphBLAS opaque data types. The value of this variable holds a reference to a GraphBLAS object but not the contents of the object itself. Hence, assigning a value of one handle to another variable copies the reference to the GraphBLAS object but not the contents of the object.

- 102 • *non-opaque datatype*: Any datatype that exposes its internal structure. This is contrasted
103 with an *opaque datatype* that hides its internal structure and can be manipulated only through
104 an API.

Notation	Description
$D_{out}, D_{in}, D_{in_1}, D_{in_2}$	Refers to output and input domains of various GraphBLAS operators.
$\mathbf{D}_{out}(*), \mathbf{D}_{in}(*),$ $\mathbf{D}_{in_1}(*), \mathbf{D}_{in_2}(*)$	Evaluates to output and input domains of GraphBLAS operators (usually a unary or binary operator, or semiring).
$\mathbf{D}(*)$	Evaluates to the (only) domain of a GraphBLAS object (usually a monoid, vector, or matrix).
f	An arbitrary unary function, usually a component of a unary operator.
$\mathbf{f}(F_u)$	Evaluates to the unary function contained in the unary operator given as the argument.
\odot	An arbitrary binary function, usually a component of a binary operator.
$\odot(*)$	Evaluates to the binary function contained in the binary operator or monoid given as the argument.
\otimes	Multiplicative binary operator of a semiring.
\oplus	Additive binary operator of a semiring.
$\otimes(S)$	Evaluates to the multiplicative binary operator of the semiring given as the argument.
$\oplus(S)$	Evaluates to the additive binary operator of the semiring given as the argument.
$\mathbf{0}(*)$	The identity of a monoid, or the additive identity of a GraphBLAS semiring.
$\mathbf{L}(*)$	The contents (all stored values) of the vector or matrix GraphBLAS objects. For a vector, it is the set of (index, value) pairs, and for a matrix it is the set of (row, col, value) triples.
$\mathbf{v}(i)$ or v_i	The i^{th} element of the vector \mathbf{v} .
$\mathbf{size}(\mathbf{v})$	The size of the vector \mathbf{v} .
$\mathbf{ind}(\mathbf{v})$	The set of indices corresponding to the stored values of the vector \mathbf{v} .
$\mathbf{nrows}(\mathbf{A})$	The number of rows in the \mathbf{A} .
$\mathbf{ncols}(\mathbf{A})$	The number of columns in the \mathbf{A} .
$\mathbf{indrow}(\mathbf{A})$	The set of row indices corresponding to rows in \mathbf{A} that have stored values.
$\mathbf{indcol}(\mathbf{A})$	The set of column indices corresponding to columns in \mathbf{A} that have stored values.
$\mathbf{ind}(\mathbf{A})$	The set of (i, j) indices corresponding to the stored values of the matrix.
$\mathbf{A}(i, j)$ or A_{ij}	The element of \mathbf{A} with row index i and column index j .
$\mathbf{A}(:, j)$	The j^{th} column of the the matrix \mathbf{A} .
$\mathbf{A}(i, :)$	The i^{th} row of the the matrix \mathbf{A} .
\mathbf{A}^T	The transpose of the matrix \mathbf{A} .
$\neg \mathbf{M}$	The complement of \mathbf{M} .
$\tilde{\mathbf{t}}$	A temporary object created by the GraphBLAS implementation.
$< type >$	A method argument type that is <code>void *</code> or one of the types from Table ??.
<code>GrB_ALL</code>	A method argument literal to indicate that all indices of an input array should be used.
<code>GrB_Type</code>	A method argument type that is either a user defined type or one of the types from Table ??.
<code>GrB_Object</code>	A method argument type referencing any of the GraphBLAS object types.
<code>GrB_NULL</code>	The GraphBLAS NULL.

2.3 Error model

2.4 Execution Model

A program using the GraphBLAS C API constructs GraphBLAS objects, manipulates them to implement a graph algorithm, and then extracts values from the GraphBLAS objects as the result of the algorithm. Functions defined within the GraphBLAS C API that manipulate GraphBLAS objects are called *methods*. If the method corresponds to one of the operations defined in the GraphBLAS mathematical specification, we refer to the method as an *operation*.

Graph algorithms are expressed as an ordered collection of GraphBLAS method calls defined by the order they are encountered in a program. This is called the *program order*. Each method in the collection uniquely and unambiguously defines the output GraphBLAS objects based on the GraphBLAS operation and the input GraphBLAS objects. This is the case as long as there are no execution errors, which can put objects in an invalid state (see Section 2.5).

The GraphBLAS method calls in program order are organized into contiguous and nonoverlapping *sequences*. A sequence is an ordered collection of method calls as encountered by an executing thread. (For more on threads and GraphBLAS, see Section ??.) A sequence begins with either (1) the first GraphBLAS method called by a thread, or (2) the first method called by a thread after the end of the previous sequence. A sequence can end (terminate) in a variety of ways. A call to the GraphBLAS `GrB_wait()` method (Section ??) always ends a sequence. The GraphBLAS `GrB_finalize()` method (Section ??) also implicitly ends a sequence. Finally, in blocking mode (see below), each GraphBLAS method starts and ends its own sequence.

The GraphBLAS objects are fully defined at any point in a sequence by the methods in the sequence as long as there are no execution errors. In particular, as soon as a GraphBLAS method call returns, its output can be used in the next GraphBLAS method call. However, individual operations in a sequence may not be *complete*. We say that an operation is complete when all the computations in the operation have finished and all the values of its output object have been produced and committed to the address space of the program. Furthermore, no additional execution time can be charged to a completed operation and no additional errors can be attributed to a completed operation.

The opaqueness of GraphBLAS objects allows execution to proceed from one method to the next even when operations are not complete. Processing of nonopaque objects is never deferred in GraphBLAS. That is, methods that consume nonopaque objects (e.g., `GrB_Matrix_build()`, Section ??) and methods that produce nonopaque objects (e.g., `GrB_Matrix_extractTuples()`, Section ??) always finish consuming or producing those nonopaque objects before returning.

2.5 Error Model

All GraphBLAS methods return a value of type `GrB_Info` to provide information available to the system at the time the method returns. The returned value can be either `GrB_SUCCESS` or one of the defined error values shown in Table 2.1. The errors fall into two groups: API errors

(Table 2.1(a)) and execution errors (Table 2.1(b)).

An API error means that a GraphBLAS method was called with parameters that violate the rules for that method. These errors are restricted to those that can be determined by inspecting the types and domains of GraphBLAS objects, GraphBLAS operators, or the values of scalar parameters fixed at the time a method is called. API errors are deterministic and consistent across platforms and implementations. API errors are never deferred, even in nonblocking mode. That is, if a method is called in a manner that would generate an API error, it always returns with the appropriate API error value. If a GraphBLAS method returns with an API error, it is guaranteed that none of the arguments to the method (or any other program data) have been modified.

Execution errors indicate that something went wrong during the execution of a legal GraphBLAS method invocation. Their occurrence may depend on specifics of the executing environment and data values being manipulated. This does not mean that execution errors are the fault of the GraphBLAS implementation. For example, a memory leak could arise from an error in an application's source code (a "program error"), but it may manifest itself in different points of a program's execution (or not at all) depending on the platform, problem size, or what else is running at that time. Index-out-of-bounds and insufficient space execution errors always indicate a program error.

In blocking mode, where each method executes to completion, a returned execution error value applies to the specific method. If a GraphBLAS method, executing in blocking mode, returns with any execution error from Table 2.1(b) other than `GrB_PANIC`, it is guaranteed that no argument used as input-only has been modified. Output arguments may be left in an invalid state, and their use downstream in the program flow may cause additional errors. If a GraphBLAS method returns with a `GrB_PANIC` execution error, no guarantees can be made about the state of any program data.

In nonblocking mode, execution errors can be deferred. A return value of `GrB_SUCCESS` only guarantees that there are no API errors in the method invocation. If an execution error value is returned by a method in nonblocking mode, it indicates that an error was found during execution of the sequence, up to and including the `GrB_wait()` method (Section ??) call that ends the sequence. When possible, that return value will provide information concerning the cause of the error.

As discussed in Section ??, a `GrB_wait(obj)` on a specific GraphBLAS object `obj` does not necessarily end a sequence. However, no additional errors on the methods of the sequence that have `obj` as an `OUT` or `INOUT` argument can be reported. From a GraphBLAS perspective, those methods are *complete*.

If a GraphBLAS method, executing in nonblocking mode, returns with any execution error from Table 2.1(b) other than `GrB_PANIC`, it is guaranteed that no argument used as input-only through the entire sequence has been modified. Any output argument in the sequence may be left in an invalid state and its use downstream in the program flow may cause additional errors. If a GraphBLAS method returns with a `GrB_PANIC`, no guarantees can be made about the state of any program data.

After a call to any GraphBLAS method, the program can retrieve additional error information (beyond the error code returned by the method) through a call to the function `GrB_error()`. The signature of that function is shown in Figure 2.1. The function returns a pointer to a NULL-terminated string, and the contents of that string are implementation dependent. In particular, a

```
const char *GrB_error();
```

Figure 2.1: Signature of `GrB_error()` function.

186 null string (not a `NULL` pointer) is always a valid error string. The pointer is valid until the next
187 call to any GraphBLAS method by the same thread. `GrB_error()` is a thread-safe function, in the
188 sense that multiple threads can call it simultaneously and each will get its own error string back,
189 referring to the last GraphBLAS method it called.

Table 2.1: Error values returned by GraphBLAS methods.

(a) API errors

Error code	Description
GrB_UNINITIALIZED_OBJECT	A GraphBLAS object is passed to a method before <code>new</code> was called on it.
GrB_NULL_POINTER	A NULL is passed for a pointer parameter.
GrB_INVALID_VALUE	Miscellaneous incorrect values.
GrB_INVALID_INDEX	Indices passed are larger than dimensions of the matrix or vector being accessed.
GrB_DOMAIN_MISMATCH	A mismatch between domains of collections and operations when user-defined domains are in use.
GrB_DIMENSION_MISMATCH	Operations on matrices and vectors with incompatible dimensions.
GrB_OUTPUT_NOT_EMPTY	An attempt was made to build a matrix or vector using an output object that already contains valid tuples (elements).
GrB_NO_VALUE	A location in a matrix or vector is being accessed that has no stored value at the specified location.

(b) Execution errors

Error code	Description
GrB_OUT_OF_MEMORY	Not enough memory for operations.
GrB_INSUFFICIENT_SPACE	The array provided is not large enough to hold output.
GrB_INVALID_OBJECT	One of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error.
GrB_INDEX_OUT_OF_BOUNDS	Reference to a vector or matrix element that is outside the defined dimensions of the object.
GrB_PANIC	Unknown internal error.

Chapter 3

Objects

The LAGraph library depends on a number of objects to represent graphs, vectors, and other types associated with graph algorithms in LAGraph. Other objects are not directly associated with the input and output variables, but instead modify the behavior of the LAGraph functions. These are related to the descriptors in the GraphBLAS.

In this chapter, we need to describe all of the objects a user of LAGraph needs to understand. This is also where we describe the types of LAGraph objects and any constraints on those types.

Chapter 4

Functions

The LAGraph library is composed of the following groups of functions:

- Context: Functions that manage the context or environment of an instance of the LAGraph library.
- Graph Algorithms: Functions that implement a Graph Algorithm.
- Utilities: Functions that support implementation of Graph Algorithms or support users of LAGraph.

We need to discuss the rules used in naming the functions and defining their argument lists.

4.1 Context

LAGraph init, finalize and other functions that manage the environment of an instance of LAGraph.

4.2 Graph Algorithms

List the algorithms here. Then have a subsection with the definition of each algorithms.

4.2.1 vxm: Vector-matrix multiply

Multiplies a (row) vector with a matrix on an semiring. The result is a vector.

C Syntax

```
GrB_Info GrB_vxm(GrB_Vector w,  
                 const GrB_Vector mask,
```

```

216         const GrB_BinaryOp      accum,
217         const GrB_Semiring      op,
218         const GrB_Vector        u,
219         const GrB_Matrix        A,
220         const GrB_Descriptor    desc);

```

221 Parameters

222 **w** (INOUT) An existing GraphBLAS vector. On input, the vector provides values
223 that may be accumulated with the result of the vector-matrix product. On output,
224 this vector holds the results of the operation.

225 **mask** (IN) An optional “write” mask that controls which results from this operation are
226 stored into the output vector **w**. The mask dimensions must match those of the
227 vector **w**. If the **GrB_STRUCTURE** descriptor is *not* set for the mask, the domain
228 of the **mask** vector must be of type **bool** or any of the predefined “built-in” types
229 in Table ???. If the default mask is desired (i.e., a mask that is all **true** with the
230 dimensions of **w**), **GrB_NULL** should be specified.

231 **accum** (IN) An optional binary operator used for accumulating entries into existing **w**
232 entries. If assignment rather than accumulation is desired, **GrB_NULL** should be
233 specified.

234 **op** (IN) Semiring used in the vector-matrix multiply.

235 **u** (IN) The GraphBLAS vector holding the values for the left-hand vector in the
236 multiplication.

237 **A** (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the
238 multiplication.

239 **desc** (IN) An optional operation descriptor. If a *default* descriptor is desired, **GrB_NULL**
240 should be specified. Non-default field/value pairs are listed as follows:

241

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask .
A	GrB_INP1	GrB_TRAN	Use transpose of A for the operation.

242

243 Return Values

244 **GrB_SUCCESS** In blocking mode, the operation completed successfully. In non-
 245 blocking mode, this indicates that the compatibility tests on di-
 246 mensions and domains for the input arguments passed successfully.
 247 Either way, output vector **w** is ready to be used in the next method
 248 of the sequence.

249 **GrB_PANIC** Unknown internal error.

250 **GrB_INVALID_OBJECT** This is returned in any execution mode whenever one of the opaque
 251 GraphBLAS objects (input or output) is in an invalid state caused
 252 by a previous execution error. Call **GrB_error()** to access any error
 253 messages generated by the implementation.

254 **GrB_OUT_OF_MEMORY** Not enough memory available for the operation.

255 **GrB_UNINITIALIZED_OBJECT** One or more of the GraphBLAS objects has not been initialized by
 256 a call to **new** (or **dup** for matrix or vector parameters).

257 **GrB_DIMENSION_MISMATCH** Mask, vector, and/or matrix dimensions are incompatible.

258 **GrB_DOMAIN_MISMATCH** The domains of the various vectors/matrices are incompatible with
 259 the corresponding domains of the semiring or accumulation opera-
 260 tor, or the mask's domain is not compatible with **bool** (in the case
 261 where **desc[GrB_MASK].GrB_STRUCTURE** is not set).

262 Description

263 **GrB_vxm** computes the vector-matrix product $\mathbf{w}^T = \mathbf{u}^T \oplus . \otimes \mathbf{A}$, or, if an optional binary accu-
 264 mulation operator (\odot) is provided, $\mathbf{w}^T = \mathbf{w}^T \odot (\mathbf{u}^T \oplus . \otimes \mathbf{A})$ (where matrix **A** can be optionally
 265 transposed). Logically, this operation occurs in three steps:

266 **Setup** The internal vectors, matrices and mask used in the computation are formed and their
 267 domains/dimensions are tested for compatibility.

268 **Compute** The indicated computations are carried out.

269 **Output** The result is written into the output vector, possibly under control of a mask.

270 Up to four argument vectors or matrices are used in the **GrB_vxm** operation:

- 271 1. $\mathbf{w} = \langle \mathbf{D}(\mathbf{w}), \mathbf{size}(\mathbf{w}), \mathbf{L}(\mathbf{w}) = \{(i, w_i)\} \rangle$
- 272 2. $\mathbf{mask} = \langle \mathbf{D}(\mathbf{mask}), \mathbf{size}(\mathbf{mask}), \mathbf{L}(\mathbf{mask}) = \{(i, m_i)\} \rangle$ (optional)
- 273 3. $\mathbf{u} = \langle \mathbf{D}(\mathbf{u}), \mathbf{size}(\mathbf{u}), \mathbf{L}(\mathbf{u}) = \{(i, u_i)\} \rangle$
- 274 4. $\mathbf{A} = \langle \mathbf{D}(\mathbf{A}), \mathbf{nrows}(\mathbf{A}), \mathbf{ncols}(\mathbf{A}), \mathbf{L}(\mathbf{A}) = \{(i, j, A_{ij})\} \rangle$

275 The argument matrices, vectors, the semiring, and the accumulation operator (if provided) are
 276 tested for domain compatibility as follows:

- 277 1. If `mask` is not `GrB_NULL`, and `desc[GrB_MASK].GrB_STRUCTURE` is not set, then $\mathbf{D}(\text{mask})$
 278 must be from one of the pre-defined types of Table ??.
- 279 2. $\mathbf{D}(\mathbf{u})$ must be compatible with $\mathbf{D}_{in_1}(\text{op})$ of the semiring.
- 280 3. $\mathbf{D}(\mathbf{A})$ must be compatible with $\mathbf{D}_{in_2}(\text{op})$ of the semiring.
- 281 4. $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{out}(\text{op})$ of the semiring.
- 282 5. If `accum` is not `GrB_NULL`, then $\mathbf{D}(\mathbf{w})$ must be compatible with $\mathbf{D}_{in_1}(\text{accum})$ and $\mathbf{D}_{out}(\text{accum})$
 283 of the accumulation operator and $\mathbf{D}_{out}(\text{op})$ of the semiring must be compatible with $\mathbf{D}_{in_2}(\text{accum})$
 284 of the accumulation operator.

285 Two domains are compatible with each other if values from one domain can be cast to values in
 286 the other domain as per the rules of the C language. In particular, domains from Table ?? are
 287 all compatible with each other. A domain from a user-defined type is only compatible with itself.
 288 If any compatibility rule above is violated, execution of `GrB_vxm` ends and the domain mismatch
 289 error listed above is returned.

290 From the argument vectors and matrices, the internal matrices and mask used in the computation
 291 are formed (\leftarrow denotes copy):

- 292 1. Vector $\tilde{\mathbf{w}} \leftarrow \mathbf{w}$.
- 293 2. One-dimensional mask, $\tilde{\mathbf{m}}$, is computed from argument `mask` as follows:
 294 (a) If `mask = GrB_NULL`, then $\tilde{\mathbf{m}} = \langle \text{size}(\mathbf{w}), \{i, \forall i : 0 \leq i < \text{size}(\mathbf{w})\} \rangle$.
 295 (b) If `mask \neq GrB_NULL`,
 296 i. If `desc[GrB_MASK].GrB_STRUCTURE` is set, then $\tilde{\mathbf{m}} = \langle \text{size}(\text{mask}), \{i : i \in \text{ind}(\text{mask})\} \rangle$,
 297 ii. Otherwise, $\tilde{\mathbf{m}} = \langle \text{size}(\text{mask}), \{i : i \in \text{ind}(\text{mask}) \wedge (\text{bool})\text{mask}(i) = \text{true}\} \rangle$.
 298 (c) If `desc[GrB_MASK].GrB_COMP` is set, then $\tilde{\mathbf{m}} \leftarrow \neg \tilde{\mathbf{m}}$.
- 299 3. Vector $\tilde{\mathbf{u}} \leftarrow \mathbf{u}$.
- 300 4. Matrix $\tilde{\mathbf{A}} \leftarrow \text{desc}[\text{GrB_INP1}].\text{GrB_TRAN} ? \mathbf{A}^T : \mathbf{A}$.

301 The internal matrices and masks are checked for shape compatibility. The following conditions
 302 must hold:

- 303 1. $\text{size}(\tilde{\mathbf{w}}) = \text{size}(\tilde{\mathbf{m}})$.
- 304 2. $\text{size}(\tilde{\mathbf{w}}) = \text{ncols}(\tilde{\mathbf{A}})$.
- 305 3. $\text{size}(\tilde{\mathbf{u}}) = \text{nrows}(\tilde{\mathbf{A}})$.

306 If any compatibility rule above is violated, execution of `GrB_vxm` ends and the dimension mismatch
 307 error listed above is returned.

308 From this point forward, in `GrB_NONBLOCKING` mode, the method can optionally exit with
 309 `GrB_SUCCESS` return code and defer any computation and/or execution error codes.

310 We are now ready to carry out the vector-matrix multiplication and any additional associated
 311 operations. We describe this in terms of two intermediate vectors:

- 312 • $\tilde{\mathbf{t}}$: The vector holding the product of vector $\tilde{\mathbf{u}}^T$ and matrix $\tilde{\mathbf{A}}$.
- 313 • $\tilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

314 The intermediate vector $\tilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathbf{op}), \mathbf{ncols}(\tilde{\mathbf{A}}), \{(j, t_j) : \mathbf{ind}(\tilde{\mathbf{u}}) \cap \mathbf{ind}(\tilde{\mathbf{A}}(:, j)) \neq \emptyset\} \rangle$ is created.
 315 The value of each of its elements is computed by

$$316 \quad t_j = \bigoplus_{k \in \mathbf{ind}(\tilde{\mathbf{u}}) \cap \mathbf{ind}(\tilde{\mathbf{A}}(:, j))} (\tilde{\mathbf{u}}(k) \otimes \tilde{\mathbf{A}}(k, j)),$$

317 where \oplus and \otimes are the additive and multiplicative operators of semiring `op`, respectively.

318 4.3 Utilities

319 Import, Export, and other functions to support users and LAGraph algorithm developers.

Appendix A

Revision History

Changes in 1.3.1:

- (Issue 70,67) [PENDING] changes to GrB_wait(obj).
- (Issue 69) Made names/symbols containing underscores searchable in PDF.
- Typographical change to eWiseAdd Description to be consistent in order of set intersections.

³²⁶ **Appendix B**

³²⁷ **Examples**

B.1 Example: level breadth-first search (BFS) in GraphBLAS

```

1  #include <stdlib.h>
2  #include <stdio.h>
3  #include <stdint.h>
4  #include <stdbool.h>
5  #include "GraphBLAS.h"
6
7  /*
8   * Given a boolean  $n \times n$  adjacency matrix  $A$  and a source vertex  $s$ , performs a BFS traversal
9   * of the graph and sets  $v[i]$  to the level in which vertex  $i$  is visited ( $v[s] == 1$ ).
10  * If  $i$  is not reachable from  $s$ , then  $v[i] = 0$ . (Vector  $v$  should be empty on input.)
11  */
12  GrB_Info BFS(GrB_Vector *v, GrB_Matrix A, GrB_Index s)
13  {
14      GrB_Index n;
15      GrB_Matrix_nrows(&n,A);                //  $n = \#$  of rows of  $A$ 
16
17      GrB_Vector_new(v,GrB_INT32,n);          // Vector<int32_t>  $v(n)$ 
18
19      GrB_Vector q;                          // vertices visited in each level
20      GrB_Vector_new(&q,GrB_BOOL,n);          // Vector<bool>  $q(n)$ 
21      GrB_Vector_setElement(q,(bool)true,s);  //  $q[s] = \text{true}$ , false everywhere else
22
23      /*
24       * BFS traversal and label the vertices.
25       */
26      int32_t d = 0;                          //  $d = \text{level in BFS traversal}$ 
27      bool succ = false;                      //  $\text{succ} == \text{true}$  when some successor found
28      do {
29          ++d;                                // next level (start with 1)
30          GrB_assign(*v,q,GrB_NULL,d,GrB_ALL,n,GrB_NULL); //  $v[q] = d$ 
31          GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL,
32                 q,A,GrB_DESC_RC);           //  $q[!v] = q \parallel A$ ; finds all the
33                                              // unvisited successors from current  $q$ 
34          GrB_reduce(&succ,GrB_NULL,GrB_LOR_MONOID_BOOL,
35                   q,GrB_NULL);              //  $\text{succ} = \parallel(q)$ 
36      } while (succ);                         // if there is no successor in  $q$ , we are done.
37
38      GrB_free(&q);                          //  $q$  vector no longer needed
39
40      return GrB_SUCCESS;
41  }

```

Bibliography

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- [2] Jeremy Kepner and John Gilbert. *Graph Algorithms in the Language of Linear Algebra*. SIAM Press, 2011.