LAGraph Algorithms

LAGraph Working Group

October 25, 2021

Abstract

Theoretical documentation for LAGraph.

1 Notation

Table 1 contains the notation of GraphBLAS operations

Additionally, we use $\mathbf{D} = diag(J, n)$ to construct a diagonal matrix $\mathbf{D} \leftarrow \{J, J, [1, 1, \dots, 1]\}$. The elements of the matrix are $\mathbf{D}(j, j) = 1$ for $j \in J$.

Initializing scalars, vectors, and matrices (GraphBLAS methods):

- let: $s \in \mathbb{Q}_{64}$

- let: $\mathbf{u} \in \mathbb{Q}_{32}^n$

- let: $\mathbf{A} \in \mathbb{N}_{16}^{m \times n}$

- let: $\mathbf{A} \in \mathbb{Z}_{64}^{k \times m}$

2 Algorithms

LAGraph [12] implements graph algorithms using the GraphBLAS C API [13].

An incomplete list of GraphBLAS algorithms:

- MSBFS [8], bidirectional BFS [8], pushpull BFS [19]

- DFS [16]

- weakly connected components [21]

- SCC (LAGraph)

- SSSP, delta-stepping [17]

- triangle count [1, 5], triangle enumeration [1], item local clustering coefficient [4]

- k-truss [5]

- betweenness centrality [12]

- closeness centrality [8]

- DNN algorithm [10, 7]

- PageRank variants (at least 2) [4], IISWC paper, ...

- Louvain [11]

- property graphs: incremental TTC case [9], SIGMOD 2014 Contest [8] Roi Lipman's talk[1]

- CFPQs based on a string of GRADES/ADBIS/other papers [2, 3, 14, 18, 15]

- Implementations: GBTL[2], SuiteSparse:GraphBLAS [6], GraphBLAST [20]

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[1] http://wiki.ldbcouncil.org/pages/viewpage.action?pageId=106233859&preview=/106233859/111706128/LDBC-July-2019.pdf

[2] https://github.com/cmu-sei/gbtl

op./method	name	notation
`mxm`	matrix-matrix multiplication	$\mathbf{C}\langle\mathbf{M}\rangle\odot=\mathbf{A}\oplus.\otimes\mathbf{B}$
`vxm`	vector-matrix multiplication	$\mathbf{w}\langle\mathbf{m}\rangle\odot=\mathbf{u}\oplus.\otimes\mathbf{A}$
`mxv`	matrix-vector multiplication	$\mathbf{w}\langle\mathbf{m}\rangle\odot=\mathbf{A}\oplus.\otimes\mathbf{u}$
`eWiseAdd`	element-wise addition	$\mathbf{C}\langle\mathbf{M}\rangle\odot=\mathbf{A}\oplus\mathbf{B}$
	set union of patterns	$\mathbf{w}\langle\mathbf{m}\rangle\odot=\mathbf{u}\oplus\mathbf{v}$
`eWiseMult`	element-wise multiplication	$\mathbf{C}\langle\mathbf{M}\rangle\odot=\mathbf{A}\otimes\mathbf{B}$
	set intersection of patterns	$\mathbf{w}\langle\mathbf{m}\rangle\odot=\mathbf{u}\otimes\mathbf{v}$
`extract`	extract submatrix	$\mathbf{C}\langle\mathbf{M}\rangle\odot=\mathbf{A}(I,J)$
	extract column vector	$\mathbf{w}\langle\mathbf{m}\rangle\odot=\mathbf{A}(:,j)$
	extract row vector	$\mathbf{w}\langle\mathbf{m}\rangle\odot=\mathbf{A}(i,:)$
	extract subvector	$\mathbf{w}\langle\mathbf{m}\rangle\odot=\mathbf{u}(I)$
`assign`	assign matrix to submatrix with mask for $\mathbf{C}$	$\mathbf{C}\langle\mathbf{M}\rangle(I,J)\odot=\mathbf{A}$
	assign scalar to submatrix with mask for $\mathbf{C}$	$\mathbf{C}\langle\mathbf{M}\rangle(I,J)\odot=s$
	assign vector to subvector with mask for $\mathbf{w}$	$\mathbf{w}\langle\mathbf{m}\rangle(I)\odot=\mathbf{u}$
	assign scalar to subvector with mask for $\mathbf{w}$	$\mathbf{w}\langle\mathbf{m}\rangle(I)\odot=s$
`subassign (GxB)`	assign matrix to submatrix with submask for $\mathbf{C}(I,J)$	$\mathbf{C}(I,J)\langle\mathbf{M}\rangle\odot=\mathbf{A}$
	assign scalar to submatrix with submask for $\mathbf{C}(I,J)$	$\mathbf{C}(I,J)\langle\mathbf{M}\rangle\odot=s$
	assign vector to subvector with submask for $\mathbf{w}(I)$	$\mathbf{w}(I)\langle\mathbf{m}\rangle\odot=\mathbf{u}$
	assign scalar to subvector with submask for $\mathbf{w}(I)$	$\mathbf{w}(I)\langle\mathbf{m}\rangle\odot=s$
`apply`	apply unary operator	$\mathbf{C}\langle\mathbf{M}\rangle\odot=f(\mathbf{A})$
		$\mathbf{w}\langle\mathbf{m}\rangle\odot=f(\mathbf{u})$
`select (GxB)`	apply select operator	$\mathbf{C}\langle\mathbf{M}\rangle\odot=select(\mathbf{A},f(k))$
		$\mathbf{C}\langle\mathbf{M}\rangle\odot=select(low\le\mathbf{A}\le up)$
		$\mathbf{w}\langle\mathbf{m}\rangle\odot=select(\mathbf{u},f(k))$
		$\mathbf{w}\langle\mathbf{m}\rangle\odot=select(low\le\mathbf{u}\le up)$
`reduce`	reduce matrix to column vector	$\mathbf{w}\langle\mathbf{m}\rangle\odot=[\oplus_j\mathbf{A}(:,j)]$
	reduce matrix to scalar	$s\odot=[\oplus_{ij}\mathbf{A}(i,j)]$
	reduce vector to scalar	$s\odot=[\oplus_i\mathbf{u}(i)]$
`transpose`	transpose	$\mathbf{C}\langle\mathbf{M}\rangle\odot=\mathbf{A}^\top$
`kronecker`	Kronecker multiplication	$\mathbf{C}\langle\mathbf{M}\rangle\odot=kron(\mathbf{A},\mathbf{B})$
`new`	new matrix	let: $\mathbf{A}\in\mathsf{TYPE}_{\mathrm{PRECISION}}^{n\times m}$
	new vector	let: $\mathbf{u}\in\mathsf{TYPE}_{\mathrm{PRECISION}}^{n}$
`build`	build matrix from index/value arrays	$\mathbf{C}\leftarrow\{I,J,X\}$
	build vector from index/value arrays	$\mathbf{w}\leftarrow\{I,X\}$
`extractTuples`	extract index/value arrays	$\{I,J,X\}\leftarrow\mathbf{A}$
		$\{I,X\}\leftarrow\mathbf{u}$
`dup`	duplicate matrix	$\mathbf{C}\leftarrow\mathbf{A}$
	duplicate vector	$\mathbf{w}\leftarrow\mathbf{u}$
`extractElement`	extract scalar element	$s=\mathbf{A}(i,j)$
		$s=\mathbf{u}(i)$
`setElement`	set element	$\mathbf{C}(i,j)=s$
		$\mathbf{w}(i)=s$

Table 1: GraphBLAS operations and methods based on [6]. *Notation:* Matrices and vectors are typeset in bold, starting with uppercase ($\mathbf{A}$) and lowercase ($\mathbf{u}$) letters, respectively. Scalars including indices are lowercase italic (s, i, j) while arrays are uppercase italic (X, I, J). $\oplus$ and $\otimes$ are the addition and multiplication operators forming a semiring and default to conventional arithmetic $+$ and $\times$ operators. $\odot$ is the apply operator. Masks $\langle\mathbf{M}\rangle$ and $\langle\mathbf{m}\rangle$ are used to selectively write to the result matrix/vector. The complements of masks $\langle\mathbf{M}\rangle$, $\langle\mathbf{m}\rangle$ can be selected with the negation symbol, denoted with $\langle\neg\mathbf{M}\rangle$, $\langle\neg\mathbf{m}\rangle$, respectively. Masks with "replace" semantics (annihilating all elements outside the mask) are denoted with $\langle\!\langle\mathbf{M}\rangle\!\rangle$, $\langle\!\langle\neg\mathbf{M}\rangle\!\rangle$, $\langle\!\langle\mathbf{m}\rangle\!\rangle$, and $\langle\!\langle\neg\mathbf{m}\rangle\!\rangle$, The structure of the mask is denoted with $\langle\{\mathbf{M}\}\rangle$, $\langle\neg\{\mathbf{M}\}\rangle$, $\langle\{\mathbf{m}\}\rangle$, and $\langle\neg\{\mathbf{m}\}\rangle$.

Algorithm 1: Breadth-first search.

---

1 Function *BFS*
    2   frontier(s) = TRUE
    3   **for** $level = 1$ **to** $n - 1$ **do**
    4   seen$\langle$frontier$\rangle$ = $level$
    5   frontier$\langle\langle\neg$seen$\rangle\rangle$ = frontier any.pair **A**

Algorithm 2: Breadth-first search (push/pull).

---

1 Function *BFS*
    2   frontier(s) = TRUE
    3   **for** $level = 1$ **to** $n - 1$ **do**
    4   seen$\langle$frontier$\rangle$ = $level$
    5 $push$ = use some heuristics to determine whether to push/pull
    6   **if** $push$ **then**
    7   frontier$\langle\langle\neg$seen$\rangle\rangle$ = frontier any.pair **A**
    8   **else**
    9   frontier$\langle\langle\neg$seen$\rangle\rangle$ = **A** any.pair frontier

---

Algorithm 3: Multi-source breadth-first search.

  Data: ...
  Result: ...
1 Function *MSBFS*
    2   **Frontier** $\leftarrow$ diag(S, n)
    3   **for** $level = 1$ *to* $n - 1$ **do**
    4   **Seen**$\langle$**Frontier**$\rangle$ = $level$
    5   **Frontier**$\langle\langle\neg$**Seen**$\rangle\rangle$ = **Frontier** any.pair **A**

Algorithm 4: Betweenness centrality.

---

1 Function *MSBFS*

2   // The **NumSp** structure holds the number of shortest paths for each node and
        starting vertex discovered so far.

3   // Initialized to source vertices.

4 $\mathbf{NumSp} \leftarrowtail \{s, [1, 1, \ldots, 1]\}$

5   // The **Frontier** holds the number of shortest paths for each node and starting vertex
        discovered so far.

6   // Initialized to source vertices.

7 $\mathbf{Frontier}\langle\mathbf{NumSp}\rangle = \mathbf{A}(s, :)$

8 $d = 0$

9   // The **Sigmas** matrices store frontier information for each level of the BFS phase.

10 // BFS phase (forward sweep)

11 **do**

12 | // $\mathbf{Sigmas}[d](:, s) = d^{\text{th}}$ level frontier from source vertex s

13 | let: $\mathbf{Sigmas}[d] \in \mathbb{B}^{n \times nsver}$

14 | $\mathbf{Sigmas}[d](:, :) = \mathbf{Frontier}$ // Convert matrix to Boolean

15 | $\mathbf{NumSp} = \mathbf{NumSp} \oplus \mathbf{Frontier}$ // Accumulate path counts

16 | $\mathbf{Frontier}\langle\!\langle\mathbf{NumSp}\rangle\!\rangle = \mathbf{A}^{\top} \oplus.\otimes \mathbf{Frontier}$ // Update frontier

17 **while** *nvals*($\mathbf{Frontier}$) > 0

18 let: $\mathbf{NumSpInv} \in \mathbb{Q}_{32}^{n \times nsver}$

19 $\mathbf{NumSpInv} = 1.0 \oslash \mathbf{NumSp}$

20 let: $\mathbf{BCU} \in \mathbb{Q}_{32}^{n \times nsver}$

21 $\mathbf{BCU}(:) = 1.0$ // Make **BCU** dense, initialize all elements to 1.0

22 let: $\mathbf{W} \in \mathbb{Q}_{32}^{n \times nsver}$

23 // Tally phase (backward sweep)

24 **for** $i = d - 1$ **downto** 0 **do**

25 | $\mathbf{W}\langle\!\langle\mathbf{Sigmas}[i]\rangle\!\rangle = \mathbf{NumSpInv} \oslash \mathbf{BCU}$

26 | $\mathbf{W}\langle\!\langle\mathbf{Sigmas}[i-1]\rangle\!\rangle = \mathbf{A} \oplus.\otimes \mathbf{W}$ // Add contributions by successors and mask with that
        BFS level's frontier.

27 | $\mathbf{BCU} \oplus= \mathbf{W} \otimes \mathbf{NumSp}$

28 // Row reduce **BCU** and subtract *nsver* from every entry to account for 1 extra value
        per **BCU** row element

29 $\mathbf{delta} = [\oplus_j \mathbf{BCU}(:, j)]$

30 $\mathbf{delta} \ominus= nsver$

Algorithm 5: PageRank (used in Graphalytics).

Data: α constant (damping factor)

Result: ...

1 Function *PageRank*

2 | $\mathbf{pr}(:) = 1/n$

3 | $\mathbf{outdegrees} = [\oplus_j \mathbf{A}(:, j)]$

4 | **for** $k = 1$ **to** *numIterations* **do**

5 | | $\mathbf{importance} = \mathbf{pr} \oslash \mathbf{outdegrees}$

6 | | $\mathbf{importance} = times(\mathbf{importance}, \alpha)$ // apply the $times(x, s) = x \cdot s$ operator

7 | | $\mathbf{importance} = \mathbf{importance} \oplus.\otimes \mathbf{A}$

8 | | $\mathbf{danglingVertexRanks}\langle\neg\mathbf{outdegrees}\rangle = \mathbf{pr}(:)$

9 | | $totalDanglingRank = [\oplus_i \mathbf{danglingVertexRanks}(i)(\otimes)]\frac{\alpha}{n}$

10 | | $\mathbf{pr}(:) = \frac{1-\alpha}{n} \oplus totalDanglingRank$

11 | | $\mathbf{pr}\langle\mathbf{importance}\rangle = \mathbf{pr} \oplus \mathbf{importance}$

Algorithm 6: Algebraic Bellman-Ford for SSSP.

---

1 Function *SSSP*
2 $\mathbf{d}(s) = 0$
3   **for** $k = 1$ **to** $n-1$ **do**
4 $\mathbf{dtmp} = \mathbf{d}$ min.plus $\mathbf{A}$
5   **if** $\mathbf{dtmp} = \mathbf{d}$ **then**
6   **break**
7 $\mathbf{d} \hookleftarrow \mathbf{dtmp}$

Algorithm 7: Delta-stepping SSSP.

Data:
   $\mathbf{A}, \mathbf{A_H}, \mathbf{A_L} \in \mathbb{Q}^{|V| \times |V|}$
   $s, i \in \mathbb{N}$
   $\Delta \in \mathbb{Q}$
   $\mathbf{t}, \mathbf{t_{Req}} \in \mathbb{Q}^{|V|}$
   $\mathbf{t_{B_i}}, \mathbf{e} \in \mathbb{N}^{|V|}$

1 Function *DeltaStepping*
2 $\mathbf{A_L} = select(0 < \mathbf{A} \leq \Delta)$
3 $\mathbf{A_H} = select(\Delta < \mathbf{A})$
4 $\mathbf{t}(:) = \infty$
5 $\mathbf{t}(s) = 0$
6   **while** $nvals(select(i\Delta \leq \mathbf{t})) \neq 0$ **do**
7 $s = 0$
8 $\mathbf{t_{B_i}} = select(i\Delta \leq \mathbf{t} < (i+1)\Delta)$
9   **while** $\mathbf{t_{B_i}} \neq 0$ **do**
10 $\mathbf{t_{Req}} = \mathbf{A_L^\top} \oplus.\otimes (\mathbf{t} \otimes \mathbf{t_{B_i}})$
11 $\mathbf{e} = select(0 < \mathbf{e} \oplus \mathbf{t_{B_i}})$
12 $\mathbf{t_{B_i}} = select(i\Delta \leq \mathbf{t_{Req}} < (i+1)\Delta) \otimes (\mathbf{t_{Req}} \underset{\oplus}{\min} \mathbf{t})$
13 $\mathbf{t_{Req}} = \mathbf{A_H^\top} \oplus.\otimes (\mathbf{t} \otimes \mathbf{e})$
14 $\mathbf{t} = \mathbf{t}$ min $\mathbf{t_{Req}}$
15 $i = i + 1$

Algorithm 8: All-pairs shortest path (Floyd–Warshall algorithm).

---

1 Function *FloydWarshall*
2 $\mathbf{D} \hookleftarrow \mathbf{A}$
3   **for** $k = 1$ **to** n **do**
4 $\mathbf{D} = \mathbf{D}$ min$[\mathbf{D}(:, k)$ min.plus $(\mathbf{D}(k, :)]$

Algorithm 9: FastSV algorithm.

```
1  Function FastSV
2  |    n = nrows(A)
3  |    gf = f
4  |    dup = gf
5  |    mngf = gf
6  |    {I, X} ↤ f
7  |    repeat
8  |    |    // Step 1: Stochastic hooking
9  |    |    mngf = mngf min A
10 |    |    mngf = mngf second.min gf
11 |    |    f(X) = f min mngf
12 |    |    // Step 2: Aggressive hooking
13 |    |    f = f min mngf
14 |    |    // Step 3: Shortcutting
15 |    |    f = f min gf
16 |    |    // Step 4: Calculate grandparents
17 |    |    {I, X} ↤ f
18 |    |    gf = f(X)
19 |    |    // Step 5: Check termination
20 |    |    diff = dup ≠ gf
21 |    |    sum = [⊕ᵢ diff(i)]
22 |    |    dup = gf
23 |    until sum == 0
```

Algorithm 10: Triangle count (Cohen's algorithm).

```
1  Function TriangleCount
2  |    L = tril(A)
3  |    U = triu(A)
4  |    B = L ⊕.⊗ U
5  |    C = B ⊗ A
6  |    t = [⊕ᵢⱼ C(i, j)]/2
```

Algorithm 11: Triangle count (Sandia).

```
1  Function TriangleCount
2  |    L = tril(A)
3  |    C⟨L⟩ = L ⊕.⊗ L
4  |    t = [⊕ᵢⱼ C(i, j)]
```

Algorithm 12: Triangle count (FLAME).

```
1  Function TriangleCountFlame
2  |    for i = 2 to n − 1 do
3  |    |    A₂₀ = A(i + 1 : n, 0 : i − 1)
4  |    |    a₁₀ = A(0 : i − 1, i)
5  |    |    a₁₂ = A(i, i + 1 : n)
6  |    |    t ⊕= a₁₀ ⊕.⊗ A₂₀ ⊕.⊗ a₁₂
```

Algorithm 13: Local clustering coefficient.

```
1  Function PageRank
2  |    Tri⟨A⟩ = A ⊕.⊗ A                          // compute triangle count matrix
3  |    tri = [⊕ⱼ Tri(:, j)]                       // reduce to triangle count vector
4  |    deg = [⊕ⱼ A(:, j)]                         // reduce to vertex degree vector
5  |    wed = perm2(deg)          // apply perm2(x) = x · (x − 1) to get wedge count vector
6  |    lcc = tri ⊘ wed                                             // LCC vector
```

Algorithm 14: k-truss algorithm.

1 Function *KTruss*
  2 $\mathbf{C} \leftarrow \mathbf{A}$
  3 $nonzeros \leftarrow nvals(C)$
  4   **for** $i = 1$ **to** $n - 1$ **do**
  5 $\mathbf{C}\langle\mathbf{C}\rangle = \mathbf{C} \oplus.\,\mathsf{land}\ \mathbf{C}$
  6 $\mathbf{C} = select(\mathbf{C} \geq k - 2)$
  7   **if** $nonzeros == nvals(C)$ **then**
  8   **break**
  9 $nonzeros \leftarrow nvals(C)$

Algorithm 15: Louvain algorithm (WIP).

1 Function *Louvain*
  2 $\mathbf{G} \oplus= \mathbf{G}^{\top}$
  3 $\mathbf{k} = [\oplus_j \mathbf{A}(:,j)]$
  4 $m = \frac{1}{2}[\oplus_i \mathbf{k}(i)]$
  5 $\mathbf{S} \leftarrow \mathbf{I}$
  6
  7 $vertices\_changed \leftarrow nvals(\mathbf{k})$
  8   **while** $vertices\_changed > 0$ **do**
  9   **for** $j \in range(|V|)$ **do**
  10 $\mathbf{v} = \mathbf{G}(j,:)$
  11 $\mathbf{t_q} = \mathbf{v}\ \mathsf{any.pair}\ \mathbf{S}$
  12 $\mathbf{sr} = \mathbf{S}(j,:)$
  13 $\mathbf{S}(j,:) = \text{empty}$
  14
  15 $\mathbf{q} \leftarrow \mathbf{k}$
  16 $\mathbf{q}\langle\mathbf{k}\rangle \otimes= -\mathbf{k}(j)/m$
  17 $\mathbf{q} \oplus= \mathbf{v}$
  18 $\mathbf{q_1}\langle\mathbf{t_q}\rangle = \mathbf{q} \oplus.\otimes \mathbf{S}$
  19
  20 $\mathbf{t} = (\mathbf{q_1} == [\mathsf{max}_i\ \mathbf{q_1}(i)])$
  21   **while** $nvals(t) \neq 1$ **do**
  22 $\mathbf{p} = random() \otimes \mathbf{t}$
  23 $\mathbf{t} = (\mathbf{p} == [\mathsf{max}_i\ \mathbf{p}(i)])$
  24 $\mathbf{S}(j,:) = \mathbf{t}$
  25
  26   **if** $nvals(\mathbf{sr} \otimes \mathbf{t}) == 0$ **then**
  27 $vertices\_changed = nvals(\mathbf{k})$
  28 $vertices\_changed = vertices\_changed - 1$

Algorithm 16: Community detection using label propagation (for undirected graphs).

1 Function *CDLP*
  2 $\mathbf{L} \leftarrow diag([0, 1, \ldots, n - 1])$
  3   **for** $k = 1$ **to** t **do**
  4 $\mathbf{F} = \mathbf{A}\ \mathsf{any.second}\ \mathbf{L}$ // Frequency matrix
  5 $\{I, \_, X\} \leftarrow \mathbf{F}$
  6 $merge\_sort\_pairs(I, X)$
  7 $\mathbf{labels} = $ for each row in I, select min mode value from X

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