What is the role of the bias in neural networks? [closed]

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896

452

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I'm aware of the gradient descent and the back-propagation algorithm. What I don't get is: when is using a bias important and how do you use it?

For example, when mapping the `AND` function, when I use two inputs and one output, it does not give the correct weights. However, when I use three inputs (one of which is a bias), it gives the correct weights.

machine-learning neural-network artificial-intelligence backpropagation

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edited Mar 28 at 22:59 Peter Mortensen **29k** 21 96 124

asked Mar 19 '10 at 21:18 Karan **10.8k** 6 32 38

19 Check out this question: stackoverflow.com/questions/1697243/help-with-perceptron for an example of an actual problem where the OP was not using a bias term – Amro Mar 23 '10 at 23:28

7 And here is a nice example in Python of **why the bias is important** :) stackoverflow.com/questions/38248657/… – minerals Jul 7 '16 at 22:31

3 here's a great article full of of backprop math, also covering bias updates: theclevermachine.wordpress.com/2014/09/06/… – Andy Dec 25 '16 at 17:31

19 Answers

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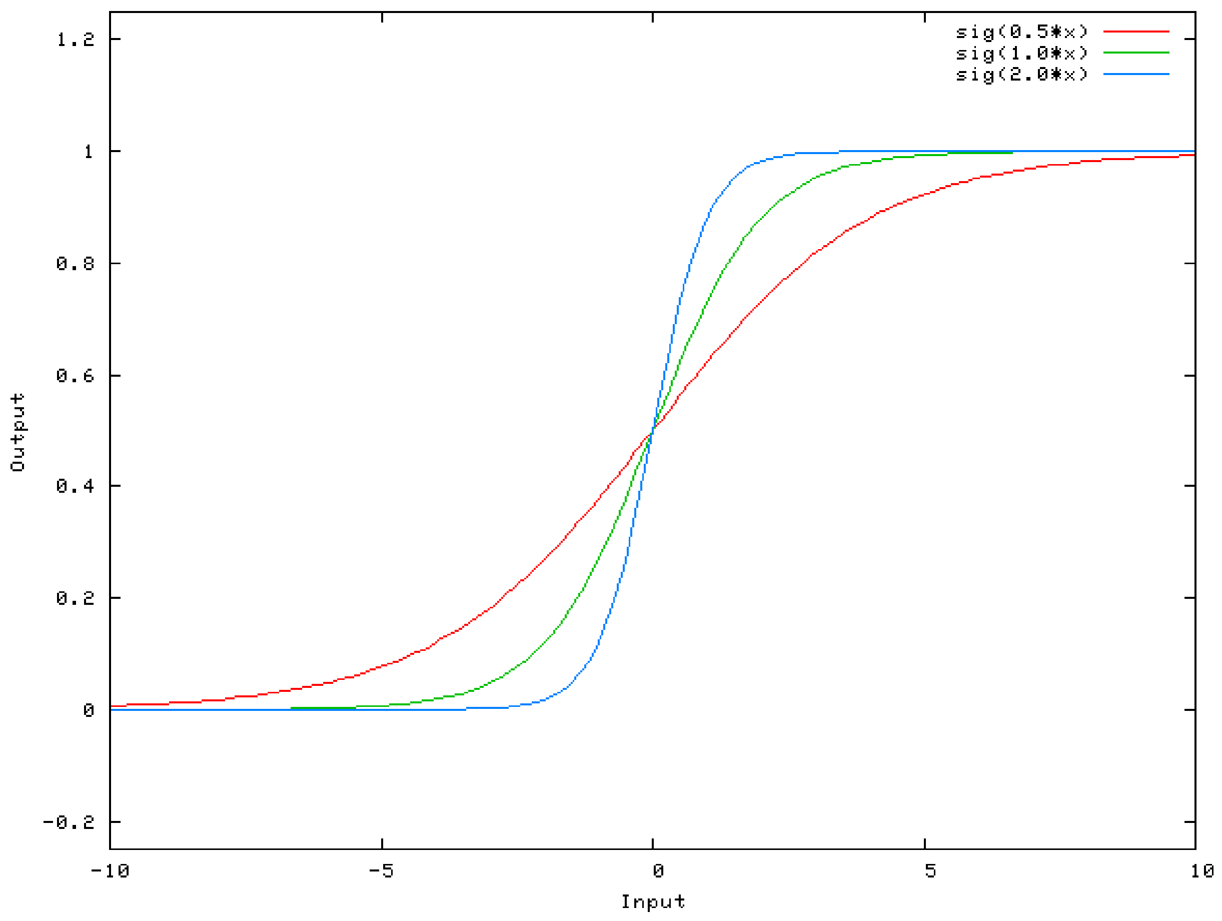
1483

I think that biases are almost always helpful. In effect, **a bias value allows you to shift the activation function to the left or right**, which may be critical for successful learning.

It might help to look at a simple example. Consider this 1-input, 1-output network that has no bias:

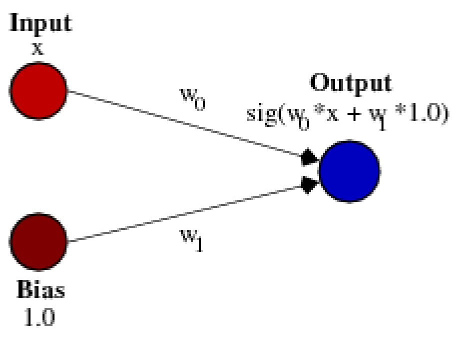
The output of the network is computed by multiplying the input (x) by the weight (w_0) and passing the result through some kind of activation function (e.g. a sigmoid function.)

Here is the function that this network computes, for various values of w_0:

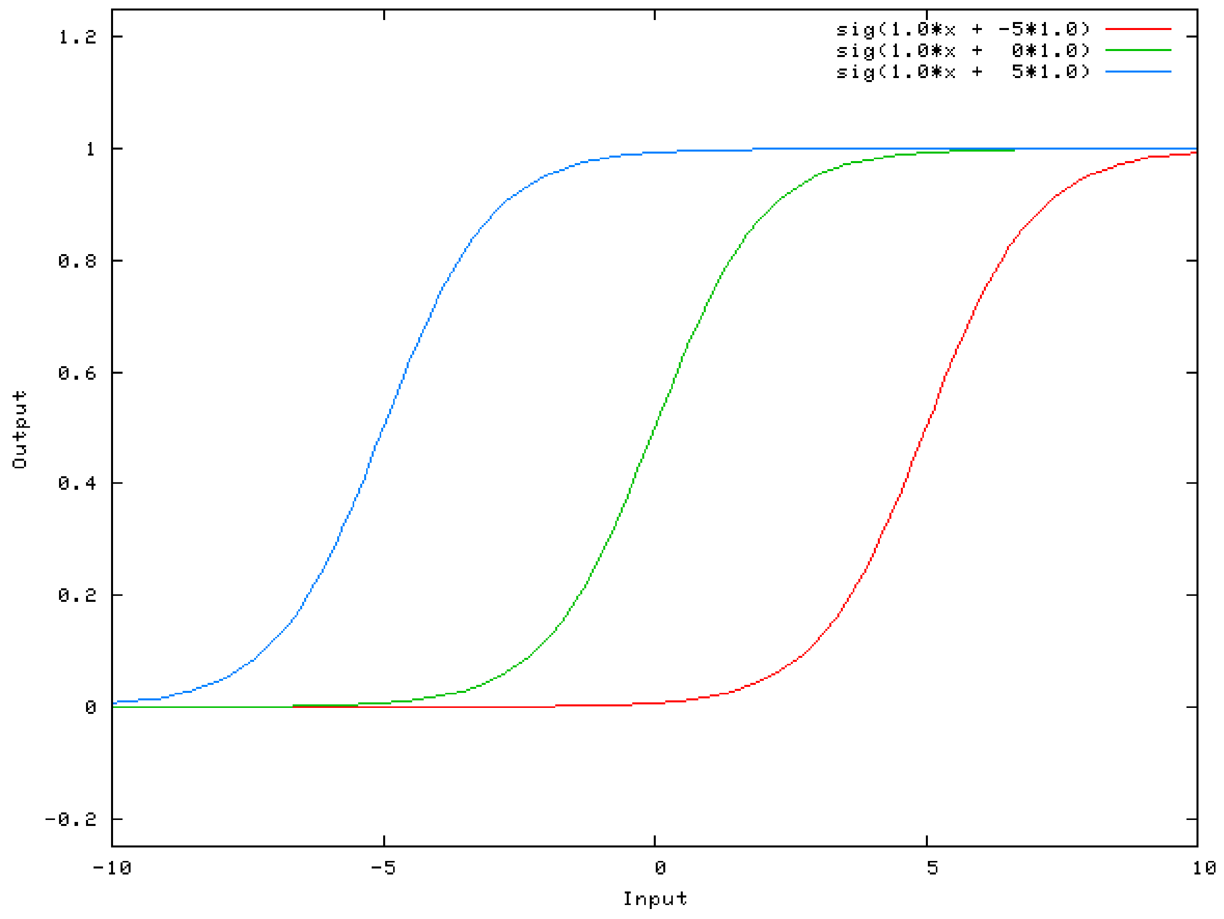


Changing the weight w_0 essentially changes the "steepness" of the sigmoid. That's useful, but what if you wanted the network to output 0 when x is 2? Just changing the steepness of the sigmoid won't really work -- **you want to be able to shift the entire curve to the right**.

That's exactly what the bias allows you to do. If we add a bias to that network, like so:



...then the output of the network becomes sig(w_0*x + w_1*1.0). Here is what the output of the network looks like for various values of w_1:



Having a weight of -5 for w_1 shifts the curve to the right, which allows us to have a network that outputs 0 when x is 2.

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edited Mar 22 '17 at 0:49
Archy Will He 何魏奇
9,139 4 29 48

answered Mar 23 '10 at 12:50
Nate Kohl
33.6k 10 41 52

6 @user1621769: The simplest approach is a network with a single bias node that connects to all non-input nodes in the network. – Nate Kohl Mar 23 '13 at 19:02

75 @user1621769: The main function of a bias is to provide every node with a trainable constant value (in addition to the normal inputs that the node recieves). You can achieve that with a single bias node with connections to N nodes, or with N bias nodes each with a single connection; the result should be the same. – Nate Kohl Mar 24 '13 at 13:24

4 @user1621769: You might be failing to take weightings into account? Each connection has a trainable weighting, and the bias node has a fixed value. – Dimpl Dec 18 '15 at 1:11

4 @user132458, if the training algorithm figures out that you don't need the bias shift, the bias weights will probably approach 0. Thus eliminating the bias signal. – jorgenkg Aug 5 '16 at 6:01

9 @Gabriel: There should be one bias per hidden neuron. – user4846835 Jan 4 '18 at 5:20

A simpler way to understand what the bias is: it is somehow similar to the constant *b* of a linear function

It allows you to move the line up and down to fit the prediction with the data better.

Without *b*, the line always goes through the origin (0, 0) and you may get a poorer fit.

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edited Mar 29 at 0:21
Peter Mortensen
29k 21 96 124

answered Nov 4 '14 at 0:32
zfy
4,117 1 9 7

8 nice anology but if we set the bias to 1 then why does it make difference to the fit now that every line will now go through (0,1) instead of (0,0)?As all lines are now biased to y=1 instead of y=0 why is this helpful? – blue-sky May 10 '16 at 22:40

41 @blue-sky Because by multiplying a bias by a weight, you can shift it by an arbitrary amount. – Carcigenicate Jun 20 '16 at 15:50

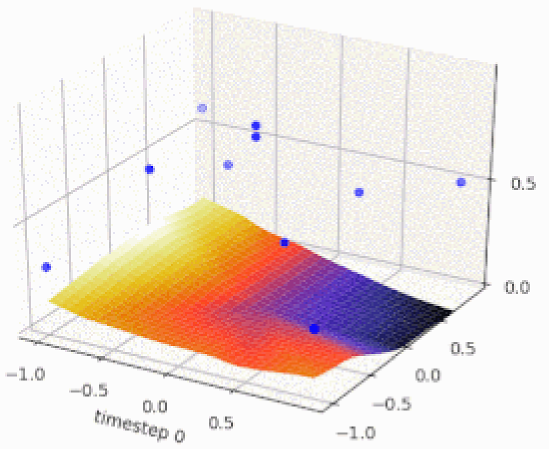
3 Is it correct to call *b* a "coefficient"? Isn't a "coefficient" a number used to multiply a variable? – Ben Jul 18 '16 at 23:33

11 b is not "coefficient" rather it is intercept. – Espanta Aug 25 '16 at 23:47

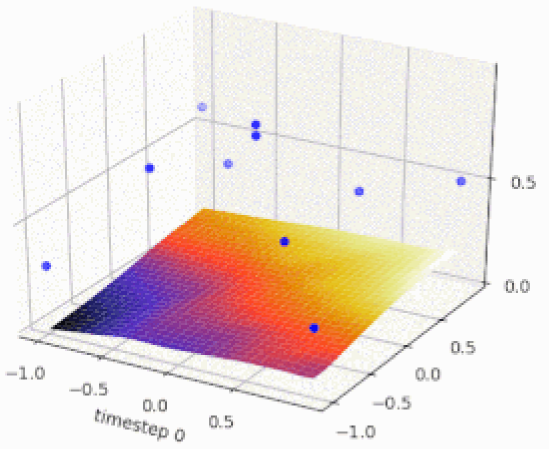
27 b is the coefficient of x^0. a is the coefficient of x^1 – user2918461 Feb 14 '17 at 15:03

99

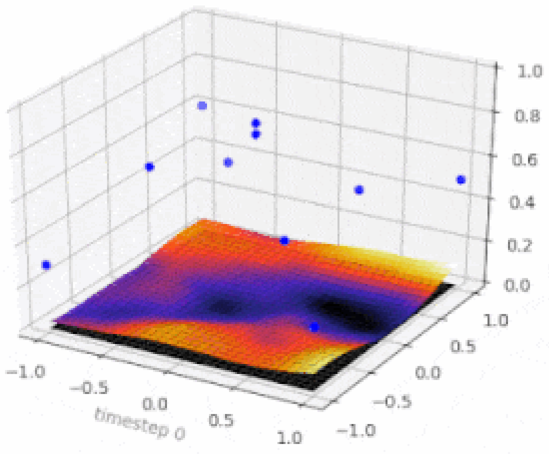
Here are some further illustrations showing the result of a simple 2-layer feed forward neural network with and without bias units on a two-variable regression problem. Weights are initialized randomly and standard ReLU activation is used. As the answers before me concluded, without the bias the ReLU-network is not able to deviate from zero at (0,0).



no bias units



with bias units



with bias units and Adam optimizer

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edited Mar 20 at 1:24
desertnaut
48.7k 19 114 147

answered Nov 5 '18 at 9:34
JP K.
1,071 7 10

Sorry, the points are just randomly chosen. There is no real function behind them. – JP K. Nov 6 '20 at 17:16

1 Can you share the code doing the animation? – mon Mar 14 at 6:10

Are you still interested in this? What would be the best way to share code on stackoverflow? – JP K. Apr 27 at 16:20

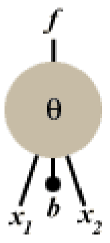
@JPK. share it as github link please – Syntax Hacker Oct 14 at 4:15

54

> Two different kinds of parameters can be adjusted during the training of an ANN, the weights and the value in the activation functions. This is impractical and it would be easier if only one of the parameters should be adjusted. To cope with this problem a bias neuron is invented. The bias neuron lies in one layer, is connected to all the neurons in the next layer, but none in the previous layer and it always emits 1. Since the bias neuron emits 1 the weights, connected to the bias neuron, are added directly to the combined sum of the other weights (equation 2.1), just like the t value in the activation functions.[1]

The reason it's impractical is because you're simultaneously adjusting the weight and the value, so any change to the weight can neutralize the change to the value that was useful for a previous data instance... adding a bias neuron without a changing value allows you to control the behavior of the layer.

Furthermore the bias allows you to use a single neural net to represent similar cases. Consider the AND boolean function represented by the following neural network:



(source: aihorizon.com)

- *w0* corresponds to *b*.
- *w1* corresponds to *x1*.
- *w2* corresponds to *x2*.

> A single perceptron can be used to represent many boolean functions.
>
> For example, if we assume boolean values of 1 (true) and -1 (false), then one way to use a two-input perceptron to implement the AND function is to set the weights w0 = -3, and w1 = w2 = .5. This perceptron can be made to represent the OR function instead by altering the threshold to w0 = -.3. In fact, AND and OR can be viewed as

> function to m = n. Any m-of-n function is easily represented using a perceptron by setting all input weights to the same value (e.g., 0.5) and then setting the threshold w0 accordingly.
>
> Perceptrons can represent all of the primitive boolean functions AND, OR, NAND (1 AND), and NOR (1 OR). Machine Learning- Tom Mitchell)

The threshold is the bias and *w0* is the weight associated with the bias/threshold neuron.

Share Improve this answer Follow edited Jul 3 '19 at 13:06 Glorfindel **20.1k** 13 67 96 answered Mar 19 '10 at 21:38 Kiril **38.1k** 30 163 219

8 Think of it as a general rule of thumb: add bias! Neural networks are *"unpredictable"* to a certain extent so if you add a bias neuron you're more likely to find solutions faster then if you didn't use a bias. Of course this is not mathematically proven, but it's what I've observed in literature and in general use. – Kiril Mar 19 '10 at 22:02

32

The bias is not an *NN* term. It's a generic algebra term to consider.

`Y = M*X + C` (straight line equation)

Now if `C(Bias) = 0` then, the line will always pass through the origin, i.e. `(0,0)`, and depends on only one parameter, i.e. `M`, which is the slope so we have less things to play with.

`C`, which is the bias takes any number and has the activity to shift the graph, and hence able to represent more complex situations.

In a logistic regression, the expected value of the target is transformed by a link function to restrict its value to the unit interval. In this way, model predictions can be viewed as primary outcome probabilities as shown:

Sigmoid function on Wikipedia

This is the final activation layer in the NN map that turns on and off the neuron. Here also bias has a role to play and it shifts the curve flexibly to help us map the model.

Share Improve this answer Follow edited Mar 29 at 0:24 Peter Mortensen **29k** 21 96 124 answered Mar 13 '17 at 1:08 Pradi KL **558** 5 12

29

A layer in a neural network without a bias is nothing more than the multiplication of an input vector with a matrix. (The output vector might be passed through a sigmoid function for normalisation and for use in multi-layered ANN afterwards, but that's not important.)

Using a bias, you’re effectively adding another dimension to your input space, which always takes the value one, so you’re avoiding an input vector of all zeros. You don’t lose any generality by this because your trained weight matrix needs not be surjective, so it still can map to all values previously possible.

2D ANN:

For a ANN mapping two dimensions to one dimension, as in reproducing the AND or the OR (or XOR) functions, you can think of a neuronal network as doing the following:

On the 2D plane mark all positions of input vectors. So, for boolean values, you’d want to mark (-1,-1), (1,1), (-1,1), (1,-1). What your ANN now does is drawing a straight line on the 2d plane, separating the positive output from the negative output values.

Without bias, this straight line has to go through zero, whereas with bias, you’re free to put it anywhere. So, you’ll see that without bias you’re facing a problem with the AND function, since you can’t put both (1,-1) *and* (-1,1) to the negative side. (They are not allowed to be *on* the line.) The problem is equal for the OR function. With a bias, however, it’s easy to draw the line.

Note that the XOR function in that situation can’t be solved even with bias.

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edited Mar 29 at 0:20
Peter Mortensen
29k 21 96 124

answered Mar 19 '10 at 22:24
Debilski
64.2k 11 107 132

4 If you use a sigmoid transfer function, you introduce non-linearity. Stating that this is a linear function is wrong and also somehow dangerous, as the non-linearity of the sigmoid is key to the solution of several problems. Also, sigmoid(0) = 0.5, and there is no x for which sigmoid(x) = 0. – bayer Mar 21 '10 at 21:35

2 Yeah, but it is 0.5 for any input of 0 without a bias, regardless of what the linear function before looks like. And that’s the point. You don’t normally train your sigmoid function, you just live with it. The linearity problem happens well before the sigmoid function. – Debilski Mar 21 '10 at 22:05

I get your point: the layer is not able to learn a different output for 0 than the one it started out with. That's correct and important. However, the "linear function argument" just does not apply in my opinion. Even with a bias, the function is still linear. The linearity property is misleading here. (Yes, I might be nitpicking.) – bayer Mar 22 '10 at 16:28

I’d say, that with a bias it’s *affine*. (en.wikipedia.org/wiki/Affine_transformation#Representation) – Debilski Mar 22 '10 at 16:33

Yes, you're correct. Thanks for pointing out that difference to me. (Why do we call it linear regression then, btw, although it's affine?) – bayer Mar 22 '10 at 17:58

24

When you use ANNs, you rarely know about the internals of the systems you want to learn. Some things cannot be learned without a bias. E.g., have a look at the following data: (0, 1), (1, 1), (2, 1), basically a function that maps any x to 1.

In an ideal setting, a bias could also map all points to the mean of the target points and let the hidden neurons model the differences from that point.

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answered Mar 21 '10 at 21:40
bayer
6,625 20 35

22

Modification of neuron WEIGHTS alone only serves to manipulate the *shape/curvature* of your transfer function, and not its *equilibrium/zero* crossing point.

The introduction of **bias** neurons allows you to shift the transfer function curve horizontally (left/right) along the input axis while leaving the shape/curvature unaltered. This will allow the network to produce arbitrary outputs different from the defaults and hence you can customize/shift the input-to-output mapping to suit your particular needs.

See here for graphical explanation: http://www.heatonresearch.com/wiki/Bias

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edited Oct 8 '18 at 7:49
Oke Uwechue
128 1 12

answered Jan 23 '14 at 22:56
Oke Uwechue
409 4 4

The link is dead. – Burak Kaymakci Dec 5 '20 at 15:56

20

If you're working with images, you might actually prefer to not use a bias at all. In theory, that way your network will be more independent of data magnitude, as in whether the picture is dark, or bright and vivid. And the net is going to learn to do it's job through studying relativity inside your data. Lots of modern neural networks utilize this.

For other data having biases might be critical. It depends on what type of data you're dealing with. If your information is magnitude-invariant --- if inputting [1,0,0.1] should lead to the same result as if inputting [100,0,10], you might be better off without a bias.

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edited Jan 30 at 22:37
desertnaut
48.7k 19 114 147

answered Sep 20 '16 at 19:55
Íhor Mé
821 9 13

you're probably better off with normalization. What's an example of a modern network that uses "lack of bias" to produce magnitude invariaance? – AwokeKnowing Jan 10 '17 at 0:29

@AwokeKnowing , I believe, the usual ResNet utilizes that, as it's a part of it's "initialization", but I'm not exactly sure they did this for strictly this purpose, or, maybe for considerations of model size/efficiency and I'm not sure this concept is published anywhere. But I think it's completely understandable on a theory level. If you don't have a bias that doesn't scale, when you scale values, all of the outputs simply scale accordingly. Aware of this concept, or not, large part of modern architectures don't have biases at least in a large part of their structures. – Íhor Mé Feb 26 '17 at 1:06

19

In a couple of experiments in [my masters thesis](#) (e.g. page 59), I found that the bias might be important for the first layer(s), but especially at the fully connected layers at the end it seems not to play a big role.

This might be highly dependent on the network architecture / dataset.

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answered Aug 1 '17 at 17:09
Martin Thoma
97.5k 124 520 792

That sounds completely analogous to the process of modeling systems "by hand" with linear regression. The simplest model would be Y_bar=mean(Y). Then you add complexity by including various X terms, stopping when there is no significant information gain. – IRTFM Apr 7 '20 at 15:27

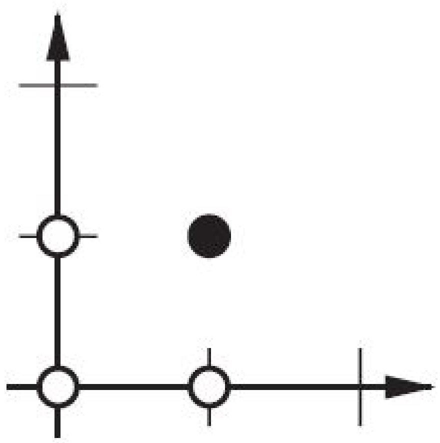
18

Bias determines how much angle your weight will rotate.

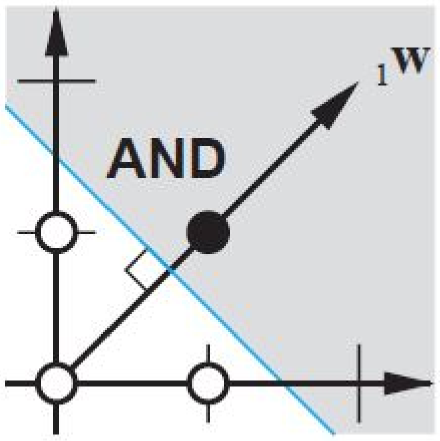
In a two-dimensional chart, weight and bias can help us to find the decision boundary of outputs.

Say we need to build a AND function, the input(p)-output(t) pair should be

{p=[0,0], t=0},{p=[1,0], t=0},{p=[0,1], t=0},{p=[1,1], t=1}

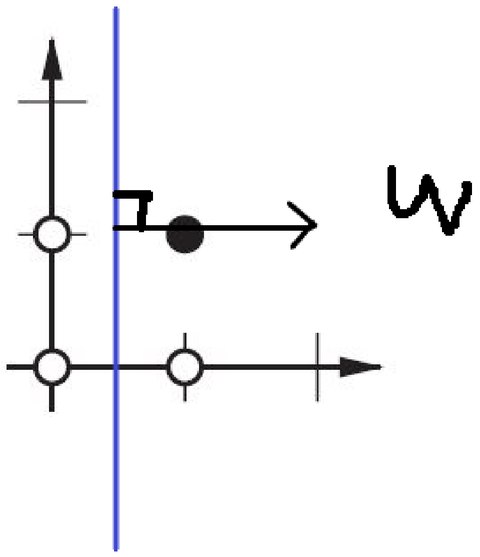


Now we need to find a decision boundary, and the ideal boundary should be:



See? W is perpendicular to our boundary. Thus, we say W decided the direction of boundary.

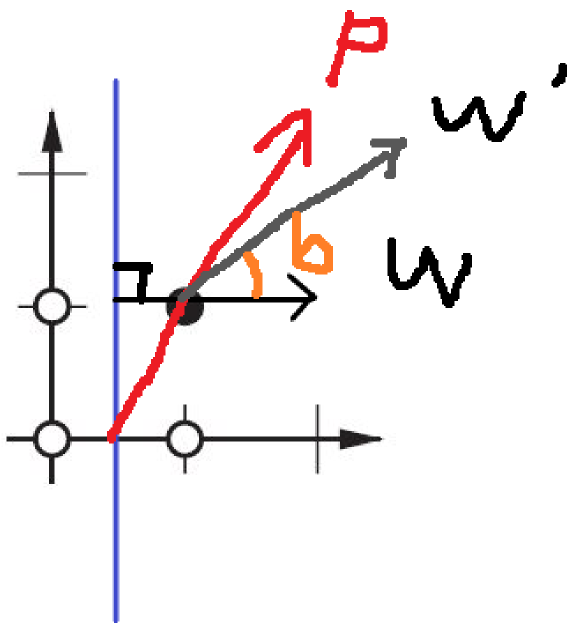
However, it is hard to find correct W at first time. Mostly, we choose original W value randomly. Thus, the first boundary may be this:



Now the boundary is parallel to the y axis.

We want to rotate the boundary. How?

By changing the W.



W'=W+P is equivalent to W' = W + bP, while b=1.

Therefore, by changing the value of b(bias), you can decide the angle between W' and W. That is "the learning rule of ANN".

You could also read Neural Network Design by Martin T. Hagan / Howard B. Demuth / Mark H. Beale, chapter 4 "Perceptron Learning Rule"

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edited Mar 29 at 0:39
Peter Mortensen
29k 21 96 124

answered May 4 '18 at 8:12
user8396969

14

In simpler terms, biases allow for **more and more variations** of weights to be learnt/stored... (**side-note**: sometimes given some threshold). Anyway, **more variations** mean that biases add **richer representation** of the input space to the model's learnt/stored weights. **(Where better weights can enhance the neural net’s guessing power)**

For example, in learning models, the hypothesis/guess is desirably bounded by y=0 or y=1 given some input, in maybe some classification task... i.e some y=0 for some x=(1,1) and about above. Note that my examples setup inputs X to be each x=a double or 2 valued-vector, instead of Nate's single valued x inputs of some collection X).

If we **ignore** the bias, **many inputs may end up being represented by a lot of the same weights** (i.e. the learnt weights **mostly occur close to the origin** (0,0). The model would then be limited to poorer quantities of good weights, instead of the many many more good weights it could better learn with bias. **(Where poorly learnt weights lead to poorer guesses or a decrease in the neural net's guessing power)**

So, it is optimal that the model learns both close to the origin, but also, in as many places as possible inside the threshold/decision boundary. **With the bias we can enable degrees of freedom close to the origin, but not limited to origin's immediate region.**

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edited Mar 20 at 1:25
desertnaut
48.7k 19 114 147

answered Nov 5 '17 at 3:55
Jordan Bennett
191 2 8

12

Expanding on zfy's explanation:

The equation for one input, one neuron, one output should look:

```
y = a * x + b * 1    and out = f(y)
```

where x is the value from the input node and 1 is the value of the bias node; y can be directly your output or be passed into a function, often a sigmoid function. Also note that the bias could be any constant, but to make everything simpler we always pick 1 (and probably that's so common that zfy did it without showing & explaining it).

Your network is trying to learn coefficients a and b to adapt to your data. So you can see why adding the element `b * 1` allows it to fit better to more data: now you can change both slope and intercept.

If you have more than one input your equation will look like:

```
y = a0 * x0 + a1 * x1 + ... + aN * 1
```

Note that the equation still describes a one neuron, one output network; if you have more neurons you just add one dimension to the coefficient matrix, to multiplex the inputs to all nodes and sum back each node contribution.

That you can write in vectorized format as

```
A = [a0, a1, .., aN] , X = [x0, x1, ..., 1]
Y = A . XT
```

correct, I wrote XT a 'X transposed')

So in the end you can also see your bias as is just one more input to represent the part of the output that is actually independent of your input.

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edited Mar 29 at 0:23
Peter Mortensen
29k 21 96 124

answered Sep 10 '16 at 6:39
RobMcZag
507 5 13

I seem to remember from Andrew Ng's class that the bias was left out in part of the training process. could you update your answer to explain that considering your conclusion that it's "just another input"? – AwokeKnowing Jan 10 '17 at 0:32

@AwokeKnowing I do not remember that from Andrew Ng's class, but that was a few years ago. Also Bias can be on or off depending what you are trying to learn. I read that in image processing they do not use it to allow scaling. To me if you use it, you use it also in training. The effect is to stabilize coefficients when all or part of the inputs is null or almost null. Why would you not use bias during training and then use it when using the NN to predict outputs for new inputs? How could that be useful? – RobMcZag Jan 17 '17 at 19:15

No, it was more like, use it in the forward pass, but don't use it when calculating the gradient for backprop, or something like that. – AwokeKnowing Jan 18 '17 at 7:37

@AwokeKnowing I suppose that is a way to save some memory and time. You can decide you do not care to learn coefficients for the bias units. That can be fine if you have at least one hidden layer as the bias will provide some input to that layer and the output can be learned by the coefficients from the first to the second layer. I am not sure if the convergence speed will change. In my one layer example you are forced to learn also the bias coefficient as it is applied to the output. – RobMcZag Jan 18 '17 at 20:53

In many simple problems, the target data has been demeaned and scaled, so no bias is needed.and the potential for excessive outlier influence in variables with large ranges is reduced. – IRTFM Apr 7 '20 at 15:29

11

In neural networks:

1. Each neuron has a bias
2. You can view bias as a threshold (generally opposite values of threshold)
3. Weighted sum from input layers + bias decides activation of a neuron
4. Bias increases the flexibility of the model.

In absence of bias, the neuron may not be activated by considering only the weighted sum from the input layer. If the neuron is not activated, the information from this neuron is not passed through rest of the neural network.

The value of bias is learnable.

$$output = \begin{cases} 0 \text{ if } \sum_i w_i x_i + bias < 0 \\ 1 \text{ if } \sum w_i x_i + bias > 0 \end{cases}$$

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Effectively, bias =—threshold. You can think of bias as how easy it is to get the neuron to output a 1—with a really big bias, it's very easy for the neuron to output a 1, but if the bias is very negative, then it's difficult.

In summary: ***bias helps in controlling the value at which the activation function will trigger.***

Follow this video for more details.

Few more useful links:

geeksforgeeks

towardsdatascience

Share Improve this answer Follow edited Mar 29 at 0:44 Peter Mortensen **29k** 21 96 124 answered Feb 12 '19 at 14:00 Ravindra babu **44k** 8 218 199

1 Nice interpretation. But I am not clear how the bias is learn-able (or trainable?). In a simple case where loss = ReLU(omega * x + a), the weight omega can be trained through the chain-rule, but how can the bias `a` be trained when the gradient to `a` is always a constant? – Libin Wen Aug 31 '19 at 11:26

10

To think in a simple way, if you have **y=w1*x** where **y** is your output and **w1** is the weight, imagine a condition where **x=0** then **y=w1*x equals to 0**.

If you want to update your weight you have to compute how much change by **delw=target-y** where target is your target output. In this case **'delw'** will not change since **y** is computed as 0. So, suppose if you can add some extra value it will help **y = w1*x* + *w0*1**, where bias=1 and weight can be adjusted to get a correct bias. Consider the example below.

In terms of line *slope*, intercept is a specific form of linear equations.

y = mx + b

Check the image

image

Here b is (0,2)

If you want to increase it to (0,3) how will you do it by changing the value of b the bias.

Share Improve this answer Follow edited Mar 29 at 1:14 answered Apr 24 '17 at 16:55 Yumlembam Rahul **121** 1 7

9

The stronger the signals will be transmitted from the firing neuron to the target neuron or Y = w * X as a result to maintain the biological character of neurons, we need to keep the 1 >=W >= -1, but in the real regression, the W will end up with |W| >=1 which contradicts how the neurons are working.

As a result, I propose W = cos(theta), while 1 >= |cos(theta)|, and Y= a * X = W * X + b while a = b + W = b + cos(theta), b is an integer.

Share Improve this answer Follow edited Mar 29 at 0:34 Peter Mortensen 29k 21 96 124 answered Sep 10 '17 at 20:19 Edward 109 1 2

8

Bias acts as our anchor. It's a way for us to have some kind of baseline where we don't go below that. In terms of a graph, think of like y=mx+b it's like a y-intercept of this function.

output = input times the weight value and added a ***bias value*** and then apply an activation function.

Share Improve this answer Follow edited Mar 20 at 1:26 desertnaut 48.7k 19 114 147 answered Oct 8 '17 at 10:51 Naren Babu R 316 7 32

7

The term bias is used to adjust the final output matrix as the y-intercept does. For instance, in the classic equation, y = mx + c, if c = 0, then the line will always pass through 0. Adding the bias term provides more flexibility and better generalisation to our neural network model.

Share Improve this answer Follow edited Mar 29 at 0:45 Peter Mortensen 29k 21 96 124 answered Mar 11 '19 at 10:30 Aman pradhan 248 5 11

2

The bias helps to get a better equation.

Imagine the input and output like a function *y* = *ax* + *b* and you need to put the right line between the input(x) and output(y) to minimise the global error between each point and the line, if you keep the equation like this *y* = *ax*, you will have one parameter for adaptation only, even if you find the best `a` minimising the global error it will be kind of far from the wanted value.

You can say the bias makes the equation more flexible to adapt to the best values

Share Improve this answer Follow edited Mar 29 at 0:48 Peter Mortensen 29k 21 96 124 answered May 21 '20 at 0:09 Karam Mohamed 833 1 5 13