Chapter 1

Chi-Squared χ^2 Tests

1.1 The χ^2-Distribution

Definition 1.1

A random variable X is said to follow a χ^2-distribution, with degree of freedom ν, iff its probability density function is given by

$$f(x) = \begin{cases} \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{(\nu/2)-1} e^{-x/2} & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

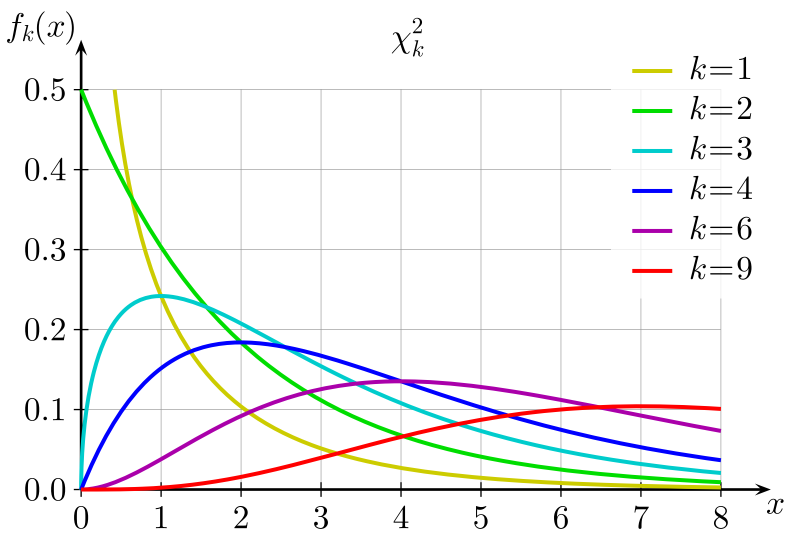


Figure 1.1: Illustration of how the $\chi^2_{(\nu)}$ distribution looks with increasing degree of freedom ν.

General Information

- Properties of chi-squared distributions.
  - $\mathrm{E}(X) = \nu$ and $\mathrm{Var}(X) = 2\nu$.
  - The $\chi^2_{(\nu)}$ distribution tends to a normal distribution as $\nu \to \infty$.
  - Suppose $Z_i \sim \mathrm{N}(0, 1)$ are independent. Then, $Z_1^2 + \cdots + Z_n^2 \sim \chi^2_{(n)}$.
  - If $X \sim \chi^2_{(\nu)}$ and $Y \sim \chi^2_{(\upsilon)}$, then $X + Y \sim \chi^2_{(\nu+\upsilon)}$.

1.2 A Goodness-of-Fit Test

General Information

1. Let [X in context].

2. *Note.* Use a pen to draw any necessary tables.

Test	H_0: [X follows the distribution in context]
against	H_1: [X does not follows the distribution in context]
at the $100\alpha\%$ significance level.	

3.

x	x_1	x_2	$\cdots$	x_n
f_i	f_1	f_2	$\cdots$	f_n
e_i	e_1	e_2	$\cdots$	e_n
$\frac{(f_i-e_i)^2}{e_i}$	$\frac{(f_1-e_1)^2}{e_1}$	$\frac{(f_2-e_2)^2}{e_2}$	$\cdots$	$\frac{(f_n-e_n)^2}{e_n}$

Table 1.1: Observed and expected frequencies for a goodness-of-fit test

4. Check whether $e_i \geq 5$ for each of the n classes. If it isn't, we need to combine *just enough* adjacent classes, till they do. Working-wise, use some underbraces/overbraces to indicate the combined values.

5. Under H_0, the test statistic is

$$\chi^2 = \sum \frac{(F_i - E_i)^2}{E_i} \sim \chi^2_{(\nu)}.$$

Here, $n :=$ #classes and $\nu =$ (#classes $-$ #estimated parameters) $- 1$.

6. Continue as per usual, calculating the critical region $\chi^2_{(\nu)} > \chi^2_{(\nu,1-\alpha)}$ or the p-value.

G.C. Skills

- To find the value of $\chi^2_{(\nu,1-\alpha)}$, which satisfies $\mathrm{P}\left(X > \chi^2_{(\nu,1-\alpha)}\right) = \alpha$, we use the table in the MF26 formula sheet (Page 9). Unfortunately, there is no inverse χ^2 function available.

- For the p-value:

$$\texttt{stat} \implies \texttt{TESTS} \implies \texttt{D:}\chi^2\texttt{GOF-Test...}$$

Note

If X follows a *discrete* uniform distribution, we must state it out in words. We cannot write $X \sim \mathrm{U}(\mu, \sigma^2)$ as this would denote that X is a *continuous* random variable. But if $X \sim \mathrm{B}(n,p)$ (or $X \sim \mathrm{Po}(\lambda)$, etc), then we can just denote it as such.

Example 1.1: #estimated parameters $= 0$

Given $X \sim \mathrm{N}(0,1)$ (note how the *population parameters* that define the distribution are *known*), the degree of freedom $\nu =$ #estimated parameters $=: n$.

Example 1.2: #estimated parameters = 1

Consider when $X \sim \mathrm{B}(m, p)$, such that the expected frequency for each of the n classes is at least 5, but we do not know the exact value of p. So, we *estimate* it according to the sample given. Then, the degree of freedom is $\nu = n - 1 - 1 = n - 2$.

Example 1.3: #estimated parameters = 2

Similarly, suppose $X \sim \mathrm{N}(\mu, \sigma^2)$, such that the expected frequency of each of the n classes is at least 5, and the true values of μ and σ^2 are unknown. In this case, the degree of freedom $\nu = n - 2 - 1 = n - 3$.

Note

Consider when we are testing

Test	$H_0 : X \sim \mathrm{N}(\mu, \sigma^2)$
against	$H_1 : X \not\sim \mathrm{N}(\mu, \sigma^2)$
at the $100\alpha\%$ significance level.	

So, we want to fill up the values of e_i below.

x	$a_1 \leq x_1 \leq a_2$	$a_2 \leq x_2 \leq a_3$	$\cdots$	$a_n \leq x_n \leq a_{n+1}$
f_i	f_1	f_2	$\cdots$	f_n
e_i	e_1	e_2	$\cdots$	e_n

Table 1.2: Observed and expected frequencies when testing goodness-of-fit with a normal distribution.

Let the sample size $\sum f_i$ be m. Then, we should calculate $e_1 = m\,\mathrm{P}(-\infty < X \leq a_2)$ and $e_n = m\,\mathrm{P}(a_n \leq X < \infty)$, instead of $e_1 = m\,\mathrm{P}(a_1 \leq X \leq a_2)$ or $e_n = m\,\mathrm{P}(a_n \leq X \leq a_{n+1})$. Similarly, for goodness-of-fit tests with Poisson and Geometric distributions, we must also be careful in ensuring that we account for *all* possible values which X can take on, in calculating e_i.

Note

Suppose we are given a question of the following form.

Some context...

x_i	x_1	x_2	$\cdots$	x_n
f_i	f_1	f_2	$\cdots$	f_n

Table 1.3: Some data.

(i) Show, at the $100\alpha\%$ significance level, that the data does not support the hypothesis of $X \sim \mathrm{Geo}(p)$ with $p = 0.5$.

(ii) State how the test in (i) would have to be amended to test the hypothesis of a geometric distribution for an *unspecified value of* p.

Then, for (ii), two main changes have to be made:

1. Estimate the value of p by computing the sample mean $\bar{x}$ and letting $p = 1/\bar{x}$.
2. Adjust the degree of freedom from 4 to $4 - 1 = 3$, as there is one more restriction, that the mean must agree.

(The phrasing is similar for gof tests for other distributions; simply use the appropriate estimators for the unknown population parameters.)

1.3 Tests of Independence

General Information

1. Let [X in context].

2. | | |
   |---|---|
   | Test | H_0: [X in context] is independent of [Y in context] |
   | against | H_1: [X in context] is dependent on [Y in context] |
   | at the $100\alpha\%$ significance level. | |

3. *Note.* Unless the question asks for it, we do not need to write $\left[\frac{(f_i-e_i)^2}{e_i}\right]$ or its corresponding values, in the following table.

$f_i\ (e_i)\ \left[\frac{(f_i-e_i)^2}{e_i}\right]$		X				
		x_1	x_2	$\cdots$	x_n	Total
Y	y_1					t_{r_1}
	y_2					t_{r_2}
	$\vdots$					$\vdots$
	y_m					t_{r_m}
	Total	t_{c_1}	t_{c_2}	$\cdots$	t_{c_n}	$S = \sum t_{r_i} + \sum t_{c_i}$

Table 1.4: *Expected* frequencies for a test of independence.

Remark

The expected frequencies are given by $e_{ij} = \dfrac{\text{row total} \cdot \text{column total}}{\text{total number of observations}} = \dfrac{t_{r_i} t_{c_j}}{S}$.

4. Check whether $e_i \geq 5$ for each of the mn cells. If it isn't, we need to combine *just enough* adjacent classes, till they do. Working-wise, use some underbraces/overbraces/side braces to indicate the combined values.

5. Under H_0, the test statistic is

$$\chi^2 = \sum \frac{(F_i - E_i)^2}{E_i} \sim \chi^2_{(\nu)}.$$

   Here, $n :=$ #cols and $\nu = (\#\text{rows} - 1)(\#\text{cols} - 1)$.

6. Continue as per usual, calculating the critical region $\chi^2_{(\nu)} > \chi^2_{(\nu, 1-\alpha)}$ or the p-value.

G.C. Skills

Key in the matrix of *observed frequencies* (not Table 1.2 of *expected* frequencies):

$$\texttt{2nd} \implies \texttt{x}^{-1} \implies \texttt{EDIT} \implies \texttt{[A]}.$$

Then, conduct the test for independence:

$$\texttt{stat} \implies \texttt{TESTS} \implies \texttt{C:}\chi^2\texttt{-Test...}$$

Note

If it's unclear as to what is to be stated as independent/dependent in the hypotheses, consider the expected values and how they relate to the context.

Example 1.4

Consider the following context:

Statement	Independent/Dependent?
There is consistency in the marking of the two T.A.s.	?
There is no consistency in the marking of the two T.A.s.	?

Table 1.5: Two statements on the relationship between the marks awarded and the T.A. marking.

Then, under H_0 — the independence claim — the expected frequencies are as stated below.

e_{ij}		Grade A	Grade B	Grade C
T.A.	X	a	b	c
T.A.	Y	a	b	c

Table 1.6: Expected frequencies.

Since $e_{1j} = e_{2j}$ for all $1 \leq j \leq 3$, we infer the following.

Statement	Independent/Dependent?
There is consistency in the marking of the two T.A.s.	Independent
There is no consistency in the marking of the two T.A.s.	Dependent

Table 1.7: Which statement corresponds to independence and which coresponds to dependence.

Note

If the question says to "use an approximate χ^2-statistic…", then we must use the critical region method. It is incorrect to use the p-value.

Note

Consider when we are asked to state which cells correspond to the highest contributions to the test statistic, and relate that back to the context of the question. Then:

1. State the cells in the form (___, ___). E.g. (High, Good) and (Low, Good).
2. In table 1.4, add an asterisk to each of these cells. E.g. $\boxed{1\ (5)\ [10.1]^*}$.
3. Use words that imply correlation and *not* causation. E.g. directly associated, correlates with, etc.

Note

On a similar note, if the question asks "Can it can be concluded that…", but is unclear about whether it's implying correlation or causation, it may be safer to explain both ways. i.e. what correlation is there and why is there no causation.

Note

Explain why we cannot conclude any casual relationships from a test of independence.

> No, the above test does not reflect the actual casual relationship between the two factors, if it exists. Rather, it merely suggests that they are not independent.

Note

Explain why we cannot apply a χ^2-test for independence using the data given.

> The expeceted frequency for (___, ___) is ___ < 5. If we combine the columns, the degree of freedom $\nu = 1 \cdot 0 = 0$. If we combine the rows, $\nu = 0 \cdot 1 = 0$. Thus, we cannot apply a χ^2-test for independence.

Chapter 2

Correlation and Linear Regression

2.1 Scatter Diagrams

Note

Guidelines for drawing a scatter diagram

- The relative position of each point on the scatter diagram should be clearly shown.
- The range of values for the set of data should be clearly shown by marking out the extreme x and y values on the corresponding axis.
- The axes should be labeled clearly with the variables.

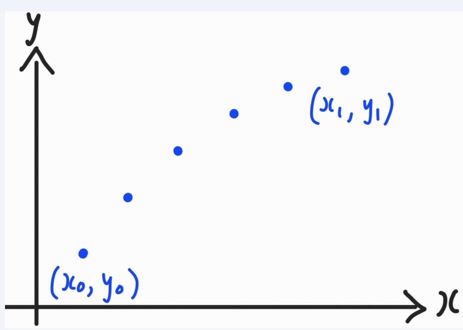


Figure 2.1: An illustration of a scatter plot.

Note. We do not need to start from the origin.

G.C. Skills

To show a scatter plot on the G.C.:

$$\texttt{2nd} \implies \texttt{y=} \implies \texttt{1:Plot1...on} \implies \texttt{enter} \implies \texttt{on}.$$

Note. When we no longer need a scatter plot, turn the scatter plot(s) *off* in the G.C., lest it erroneously interferes with other functionalities of the G.C.

Example 2.1

One of the values of t appears to be incorrect. Indicate the corresponding point on your diagram by labelling it P and explain why the scatter diagram for the remaining points may be consistent with a model of the form $y = a + bf(x)$.

With P removed, the remaning points seem to lie, on a curve that [e.g. increases at a decreasing rate], suggesting consistency with the model $y = a + bf(x)$.

2.2 Product Moment Correlation Coefficient r

Definition 2.1

The Product Moment Correlation Coefficient is a measure of the linear correlation between two variables. It is defined by

$$r := \frac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-\bar{y})^2}} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left[\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right]\left[\sum y^2 - \frac{\left(\sum y\right)^2}{n}\right]}},$$

and takes on a value from 0 to 1. See Figure 2.2 for some scatter plots of various r values.

Note

Explain whether your estimate using the regression line of y on x is reliable.

Since the $|r|$ value of ___ is close to 1, *and* $x =$ ___ is within the data range of ___ $\leq x \leq$ ___, the estimate is reliable.

Note

Explain why the estimate using the regression line y on x is not reliable.

Since $x =$ ___ falls outside of the range of data ___ $\leq x \leq$ ___, we would be extrapolating the observed data points. This makes the estimate of the value of y at $x =$ ___ unreliable.

Note

Explain which dataset would result in a larger absolute value of the product moment correlation coefficient.

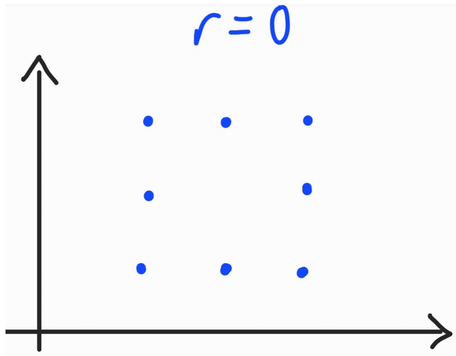
- Set A will have a larger $|r|$ value, because its data points lie relatively *closer to a straight line* (with positive/negative gradient), suggesting a stronger linear correlation.
- Set B's $|r|$ value will be closer to zero, since its data points are *more scattered*, suggesting a weaker linear correlation.

2.3 Regression Lines

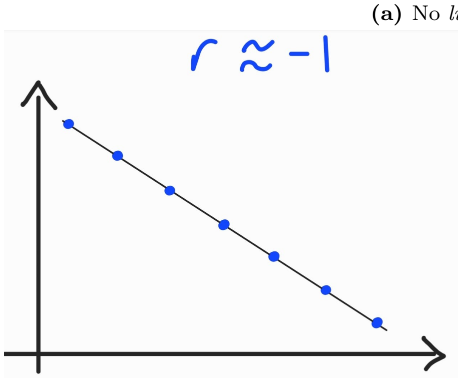
General Information

The regression line of y on x minimises the sum of squares deviation (error) in the y-direction — we assume that x is the independent variable whose values are known exactly. It is given by

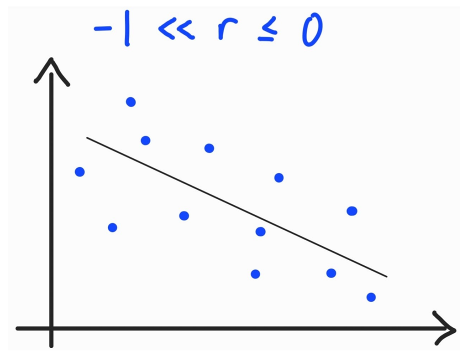
$$y = \bar{y} + b(x-\bar{x}), \qquad \text{where} \qquad b := \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}.$$



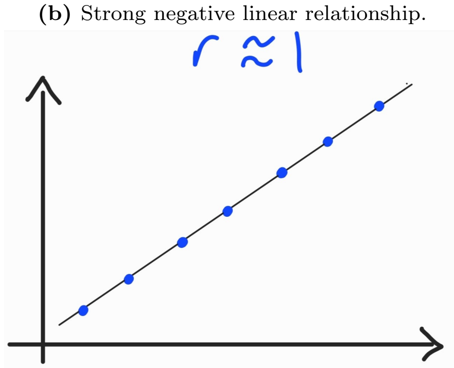
(a) No *linear* relationship.



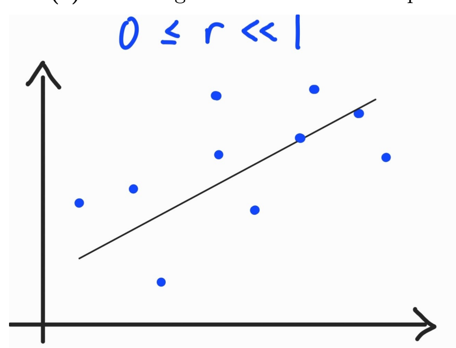
(b) Strong negative linear relationship.



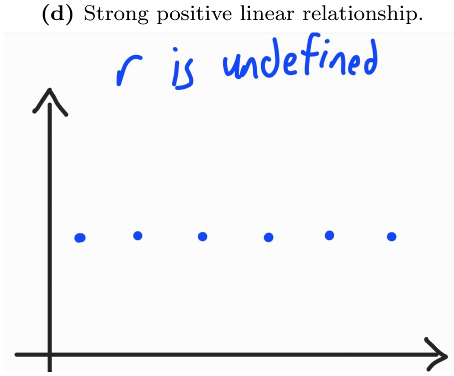
(c) Weak negative linear relationship.



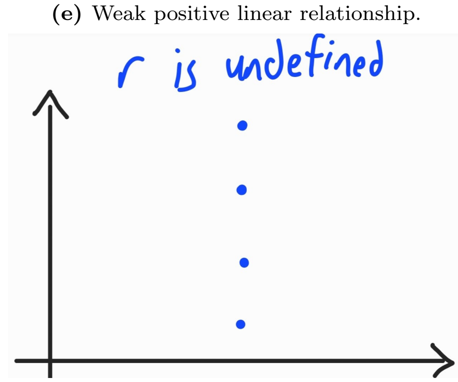
(d) Strong positive linear relationship.



(e) Weak positive linear relationship.



(f) $\sum(y-\bar{y})^2 = 0$.



(g) $\sum(x-\bar{x})^2 = 0$.

Figure 2.2: Example scatter plots with different values of r.

Note. Even though there is no *linear* relationship when $r = 0$, there might be a *nonlinear* relationship present.

G.C. Skills

To find the r-value, or the regression line of y on x, for a given dataset:

`stat` $\Longrightarrow$ `CALC` $\Longrightarrow$ `4:LinReg(ax+b)` or `8:LinReg(a+bx)`

Example 2.2

Suppose that we are given pairs of data for x and y, as shown below:

x	x_1	x_2	$\cdots$	x_n
y	y_1	y_2	$\cdots$	y_n

Table 2.1: A dataset of n pairs of x and y values.

Let Y be the value obtained by substituting a sample value of x into the equation of the regression line of y on x, given by $Y = ax + b$. Consider any $Y' = \alpha x + \beta$. What can you say about the value of $\sum (y - Y')^2$?

Since $\sum (y - Y')^2$ is minimised when $Y' = ax + b$, we see that $\sum (y - Y')^2 \geq \sum (y - Y)^2$ for any $Y' = \alpha x + \beta$.

Note

The regression lines of y on x and x on y intersect at $(\bar{x}, \bar{y})$.

Note

To estimate the value of a variable y, given a the value of another variable x, we always use the regression line of the *dependent* variable on the *independent* variable.

Independent variable	Dependent variable	Regression line
x	y	y on x
y	x	x on y

Table 2.2: Regression line to use for estimations.

Note

Explain why a linear model would not be appropriate. Choose any relevant ones.

- The scatter diagram/data indicates that, as x increases, y [e.g. increases at a decreasing rate], which is *not* a linear relationship.
- A linear model will increase *indefinitely* with more [x in context]. This is contextually *unrealistic*, as [reason in context].
- A linear model would imply that, in the long run, the [e.g. time taken] would be negative, which is impossible.

Note

By calculating the product moment correlation coefficients, explain whether model $y = ax + b$ or model $\mathbf{y} = \mathbf{ax} + \mathbf{b}$ is more appropriate.

The $|r|$ value for the model $y = ax + b$ is higher at ___, compared to ___ for the model $\mathbf{y} = \mathbf{ax} + \mathbf{b}$. Thus, there is a stronger (positive/negative) correlation between x and y. As such, the model $y = ax + b$ is more appropriate.

Note

Let x be in unit$_x$ and $\mathbf{x}$ be in unit$_{\mathbf{x}}$. Suppose that the c unit$_x$ $= 1$ unit$_{\mathbf{x}}$, where c is a constant. Then, if $y = ax + b$, we have $y = ac\mathbf{x} + b$.

2.4 Other Notes

Note

Explain whether it is valid to conclude that a higher value of x will *result in* a lower/higher value of y.

> No. While a higher value of x is *correlated* with a higher value of y, this does not imply any *causal* relationship between x and y.

Note. "result in" tends to refer to a *causal* relationship.

Example 2.3

Suggest an improvement to the data collection process so that the results could provide a fairer gauge of the expected outcome.

> The randomly selected [members of population] might have been of different [category 1; e.g. gender] and [category 2; e.g. age]. To make the results fairer, the data could have been separated based on [category 1] and [category 2].

Chapter 3

Non-Parametric Tests

3.1 Sign Test

General Information

- A *sign test*.

1. Let m be the population median of $D =$ _________ $-$ _________.
2. Test $H_0: m = m_0$ against $H_1:$ (a) $m < m_0$, (b) $m \neq m_0$, or (c) $m > m_0$, at the $100\alpha\%$ significance level.
3.

[label in context]	1	2	3	$\cdots$	N
Sign	$+$	0	$-$	$\cdots$	$+$

Table 3.1: The signs of $d_1, d_2, \ldots, d_N$, for a sigm test. Instead of $1, 2, \ldots, N$ the labeling/column headers can differ in the given context. E.g. $A, B, \ldots, K$. Similarly, the signs here are mere examples; the ith sign cell should be filled with $+$ $(-)$ [0] if $\operatorname{sgn}(d_i) = 1$ $(= -1)$ $[= 0]$.

4. Let X_+ be the number of '$+$'. Under H_0, $X_+ \sim \mathrm{B}(n, 1/2)$, $x_+ = 11$. (Alternatively, X_- can also be used.)
5. Since p-value $=$ ______ $< 100\alpha\%$ $(\geq 100\alpha\%)$, there is sufficient (insufficient) evidence, at the $100\alpha\%$ significance level, to conclude that [H_1 in context].

- *Note.* The p-value for a sign test is given by

H_1	$m < m_0$	$m > m_0$	$m \neq m_0$
X_+	$\mathrm{P}(X_+ \leq x_+)$	$\mathrm{P}(X_+ \geq x_+)$	$2\min\{\mathrm{P}(X_+ \geq x_+), \mathrm{P}(X_+ \leq x_+)\}$
X_-	$\mathrm{P}(X_- \geq x_-)$	$\mathrm{P}(X_- \leq x_-)$	$2\min\{\mathrm{P}(X_- \geq x_-), \mathrm{P}(X_- \leq x_-)\}$

Table 3.2: The p-value for a sign test.

Note

Sign test. Suppose we have $H_1: m \neq m_0$. To find the range of values of x_+ that result in the rejection of H_0, use GC to compute the following tables.

x_+	$\alpha/2 - 2\,\mathrm{P}(X_+ \leq x_+)$
$n-1$	___ > 0
n	___ > 0
$n+1$	___ < 0

x_+	$\alpha/2 - 2\,\mathrm{P}(X_+ \geq x_+)$
$N-1$	___ < 0
N	___ < 0
$N+1$	___ > 0

Then, we conclude that $x_+ \leq n$ or $x_+ \geq N$.

3.2 Wilcoxon Matched-Pairs Signed Rank Test

Note

Assumptions needed for the Wilcoxon Matched-Pairs Signed Rank Test:

1. The data within each pair are dependent on each other, but pairs are independent of each other.
2. The distribution of the differences is continuous and symmetrical.

General Information

- A Wilcoxon matched-pairs signed rank test.
  1. Let m be the population median of $D =$ ________ $-$ ________.
  2. Test $H_0: m = 0$ against H_1: (a) $m < 0$, (b) $m \neq 0$, or (c) $m > 0$, at the $100\alpha\%$ significance level.
  3.

[label in context]	1	2	3	$\cdots$	N
D	d_1	0	d_3	$\cdots$	d_N
Rank	1	0	5	$\cdots$	2

Table 3.3: The value of the differences $d_1, d_2, \ldots, d_N$, which are then ranked according to their absolute size $|d_i|$. For our syllabus, each d_i is always distinct.

  4.
     - $t_- =$ ___ $+$ ___ $+ \cdots +$ ___ $=$ ___
     - $t_+ =$ ___ $+$ ___ $+ \cdots +$ ___ $=$ ___
     - The test statistic is $T := \min\{T_-, T_+\} =$ ___.
     - Reject H_0 if $T =$ ___. (see table 3.4)

Remark

$$\frac{n(n+1)}{2} = t_- + t_+.$$

  5. Since $t =$ ___ $\square$ ___, there is sufficient/insufficient evidence, at the $100\alpha\%$ significance level, to conclude that [H_1 in context].

- The test statistics T_+ and T_- can also be used, depending on our preference.
- The critical regions for a Wilcoxon test, for each alternative hypothesis and test statistic T_- or T_+. The value of c is obtained from MF26*.

  Note. the value of c may differ for a one-tail vs a two-tail test, so look at the table carefully, to obtain the correct value.

H_1	$m < m_0$	$m > m_0$	$m \neq m_0$
T_+	$T_+ \leq c$	$T_+ \geq \dfrac{n(n+1)}{2} - c$	$T_+ \leq c$ or $T_+ \geq \dfrac{n(n+1)}{2} - c$
T_-	$T_- \geq \dfrac{n(n+1)}{2} - c$	$T_- \leq c$	$T_- \leq c$ or $T_- \geq \dfrac{n(n+1)}{2} - c$
T	$T \leq c$[1]		$T \leq c$ or $T \geq \dfrac{n(n+1)}{2} - c$

Table 3.4: The critical regions for Wilcoxon tests.

[1]Assuming $T_- \geq T_+$ for $m < m_0$, and $T_+ \geq T_-$ for $m > m_0$.

- For large sample sizes $n \geq 21$, we use the approximation

$$T \sim \mathrm{N}\left(\frac{n(n+1)}{4}, \frac{n(n+1)(2n+1)}{24}\right)$$

and conduct a one/two-tailed z-test.

G.C. Skills

fter calculating our list of differences $\mathtt{L}_3$, we can calculate $\mathtt{L}_4 = |\mathtt{L}_3|$ and use the G.C. to rank this list in ascending order:

$$\texttt{stat} \implies \texttt{2:SortA} \implies \mathtt{L}_4.$$

This allows us to easily compute the ranks associated with each difference.

Note

The value of n for the test statistic/MF26 critical region in both tests should be the number of columns with nonzero difference d. i.e.

$$n := \#\{i \mid d_i \neq 0\} = \#\text{cols} - \#\{i \mid d_i = 0\}.$$

Note

If we need to use both the sign test and a Wilcoxon test on the same sample, then consider creating just a single table, as shown below.

[label in context]	1	2	3	$\cdots$	n
D	d_1	0	d_3	$\cdots$	d_n
Sign	$+$	0	$-$	$\cdots$	$+$
Rank	1	0	5	$\cdots$	2

Table 3.5: Combined table for both the sign test and Wilcoxon test.

Note

How do you improve the Wilcoxon test used in [the previous part]?

Increase the sample size for the test.

Note

State the circumstances under which a non-parametric test would be used rather than a parametric test.

We use a non-parametric test, rather than a parametric test, when:

1. The population is not known to be normally distributed.
2. The population mean is not the best way to measure central tendency.
3. The measurement scale has no predetermined rank or ordering.

Note

Why is it not appropriate to use a paired-sample t-test?

There is no contextual evidence to support the assumption that $D_1, D_2, \ldots, D_n$ are normally distributed. So, conducting a paired-sample t-test may result in unreliable results, given our small sample size n.

Note

State the precautions that should be taken to avoid (statistical) bias.

Choose any approperate ones.

1. The test should be *'blind'*. [Testers in context] should not know which of the [two variations involved in the test, in context] they are [tasting/wearing/etc, in context]. If the [testers] knew, their preconceptions may affect __________.
2. Pick a random sample of n [testers].
3. The *order* of the test — whether the [first variation] or [second variation] comes first — should be randomised.
4. The [testers] should not communicate with each other.
5. There should be sufficient rest time between the two runs, so that the running timing of the second run would not be affected due to fatigue.

Note

Explain why it is better to conduct a Wilcoxon test than a sign test.

While a sign test only considers the sign of the differences, a Wilcoxon test takes into account both the sign and *magnitude* of the differences. Therefore, a Wilcoxon test is more reliable, as it incorporates more information about the data.

Note

Explain why a sign test is more suitable/a Wilcoxon test is inappropriate.

Choose any appropreiate ones

1. The data here is non-numeric and is not meausred on an ordinal scale. Hence, it is inappropreiate to conduct a Wilcoxon test. A sign test is better, as the data can still be represented by positie and negative responses — denoting ______ and ______, respectively.
2. The magnitude of the differences is irrelevant because ______. So, a sign test — which only accounts for the sign of the differences — is more appropriate.
3. In this case, the data has too many *tied ranks*. Thus, the conclusion obtained from a Wilcoxon test may not be reliable.
4. An additional assumption that the distribution of the differences $D = \_\_\_ - \_\_\_$ must be continuous and symmetric about the median.

Example 3.1: A tricker question, involving an unknown in the data provided.

Let m be the median of $D\colon X - Y$. For the data in Table 3.6, assume that there are no tied ranks, and $x_i \neq y_i$ for each $1 \leq i \leq 7$. Carry out a Wilcoxon test, at the 5% significance level, to determine if the data supports the alternative hypothesis $H_1: m > 0$.

Index	1	2	3	4	5	6	7
x_i	4	8	7	7	1	9	9
y_i	6	9	3	4	a	1	2

Table 3.6: Data with an unknown variable $a \in \mathbb{Z}^+$.

First, we calculate the differences. Since $x_i \neq y_i$, we have $a \neq 1$. In fact, $a \neq 1, 2, 3, 4, 7, 8$ because $d_i \neq d_j$, for $i \neq j$. Thus, $a = 6, 7$ or $a \geq 10$. The corresponding rank r_5 is hence 5 or 7.

Index	1	2	3	4	5	6	7
d_i	-2	-1	4	3	$1-a$	8	7
$\|d_i\|$	2	1	4	3	$a-1$	8	7
rank r_i	2	1	4	3	r_5	r_6	r_7

Table 3.7: The values of the differences d_i and the associated ranks. The columns highlighted in grey are those with negative differences d_i.

Now,

$$t_- = 2 + 1 + r_5 = 8, 10 \qquad \text{and} \qquad t_+ = 7(7+1)/2 - t_- = 25 - r_5 = 20, 18.$$

Hence, the test statistic $T \coloneqq \min\{T_-, T_+\} = T_-$, where we reject H_0 if $T \leq 3$. So, since $t_- = 3 + r_5 > 3$, we do not reject H_0.