

A-Levels Physics Notes

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Kinematics



Figure 1.1: [1] Parabolic path travelled by balls thrown at varying angles.

- *Distance* is defined as the total length of *path* travelled.
- *Displacement* is defined as the distance of a point from some reference point *in a specified direction*.
- *Velocity* is defined as the rate of change of displacement.
- *Acceleration* is defined as the rate of change of velocity.
- Four key formulas for kinematics. When the acceleration a is constant:
 1. $v = u + at$,
 2. $s = ut + \frac{1}{2}at^2$,
 3. $s = \frac{1}{2}(u + v)t$,
 4. $v^2 = u^2 + 2as$.

Dynamics

- *Newton's First Law of Motion* states that an object at rest will remain at rest and an object in motion will remain in motion at constant velocity in a straight line in the absence of an *external* resultant force.
- The *linear momentum* of a body is the product of its mass and velocity. The linear momentum is in the *same direction* as its velocity.
- *Newton's Second Law of Motion* states that the rate of change of momentum of a body is directly proportional to the resultant force acting on the body and occurs *in the direction* of the resultant force.
- *Newton's Third Law of Motion* states that if body A exerts a force on body B, then body B exerts a force of the *same type* that is equal in magnitude and opposite in direction on body A.
- *Impulse* is defined as the product of *average* force acting on an object and the time for which the force acts.
- The *Principle of Conservation of Linear Momentum* states that the total momentum of a system remains constant provided no *external* resultant force acts on the system. i.e.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$

- A elastic collision is one where kinetic energy is conserved. i.e.

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 v_2^2.$$

- The below table provides a quick shortcut, that is particularly helpful in solving multiple choice questions fast. We assume a mass m_1 is moving with velocity u towards a *stationary* ($u_2 = 0$) mass m_2 .

Case	v_1	v_2
$m_1 = m_2$	0	u
$m_1 \ll m_2$	$\approx -u$	≈ 0
$m_1 \gg m_2$	$\approx u$	$\approx 2u$

Table 2.1: Special cases of elastic collisions.

- For an elastic collision, the relative speed of approach is equal to the relative speed after collision:

$$u_1 - u_2 = v_2 - v_1.$$

Note. The formula is *not* $u_1 - u_2 = v_1 - v_2$.

- An inelastic collision is one where kinetic energy is not conserved.
- In a perfectly inelastic collision, the masses coalesce (become one) and move off with the same velocity. In other words,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)v.$$

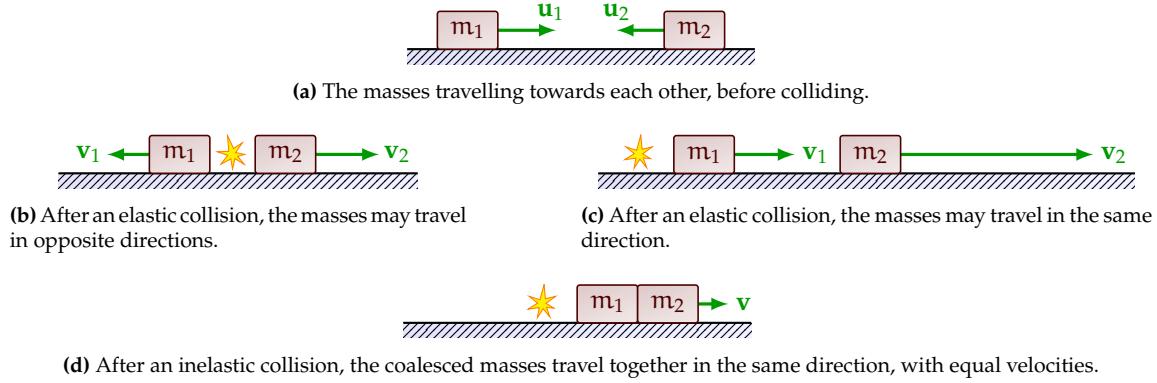


Figure 2.1: [2] An illustration of (perfectly) elastic and inelastic collisions.



Figure 2.2: [2] Another illustration of a (perfectly) inelastic collision.

- An explosion does not conserve kinetic energy. But momentum is still conserved. Suppose a (stationary) mass splits into n masses m_1, m_2, \dots, m_n . Then we have

$$m_1 v_1 + m_2 v_2 + \dots + m_n v_n = 0.$$

Forces

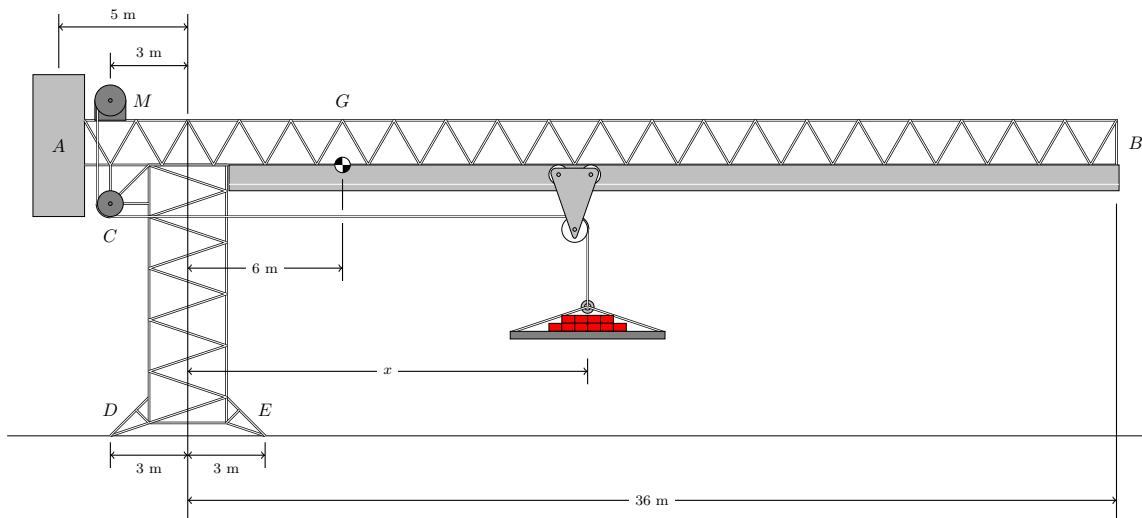


Figure 3.1: [3] Forces acting on a crane.

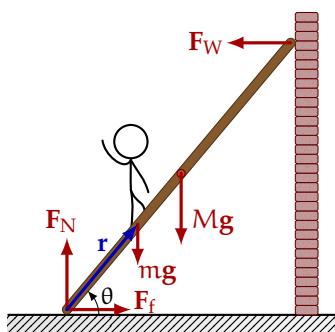


Figure 3.2: [4] The free body diagram for a ladder, on which a person stands, and which is leaning against a wall.

- A field of force due to a body's property (e.g. charge, mass) is a region in space in which a body carrying that property experiences a force when it is placed in the field.
 - *Hooke's Law* states that the force acting on a material directly proportional to the extension in the material, provided that the *limit of proportionality* is not exceeded.
 - The *center of gravity* of an object is the point at which the entire weight of a body may be considered to act.
 - For a mass placed on a ramp, the forces it experiences are as shown below.

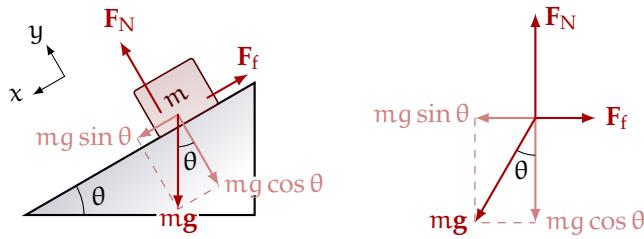


Figure 3.3: [5] Mass m placed on a ramp whose top surface makes an angle θ with the horizontal.

- The *moment* of a force is equal to the product of the force and the *perpendicular* distance of the *line of action* of the force from the pivot. It is also the turning effect of a force.
- Torque of a couple* is defined as the product of one of the forces and the *perpendicular* distance between the *lines of action* of the forces.

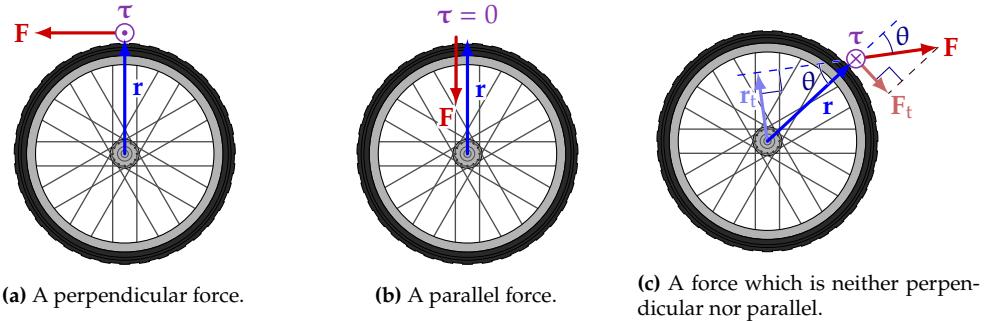


Figure 3.4: [6] The torque of a force acting on a wheel

- The *Principle of Moments* states that if a body is in rotational equilibrium, the (vector) sum of all the clockwise moments about *any axis* must be equal in magnitude to the (vector) sum of anticlockwise moments about the *same axis*.

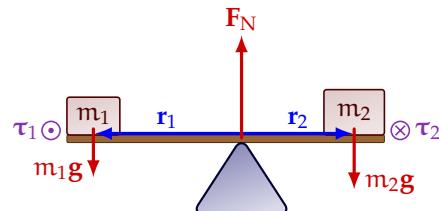


Figure 3.5: [6] The free body diagram of a lever, which is in equilibrium iff $m_1 g r_1 = m_2 g r_2$.

- Note.* When using the Principle of Moments, always make clear which pivot/axis of rotation is being considered. E.g. “Taking moments about ___, by the principle of moments, . . .”
- Density* is defined as the mass per unit volume of a substance.
- Pressure* is defined as force per unit area, where the force is *acting perpendicularly* to the area.



- Deriving $p = \rho gh$:

- Consider a point at a depth h below the surface of a liquid of density ρ .
- The force F acting perpendicularly on a surface area A at depth h is due to the weight of the liquid column above A to give pressure p . Thus, $p = \frac{F}{A} = \frac{mg}{A} = \frac{\rho A h}{g} = \rho gh$.

Figure 3.6: [7] A column of fluid.

- Upthrust* is the upward force exerted by a fluid on a body immersed in the fluid (due to pressure difference in the fluid).
- The origin of upthrust:* Upthrust is a result of the pressure difference between top and bottom surfaces of the body, resulting in a net upwards force being exerted on the body by the third medium in which the body is located.

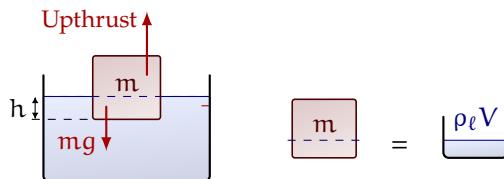


Figure 3.7: [8] An illustration of Archimedes' Principle.

- Archimedes' Principle* states that when a body is totally or partially immersed in a fluid, it experiences an upward force (upthrust) equal to the weight of fluid displaced.
- The Principle of Floatation* states that, for any object floating in *equilibrium*, the upthrust is equal to the weight of the object.

- Remember to account for atmospheric pressure, where necessary.
- Suppose a body is subjected to forces acting at three points. Then, for it to be in equilibrium, the lines of action must be *concurrent* (intersection at a common point) or *parallel*.

Example 3.1

A uniform strip of steel is clamped at one end. A metal block of mass M is fixed to the strip, a distance L from the clamp. The mass causes the end of the steel strip to be depressed.

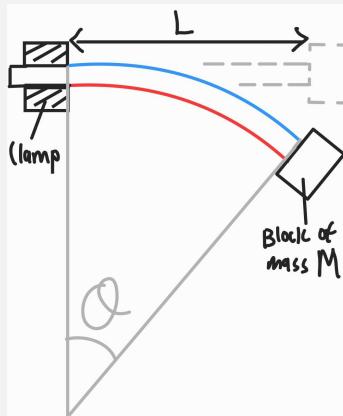


Figure 3.8: [22]

Explain why the part of the strip is compressed and part of the strip is extended.

[3]

- For the bent strip, the upper and lower surfaces form two arcs subtended by the same angle.

- Since the upper surface has a larger radius of curvature, it is bent longer than L; it is extended.
- Since the lower surface has a shorter radius of curvature, it is bent shorter than L; it is compressed.

Each school's solution has a different answer to this question. So, its hard to know what exactly is required. I just picked the one that seems the most value-adding — i.e. does not just subtly paraphrase the question.

Work, Energy, and Power

- *Work done* is defined as the product of a force and the displacement in the direction of the force.
- *One joule of work* is defined as the work done by a one Newton force when its *point of application* is displaced one metre in the direction of the force.
- *Energy* is defined as the ability to do work.
- *The Principle of Conservation of Energy* states that energy can neither be created or destroyed in *any process*. It can be transformed from one form to another, and transferred from one body to another.
- Deriving $E_k = \frac{1}{2}mv^2$:
 1. Consider a constant horizontal applied force F acting on an object of mass m travelling with initial velocity u to reach a final velocity v over a displacement s .
 2. For uniform acceleration, $v^2 = u^2 + 2as$ so $as = \frac{1}{2}(v^2 - u^2)$. Combined with Newton's Second Law, $W = Fs = mas = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$. When the object starts from rest, $u = 0$.
 3. By conservation of energy, *the work done by force F must be converted into the kinetic energy E_k of the object*. Hence, $E_k = W = \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}mv^2$.
- $E_k = \frac{p^2}{2m}$.
- The *Work-Energy Theorem* states that the *net* work done by *external* forces acting on a particle is equal to the *net* change in kinetic energy of the particle.
- Deriving $E_p = mgh$:
 1. Consider an object from the Earth's surface — which is taken as the reference for zero gravitational potential energy — raised up by a *constant force F equal to and opposite to the weight mg* of the object such that the object moves up at *constant velocity* to a height h .
 2. Thus, the object moves at constant speed so $\Delta E_k = 0$. Therefore,

$$\begin{aligned}\Delta E_p &= W \\ E_p - 0 &= Fs \\ E_p &= mgh, \quad \text{by Newton's Second Law.}\end{aligned}$$

where E_p is the gravitational potential energy at height h above the Earth's surface.

- Know how to derive $\Delta E_p = \frac{1}{2}kx^2$ from area under graph.
- *Power* is defined as the rate of doing work.
- Derive $P = Fv$: $P = \frac{dW}{dt} = \frac{d(Fs)}{dt} = F\frac{ds}{dt} = Fs$ for a constant force F .

Temperature and Ideal Gases

- From empirical results, we know that there appears to be a *linear* trend, wherein pressure and volume decrease, when temperature decreases. So, there must be a temperature where pressure is zero, and a temperature where volume is zero. Empirical data shows that the temperature at which this occurs is the same for all gases under all experimental conditions. This temperature is known as *absolute zero*.
- The *Zeroth Law of Thermodynamics* states that if bodies A and B are separately in thermal equilibrium with body C, then bodies A and B are in thermal equilibrium with each other.
- One mole* is defined as the amount of substance that contains as many elementary particles as there are atoms in 0.012kg of carbon-12.
- Avogadro's Constant* N_A is the number of atoms in 0.012kg of carbon-12.
- Celsius-Kelvin conversion: $T \text{ } ^\circ\text{C} = (T + 273.15) \text{ K}$.

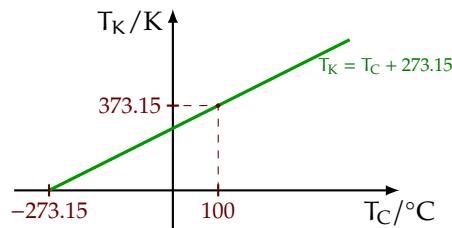


Figure 5.1: [9] Kelvin against celsius graph.

- An *ideal gas* is a *hypothetical gas* that *perfectly obeys* the *equation of state* for an ideal gas, at *all pressures, temperatures, and volumes*. The equation of state is $pV = nRT$, where
 - p : pressure of ideal gas
 - V : volume of ideal gas
 - n : number of moles of ideal gas
 - R : molar gas constant
 - T : temperature of ideal gas.
- There is no ideal gas in nature, but at *low pressures and high temperatures*, real gases behave like ideal gases.
- Boyle's Law*. When the temperature of an ideal gas remains constant, the product of its pressure p and volume V is constant. i.e. $p_1V_1 = p_2V_2$.
- Charles' Law*. When the pressure of an ideal gas remains constant, its volume V is directly proportional to its thermodynamic temperature T/K . i.e. $V_1/T_1 = V_2/T_2$.
- Gay-Lussac's Law*. When the volume of an ideal gas remains constant, its pressure p is directly proportional to its thermodynamic temperature T/K . i.e. $p_1/T_1 = p_2/T_2$.
- Let n be the amount of gas in moles and the integer N be the number of gas molecules. Then, for the molar gas constant $R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$ and Boltzmann's constant $k = 1.38 \cdot 10^{-23} \text{ J K}^{-1}$, the equation of state for an ideal gas is $pV = nRT = NkT$.
- Some useful conversions. For $\frac{AX}{Z}$, we have the ratio 1 mole : Z grams and $n = \frac{N}{N_A} = \frac{m/g}{Z}$.

	Assumptions of the Kinetic Theory of Gases
M	The molecules of the gas are in <i>rapid</i> and <i>random</i> motion.
A	There are <i>no intermolecular</i> attractive forces.
N	Any gas consists of a <i>very large number</i> of molecules.
T	The duration of collisions is negligible compared to the time interval between collisions.
E	The collisions between gas molecules, and between gas molecules and the container walls are <i>perfectly elastic</i> .
V	The volume of the gas molecules themselves is negligible compared to the volume of the container.

- How would the force exerted by gas differ in the presence of intermolecular forces, compared to the ideal case? Particles would attract each other, so the average force on the wall will decrease.
- Real gases are approximately ideal at high temperatures and low pressures.
- Deriving $p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle$ for an ideal gas:
 - Consider a cubic container of side l containing N molecules, each of mass m .
 - The change in momentum of one molecule due to its *elastic* collision with the container wall is $2mc_x$.
 - The time interval between collisions of this molecule with the same wall is $\Delta t = \frac{2l}{c_x}$.
 - By Newton's 2nd Law, the net force F experienced by one molecule is equal to the rate of change of momentum it experiences. i.e. $F = \frac{2mc_x}{\frac{2l}{c_x}} = \frac{mc_x^2}{l}$.
 - By Newton's 3rd Law, since the molecule and the container wall form an action-reaction pair, the net force experienced by the container wall (due to this one molecule) is F .
 - The pressure experienced by the wall due to this one molecule is $p = \frac{mc_x^2}{l^3} = \frac{mc_x^2}{V}$.
 - The pressure experienced by the wall due to N molecules is $p_N = \frac{Nmc_x^2}{V}$.
 - The molecules can move in three directions, so $c^2 = c_x^2 + c_y^2 + c_z^2$. Since the molecules' motion is random, $\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$. Hence $\langle c^2 \rangle = 3\langle c_x^2 \rangle$.
 - Now, $p_N = \frac{Nm\langle \frac{1}{3}c^2 \rangle}{V} = \frac{1}{3} \frac{Nm\langle c^2 \rangle}{V}$.
- The mean translational kinetic energy of an ideal gas molecule is $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$.
- The total translational kinetic energy of N ideal gas molecules is $\frac{3}{2}NkT = \frac{3}{2}nRT = \frac{3}{2}pV$.

First Law of Thermodynamics

Difference	Evaporation	Boiling
Temperature	Occurs at all temperatures	At each pressure, there is exactly one fixed temperature at which boiling occurs.
Location	Occurs only at the surface of a liquid	Occurs throughout the liquid

Table 6.1: Differences between evaporation and boiling.

- The *heat capacity* of a body is defined as the amount of thermal energy required to raise its temperature by one Kelvin / degree Celsius.
 - The specific *heat capacity* of a body is defined as the amount of thermal energy required to raise the temperature of one unit mass of the substance by one Kelvin / degree Celsius.
 - For a substance of mass m , heat capacity C , and specific heat capacity c , we write $Q = C\Delta\theta = mc\Delta\theta$.
 - Note.* In the case that the mass of gas present is not explicitly given, check whether the question specified what the gas is. For example, there might have been a sneaky mention that the gas is neon-20, so $m = 20N$ unified atomic masses can be deduced
 - The *specific latent heat* of a body is defined as the thermal energy required to change *phase* of one unit mass of a substance, *without a change in temperature*.
 - For a substance of specific latent heat of fusion (vaporisation), we write $Q = mL_f$ ($Q = mL_v$).
 - Internal energy* of a system is a sum of *random distribution* of kinetic and potential energies associated with the *molecules* of the system. Microscopic potential energy is due to the intermolecular forces between the molecules. Microscopic kinetic energy is due to the random motion of the molecules.
 - The *internal energy of an ideal gas* is the sum of microscopic kinetic energies due to the random motion of the ideal gas molecules.
- Note.* The inclusion of microscopic potential energy is forbidden, since ideal gases have zero microscopic potential energy.
- Words such as “insulated” implies that there is no heat supplied to the system. i.e. $Q = 0$.
 - Note.* Work done versus heat supplied. Work done is the transfer of energy by force/macrosopic changes, while heat supplied is the direct transfer of thermal energy through microscopic changes. E.g. consider a liquid that is isolated from all other sources of heat. When stirring leads to an increase in temperature (via frictional heating), it is considered work done, even though the liquid experiences an increase in thermal energy.
 - An ideal monoatomic gas of N atoms only has translational motion and no intermolecular potential energy, $U = E_k = N \cdot$ translational E_k of one atom $= \frac{3}{2}NkT$.
 - For non-monoatomic ideal gases, $U \neq \frac{3}{2}NkT$. But we know that $U_{\text{Total}} \propto NT$ and $U_{\text{one molecule}} \propto T$.
 - The *First Law of Thermodynamics* states that the *increase* in internal energy of a closed system is the *sum* of heat *supplied* to the system and the work done *on* the system. i.e. $\Delta U = Q + W$.

- The work done W on a gas against a constant external pressure p is such that $W = -p\Delta V$.
- In general, the work done on a gas is $W = - \int_{V_i}^{V_f} p \, dV$.
- An *isothermal* process is one that takes place without a change in temperature, i.e. $\Delta T = 0$. Thus, for an ideal gas (possibly non-monoatomic), $\Delta U = 0$ since $U \propto T$.
- An *iso-volumetric/isochoric* process is one that takes place without a change in volume, i.e. $\Delta V = 0$. Hence, when the external pressure is constant, $W = 0$.
- An *isobaric* process is one that takes place without a change in pressure, i.e. $\Delta p = 0$.
- An *adiabatic* process is one that takes place without heat transfer, i.e. $Q = 0$. Therefore, $U = W$.

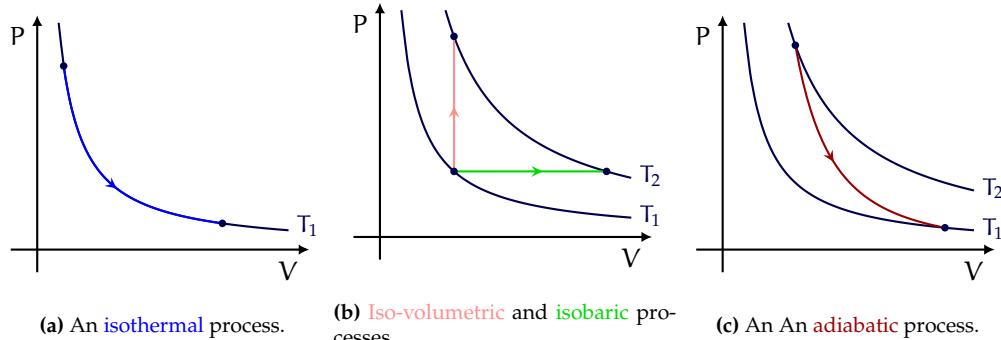


Figure 6.1: [10] Illustrations for isothermal, iso-volumetric/isochoric, isobaric, and adiabatic processes.

- A thermodynamic cyclic process is one that eventually returns to its initial state.
 - $\Delta U = 0$ so $Q = -W$.
 - The net work done by/on the gas is given by the area enclosed by the closed loop.

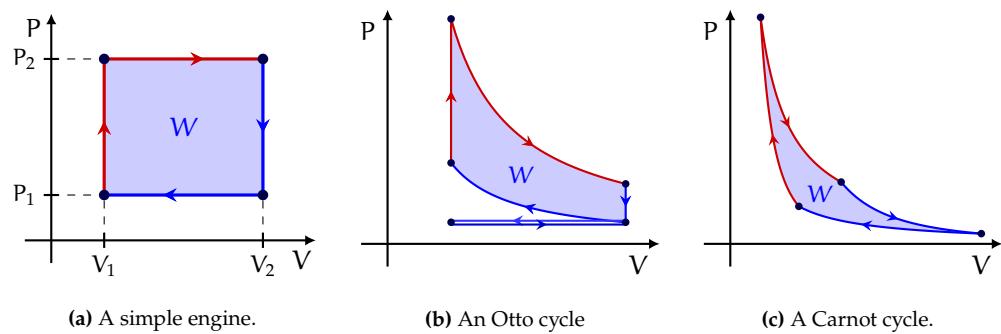


Figure 6.2: [10] Some thermodynamic cyclic processes.

Circular Motion

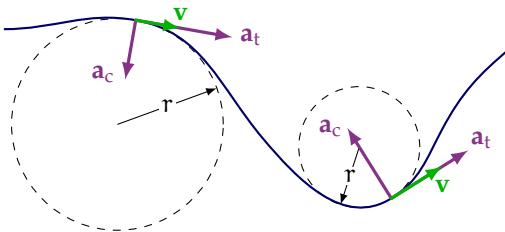


Figure 7.1: [11] An illustration of circular motion.

- *Angular displacement* is the angle through which an object turns *with respect to the center* of its circular path.
- *The radian* is defined as the angle *subtended* at the *center* of a circle by an *arc* of length equal to the radius of the circle.
- *Angular velocity* is the rate of change of angular displacement.

$\omega = \frac{2\pi}{T} = 2\pi f$	$v = r\omega$	$a_c = \frac{v^2}{r} = r\omega^2 = v\omega$	$F_c = ma_c$
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□ Common formulae: $\tan(\theta) = \left(\frac{v^2}{rg}\right)$ and $v = \sqrt{rg}$.

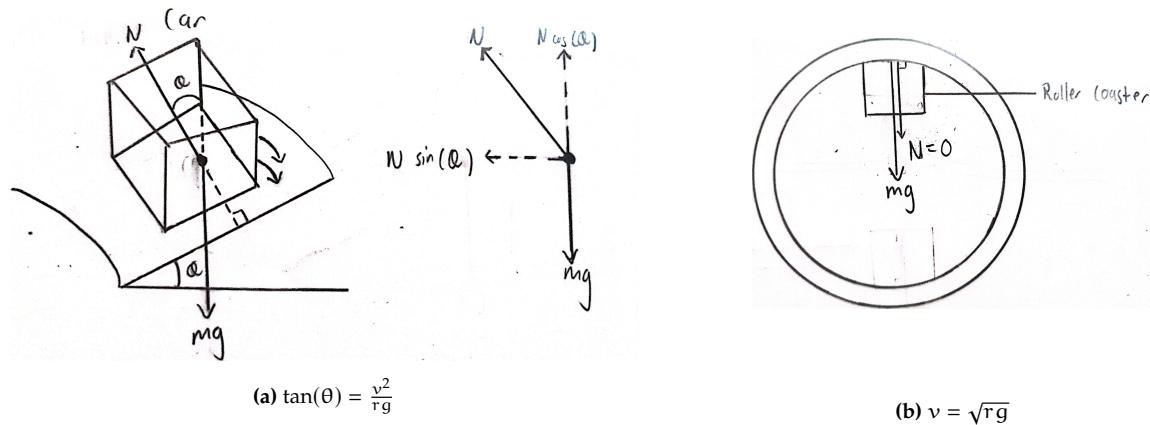


Figure 7.2: [22] Example situations where we will use $\tan(\theta) = \frac{v^2}{rg}$ and $v = \sqrt{rg}$.

- Water in bucket at top position: $F_c = N + W$ (where $N \geq 0$) so $\omega \geq \sqrt{\frac{g}{r}}$.
- We need to write “Centripetal force is provided by _____”
- Why are teardrop designs preferred over full circles for roller coaster tracks? It increases the radius of the circular path (decreases the curvature) when entering the loop, reducing the acceleration experienced by the passengers.

Example 7.1

Explain why the path of a charged particle in the uniform magnetic field is circular. [2-3]

- The magnetic force F_B acting on the charged particle is always *perpendicular* to the direction of its motion, throughout its time in the field. [1]
- No displacement occurs in the direction of F_B . It does no work on the particle. So, the particle has constant kinetic energy, i.e. constant speed. Hence, the *magnitude* of the magnetic force is *constant*. [1]
- This force provides the centripetal force for the circular motion of the particle within the region of the magnetic field. [0-1]

Gravitational Fields

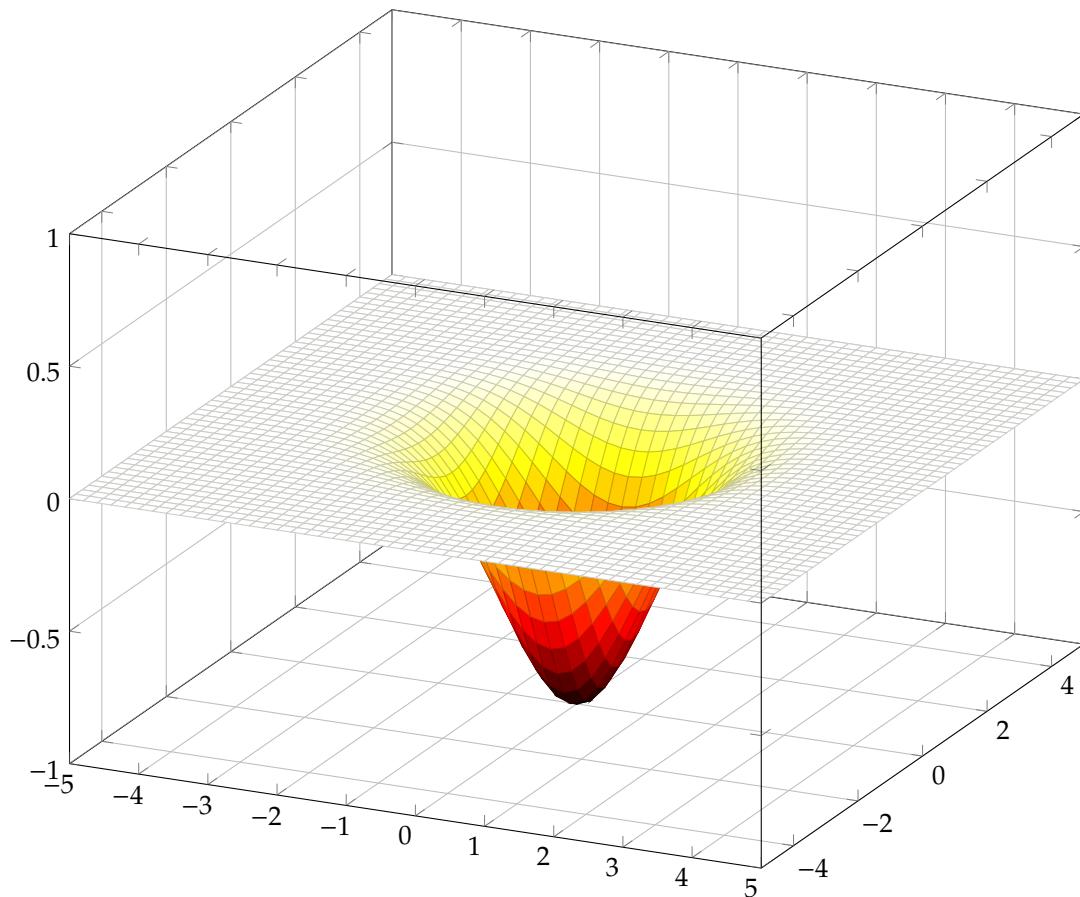


Figure 8.1: [12] A gravitational potential well.

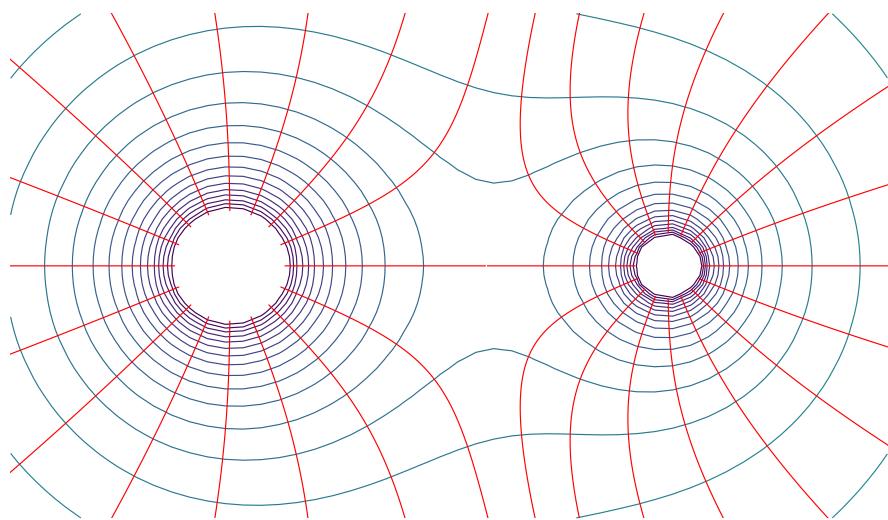


Figure 8.2: [13] The gravitational field and equipotential lines produced by two masses.

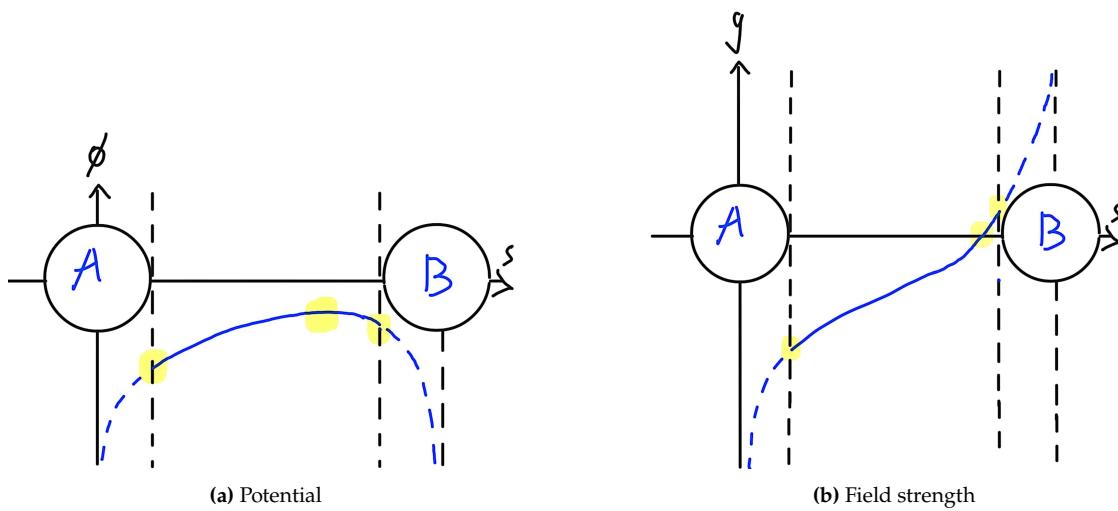


Figure 8.3: The gravitational potential and gravitational field strength between two masses.

- *Newton's Law of Gravitation* states that the force of attraction between any two point masses is directly proportional to the product of their masses and inversely proportional to the square of their separation.
 - A *gravitational field* is a region in space where mass experiences a gravitational force acting on it.
 - Gravitational field strength at a point is defined as the gravitational force per unit mass acting on a small mass placed at that point
 - The *gravitational potential energy* of a mass at a point is defined as the work done by an *external agent* in bringing the mass *from infinity* to that point (without any net change in kinetic energy).
 - *Gravitational potential* at a point is defined as the work done per unit mass by an *external agent* in bringing a mass *from infinity* to that point (without a net change in kinetic energy).

Example 8.1

The gravitational potential ϕ at a distance x from a point mass M is given by the expression

$$\phi = -\frac{GM}{x}$$

where G is the gravitational constant. Explain why gravitational potential is a negative quantity [3]

- Since gravitational potential energy is *attractive*, [1]
 - an external force that brings a (small test) mass from infinity to a finite distance x from M , with no net change in kinetic energy, is *opposite in direction* to the displacement of the mass. So, *negative work* is done by the external force. [1]
 - As *gravitational potential at infinity is zero*, the gravitational potential at distance x from M is negative. [1]

- Escape velocity is the *minimum* velocity a mass needs to be projected from the *surface* of the Earth in order to have sufficient kinetic energy to overcome the gravitational field it experiences and *move to infinity*.

- Escape velocity $v_{\min} = \sqrt{\frac{2GM}{r}}$ (where Min E_k needed is the gain in E_p to reach infinity).

□

$$\begin{array}{ccc} U_G = -\frac{GMm}{r} & \xrightarrow{-\frac{d}{dr}} & F_G = -\frac{GMm}{r^2} \\ \downarrow \frac{1}{m} & & \downarrow \frac{1}{m} \\ \phi = -\frac{GM}{r} & \xrightarrow{-\frac{d}{dr}} & g = -\frac{GM}{r^2} \end{array}$$

□ $U_G = m\phi$ & $\Delta U_G = m\Delta\phi$.

□ Gravitational force provides the centripetal force:

$$\begin{aligned} F_G &= F_C \\ \frac{GMm}{r^2} &= \frac{mv^2}{r} \\ E_k &= \frac{GMm}{2r} \end{aligned}$$

Thus, $E_T = U_G + E_K = -\frac{GMm}{2r}$.

□ Gravitational force provides the centripetal force:

$$\begin{aligned} F_G &= F_c \\ \frac{GMm}{r^2} &= mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2 \\ T^2 &= \frac{4\pi^2}{GM}r^3 \\ T^2 &\propto r^3 \end{aligned}$$

□ Gravitational force provides the centripetal force:

$$\begin{aligned} F_G &= F_c \\ \text{For A: } \frac{Gm_A m_B}{(r_A + r_B)^2} &= m_A r_A \omega^2 \\ \text{For B: } \frac{Gm_A m_B}{(r_A + r_B)^2} &= m_B r_B \omega^2 \end{aligned}$$

The center of mass of the system is at point P where

$$m_A r_A = m_B r_B$$

such that both stars have the same angular velocity ω .

□ For binary star systems, notice that *orbital radius is replaced by the stars' separation*:

$$\begin{aligned} m_A r_A &= m_B r_B & \frac{Gm_A m_B}{(r_A + r_B)^2} &= m_B r_B \omega^2 \\ r_A + r_B &= \frac{m_B}{m_A} r_B + r_B & \text{so} &= \frac{m_A m_B}{m_A + m_B} (r_A + r_B) \omega^2 \\ r_B &= \frac{m_A}{m_A + m_B} (r_A + r_B) \end{aligned}$$

So, rearranging, we have

$$\omega^2 = \frac{G(m_A + m_B)}{(r_A + r_B)^3} = \frac{Gm_A}{r_B(r_A + r_B)^2} \quad \text{and} \quad T^2 = \frac{4\pi^2}{G(m_A + m_B)} (r_A + r_B)^3.$$

Example 8.2

A binary star system has the plane of its orbits parallel to the line of sight from Earth. This binary star system is so far from Earth that the individual stars cannot be distinguished. Suggest and explain what observation can be made to determine the period of the orbit of the stars.

- Observe the light intensity of the binary star system across time.
- The minimum intensity occurs when the line joining both stars is parallel to the line of sight from Earth — one star is directly in front or behind the other so the maximum amount of light from the star behind is blocked by the star in front.
- Determine the time t between two minimum intensities, which is half a period. The orbital period is $T = 2t$.

 Geostationary orbit facts:

1. Only one such orbit at a *fixed* distance of 4.2×10^7 m from Earth's center,
 2. Orbital period of 24 hours,
 3. Satellite's plane of orbit coincides with the equatorial plane of the Earth,
 4. Orbits West to East (in the same direction as Earth's rotation).
- Equipotential lines are not equally spaced because gravitational field strength is not constant but decreases as one goes away from the Earth.
- Assumptions made in the theory (e.g. in deriving $g = -\frac{GM}{r^2}$):
1. The bodies are separated by distances so large they can be considered as point particles (i.e. separation » radius).
 2. The bodies are homogenous spheres (constant density throughout the sphere).
 3. The bodies have masses distributed symmetrically around their centers in uniform layers.
 4. In the absence of other masses.

Example 8.3

State and explain the effect, if any, on the graph — of gravitational field strength against the distance from the center of the sphere — you have sketched on Fig 9.2 when mass is lost uniformly from the surface of the sphere. [2]

- The gravitational field strength g still obeys the inverse square law. But since g is directly proportional to the mass of the sphere, g decreases. [1]
- Hence, the new graph will be shifted down and entirely below the original graph. [1]

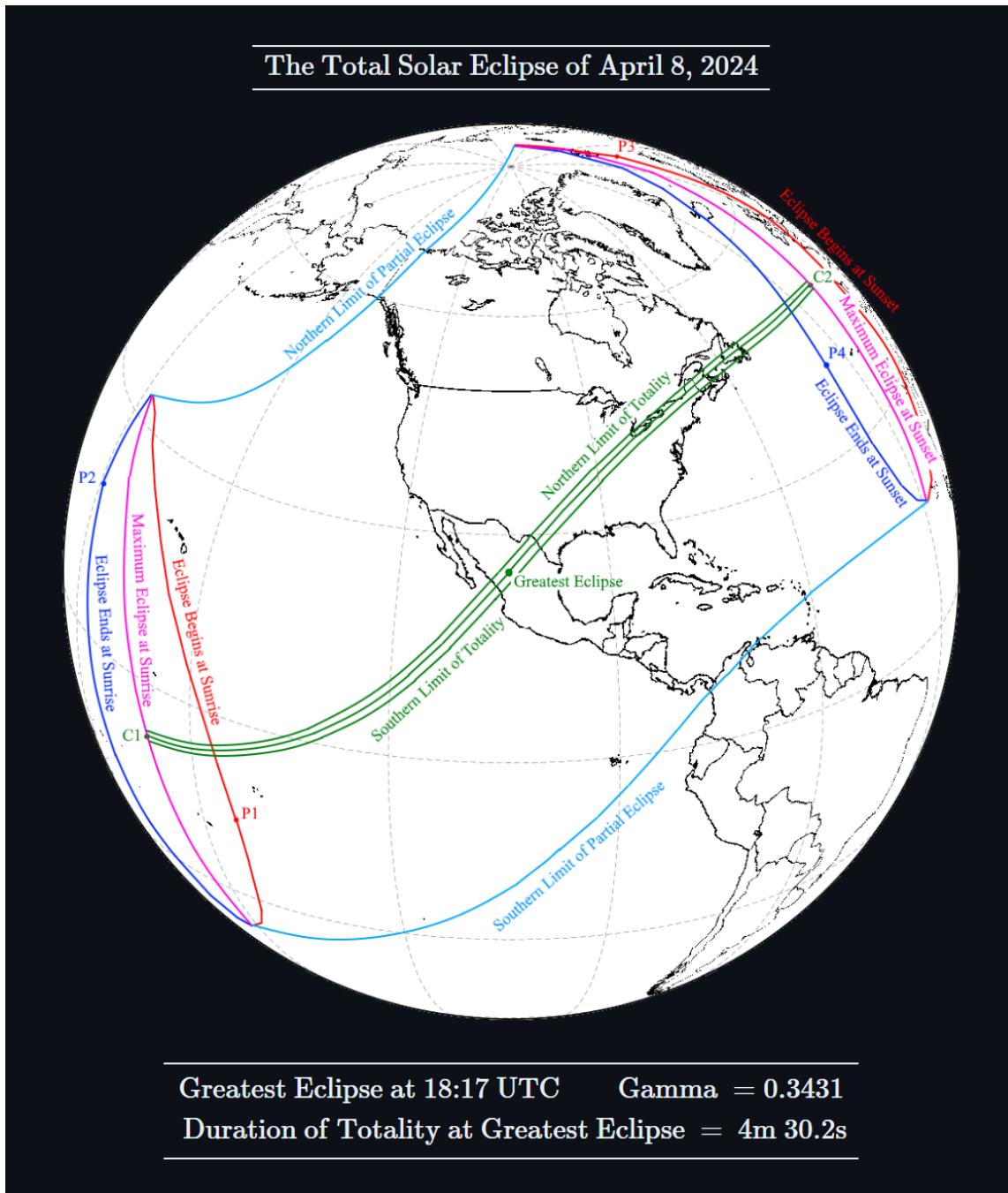


Figure 8.4: [14] Forces acting on a crane.

Note. According to the author of this image, when compared to their above prediction,

1. The duration of eclipse at greatest eclipse is 2 seconds shorter.
2. The moments of contact are a couple seconds off.

Oscillations

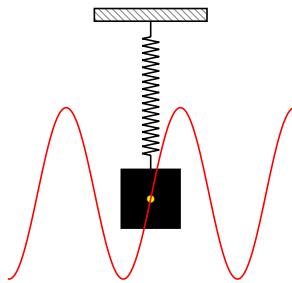


Figure 9.1: [15] Simple harmonic motion.

- *Simple harmonic motion* is defined as the motion of a body whose acceleration is directly proportional to its displacement from a fixed point (equilibrium position) and is always directed towards that fixed point.

Example 9.1

Describe the restoring force that gives rise to oscillations in a simple pendulum.

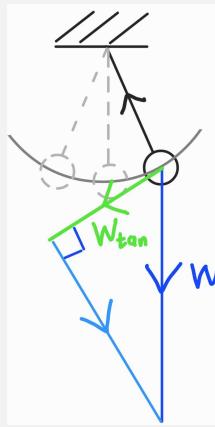


Figure 9.2: A simple pendulum.

The restoring force is the **tangential component of the weight** of the bob, which is normal to the string and points towards the equilibrium position — where the pendulum is hanging vertically.

Note. the **tension** in the string does not provide the restoring force, since there is no component of tension along the tangential direction of motion.

- A *freely oscillating system* oscillates at its own *natural frequency* without *external influences* other than the *initial impulse when displaced* from its equilibrium position, with *no dissipation* of energy.
- *Damped oscillations* are oscillations in which the amplitude diminishes with time as a result of *dissipative forces* that reduce the total energy of the oscillations.
- A *forced oscillating system* is one that is driven by an *external periodic driver*. There is

continuous energy input to the oscillating system by the driver, and the system *oscillates at the driver's frequency*.

- **Resonance** is a phenomenon in which there is *maximum transfer of energy* from the driver to the oscillating system in a *forced oscillation*. The oscillating system achieves *maximum amplitude* during resonance. For an *undamped system*, resonance occurs when the *driving frequency* is *equal* to its *natural frequency* of vibration.

$v = \pm\omega\sqrt{x_0^2 - x^2}$	$a = -\omega^2x$	Spring-Mass $\omega = \sqrt{\frac{k}{m}}$	Pendulum $\omega = \sqrt{\frac{g}{l}}$
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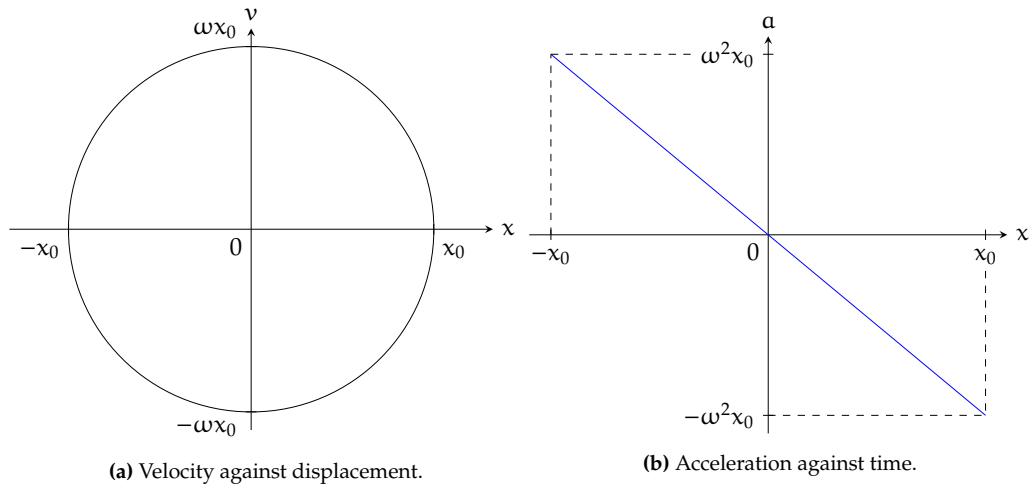


Figure 9.3: [16] Graphs for velocity and acceleration.

Variable	E_k	E_p	E_T
Time t	$\frac{1}{2}m\omega^2x_0^2\cos^2(\omega t)$	$\frac{1}{2}m\omega^2x_0^2\sin^2(\omega t)$	$\frac{1}{2}m\omega^2x_0^2$
Displacement x	$\frac{1}{2}m\omega^2(x_0^2 - x^2)$	$\frac{1}{2}m\omega^2x^2$	

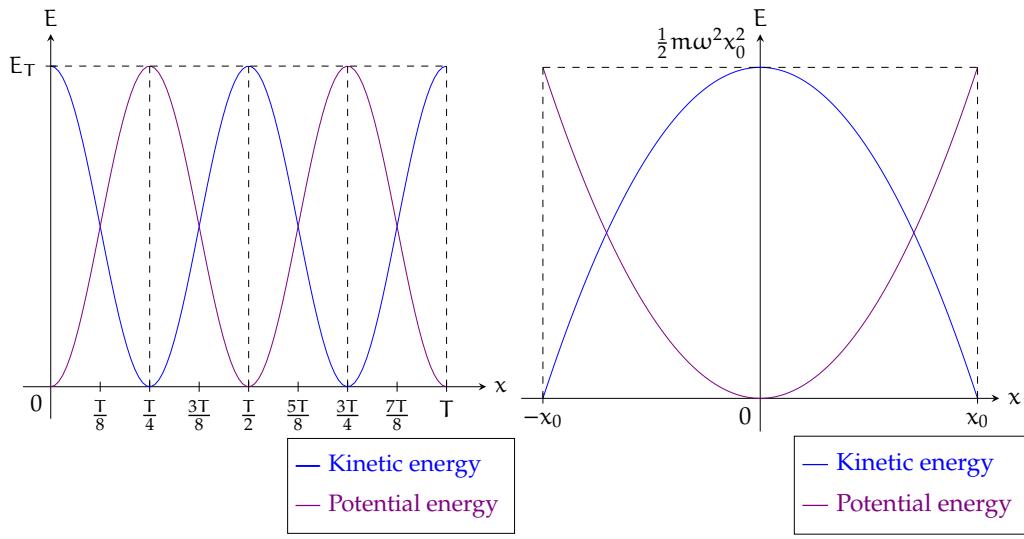
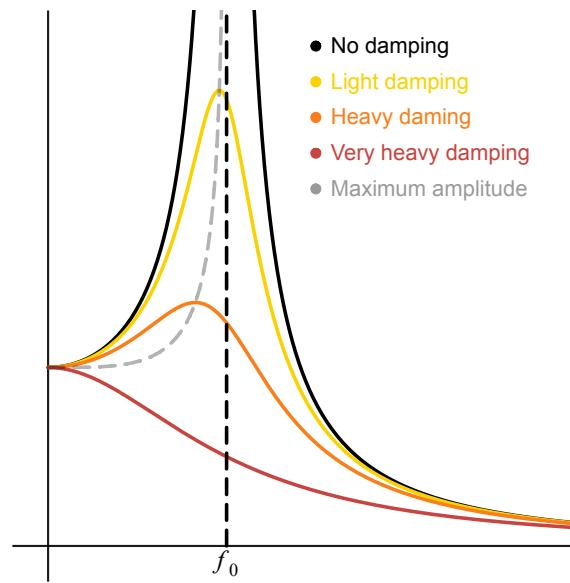


Figure 9.4: [16] Graphs for kinetic/potential energies.

- Simple pendulums and mass spring systems can be approximated to be SHM when the angle of oscillation ($\leq 20^\circ$) and oscillation amplitude are small, respectively.

	In Phase	Antiphase	Out of Phase
$\Delta\phi/\text{rad}$	0	π	nonzero

- When damping increases:
 - Amplitude at *all* frequencies decreases.
 - (Resonance) frequency at max amplitude shifts gradually to lower frequencies.
 - Peak (max amplitude) becomes flatter.

Figure 9.5: [17] The graph of amplitude against frequency, as damping increases. The resonance frequency is denoted by f_0 .

- When drawing the displacement-time graph of a underdamped oscillator, we must show the exponential decay in amplitude:

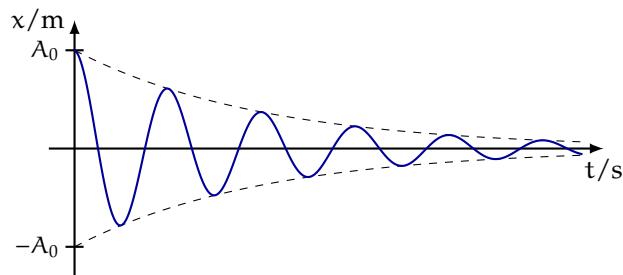


Figure 9.6: [18] The displacement-time graph of an underdamped oscillator

Example 9.2

The variation with time t of the total vertical displacement y of the mass M is shown in Figure 9.7, when the oscillations are undamped. On Figure 9.7, sketch the variation of time t of the total vertical displacement y of mass M for **three** complete, lightly damped oscillations of the mass.

- Same constant period [1]
- Gradual/continuous decrease in amplitude [1]
- Three complete cycles

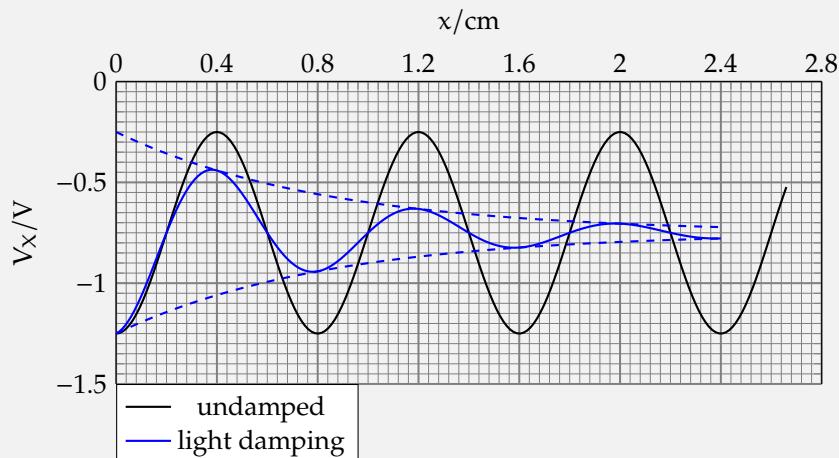


Figure 9.7: [22] Changes to the displacement-time graph going from no damping to **light damping**.

(Ideal) Isolated spring systems are in simple harmonic motion, but spring systems in general need not be in simple harmonic motion.

Example 9.3: spring systems are not always in simple harmonic motion.

Consider when a mass m is dropped onto a massless spring, that is at its equilibrium position. Assume that the mass does not rebound; the spring-mass system moves as one body after the initial collision occurs.

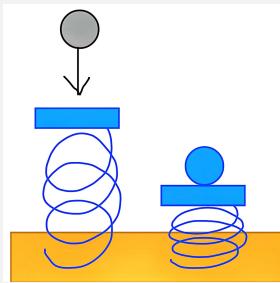


Figure 9.8: An illustration of the spring-mass system aforementioned.

Then, the net force on the spring-mass system is $W - F_{\text{spring}} = mg - kx$. Hence, the acceleration of the spring-mass system is $g - (kx/m) \not\propto x$. i.e. the system does not move in simple harmonic motion.

(Regardless, the maximum speed occurs at the point where net $F = 0$.)

Wave Motion

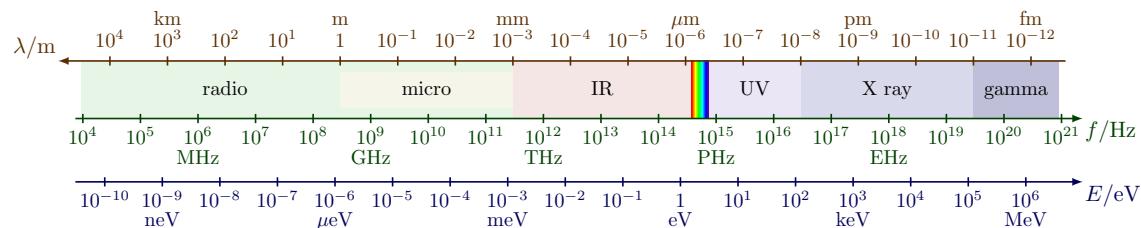


Figure 10.1: [20] The electromagnetic spectrum.

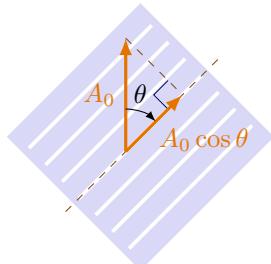
- Note. When deducing the period T of a waveform based off a graph, try to take the total time taken nT for the largest number of oscillations n possible, and then, calculate $T = nT/n$. (Avoid directly calculating the period with $n = 1$.) This is especially important if the period is small relative to the time scale given/varies slightly with time.
- A *progressive wave* is a wave in which *energy is carried* from one point to another by means of *vibrations or oscillations* within the wave. Particles within the wave are *not transported along* the wave.
- A *transverse wave* is a wave in which the direction of *oscillation* of the particles (or wave medium) in the wave is *normal* to the *direction of energy transfer* of the wave.
- A *phase* is an angle which gives a measure of the *fraction of a cycle* that has been *completed* by an oscillating particle or by a wave.
- *Intensity* of a wave is the wave energy incident per unit time per unit area *normal* to the wave.
- *Polarisation* is a process where the *oscillations* of the particles in a *transverse wave* is restricted to a *single direction*, in the plane *normal* to the direction of energy transfer.

Example 10.1

Explain why two (coherent) sources of light that are polarised in planes perpendicular to each other will not produce observable interference fringes.

The displacements s_1 and s_2 , due to each of the two waves, are in perpendicular axes. No component of s_1 can ever be in the same or opposite direction as that s_2 . Hence, the vector sum $s_1 + s_2$ will not be able to produce distinct maxima and minima; no bright or dark fringes are formed.

- A longitudinal wave *cannot be polarised* because the direction of oscillation of particles in the wave is *parallel* to the direction of energy transfer in the wave.



- Malus' Law states that the intensity of a beam of *plane polarised light* after passing through a rotatable polariser is directly proportional to the square of the cosine of the angle through which the polariser is rotated from the position that gives maximum intensity. ($I = I_{\max} \cos^2(\theta)$)

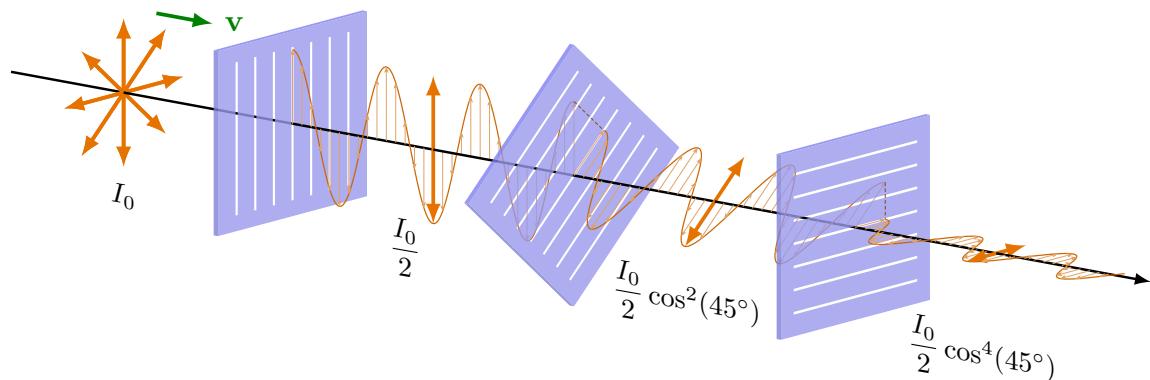


Figure 10.2: [21] An illustration of Malus' Law.

- ☆ When unpolarised light passes through a polariser, the average value of $\cos^2(\theta)$ is $\frac{1}{2}$ so $I_{\text{new}} = \frac{1}{2} I_{\text{max}}$.

	Phase Angle ϕ	$2\pi \cdot \frac{x}{\lambda}$	$2\pi \cdot \frac{t}{T}$
□	Phase Difference $\Delta\phi$	$2\pi \cdot \frac{\Delta x}{\lambda}$	$2\pi \cdot \frac{\Delta t}{T}$

	Amplitude	Intensity		
		Spherical	Circular	Plane
□	$I \propto A^2$	$I \propto \frac{1}{r^2}$	$I \propto \frac{1}{r}$	I is constant (No spreading of waves)

- When a wave travels from one medium to another, its frequency remains unchanged. The intuition: think of two ropes of different (linear) densities connected in series. It would snap if they oscillated at different frequencies.

Superposition

- *Principle of Superposition:* When two or more waves of the *same type*, meet at a point in space, the resultant displacement of the waves at that point is the *vector sum* of the *displacements* due to each wave acting independently at that point.
- *Stationary waves* are waves whose *waveforms do not advance* and where there is *no net translation of energy*. The *amplitude* of the waves varies according to *position* from zero at the nodes to a maximum at the antinodes.
- A stationary wave is formed when two *progressive* waves
 1. Having the *same frequency* and *same speed*
 2. Travel in *opposite directions* towards each other
 3. Have *similar amplitudes*
 4. Are unpolarised, or polarised along the same axis
 5. Are *superposed*

Example 11.1

State how a stationary wave is formed.

A stationary wave is formed by the superposition of two progressive waves of the same type, frequency, amplitude and speed travelling in opposite directions towards each other.

	Properties	Reflection Surface	
		Loose End ¹	Fixed End
•	Allows for Oscillations?	Yes	No
	Will Reflected Wave be Inverted (phase change of π)?	No	Yes

- Characteristics of Stationary Waves:

1. Displacement node = Pressure antinode
2. Displacement antinode = Pressure node

Properties	Stationary Wave	Progressive Wave
Energy	No net transfer of energy	Energy is transferred in the direction of propagation of the wave.
Phase	<input type="checkbox"/> Adjacent nodes: In phase <input type="checkbox"/> Adjacent segments: Antiphase. (Fig 12.1)	All points within one wavelength have different phases.
Amplitude	Varies: 0 at nodes to max at antinodes.	Same for all particles.
Wavelength	Twice the distances between adjacent nodes or adjacent antinodes.	Distance between adjacent in-phase particles.
Frequency	Same for all particles	
Nodes ² /Antinodes ³	✓	✗

¹Particles of the wave can move about freely.

²At which particles don't oscillate/amplitude = 0.

³At which particles have the largest amplitude.

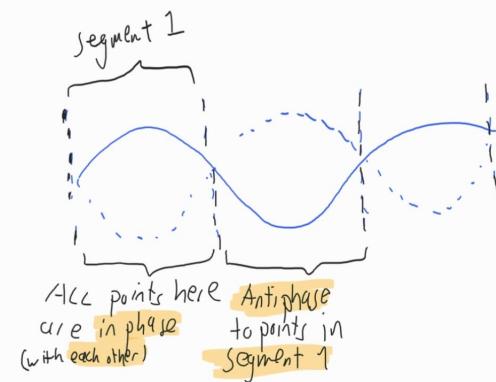


Figure 11.1: [22] Phases in stationary waves

$n \geq 0$	Modes		Diagrams	Wavelength	Frequency
	Overtone	Harmonic			
Strings/Open Pipes	nth	(n + 1)th	12.2 & 12.3	$\lambda = \frac{2L}{n+1}$	$f = (n+1) \frac{v}{2L}$
		(2n + 1)th		$\lambda = \frac{4L}{2n+1}$	$f = (2n+1) \frac{v}{4L}$

Table 11.2: The possible wavelengths and frequency of stationary waves in a string/pipe of fixed length L, and the corresponding overtone/harmonic.

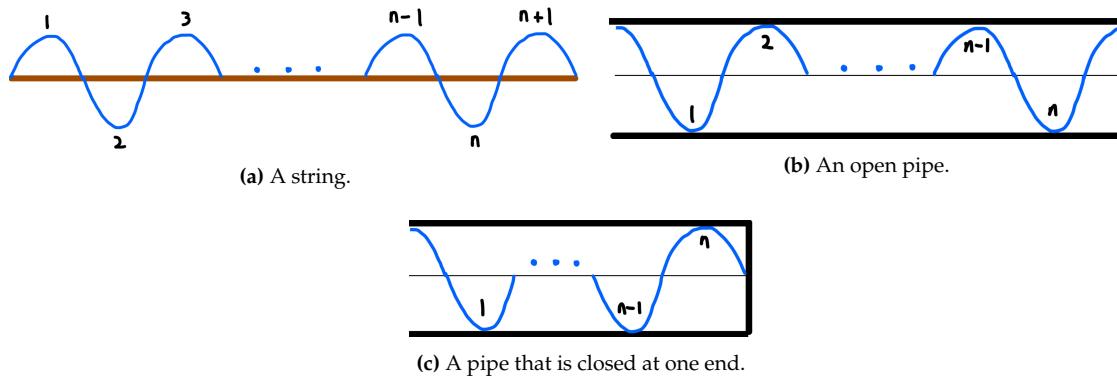


Figure 11.2: [22] How stationary waves look for a string, an open pipe, and a pipe that is closed at one end.

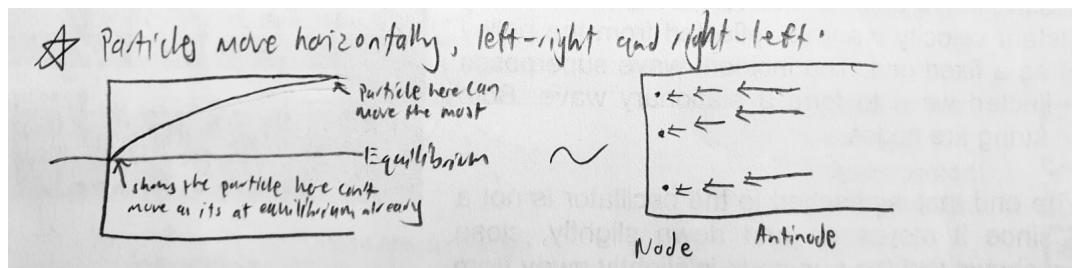
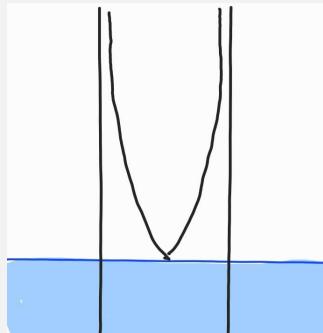


Figure 11.3: [22] Movement of particles in pipe.

Example 11.2**Figure 11.4**

Describe how the representation in Figure 11.4 illustrates a longitudinal wave. [2]

- The representation shows the maximum and minimum displacements of the particles' oscillations along the length of the tube. [1]
- It does not show the actual positions of the particles' oscillations, as the oscillations are parallel to the length of the tube and hence parallel to the direction of travel of waves, making it a longitudinal wave. [1]

- Resonance length² with pipes closed at one end

$$L = \frac{\lambda}{4} = \frac{v}{4f}.$$

Example 11.3

Explain, with reference to resonance, why the loudness of sound changes as the water level changes.

1. Natural frequency of vibration depends on length of air column.
2. When fork frequency is equal to natural frequency/odd multiple of fundamental frequency, resonance occurs. There is maximum energy transfer and maximum amplitude of vibrations, leading to maximum loudness.
3. When fork frequency is not equal to natural frequency, no resonance occurs and loudness drops.

- If a tube achieves stationary waves at fundamental frequency f , then reducing f /increasing λ will not result in stationary waves.

Example 11.4

Sound from a loudspeaker reaches a point by two path of differing lengths. Explain why the amplitude of the sound varies from a maximum to a minimum, and back to a maximum, when the frequency is gradually increased. [2]

- When the frequency is gradually increased, the wavelength gradually decreases, causing

²End correction: Actual length of vibration is $L + 2c$ for open pipes, and $L + c$ for closed pipes.

the sound wave to vary from meeting in phase, to antiphase, and back in phase. [1]

- When they are in phase, constructive interference occurs, producing a maximum. When both waves are in antiphase, destructive interference occurs, resulting in a minimum.

[1]

- *Diffraction* is the bending or spreading out of waves when they travel through a small opening or when they pass round a small obstacle.
- Large amounts of diffraction occur when the slit width is about the same as the wavelength.
- The wavelength before and after diffraction should be around the same.
- Single Slit Diffraction: Let b be the slit width, and L the slit-screen distance.

1. For all nonzero integers m , the angular positions θ of the $1 \leq m \leq m$ th order minima satisfies

$$\sin(\theta) = \frac{m\lambda}{b}.$$

2. Distance y_1 of the first minima from either side of the central bright fringe is

$$y_1 = \frac{\lambda L}{b}.$$

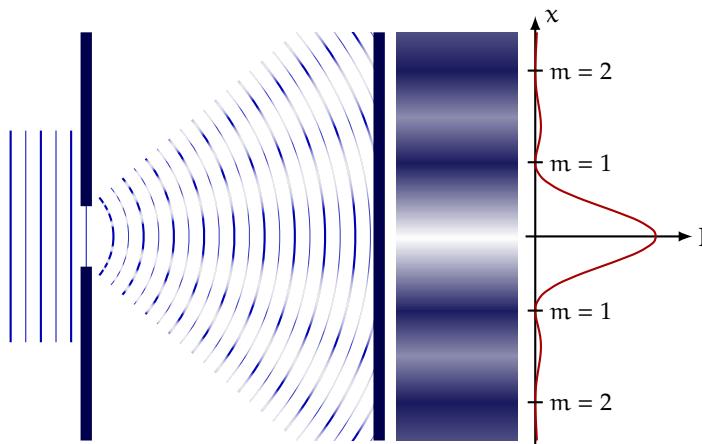


Figure 11.5: [23] Single-slit diffraction.

- Circular aperture: $\theta \approx \frac{\lambda}{b}$.
- *Rayleigh's Criterion* is the minimum separation between two objects in order to be distinguished as two distinct objects:

$$\theta \approx \frac{\lambda}{b}.$$

- *Rayleigh's Criterion* states that the images of two point sources are considered just resolved when the central maximum of the diffraction pattern of one image coincides with the first order minimum of the diffraction pattern of the other.
- Sources are *coherent* if they have a constant phase difference with respect to time.
- *Interference* is the superposing of two or more waves to produce regions of maxima and minima in space, according to the Principle of Superposition.

- Conditions for an *observable* interference pattern:
 1. The waves must *overlap* to produce regions of maxima and minima.
 2. The *sources* must be *coherent*.
 3. The waves must have approximately the *same amplitude*.
 4. The waves, if transverse, must be *unpolarised* or have the *same plane of polarisation*.

- For $n \in \mathbb{Z}_0^+$, representing the n th order max/min, we have

Sources' Phase Difference	Path Difference	
	Constructive Interference (maxima)	Destructive Interference (minima)
In phase	$\Delta = n\lambda$	$\Delta = (n + 1/2)\lambda$
Antiphase	$\Delta = (n + 1/2)\lambda$	$\Delta = n\lambda$

- We always need to take the path difference starting from the actual source itself, even when the source travels through two slits onto a screen, for instance.
- Double-slits: For³ a (*center-to-center*) slit separation a and slit-screen distance D , the fringe separation (between two adjacent minima, or two adjacent maxima) is

$$x = \frac{\lambda D}{a}.$$

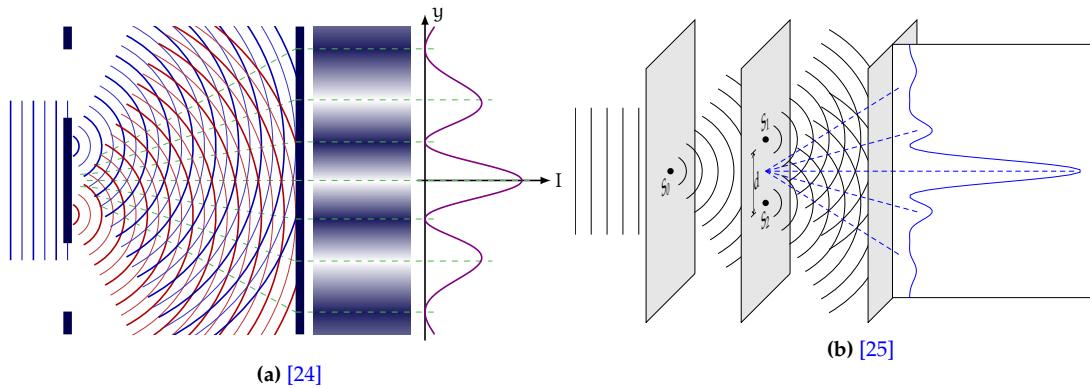


Figure 11.6: Double-slit diffraction.

- Why does the central bright fringe of a double-slit diffraction pattern have a higher intensity than other maxima?
 - Light passing through each slit undergoes single-slit diffraction, where the intensity is maximum at the central bright fringe.
 - The central bright fringe formed due to double-slit diffraction has a higher intensity because it was formed from the superposition of two single slit diffraction patterns coming from each slit.

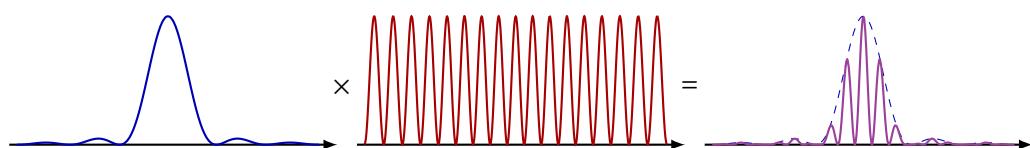


Figure 11.7: [23] The combined effects of diffraction and interference.

³Typical values: $a \approx 0.5\text{mm}$, $D \approx 1\text{m}$, and $\lambda \approx 600\text{nm}$. In any case, using the equation requires $a \ll D$.

- Diffraction grating: For a slit-separation d and $n \in \mathbb{Z}_0^+$, the angular positions for the n th order maxima satisfies

$$d \sin(\theta_n) = n\lambda.$$

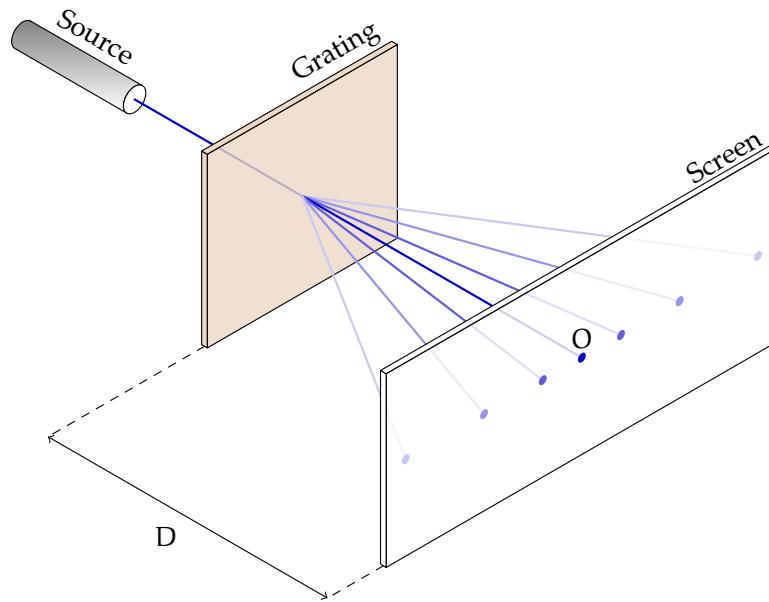


Figure 11.8: [26] Diffraction grating.

- Check answer: For visible light, $400\text{nm} \leq \lambda \leq 700\text{nm}$.
- Total number of bright regions/maximas = $2n + 1$, where n is the highest order maxima possible.
- Total number of dark regions/minimas = $2m$, where m is the highest order minima possible.
- When slit width is reduced, intensity is reduced.
- To calculate *resultant intensity*, first take the sum of the amplitudes and use proportionality ($I \propto A^2$).
- Every other line means half of the lines are covered.

Example 11.5

Describe and explain the appearance of the central fringe if the light is now replaced with white light.

- The central bright fringe is generally white.
- The zeroth order fringes of all the wavelengths coincide at the center where the path difference from the two slits is zero for all wavelengths. The combined central fringes remains white.
- The sides of the central fringes are likely more reddish.
- This is because the wider red fringe extends beyond the narrower blue fringe.

Example 11.6

A beam of white light is shone onto a diffraction grating with slit separation d . The light source consists of wavelength between l m to u m. Determine whether the first and second spectra overlap.

This means that we want to see whether the outer end of the first order diffracted light will touch the inner end of the second order diffracted light. So, we calculate

$$\begin{aligned} \text{For } u \text{ m} \\ d \sin(\theta_1) = u \\ \theta_1 = \sin^{-1}(u/d) \end{aligned}$$

$$\begin{aligned} \text{For } l \text{ m} \\ d \sin(\theta_2) = l \\ \theta_2 = \sin^{-1}(l/d) \end{aligned}$$

If $\theta_1 > \theta_2$, the spectra overlap. Otherwise, when $\theta_2 > \theta_1$, the spectra do not overlap.

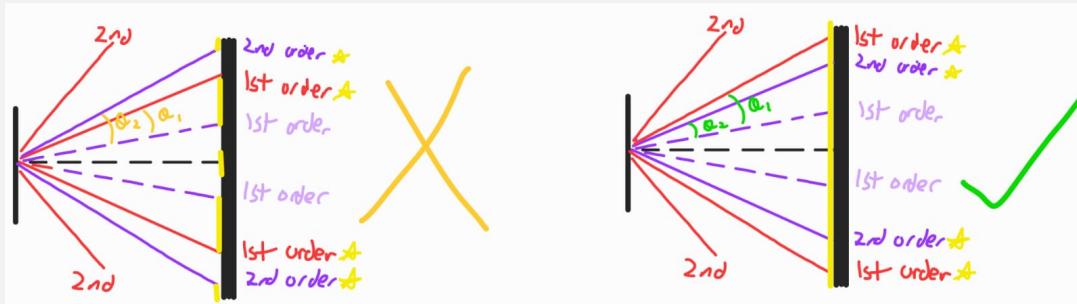


Figure 11.9: [22] Spectra overlap.

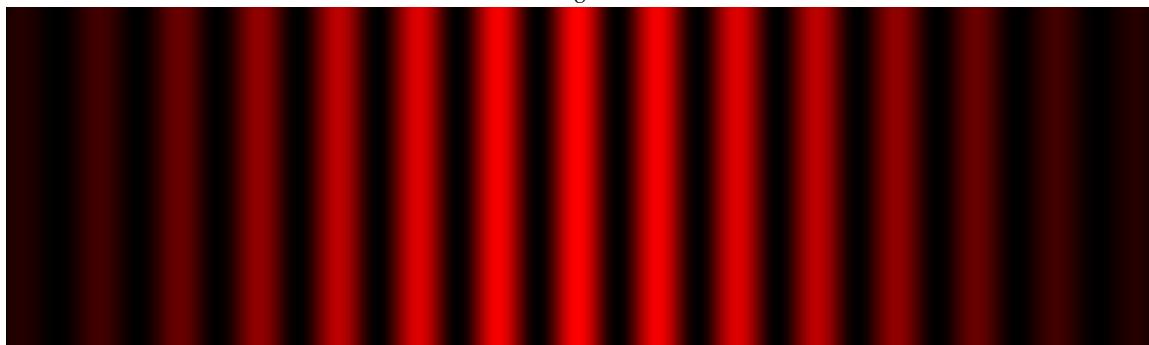
Example 11.7: Rotating a diffraction grating.

The diffraction grating is now rotated through a small angle so that the incident light is no longer normal to the grating. The resultant diffraction pattern is no longer symmetrical. Suggest qualitatively, any other effects on the appearance of the diffraction pattern.

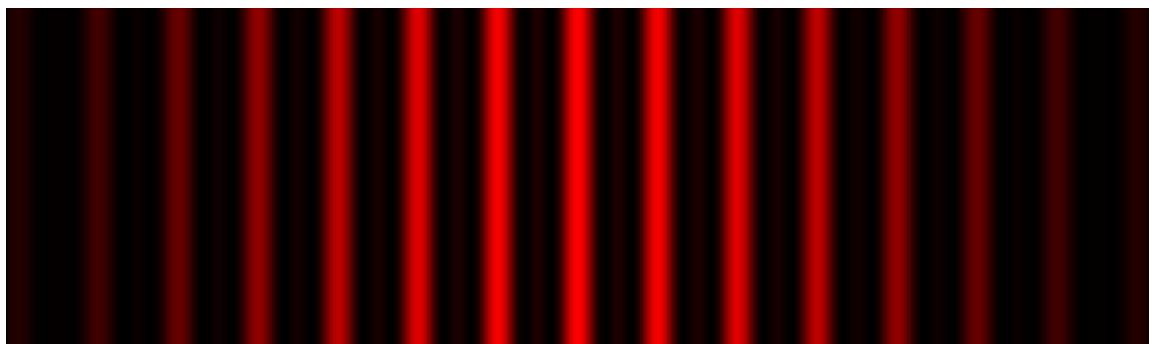
- The zeroth order will not be along the normal to the plane of the grating.
- The number of maxima observable will not be symmetrical about the zeroth order. There will be more orders, and hence more diffraction fringes, observed on the side of the zeroth order where the rotation is proceeding than on the other side.



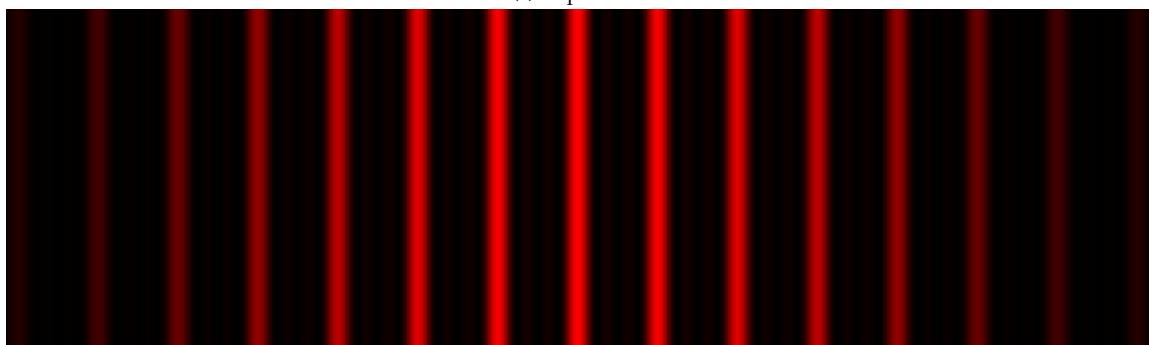
(a) Single slit.



(b) Double slits.



(c) Triple slits.



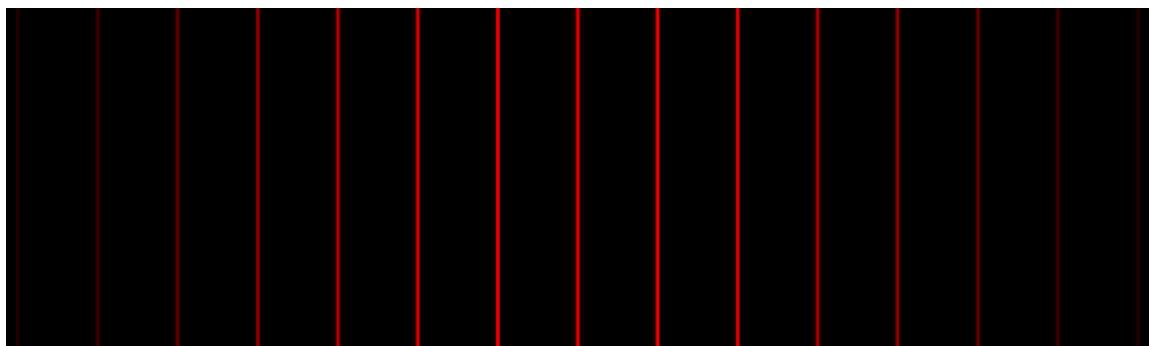
(d) Quadruple slits.



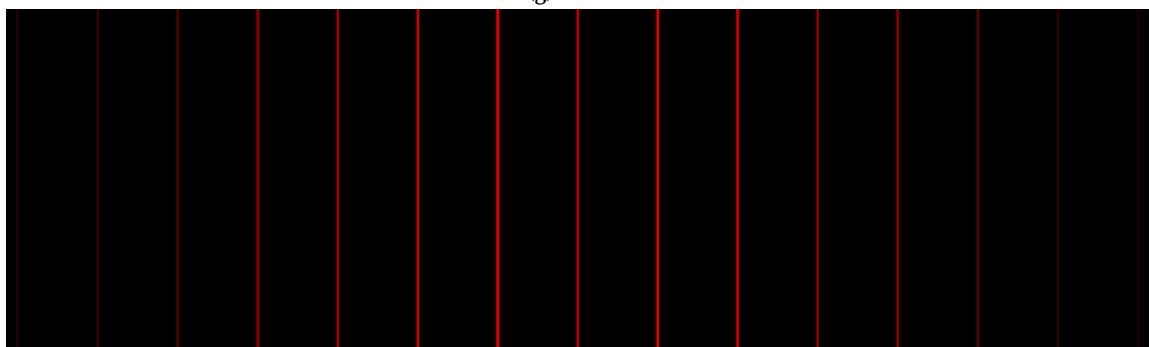
(e) Quintuple slits.



(f) 10 slits.



(g) 20 slits.



(h) 30 slits.

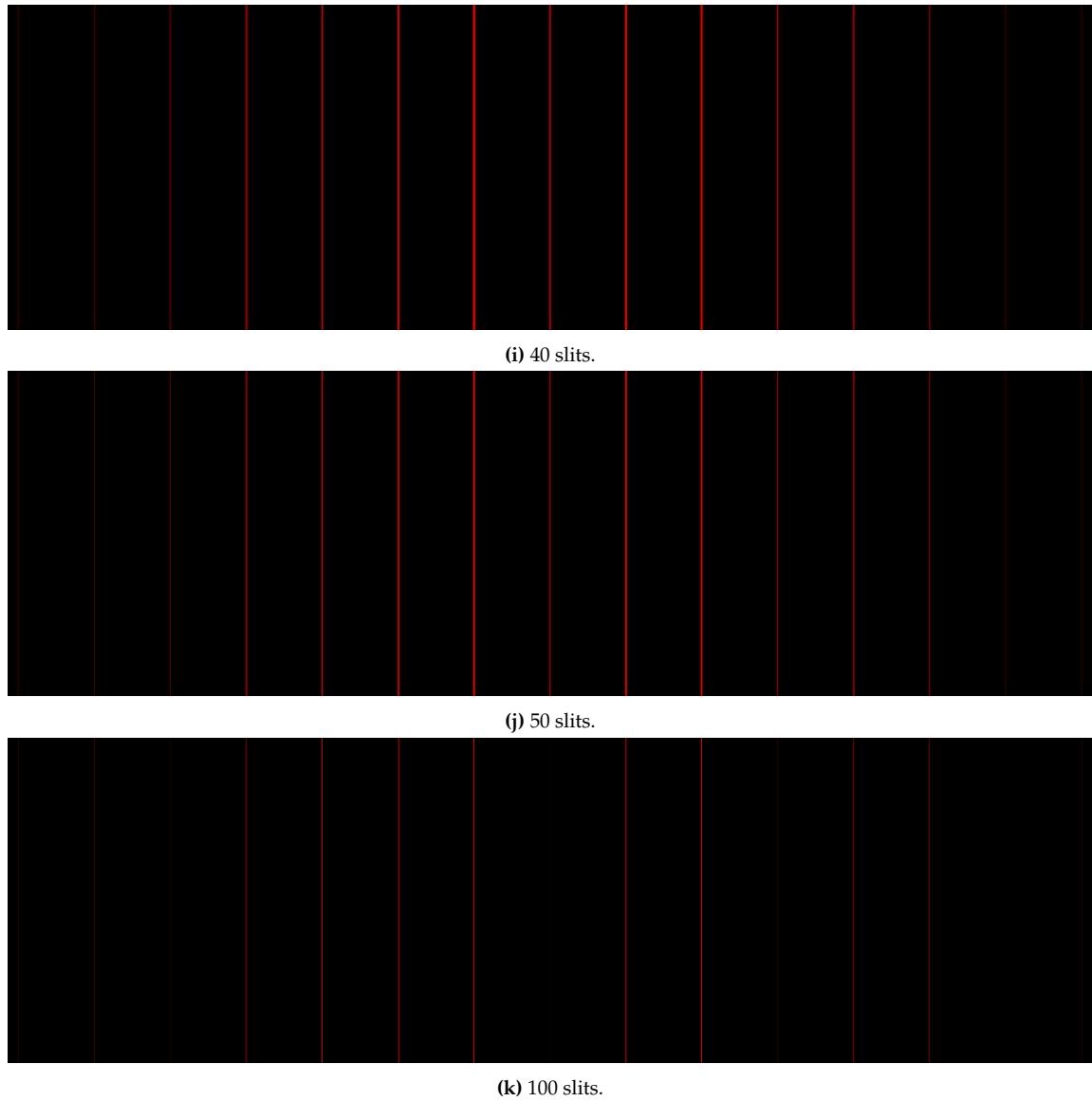


Figure 11.10: [27] Interference patterns generated by pgf-interference

Currents of Electricity

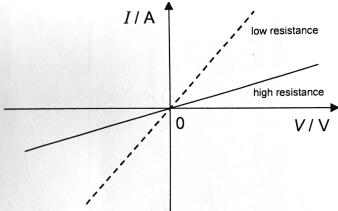
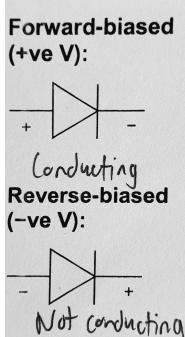
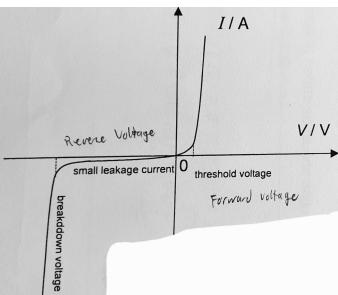
- The *number density* n is defined as the number of particles per unit volume.
- The *drift velocity* v is the *average* velocity at which *charge carriers* move through a *conductor* when there is *electric current in the conductor*.
- Deriving the equation $I = nAvq$:
 1. Assume that the *current is constant*. Then, $I = \frac{Q}{t}$. [n]
 2. Assume that there are N charge carriers passing through a cross-sectional area A in time t , and that *each* of them carries an *identical amount of charge* q . Then, the *total charge* that passing through A in time t is $Q = Nq$. [N, A(t), Q(t)]
 3. Assume that the *number density* n of charge carriers is *uniform*, and let V be the volume covered by the current in time t . Thus,
$$N = nV. \quad [n, V(t)]$$
 4. Furthermore, since the current travels at some velocity v ,

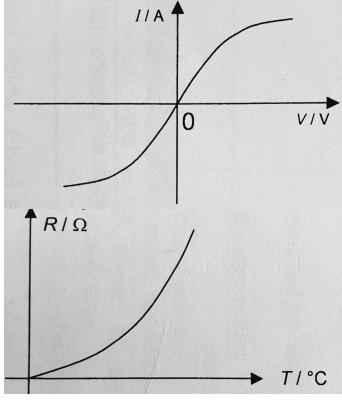
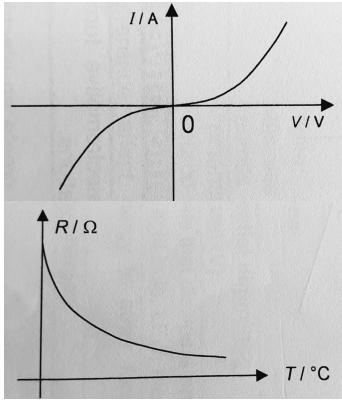
$$V = A\Delta x = Avt. \quad [v]$$

- 5. Therefore,
- $$I = \frac{Q}{t} = \frac{Nq}{t} = \frac{nVq}{t} = \frac{nAvtq}{t} = nAvq.$$
- Elementary charge $e = 1.6 \times 10^{-19} C$ (Charge of an electron/proton).
 - The *potential difference* between *two points* of a circuit is defined as the amount of electrical energy *converted* to other forms of energy *per unit charge* moved *between* the two points.
 - *Ohm's Law* states that the *current* flowing in a *conductor* is *directly proportional* to the *potential difference across it* under *constant physical conditions*.
 - *Resistance* is defined as the *ratio* of the *potential difference* across the conductor to the *current* flowing through it.
 - *Resistivity* ρ is the *proportionality constant* between the *dimensions of a specimen of a material* and its *resistance* such that

$$R = \frac{\rho L}{A}.$$

- Electrical Components

Types of Conductors	Changes to Resistance	Reason
Metallic conductor at constant temperature	<ul style="list-style-type: none"> At <i>constant temperature</i> this is an <i>Ohmic conductor</i>. Has <i>constant resistance</i>. Ratios of V/I is constant. 	<ul style="list-style-type: none"> At <i>constant temperature</i>, the <i>number of free electrons</i> and the <i>rate of atomic vibration</i> is constant. A resistor at a different constant temperature will have a different resistance, and hence, V/I ratio.
Semiconductor Diode	 <ul style="list-style-type: none"> Forward-biased ($+ve\ V$): Conducting Reverse-biased ($-ve\ V$): Not conducting 	<ul style="list-style-type: none"> Forward-biased: <i>Resistance decreases when p.d. increases</i>. In fact, if the forward-biased p.d. goes past its <i>threshold voltage</i>, <i>resistance becomes very low</i>. Reverse-biased: <i>Very high resistance</i>. But, there will be a <i>small leakage current</i>. If the reverse-biased p.d. is so high that it exceeds the <i>breakdown voltage</i>, the diode will <i>break down</i> and <i>conduct electricity</i>. <ul style="list-style-type: none"> When connected in <i>forward bias</i>, the circuit's <i>electric field set up</i> allows for available <i>charge carriers</i> to flow through, allowing it to conduct with <i>low resistance</i>. When connected in <i>reverse bias</i>, the circuit's <i>electric field set up</i> creates a <i>widened 'depletion region'</i> in the diode which <i>impedes charge carriers</i> from flowing through the region, creating a <i>large resistance</i>.

<p>Filament Lamp</p>	<ul style="list-style-type: none"> - When p.d. increases, current increases, with decreasing I-V ratio. - The resistance of a metallic conductor increases with an increase in temperature. 	<ul style="list-style-type: none"> - As current increases, power dissipated increases since $P = I^2R$. Heat is generated so equilibrium temperature rises—as electrons drift through the metal, they collide with the metal lattice and transfers energy to it. - The lattice ions vibrate more vigorously. This hinders the flow of 'charge carriers'. Therefore, resistance is increased. - Ohmic conductors hence do not obey Ohm's Law at high voltages/currents.
<p>Negative Temperature Coefficient (NTC) Thermistor</p>	<ul style="list-style-type: none"> - When p.d. increases, current through the thermistor also increases, with increasing I-V ratio. - The resistance of a thermistor decreases with an increase in temperature (This is the meaning of NTC). 	<ul style="list-style-type: none"> - As current increases, power dissipated increases. More heat is generated, leading to a rise in equilibrium temperature. - Thus, the mean kinetic energy of the electrons and lattice ions increases. So, <ol style="list-style-type: none"> 1. The bonded electrons break free from bonds, increasing the number of 'mobile charge carriers'. Therefore, resistance decreases. 2. The lattice ions vibrate more vigorously, hindering the flow of 'mobile charge carriers'. Thence, resistance increases. - The first effect is much more significant than the second. So, there is a net decrease in resistance.

The images above come from [19].

Example 12.1

Explain, in *microscopic terms*, why the resistance of a wire increases as the current through it increases. [2]

- As current increases, the rate at which electrons collide into the metal lattice increases; temperature increases. [1]
- As temperature increases, lattice ions vibrate more vigorously, hindering the flow of electrons; resistance increases. [1]

- The *electromotive force* (e.m.f) of a source is defined as the amount of *energy converted from non-electrical forms of energy to electrical energy per unit charge* as the *charge passes through a complete circuit*.
- Internal resistance: the p.d. across a component is given by $V = E - Ir$.
- Power dissipated $P = IV$.
- Power delivered is *maximum* when $R = r$, such that

$$P_{\max} = \frac{E^2}{4r}$$

Note. Unless specified by the question, try to avoid assuming that the internal resistance r of the internal resistance of a cell is constant, if possible.

- Efficiency of the source
 - Increases when external load/resistance increases.
 - Is halved when $R = r$.
- Note:* Maximum power \neq maximum efficiency.

Electric Fields

- Coulomb's Law states that the *magnitude* of the *electric force* between two *point charges* is directly proportional to the product of the charges, and inversely proportional to the square of their separation.
- The constant ϵ_0 is the permittivity of *free space* (ϵ_0 is applicable only in air or a vacuum).
- Sign of F_E :

F_E	Direction
Positive	Repulsive
Negative	Attractive

- The sign of the electric force does *not* represent the *direction* of the electric force. It only informs us whether the force is attractive or repulsive. So, when calculating the *resultant* electric force, we need to account for the direction ourselves.
- Comparison between E-fields and G-fields:

Sim/Diff	E-field	G-field
S	For both Coulomb's Law and Newton's Law of Gravitation, r is the <i>center to center separation</i> of the objects.	
S	By Newton's Third Law, the <i>forces</i> that masses and charges exert on each other is <i>equal in magnitude and opposite in direction</i> .	
D	Electric Forces can be <i>attractive or repulsive</i> .	Gravitational forces are <i>always attractive</i> .

- An *electric field* is a *region in space* where a *charge experiences an electric force*.

- How to draw electric field lines:

- Lines cannot intersect one another.
- Lines must begin from positive charges and end on negative charges.
- Arrows show the *direction of force* exerted on a positive test charge.
- The *greater* the electric field strength, the *closer together* field lines are drawn.
- Lines leave/end on *conducting surfaces* at *right angles*

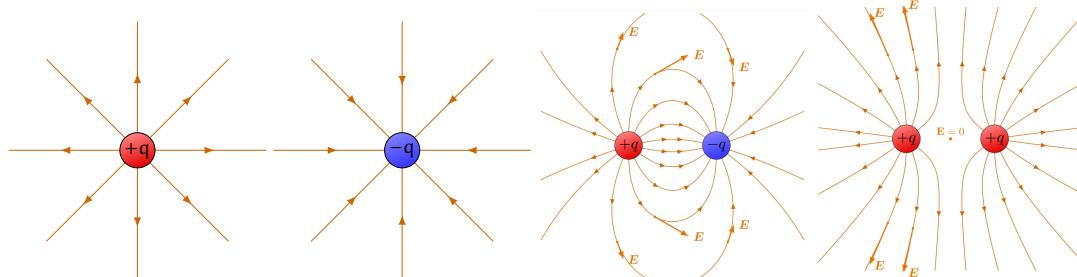


Figure 13.1: [28], [29] Electric field lines of point charges and two interacting charges.

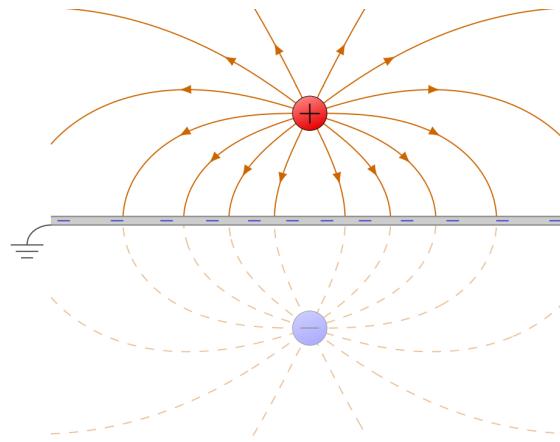


Figure 13.2: [30] Interaction of a point charge with a charged plate.

- The *electric field strength* at a point is defined as the *electric force per unit positive charge* acting on a small stationary test charge placed at that point.

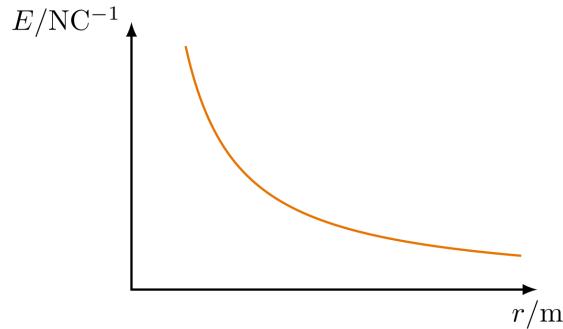


Figure 13.3: [31] Electric field strength of a positive point charge

- The *electric potential energy* of a point charge in an electric field is defined as the *work done by an external agent* in bringing the point charge from infinity to that point (without any net change in kinetic energy).
- The *electric potential* at a point in an electric field is defined as the *work done per unit positive charge*, by an external agent, in bringing a *small* test charge from infinity to that point (without any net change in kinetic energy).
- Let U be the electric potential energy, and V the electric potential. Then,

$$U = qV \quad \text{and} \quad \Delta U = q\Delta V.$$

$$\begin{aligned} U &= \frac{Qq}{4\pi\epsilon_0 r} \xrightarrow{-\frac{d}{dr}} F_E = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \\ &\downarrow \frac{1}{q} \qquad \qquad \downarrow \frac{1}{q} \\ V &= \frac{Q}{4\pi\epsilon_0 r} \xrightarrow{-\frac{d}{dr}} E = \frac{Q}{4\pi\epsilon_0 r^2} \end{aligned}$$

- A “singly charged” object is one which has charge $e = 1.60 \cdot 10^{-19}$.
- The *electric potential* at a point X due to a *system* of point charges q_i is the algebraic sum of the electric potential V_i due to each individual charge q_i which is distance r_i away from X . That is,

$$V = \sum_i V_i = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i}.$$

- The *potential energy* of a system of charges q_i is the work done to assemble it. This is the sum of energies U_{ij} needed to bring each charge q_i to the charges q_j (for $i > j$) already present. In other words letting r_{ij} be the distance of q_i from q_j , we have

$$U = \sum_{i>j} U_{ij} = \sum_{i>j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}.$$

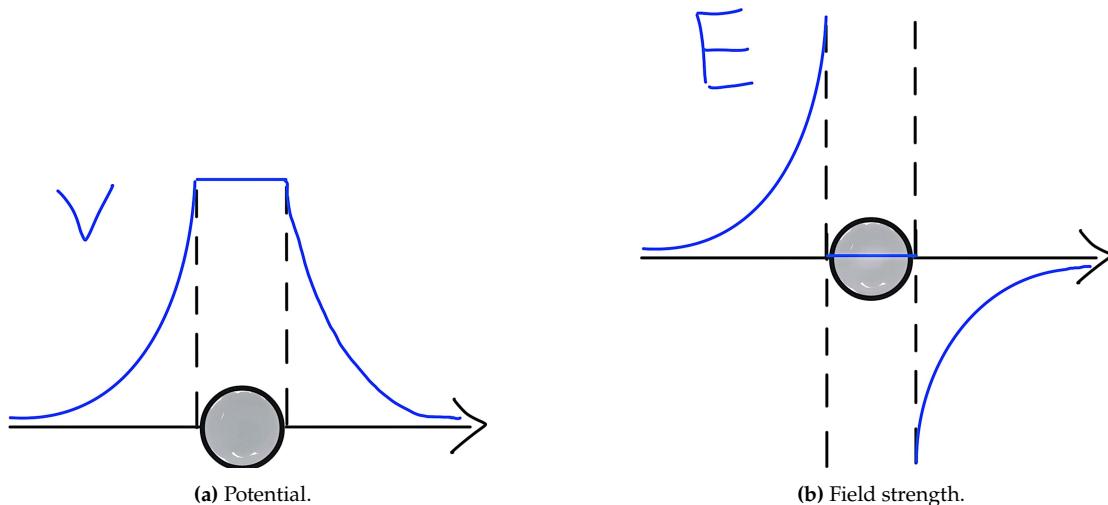


Figure 13.4: The electric potential and electric field strength around an isolated conductor.

- Charge distribution on a conducting sphere:
 - Excess charges are forced to the surface of the conductor until the electric field inside the conductor is zero.
 - Outside the conductor, the electric field is the same as that of an isolated point charge equal to the excess charge.
- Properties of conductors in *electrostatic equilibrium*:
 - The electric field is zero inside a conductor (regardless of shape).
 - So, the entire conductor is at the same potential.
 - Just outside the conductor, the e-field lines are perpendicular to its surface, starting and ending on charges on the surface.
 - Excess charges resides exclusively on the surfaces of a conductor.
- Electric field strength between two charged parallel plates is uniform everywhere between the plates, except at both ends of the plates.
- Also, charges between the plates experience uniform acceleration.

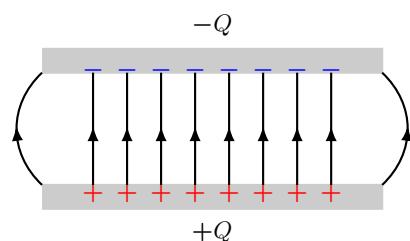


Figure 13.5: [32] Electric field lines between parallel plates.

- Magnitude of electric field strength between the plates:

$$E = \frac{dV}{dr} = \frac{\Delta V}{d}$$

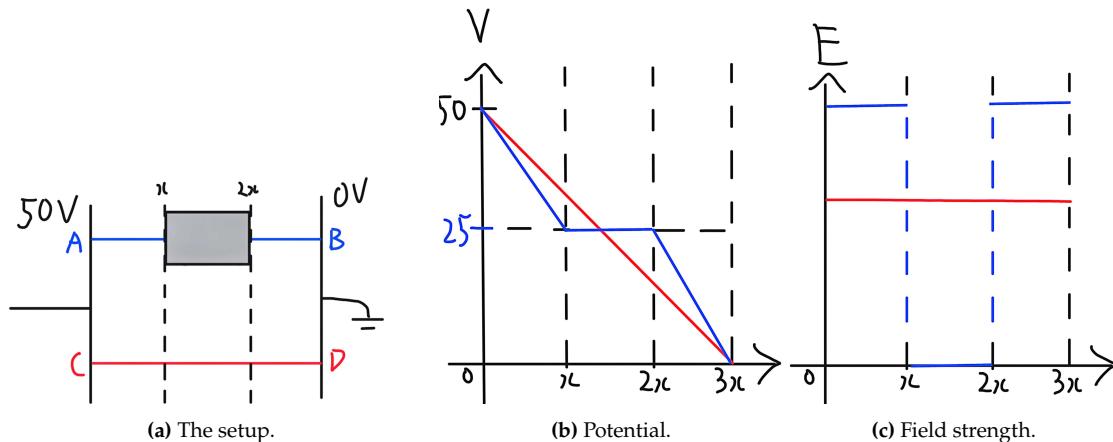


Figure 13.6: The electric potential and electric field strength between two parallel plates containing a conductor.

Example 13.1

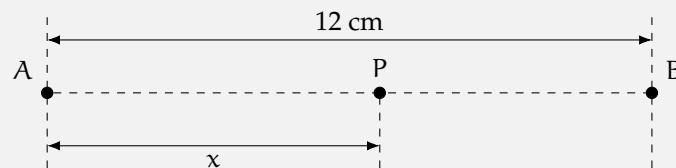


Figure 13.7: [22] Two point charges, A and B, are separated in a vacuum by a distance of 12cm.

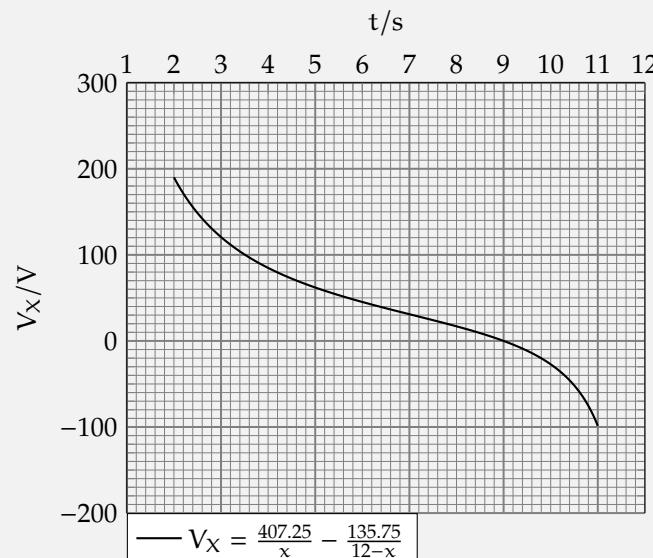


Figure 13.8: [22] The variation with distance x of the electric potential V_x at point P.

Explain whether the charges have the same, or opposite, signs.

- The potential due to two charges at a point is the scalar sum of the potentials due to each charge at the same point.^a
- Since potential is positive near to A and negative close near to B, charges A and B are positive and negative, respectively.
- i.e. the charges have opposite signs.

^aThis point can be skipped, if the question is one mark and time is running short.

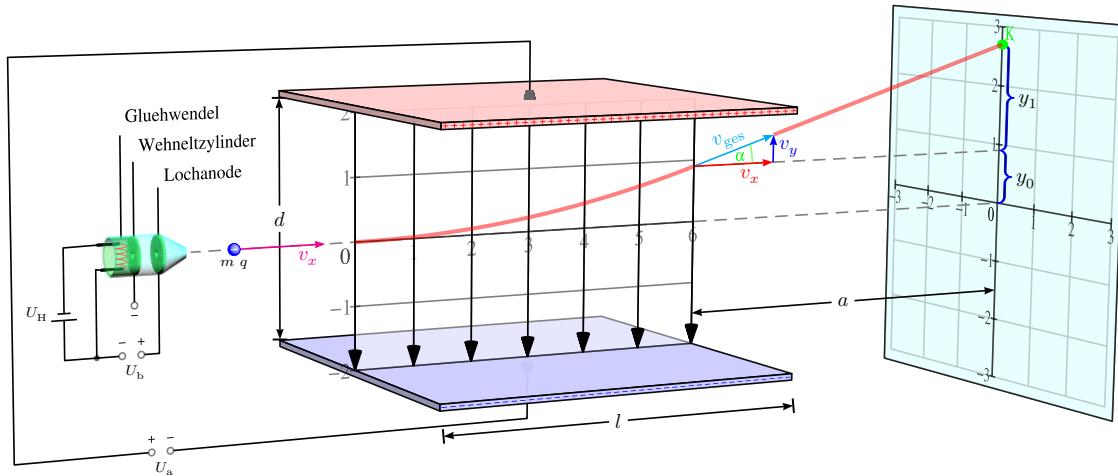


Figure 13.9: [33] Deflection of an electron.

D.C. Circuits

- An explanation for how energy is transferred in a circuit: [Stack Exchange](#).

Property	Series	Parallel
Current	$I_1 = I_2 = \dots = I_n$	$I_i = \frac{E}{R_i}$
Resistance	$R_{\text{effective}} = R_1 + R_2 + \dots + R_n$	$\frac{1}{R_{\text{effective}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$
Voltage	$V_i = \frac{R_i}{R_T} \cdot E$	$E = V_1 = V_2 = \dots = V_n.$

- For n identical resistors in parallel, which are each of resistance R , we have $R_{\text{effective}} = R/n$. Furthermore, the effective resistance is at most the resistance of the smallest resistor, i.e. $R_{\text{effective}} \leq R_i$ for each i . In fact, the inequality is strict when the number of resistors in parallel $n \geq 2$.
- To resolve tricky systems of resistors, use the fact that electric potential is constant along a wire.

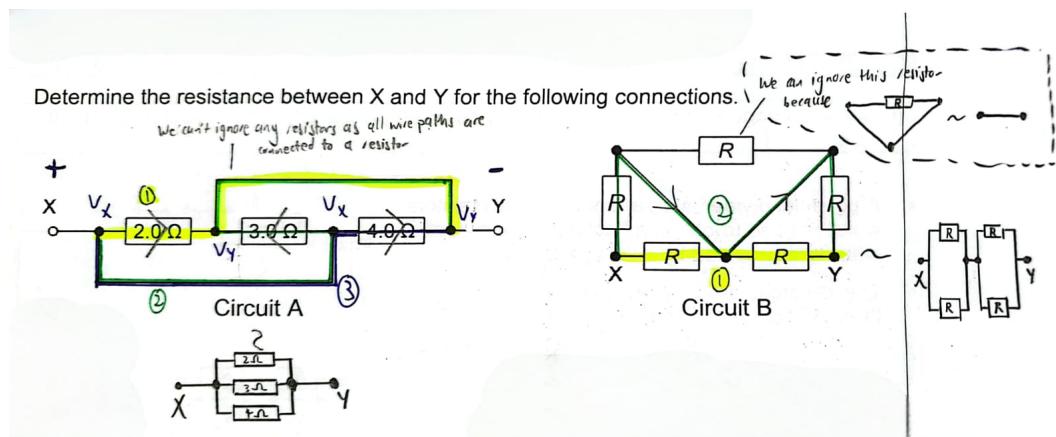


Figure 14.1: [19] Some tricky circuits.

- Potentiometer:
 - E.m.f. of driver cell is more than that of the unknown cell, E .
 - The direction of charge flow for the primary and secondary circuits are opposite. i.e. the positive/negative terminals should 'point' towards each other.
 - The potential difference V across length L of a resistance wire is directly proportional to L .
 - Consider the following circuit. When the galvanometer reads zero, the e.m.f. of the unknown cell is E_{AJ} , where

$$\frac{E_{AJ}}{E} = \frac{L_{AJ}}{L_{AB}} = \frac{R_{AJ}}{R_{AB}}.$$

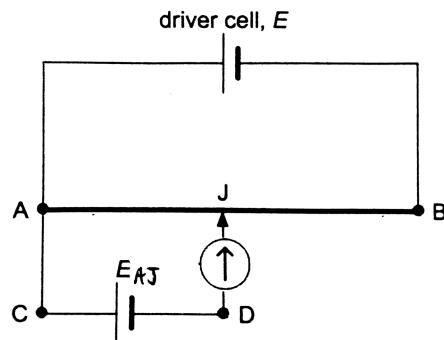


Figure 14.2: [19] An illustration of a potentiometer.

- In the circuit below, the internal resistance r satisfies

$$\frac{L_{AJ_2}}{L_{AJ_1}} = \frac{R}{R+r}.$$

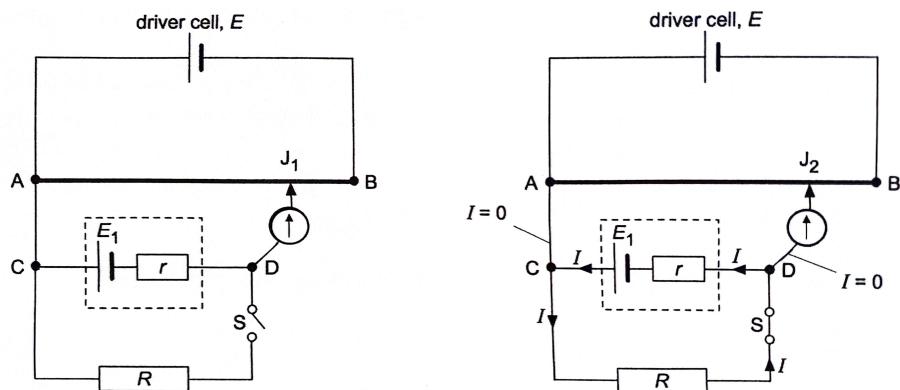


Figure 14.3: [19] Some more illustrations of potentiometers.

For the diagram to the left, why $V_{CD} = V_{AJ_1}$ is easily seen from the diagram below.

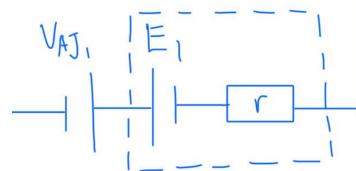


Figure 14.4: [22] Part CD of the circuit.

Electromagnetism

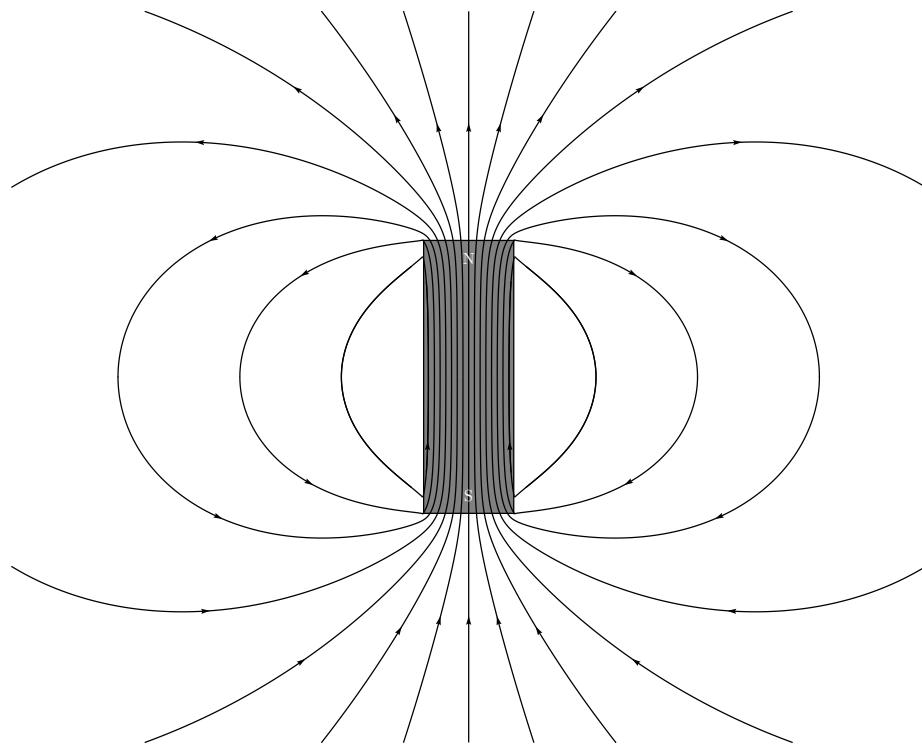


Figure 15.1: [34] Magnetic field produced by a bar magnet.

- A *magnetic field* is a *region of space* where a magnetic pole, moving charged particle or current-carrying conductor will *experience a magnetic force*.
- Normally, a magnetic field points from north-to-south. However, *inside* a magnet, the magnetic field points south-to-north.
- A compass points in the same direction as the B-field.

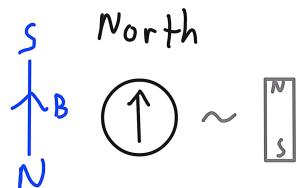


Figure 15.2: [22] Compass points in the same direction as the B-field

- *Magnetic flux density* is defined as the *force per unit current per unit length* acting on an *infinitely long current-carrying conductor* placed *perpendicularly* to the magnetic field.
- Dots and crosses as indicators of direction:

	Into the page
	Out of the page

- Left and right hand rules:

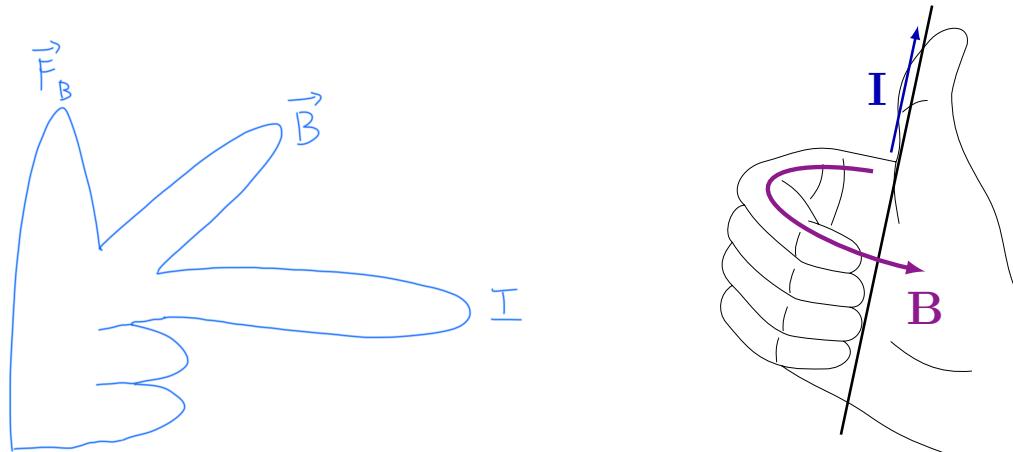


Figure 15.3: [36] Left and right hand rules.

Example 15.1

Mark with a letter X the position where the resultant magnetic flux density is zero. Explain your reasoning [2]

- (Explain) To obtain zero magnetic flux density, the direction of the magnetic flux density due to current must be opposite that of the Earth.
- (Direction) Using the right hand grip rule, this is to the east of the wire.
- (Distance) To have the same magnitude as the Earth's magnetic flux density, the position should be _____ (e.g. nearer than 19cm).

- At any point some perpendicular distance d from the center of an *infinitely long straight* current-carrying conductor, the magnitude of the magnetic flux is given by

$$B = \frac{\mu_0 I}{2\pi d}$$

Here, $\mu_0 = 4\pi \cdot 10^{-7}$ is the permeability of free space.

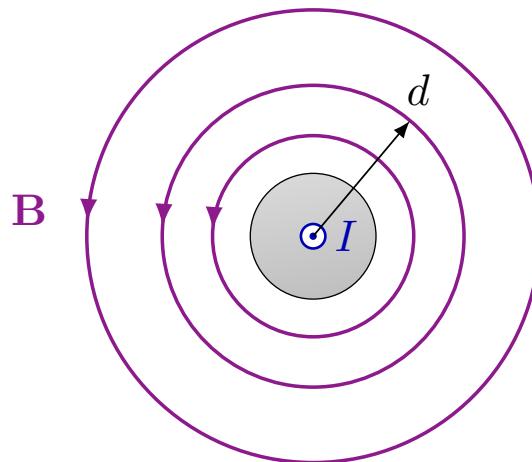


Figure 15.4: [35] Current in a wire.

- At the center of a flat circular coil with N turns, radius r , and current I flowing through it, the magnitude of the magnetic flux density is given by

$$B = \frac{\mu_0 NI}{2r}.$$

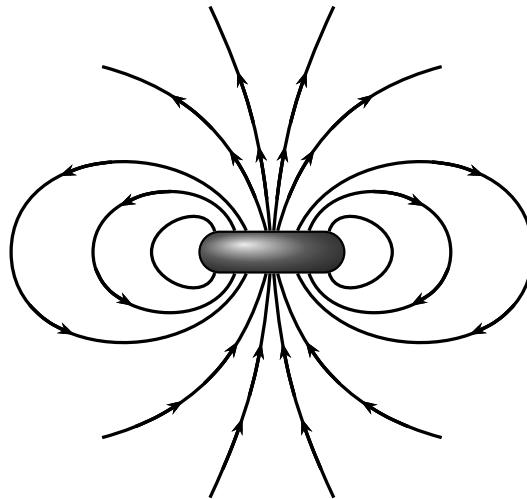


Figure 15.5: [37] Current in a flat circular coil.

- Suppose we have an ideal (having infinite length) solenoid of $n = N/L$ number of turns per unit length, which has a current I flowing through it. Then, the magnitude of the uniform magnetic flux density at its center is given by

$$B = \mu_0 n I.$$

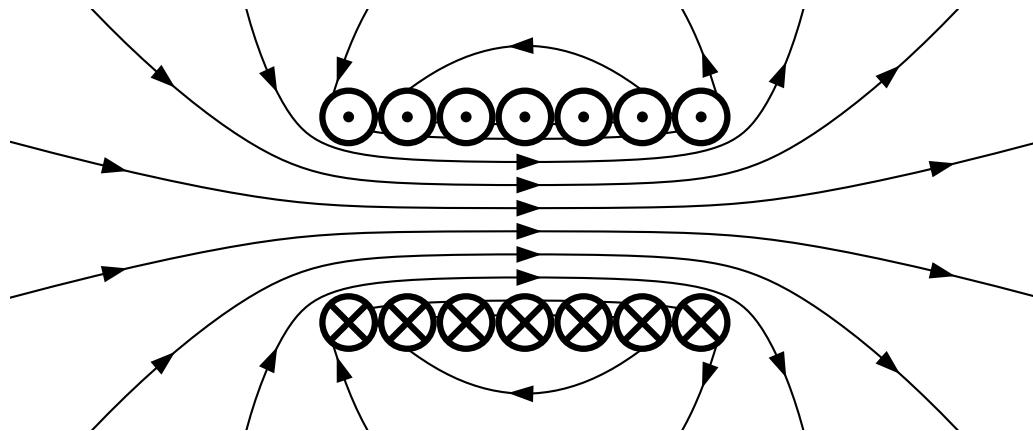


Figure 15.6: [38] Current in a solenoid and the magnetic field produced.

- A set of coils can be considered to be a solenoid when the radius of said coils is negligible compared to their length.
- The addition of a ferrous core, of permeability μ , into a solenoid increases the magnetic flux density there, which is given by

$$B = \mu n I.$$

- Say we have a straight current-carrying conductor of length l with current I_{\perp} flowing perpendicular to the magnetic field. Then for any point on that conductor that experiences a magnetic flux of density B ,

$$F_B = BI_{\perp}l.$$

•

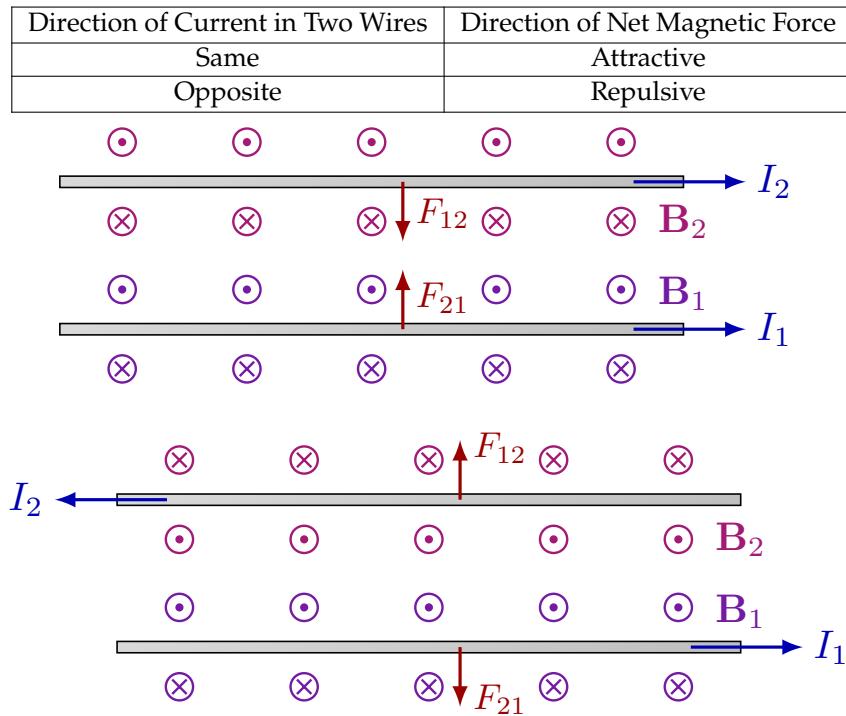


Figure 15.7: [39] Current carrying conductors in parallel.

- A charge q moving at speed v_{\perp} perpendicular to the magnetic field, of flux density B , experiences a magnetic force of magnitude

$$F_B = qv_{\perp}B.$$

- Notice that the charged particle above travels in circular motion, with radius and period

$$r = \frac{mv}{qB} \quad \text{and} \quad T = \frac{2\pi m}{qB}.$$

- Velocity selector. Suppose we have a velocity selector with a uniform magnetic field, of flux density B out of (or into) the paper. Further assume there is an electric field of strength E rightwards (or leftwards) in the selector. This is illustrated in Fig 15.7. Then, for a charged particle perpendicular to both fields to pass (undeflected) through the slits, $F_E = F_B$. This simplifies to

$$v = \frac{E}{B}.$$

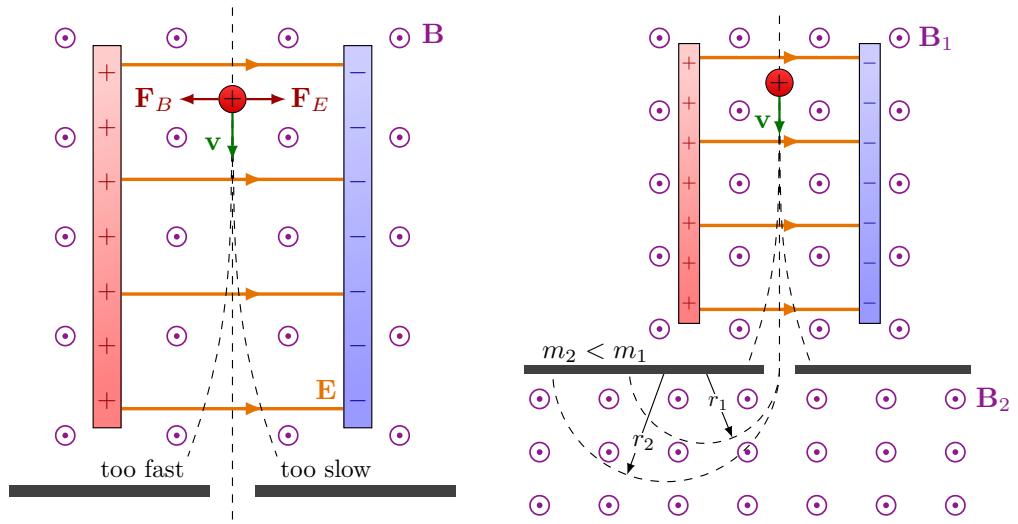


Figure 15.8: [40] An illustration of a velocity selector.

Some nice images:

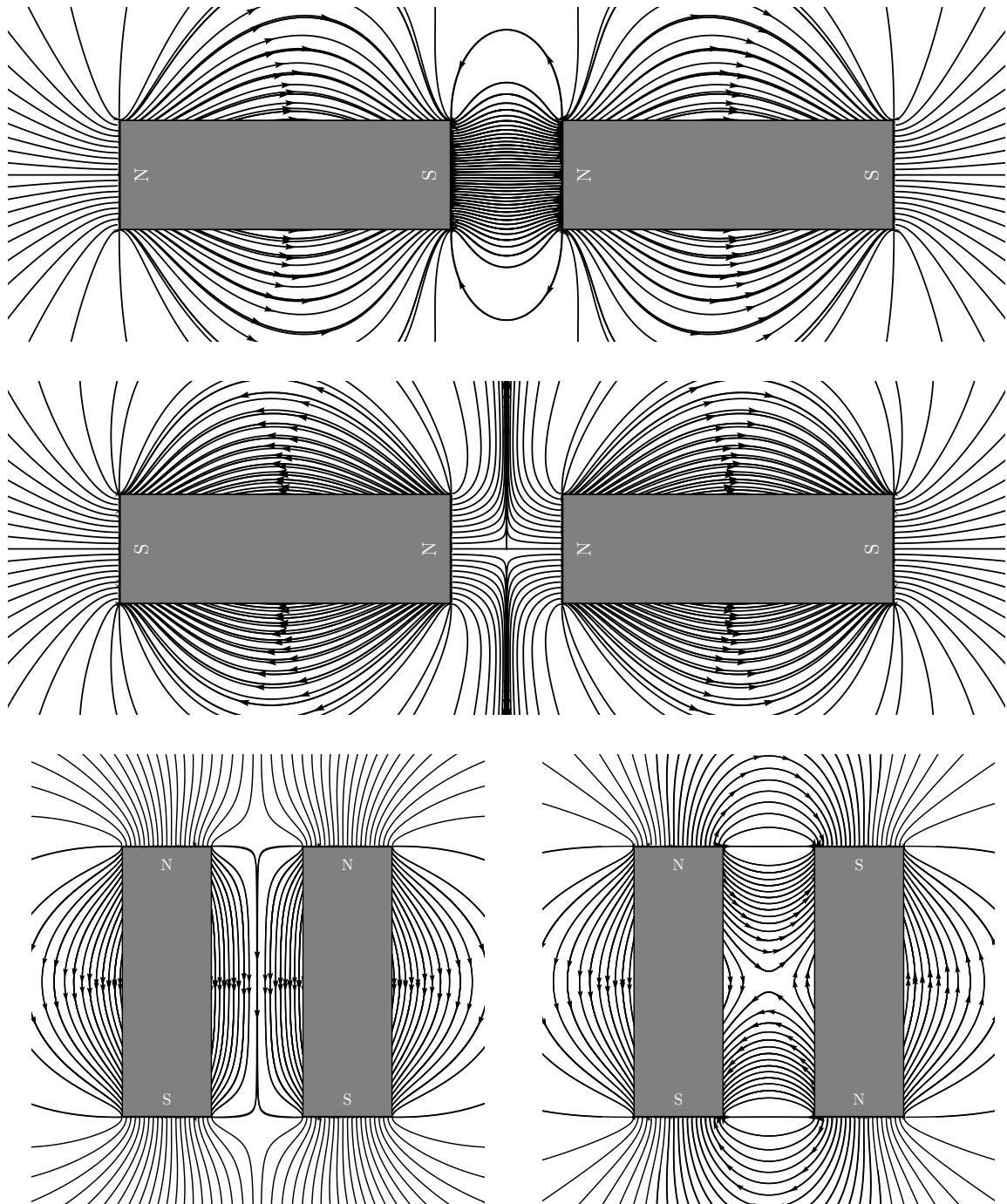


Figure 15.9: [34] Magnetic fields produced by two bar magnets.

Electromagnetic Induction

- Magnetic flux is the product of an area and the component of magnetic flux density perpendicular to that area.
- In other words, let A be an area in a uniform magnetic field, of flux density B_{\perp} perpendicular to A . Then, the magnetic flux Φ through A is

$$\Phi = B_{\perp}A.$$

- The area A here is a *vector*! When we flip it through π radians, the magnetic flux through it is now $-\Phi = -B_{\perp}A$.
- One weber is the *magnitude* of magnetic flux through an *area* of $1m^2$ when a *magnetic field* of $1T$ acts *perpendicularly into* the area.
- Magnetic flux linkage through a coil is defined as the *product* of the *number of turns* of the coil and the magnetic flux through each turn of the coil.
- The magnetic flux linkage through a coil of N turns is hence

$$N\Phi = NB_{\perp}A.$$

- Faraday's Law of Electromagnetic Induction states that the e.m.f. induced is directly proportional to the rate of change of magnetic flux linkage.
- Lenz's Law states that the direction of the induced e.m.f. is such that it may produce an effect that opposes the change causing it.
- Lenz's Law is a consequence of conservation of energy. Mechanical work done to enable the change in magnetic flux linkage is converted into electrical energy.
- Faraday's and Lenz's Laws imply that the e.m.f generated is

$$E = -(N\Phi)' = -(NB_{\perp}A)'.$$

(The negative sign indicates that the induced e.m.f. opposes the change in magnetic flux linkage.)

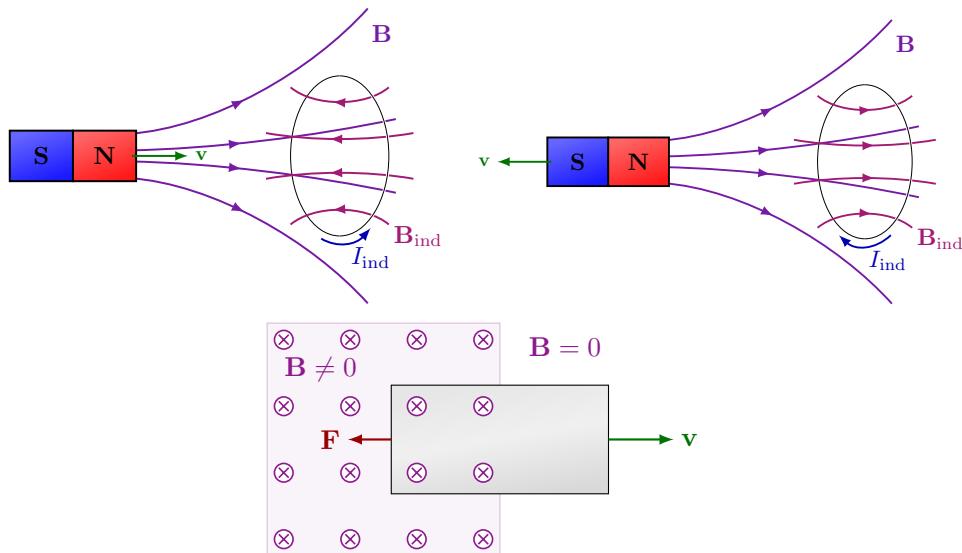


Figure 16.1: [41] Examples of Lenz's Law in action.

- Motional e.m.f.: Suppose we have a circuit as shown in the bottom left of the following figure. Then, by Faraday's Law,

$$|E| = \Phi' = BA' = Blv.$$

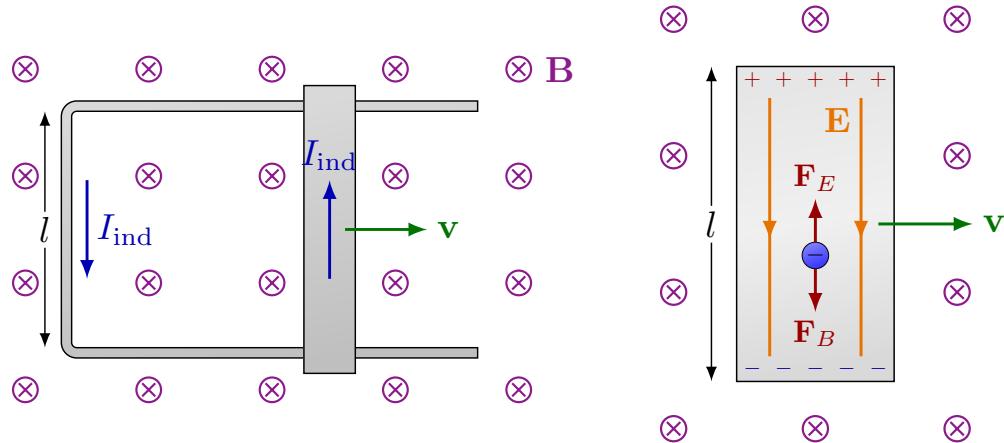


Figure 16.2: [41] Example of motional emf.

- Conventions for polarity and when to use them.

Energy conversion	Function in a circuit	Convention for polarity
Electrical to others	Resistor/Wire	Higher potential = relatively positive
Others to electrical	Battery	Lower potential = relatively positive

- Faraday's Paradox. Consider a metal disc with area A rotating at frequency f in a uniform magnetic field, of flux density B . Then, let O be the centre of the disk, and pick any point P along the circumference of the disk. We see that A' , the area swept by OP per unit time, is $\pi r^2 f$. So, by Faraday's Law,

$$E = B\pi r^2 f.$$

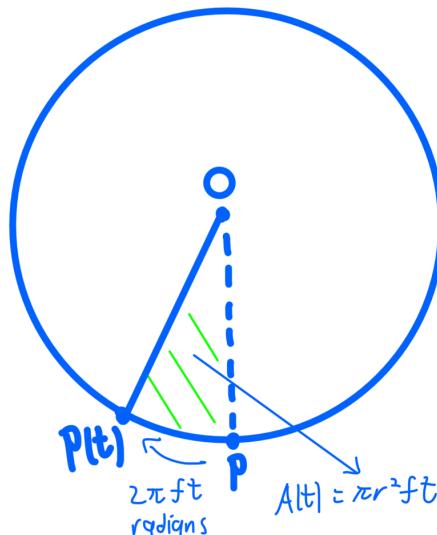


Figure 16.3: [22] An illustration of Faraday's Paradox.

Example 16.1

Initially, the switch is open. The switch is then closed for a short time and then reopened. Use Faraday's law of electromagnetic induction to explain what happens to the reading on the voltmeter. [5]

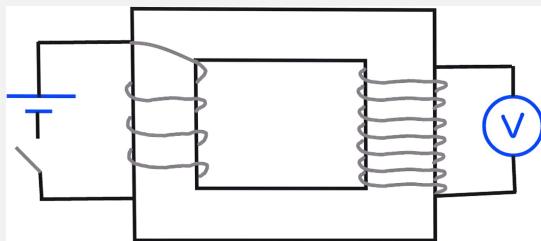


Figure 16.4: [22]

- When the switch is closed for a short time, the circuit is closed and current flows through the primary coil. This causes the magnetic flux density in the primary coil (and hence the core) to increase. [1]
 - The magnetic field is directed to the secondary coil by the iron core. [1]
 - This causes the magnetic flux linkage through the secondary coil to increase. Hence, by Faraday's Law, e.m.f. is induced in the secondary coil; the voltmeter registers a nonzero reading. [1]
 - When the current in the primary coil reaches a constant value, the magnetic flux linkage through the secondary coil is constant. Thus, no e.m.f. is induced in the secondary coil. Accordingly, the voltmeter reading goes back to zero after a short time. [1]
 - When the switch is re-opened, the decrease in current and magnetic flux density in the primary coil causes the magnetic flux linkage through the secondary coil to decrease; e.m.f. is again induced momentarily in the secondary coil, but in the opposite direction, before returning to zero [1].
- If asked to explain why an alternating current/e.m.f. is produced in the secondary coil, consider writing what follows.

Since the rate of change of magnetic flux linkage alternates due to the alternating (sinusoidal) primary current, the induced e.m.f. in the secondary coil also alternates with the same frequency as the primary current.

Example 16.2

Assume that the transformer in the setup below is ideal.

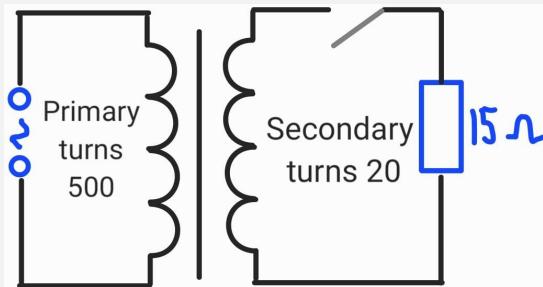


Figure 16.5: [22]

Use Faraday's law of electromagnetic induction to explain why the e.m.f. is less than the potential difference across the primary coil.

- The soft-iron core provides a closed magnetic circuit which ensures that both the primary and secondary coils experience the same changing magnetic flux Φ .
- Since the secondary coil has a lower number of turns N , by Faraday's Law, induced e.m.f

$$E = \frac{d(N\Phi)}{dt} = N \frac{d\Phi}{dt}$$

is smaller for the secondary coil.

Alternating Current

- An *alternating current (a.c.) source* creates an electrical *current* that varies in magnitude *and direction periodically with time*, as opposed to a *direct current (d.c.) source* where the *direction of the current stays constant*.
- Alternating current *must change direction*. For instance, the following two functions, namely $I = \sin(t) + 1$ and $I = \cos(t) + 1$, are both (varying) direct currents.

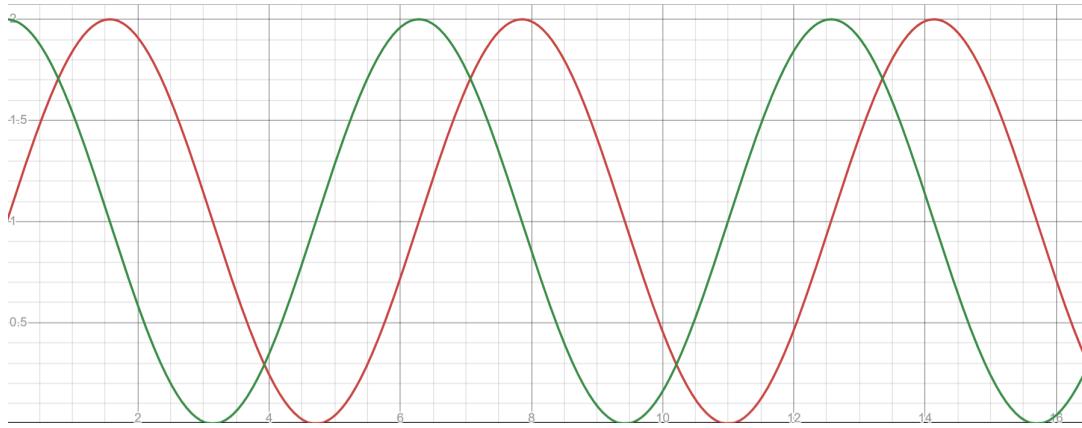


Figure 17.1: [42] Direct currents.

- A $kV f \text{ Hz}$ a.c. supply is one which has $V_{\text{rms}} = k$ and frequency f .
 - The *root-mean-square (r.m.s.)* value I_{rms} (or V_{rms}) of an *alternating current* (or alternating voltage) is the value of a *steady direct current* (or direct voltage) that would produce the *same average power* in a given resistor.
 - We denote the mean value of a quantity x by $\langle x \rangle$. So, $x_{\text{rms}} = \sqrt{\langle x^2 \rangle}$, and it also holds that
- $$\langle P \rangle = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}.$$
- Steps to determine the rms value: square → mean → root.
 - Square all values of I (or V).
 - Find the mean square value $\langle I^2 \rangle$ (or $\langle V^2 \rangle$). This is just the area under the graph of I^2 (or V^2) against t in one period.
 - Square root the value obtained above.
 - Note that for a *full wave sinusoidal* alternating current (which can be assumed unless otherwise stated),

$$\langle P \rangle = \frac{1}{2} P_0, \quad V_{\text{rms}} = \frac{1}{\sqrt{2}} V_0, \quad \text{and} \quad I_{\text{rms}} = \frac{1}{\sqrt{2}} I_0.$$

- Explain why in the selection of a suitable fuse, the r.m.s. current is considered instead of the peak current. [2]
 - The fuse will only be blown if its rating is *lower* than the *r.m.s. current*. [1]

- The peak current only occurs for a very short time. Hence, the *heat dissipated* by the peak current is *insufficient* to blow the fuse. [1]
 - An *ideal transformer* is 100% efficient with no power loss, such that the input and output power are equal.
 - Let N_P , V_P , and I_P be the number of turns, rms voltage, and rms current, respectively, in the primary winding. (The peak voltage/current works too, for a sinusoidal wave form, by proportionality.) Similarly define N_S , V_S , and I_S for the secondary winding. Then, the turns ratio is defined as
- $$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}.$$
- (To quickly determine which side carries a greater voltage, we can use the principle of ‘more turns, more voltage’).
- A step up transformer is one that increases voltage, i.e. $V_S > V_P$ (or $N_S > N_P$).
 - A step down transformer is one that decreases voltage, i.e. $V_P > V_S$ (or $N_P > N_S$).

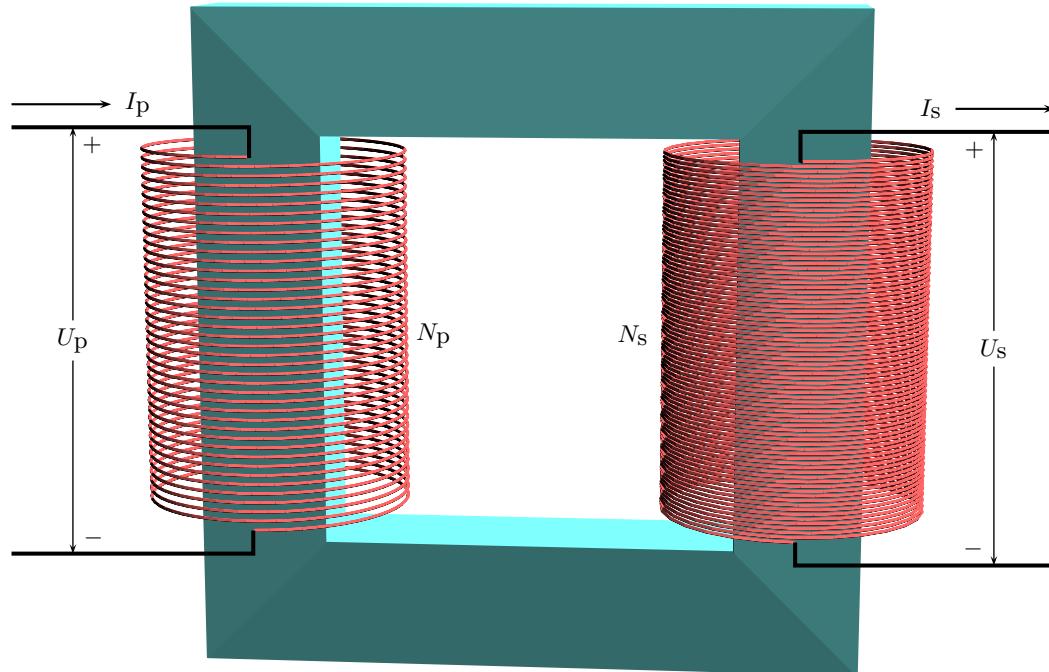


Figure 17.2: [43] A step-up transformer with $N_P = 40$ and $N_S = 80$.

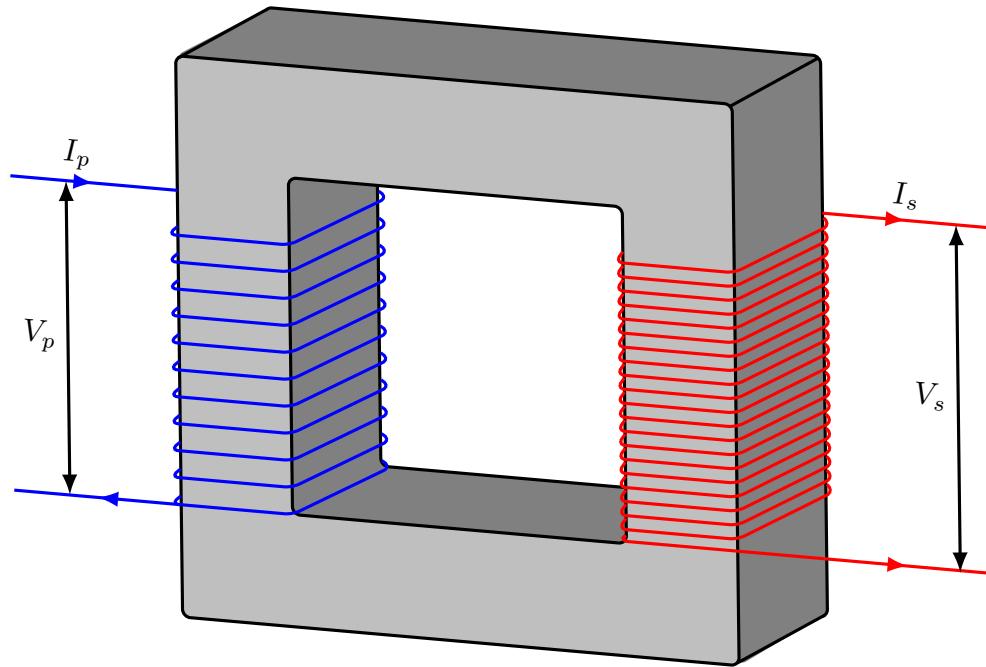


Figure 17.3: [44] Another step-up transformer.

- Transformers cannot be used for *constant* direct current. Since there is no change in voltage/current, no emf will be (constantly) induced in the secondary coil.
- When the direct current varies, however, there will be an *alternating* current induced.
- Energy loss in a transformer.
 1. Winding resistance. Current flowing through the windings causes *resistive heating* of the conductors.
 2. Hysteresis losses. Each time the *magnetic field is reversed*, a small amount of energy is lost due to hysteresis within the core.
 3. Magnetostriction. Magnetic flux in a ferromagnetic core causes it to physically expand and contract slightly with each cycle of the magnetic field. This produces a *buzzing sound* and can cause losses due to *frictional heating*.
 4. Mechanical losses. The alternating magnetic field causes fluctuating forces between the primary and secondary windings. These cause vibrations within nearby metalwork, adding to the *buzzing noise* and *consuming a small amount of power*.

Example 17.1

State two reasons why soft iron, which is easily magnetised and demagnetised, is used as the core.

- Soft iron has low hysteresis losses; energy dissipated each cycle of magnetisation is reduced.
- Soft iron's quick magnetisation and demagnetisation allows for a high frequency alternating current.
- Quick magnetisation means that there are no residual effects if the transformer is not in use.

Example 17.2

Explain how the lamination of the core reduces energy losses.

- The changing magnetic flux linkage induces eddy currents.
- The laminated sheets block the path of eddy currents, so smaller loops of eddy currents form within each laminate.
- Hence, thermal energy losses are reduced.

- Power is typically transmitted at high voltages to minimise the power lost during transmission, which is proportional to $1/V^2$ (r.m.s.) as seen below.

Example 17.3

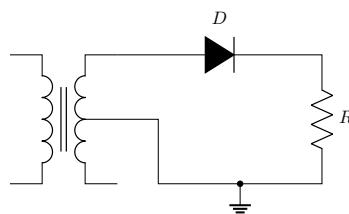
A power plant sends an average of $\langle P \rangle$ watts of electrical power to a town by electrical cables, that have a total resistance of R ohms. What is the power lost in the cables when the power was transmitted at V volts?

✓ Power lost in the cables = $I^2R = \left(\frac{\langle P \rangle}{V}\right)^2 R$.

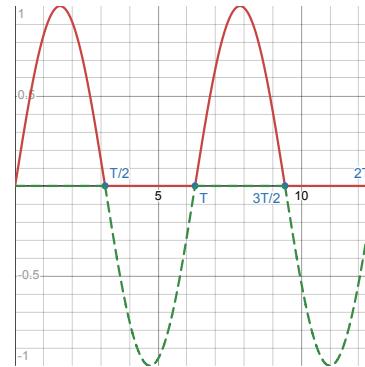
✗ Power lost in the cables = V^2/R . This is incorrect because V is not the voltage across the cables. Rather, it is the rms voltage of the power plant (a.c. source).

- Rectification.

- A *half-wave rectification* converts only *half* of the a.c. into d.c. by allowing current flow in only one direction.



(a) [45] Circuit diagram.

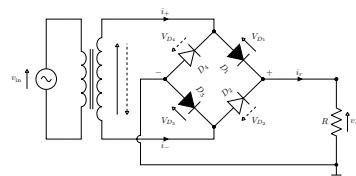


(b) [46] The red and green lines denote the $V_R - t$ and $V_D - t$ graphs, respectively.

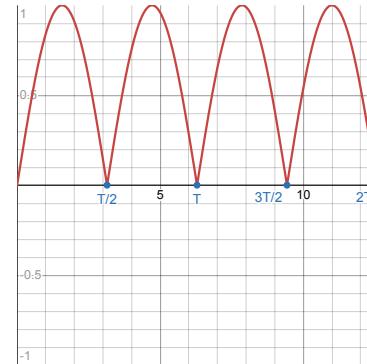
Figure 17.4: Half-wave rectification.

- * First half cycle ('positive' V and I)
 - The diode is *forward-biased* and has almost zero resistance, allowing the flow of current through the circuit.
 - There is negligible p.d. (approximately zero) across the diode due to the low resistance.
 - Thus, p.d. V_R across the resistor R is equal to the a.c. input voltage.
 - So, when the diode is forward-biased, V_R follows the a.c. supply.

- * Latter half cycle ('negative' V and I)
 - The diode is now *reverse-biased* and has almost infinite resistance, such that only a negligible current flows through it.
 - The p.d. across the diode is equal to the a.c. input supply voltage as its resistance is very much larger than R.
 - Thus, the p.d. V_R across R is negligible.
 - So, when the diode is reverse-biased, $V_R = 0$.
- A *full-wave rectification* converts *all* of the a.c. into d.c. by inverting the negative current flow into positive current flow.



(a) [47] Circuit diagram.

(b) [48] The $V_R - t$ graph.**Figure 17.5:** Full-wave rectification.

Quantum Physics

- * A photon is defined as a quantum of electromagnetic energy. A photon of frequency f has energy $E = hf$, where h is Planck's constant.
- Planck's constant $h = 6.63 \cdot 10^{-34}$ Js. A useful conversion: $1 \text{ eV} = 1.60 \cdot 10^{-19} \text{ J}$
- * The photoelectric effect refers to the emission of electrons from a metal surface when the surface is irradiated with electromagnetic radiation of a high enough frequency.

Experimental observations	Predictions of classical theory	Explanation via quantum theory
<ul style="list-style-type: none"> • Emission of photoelectrons is almost instantaneous, with a negligible time lag. • Is unaffected by the intensity or brightness of the light if the light is above the threshold frequency. 	<ul style="list-style-type: none"> • Electrons should absorb energy over a period of time before it gains enough energy to escape the metal. • A dim light after some delay would transfer sufficient energy to the electron for ejection, whereas a very bright light would eject electrons after a short while. 	<ul style="list-style-type: none"> • Photon-electron interaction is one-to-one. • Energy of a single photon cannot be shared; it either gives up all of its energy to the electron, or none at all. • This absorption of the photon then leads to a gain in energy for the electron. • The time taken for the electron to gain sufficient energy is negligible. Hence, photoemission is instantaneous.
<ul style="list-style-type: none"> • Electrons are emitted when the frequency of light is above the threshold frequency f_0 (or below the threshold wavelength λ_0). • No electron is emitted below f_0, regardless of the light intensity. 	<ul style="list-style-type: none"> • Energy of the wave is dependent on the square of its amplitude. • Electrons will absorb enough energy to escape given sufficient time. • There should not be any threshold frequency. 	<ul style="list-style-type: none"> • The existence of the threshold frequency again traces back to the one-to-one interactions between electrons and photons. * The work function ϕ is the minimum energy required to eject/free an electron from the surface of a metal. • Note. The energy $E \geq \phi$ required to free an electron varies from electron to electron.

<p>The <i>maximum kinetic energy</i> of the ejected electrons is independent of the <i>intensity</i> of the light, but dependent on the <i>frequency</i> of the light.</p>	<p>The higher the light <i>intensity</i>, the higher the <i>energy</i> of the light wave striking the metal surface. Electrons should be ejected with <i>greater</i> kinetic energy. It <i>cannot</i> explain why the maximum kinetic energy is dependent on frequency and independent of intensity.</p>	✗	<ul style="list-style-type: none"> • By the conservation of energy, energy imparted by a photon is used to <i>free</i> the electron and becomes the <i>kinetic energy</i> of the emitted electron. • For an electron requiring the <i>minimum</i> energy ϕ to escape from the metal, its kinetic energy will be a <i>maximum</i>. i.e. $hf = \phi + E_{k,\max}$. • When the light source increases in <i>frequency</i>, the <i>energy</i> $E_{\text{photon}} \propto f$ of the emitted <i>photons</i> increases proportionately. So, the <i>maximum</i> kinetic energy $E_{k,\max} = E_{\text{photon}} - \phi$ of the electrons emitted also increases.
<p>The <i>rate</i> at which electrons were ejected, and therefore, the <i>photocurrent</i> produced is proportional to the <i>intensity</i> of the light.</p>	<p>A higher light intensity should be able to simultaneously free more electrons from the metal at any instant. This results in a higher current.</p>	✓	<p>The <i>intensity</i> of the <i>monochromatic</i> light beam is directly proportional to the number of <i>photons</i> passing through a cross-sectional <i>area</i> per unit <i>time</i>. In fact,</p> $I = \frac{N}{t} \cdot \frac{hf}{A} = \frac{Nhf}{tA}.$ <p>The higher the light <i>intensity</i>, the higher the <i>number of photons per unit time</i> hitting the metal surface, and hence, its electrons.</p>

Table 18.1: The photoelectric effect, in summary.

Example 18.1

State **two** pieces of evidence provided by the photoelectric effect for the particulate nature of electromagnetic radiation.

- The emission of electrons from the metal surface is instantaneous, regardless of how low the intensity of incident light is, as long as frequency of incident light is sufficiently high (above the threshold frequency). [1]
- There exists a threshold frequency, which is the minimum frequency at which light must be incident for electrons to be freed from the metal surface. It is independent of intensity of incident light. [1]
- The maximum kinetic energy of an emitted electron is dependent on the frequency of the incident light. [1]
- The maximum kinetic energy of an emitted electron is independent on the intensity of incident light. [1]

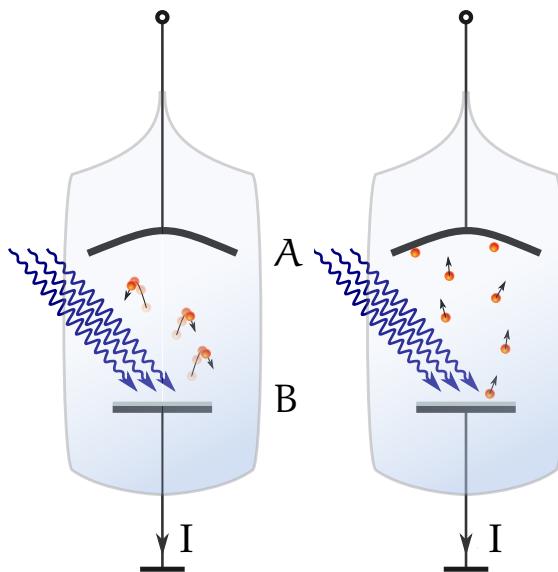


Figure 18.1: [52] An illustration of the photoelectric effect.

- A is negative relative to B ($V = V_A - V_B < 0$).

- Electrons decelerate when moving towards A.
- If the kinetic energy E_k of an electron exceeds eV , then it will reach electrode A.
- And when $E_k < eV$, the electron will be repelled back to electrode A.
- As V increases, more electrons have insufficient E_k to reach electrode A. i.e., less electrons reach A. So, the photocurrent is reduced.
- At the stopping potential V_s , even the most energetic electrons have insufficient E_k to reach A. i.e. $eV_s = E_{k,max}$. Hence, there is no photocurrent.

- A is positive relative to B ($V = V_A - V_B > 0$).

- All electrons accelerate towards A.
- The saturation current is not immediately reached because some electrons' path are such that they do not hit electrode A. So, a sufficiently strong electric field is required for them to hit A.

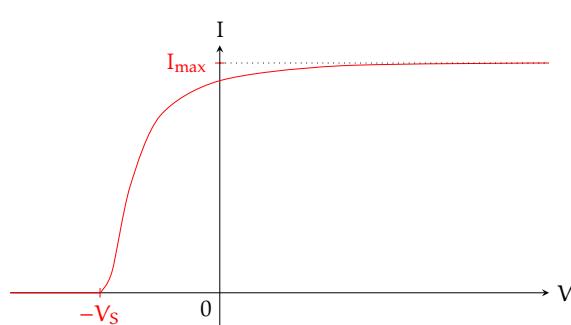
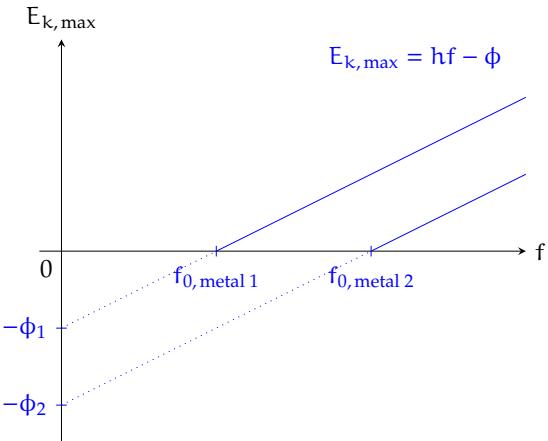
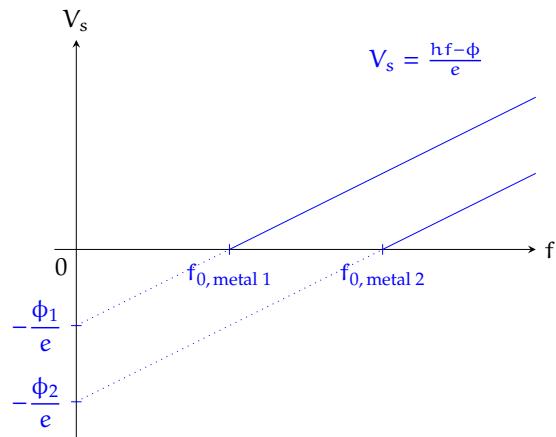


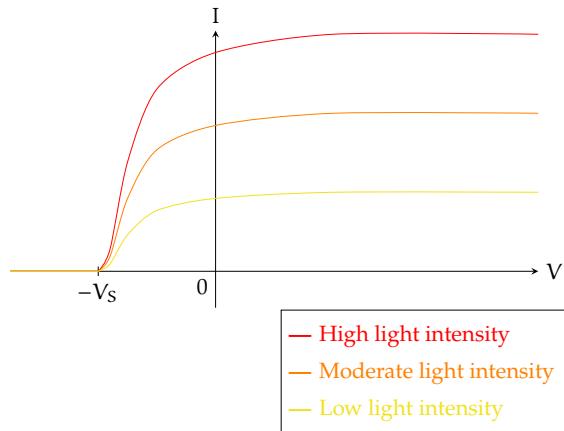
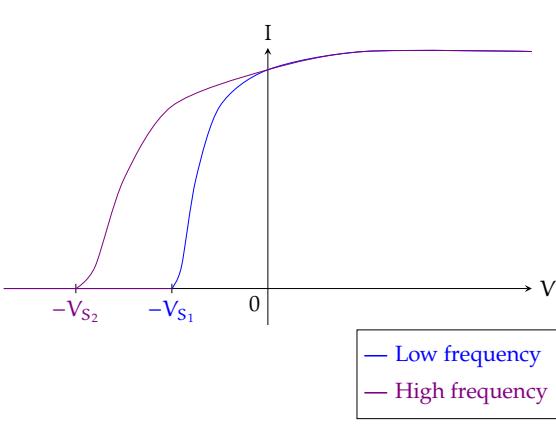
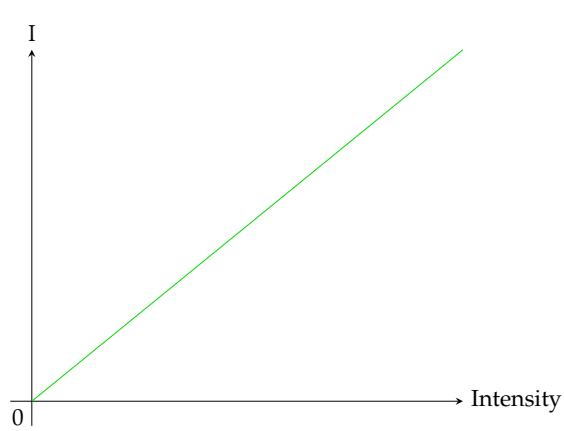
Figure 18.2: [53] An illustration of the photoelectric effect.



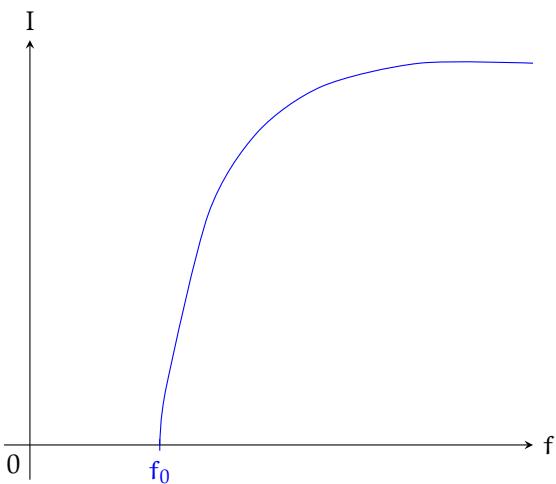
(a) The maximum kinetic energy of a photoelectron plotted against the frequency of light used.



(b) The stopping potential for a metal surface plotted against the frequency of light used.

(c) The graph of photocurrent I against potential difference $V = V_A - V_B$, under (a constant frequency but) varying intensities of light.(d) The graph of photocurrent I against potential difference $V = V_A - V_B$, under (a constant rate of photon emission but) varying frequencies of light.

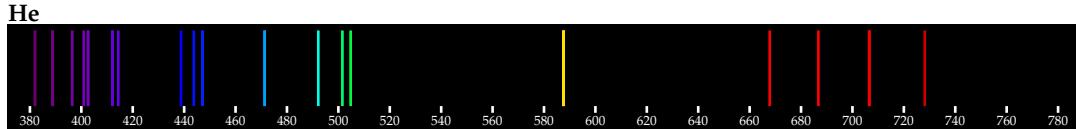
(e) The graph of photocurrent I against the intensity of light used (under a constant frequency, which exceeds the threshold).



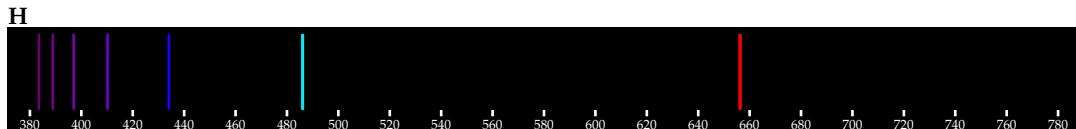
(f) The graph of photocurrent I against the frequency of light used (under a constant rate of photon emission).

Figure 18.3: [53] The photoelectric effect: the relationship between the maximum kinetic energy $E_{k,\max}$, the stopping potential V_s , photocurrent I , potential difference $V = V_A - V_B$, frequency f of light used, intensity of light used.

- The *de Broglie wavelength* of any particle is given by $\lambda = h/p$.
- The Heisenberg Uncertainty Principle states that $\Delta p \Delta x \gtrsim h$, where p and x are the momentum and position, respectively, of a particle.
- Note. The above applies only when considering the momentum p and position x *in the same direction*. So, it is possible that, for example, $\Delta x \Delta p_y = 0$ or $\Delta y \Delta p_x = 0$.
- The energy of a photon involved in the transition between energy levels obeys energy conservation. i.e. $hf = E_f - E_i$ where f is the photon's frequency.
- Some terminology.
 1. The *ground state* of an atom is the lowest state an atom can be at, such that all electrons in the atom occupy the lowest possible energy levels.
 2. An atom is said to be *excited* iff one or more of its electrons are not occupying the lowest possible energy states.
 3. For this topic, *ionisation* is the removal of an electron to create an ion.
 4. The *ionisation energy* of an atom is the energy required for the atom to transition from the $n = 1$ to $n = \infty$ state.
- Why do energy levels have negative values? Electrons have *zero potential energy at infinity*, and so have *less potential energy* (i.e. negative potential energy) near the *positively charged nucleus*.
- An *emission line spectrum* is a series of distinctly coloured lines against a dark background.



(a) The emission line spectrum of Helium.



(b) The emission line spectrum of Hydrogen.

Figure 18.4: [56] Some emission line spectra.

- Excited atoms are unstable and de-excite by *emitting* photons, eventually reaching the ground state (it does not have to immediately de-excite to the ground state).
- The frequency of these photons is such that $hf = E_i - E_f$, where E_i and E_f are the initial and final energy states, respectively (f can be nonzero).
- Each emission spectrum is unique to each element.
- Given n energy states, there will be $(n)_2$ emission lines.
- Do not use say that an atom transits from the $n = \underline{\hspace{2cm}}$ state to the $n = \underline{\hspace{2cm}}$ state, if the question does *not* define n .
- An *absorption line spectrum* is a series of distinct dark lines against a continuous spectrum.

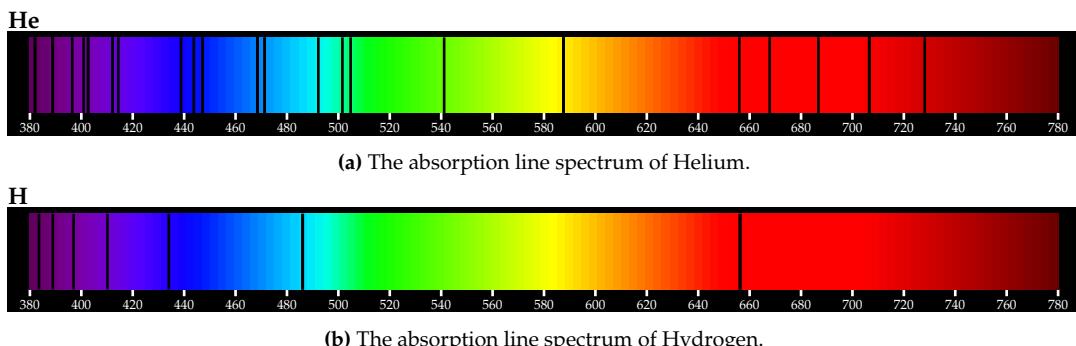


Figure 18.5: [56] Some absorption line spectra.

- Gaseous atoms *absorb* photons with (exactly) the energies they need to transit to a higher energy state, *from the ground state*. (We do not consider further excitations from above a higher energy level.)
- Given n energy states, there will be $n - 1$ absorption lines.

Example 18.2

State how line spectra, together with the concept of a photon, provide evidence for discrete energy levels in isolated atoms. [2]

- Since the emission line spectrum of an atom comprises of discrete lines, where each line corresponds to one specific wavelength λ , the energies of photons $E \propto 1/\lambda$ emitted are discrete. [1]
- Each photon is emitted when one electron transits from a higher energy level to a lower energy level. Since the energy level difference is equal to the energy of the emitted photon, the energy levels must be discrete. [1]

Example 18.3

In the visible section of the spectrum of electromagnetic radiation, purple light has the shortest wavelength.

1. Calculate the energy, in eV, of a photon of purple light of wavelength $340 \cdot 10^{-9}$ m.

$$E = hc/\lambda = \dots = 3.66\text{eV}.$$

2. By reference to your answer in (i), explain why a visible line spectrum does **not** result from electrons de-exciting to the -13.6 eV energy level. [2]

- The de-excitation of an electron to the -13.6 eV energy level produces a photon of minimum energy $13.6 - \dots = \dots > 3.66\text{eV} =$ energy of one purple photon. [1]
- So, an emitted photon has shorter wavelength than purple light. Since purple light has the shortest wavelength in the visible light spectrum, an emitted photon will not be in the visible light spectrum. [1]

- All the light absorbed by atoms is re-emitted. Suggest why dark spectral lines are still seen: Incident light is directional, but light is re-emitted in all directions.
- If an element ${}^A_Z X$ lies on the m th row and in the n th group of the periodic table, then an atom of X at ground state has m electron shells and n valence electrons.

- The maximum number of electrons that can occupy the n th shell of an atom is $2n^2$.

Shell		Maximum number of electrons
Number	Letter	
1st	K	2
2nd	L	8
3rd	M	18
4th	N	32

Table 18.2: The maximum number of electrons that can occupy the first four shells of an atom.

- Similarity: All distinct lines in an absorption line spectrum are present and of the *same wavelengths* in an emission line spectrum for the same atoms.
- Difference: There are at least as many emission lines as there are absorption lines. In general, there are *more* emission lines.
- Note.* It is critical to accurately represent the distance between the each emission line (or absorption line), especially the one furthest away from adjacent lines. E.g. the line near the 660 nm mark in Figure 18.4b or 18.5b.



Figure 18.6: [57] The energy levels of Hydrogen (drawn to scale).

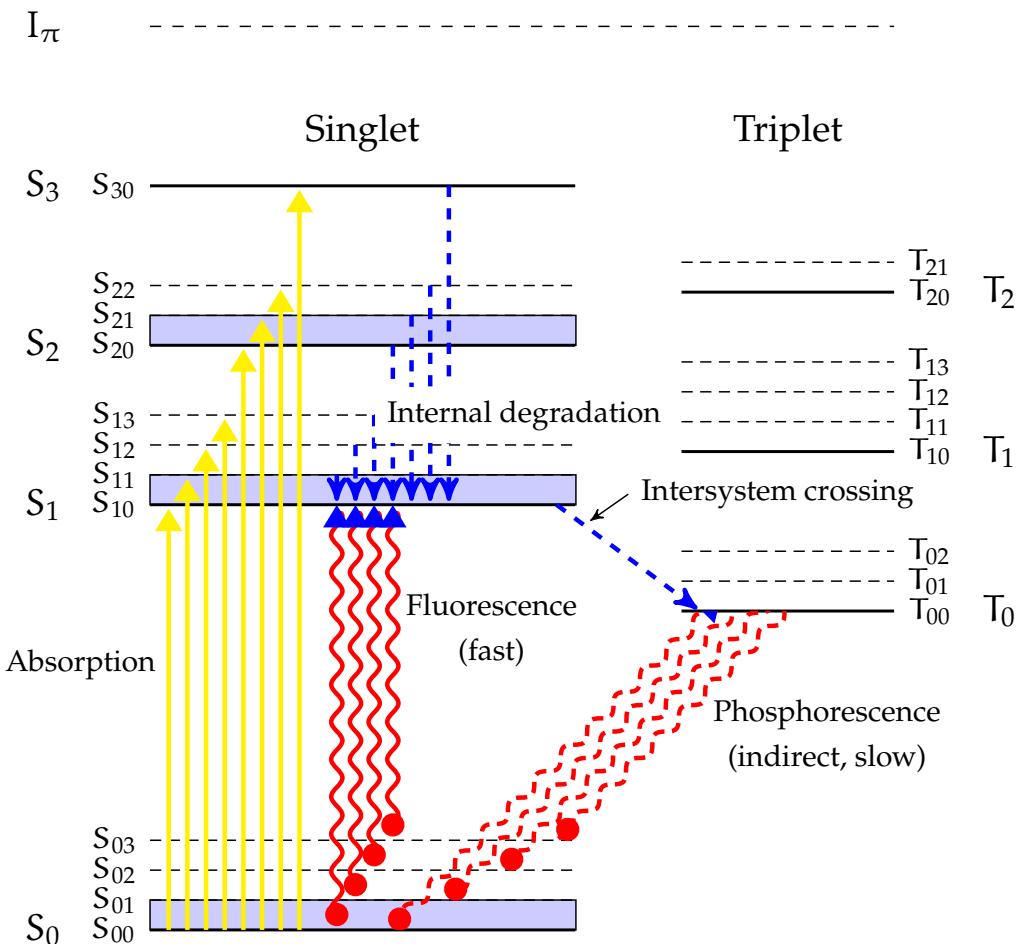


Figure 18.7: [54] (Pretty diagram; *out of syllabus*) Typical energy levels for π -orbitals of a fluor molecule. Spin singlet (S) and triplet (T) states are separated for clarity. The ionization level I_π is shown at the top. Excited states as well as vibrational sublevels (dashed horizontal lines) are shown. Internal degradation is a non-radiative process, while fluorescence and phosphorescence are radiative decays. The decay $T_0 \rightarrow S_0$, however, is indirect, by interactions with other molecules.

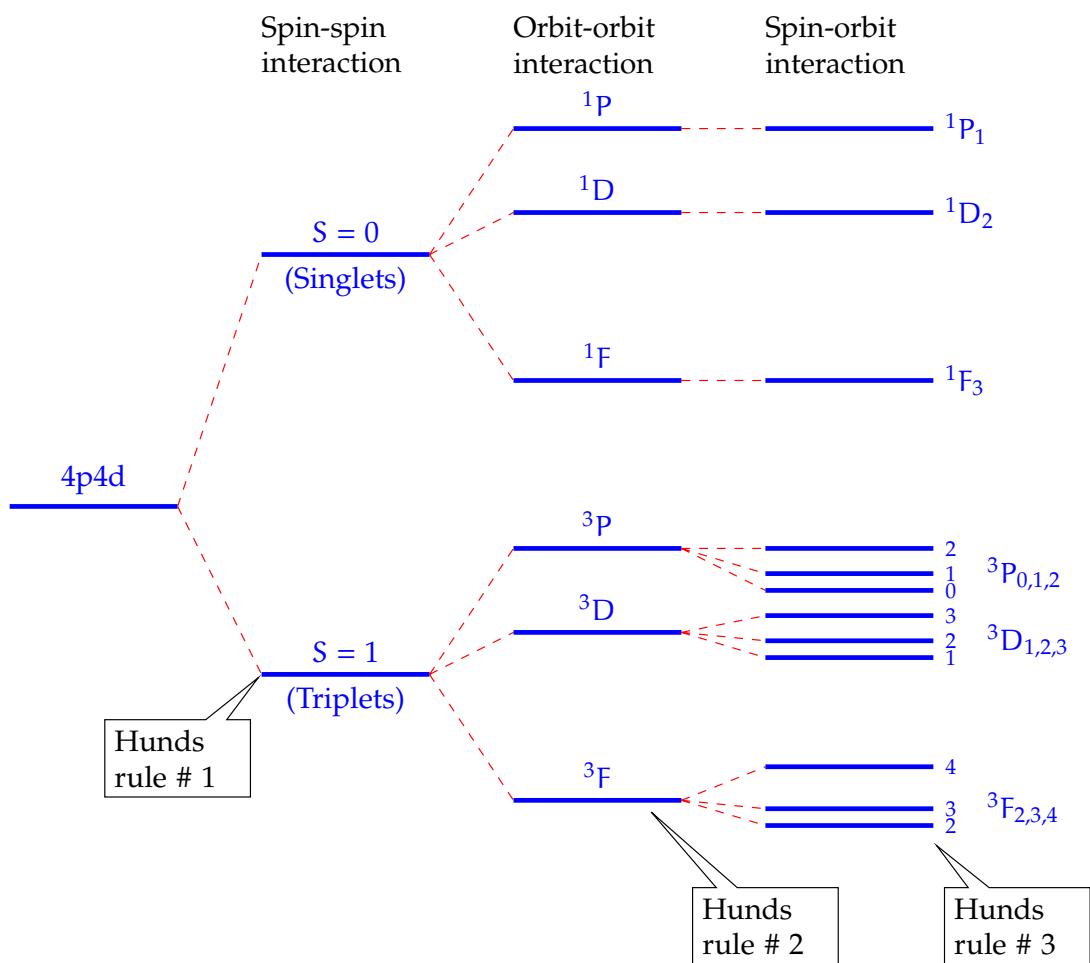


Figure 18.8: [55] (Another pretty diagram; *out of syllabus*) An illustration of Hund's rule.

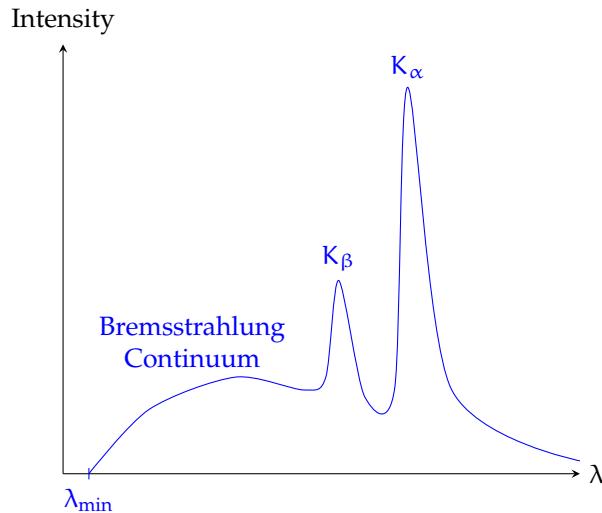


Figure 18.9: [58] X-ray spectrum.

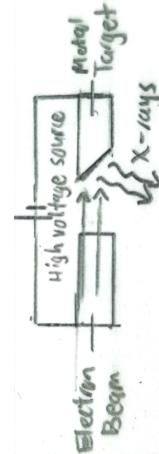


Figure 18.10: [59] Experimental set-up to produce X-rays

Features	Explanation
A sharp cut-off / minimum wavelength λ_{\min}	<ul style="list-style-type: none"> Incident electrons are accelerated through an electric potential V, giving them a maximum kinetic energy of $E_k = eV$. When such an electron is decelerated and loses all its E_k, it emits a single X-ray photon of maximum possible energy, corresponding to the minimum wavelength seen on the spectrum. By the conservation of energy, $eV = \frac{hc}{\lambda_{\min}}$ so $\lambda_{\min} = \frac{hc}{eV}$.
A broad continuous spectrum known as the <i>Bremsstrahlung continuum</i> . (Bremsstrahlung is braking radiation in German.)	<ul style="list-style-type: none"> When high speed incident electrons pass by the (positively) charged nuclei of the high atomic mass lead atoms, the strong (attractive) electric forces lead to a sudden, large deceleration of the electrons. Their kinetic energies are lost through the emission of Bremsstrahlung X-ray photons. Since the magnitude of the deceleration, and hence the magnitude of change in kinetic energy, experienced by the incident electrons varies continuously and is not discrete, the wavelengths of the emitted photons have a continuous distribution. Hence, Bremsstrahlung produces a continuous spectrum of electromagnetic radiation.

- An accelerated incident electron ‘collides’ with an electron from the innermost K-shell, kicking it out of the shell.
- The atom is excited due to the vacancy in the K-shell.
- An electron from the higher energy shell transits to the K-shell, emitting a photon.

Sharp peaks K_{α} and K_{β} called characteristic lines/X-rays.

	Series	Line	Initial shell (m)	Final shell (n)	$m - n$
•	K series	K_{α}	L-shell (2nd)	K-shell (1st)	1
		K_{β}	M-shell (3rd)		2
	L series	L_{α}	M-shell (3rd)	L-shell (2nd)	1
		L_{β}	N-shell (4th)		2

- See Figure 18.11 for an illustration.
- Every element has a unique set of energy levels, and hence, characteristic lines.

Table 18.3: The features of an X-ray spectrum.

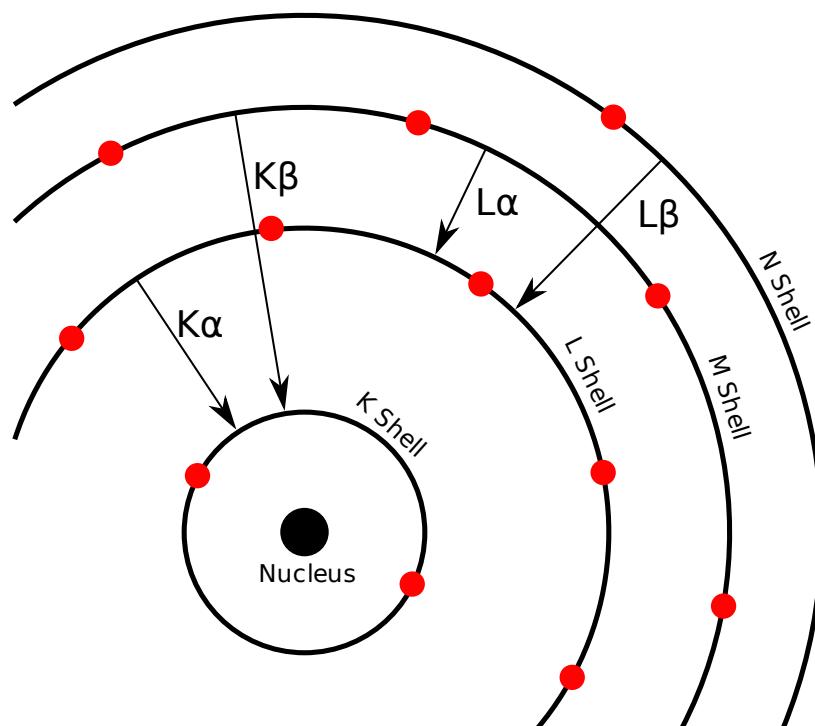


Figure 18.11: [60] X-ray series

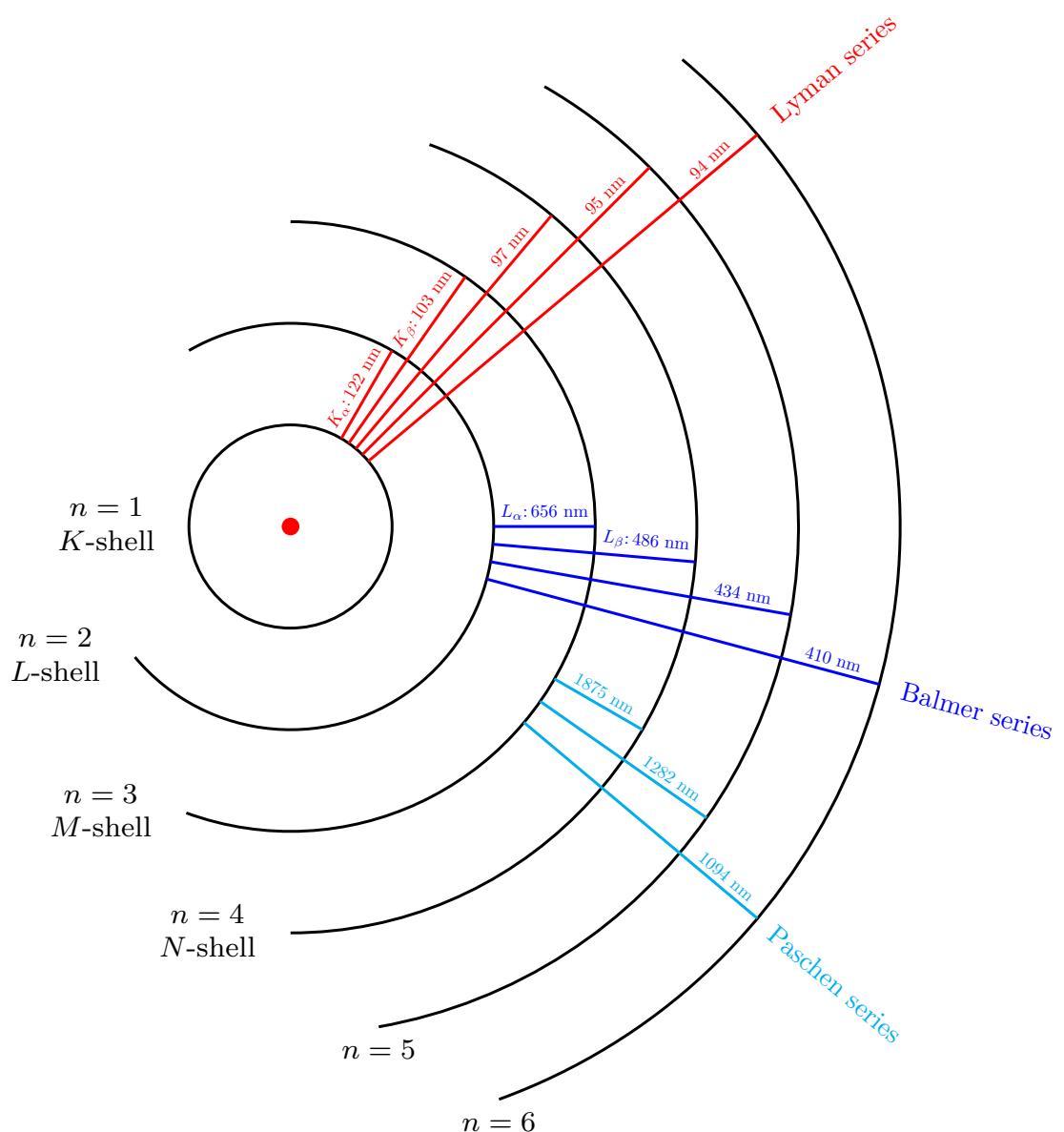


Figure 18.12: [61] The spectral emissions of the hydrogen atom.

Nuclear Physics

- The Rutherford-Geiger-Marsden experiment.

Observation	Explanation
Most α -particles passed straight through the foil, <i>undeflected</i> .	An atom comprises of mostly empty space, with <i>most of its mass concentrated in its nucleus which is very small in size compared to the atom</i> . Hence, most pass through the empty space in the metal foil's atoms with minimal deflection.
Less than 1% of the α -particles were <i>scattered backwards</i> , with an angle of <i>deflection</i> exceeding 90°.	Only a very small proportion of α -particles — that pass sufficiently close to or are incident on the nucleus of a metal foil atom — experience <i>strong repulsive electric forces</i> by the nucleus and are hence deflected by large angles $> 90^\circ$.

Table 19.1: The Rutherford-Geiger-Marsden experiment: observations and explanations.

Example 19.1

In the α -particle scattering experiment, only a small proportion of the α -particles incident on the metal foil are deflected through angles greater than 90°. [3]

- * A *isotope* is one of two or more atoms with the *same atomic number* but *different number of neutrons*.
- * The *unified atomic mass unit u* is equivalent to *one-twelfth* of the mass of a carbon-12 atom. i.e. $1 \text{ u} = 1.66 \cdot 10^{-27} \text{ kg}$.
- * The *mass defect* is the amount by which the mass of an atomic *nucleus* is less than the sum of the masses of its *constituent particles*. i.e.

$$\text{mass defect } \Delta m = \text{sum of masses of nucleons} - \text{mass of nucleus.}$$

- * The *binding energy* of a nucleus is the amount of energy that is required to *break a nucleus* into its *constituent nucleons*.
 - Binding energy is not a form of stored energy.
 - Mass-energy equivalence: $E = mc^2$.
- * The *binding energy per nucleon* is defined as the *binding energy* of a nucleus divided by the *number of nucleons* in the nucleus.

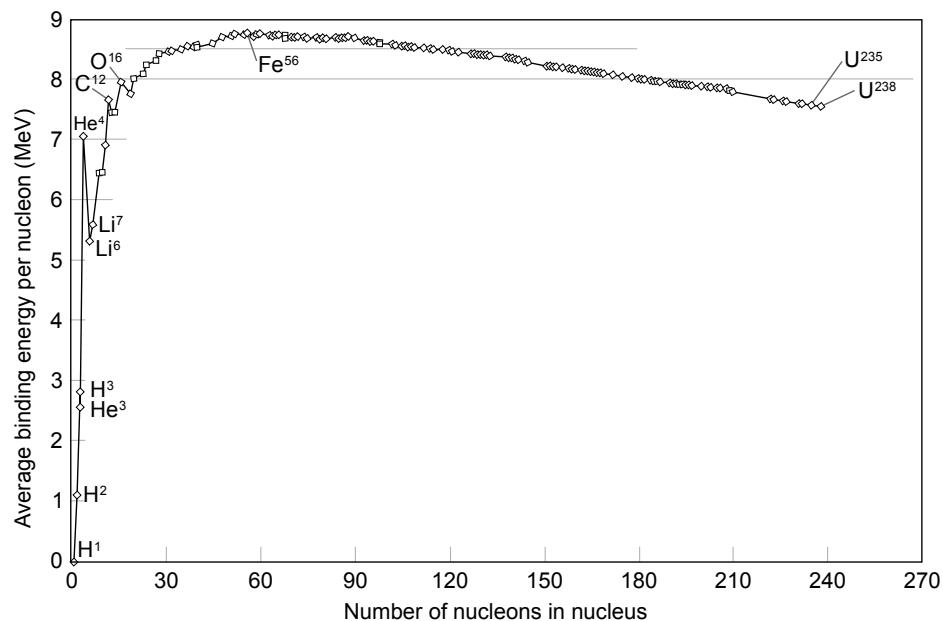


Figure 19.1: [49] The variation of binding energy per nucleon with nucleon number.

- Remember that, Fe-56 has a binding energy per nucleon of 8.8 MeV and is located near (slightly to the right of) the peak.
- The binding energy per nucleon can be seen as a measure of stability: a higher binding energy per nucleon corresponds to a higher stability of the nucleus.

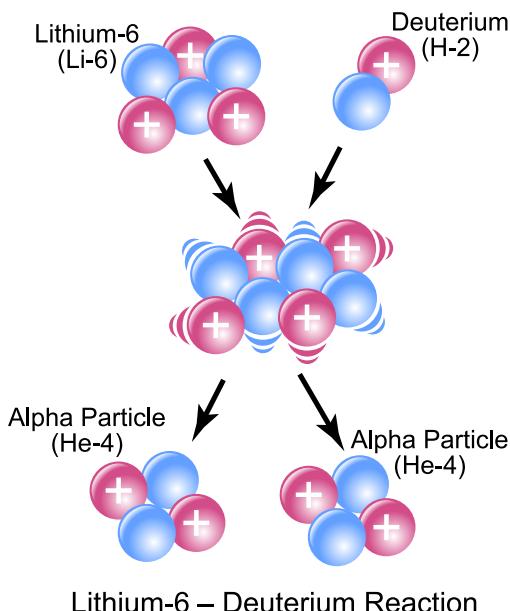


Figure 19.2: [50] Nuclear fusion followed by fission.

- * Nuclear fission occurs when an atomic nucleus of greater mass splits into two nuclei, each of smaller masses and approximately of the same nucleon number as each other.
- * Nuclear fusion occurs when two nuclei of smaller nucleon numbers fuse to form a single nucleus of a larger nucleon number.

- Useful equations for calculating the energy liberated in a nuclear reaction.

- energy released = (mass of reactants – mass of products)c².

- energy released = B.E. of products – B.E. of reactants.

- Conserved quantities in nuclear reactions.

- Nucleon number.
- Charge. The atomic numbers of reactants and products each sum to the same total. But, the total number of protons may change due to beta decay. i.e.

$${}^A_Z X \rightarrow {}^{A+1}_{Z+1} X + {}^0_{-1} e.$$
- Mass-energy.

* Radioactive decay is the spontaneous and random disintegration of atomic nuclei into another

(nuclear) species through the *emission* of alpha or beta particles or the de-excitation of nuclei to lower energy states through the emission of gamma radiation.

- *Spontaneous* means that the nucleus decays because it is *unstable*, not because of *external conditions*.
- *Random* means that it is impossible to predict *which* nucleus will decay or *when* a particular nucleus will decay.

Example 19.2

State *experimental evidence* to suggest that the process of radioactive decay is

1. random
2. spontaneous.

Answer:

1. When using a Geiger-Muller tube connected to a ratemeter, the count rate shown on the ratemeter shows unpredictable fluctuations that vary significantly each time the experiment is conducted.
2. When using a Geiger-Muller tube connected to a ratemeter, the average count rate is not affected by external conditions such as temperature and pressure.

* *Background radiation* is the radiation detected by a *radiation counter* when *no radioactive source* is nearby.

* The *activity A* of a radioactive isotope is defined as the number of *nuclear disintegrations per unit time*.

- The count rate C is the rate at which counts are triggered on a ratemeter, by ionising radiation.
- Note. The count rate is *not the same* as activity. In fact, $C \leq A$ because not all emissions — stemming from radioactive decay — are counted. Some reasons include:

1. Not all the radiation is directed towards the detector.
2. Some of the radiation will be absorbed in the source itself, and by the air between the source and the detector.

* The *decay constant λ* of a *sample of a radioactive nuclide* is the *probability* that a nucleus will *decay per unit time*.

- The number N of undecayed nuclei is such that $A = -\frac{dN}{dt} = \lambda N$.
- The number N of undecayed nuclei, mass m of radioactive sample, activity A, and count rate C all follow the equation $y = y_0 e^{-\lambda t}$ ($y = N, m, A, C$).
- When we are given a set of plotted points and asked to determine a value for half-life, do the following.

- Draw the curve of best-fit.
- Calculate two values for half-life and take their mean:

Time to half count rate from ____ to ____ = x.

Time to half count rate from ____ to ____ = y.

Average half-life = $\frac{x+y}{2} = ____$.

* The *half-life $t_{1/2}$* of a *radioactive isotope* is the *average time* taken for its *activity to be halved*. Note. $\lambda t_{1/2} = \ln(2)$.

Example 19.3: the importance of semantics.

A student defines half-life of a radioactive isotope as the *time taken* for its activity to be halved. Explain why this is inappropriate. [2]

- Radioactive decay is a random process. [1]
- The time taken to decay by half will fluctuate; we should instead consider the *average* time taken. [1]

Type of decay	Equation	Ionising power ↓	Penetration power ↑	Velocity
Alpha decay	α -particle (zero electrons) ${\text{A}}_{Z}^{\text{X}} \rightarrow {\text{A}}_{Z-2}^{-4}\text{Y} + {\overset{\sim}{\text{He}}}_{2}^{4}$	Strong (10^5 ions per cm)	Stopped by a few cm of air or paper	10^7 ms^{-1} or $0.1c$
Beta decay	β -particle ${\text{A}}_{Z}^{\text{X}} \rightarrow {\text{A}}_{Z+1}^{-1}\text{Y} + {\overset{\sim}{\text{e}}}^{-}_0$	(10^3 ions per cm)	5 mm of aluminum	Continuous range up to 10^8 ms^{-1} or $0.9c$
Gamma decay	γ -ray $\text{---} \rightarrow \text{---} + {\overset{\sim}{\gamma}}_0^0$	Weak (indirect ionisation)	Several cm of lead	c

Table 19.2: Types of radioactive decay.

Example 19.4

The isotope beryllium-7 is radioactive with a half-life of 53 days. A beryllium-7 nucleus decays by the emission of a γ -ray photon. State why, although the beryllium-7 is radioactive, the number of beryllium-7 nuclei in the sample does not change. [2]

- The γ -ray photon emitted in the decay of a beryllium-7 nucleus is a quantum of energy that contains no protons or neutrons. [1]
- So the daughter nucleus is still a beryllium-7 nucleus. Hence, the number of beryllium-7 nuclei is constant. [1]

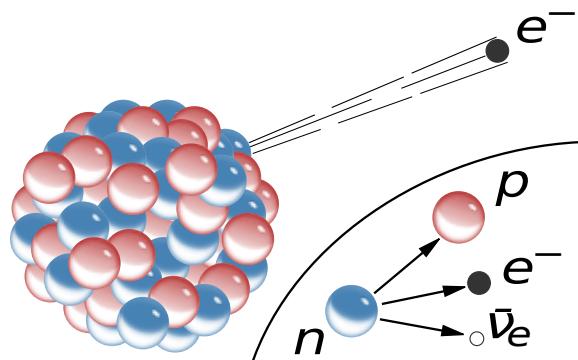


Figure 19.3: [51] Beta decay.

- In a beta decay, a *neutrino* is emitted along with the β -particle, carrying away some kinetic energy and momentum. Hence, the electron emission may not be collinear, in the opposite direction of, the recoil of the nucleus.
- While a couple mm or cm of metal suffices to stop β -particles, X-rays (bremsstrahlung) may be produced outside the container:

- When β -particles — electrons moving at *high speeds* — pass by the positively charged aluminum nuclei, they experience a sudden, *large* deceleration.

- The kinetic energy of the β -particles are lost through the emission of Bremsstrahlung X-ray photons.
- X-ray photons have a higher penetrating power than β -particles. So, while β particles are absorbed, X-ray photons pass through the aluminum walls.

So, thicker layers of lower atomic mass materials — plastic, wood, water, etc — may be preferable.

- Biological effects of radiation.

- Direct effects. When radiation interacts with DNA, affecting (or even breaking) the bonds within, it impacts the ability of a cell to reproduce and survive. Mutations may also occur, eventually leading to cancer.
- Indirect effects. When radiation interacts with and breaks the bonds in water molecules, it creates free radicals: molecules which are highly reactive due to the presence of unpaired electrons. These free radicals form compounds which initiates harmful chemical reactions within cells, causing them to be destroyed.

Example 19.5

Explain why, although the count rates are too low for the radiation to cause immediate symptoms in the student, careful shielding of the source is still necessary.

- Ionising radiation damages DNA and causes biological harm to the body. Even though no immediate symptoms may arise, biological damage can build up with *prolonged close-proximity* exposure. In the long run, the student might experience increased risk of cancer, among other health impacts.
- It also prevents radioactive contamination due to accidental inhalation or ingestion of the residues of the radioactive source — especially if especially if the source is powdery — which may lead to irreversible long run exposure to radioactive material.

Example 19.6: radioactive dating.

To find the age of a relic, a recreation of it is made. Suppose that

1. The recreation is perfect. i.e. it is identical to the relic when it was first made.
 2. The background count rate has remained constant throughout time at C_B .
 3. The only radioactive process that has affected the relic in its entire lifetime is ${}^A_Z X \rightarrow {}^A_Z Y + \dots$
 4. The number of ${}^A_Z Y$ nuclei only changes due to the reaction ${}^A_Z X \rightarrow {}^A_Z Y + \dots$
- (a) In the absence of other radioactive samples, the relic and its recreation are found to have count rate C' and C respectively. Find the age T of the relic, in terms of the half life $t_{1/2}$ of ${}^A_Z X$.

$$\begin{aligned} C &= C_0 e^{-\lambda t} \\ C' &= C e^{-\frac{T \ln(2)}{t_{1/2}}} \\ T &= \frac{\ln(C/C')}{\ln(2)} \cdot t_{1/2}. \end{aligned}$$

- (b) The ratio of ${}^A_Z X$ to ${}^A_Z Y$ for the relic and its recreation are k' and k , respectively. Find the age T of the relic.

Let M denote the initial number of $_{\mathcal{Z}}^{\mathcal{A}}Y$ nuclei. Since $N_0/M = k$, we have $M = N_0/k$. Therefore,

$$\begin{aligned} \frac{N}{M + (N_0 + N)} &= k' \\ \frac{N}{(1/k + 1)N_0 - N} &= k' \\ (k' + 1)N &= k'(1/k + 1)N_0 \\ N_0 e^{-\frac{T \ln(2)}{t_{1/2}}} &= \frac{k'(1/k + 1)}{k' + 1} \cdot N_0 \\ T &= \frac{t_{1/2}}{\ln(2)} \cdot \ln \left(\frac{k' + 1}{k'(1/k + 1)} \right). \end{aligned}$$

(c) What are some limitations of radioactive dating?

- It cannot be used for very old samples, as the activity will be very low so the calculated age may not be accurate.
- We are assuming that the recreation has the same number of $_{\mathcal{Z}}^{\mathcal{A}}X$ and $_{\mathcal{Z}}^{\mathcal{A}}Y + \dots$ nuclei as the relic when it was first made.
(The exact assumption depends on the question. It might instead be the concentration of $_{\mathcal{Z}}^{\mathcal{A}}X$ to $_{\mathcal{Z}}^{\mathcal{A}}Y$, instead of the exact number.)
- It cannot be used for living things, if $_{\mathcal{Z}}^{\mathcal{A}}X$ or $_{\mathcal{Z}}^{\mathcal{A}}Y + \dots$ is carbon-14.
(Living things will continue to absorb carbon-14 from their surroundings into their tissue. This process stops, allowing the number/concentration of carbon-14 to decrease, only after death.)

Note. Chemical processes do not change the *ratio* of two isotopes; both are used in equal proportions as the chemical behavior of isotopes are nearly identical.

Practical

20.1 A-Level Trend

- One question on electrical circuits.
- One question on oscillations.
- One ‘wild card’ question on any topic.
- One planning question on any topic.

20.2 Measurements & Calculations

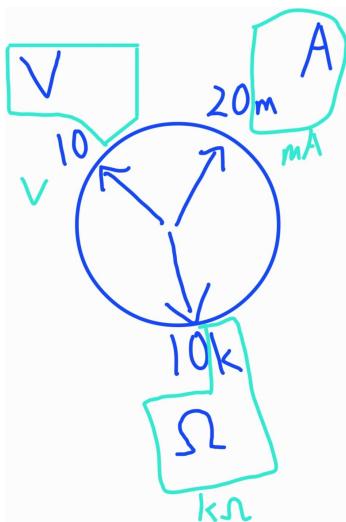


Figure 20.1: [22] The units that a dmm is measuring in, given a particular config.

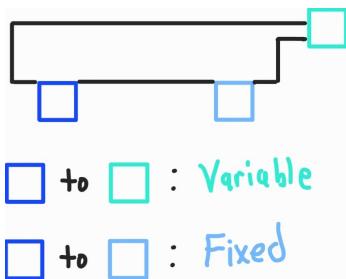


Figure 20.2: [22] How to connect a rheostat to obtain a variable resistor or a fixed resistor.

- Accuracy refers to *how closely* a measured value *agrees with its true value*.
- Precision refers to *how closely* individual measurements *agree with each other*, without reference to a true value.

Instrument	Measurement	Readings	Precision of a measurement	
Analogue Instruments	in general	2	smallest division	
Metre rule	length	2	0.1 cm	1 mm
Vernier calipers	length	2	0.01 cm	0.1 mm
Micrometer screw gauge	length	2	0.001 cm	0.01 mm
Stopwatch (ensure $t \geq 20$ s)	time interval	2	0.01 s (but $\Delta t = 0.2 - 0.4$ s)	
Thermometer	temperature	2	1 °C	
Voltmeter	p.d.	2	smallest division	
Ammeter	current	2	smallest division	
Protractor	angle	2	1°	
Measuring cylinder	volume	1	0.5 cm ³	
Digital instruments	in general	1	smallest increment displayed E.g. $x = 5.02$ g $\Rightarrow \Delta x = 0.01$ g	

Table 20.1: The precision of some standard instruments.

Operation	D.P. / S.F.	
	Calculated value	Data used
Addition and Multiplication		
Addition	d.p.	least d.p.
Multiplication	s.f.	least s.f.
Multiplication by a fixed factor (e.g. π)	s.f.	s.f.
Special Operations		
Logarithms of base 10 or e	d.p.	s.f.
Trigonometric functions	s.f.	s.f.

Table 20.2: D.P.S.F.

Equation	Rules
$Z = aX \pm bY$	$\Delta Z = a \Delta X + b \Delta Y$
$Z = cX^aY^b$	$\frac{\Delta Z}{Z} = a \frac{\Delta X}{X} + b \frac{\Delta Y}{Y}$

Table 20.3: Uncertainty rules

- Always take the average of three measurements (and show the working for the averaging explicitly) wherever possible. This reduces the effect of random error on the value of ___, hence making it more precise.
- Always write the associated units for angles, be it θ° or θ rad.
- After finishing calculations for a question, always check whether the number of s.f. or d.p. is correct. See table 20.2.
- Note.* Whenever a physical quantity is given — even a resistor clearly labelled “10Ω” — we still must account for its dpsf. However, if it’s the number of turns/number of oscillations/etc, do not account for its dpsf.
- Note.* Even for physical constants — such as $e = 1.60 \cdot 10^{-19}$ — their dpsf should also be taken into account! However, mathematical constants, like π , can be disregarded in dpsf considerations.

- In any set of repeated readings for time, the total time taken (e.g. for n oscillations) should not usually differ by more than human reaction time ($0.2 - 0.4\text{s}$).
- Percentage uncertainty of stopwatch,

$$\frac{\Delta t}{t} = \frac{0.2}{\langle t \rangle} \cdot 100\% = \underline{\hspace{2cm}} \% \text{ (1 s.f.)}$$

- Be careful of which quantities you are calculating. For example, in determining the radius r of an object, we might first measure its diameter d . Take caution to remember to calculate $r = d/2$, and *not* carelessly write $r = d$.
- If it's difficult to measure a quantity accurately, we should account for it when calculating uncertainties. E.g.
 1. The base of a beaker is not flat. So its height h (measured with vernier calipers) has uncertainty $\Delta h = 0.2\text{ cm}$, rather than the usual 0.01 cm .
 2. It may be difficult to precisely measure the height h of a suspended object (with a metre rule). So, we use $\Delta h = 0.2\text{ cm}$, instead of the usual 0.1 cm .
 3. It is difficult to maintain protraction when measuring an angle θ (freehand) between, say, a metre rule and a string. Therefore, we account for this by taking $\Delta\theta = 3^\circ$, in place of the usual 1° .

In general, when expecting such sources of error, a safe bet is to take the absolute uncertainty to be twice the precision of the instrument.

- Make the retort stands face inwards to increase stability

20.3 Tables & Graphing

- Define new variables — i.e. ones that the question has not defined — clearly. E.g. let n be the number of oscillations.
- Place all raw data on the left and leave space on the right to add extra columns.
- For the raw data for n oscillations, we can write it in this way:

n	nT_1/s	nT_2/s	nT_3/s	T/s

Table 20.4: Presentation of data for oscillations.

- Look out for
 1. Correct column headings
 2. Dpsf of raw data
 3. Dpsf of calculated data
 4. Number of datasets (see Table 20.6)
- The dependent variable is always plotted as the horizontal axis.
- Always include axis labels for every big square / every 2.0 cm .
- When drawing a best fit line, take note of the following:
 1. The number of points above and below the line should differ by at most one.

2. Do not draw a '**lopsided**' graph, as that is unlikely to be the **best fit line**.

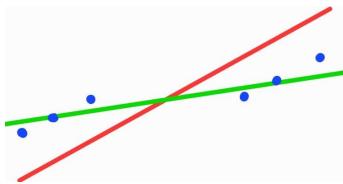


Figure 20.3: A '**lopsided**' line and the corresponding **best fit line**.

- On anomalous points:

Is there an anomalous point?	Graph	Anomaly statement
✗	Do nothing.	There is no anomalous point as all plotted points follow the trend of the best fit line.
✓	Circle the point and write "anomaly" next to it.	There is one anomalous point as it does not follow the trend of the best fit line. There is one anomalous point as it lies relatively far from the best fit line, <i>compared to other points</i> .

Table 20.5: Anomalous points.

Type	Total number of points	Anomalous points allowed
Straight line	6	1
	7	2
Curve	8	1

Table 20.6: Minimum number of points/maximum number of anomalous points.

- Linearisation statement:
 - The equation can be rewritten as $y = mx + c$. E.g. $\frac{1}{L} = A + Ar \cdot \frac{1}{R_2}$.
 - Plot y against x . Then, if a straight best-fit line is obtained, the vertical intercept is c and the gradient is m .
- Draw a gradient triangle using dotted lines and label the selected points (do not mark these points out using a dot/cross/etc).
- Always check that the two points selected for calculating gradient are stated to the correct number of decimal places, corresponding to half the smallest square.
- Follow the dpsf of the graph when calculating gradient. But follow the least sf of the graph and table when relating the gradient to a physical quantity.

Example 20.1

Consider when are given that $y = mx + c$, where m and c are physical constants, and have the following table of values.

...	x	y	...
	$(n_x \text{ s.f.})$	$(n_y \text{ s.f.})$	

Table 20.7: The dpsf of the quantities x and y .

Suppose that, using values from our graph, $y_2 - y_1$ and $x_2 - x_1$ have m_y and m_x s.f., respectively. Then,

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \underline{\quad} (\min\{m_x, m_y\} \text{ s.f.})$$

but

$$a = \text{gradient} = \underline{\quad} (\min\{n_x, n_y, m_x, m_y\} \text{ s.f.}).$$

- Suppose we want to draw a new line such that its vertical intercept is the same as our best fit line, but our horizontal axis does not start from zero. Then, just ensure that the lines do not intersect.

20.4 Questions post-graphing

- State/explain whether or not the results of your experiment supports the relationship that $\underline{\quad}$.

$$\text{Percentage difference} = \frac{k_{\text{larger}} - k_{\text{smaller}}}{k_{\text{larger}}} \cdot 100\%.$$

- Since the percentage difference of $k = \underline{\quad}\% > \underline{\quad}\% =$ the percentage uncertainty of $\underline{\quad}$, the results of the experiment does not support the suggested relationship.
- Since the percentage difference of $k = \underline{\quad}\% \leq \underline{\quad}\% =$ the percentage uncertainty of $\underline{\quad}$, the results of the experiment supports the suggested relationship.
- Use your values in (f)(ii) to plot another point Z on your graph. State whether this point agrees with the pattern of the other points on your graph, with reference to your values in (a)(v) — where we obtained two values for y (e.g. current I).

- Calculate the difference in current Δy_1 between the y value obtained from the graph and the y value obtained in (f)(ii).
- Calculate half the range

$$\Delta y_2 := \frac{y_{\text{higher}} - y_{\text{smaller}}}{2}$$

between the two values of y obtained in (a)(v).

- Since $\Delta y_1 > \Delta y_2$, this point Z does not agree with the pattern of the other points on the graph.
- Since $\Delta y_1 < \Delta y_2$, this point Z agrees with the pattern of the other points on the graph.
- Avoid writing “... because $\underline{\quad}$ is not equal to $\underline{\quad}$ ”. Rather, we should say “... because $\underline{\quad}$ is far from/near to/close to $\underline{\quad}$ ”.
- NY’s suggested format for writing sources of error:
 - What is error is/what causes the error,
 - Which raw data are affected.

An example. It is difficult to release the ball from the exact position (what the error is), as the ball is only released by a free hand, which is unsteady (what causes the error). This results in imprecise values of h (which raw data are affected).

- Examples of (i) significant sources of error and (ii) the ways to remedy them.

Source of error	Solution
RV J1 practical sessions	
(J1 P01) It is difficult to maintain protraction, and hence, hard to measure θ precisely.	Clamp the protractor using another retort stand.
(J1 P04) There is significant friction between the hooks and the thread. (This adds systematic error into the measured values of t .)	Use rollers instead of hooks, to minimise the friction experienced by the moving thread.
(J1 P06) The values for current and voltage constantly fluctuate throughout the experiment. This introduces significant random error, which makes the value of c_2 obtained less precise.	Increase the mass of the oil added into the metal significantly. (So the measured values of t all increase in magnitude, reducing the effect of the random error...)
(J1 P07)	<ol style="list-style-type: none"> Conduct the experiment using a sufficiently large hanger mass, such that the circular motion is more horizontal. Take a video of the experiment and analyse the data collected using tracking software.
(J1 P08) It is difficult to track the oscillating mass when the amplitude of the oscillations is small and the period is short.	<ol style="list-style-type: none"> Change the two springs to one equivalent spring of similar effective spring constant. Use a heavier mass but which does not cause the springs to exceed their limit of proportionality. Provide a visual mark near the oscillating mass to help in tracking its movement.
RV J2 practical sessions	
(J2 P04) The precise point where resonance occurs is not easily determined when the vibration of the wire is not significant enough to be observed.	<ol style="list-style-type: none"> Increase the magnetic flux density by using a stronger magnet, e.g. neodymium magnets. Add a white grid (e.g. graph paper) behind the wire, so that it is easier to estimate and compare the magnitude of oscillation.
(J2 P06) <ol style="list-style-type: none"> It is hard to ensure that the oscillation is only about the vertical axis. Moreover, the ruler tends to swing back and forth, affecting the measurement of T. It is hard to measure the angle freehand using a protractor without disturbing the equilibrium of the system. 	<ol style="list-style-type: none"> Keeping the ruler in the horizontal position will make it rotate more easily about the vertical axis. Secure the protractor to another retort stand, so that the protractor can be brought near to the system to read off the angle.
NY practicals	
The large size of the object resulted in a larger uncertainty in the measurement.	Use a thinner/smaller object

The mark is larger than the smallest division of ____. It is difficult for the naked eye to pinpoint the exact moment when the object crosses the mark.	Use a smaller mark. Take a video of the experiment. Then, using tracking software, find the exact time when the object crosses the mark.
(Experiment 01) 1. Human reaction time. 2. The rubber band is thicker than the smallest division of the ruler used and this adds error to the measured values of s.	1. Place two photogates, held by retort stands, at [location of gates]. Then, connect both photogates to an electronic timer. We use this photogates set-up to start and stop the measurement of the time t. 2. Replace the rubber band with a brightly coloured thin string and tie a loop around the cylinder.
(Experiment 02) One's hand is not steady and cannot release the object from the same velocity/position consistently.	Use a ruler/retort stand/etc to hold the object in a stable position before release so the initial velocity is constant.
(Experiment 05) The upper string might not be parallel to the ground as it is just an estimate by the human eye. Hence, the value of h would be inaccurate.	A ruler and a set square can be used to ensure that the string is perpendicular to the retort stand, which is perpendicular to the tabletop. Hence, this makes the values of h more accurate.
(Experiment 07) 1. It is difficult to ascertain the extreme position of the ball accurately as it is only at rest instantaneously. This results in imprecision. 2. It is difficult to release the ball with constant height.	1. Use of a motion sensor/video. 2. Use of a release mechanism.
(Experiment 13) Kinks in the wire.	Use a wingbolt to tighten the wire.
(Experiment 15) The paper cone is flimsy and cannot hold a perfect circular shape at the base of the cone. Hence, this causes the values at θ to be smaller than expected. So, precision is lost for the values of h, y, and x.	Use a plastic cone, so that the measurements for h, x, and y are more precise, as the shape of the cone cannot change.

Table 20.8: Additional examples of sources of errors and the corresponding solutions.

20.5 Planning questions

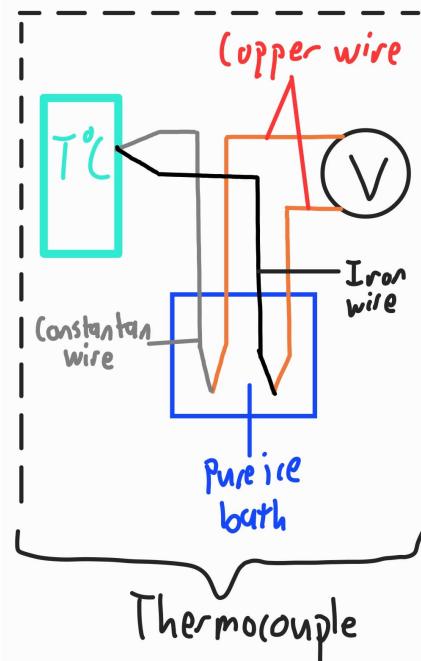
Context	Sentence
A solar panel heats water by absorbing intra-red radiation from the Sun. It consists of an array of pipes, through which water is passed... Design an experiment to determine the efficiency of a model solar panel.	Check that there is a significant temperature difference between the two thermometers. If not, adjust the power of the lamp. This will help us to determine the range of suitable values of intensity that can be used.
The efficiency of a motor is thought to depend on the angular velocity ω of the motor. The relation between the efficiency and the angular velocity ω of the motor may be written in the form $\eta = a\omega^b$, where a and b are constants...	Check for the appropriate masses and heights such the mass can be raised when adjusting the angular speed of the motor using the variable power supply.

Table 20.9: Examples on preliminary readings.

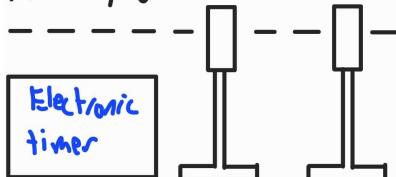
- For *short* planning questions, we can reference the question's diagram(s). But we must include *our own* for *long* planning questions.
- Always include a bench to avoid floating equipment.
- In testing for a linear relationship, we must state that the best fit line should be a *straight line*. If we're testing for direct proportionality, this line should also *cut through the origin*.
- A “○” denotes a point outside of accuracy/precision/safety precautions.

Topic	Accuracy/Precision	Safety
General	<ul style="list-style-type: none"> ✓ Always take preliminary readings. ✓ Measure t thrice and take the average. ✓ Remove all other sources of _____ (sound/light/etc) before conducting the experiment, to reduce systematic/random error. (circuits e.g.) (sound e.g.) 	

- | | | |
|----------------|--|---|
| <p>Nuclear</p> | <ul style="list-style-type: none"> ✓ Preliminary readings. E.g. Determine the range of distances d and thicknesses t of front plates that will give a measurable difference in the count rate. ✓ Repeat the measurements for C_b and C_T thrice and take the average. ✓ Measure the background count rate C_b and subtract it from the total count rate C_T, to obtain the count rate $C = C_T - C_b$ of the sample. | <ul style="list-style-type: none"> ✓ The source is handled with a pair of tongs, to prevent contact with the radioactive material. ✓ The source is stored in a lead lined box when not in use, to prevent contact with the radioactive material. ✓ Cordon off the area with tape and put up a warning sign, to prevent others from being exposed to radioactive material. ✗ Avoid pointing the source at people. ✗ Do not look directly at the source. ✗ Protective clothing, lead suits, lead gloves, goggles. |
|----------------|--|---|
-
- | | | |
|----------------|--|---|
| <p>Thermal</p> | <ul style="list-style-type: none"> ✓ Check calibration of thermometer (mercury-in-glass/temperature sensor) against melting pure ice and pure steam. ✓ A constant temperature environment can be set up using a sand bath, water bath, or putting the object into an oven. ✓ Enclose the object to prevent convection currents from forming in the air. ○ Connect the thermocouple to a voltmeter. Calibrate The thermocouple by placing one junction of the thermocouple into water at ice point (pure ice-water mixture) and the other into steam point (boiling pure water). Allow the voltage to stabilise and record the voltage reading as V_0. Remove the junction at steam point and place it on the _____. Record the voltage reading as V. Then, | <ul style="list-style-type: none"> ✓ Wear heat insulating gloves. ✓ Avoid touching the hot objects. |
|----------------|--|---|



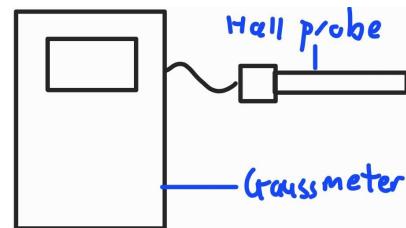
- $$T = \frac{V}{V_0} \cdot 100^\circ\text{C.}$$
- Place the temperature sensor in the _____. Connect the temperature sensor to a datalogger and computer to generate a graph of temperature against time.

	– Light
	✓ Do not look directly at the light source, or point it directly at anyone.
	✓ Wear a pair of laser safety glasses (or shades for non-lasers) to protect the eyes.
	✓ Wear heat insulating gloves when moving the hot light source.
	– Sound
Waves	<ul style="list-style-type: none"> <li style="margin-bottom: 10px;">✓ Conduct the experiment in a soundproofed room, to minimise random error due to environmental noise.
	– Sound
	<ul style="list-style-type: none"> <li style="margin-bottom: 10px;">✓ Wear ear muffs to prevent hearing damage. <li style="margin-bottom: 10px;">✓ Switch on the sound source for a short period of time; turn it off when the apparatus is not in use.
	<hr/>
	✓ Explain clearly how to find the frequency of vibration f . Starting with the highest available frequency, the frequency of strobing is gradually reduced till the vibrating object appears stationary. Record the frequency of vibration f as the frequency of strobing.
Mechanics	<ul style="list-style-type: none"> <li style="margin-bottom: 10px;">✓ Explain clearly how to use a pair of photogates with an electronic timer to obtain the time taken t. The object is projected so that it passes through both photogates. Record the times at which it passes through each gate as t_1 and t_2. Calculate the time taken $t = t_2 - t_1$. <p>Path of projectile</p>  <p>When the distance between the gates is small, the average velocity \approx instantaneous velocity.</p>
	– Water/oil
	✓ Mop up any spillage of oil/water to avoid injuries due to falling.
	– Load
	✓ Wear safety boots to reduce possible injuries to your feet.
	– Moving parts
	✓ Avoid the moving blades of the fan using a safety screen. Switch off the fan when the apparatus is not in use.
	– Glass
	✓ Wear thick safety goggles and thick gloves to prevent injury when handling the glass.

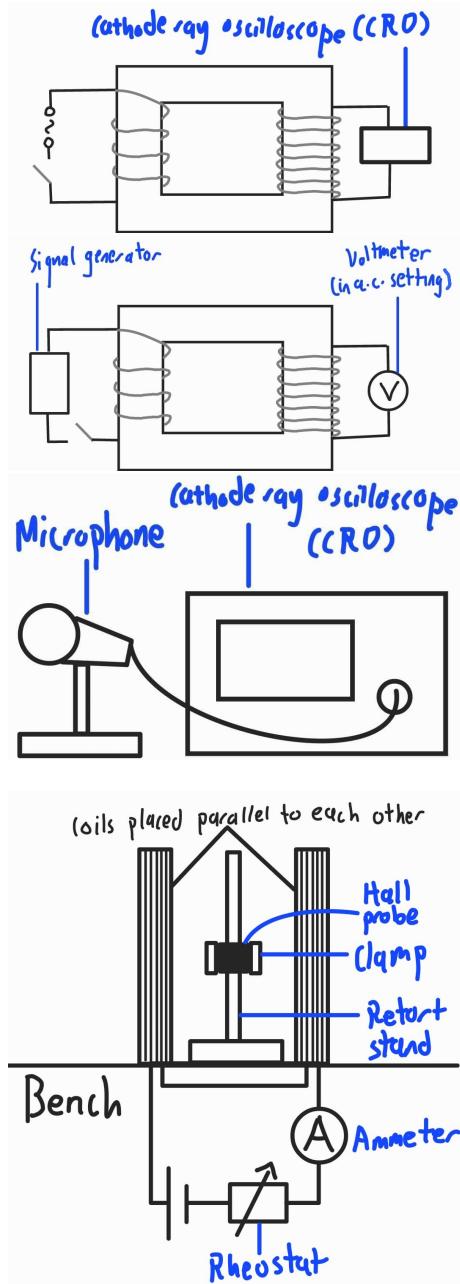
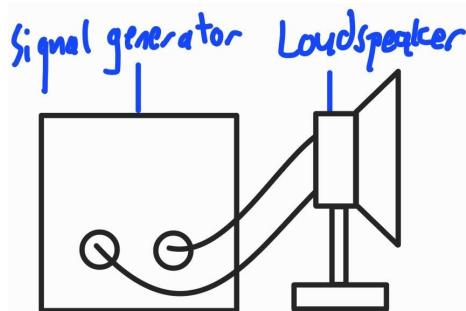
E&M

-
- ✓ Preliminary readings. E.g. Determine the range of frequencies f of e.m.f. and cross sectional area of the coil X that lead to a measurable range of induced e.m.f. ε .
 - ✓ Use an iron core to increase the e.m.f. induced.
 - ✓ Detailed description on how to use the cathode ray oscilloscope (c.r.o.) to obtain raw data. E.g.
 1. Using the grid on the screen, measure the maximum vertical distance y occupied by a complete waveform. Multiply y by the scale indicated on the Y-gain to obtain the peak voltage V_0 .
 2. Using the grid on the screen, measure the horizontal distance x occupied by a complete waveform. Multiply x by the scale indicated on the time base to obtain the period T .

(The c.r.o. should be connected in parallel for the above.)
 - ✓ Remove all other sources of alternating magnetic fields, which lead to systematic error, before starting the experiment.
 - ✓ Connect a Hall probe to a Gaussmeter. Calibrate the probe using a magnetic field of known strength. Then, orient the hall probe till a maximum voltage reading V_0 is obtained. Repeat this a few times at different positions; the magnetic field is uniform if V_0 remains approximately constant.



- o Specify which setting, d.c. or a.c., an ammeter or voltmeter should be used in.
- o A c.r.o. can either be connected directly to an electrical circuit, or to a microphone.
- o A signal generator is an a.c. source with variable frequency, amplitude, and waveform. It can be connected to a loudspeaker.



- A uniform magnetic field can be produced using a Helmholtz coil. If necessary, a hall probe can be placed, perpendicular to the magnetic field/perpendicular to the plane of the paper.

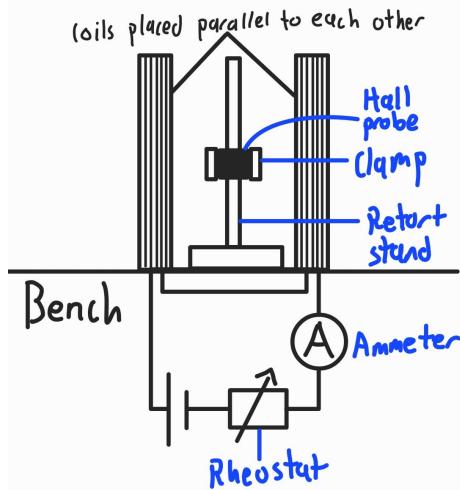


Table 20.10: Precautions and some miscellaneous notes.

(All figures in table 20.10 are attributed to [22].)

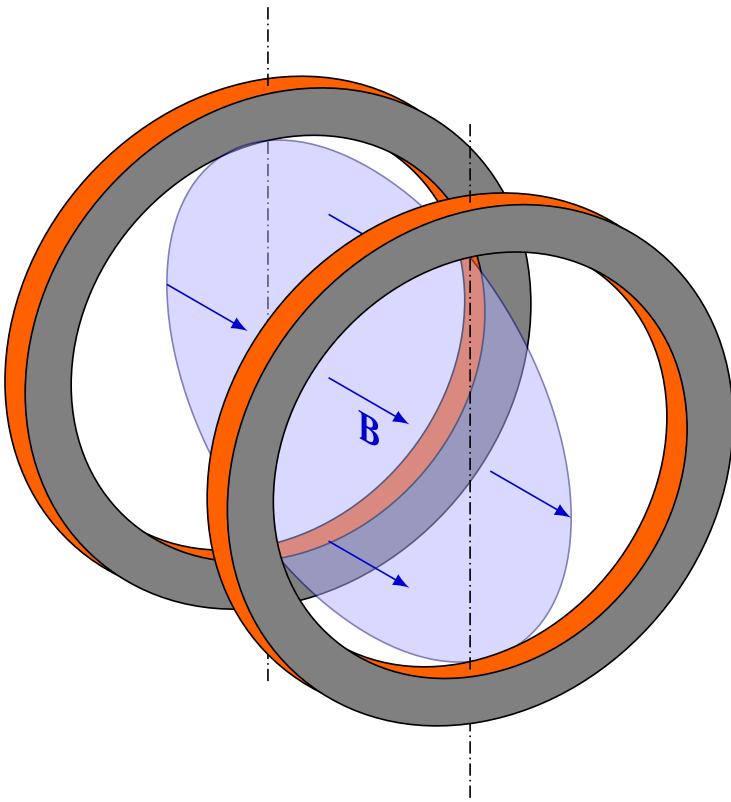


Figure 20.4: [62] A Helmholtz coil.

- Other instruments to know: newton meter, flow rate meter (dV/dt), mass flow meter (dm/dt), pressure gauge, light intensity meter, sound level meter.

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