1. *Let $P(x) = \sum_{i=0}^{n} a_i x^i$ be a polynomial of even degree n, such that a_n is positive and a_0 is negative. Prove that $P(x)$ has at least one positive real root and one negative real root.*

Proof. Recall that $\lim_{x\to\pm\infty} 1/x = 0$. By limit laws, $\lim_{x\to\pm\infty} \sum_{i=0}^{n} a_i x^{i-n} = 0$. Therefore, there exists $M > 0$ so $a_n > \sum_{i=0}^{n-1} -a_i M^{i-n}$. That is, $P(M) > 0$. Since $P(0) < 0$ and all polynomials are continuous, there is a positive root of $P(y) = 0$ by the Intermediate Value Theorem. Similarly there is also a negative root. □