Physics

Cambridge GCE A-Levels

Notes by Grass

Contents

Kinematics

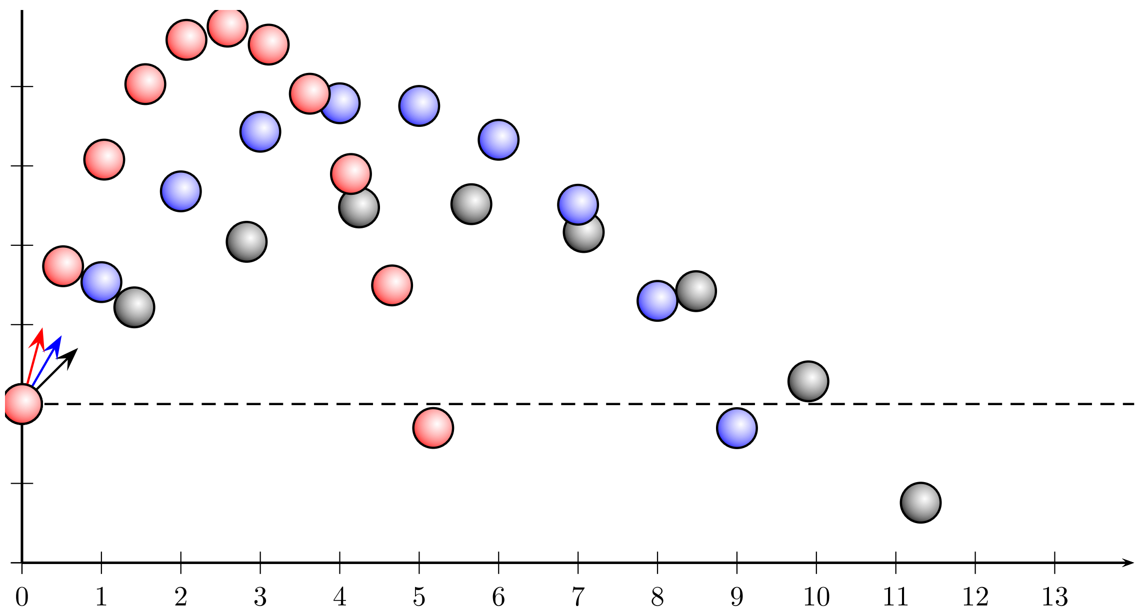


Figure 1.1: [1] Parabolic path travelled by balls thrown at varying angles.

- *Distance* is defined as the total length of *path* travelled.
- *Velocity* is defined as the rate of change of displacement.
- *Acceleration* is defined as the rate of change of velocity.

Dynamics

- *Newton's First Law of Motion* states that an object at rest will remain at rest and an object in motion will remain in motion at constant velocity in a straight line in the absence of an *external* resultant force.
- The *linear momentum* of a body is the product of its mass and velocity. The linear momentum is in the *same direction* as it velocity.
- *Newton's Second Law of Motion* states that the rate of change of momentum of a body is directly proportional to the resultant force acting on the body and occurs *in the direction* of the resultant force.
- *Newton's Third Law of Motion* states that if body A exerts a force on body B, then body B exerts a force of the *same type* that is equal in magnitude and opposite in direction on body A.
- *Impulse* is defined as the product of *average* force acting on an object and the time for which the force acts.
- The *Principle of Conservation of Linear Momentum* states that the total momentum of a system remains constant provided no *external* resultant force acts on the system.

Forces

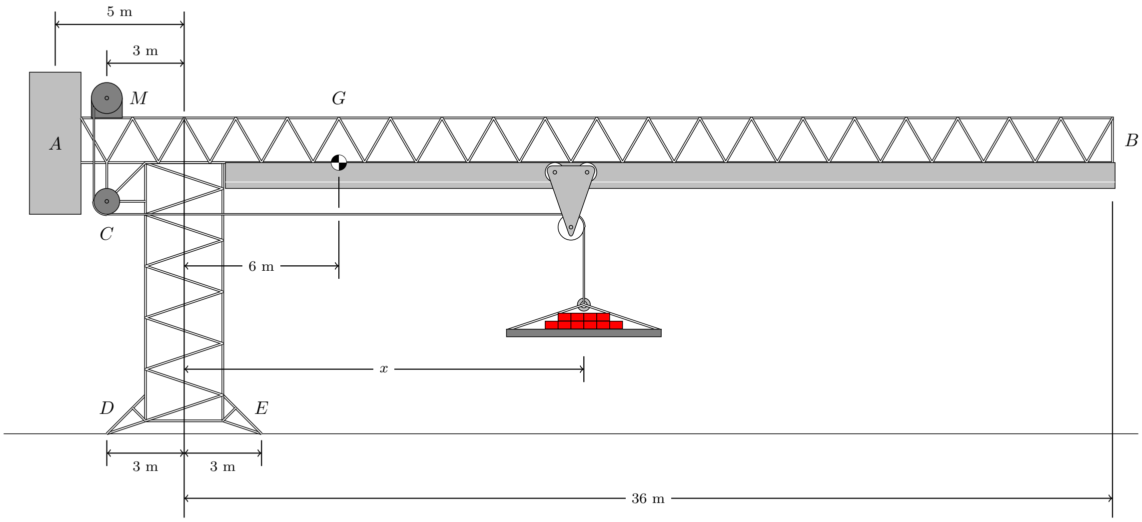


Figure 3.1: [2] Forces acting on a crane.

- *Hooke's Law* states that the force is directly proportional to the extension in a material if its *limit of proportionality* is not exceeded.
- The *center of gravity* of an object is the point at which the entire weight of a body may be considered to act.
- The *moment* of a force is equal to the product of the force and the *perpendicular* distance of the *line of action* of the force from the pivot. It is also the turning effect of a force.
- *Torque of a couple* is defined as the product of one of the forces and the *perpendicular* distance between the *lines of action* of the forces.
- The *Principle of Moments* states that if a body is in equilibrium, the sum of all the clockwise moments about *any axis* must be equal to the sum of anticlockwise moments about the *same axis*.
- *Density* is defined as the mass per unit volume of a substance.
- *Pressure* is defined as force per unit area, where the force is *acting perpendicularly* to the area.
- Deriving $p = \rho g h$:
  1. Consider a point at a depth h below the surface of a liquid of density ρ.
  2. The force F acting perpendicularly on a surface area A at depth h is due to the weight of the liquid column above A to give pressure p. Thus, $p = \frac{F}{A} = \frac{mg}{A} = \frac{\rho A h}{g} = \rho g h$.
- *Upthrust* is the upward force exerted by a fluid on a body immersed in the fluid (due to pressure difference in the fluid).
- *The origin of upthrust:* Upthrust is a result of the pressure difference between top and bottom surfaces of the body, resulting in a net upwards force being exerted on the body by the third medium in which the body is located.
- *Archimedes' Principle* states that when a body is totally or partially immersed in a fluid, it experiences an upward force (upthrust) equal to the weight of fluid displaced.

- *The Principle of Floatation* states that, for any object floating in *equilibrium*, the upthrust is equal to the weight of the object.

Work, Energy, and Power

- *Work done* is defined as the product of a force and the displacement in the direction of the force.
- *One joule of work* is defined as the work done by a force of 1 Newton when its *point of application* moves through a distance of 1 metre in the direction of the force.
- *Energy* is defined as the ability to do work.
- *The Principle of Conservation of Energy* states that energy can neither be created or destroyed in *any process*. It can be transformed from one form to another, and transferred from one body to another.
- Deriving $E_k = \frac{1}{2}mv^2$:
  1. Consider a constant horizontal applied force F acting on an object of mass m travelling with initial velocity u to reach a final velocity v over a displacement s.
  2. For uniform acceleration, $v^2 = u^2 + 2as$ so $as = \frac{1}{2}(v^2 - u^2)$. Combined with Newton's Second Law, $W = Fs = mas = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$. When the object starts from rest, $u = 0$.
  3. By conservation of energy, *the work done by force* F *must be converted into the kinetic energy* E_k *of the object*. Hence, $E_k = W = \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}mv^2$.
- The *Work-Energy Theorem* states that the net work done by *external* forces acting on a particle is equal to the change in kinetic energy of the particle.
- Deriving $E_p = mgh$:
  1. Consider an object from the Earth's surface — which is taken as the reference for zero gravitational potential energy — raised up by a *constant force* F *equal to and opposite to the weight* mg of the object such that the object moves up at *constant velocity* to a height h_2.
  2. Thus, the object moves at constant speed so $\Delta E_k = 0$. Therefore,

     $$\begin{aligned} \Delta E_p &= W \\ E_p - 0 &= Fs \\ E_p &= mgh. \end{aligned}$$

     Where E_p is the gravitational potential energy at height h above the Earth's surface.
- Know how to $\Delta E_p = \frac{1}{2}kx^2$ from area under graph.
- *Power* is defined as the rate of doing work.
- Derive $P = Fv$: $P = \frac{dW}{dt} = \frac{Fds}{dt} = Fv$.

Temperature and Ideal Gases

- The *Zeroth Law of Thermodynamics* If bodies A and B are separately in thermal equilibrium with body C, then bodies A and B are in thermal equilibrium with each other.
- *One mole* is defined as the amount of substance that contains as many elementary particles as there are atoms in 0.012kg of carbon-12.
- *Avogadro's Constant* N_A is the number of atoms in 0.012kg of carbon-12.
-

	Assumptions of the Kinetic Theory of Gases
M	The molecules of the gas are in *rapid* and *random* motion.
A	There are *no intermolecular* attractive forces.
N	Any gas consists of a *very large number* of molecules.
T	The duration of collisions is negligible compared to the time interval between collisions.
E	The collisions between gas molecules, and between gas molecules and the container walls are *perfectly elastic*.
V	The volume of the gas molecules themselves is negligible compared to the volume of the container.

- When do real gases behave like ideal gases: At high temperatures and low pressures.
- Deriving $p = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$:
  1. Consider a cubic container of side l containing N molecules, each of mass m.
  2. Change in momentum due to *elastic* collision between wall and molecule = $2mc_x$
  3. Time interval between collisions, $\Delta t = \frac{2l}{c_x}$.
  4. By Newton's 2nd Law, $F = \frac{2mc_x}{\frac{2l}{c_x}} = \frac{mc_x^2}{l}$.
  5. Since $A = l^2$, Pressure due to 1 particle, $p = \frac{mc_x^2}{l^3} = \frac{mc_x^2}{V}$.
  6. Pressure due to N particles, $p_N = \frac{Nmc_x^2}{V}$.
  7. By Pythagoras'Theorem, $c^2 = c_x^2 + c_y^2 + c_z^2$. The average speed in the x, y, and z directions can be taken to be $c_x = c_y = c_z$ so $c^2 = 3c_x^2$. Now, $p_N = \frac{Nm\langle\frac{1}{3}c^2\rangle}{V} = \frac{1}{3}\frac{Nm\langle c^2\rangle}{V}$.

First Law of Thermodynamics

- The *heat capacity* of a body is defined as the amount of thermal energy required to raise its temperature by one Kelvin / degree Celsius.
- The specific *heat capacity* of a body is defined as the amount of thermal energy required to raise the temperature of one unit mass of the substance by one Kelvin / degree Celsius.
- The *specific latent heat* of a body is defined as the thermal energy required to change *phase* of one unit mass of a substance, *without a change in temperature*.
- *Internal energy* of a system is a sum of *random distribution* of kinetic and potential energy *associated with the molecules* of the system.
- The *First Law of Thermodynamics* states that the *increase* in internal energy of a closed system is the *sum* of heat *supplied* to the system and the work done *on* the system.

Circular Motion

- *Angular displacement* is the angle through which an object turns *with respect to the center* of the circular path.
- *The radian* is defined as the angle *subtended* at the *center* of a circle by an *arc* of length equal to the radius of the circle.
- *Angular velocity* is the rate of change of angular displacement.

□

$\omega = \dfrac{2\pi}{T} = 2\pi f$	$v = r\omega$	$a_c = \dfrac{v^2}{r} = r\omega^2 = v\omega$	$F_c = ma_c$

□ Common formulae: $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$, $v = \sqrt{rg}$.

□ Water in bucket at top position: $F_c = N + W$ (where $N \geqslant 0$) so $\omega > \sqrt{\frac{g}{r}}$.

□ Need to write "Centripetal force is provided by ______"

Gravitational Fields

- *Newton's Law of Gravitation* states that the force of attraction between any two point masses is directly proportional to the product of their masses and inversely proportional to the square of their separation.
- A *gravitational field* is a region in space where mass experiences a gravitational force acting on it.
- Gravitational field strength at a point is defined as the gravitational force per unit mass acting on a small mass placed at that point
- The *gravitational potential energy* of a mass at a point is defined as the work done by an *external agent* in bringing the mass *from infinity* to that point (without any net change in kinetic energy).
- *Gravitational potential* at a point is defined as the work done per unit mass by an *external agent* in bringing a mass *from infinity* to that point (without a change in kinetic energy).
- Escape velocity is the *minimum* velocity a mass needs to be projected from the *surface* of the moon in order to have sufficient kinetic energy to overcome the gravitational field it experiences and *move to infinity*.
- Escape velocity $v_{min} = \sqrt{\frac{2GM}{r}}$ (where Min E_k needed is the gain in E_p to reach infinity).

□

$$\begin{tikzcd} U_G = -\frac{GMm}{r} \arrow[r, "-\frac{d}{dr}"] \arrow[d, "\frac{1}{m}"'] & F_G = -\frac{GMm}{r^2} \arrow[d, "\frac{1}{m}"'] \\ \phi = -\frac{GM}{r} \arrow[r, "-\frac{d}{dr}"] & g = -\frac{GM}{r^2} \end{tikzcd}$$

□ $U_G = m\phi$ & $\Delta U_G = m\Delta\phi$.

□ U_G is negative because infinity is taken as the reference point for zero potential energy. The work done against gravitational force in bringing a mass from infinity to that point is negative.

□ Gravitational force provides the centripetal force:

$$F_G = F_c$$
$$\frac{GMm}{r^2} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$
$$T^2 = \frac{4\pi^2}{GM}r^3$$
$$T^2 \propto r^3$$

□ Gravitational force provides the centripetal force:

$$F_G = F_c$$

For A: $$\frac{Gm_A m_B}{(r_A + r_B)^2} = m_A r_A \omega^2$$

For B: $$\frac{Gm_A m_B}{(r_A + r_B)^2} = m_B r_B \omega^2$$

The center of mass of the system is at point P where

$$m_A r_A = m_B r_B$$

such that both stars have the same angular velocity ω.

- For binary star systems, notice that *orbital radius is replaced by the stars' separation*:

$$\begin{aligned} m_A r_A &= m_B r_B \\ r_A + r_B &= \frac{m_B}{m_A} r_B + r_B \\ r_B &= \frac{m_A}{m_A + m_B}(r_A + r_B) \end{aligned} \qquad \text{so} \qquad \begin{aligned} \frac{G m_A m_B}{(r_A + r_B)^2} &= m_B r_B \omega^2 \\ &= \frac{m_A m_B}{m_A + m_B}(r_A + r_B)\omega^2 \end{aligned}$$

So, rearranging, we have

$$\omega^2 = \frac{G(m_A + m_B)}{(r_A + r_B)^3} = \frac{G m_A}{r_B (r_A + r_B)^2} \qquad \text{and} \qquad T^2 = \frac{4\pi^2}{G(m_A + m_B)}(r_A + r_B)^3.$$

- Geostationary orbit facts:
  1. Only one such orbit at a *fixed* distance of 4.2×10^7m from Earth's center,
  2. Orbital period of 24 hours,
  3. Satellite's plane of orbit coincides with the equatorial plane of the Earth,
  4. Orbits West to East (in the same direction as Earth's rotation).

- Equipotential lines are not equally spaced because the gravitational field strength is not constant but decreases as one goes away from the Earth.

- Assumptions made in the theory (e.g. in deriving $g = -\frac{GM}{r^2}$):
  1. The bodies are separated by distances so large they can be considered as point particles (i.e. separation » radius).
  2. The bodies are homogenous spheres (constant density throughout the sphere).
  3. The bodies have masses distributed symmetrically around the center in uniform layers.
  4. In the absence of other masses.

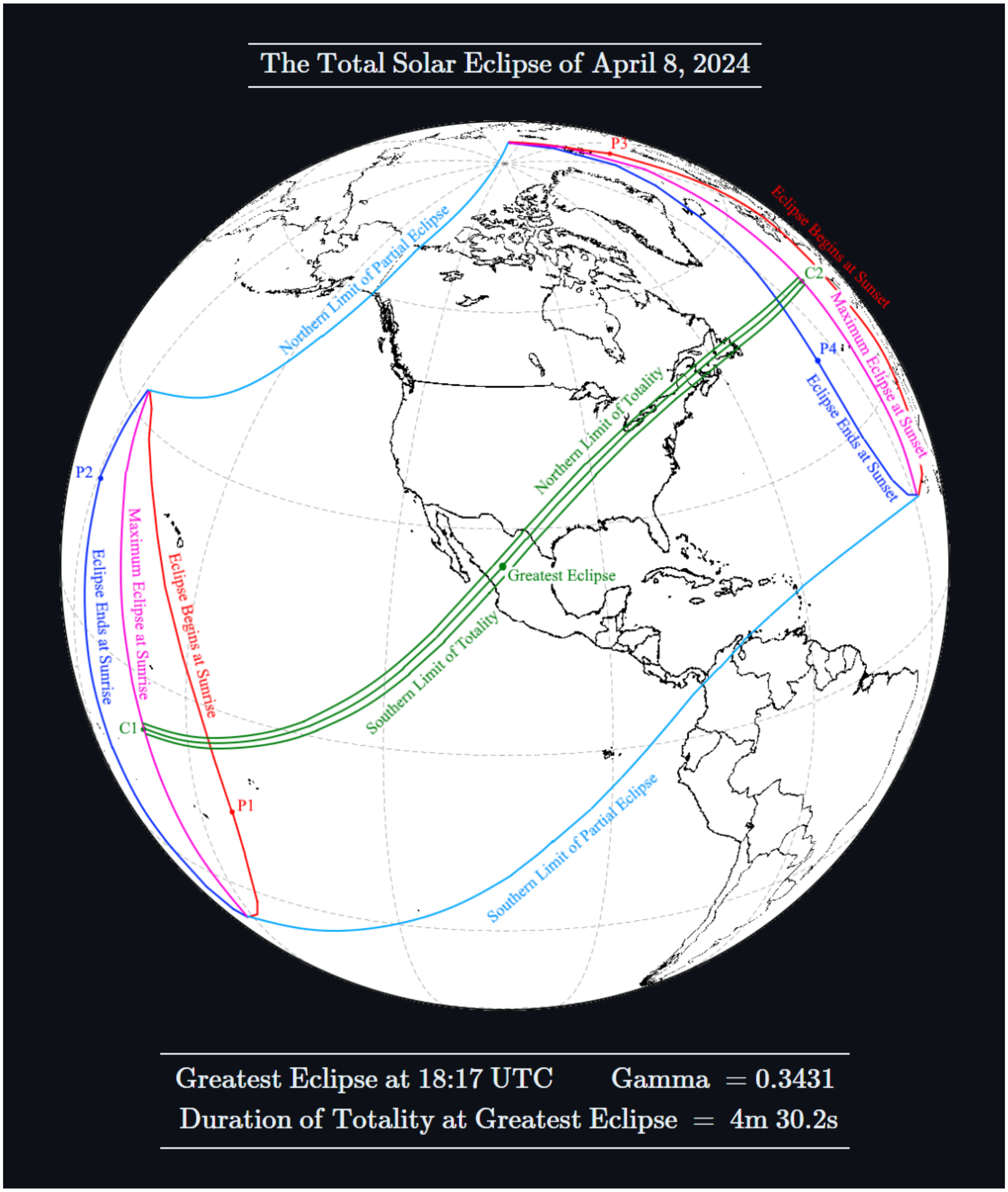


Figure 8.1: [3] Forces acting on a crane.

Note. According to the author of this image, when compared to their above prediction,

1. The duration of eclipse at greatest eclipse is 2 seconds shorter.
2. The moments of contact are a couple seconds off.

Oscillations

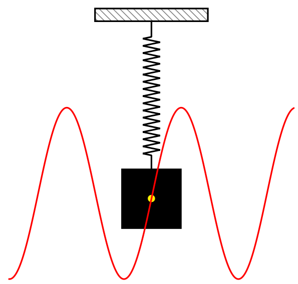


Figure 9.1: [4] Simple harmonic motion.

- *Simple harmonic motion* is defined as the motion of a body whose acceleration is directly proportional to its displacement from a fixed point (equilibrium position) and is always directed towards that fixed point.
- A *freely oscillating* system oscillates at its own *natural frequency* without *external* influences other than the *initial impulse when displaced* from its equilibrium position, with *no dissipation* of energy.
- *Damped oscillations* are oscillations in which the amplitude diminishes with time as a result of *dissipative forces* that reduce the total energy of the oscillations.
- A system is in *forced oscillations* when it is forced to oscillate at a frequency other than the natural frequency by a *periodic external* force.
- *Resonance* is a phenomenon that occurs when the frequency at which an object is being made to vibrate (the forced frequency of vibration) is equal equal to its natural frequency of vibration.

□

$v = \pm\omega\sqrt{x_0^2 - x^2}$	$a = -\omega^2 x$	Spring-Mass $T = 2\pi\sqrt{\frac{m}{k}}$	Pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

□

	E_k	E_p	E_T
t	$\frac{1}{2}m\omega^2 x_0^2\cos^2(\omega t)$	$\frac{1}{2}m\omega^2 x_0^2\sin^2(\omega t)$	$\frac{1}{2}m\omega^2 x_0^2$
m	$\frac{1}{2}m\omega^2(x_0^2 - x^2)$	$\frac{1}{2}m\omega^2 x^2$	$\frac{1}{2}m\omega^2 x_0^2$

□ Simple pendulums and mass spring systems can be approximated to be SHM when the angle of oscillation ($\leqslant 20°$) and oscillation amplitude are small, respectively.

□

	In Phase	Antiphase	Out of Phase
$\Delta\phi$/rad	0	π	nonzero

□ When damping increases:

- Amplitude at *all* frequencies decreases.
- (Resonance) frequency at max amplitude shifts gradually to lower frequencies.
- Peak (max amplitude) becomes flatter.

Wave Motion

- A *progressive* wave is a wave in which *energy is carried* from one point to another by means of *vibrations or oscillations* within the wave. Particles within the wave are *not transported along* the wave.
- A *phase* is an angle which gives a measure of the *fraction of a cycle* that has been *completed* by an oscillating particle or by a wave.
- *Intensity* of a wave is the wave energy incident per unit time per unit area *normal* to the wave.
- *Polarisation* of a wave refers to the *confinement* of oscillations in *only* one plane. The plane of oscillations is *parallel* to the direction of energy transfer.
- Malus' Law states that the intensity of a beam of *plane polarised light* after passing through a rotatable polariser is directly proportional to the square of the cosine of the angle through which the polariser is rotated from the position that gives maximum intensity.
  ($I = I_{max} \cos^2(\theta)$)

□

Phase Difference $\Delta\phi$	$\frac{2\pi}{\lambda}\Delta x$	$\frac{2\pi}{T}\Delta t$

□

Intensity			
Amplitude	Wave		
	Spherical	Circular	Plane
$I \propto A^2$	$I \propto \frac{1}{r^2}$	$I \propto \frac{1}{r}$	I is constant (No spreading of waves)

☆ When unpolarised light passes through a polariser, the average value of $\cos^2(\theta)$ is $\frac{1}{2}$ so $I_{new} = \frac{1}{2} I_{max}$.

Superposition

- *Principle of Superposition*: When two or more waves of the *same type*, meet at *a point in space*, the *resultant displacement* of the waves at any point is the *vector sum* of the *displacements* due to *each wave acting independently* at that point.
- *Stationary waves* are waves whose *waveforms does not advance* and there is *no net translation of energy*. The *amplitude* of the waves varies according to *position* from zero at the nodes to a maximum at the antinodes.
- A stationary wave is formed when two *progressive* waves
  1. Having the *same frequency* and *same speed*
  2. Travel in *opposite directions* towards each other
  3. Have *similar amplitudes*
  4. Are unpolarised, or polarised along the same axis
  5. Are *superposed*
-

Properties	Reflection Surface	
	Loose End[1]	Fixed End
Allows for Oscillations?	Yes	No
Will Reflected Wave be Inverted (phase change of π)?	No	Yes

- Characteristics of Stationary Waves:
  1. Displacement node = Pressure antinode
  2. Displacement antinode = Pressure node

Properties	Stationary Wave	Progressive Wave
Energy	No net transfer of energy	Energy is transferred in the direction of propagation of the wave.
Phase	□ Adjacent nodes: In phase □ Adjacent segments: Antiphase. (Fig 12.1)	All points within one wavelength have different phases.
Amplitude	Varies: 0 at nodes to max at antinodes.	Same for all particles.
Wavelength	Twice the distances between adjacent nodes or adjacent antinodes.	Distance between adjacent in-phase particles.
Frequency	Same for all particles	
Nodes[2]/Antinodes[3]	✓	×

[1]Particles of the wave can move about freely.
[2]At which particles don't oscillate/amplitude = 0.
[3]At which particles have the largest amplitude.

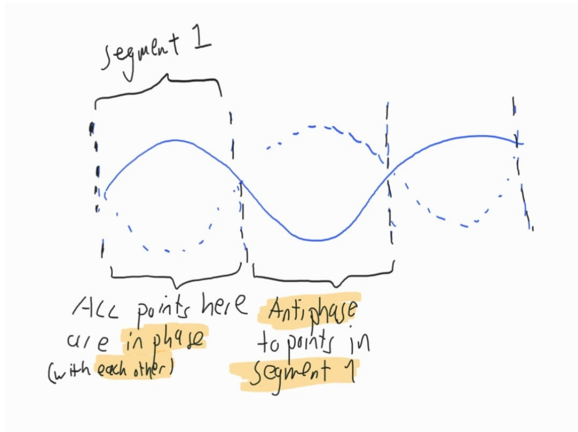


Figure 11.1: [6] Phases in stationary waves

	Modes		Diagrams	Wavelength	Frequency
	Overtone	Harmonic			
Strings/Open Pipes	nth	$(n+1)$th	12.2 & 12.3	$\lambda = \frac{2L}{n+1}$	$f = (n+1)\frac{v}{2L}$
Pipes Closed at One End		$(2n+1)$th	12.4 & 12.5	$\lambda = \frac{4L}{2n+1}$	$f = (2n+1)\frac{v}{4L}$

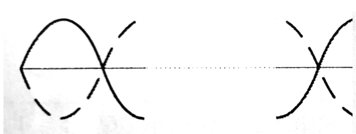


Figure 11.2: [5] String.

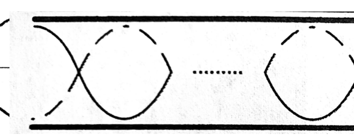


Figure 11.3: [5] Open pipe.

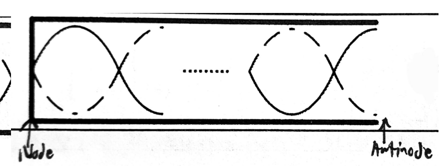


Figure 11.4: [5] Pipe closed at one end

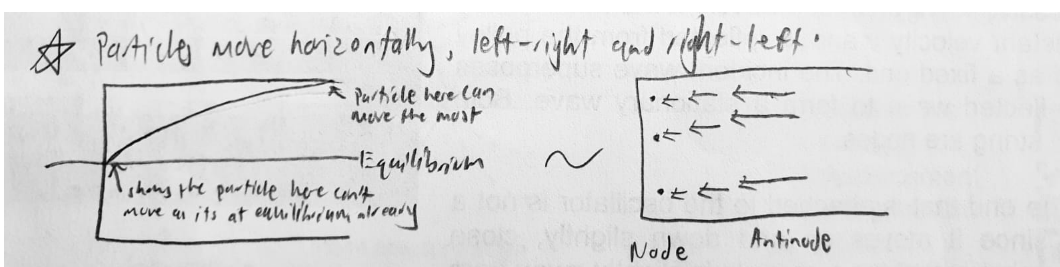


Figure 11.5: Movement of particles in pipe.

- Resonance length[2] with pipes closed at one end

$$L = \frac{\lambda}{4} = \frac{v}{4f}.$$

[2]End correction: Actual length of vibration is $L + 2c$ for open pipes, and $L + c$ for closed pipes.

Example 11.1

Explain, with reference to resonance, why the loudness of sound changes as the water level changes.

1. Natural frequency of vibration depends on length of air column.
2. When fork frequency is equal to natural frequency/odd multiple of fundamental frequency, resonance occurs. There is maximum energy transfer and maximum amplitude of vibrations, leading to maximum loudness.
3. When fork frequency is not equal to natural frequency, no resonance occurs and loudness drops.

- If a tube achieves stationary waves at fundamental frequency f, then reducing f/increasing λ will not result in stationary waves.
- *Diffraction* is the *bending or spreading out* of waves when they travel through a *small opening* or when they pass round a *small obstacle*.
- Large amount of diffraction occurs when the width of slit is about the same as the wavelength.
- The wavelength before and after diffraction should be around the same.
- Single Slit Diffraction: Let b be the slit width, and L the slit-screen distance.
  1. For all nonzero integers m, the angular positions θ of the $1 \leqslant m$th order minima satisfies
  $$\sin(\theta) = \frac{m\lambda}{b}.$$
  2. Distance y_1 of the first minima from either side of the central bright fringe is
  $$y_1 = \frac{\lambda L}{b}.$$

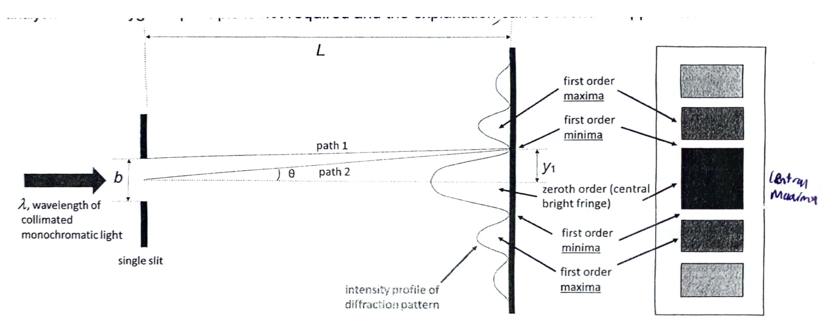


Figure 11.6: [5] Single slit diffraction.

- *Circular* aperture: $\theta \approx \frac{\lambda}{b}$.
- *Rayleigh's Criterion* is the *minimum separation* between two objects in order to be distinguished as two *distinct* objects:
$$\theta \approx \frac{\lambda}{b}.$$
- Sources are *coherent* if they have a *constant phase difference* with respect to time.
- *Interference* is the *superposing* of two or more waves to produce *regions of maxima and minima* in space, according to the *Principle of Superposition*.

- Conditions for an *observable* interference pattern:
  1. The waves must *overlap* to produce regions of maxima and minima.
  2. The *sources* must be *coherent*.
  3. The waves must have approximately the *same amplitude*.
  4. The waves, if transverse, must be *unpolarised* or have the *same plane of polarisation*.
- For $n \in \mathbb{Z}_0^+$, representing the nth order max/min, we have

Sources' Phase Difference	Path Difference	
	Constructive Interference (maxima)	Destructive Interference (minima)
In phase	$\Delta = n\lambda$	$\Delta = (n + 1/2)\lambda$
Out-of-phase	$\Delta = (n + 1/2)\lambda$	$\Delta = n\lambda$

- We always need to take the path difference starting from the actual source itself, even when the source travels through two slits onto a screen, for instance.
- Double-slits: For[3] a (*center-to-center*) slit separation a and slit-screen distance D, the fringe separation (between two adjacent minima, or two adjacent maxima) is

$$x = \frac{\lambda D}{a}.$$

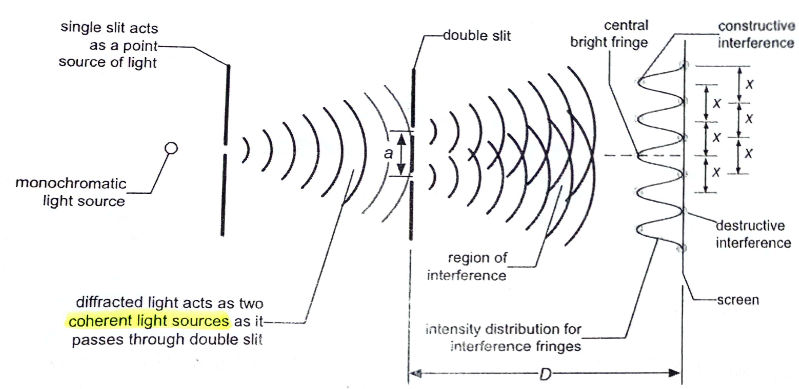


Figure 11.7: [5] Double-slit diffraction.

- Diffraction grating: For a slit-separation d and $n \in \mathbb{Z}_0^+$, the angular positions for the nth order maxima satisfies

$$d\sin(\theta_n) = n\lambda.$$

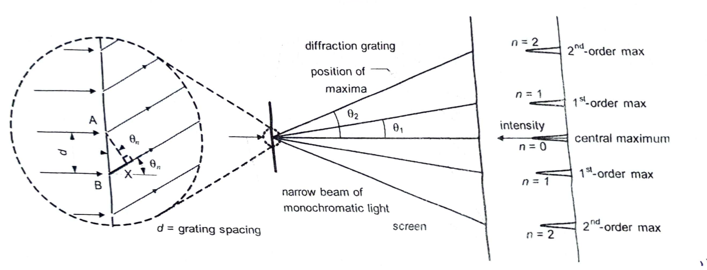


Figure 11.8: [5] Diffraction grating.

[3]Typical values: $a \approx 0.5$mm, $D \approx 1$m, and $\lambda \approx 600$nm. In any case, using the equation requires $a << D$.
[4]The intensity of the double-slit interference pattern is not constant because of single-slit diffraction effects.

- Check answer: For visible light, $400\text{nm} \leqslant \lambda \leqslant 700\text{nm}$.
- Total number of bright regions = 2·highest order+1.
- When slit width is reduced, intensity is reduced.
- To calculate *resultant intensity*, first take the sum of the amplitudes and use proportionality ($I \propto A^2$).
- Every other line means half of the lines are covered.

Example 11.2

Describe and explain the appearance of the central fringe if the light is now replaced with white light.

1. The central bright fringe is generally white.
2. The zeroth order fringes of all the wavelengths coincide at the center where the path difference from the two slits is zero for all wavelengths. The combined central fringes remains white.
3. The sides of the central fringes are likely more reddish.
4. This is because the wider red fringe extends beyond the narrower blue fringe.

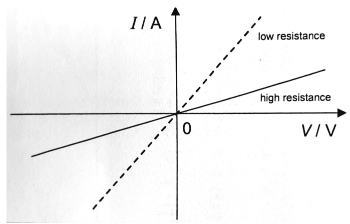
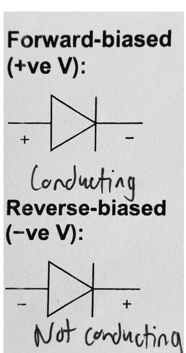
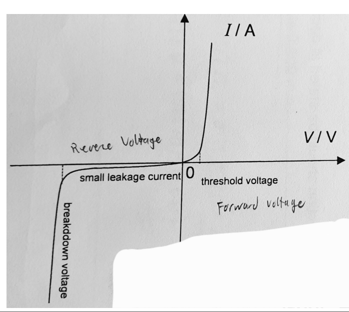
Currents of Electricity

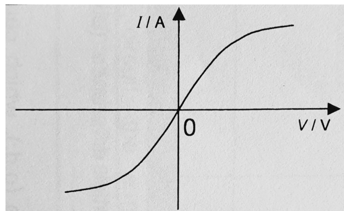
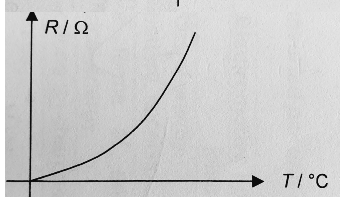
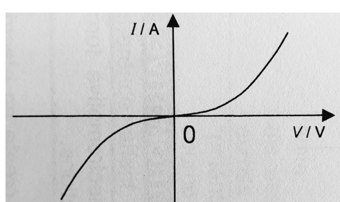
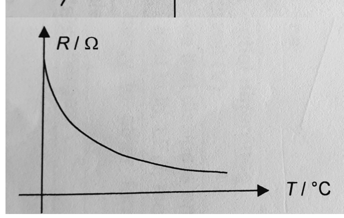
- The *number density* n is defined as the number of particles per unit volume.
- The *drift velocity* v is the *average* velocity at which *charge carriers* move through a *conductor* when there is *electric current in the conductor*.
- Deriving the equation $I = nAvq$:

  1. Assume that the *current is constant*. Then, $I = \frac{Q}{t}$.
  2. Assume that there are N charge carriers passing through a cross-sectional area A in time t, and that *each* of them carries an *identical amount of charge* q. Then, the *total charge* that passing through A in time t is
  $$Q = Nq.$$
  3. Assume that the *number density* n of charge carriers is *uniform*, and let V be the volume covered by the current in time t. Thus,
  $$N = nV.$$
  4. Furthermore, since the current travels at some velocity v,
  $$V = A\Delta x = Avt.$$
  5. Therefore,
  $$I = \frac{Q}{t} = \frac{Nq}{t} = \frac{nVq}{t} = \frac{nAvtq}{t} = nAvq.$$

- Elementary charge $e = 1.6 \times 10^{-19}$C (Charge of an electron/proton).
- The *potential difference* between *two points* of a circuit is defined as the amount of electrical energy *converted* to other forms of energy *per unit charge* moved *between* the two points.
- *Ohm's Law* states that the *current* flowing in a *conductor* is *directly proportional* to the *potential difference across it* under *constant physical conditions*.
- *Resistance* is defined as the *ratio* of the *potential difference across* the conductor to the *current* flowing through it.
- *Resistivity* ρ is the *proportionality constant* between the *dimensions of a specimen of a material* and its *resistance* such that
$$R = \frac{\rho L}{A}.$$

- Electrical Components

Types of Conductors	Changes to Resistance	Reason
Metallic conductor at constant temperature	– At *constant temperature* this is an Ohmic conductor. – Has *constant resistance*. Ratios of V/I is constant.  	– At *constant temperature*, the *number of free electrons* and the *rate of atomic vibration* is constant. – A resistor at a different constant temperature will have a difference resistance, and hence, V/I ratio.
Semiconductor Diode  	– Forward-biased: *Resistance decreases* when *p.d. increases*. In fact, if the forward-biased p.d. goes past its *threshold voltage*, *resistance* becomes very low. – Reverse-biased: *Very high resistance*. But, there will be a *small leakage current*. – If the reverse-biased p.d. is so high that it exceeds the *breakdown voltage*, the diode will *break down* and *conduct electricity*.  	– When connected in *forward bias*, the circuit's *electric field set up* allows for available *charge carriers to flow* through, allowing it to conduct with *low resistance*. – When connected in *reverse bias*, the circuit's electric field set up creates a *widened 'depletion region'* in the diode which *impedes charge carriers* from flowing through the region, creating a *large resistance*.

Filament Lamp	– When *p.d. increases*, *current increases*, with *decreasing* $\mathrm{I}-\mathrm{V}$ *ratio*. – The *resistance* of a metallic conductor *increases* with an increase in *temperature*.  	– As *current increases*, *power dissipated increases* since $\mathrm{P} = \mathrm{I}^2\mathrm{R}$. Heat is generated so *equilibrium temperature rises* — as electrons drift through the metal, they collide with the metal lattice and transfers energy to it. – The *lattice ions vibrate more vigorously*. This *hinders the flow* of 'charge carriers'. Therefore, *resistance is increased*. – Ohmic conductors hence do *not* obey Ohm's Law at *high* voltages/currents.
Negative Temperature Coefficient (NTC) Thermistor	– When *p.d. increases*, *current* through the thermistor also *increases*, with *increasing* $\mathrm{I}-\mathrm{V}$ *ratio*. – The *resistance* of a thermistor *decreases* with an *increase* in *temperature* (This is the meaning of NTC).  	– As *current increases*, *power dissipated increases*. *More heat* is generated, leading to a *rise* in *equilibrium temperature*. – Thus, the *mean kinetic energy* of the electrons and lattice ions *increases*. So, 1. The *bonded electrons break free* from bonds, increasing the *number* of 'mobile charge carriers'. Therefore, *resistance decreases*. 2. The *lattice ions vibrate more vigorously*, *hindering the flow* of 'mobile charge carriers'. Thence, resistance increases. – The first effect is much more significant than the second. So, there is a net *decrease in resistance*.

The images above come from [5].

- The *electromotive force* (e.m.f) of a *source* is defined as the amount of *energy converted* from *non-electrical* forms of energy to *electrical* energy *per unit charge* as the *charge passes through a complete circuit*.
- Internal resistance: the p.d. across a component is given by $V = E - Ir$.
- Power dissipated $P = IV$.
- Power delivered is *maximum* when $R = r$, such that

$$P_{max} = \frac{E^2}{4r}.$$

- Efficiency of the source
  - *Increases* when external load/*resistance increases*.
  - Is halved when $R = r$.
    Note: Maximum power ≠ maximum efficiency.

Electric Fields

- *Coulomb's Law* states that the *magnitude* of the *electric force* between two *point charges* is directly proportional to the product of the charges, and inversely proportional to the square of their separation. where ε_0 is the permittivity of *free space* (ε_0 is applicable for vacuum and air only).
- Sign of F_E:

F_E	Direction
Positive	Repulsive
Negative	Attractive

- The sign of the electric force does *not* represent the *direction* of the electric force. It only informs us whether the force is attractive or repulsive. So, when calculating the *resultant* electric force, we need to account for the direction ourselves.
- Comparison between E-fields and G-fields:

Sim/Diff	E-field	G-field
S	For both Column's Law and Newton's Law of Gravitation, r is the *center to center separation* of the objects.	
S	By Newton's Third Law, the *forces* that masses and charges on each other is *equal in magnitude* and *opposite in direction*.	
D	Gravitational force is *always attractive*.	Electric Force can be *attractive or repulsive*.

- An *electric field* is a *region in space* where a *charge experiences an electric force*.
- How to draw electric field lines:
  - *Lines cannot intersect* one another.
  - Lines must *begin from positive charges* and *end on negative charges*.
  - Arrows show the *direction of force* exerted on a positive test charge.
  - The *greater* the electric field strength, the *closer* together field lines are drawn.
  - Lines leave/end on *conducting surfaces* at *right angles*

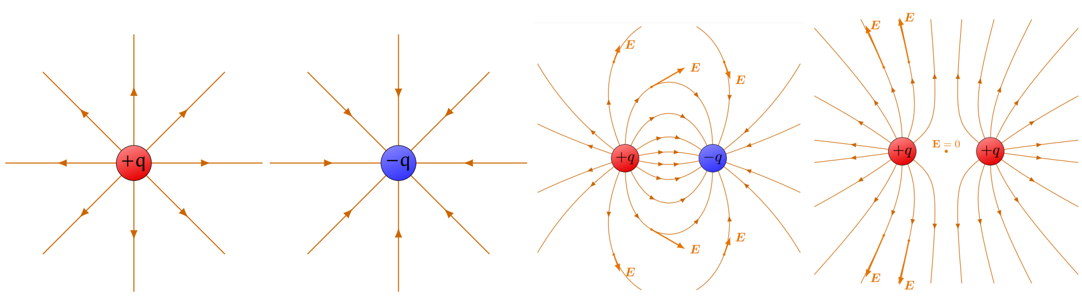


Figure 13.1: [7], [8] Electric field lines of point charges and two interacting charges.

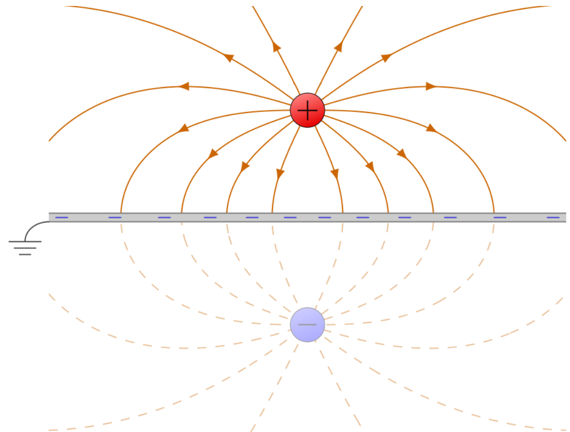


Figure 13.2: [9] Interaction of a point charge with a charged plate.

- The *electric field strength* at a point is defined as the *electric force* per *unit positive charge* placed at that point.

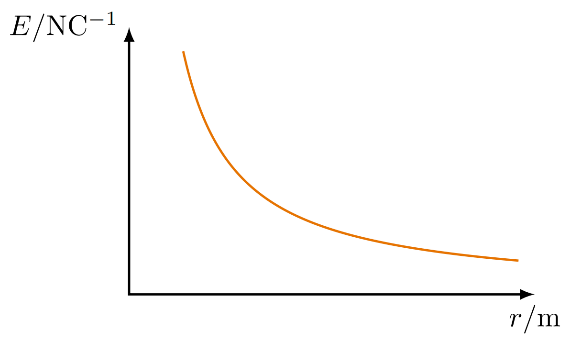


Figure 13.3: [10] Electric field strength of a positive point charge

- Charge distribution on a conducting sphere:
  - Excess charges are forced to the surface of the conductor until the electric field inside the conductor is zero.
  - *Outside* the conductor, the electric field is the *same* as that of an isolated point charge equal to the excess charge.
- Properties of conductors in *electrostatic equilibrium*:
  - The *electric field is zero inside* a conductor (regardless of shape).
  - So, the *entire conductor* is at the *same potential*.
  - Just outside the conductor, the e-field lines are *perpendicular to its surface*, starting and ending on charges on the surface.
  - *Excess charges* resides exclusively on *the surfaces* of a conductor.
- Electric field strength between two charged parallel plates is uniform everywhere between the plates, except at both ends of the plates.
- Also, *charges* between the plates experience *uniform acceleration*.

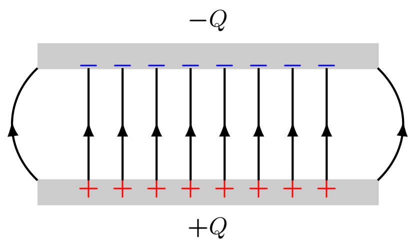


Figure 13.4: [11] Electric field lines between parallel plates.

- Magnitude of electric field strength between the plates:

$$E = \frac{dV}{dr} = \frac{\Delta V}{d}.$$

- The *electric potential energy* of a point charge in an electric field is defined as the *work done by an external agent* in bringing the point charge from infinity to that point (without any net change in kinetic energy).
- The *electric potential* at a point in an electric field is defined as the *work done* per unit *positive* charge, by an external agent, in bringing a *small* test charge from infinity to that point (without any net change in kinetic energy).
- Let U be the electric potential energy, and V the electric potential. Then,

$$U = qV \qquad \text{and} \qquad \Delta U = q\Delta V.$$

-

$$\begin{tikzcd}
U = \frac{Qq}{4\pi\varepsilon_0 r} \arrow[r, "-\frac{d}{dr}"] \arrow[d, "\frac{1}{q}"'] & F_E = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2} \arrow[d, "\frac{1}{q}"'] \\
V = \frac{Q}{4\pi\varepsilon_0 r} \arrow[r, "-\frac{d}{dr}"] & E = \frac{Q}{4\pi\varepsilon_0 r^2}
\end{tikzcd}$$

- The *electric potential* at a point X due to a *system* of point charges q_i is the algebraic sum of the electric potential due to each individual charge q_i which is distance r_i away from X. That is,

$$V = \sum_i V_i = \sum_i \frac{q_i}{4\pi\varepsilon_0 r_i}.$$

- The *potential energy* of a *system* of charges q_i is the work done to assemble it. This is the sum of energies needed to bring each charge q_i to the charges q_j (for $i > j$) already present. In other words letting r_{ij} be the distance of q_i from q_j, we have

$$U = \sum_i U_{ij} = \sum_{i>j} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}.$$

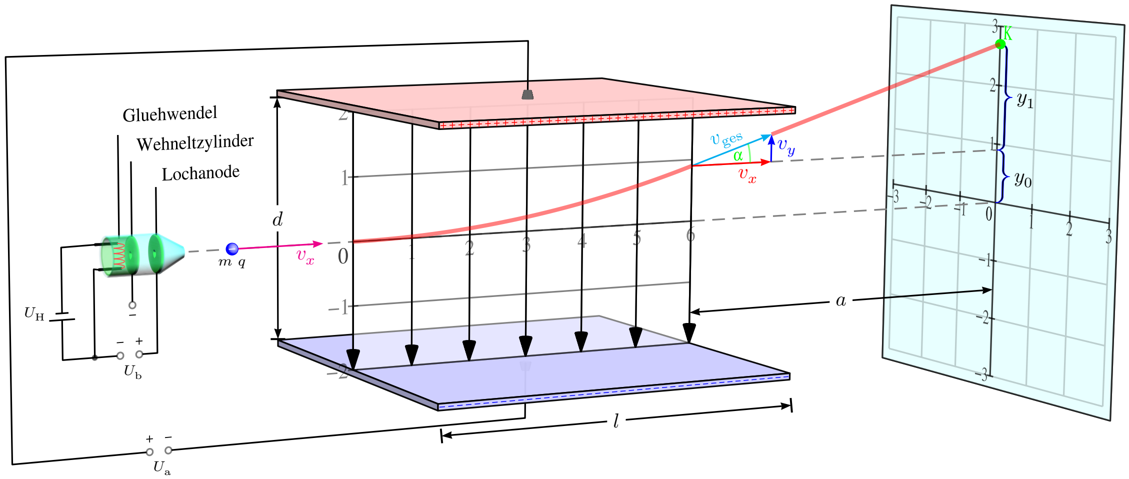


Figure 13.5: [12] Deflection of an electron.

D.C. Circuits

Property	Series	Parallel
Current	$I_1 = I_2 = \cdots = I_n$	$I_i = \dfrac{E}{R_i}$
Resistance	$R_{\text{effective}} = R_1 + R_2 + \cdots + R_n$	$\dfrac{1}{R_{\text{effective}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_n}$
Voltage	$V_i = \dfrac{R_i}{R_T} \cdot E$	$E = V_1 = V_2 = \cdots = V_n$.

- For n identical resistors in parallel, which are each of resistance R, we have $R_{\text{effective}} = R/n$. Furthermore, the effective resistance is at most the resistance of the smallest resistor, i.e. $R_{\text{effective}} \leqslant R_i$ for each i. In fact, the inequality is strict when the number of resistors in parallel $n \geqslant 2$.
- To resolve tricky systems of resistors, use the fact that electric potential is constant along a wire.

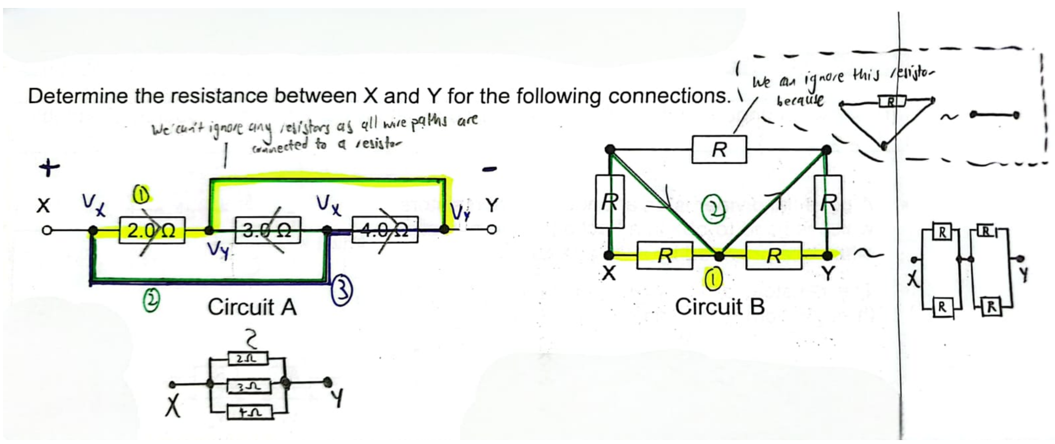


Figure 14.1: [5] Some tricky ciruits.

- Potentiometer:
  - E.m.f. of driver cell is more than that of the unknown cell, E.
  - The direction of charge flow for the primary and secondary circuits are opposite. i.e. the positive/negative terminals should 'point' towards each other.
  - The potential difference V across length L of a resistance wire is directly proportional to L.
  - Consider the following circuit. When the galvanometer reads zero, the e.m.f. of the unknown cell is E_{AJ}, where
$$\frac{E_{AJ}}{E} = \frac{L_{AJ}}{L_{AB}} = \frac{R_{AJ}}{R_{AB}}.$$

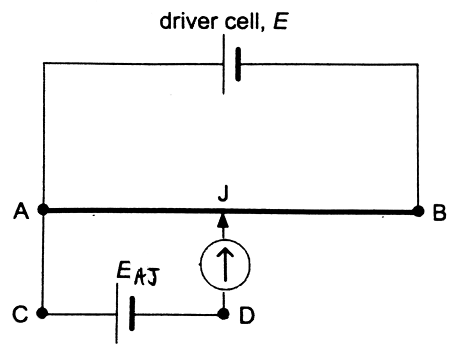


Figure 14.2: [5] An illustration of a potentiometer.

– In the circuit below, the internal resistance r satisfies

$$\frac{L_{AJ_2}}{L_{AJ_1}} = \frac{R}{R+r}.$$

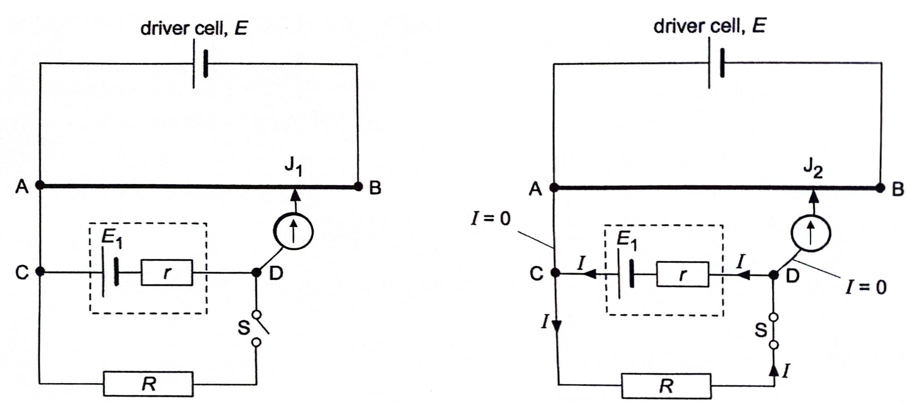


Figure 14.3: [5] Some more illustrations of potentiometers.

Electromagnetism

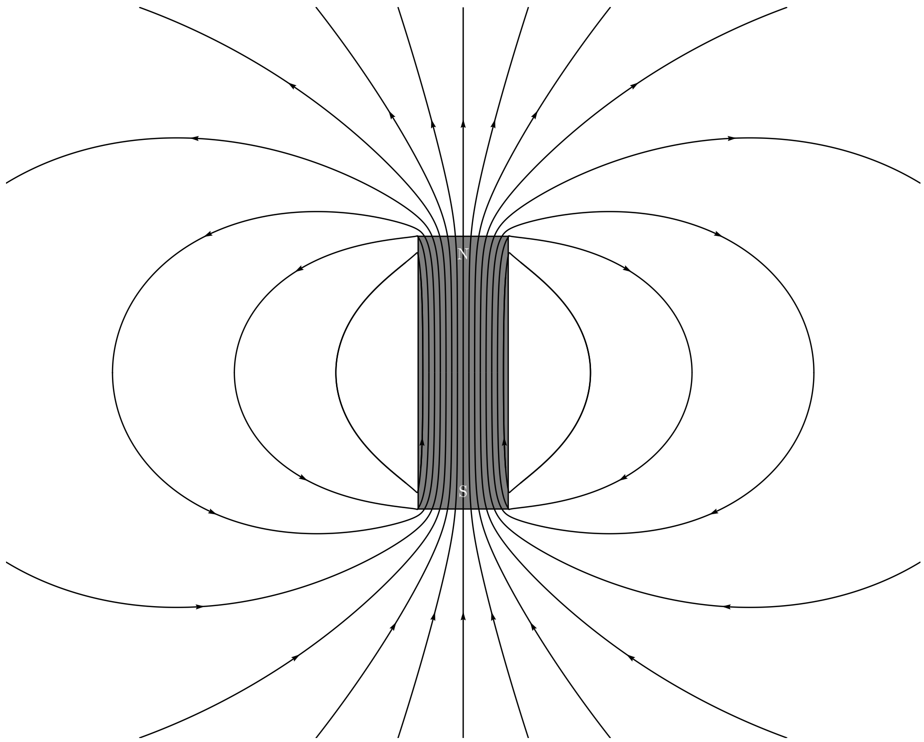


Figure 15.1: [13] Magnetic field produced by a bar magnet.

- A *magnetic field* is a *region of space* where a magnetic pole, moving charged particular or current-carrying conductor will *experience a magnetic force*.
- *Magnetic flux density* is defined as the *force per unit current per unit length* acting on an *infinitely long current-carrying conductor* placed *perpendicularly* to the magnetic field.
- Dots and crosses as indicators of direction:

$\otimes$	Into the page
$\odot$	Out of the page

- Left and right hand rules:

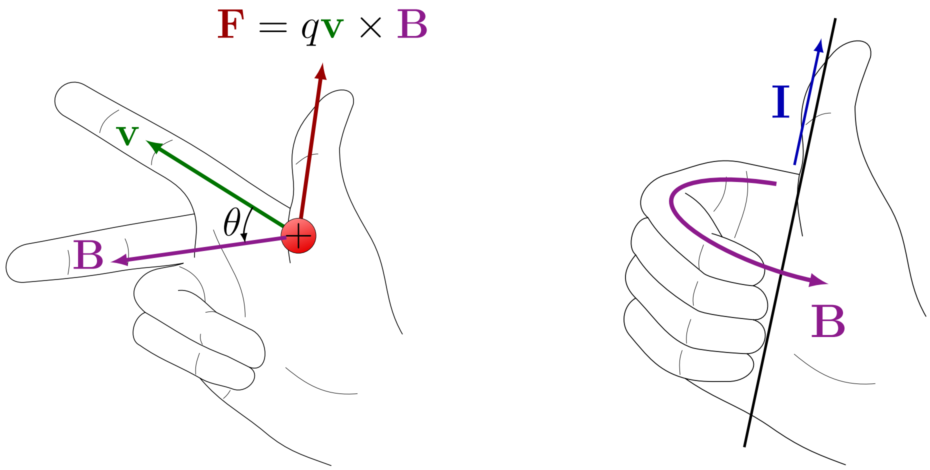


Figure 15.2: [15] Left and right hand rules.

- At any point some perpendicular distance d from the center of an *infinitely long straight* current-carrying conductor, the magnitude of the magnetic flux is given by

$$B = \frac{\mu_0 I}{2\pi d}.$$

  Here, $\mu_0 = 4\pi \cdot 10^{-7}$ is the permeability of free space.

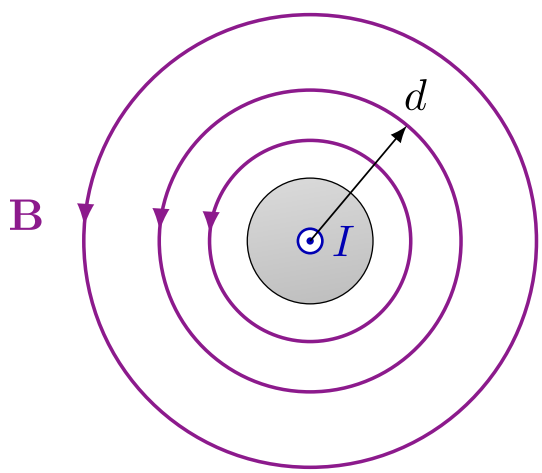


Figure 15.3: [14] Current in a wire.

- At the center of a flat circular coil with N turns, radius r, and current I flowing through it, the magnitude of the magnetic flux density is given by

$$B = \frac{\mu_0 N I}{2r}.$$

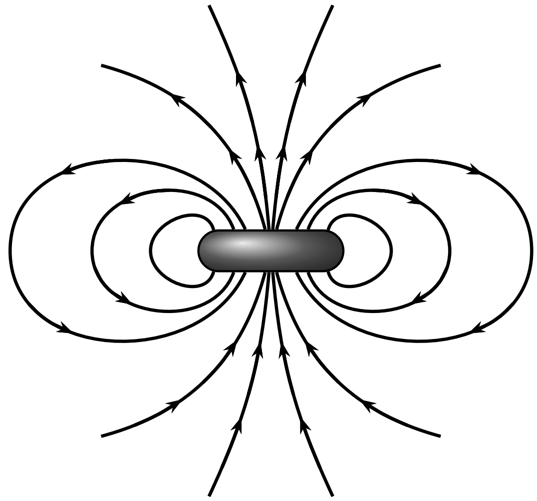


Figure 15.4: [16] Current in a flat circular coil.

- Suppose we have an ideal (having infinite length) solenoid of $n = N/L$ number of turns per unit length, which has a current I flowing through it. Then, the magnitude of the uniform magnetic flux density at its center is given by

$$B = \mu_0 n I.$$

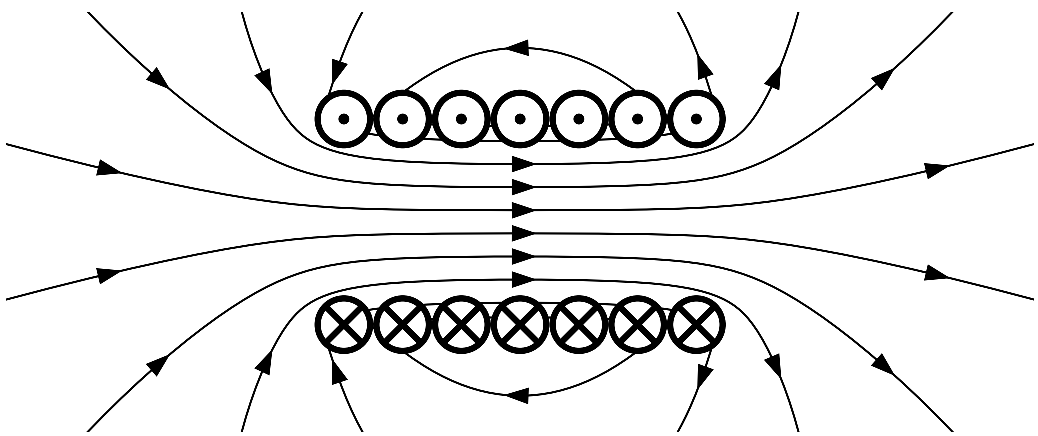


Figure 15.5: [17] Current in a solenoid and the magnetic field produced.

- A set of coils can be considered to be a solenoid when the radius of said coils is negligible compared to their length.
- The addition of a ferrous core, of permeability μ, into a solenoid increases the magnetic flux density there, which is given by
$$B = \mu n I.$$
- Say we have a straight current-carrying conductor of length l with current $I_\perp$ flowing perpendicular to the magnetic field. Then for any point on that conductor that experiences a magnetic flux of density B,
$$F_B = B I_\perp l.$$
-

Direction of Current in Two Wires	Direction of Net Magnetic Force
Same	Attractive
Opposite	Repulsive

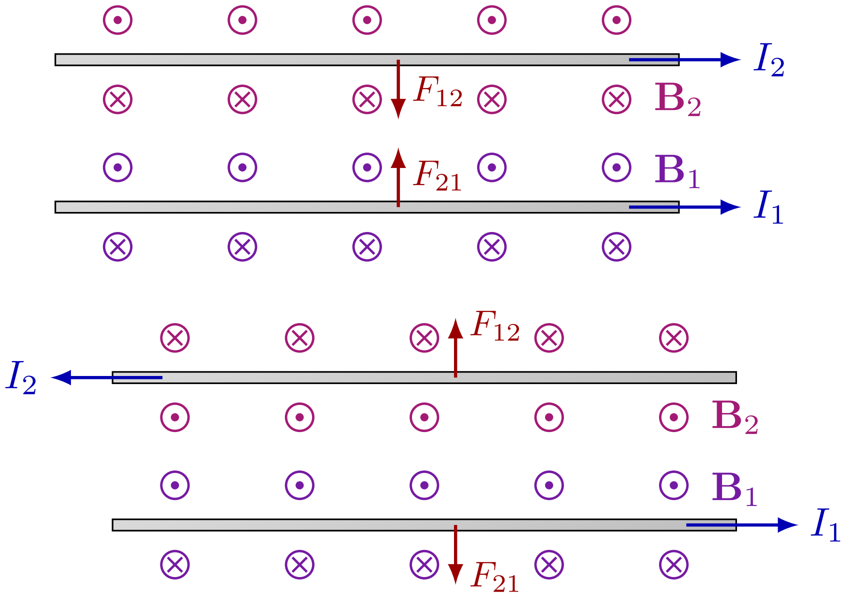


Figure 15.6: [18] Current carrying conductors in parallel.

- A charge q moving at speed $v_\perp$ perpendicular to the magnetic field, of flux density B, experiences a magnetic force of magnitude

$$F_B = qv_\perp B.$$

- Notice that the charged particle above travels in circular motion, with radius and period

$$r = \frac{mv}{Bq} \qquad \text{and} \qquad T = \frac{2\pi m}{qB}.$$

- Velocity selector. Suppose we have a velocity selector with a uniform magnetic field, of flux density B out of (or into) the paper. Further assume there is an electric field of strength E rightwards (or leftwards) in the selector. This is illustrated in Fig 15.7. Then, for a charged particle perpendicular to both fields to pass (undeflected) through the slits, $F_E = F_B$. This simplifies to

$$v = \frac{E}{B}.$$

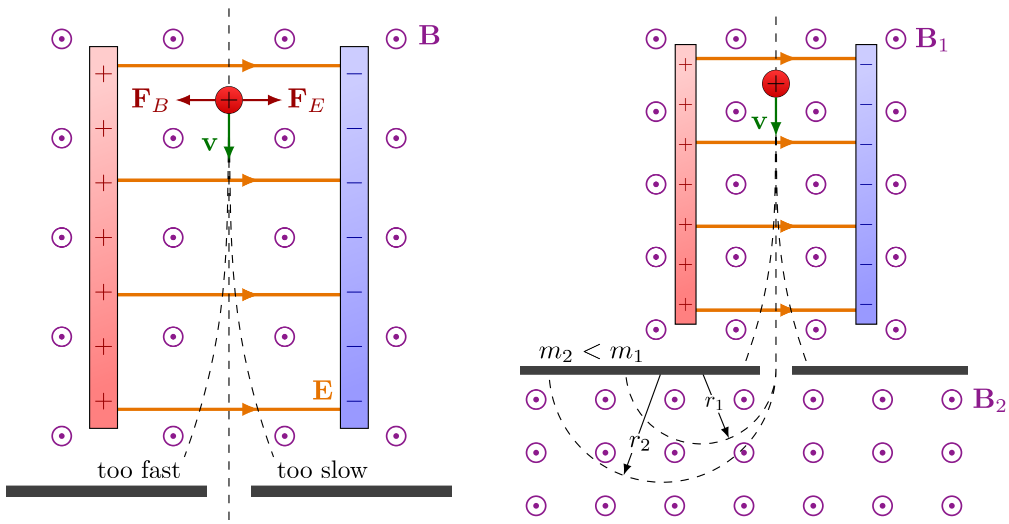


Figure 15.7: [19] An illustration of a velocity selector.

Some nice images:

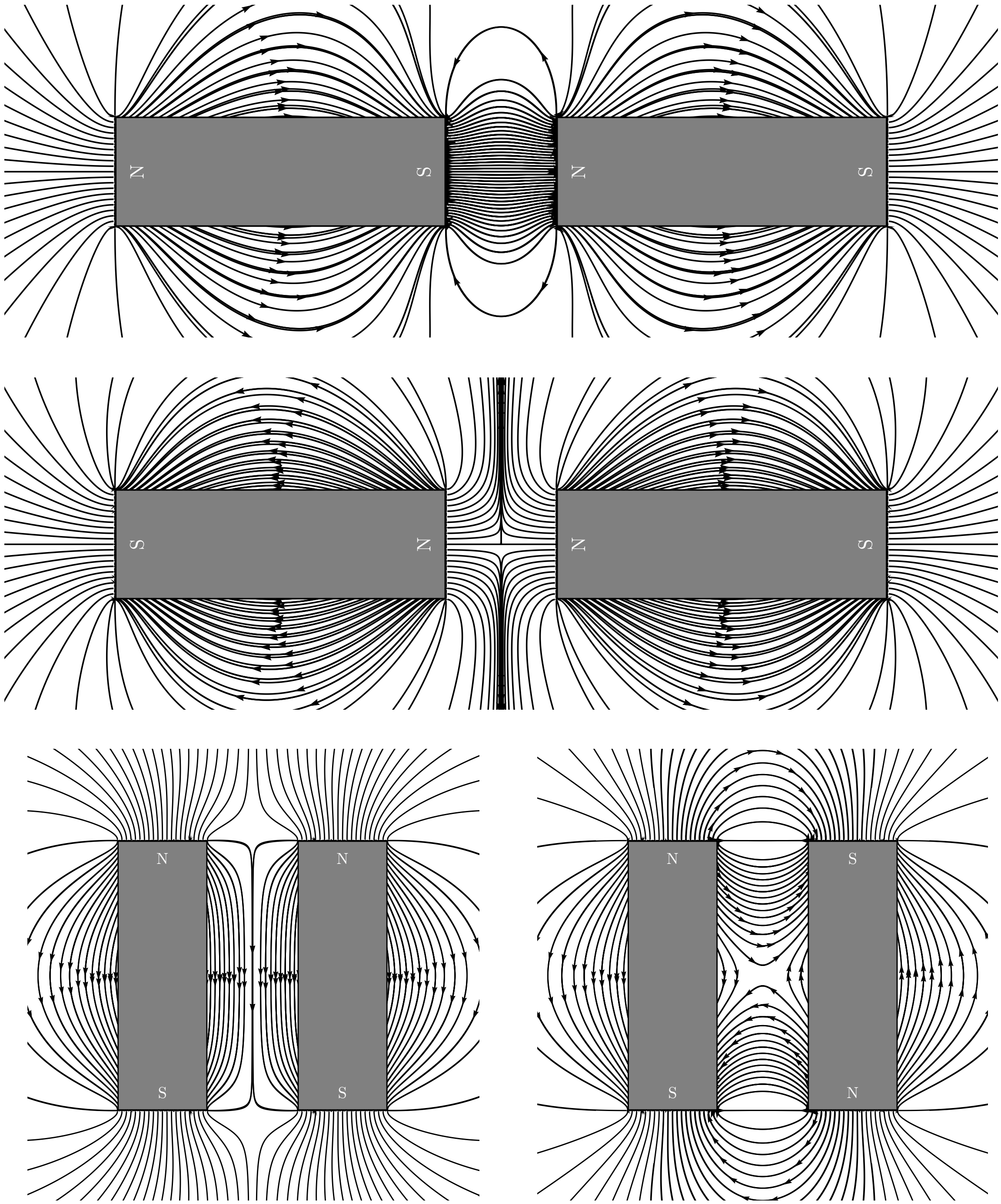


Figure 15.8: [13] Magnetic fields produced by two bar magnets.

Electromagnetic Induction

- *Magnetic flux* is the *product* of *an area* and the component of *magnetic flux density perpendicular* to that area.
- In other words, let A be an area in a uniform magnetic field, of flux density $B_\perp$ perpendicular to A. Then, the magnetic flux Φ through A is
$$\Phi = B_\perp A.$$
- The area A here is a *vector*! When we flip it through π radians, the magnetic flux through it is now $-\Phi = -B_\perp A$.
- *One weber* is the *magnitude of magnetic flux* through an *area of* $1\mathrm{m}^2$ when a *magnetic field of* 1T acts *perpendicularly into* the area.
- *Magnetic flux linkage* through a coil is defined as the *product* of the *number of turns* of the coil and the magnetic flux through each turn of the coil.
- The magnetic flux through a coil of N turns is hence
$$N\Phi = NB_\perp A.$$
- *Faraday's Law of Electromagnetic Induction* states that the *e.m.f. induced* in a *conductor* is *directly proportional* to the *rate of change of magnetic flux linkage*.
- *Lenz's Law* states that the *direction of the induced e.m.f.* is such that it may produce *an effect* that *opposes the change* causing it.
- Lenz's Law is a consequence of conservation of energy. Mechanical work done to enable the change in magnetic flux linkage is converted into electrical energy.
- Faraday's and Lenz's Laws imply that the e.m.f generated is
$$E = -(N\Phi)' = -(NB_\perp A)'.$$
(The negative sign indicates that the induced e.m.f. opposes the change in magnetic flux linkage.)

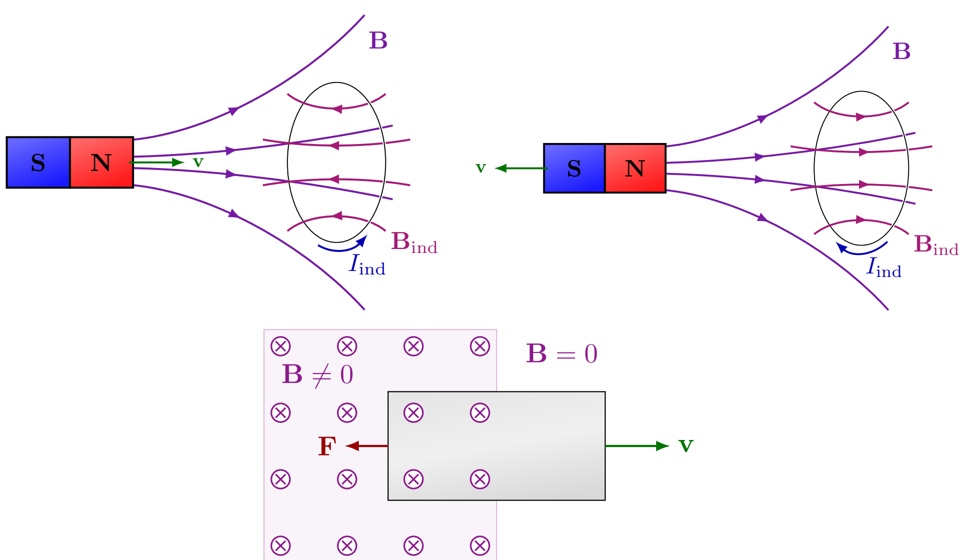


Figure 16.1: [20] Examples of Lenz's Law in action.

- Motional e.m.f.: Suppose we have a circuit as shown in the bottom left of the following figure. Then, by Faraday's Law,

$$|E| = \Phi' = B(\Delta A) = Blv.$$

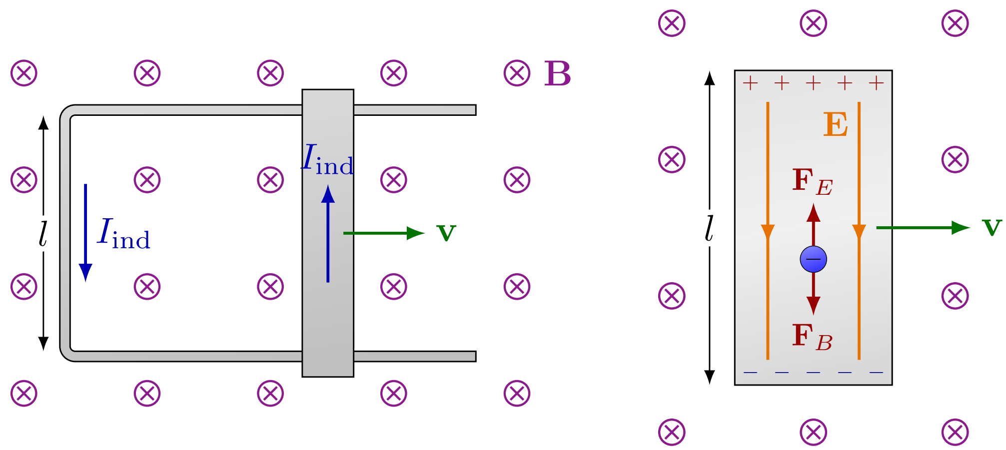


Figure 16.2: [20] Example of motional emf.

- Conventions for polarity and when to use them.

Energy conversion	Function in a circuit	Convention for polarity
Electrical to others	Resistor/Wire	Higher potential = relatively positive
Others to electrical	Battery	Lower potential = relatively positive

- Faraday's Paradox. Consider a metal disc with of area A rotating at frequency f in a uniform magnetic field, of flux density B. Then, let O be the centre of the disk, and pick any point P along the circumference of the disk. We see that A', the area swept by OP, is $\pi r^2 f$. So, by Faraday's Law,

$$E = B\pi r^2 f.$$

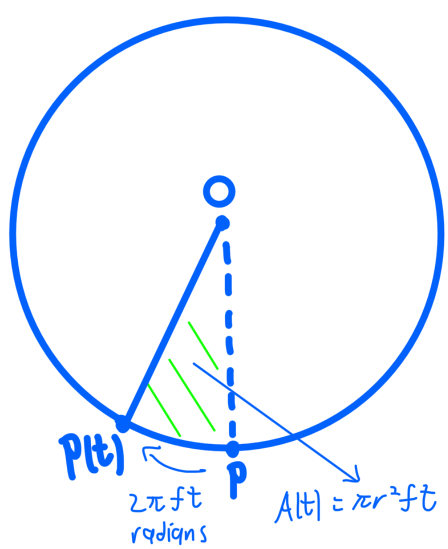


Figure 16.3: [21] An illustration of Faraday's Paradox.

Alternating Current

- An *alternating current (a.c.) source* creates an electrical *current* that varies in magnitude *and* direction *periodically with time*, as opposed to a *direct current* (d.c.) source where the *direction* of the current stays *constant*.
- Alternating current *must change direction*. For instance, the following two functions, namely $I = \sin(t) + 1$ and $I = \cos(t) + 1$, are both (varying) direct currents.

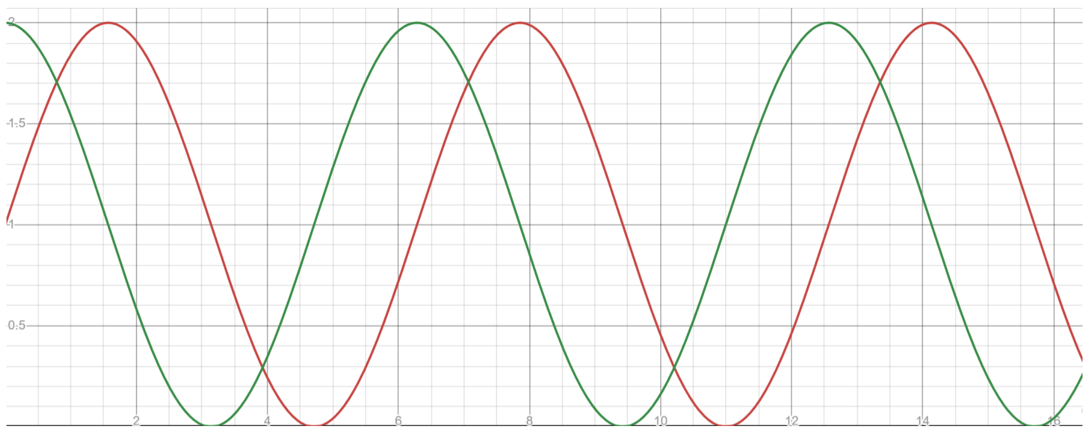


Figure 17.1: [22] Direct currents.

- The *root-mean-square* (r.m.s.) value I_{rms} (or Vrms) of an *alternating current* (or alternating voltage) is the value of a *steady direct current* (or direct voltage) that would produce the *same average power* in a given resistor.
- We denote the mean value of a quantity x by $\langle x \rangle$. So, $x_{rms} = \sqrt{\langle x^2 \rangle}$, and it also holds that

$$\langle P \rangle = I_{rms}^2 R = \frac{V_{rms}^2}{R}.$$

- Steps to determine the rms value: square $\rightarrow$ mean $\rightarrow$ root.
  1. Square all values of I (or V).
  2. Find the mean square value $\langle I^2 \rangle$ (or $\langle V^2 \rangle$). This is just the area under the graph of I^2 (or V^2) against t in one period.
  3. Square root the value obtained above.
- Note that for a *full wave sinusoidal* alternating current (which an be assumed unless otherwise stated),

$$\langle P \rangle = \frac{1}{2} P_0, \qquad V_{rms} = \frac{1}{\sqrt{2}} V_0, \quad \text{and} \quad I_{rms} = \frac{1}{\sqrt{2}} I_0.$$

- Transformers. Let N_P, V_P, and I_P be the number of turns, voltage, and current, respectively, in the primary winding. Similarly define N_S, V_S, and I_S for the secondary winding. Then,

$$\frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_S}{I_P}.$$

  (To quickly determine which side carries a greater voltage, we can use the principle of 'more turns, more voltage'.)

- A step up transformer is one that increases voltage, i.e. $V_S > V_P$ (or $N_S > N_P$).
- A step down transformer is one that decreases voltage, i.e. $V_P > V_S$ (or $N_P > N_S$).

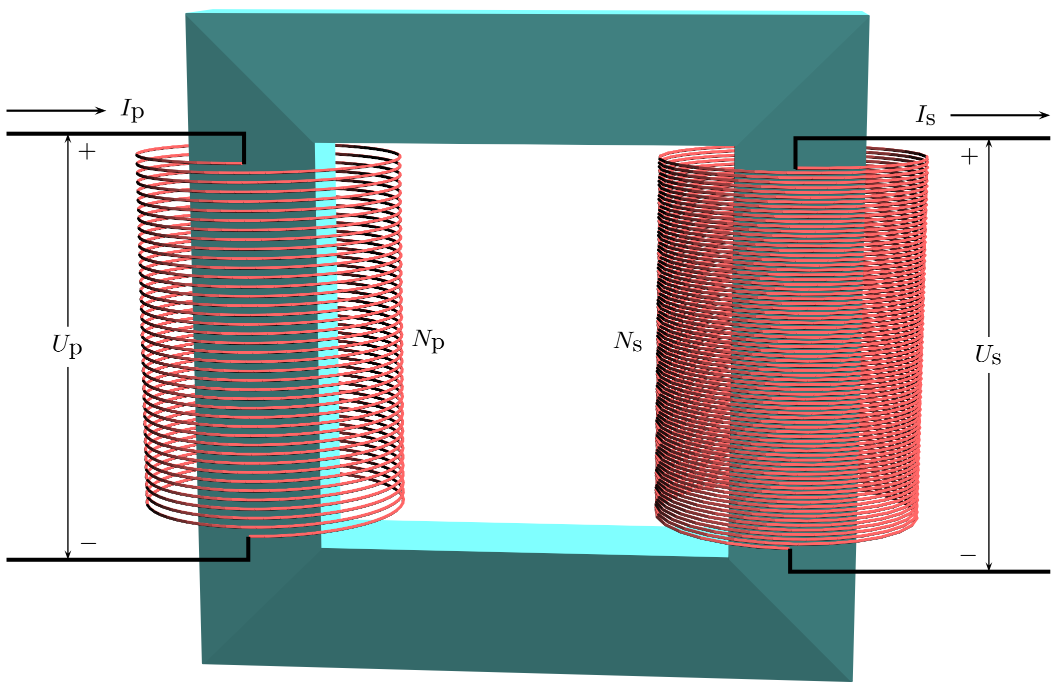


Figure 17.2: [23] A step-up transformer with $N_P = 40$ and $N_S = 80$.

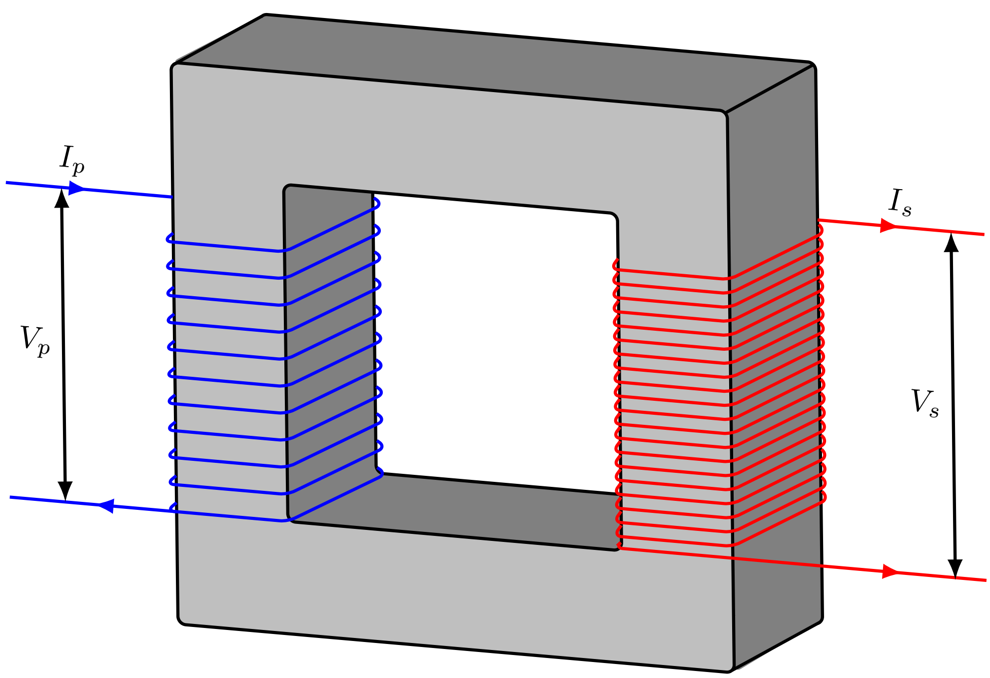


Figure 17.3: [24] Another step-up transformers.

- Transformers cannot be used for *constant* direct current. Since there is no change in voltage/current, no emf will be (constantly) induced in the secondary coil.
- When the direct current varies, however, there will be an *alternating* current induced.
- Energy loss in a transformer.
  1. Winding resistance. Current flowing through the windings causes *resistive heating* of the conductors.
  2. Hysteresis losses. Each time the *magnetic field is reversed*, a small amount of energy is lost due to hysteresis within the core.
  3. Magnetostriction. Magnetic flux in a ferromagnetic fore causes it to physically expand and contract slightly with each cycle of the magnetic field. This produces a *buzzing sound* and can cause losses due to *frictional heating*.
  4. Mechanical losses. The alternating magnetic field causes fluctuating forces between the primary and secondary windings. These cause vibrations within nearby metalwork, adding to the *buzzing noise* and *consuming a small amount of power*.
- Power is typically transmitted at high voltages to minimise the power lost during transmission.

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