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Contents

A1: Inequalities and Equations

1.1 Solving Inequalities

1.1.1 Rational Inequalities

General Methods

1. Quadratic formula for factorisation / finding roots (of polynomial).
2. Completing the square.
3. Discriminant to eliminate factors which are *always* positive or negative (e.g. removing $x^2 - 3x + 4$). *Note to include coefficient of x^2 in argument.*
4. GC (include sketch).
5. *Rational Functions*[a]: Move everything to one side (+,−), then use number line.
6. Number line (more complicated functions).

[a]Fractions of Polynomials

Important Notes

- ☐ Discriminant — include coefficient of x^2 in argument.
- ☐ Rational functions — exclude the values that causes division by zero to occur.
- ☐ Cross multiplication preserves order for $\frac{x}{y} < \frac{x'}{y'}$ iff y and y' are *both* positive or negative.[a]
- ☐ Squaring preserves order for $x < y$ iff x and y are *both* positive or negative.

[a]Otherwise, note the counterexample $\frac{1}{2} < \frac{1}{-3}$.

1.2 Modulus Inequalities

Fact. Given $x \in \mathbb{R}$, we have that

- $|x| \geq 0$,
- $|x^2| = |x|^2 = x^2$,
- $\sqrt{x^2} = |x|$.

And as long as $x \in \mathbb{R}^+$,

- $\sqrt{x}^2 = |x|$.

Useful Properties

For every $x, k \in \mathbb{R}$:

(a) $|x| < k$ iff[a] $-k < x < k$.

(b) $|x| > k$ iff $x < -k$ or $x > k$.

(of course, similarly applies for the non-strict ordering $\leq$)

[a]Notice that $k > 0$ here since $0 \leq |x| < k$.

Important Notes

- Note that when solving for $|x| = y$, $|x| < y$, etc, y must be greater than or equal to 0. In other words, there may be solutions we will need to reject.
  (For $<$, equality is of course not allowed.)

B1(A): Graphing Techniques (Part I)

General Definitions

1. **Lines of Symmetry**: A *line of symmetry* of a function is a line, such that the function is a reflection of itself about that line.

2. **Horizontal Asymptotes**: A (horizontal) line $g(x) = c$ is the *horizontal asymptote* of the curve $f(x)$ iff $\lim_{x\to\infty} f(x) = c$ (or with $-\infty$ instead of ∞).[a]

3. **Vertical Asymptotes**: A (vertical) line $x = c$ is a *vertical asymptote* of the curve $f(x)$ iff $\lim_{x\to c} f(x) = \infty \text{ or } -\infty$.

4. **Oblique Asymptotes**: A line $g(x) = mx + c$ — where $m \neq 0$ — is an *oblique asymptote* of the curve $f(x)$ iff $\lim_{x\to\infty}[f(x) - g(x)] = 0$ (or with $-\infty$ instead of ∞).

[a] Otherwise notated by $f(x) \to c$ as $x \to \infty$.