

A-Level Mathe

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A1: Inequalities and Equations

1.1 Solving Inequalities

1.1.1 Rational Inequalities

General Methods

1. Quadratic formula for factorisation / finding roots (of polynomial).
2. Completing the square.
3. Discriminant/Completing the Sq to eliminate* factors which are *always* positive or negative (e.g. removing $x^2 - 3x + 4$). *Note to include coefficient of x^2 in argument.*
4. GC (include sketch).
5. *Rational Functions*^a: Move everything to one side (+, -), then use number line.
6. Number line (more complicated functions).

^aFractions of Polynomials

Important Notes

- Eliminating Factors — *only*^a works for $c = 0$ in $f(x) \geq c$ or $f(x) \leq c$.
- Discriminant — include coefficient of x^2 in argument.
- Rational functions — exclude the values that causes division by zero to occur.
- Cross multiplication preserves order for $\frac{x}{y} < \frac{x'}{y'}$ iff y and y' are *both* positive or negative.^b
- Squaring preserves order for $x < y$ iff x and y are *both* positive or negative.

^aCounterexample: $P(x) = x(3x^2 - 9x + 10) \leq 2$ iff $x \leq 2$ is false. E.g.: $P(1.8) = 6.336 \not\leq 2$.

^bOtherwise, note the counterexample $\frac{1}{2} < \frac{1}{-3}$.

1.2 Modulus Inequalities

Fact. Given $x \in \mathbb{R}$, we have that

- $|x| \geq 0$,
- $|x^2| = |x|^2 = x^2$,
- $\sqrt{x^2} = |x|$.

And as long as $x \in \mathbb{R}^+$,

- $\sqrt{x^2} = |x|$.

Useful Properties

For every $x, k \in \mathbb{R}$:

- (a) $|x| < k$ iff^a $-k < x < k$.
- (b) $|x| > k$ iff $x < -k$ or $x > k$.

(of course, similarly applies for the non-strict ordering \leq)

^aNotice that $k > 0$ here since $0 \leq |x| < k$.

Important Notes

- Note that when solving for $|x| = y$, $|x| < y$, etc, y must be greater than or equal to 0. In other words, there may be solutions we will need to reject.
(For $<$, equality is of course not allowed.)

B1(A): Graphing Techniques (Part I)

General Definitions

1. **Lines of Symmetry:** A *line of symmetry* of a function is a line, such that the function is a reflection of itself about that line.
2. **Horizontal Asymptotes:** A (horizontal) line $g(x) = c$ is the *horizontal asymptote* of the curve $f(x)$ iff $\lim_{x \rightarrow \infty} f(x) = c$ (or with $-\infty$ instead of ∞).^a
3. **Vertical Asymptotes:** A (vertical) line $x = c$ is a *vertical asymptote* of the curve $f(x)$ iff $\lim_{x \rightarrow c} f(x) = \infty$ or $-\infty$.
4. **Oblique Asymptotes:** A line $g(x) = mx + c$ — where $m \neq 0$ — is an *oblique asymptote* of the curve $f(x)$ iff $\lim_{x \rightarrow \infty} [f(x) - g(x)] = 0$ (or with $-\infty$ instead of ∞).

^aOtherwise notated by $f(x) \rightarrow c$ as $x \rightarrow \infty$.