A-Level Mathe
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Contents

A1: Inequalities and Equations

1.1 Solving Inequalities

1.1.1 Rational Inequalities

General Methods

1. Quadratic formula for factorisation / finding roots (of polynomial).

2. Completing the square.

3. Discriminant/Completing the Sq to eliminate* factors which are *always* positive or negative (e.g. removing $x^2 - 3x + 4$). *Note to include coefficient of x^2 in argument.*

4. GC (include sketch).

5. *Rational Functions[a]*: Move everything to one side $(+,-)$, then use number line.

6. Number line (more complicated functions).

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[a]Fractions of Polynomials

Important Notes

☐ Eliminating Factors — *only[a]* works for $c = 0$ in $f(x) \geq c$ or $f(x) \leq c$.

☐ Discriminant — include coefficient of x^2 in argument.

☐ Rational functions — exclude the values that causes division by zero to occur.

☐ Cross multiplication preserves order for $\frac{x}{y} < \frac{x'}{y'}$ iff y and y' are *both* positive or negative.[b]

☐ Squaring preserves order for $x < y$ iff x and y are *both* positive or negative.

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[a]Counterexample: $P(x) = x(3x^2 - 9x + 10) \leq 2$ iff $x \leq 2$ is false. E.g.: $P(1.8) = 6.336 \nleq 2$.
[b]Otherwise, note the counterexample $\frac{1}{2} < \frac{1}{-3}$.

1.2 Modulus Inequalities

Fact. Given $x \in \mathbb{R}$, we have that

- $|x| \geq 0$,

- $|x^2| = |x|^2 = x^2$,

- $\sqrt{x^2} = |x|$.

And as long as $x \in \mathbb{R}^+$,

- $\sqrt{x^2} = |x|$.

Useful Properties

For every $x, k \in \mathbb{R}$:

(a) $|x| < k$ iff[a] $-k < x < k$.

(b) $|x| > k$ iff $x < -k$ or $x > k$.

(of course, similarly applies for the non-strict ordering $\leq$)

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[a]Notice that $k > 0$ here since $0 \leq |x| < k$.

Important Notes

○ Note that when solving for $|x| = y$, $|x| < y$, etc, y must be greater than or equal to 0. In other words, there may be solutions we will need to reject.

(For $<$, equality is of course not allowed.)

B1(A): Graphing Techniques (Part I)