A-Level Mathe

Shao Hong

Contents

A1: Inequalities and Equations

1.1 Solving Inequalities

1.1.1 Rational Inequalities

General Methods

1. Quadratic formula for factorisation / finding roots (of polynomial).
2. Completing the square.
3. Discriminant to eliminate factors which are *always* positive or negative (e.g. removing $x^2 - 3x + 4$). *Note to include coefficient of x^2 in argument.*
4. GC (include sketch).
5. *Rational Functions*[a]: Move everything to one side $(+,-)$, then use number line.
6. Number line (more complicated functions).

[a] Fractions of Polynomials

Important Notes

- ☐ Discriminant — include coefficient of x^2 in argument.
- ☐ Rational functions — exclude the values that causes division by zero to occur.
- ☐ Cross multiplication preserves order for $\frac{x}{y} < \frac{x'}{y'}$ iff y and y' are *both* positive or negative.[a]
- ☐ Squaring preserves order for $x < y$ iff x and y are *both* positive or negative.

[a] Otherwise, note the counterexample $\frac{1}{2} < \frac{1}{-3}$.

1.2 Modulus Inequalities

Fact. Given $x \in \mathbb{R}$, we have that

- $|x| \geq 0$,
- $|x^2| = |x|^2 = x^2$,
- $\sqrt{x^2} = |x|$.

And as long as $x \in \mathbb{R}^+$,

- $\sqrt{x}^2 = |x|$.

Useful Properties

For every $x, k \in \mathbb{R}$:

(a) $|x| < k$ iff[a] $-k < x < k$.

(b) $|x| > k$ iff $x < -k$ or $x > k$.

(of course, similarly applies for the non-strict ordering $\leq$)

[a] Notice that $k > 0$ here since $0 \leq |x| < k$.

Important Notes

- Note that when solving for $|x| = y$, $|x| < y$, etc, y must be greater than or equal to 0. In other words, there may be solutions we will need to reject.
  (For $<$, equality is of course not allowed.)