ISSUE #1057 - Pool Liquidity Share Lost[1]

Analysis and Fix

BEPSwap users reported two things:

1) When adding liquidity it appeared to them they got a “bonus”
2) When being in a pool for a while their ownership was eroded.

The team investigated and identified two issues in how pool ownership was allocated; the “bonus” and the “erosion”.

Bonus

The team found UniswapV1’s method to not have any issues. (UniswapV2 is not applicable for comparison since it takes the geometric mean, which will evaluate to zero if one side is zero). A fix, based on UniswapV1’s methodology, was prepared and analysed. This fixed the Bonus problem.

Erosion

Erosion of liquidity is caused by asymmetric stakers swinging the pool’s price significantly and being allocated more of the pool than if they had swapped their assets and staked. When swapping their assets they pay a slip-based fee, but the existing pool ownership function did not take this into account. To address this, the pool ownership equation is modified to include a “slip adjustment factor”, which applies a discount to inbound liquidity when it creates a change in the pool’s price before and after adding liquidity, which is the same as if they had swapped and staked. Symmetric liquidity providers do not create a slip, so receive the full allocation of ownership.

The final method:

1) Assign the first liquidity provider units equal to the amount of RUNE contributed
2) Assign subsequent liquidity providers units equal to the following equation:

r = rune added

a = asset added

R = Rune Balance (before)

A = Asset Balance (before)

P = Existing Pool Units

$$slipAdjustment = 1 - \left|\frac{Ra - rA}{(2r+R)*(a+A)}\right|$$

$$units = \frac{P(Ra + rA))}{2RA} * slipAdjustment$$

Analysis

This report will analyse pool ownership methodologies used in UniswapV1, UniswapV2, THORChainV1 and THORChainV2.

[1] https://gitlab.com/thorchain/bepswap/bepswap-web-ui/-/issues/1057

Allocating Pool Ownership

Uniswap V1

Uniswap's algorithm mints pool ownership based on the proportion of ETH deposited, compared to the existing depth, multiplied by the existing number of pool tokens.

$$s_{minted} = \frac{x_{deposited}}{x_{starting}} * s_{starting}$$

Initial Liquidity Provision

Three liquidity providers add liquidity symmetrically. Pool revenue is collected and the price changes by 30% over the course of the scenario.

	BASE	ratio	ASSET		Unit	PoolUnits
Balance (start)	0	**0.00**	0	**Balance (start)**		
ADD (LP-1)	1,000.00		1,000.00	ADD (LP-1)	1,000.00	1,000.00
Balance (after)	1,000.00	1.00	1,000.00	**Balance (after)**	100.00%	
Fees + Swaps	100.00		90.91	out		
Balance (after)	1,100.00	1.21	909.09	**Balance (after)**		
ADD (LP-2)	1,000.00		826.45	ADD (LP-2)	909.09	1,909.09
Balance (after)	2,100.00	1.21	1,735.54	**Balance (after)**	47.62%	
Fees + Swaps	100.00		75.13	out		
Balance (after)	2,200.00	1.32	1,660.41	**Balance (after)**		
ADD (LP-3)	1,000.00		754.73	ADD (LP-3)	867.77	2,776.86
Balance (after)	3,200.00	1.32	2,415.14	**Balance (after)**	31.25%	
baseAdded	**3,000.00**		**2,581.18**	**assetAdded**		

Final Pool claims

All three liquidity providers can claim back their assets in a fair proportion. The final LP does not earn any revenue.

	Units	Ownership	BASE Added	ASSET Added	BASE Claim	ASSET Claim	BASE PnL	ASSET PnL
LP-1	1,000.00	36.01%	1,000.00	1,000.00	1,152.38	869.74	152.38	-130.26
LP-2	909.09	32.74%	1,000.00	826.45	1,047.62	790.67	47.62	-35.78
LP-3	867.77	31.25%	1,000.00	754.73	1,000.00	754.73	0.00	0.00
TOTAL	**2,776.86**	**100.00%**	**3,000.00**	**2,581.18**	**3,200.00**	**2,415.14**	**200.00**	**-166.04**

THORChainV1

Bonus Problem

THORChain has to tolerate asymmetric liquidity provision since it cannot pull funds from wallets, so must deal with whatever is sent to it, and asymmetric liquidity provision is a valid arb strategy. The original algorithm took the mean of the deposited assets to find the share of the pool, then multiplied that each side, then took the mean of that. This method works fine if there is no price change.

$$share = \frac{(r/R) + (a/A)}{2} \quad units = \frac{(share * R) + (share * A)}{2}$$

$$units = \frac{(R+A)(rA + Ra)}{4RA}$$

Initial Liquidity Provision

Three liquidity providers add liquidity symmetrically. Pool revenue is collected and the price changes by 30% over the course of the scenario.

	BASE	ratio	ASSET		Unit	PoolUnits
Balance (start)	0	**0.00**	0	**Balance (start)**		
ADD (LP-1)	1,000.00		1,000.00	ADD (LP-1)	1,000.00	1,000.00
Balance (after)	1,000.00	1.00	1,000.00	**Balance (after)**	100.00%	
Fees + Swaps	100.00		90.91	out		
Balance (after)	1,100.00	1.21	909.09	**Balance (after)**		
ADD (LP-2)	1,000.00		826.45	ADD (LP-2)	913.22	1,913.22
Balance (after)	2,100.00	1.21	1,735.54	**Balance (after)**	47.73%	
Fees + Swaps	100.00		75.13	out		
Balance (after)	2,200.00	1.32	1,660.41	**Balance (after)**		
ADD (LP-3)	1,000.00		754.73	ADD (LP-3)	877.36	2,790.59
Balance (after)	3,200.00	1.32	2,415.14	**Balance (after)**	31.44%	

Final Pool claims

All three liquidity providers can claim back their assets, but the final LP gains an unfair “bonus”. This bonus is more noticeable the more drastic the price changes for a pool over the course of the liquidity being added.

	Units	Ownership	BASE Added	ASSET Added	BASE Claim	ASSET Claim	BASE PNL	ASSET PnL
LP-1	1,000.00	35.83%	1,000.00	1,000.00	1,146.71	865.46	146.71	-134.54
LP-2	913.22	32.73%	1,000.00	826.45	1,047.20	790.36	47.20	-36.09
LP-3	877.36	31.44%	1,000.00	754.73	1,006.08	759.32	6.08	4.59
TOTAL	**2,790.59**	**100.00%**	**3,000.00**	**2,581.18**	**3,200.00**	**2,415.14**	**200.00**	**-166.04**

Fixing Bonus

To address the issue found, the algorithm for assigning pool ownership is amended as follows:

The first minter is assigned pool units simply equal to the amount of RUNE deposited. This creates P.

The subsequent minters are as follows:

$$units_{rune} = \frac{r}{R} * P \quad units_{asset} = \frac{a}{A} * P$$

$$units_{final} = \frac{units_{rune} + units_{asset}}{2}$$

$$units = \frac{P(rA + Ra)}{2RA}$$

Initial Liquidity Provision

Three liquidity providers add liquidity symmetrically. Pool revenue is collected and the price changes by 30% over the course of the scenario.

	BASE	ratio	ASSET		Unit	PoolUnits
Balance (start)	0	**0.00**	0	**Balance (start)**		
ADD (LP-1)	1,000.00		1,000.00	ADD (LP-1)	1,000.00	1,000.00
Balance (after)	1,000.00	1.00	1,000.00	**Balance (after)**	100.00%	
Fees + Swaps	100.00		90.91			
Balance (after)	1,100.00	1.21	909.09	**Balance (after)**		
ADD (LP-2)	1,000.00		826.45	ADD (LP-2)	909.09	1,909.09
Balance (after)	2,100.00	1.21	1,735.54	**Balance (after)**	47.62%	
Fees + Swaps	100.00		75.13			
Balance (after)	2,200.00	1.32	1,660.41	**Balance (after)**		
ADD (LP-3)	1,000.00		754.73	ADD (LP-3)	867.77	2,776.86
Balance (after)	3,200.00	1.32	2,415.14	**Balance (after)**	31.25%	
baseAdded	**3,000.00**		**2,581.18**	**assetAdded**		

Final Pool claims

All three liquidity providers can claim back their assets in a fair proportion. There is no longer any final unfair "bonus"

	Units	Ownership	BASE Added	ASSET Added	BASE Claim	ASSET Claim	BASE PNL	ASSET PnL
LP-1	1,000.00	36.01%	1,000.00	1,000.00	1,152.38	869.74	152.38	-130.26
LP-2	909.09	32.74%	1,000.00	826.45	1,047.62	790.67	47.62	-35.78
LP-3	867.77	31.25%	1,000.00	754.73	1,000.00	754.73	0.00	0.00
TOTAL	**2,776.86**	**100.00%**	**3,000.00**	**2,581.18**	**3,200.00**	**2,415.14**	**200.00**	**-166.04**

Capital Erosion

The second issue is that of capital erosion from existing liquidity providers. The root cause is that the original algorithm did not take into account the slip generated by large asymmetric liquidity providers.

Initial Liquidity Provision

Three liquidity providers add liquidity, with the second one performing a giant asymmetrical stake.

	BASE	ratio	ASSET		Unit	PoolUnits
Balance (start)	0	**0.00**	0	**Balance (start)**		
ADD (LP-1)	1,000.00		1,000.00	ADD (LP-1)	1,000.00	1,000.00
Balance (after)	1,000.00	1.00	1,000.00	**Balance (after)**	100.00%	
ADD (LP-2)	20,000.00	0.49	40,000.00	ADD (LP-2)	30,000.00	31,000.00
Balance (after)	21,000.00	0.51	41,000.00	**Balance (after)**	96.77%	
ADD (LP-2)	40,000.00		20,000.00	ADD (LP-2)	37,084.79	68,084.79
Balance (after)	61,000.00	1.00	61,000.00	**Balance (after)**	54.47%	
ADD (LP-3)	1,000.00		1,000.00	ADD (LP-3)	1,116.14	69,200.93
Balance (after)	62,000.00	1.00	62,000.00	**Balance (after)**	1.61%	1.00
baseAdded	**22,000.00**		**42,000.00**	**assetAdded**		

All three liquidity providers claim back their assets. The first one loses to the benefit of the second provider. This needs to be fixed.

	Units	Ownership	BASE Added	ASSET Added	BASE Claim	ASSET Claim	BASE PNL	ASSET PnL
LP-1	1,000.00	1.45%	1,000.00	1,000.00	895.94	895.94	-104.06	-104.06
LP-2	67,084.79	96.94%	60,000.00	60,000.00	60,104.06	60,104.06	104.06	104.06
LP-3	1,116.14	1.61%	1,000.00	1,000.00	1,000.00	1,000.00	0.00	0.00

Fixing Capital Erosion

To fix this, slip must be accounted for and "virtualised". The intent is to transfer an equivalent amount of assets from the asymmetrical liquidity provider to all previous liquidity providers to be equivalent to the fees that would have been retained had the incoming liquidity provider done a swap instead. The pool ownership algorithm is then modified to included this slip adjustment.

$$slipAdjustment = 1 - \left|\frac{Ra - rA}{(2r+R)(a+A)}\right|$$

$$units = \frac{P(Ra + rA))}{2RA} * slipAdjustment$$

Initial Liquidity Provision

Three liquidity providers add liquidity, with the second one performing a giant asymmetrical stake.

	BASE	ratio	ASSET		ip Adjustme	Unit	PoolUnits
Balance (start)	0	**0.00**	0	**Balance (start)**			
ADD (LP-1)	1,000.00		1,000.00	ADD (LP-1)	-	1,000.00	1,000.00
Balance (after)	1,000.00	1.00	1,000.00	**Balance (after)**			
ADD (LP-2)	20,000.00	0.51	40,000.00	ADD (LP-2)	98.81%	29,643.07	30,643.07
Balance (after)	21,000.00	0.51	41,000.00	**Balance (after)**			
ADD (LP-2)	40,000.00	1.95	20,000.00	ADD (LP-2)	80.20%	29,398.83	60,041.90
Balance (after)	61,000.00	1.00	61,000.00	**Balance (after)**			
ADD (LP-3)	1,000.00	0.00	1,000.00	ADD (LP-3)	100.00%	984.29	61,026.19
Balance (after)	62,000.00	1.00	62,000.00	**Balance (after)**			
baseAdded	**22,000.00**		**42,000.00**	**assetAdded**			

The three liquidity providers then claim back their assets, but the second provider has lost out to the ones before them, because they demanded the pool's liquidity when they staked, thus transferring fees to former liquidity providers. The final provider added symmetrically and thus does not lose or gain.

	Units	Ownership	BASE Added	ASSET Added	BASE Claim	ASSET Claim	BASE PNL	ASSET PnL
LP-1	1,000.00	1.45%	1,000.00	1,000.00	1,015.96	1,015.96	15.96	15.96
LP-2	59,041.90	85.32%	60,000.00	60,000.00	59,984.04	59,984.04	-15.96	-15.96
LP-3	984.29	1.42%	1,000.00	1,000.00	1,000.00	1,000.00	0.00	0.00

Conclusion

The team did discover two issues relating to pool ownership, causing a “bonus” and liquidity erosion. The issues were separate and the root causes for them were from different reasons. The bonus was caused by incorrectly deriving pool ownership during dynamic price changes, and the erosion was caused by not factoring in slip during an asymmetrical stake. The team derived a final pool ownership equation that has addressed both issues, and should stop the bonus and erosion from occurring again.