



GAUNTLET

Gauntlet Research Report

Mechanism Design Assessment

An analysis of the THORChain Continuous Liquidity Pool and its slip-based fee



THORCHAIN

November 2020

Contents

Contents	1
I Background	4
1 Overview	4
2 Simulation Analysis	5
2.1 Scope of the Analysis	5
2.2 Goals of the Analysis	6
II Model Setup and Details	7
3 Model Setup	7
3.1 CLP Model	7
3.1.1 Providing Liquidity	8
3.1.2 Removing liquidity	8
3.1.3 Swapping	8
3.2 Price Feeds	8
3.3 Exchanges	8
3.3.1 Fixed-Fee XYK Exchange	9
3.3.2 Centralized Exchange	9
4 Agent Types	9
4.1 Liquidity Provider	9
4.2 Arbitrageur	10
4.3 Demand trader	11
5 Key Model Parameters	12
5.1 Starting LP Pool Depths	12
5.2 Trade Size	13
5.3 Trade Volumes	13
5.4 Asset Price Trajectories	13
6 Measures of Protocol Success	13
6.1 Market Share	13
6.1.1 Fee Share	13
6.1.2 Volume Share	14
6.2 LP Returns	14
6.2.1 LP ROI	14
6.2.2 LP APY	14
6.2.3 LP IL	14
6.2.4 LP Decisions	15
6.3 Other Outputs	16

6.3.1	Average Fee and Slip	17
7	Simulation Initialization	17
7.1	Price Paths	17
7.2	Agent Initialization	17
III	Simulation Results and Analysis	18
8	Price Trajectories	18
9	Liquidity and Trade Volume	20
10	Key Takeaways	23
IV	Appendix	24
A	Background on Agent-Based Simulation	24
B	Gauntlet Simulation Environment	24
C	Python Contracts	24
D	Price Inertia	25
E	References	26
F	Authors	28
G	Acknowledgements	28

Disclaimer

This report is for informational purposes only and does not constitute an offer to sell, a solicitation to buy, or a recommendation for any security, nor does it constitute an offer to provide investment advisory or other services by Gauntlet Networks Inc. No reference to any specific security constitutes a recommendation to buy, sell or hold that security or any other security. Nothing in this report shall be considered a solicitation or offer to buy or sell any security, future, option or other financial instrument or to offer or provide any investment advice or service to any person in any jurisdiction. Nothing contained in this report constitutes investment advice or offers any opinion with respect to the suitability of any security, and the views expressed in this report should not be taken as advice to buy, sell or hold any security. The information in this report should not be relied upon for the purpose of investing. In preparing the information contained in this report, we have not taken into account the investment needs, objectives and financial circumstances of any particular investor. This information has no regard to the specific investment objectives, financial situation and particular needs of any specific recipient of this information and investments discussed may not be suitable for all investors. Any views expressed in this report by us were prepared based upon the information available to us at the time such views were written. Changed or additional information could cause such views to change. All information is subject to possible correction. Information may quickly become unreliable for various reasons, including changes in market conditions or economic circumstances.

Part I

Background

1 Overview

The dramatic user growth of decentralized finance (DeFi) applications has been driven by the rise in usage of Automated Market Makers. DeFi protocols aim to replace custodial, centralized entities such as exchanges, brokers, and lenders with smart contract based mechanisms that economically incentivize fair and efficient trade. Automated Market Makers (AMMs) have captured significant cryptocurrency exchange market share, amassing daily trading volumes larger than those of their centralized counterparts. In late 2020, decentralized AMM venues became the dominant source of liquidity for a variety of Ethereum-based digital assets, such as Wrapped Bitcoin and the Dai Stablecoin [1]. One of the main drivers of this growth has been the search for sustainable cryptocurrency yields that exceed those of traditional financial products. Providing liquidity in decentralized exchanges has been a popular way to generate yield for digital asset investors, where liquidity providers earn trading fees for providing capital to AMMs. As of October 2020, there is over \$5 billion US dollars equivalent of cryptocurrency supplied by liquidity providers of Ethereum-based decentralized exchanges. The most popular type of AMM is known as the constant function market maker (CFMM), which was first popularized by Uniswap [2, 3, 4].

In AMMs, users provide liquidity for a pair of crypto-assets to a smart contract by depositing their assets in a liquidity pool that is controlled by a smart contract. When a user deposits their assets into a liquidity pool, a synthetic asset known as a liquidity provider (LP) share is minted that acts as a voucher for the user to redeem their pro-rata share of assets and fees. The quantity of each asset deposited is determined by the price that the AMM is currently quoting to prospective traders. When trader places a market order on an AMM, the protocol takes one asset from the trader and returns a quantity of the trader's desired asset from the liquidity pool. This quantity is determined by the preservation of an invariant dependent on the quantity of assets deposited into each pool and a fee. The collected fee is distributed pro-rata to LPs — e.g. if a user has 50% of LP shares before a trade takes place, they receive 50% of the fees collected for that trade. For a formal mathematical analysis of the mechanism used in Uniswap and other CFMMs, please refer to [2, 5].

THORChain is a Uniswap-like CFMM that uses its own chain to facilitate transactions. Unlike Uniswap, THORChain can facilitate cross-chain transactions, such as those between Bitcoin and Ethereum rather than providing synthetic asset swaps (e.g. trading one Ethereum asset for another). By facilitating cross-chain swaps, THORChain is able to expand the set of assets that can be traded via a CFMM-like mechanism. In particular, assets on other chains such as Bitcoin, Cosmos, and Terra can be swapped for Ethereum ERC-20 assets. You can find more info on how THORChain works in their documentation [6] and our previous analysis of THORChain system incentives [7].

While many CFMM-based exchanges use a fixed-fee, THORChain uses a fee structure that charges more for larger trades (slip-based fee). Such a fee model has the potential to increase the fees earned by liquidity providers on exchanges that use this fee structure, while also increasing the cost to execute a trade. Ideally, the increase in fees should draw more liquidity to an exchange using a slip-based fee, and this stronger liquidity would subsequently lower the cost to trade on those exchanges. On the other hand, increasing trading fees could cause a decrease in trading volume that leads to LPs removing their assets due to low yield.

However, an analytic analysis of the particular impact of a fee model is fraught with difficulties. These difficulties occur as the financial performance of the system is dependent on the behaviors, strategies, and interactions between LPs and traders. For instance, if most traders want to execute large trades but find the slip-based fee to be larger than alternatives (e.g. centralized exchanges), then the trade volume in such a CFMM will be low. If

there are not many trades executed, then LPs receive smaller fees and are liable to remove their liquidity from the CFMM pool. In order to stress test how much liquidity is present, one needs to model the risk and profit seeking behaviors of the agents in the system — the LPs and traders. Using an agent-based simulation model, we investigate how profit-maximizing LPs and traders would participate in these exchanges and find conditions under which each fee structure would be more successful.

2 Simulation Analysis

Gauntlet utilizes agent-based simulation to model the THORChain protocol and associated ecosystem of LPs and traders. Agent-based simulation has been used to study the interactions between borrowers and lenders in decentralized lending protocols [8] and Uniswap [2]. We adapt these methodologies to stress test THORChain's fee model. More information about how the Gauntlet platform works can be found in the Appendix A.

2.1 Scope of the Analysis

Three key components in THORChain are the incentive pendulum, the native currency RUNE and the continuous liquidity pool (CLP) slip-based fee. The focus of this analysis is to analyze the slip-based fee and present a comparison to the fixed fee model in isolation of the other features of THORChain. This was decided in part from having already released a prior report on the incentive pendulum [7] but more so since the numerous cross-interactions the incentive pendulum and RUNE have on the overall trading activity present challenges in making clear attributions of agent behavior towards the novel fee structure and not the other pieces of the protocol.

In particular, the incentive pendulum creates both a RUNE denominated constraint on the total value of staked assets the system can secure along with indirect exposure to inflation rewards for liquidity providers. RUNE in turn acts as the settlement currency for all liquidity pools on THORChain along with ensuring network security through incentives and serving as the governance and inflation reward token.

The multifaceted and highly active nature of RUNE within the system pose questions as to how to accurately model the liquidity and external market dynamics of the asset. As an example, a bondholder's disincentivization in RUNE to act maliciously against the network is not necessarily well parameterized by the classical assumption of a geometric Brownian motion price trajectory due to the value differential of RUNE inside and outside THORChain, the subsequent predicted illiquidity of external markets along with a general sense of altruism or belief in the network given the 1MM RUNE requirement just to run the node.

Both the valuation and associated volatility of RUNE are also not so straightforward to pinpoint. A traditional financial approach might be to take the discounted cash flows of trading fees from the pools RUNE secures but a significant portion of these fees are themselves denominated in RUNE, leading to a recursive estimate. There is also likely a nonlinearity of value with respect to quantity due to the disproportionate gains from acquiring enough RUNE to obtain early bonding rewards which also comes with the confounding issues of the market impact, slippage and potential signaling of a loss of faith in the protocol a large sale may incur.

Because of the aforementioned complications that arise in decoupling the effects of RUNE and the incentive pendulum from the slip-based fee, we ultimately removed them from our model. As both the block reward and the hub currency in a hub and spoke model, RUNE draws parallels to ETH which served a similar function in Uniswap v1 so in our simplification of THORChain which we refer to as the "CLP Exchange" there consists of a single ETH-DAI liquidity pool. In order to highlight **the effects of the slip-based fee**, we create a competitive ecosystem where this fee model is pitted head-to-head against a fixed-percentage fee model. Thus we also include a XYK exchange with an ETH-DAI liquidity pool, charging a fixed-percentage fee.¹ Keeping our observations focused

¹In much of the documentation and literature on THORChain, the constant product exchange is referred to as XYK. This name is chosen

on a single trading pair eliminates the added contrast of the hub and spoke and point to point network models. In doing so we better incorporate the market dynamics that drive individual agent behavior and influence overall metrics of protocol success such as market share or fee revenue.

2.2 Goals of the Analysis

While we have observed fixed-fee AMM protocols like Uniswap and Curve gain success in terms of dollar notional trading volume and liquidity, one can also observe users participating by providing liquidity deal with unpredictable returns. By charging a variable fee, THORChain hopes to allow liquidity providers to maintain profitability across a wider range of scenarios. In our analysis we try to assess this by answering the following questions:

1. Does the slip-based fee improve the expected returns for liquidity providers? Under what market conditions and user behaviors is THORChain's model successful?
 - (a) How does price movement in the pooled assets affect key protocol metrics?
 - (b) The slip-based fee changes with trade size and pool depth. How does variation in these factors affect system livelihood?
2. The slip-based fee is often more expensive than a fixed-fee in high value, time-sensitive circumstances. To what extent does this disincentivize arbitrageurs that keep prices on THORChain in-line with other exchanges?
3. Under what scenarios does organic demand volume favor the slip-based versus the fixed fee?

as the convex, polynomial invariant that Uniswap uses is the trading function $\phi(x, y) = xy$ [2, 5]

Part II

Model Setup and Details

3 Model Setup

The model is comprised of a few main components:

1. A Python model of the THORChain system, focused on the the CLP fee model
2. Stochastic price trajectories for asset prices
3. Models of two external exchanges — a centralized exchange and a Uniswap-style XYK Exchange
4. A set of agents that, responding to changes in price and other agents, interacts with the THORChain model and the external components

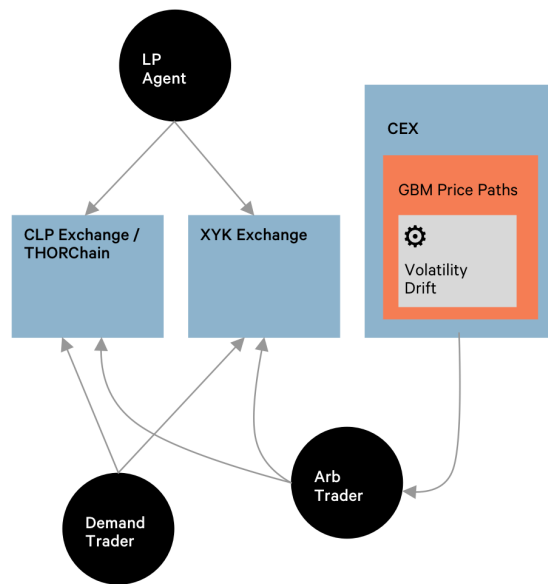


Figure 1: The Simulation Model (high-level diagram)

3.1 CLP Model

THORChain uses continuous liquidity pools (CLP) to facilitate asset swaps. Participants of the network can interact with pools in the following ways.

3.1.1 Providing Liquidity

To provide liquidity, participants provide a pair of assets to a liquidity pool and in return receive LP shares determined by

$$s = \frac{(X + Y)(xY + yX)}{4XY} \quad (1)$$

where X and Y are the pool asset balances including the newly provided asset amounts x and y ².

3.1.2 Removing liquidity

To withdraw their liquidity, participants redeem their shares for pool assets prorata relative to the total number of LP shares outstanding.

3.1.3 Swapping

To swap, participants pay a slip-based liquidity fee on the output token to the pool given by

$$\Delta y = \frac{x^2 Y}{(x + X)^2} \quad (2)$$

and receive tokens given by

$$y = \frac{xXY}{(x + X)^2} \quad (3)$$

where X and Y are the pool input and output token balances prior to the swap and x is the amount of input tokens provided. Note that

$$y + \Delta y = \frac{xXY}{(x + X)^2} + \frac{x^2 Y}{(x + X)^2} = \frac{xY(x + X)}{(x + X)^2} = \frac{xY}{x + X} \quad (4)$$

which has the same structure as the XYK formula prior to taking the fee.

3.2 Price Feeds

One of the main driving forces in the simulation model are the synthetic price feeds which agents interact with and drive trading volumes and LP returns. We use a standard Geometric Brownian motion (GBM) to simulate price trajectories. This stochastic process obeys the Itô stochastic differential equation, $dX_t = \mu S_t dt + \sigma S_t dW_t$, where dW_t is the standard Wiener measure on \mathbb{R} . GBM is also equivalent to the exponential of a randomly varying quantity following a Brownian motion, e.g. $X_t = X_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t\right)$.

3.3 Exchanges

To test the viability of THORChain, we allow agents to provide liquidity as well as trade on two other exchanges. Agents choosing to participate in the other exchanges allows us to model the competitive landscape that THORChain will operate in. We model agents trading and providing liquidity for two assets — one volatile and one not. We've chosen ETH and DAI, as those are a popular trading pair on many decentralized exchanges. The exchanges are initialized to support these two assets.

²This formula has since been modified in the protocol due to issues related to asymmetric liquidity provision. As a simplification in the model that also avoids these problems all LPing is required to be symmetric.

3.3.1 Fixed-Fee XYK Exchange

The fixed-fee is set to thirty basis points (0.30%), which is a common value used on decentralized exchanges, including Uniswap [9] and Synthetix [10]. Agents are able to execute trades and provide liquidity to this exchange. We initialize the liquidity pool to a size that we vary across simulation runs, which is specified in the outputs of Part III and in section 7

3.3.2 Centralized Exchange

The simulation emulates a centralized exchange that allows agents to buy and sell ETH and DAI in exchange for USD. It uses slippage models to represent the price impact of agent trades in the simulation environment, where larger trade sizes will incur more slippage. More specifically, the slippage s is defined as

$$s = c\sigma x^\alpha \quad (5)$$

where c is a scalar for intensity, σ is the annualized volatility, α controls the concavity of the curve, and x is defined as

$$x = \min\left(\frac{z}{d}, 1\right) \quad (6)$$

where z is the order size and d is the average daily traded volume.

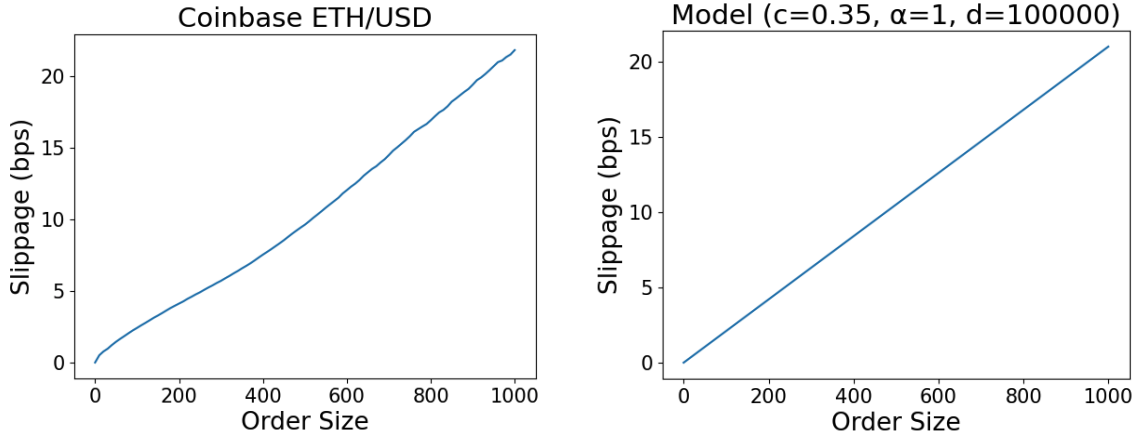


Figure 2: Coinbase ETH/USD Slippage

Using order book data from Coinbase Pro for ETH/USD and DAI/USD in Oct 2020 to fit these models we choose $c = 0.35$, $\alpha = 1$ and $d = 100000$ for ETH and $c = 0.1$, $\alpha = 0.5$ and $d = 1500000$ for DAI.

4 Agent Types

4.1 Liquidity Provider

The main decision that is modeled involves LP agents allocating funds across the XYK exchange and THORChain. Agents select whether or not to provide liquidity based on the expected returns of doing so. The agents are initialized with different "lookback windows". Lookback windows determine the timescale that agents take into

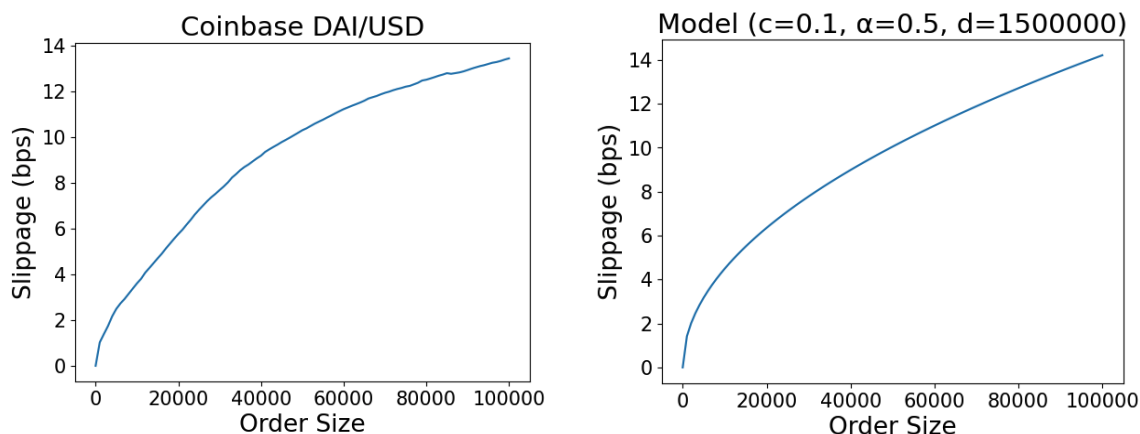


Figure 3: Coinbase DAI/USD Slippage

consideration when making the decision to provide liquidity. Agents with a shorter lookback window calculate the expected ROI based off of more recent data, and will switch more frequently. When fees (and consequently, expected ROI) on one exchange start to diminish, agents with a short lookback window will stop immediately, whereas agents with a longer time-preference will wait to see if the lowered yields persist before removing their capital. As mentioned in 2.1, modeling LP decision making is fraught with hard-to-verify assumptions, so we keep our LP agent logic relatively simple. However, the ROI metrics we expose should provide a window into the decision LPs face. These statistics demonstrate under different conditions how LPs on each exchange will be incentivized.

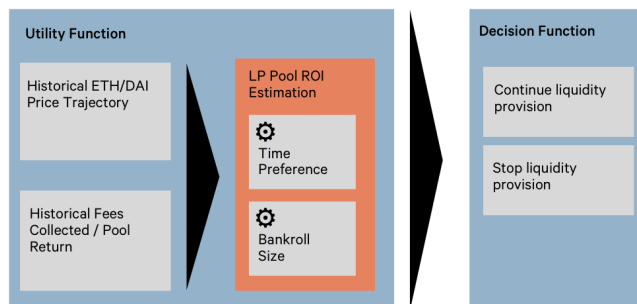


Figure 4: LP Agent Logic

4.2 Arbitrageur

Arbitrageur agents execute trades between the centralized and decentralized exchange when price differs by an amount large enough that they can realize an immediate profit.

1. This keeps the price implied by the DEX exchange pools (CLP and XYK) in line with the centralized exchange price

2. These agents are initialized with a fixed, pre-determined bankroll, which allows for realistic limits on how large of an arbitrage opportunity can be taken at one time. We note that in cryptocurrency trading, unlike traditional finance, arbitrage is often constrained by a dearth of liquidity.

One thing to note is that with a slip-based fee, arbitrageurs can try to maximize their revenue by splitting up their trades into smaller chunks to minimize slippage. This is distinct from the path deficiency property that is endemic to constant function market makers like Uniswap [2, 5]. THORChain has a mechanism called a "swap queue" which orders transactions by slip, so that the most aggressive arbitrageur (who attempts to close the largest portion of the arbitrage opportunity) has their trade selected. We approximate a competitive market in the swap queue with our arbitrageur logic, which directs the arb agents to try to swap as much as they can.

Since arbitrageurs generally prefer to be market delta neutral, the precise sequence of trades is

1. Buy asset A on centralized exchange
2. Swap asset A for asset B on cheapest to execute DEX
3. Sell asset B on centralized exchange

As such their accounts only consist of USD at the start and end of every time step. At each decision point the arbitrageur enumerates through all sequences of possible trades taking slippage, fees and market prices from the various exchanges into consideration and executes the one with highest expected profit.

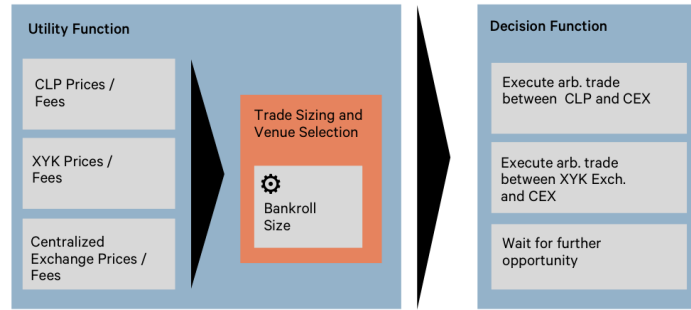


Figure 5: Arbitrage Agent Logic

4.3 Demand trader

The demand trader agent represents the demand generation process for cryptoassets simulated within the system. Agents always choose the best price for executing available trades between the exchanges in the simulation. Each agent trades a size that is fixed during a given simulation run; however, this trade size is varied across simulation scenarios. The choice of trade size parameter is critical as it can have a large affect on the simulation results. In traditional finance, market microstructure models are often specified completely by the trade size distribution and we mimic such models in our parametrization [11]. In cases where the average trade is expected to be very large or very small, there will either be a deluge or a dearth of arbitrage opportunities. The steps for trading are as follows:

1. Trader uniformly at random determines a token Y to trade in portfolio

2. Trader draws a number X from a truncated Pareto distribution with probability density function f defined by

$$f(x) = \frac{\alpha L^\alpha x^{-\alpha-1}}{1 - \left(\frac{L}{H}\right)^\alpha} \quad (7)$$

with $\alpha = 0.5$, $L = 1$ and $H = 10000$.

3. Trader swaps X basis points of their holdings in Y on the cheapest to execute pool

The mean of the truncated Pareto distribution is given by

$$\mu = \frac{L^\alpha}{1 - \left(\frac{L}{H}\right)^\alpha} \left(\frac{\alpha}{\alpha - 1} \right) \left(\frac{1}{L^{\alpha-1}} - \frac{1}{H^{\alpha-1}} \right) \quad (8)$$

for $\alpha \neq 1$ so plugging in $\alpha = 0.5$, $L = 1$ and $H = 10000$ gives $\mu = 100$ so in expectation the trader exchanges 1% of their holdings in an asset per swap. Using this distribution allows for trades of varying magnitude to be accounted for in the simulation, ensuring that there are interactions with both the CLP and XYK pools.

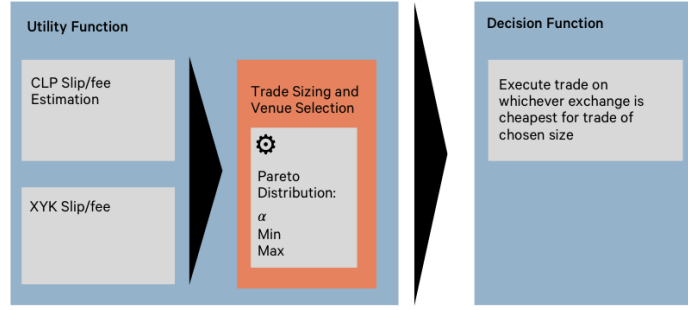


Figure 6: Demand Trader Agent Logic

5 Key Model Parameters

Within our models, we use on-chain data wherever possible to ensure the agent models used in the simulation are as representative as possible of what we expect to see on mainnet. However, it is not a guarantee that what we have seen in the past continues in the future. We still need to make assumptions in the model which will affect the results. We vary these assumptions as widely as possible so as to be sure that results remain valid given a wide variety of conditions on mainnet.

5.1 Starting LP Pool Depths

We initialize the simulation with varying amounts of capital allocated to each exchange. Since exchange liquidity has such a large impact on success, we want to understand what the minimum liquidity THORChain would need to achieve to compete with a larger XYK exchange. In the case where the CLP has much less liquidity than the XYK exchange, it will face difficulties competing with the larger XYK exchange. However as we start to increase the liquidity, measured by the ratio of pool depths between the two exchanges, we can see at what point the CLP exchange starts to be more competitive. This allows us to make sure that our assumption on the relative liquidity between the two exchanges does not create a confounding variable in the simulation results.

5.2 Trade Size

As DEXs have evolved from niche product to major trading venue, there has been a natural increase in trade sizes over time. Moreover, as crypto markets evolve, we expect to see continual changes to multiple properties of empirical trade size distributions. When analyzing fixed-fee exchanges like Uniswap and Balancer, trade size is less important, as the fee is a percentage of total volume. However with THORChain's slip-based fee, trade sizes have a large impact on the total fees generated. In both exchanges, the size of organic trades have a strong affect on arbitrage volume. In our model, we have separated the organic demand and the arbitrage trade volume with the demand trader agent and the arbitrage agent to capture this effect. This allows us to more precisely analyze the revenue split between active market participants in THORChain.

5.3 Trade Volumes

Another important assumption is the trade volume in the simulation. Both the total volume traded and the average size effects the success of each fee model. When trades are small relative to the pool size, THORchain is able to draw trades away from XYK exchanges as the lower slip fee incentivizes these trades.

5.4 Asset Price Trajectories

The price trajectories mentioned in section 3.2 have a large effect on the results of the simulation. These affect the impermanent loss experienced by LPs. We define impermanent loss in §6.2.3 and demonstrate that it is a large factor in LP returns. Unlike fixed-fee XYK exchanges, where the parameters of the price trajectory fully determine the loss, the slip-based fee adds an extra component to the analysis of impermanent loss.

6 Measures of Protocol Success

As we simulate this ecosystem under various different conditions, it naturally begs the question — how do we measure the success of the slip-based fee model? The goal of the slip-based fee is to:

1. Allow LPs to weather drifting price trajectories, which tend to create unfavorable outcomes as shown in 6.2.3.
2. Minimize the impact on trading volume and activity from multiple market participants

6.1 Market Share

There are two measures of an exchange's market share that are used in simulation. The *fee share* is the percentage of fees the total fees that each exchange earns, and the *volume share* is percentage of total volume that trades on each exchange.

6.1.1 Fee Share

The *fee share* for an exchange x is the sum of the fees paid on the exchange, divided by the total fees paid over the lifetime of the simulation.

6.1.2 Volume Share

The *volume share* for an exchange x is the sum of the trade value (*not* including fees paid) on the exchange, divided by the total trade volume in the simulation. This is also measured over the lifetime of the simulation run.

6.2 LP Returns

6.2.1 LP ROI

Liquidity provider ROI is calculated throughout the simulation by taking the current total value of the LP shares and dividing the by the initial value of the assets deposited when the LP first provided liquidity. The value of the LP shares is determined by taking the price of each asset and multiplying it by the amount of that asset in the pool.

$$\text{ROI} = \frac{\sum_{i=1}^N \text{Valuation of LP } i\text{'s Assets at Time } t}{\sum_{i=1}^N \text{Valuation of LP } i\text{'s Starting Assets at Time } 0} - 1 \quad (9)$$

6.2.2 LP APY

Since APY takes continuous compounding into account, it is computed by

$$\text{APY} = \left(\frac{\sum_{i=1}^N \text{Valuation of LP } i\text{'s Assets at Time } t}{\sum_{i=1}^N \text{Valuation of LP } i\text{'s Starting Assets at Time } 0} \right)^{\frac{\text{Year}}{t}} - 1 \quad (10)$$

6.2.3 LP IL

Ignoring gains from liquidity fees and assuming constant pool share, impermanent loss [12][13] is a term that refers to the change in portfolio value of a LP's pool share relative to simply holding tokens in wallets or centralized exchanges when prices vary from the original staking price. We note that impermanent loss has been studied in traditional finance under the guise of volatility harvesting [14]. From the XYK formula underlying the pool computations if the pool has quantities X and Y of assets x and y respectively then the product $XY = K$ is invariant for some constant K through a transaction (though over time this grows as fees are added). As an example suppose x denotes ETH, y denotes the USD Stablecoin Dai (DAI), p denotes the price of ETH in DAI and assume that the pool is efficiently priced so $Y/X = p$ then originally

$$X = \sqrt{\frac{K}{p}} \quad Y = \sqrt{Kp} \quad (11)$$

Thus the overall pool value V is

$$V = p_x X + p_y Y = p \sqrt{\frac{K}{p}} + 1 \sqrt{Kp} = 2\sqrt{Kp} \quad (12)$$

Suppose that the price moves by a factor of $c > 0$ so the new price $p' = cp$. Then the new pool value V' is

$$V' = p'_x X' + p'_y Y' = cp \sqrt{\frac{K}{cp}} + 1 \sqrt{Kcp} = 2\sqrt{Kcp} \quad (13)$$

Using the constant pool share assumption, if a staker had instead held the assets instead of staking their portfolio share scaled to the entire pool W would be worth

$$W = p'_x X + p'_y Y = cp \sqrt{\frac{K}{p}} + 1 \sqrt{Kp} = (c+1) \sqrt{Kp} \quad (14)$$

Note that $c+1 \geq 2\sqrt{c}$ by the AM-GM inequality so there is always a relative drop in value in this no fee constant pool share context. Some intuition for this comes from

$$\frac{dX}{dp} = \frac{-1}{2} \sqrt{\frac{K}{p^3}} < 0 \quad \frac{dY}{dp} = \frac{1}{2} \sqrt{\frac{K}{p}} > 0, \quad (15)$$

so as p increases so does Y while X decreases and as p decreases Y does as well while X increases. More generally, changes to p increase the delta to the weaker performing asset since initial stake and decrease the delta to the stronger asset.

$$\frac{d^2V}{dp^2} = \frac{d}{dp} \left(\frac{dV}{dp} \right) = \frac{d}{dp} \left(\sqrt{\frac{K}{p}} \right) = \frac{-1}{2} \sqrt{\frac{K}{p^3}} < 0 \quad (16)$$

shows that V is concave and the portfolio delta is inversely related to price, akin to the gamma of a short option position. Holding the assets has a linear delta to price and the holding value W is tangent to the pool value V at the staking price and from concavity it follows that $W \geq V$. This is seen in Figure 7. We define the *Impermanent Loss* as

$$V' - W = -(c+1 - 2\sqrt{c}) \sqrt{Kp} \quad (17)$$

6.2.4 LP Decisions

Two important factors in determining an LP agent's actions in a time step are the rate of return of fee income r_{LP} as an LP in a pool and the historical rate of return of ETH/DAI $r_{ETH/DAI}$. The former is given by

$$r_{LP} = \frac{\text{LP Income}}{\text{LP Asset Value}} \quad (18)$$

where an annualized estimate of an LP's fee income from the pool is given by

$$\text{LP Income} = \frac{\text{Year}}{w \Delta t} \sum_{t=T-w}^{T-1} \frac{\text{Pool Fee Income}_t \cdot \text{Agent LP Shares}}{\text{Total LP Shares}} \quad (19)$$

where Δt is the length of a time step, w denotes the length of the lookback window in time steps and T is the current environment time step.

We determine $r_{\text{ETH/DAI}}$ by annualizing a time weighted exponential moving average (EMA) of the returns of ETH/DAI at every time step over the lookback window. This is given by

$$r_{\text{ETH/DAI}} = (1 + X_t)^{\frac{\text{Year}}{\Delta t}} - 1 \quad (20)$$

for $t = T - 1$ where X_t is the EMA of the returns at time step t defined through the recursion

$$X_t = \begin{cases} \frac{r_t + (w-1)X_{t-1}}{w} & T - w \leq t < T \\ 0 & t < T - w \end{cases} \quad (21)$$

In the simulation we specify the impermanent loss adjusted LP ROI $r_{\text{LP adj}}$ as a function of r_{LP} and $r_{\text{ETH/DAI}}$ and setting the price change factor from Equation 17 as $c = 1 + r_{\text{ETH/DAI}}$ we get

$$r_{\text{LP adj}} = \frac{2\sqrt{1 + r_{\text{ETH/DAI}}}}{2 + r_{\text{ETH/DAI}}} (1 + r_{\text{LP}}) - 1 \quad (22)$$

At every time step an LP agent computes $r_{\text{LP adj}}$ and if $r_{\text{LP adj}} > r_f$ they add liquidity to the pools and if $r_{\text{LP adj}} \leq r_f$ then they withdraw their liquidity where r_f is the risk free rate of return set in the simulation as 1%.

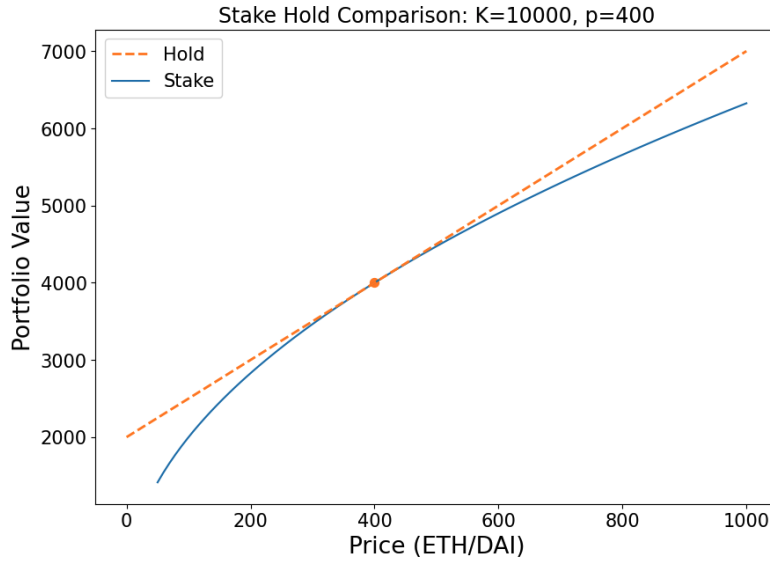


Figure 7: Impermanent Loss

6.3 Other Outputs

We also expose a few other outputs from the simulation to provide more insight into the results. The average fee size and average trade slippage are not measures of success per se, though they do provide a window into what drives the XYK and CLP exchange's success.

Parameter	Default Value
ETH/USD Initial	400
ETH/USD Drift	0%
ETH/USD Volatility	60%
DAI/USD Initial	1
DAI/USD Drift	0%
DAI/USD Volatility	5%

Table 1: Default price path values

Parameter	Default Value
No. of LP Agents on CLP	10
No. of LP Agents on XYK	10
No. of Arbitrage Agents	10
No. of Demand Trader Agents	10

Table 2: Agents

6.3.1 Average Fee and Slip

The fee is always 30 bps for the XYK exchange, whereas will vary for the CLP, depending on liquidity and trade size. Slippage has a large impact on trader behaviour, with demand traders always utilizing the exchange with the best price. This means that the exchange with a larger pool will be able to draw more trade volume because of better executed trade price. It should be noted that similar liquidity bounds for trade volume were computed for Uniswap [2, App. C] and our simulation aims to test the hypothesis of whether the CLP's liquidity bound is better than that of a standard XYK exchange. By exposing these metrics, we provide an insight into *how* the exchange achieves success.

7 Simulation Initialization

7.1 Price Paths

We vary the volatility and drift of the price paths to ensure that they are not skewing our analysis. However, on runs where we vary other parameters, like the pool liquidity, we use a default value for the volatility and drift of the GBM, shown in 1 where volatility and drift are quoted in annualized percentage terms and 0% drift indicates neither positive nor negative bias.

7.2 Agent Initialization

The number of agents and their individual bankrolls are also key parameters in determining the outcome of the simulation. In particular the demand trader and LP initialized wealth are varied throughout to account for a wide range of scenarios. The default values are shown in 2 and 3. The arbitrage agents have smaller bankrolls since they perform all legs of the arbitrage with their entire bankroll immediately as presented and the largest DAI order size we trained for in our slippage models was 100 000. To ensure that they are not capital constrained arbitrage agents are allowed to trade up to 10 times in a time step.

While the demand trader and LP bankrolls are listed in USD their holdings only ever consist of ETH and DAI with the default values obtained from marking their starting wallet to initial token prices. LPs start with an equal 50/50 notional split of ETH and DAI to allow for efficient liquidity provision whereas demand traders begin with an 80/20 split of ETH and DAI to more accurately reflect general market holdings. As a default the CLP and XYK LPs have the same amount of available capital though we progressively increase the XYK LP wealth to highlight boundaries where added pool depth cancels out some of the advantages of the slip-based fee.

Parameter	Default Value
Arb Agent Bankroll Size	\$100 000
Demand Trader Bankroll Size	\$20 000 000
LP Agent Bankroll Size	\$8 000 000

Table 3: Agent Parameters

Part III

Simulation Results and Analysis

8 Price Trajectories

ETH/USD annualized drift and volatility provide the impetus for arbitrageur volume and introduce impermanent loss for LPs. Since their variation affect nearly every measure of protocol success, we investigate by running a total of 3200 simulations across a search space of

$$\begin{aligned} \text{ETH Volatility Annual (\%)} &\in \{10, 30, 60, 100, 150, 210, 280, 360\} \\ \text{ETH Drift Annual (\%)} &\in \{-100, -50, -25, 0, 25, 50, 100, 200\} \end{aligned}$$

LPs in both the CLP exchange and the XYK exchange are making a bet that fees collected on the exchange exceed the impermanent loss (eqn. 17). Since price paths mostly determine the IL experienced by LPs, we show how the different price trajectories effect the returns in the XYK and CLP exchanges in Figures 8 and 9.

These figures show the value of becoming an LP — the ROI of changing your assets into LP shares. Figure 9 shows a result most readers are familiar with - LPs in the XYK exchange are heavily effected by price swings. One thing you will notice is that there is strong demand volume in this simulation, with annualized ROIs reaching 40%. However when drift reaches 200% (expected ETH appreciation is 3x within the sim), returns start to go negative.

In the CLP exchange however, returns are pretty constant. Since the fees taken from arbitrageurs scale with price drift, this allows the returns from organic demand to dominate. Impermanent loss doesn't go to zero, however it is much lower than in the XYK exchange. You'll notice that in 11, the CLP actually doesn't capture a dominant market share of trades since the XYK can be cheaper for traders. However since LPs have much better ROI regardless of price path you would expect LP capital to start to move to the CLP exchange. Once the CLP exchange is able to draw in more capital, the liquidity will improve, as will the prices for traders, followed by market share.

Key Takeaway

The slip-based fee improves the expected returns for liquidity providers in most market conditions. Impermanent loss is greatly reduced, but not eliminated.

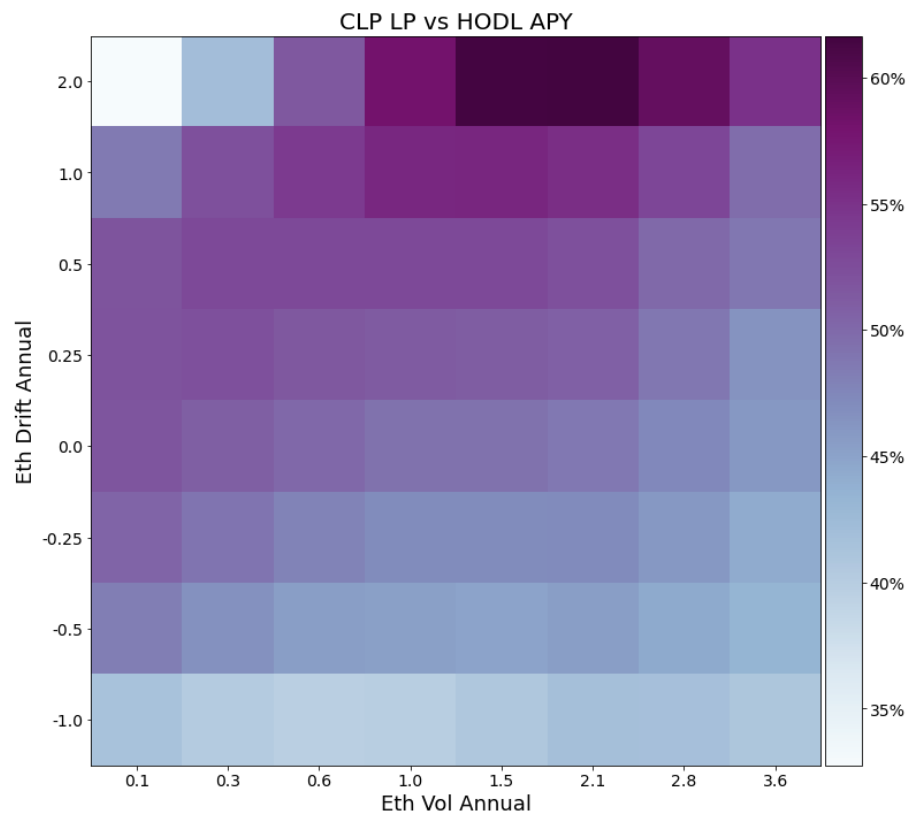


Figure 8: CLP LP Returns, Adjusted for Impermanent Loss

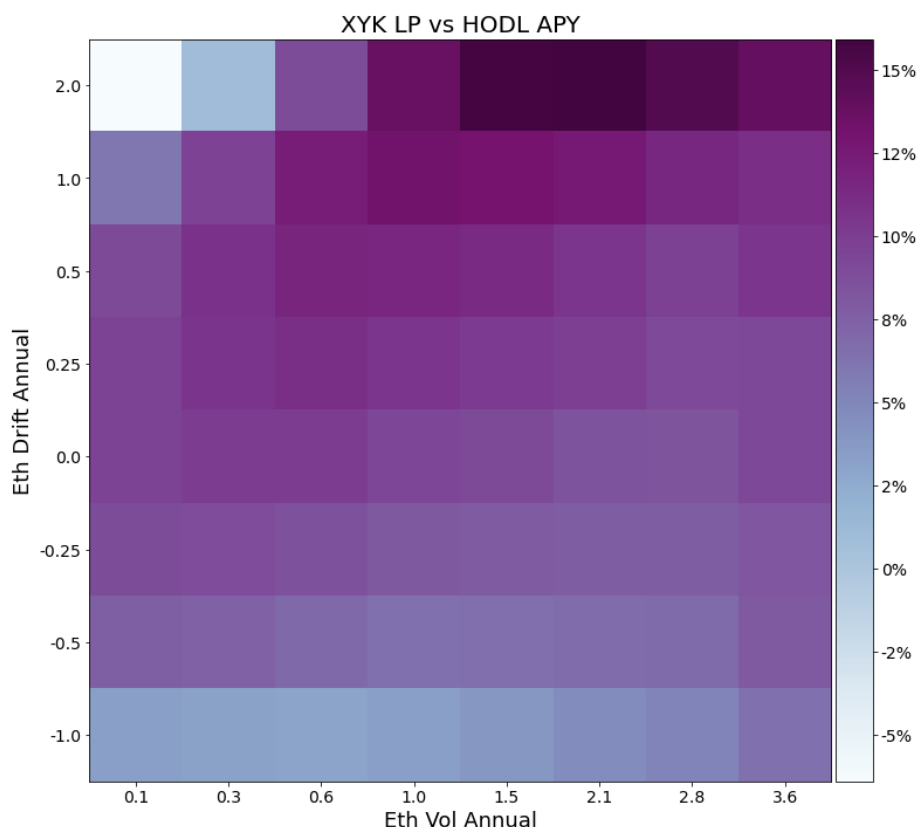


Figure 9: XYK LP Returns, Adjusted for Impermanent Loss

9 Liquidity and Trade Volume

To see the effect of these parameters on agent decision making we run a total of 2560 simulations across a search space of

Average Daily Demand Volume (MM USD) $\in \{1, 2, 5, 10, 20, 30, 40, 50\}$
 XYK/CLP Depth Ratio $\in \{1, 2, 3, 4, 5, 6, 7, 8\}$

The Y-axis of 10 and 11 shows the ratio of the starting liquidity for the XYK exchange to CLP exchange. The bottom row shows what happens when they start with same amount of capital, and the CLP is able to take in 60-90% of the fees, depending on trade volumes and sizes. There are similar results for the total market share, though it varies more widely — from 30-85%. The goal of the CLP is to drive higher fees to the protocol to offset price risk, and these results show that it is likely to succeed in this endeavor. The fee share tends to exceed the market share, showing that the CLP is able to generate fees more effectively than a standard fixed-percentage fee model.

As you vary the starting capital between the two exchanges (moving up the y-axis), and the XYK exchange starts with more capital, you can see the fee / volume shift to the XYK exchange. The greater pool depth provides

a better price for traders and they select it over the CLP exchange. However, even with fairly small pools the CLP is still able to capture a large amount of fees, mostly from arbitrageurs.

Another important factor, trade size is varied along the x-axis. On the left, we have the case where trades are small and overall volume is low, and as you progress to the right, the trade size and volume increases. You can see with small trade sizes, the CLP does pretty well, as many of the trades are small enough that the slip (therefore the fee) is less than 30 bps. As the trade size increases, the XYK fee becomes cheaper and the CLP loses out on demand trades, but continues to get revenue from arbitrage trades.

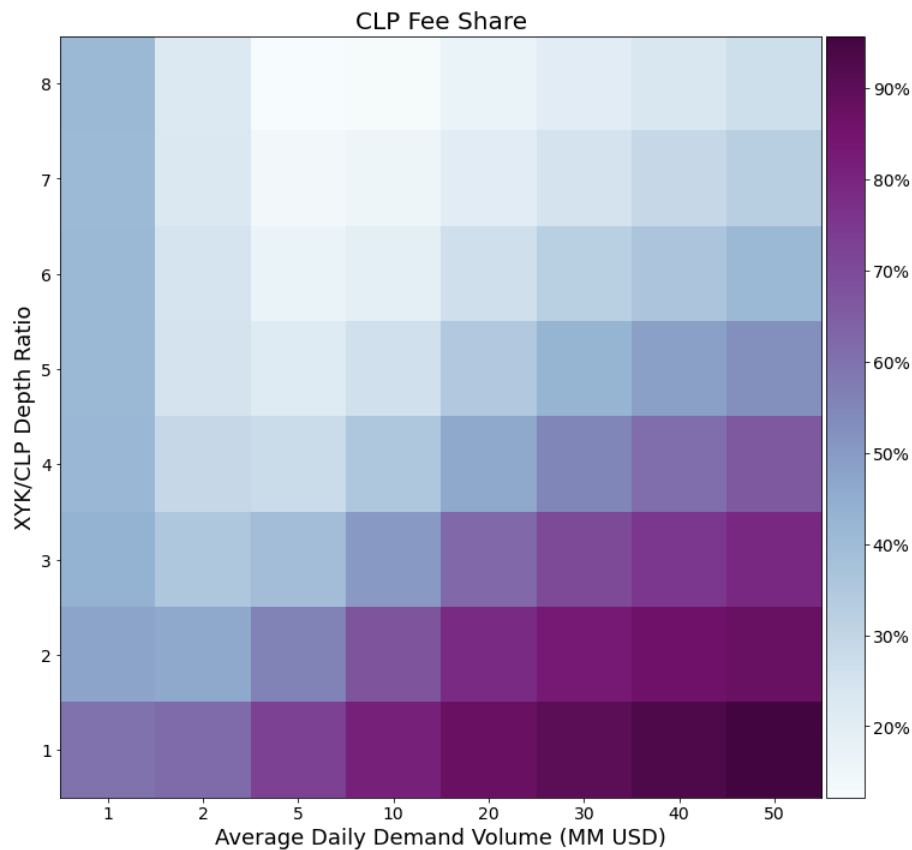


Figure 10: CLP Fee Share vs. Liquidity and Trade Volume

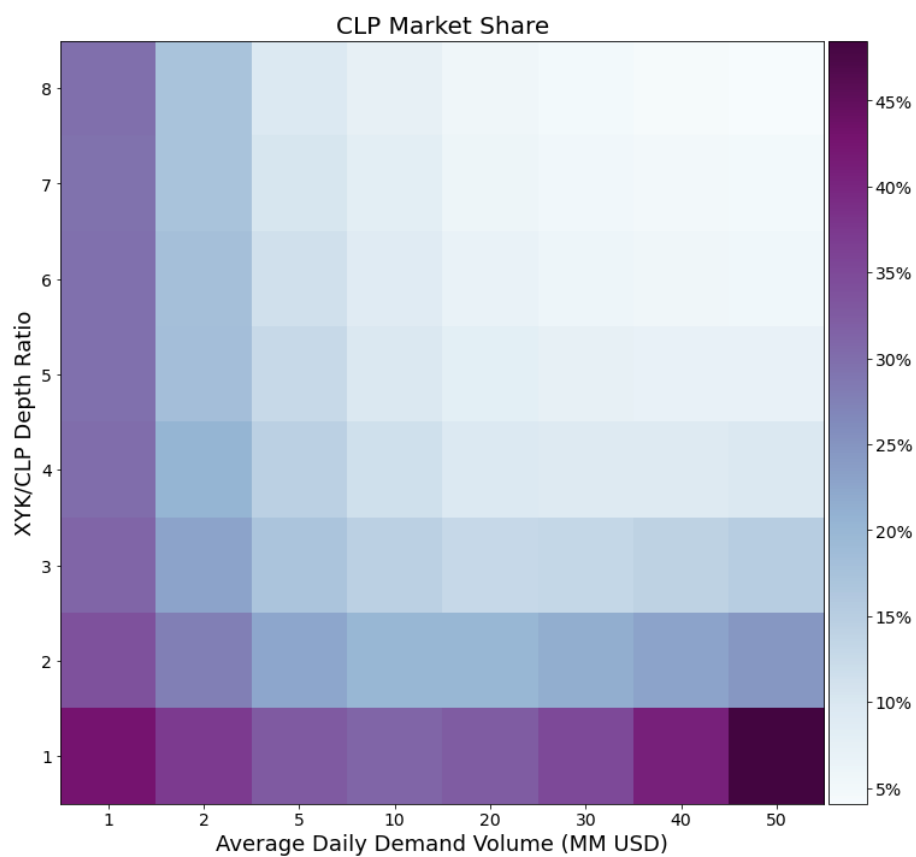


Figure 11: CLP Market Share vs. Liquidity and Trade Volume

Key Takeaway

Returns to LPs appear to be much more consistent in the CLP fee model. Because of this, the liquidity on THORChain should improve to the point where it offsets the fee for larger and larger trades. If THORChain is able to attract sufficient liquidity, organic demand will favor trading on THORChain.

One other factor that effects the fee and market share is how closely the price on each exchange tracks centralized exchange prices. We leave a detailed investigation of this for future analysis, but have included some preliminary results in Appendix [D](#).

10 Key Takeaways

As you can see from the heat maps in the previous section, **the slip-based fee improves the expected returns for liquidity providers in most market conditions**. The fee model does well when there is high volatility between tradings pairs, and is able to withstand substantial price drift. Almost all price movement that is not mean-reverting is bad for a standard XYK exchange. However there are many price trajectories where THORChain LPs are still able to make a profit despite lack of mean reversion. **Impermanent loss is greatly reduced, but not eliminated.**

The slip-based fee does disadvantage THORChain in a highly competitive market for trading volume, as large trades will be cheaper with a fixed fee. Prices on the exchange still remain competitive for smaller trades, with arbitrageurs able to keep prices close to the values on other exchanges. Arbitrage profits are better in the XYK exchange, but that's the whole point of the slip-based fee and we still see prices stay in sync as arbitrageurs can still make money, though not as much. As returns to LP providers appear to be much stronger, the liquidity on THORChain should improve to the point where it offsets the fee for larger and larger trades. **If THORChain is able to attract sufficient liquidity, organic demand will favor trading on THORChain.**

Part IV

Appendix

A Background on Agent-Based Simulation

The main tool that we use to analyze THORChain's protocol is agent-based simulation (ABS). ABS has been used in a variety of contexts in quantitative finance, including to estimate censorship in cryptocurrency protocols [15], detect fraudulent trading activity in CFTC exchanges [16], and in stress testing frameworks from the European Central Bank [17, 18] and the Federal Reserve [19, 20]. These models can provide invaluable information on the behavior of complex systems. This has made ABS widely used in industries such as algorithmic trading and self-driving car deployment.

B Gauntlet Simulation Environment

The Gauntlet platform, which was used for all simulations and results in this report, provides a modular, generic ABS interface for running simulations directly against Ethereum smart contracts as well as Python representations of contracts. Here we use the latter as the THORChain is not built on Ethereum. In this system, the agent models are specified via a Python domain-specific language (DSL), akin to Facebook's PyTorch[21]. Agents can also interact with non-blockchain modules, such as historical or synthetic market data and/or other off-chain systems. The DSL hides the blockchain-level details from the analyst, allowing the end-user to develop strategies that can migrate from one smart contract to another, should they have similar interfaces. Most of the platform's design is inspired by similar platforms in algorithmic trading that allow for quantitative researchers to develop strategies that execute over multiple exchanges (with varying order books, wire protocols, slippage models, etc.) without having to know these low-level details. Moreover, the non-blockchain portions of the simulation are analogous to trading back-testing environments,[22] so that agents are interacting with realistic order books and financial data.

C Python Contracts

There are several contracts in the simulation environment in addition to our agent based simulation platform:

1. THORChain Contract: A modification of the Python implementation of THORChain found [here](#) fit towards our simulation backend. Handles all protocol logic (churning old nodes, managing the standby queue, allocating inflation and liquidity rewards, redistributing system income etc.) and executes agent transactions such as bonding, staking and swapping.
2. XYK Contract: An implementation of a traditional constant product market maker with a fixed fee on the input token which serves as a decentralized liquidity pool alternate to THORChain.
3. CLP Contract: An implementation of a constant product market maker with a slip based fee on the output token.
4. Exchange Contract: A centralized venue for agents to buy and sell liquid tokens such as ETH and DAI through a traditional order book. Simulates the price impact effects of trades by updating the token prices according to impact model parameters. Acts as an external price feed.

5. Portfolios Contract: A helper contract that tracks all the agent coin balances and enables transferring, minting and burning tokens.

D Price Inertia

If arbitrageurs are not able to push the price back to the CEX price, the DEX risks becoming non-competitive. Without fees, there is a clear path to price parity as discussed in [2]. However as you introduce fees, the DEX price will now track within an error bounds determined by those fees:

$$(1 - \tau)m_p \leq m_{\text{DEX}} \leq (1 + \tau)m_p.$$

where τ is the fee, m_{DEX} is the price on the DEX and m_p is the market price.

In the XYK exchange, τ is simply 30 basis points, but for the CLP it is determined by the equations given in §3.1.3, which vary with trade size. Since arbitrageurs can execute arbitrarily small trades, prices in a CLP should track within a smaller range than on the XYK exchange (though gas costs / network fees impose a limit to how closely). In our simulation, the exchanges both tracked price well, as seen in Figures 12 and 13.

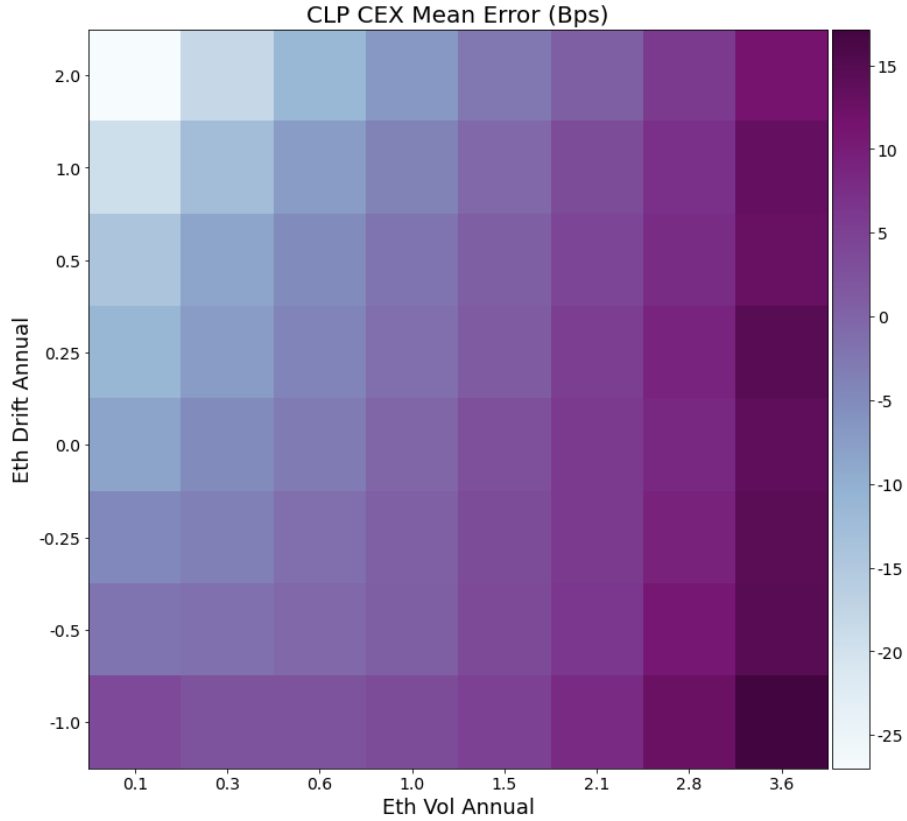


Figure 12: CLP Mean Tracking Error from CEX

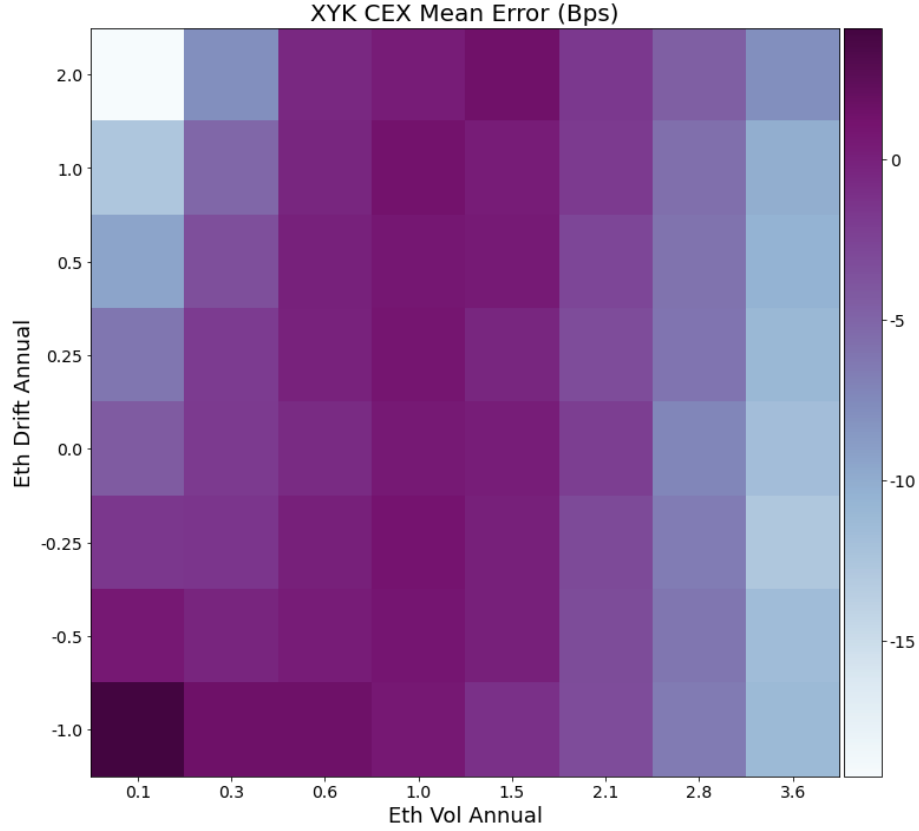


Figure 13: XYK Mean Tracking Error from CEX


Since our demand trading agents take this "quoted" price into consideration (the price implied by the pool sizes on the DEX), these variations in price are considered in the simulation, but we leave a detailed investigation of this to a future analysis. To investigate this in detail, it would make sense to look at much shorter time frame simulations than the time frame on which we investigated (6 months).



E References

- [1] Arian Klages-Mundt and Andreea Minca. (in) stability for the blockchain: Deleveraging spirals and stablecoin attacks. *arXiv preprint arXiv:1906.02152*, 2019.
- [2] Guillermo Angeris, Hsien-Tang Kao, Rei Chiang, Charlie Noyes, and Tarun Chitra. An analysis of uniswap markets. *arXiv preprint arXiv:1911.03380*, 2019.
- [3] Hayden Adams, Noah Zinsmeister, and Dan Robinson. Uniswap v2 core. *URI: <https://uniswap.org/whitepaper.pdf>*, 2020.
- [4] Mika Honkasalo. Uniswap's monthly trade volume exceeded coinbase's in september, Oct 2020.

- [5] Guillermo Angeris and Tarun Chitra. Improved price oracles: Constant function market makers. *Proceedings of the 2nd ACM Conference on Advances in Financial Technologies*, to appear.
- [6] Thorchain. Thorchain documentation. <https://docs.thorchain.org/>.
- [7] Victor Xu, Hsien-Tang Kao. Thorchain protocol assessment. <https://gauntlet.network/reports/thorchain>, June 2020.
- [8] Hsien-Tang Kao, Tarun Chitra, Rei Chiang, and John Morrow. An analysis of the market risk to participants in the compound protocol, 2020.
- [9] Uniswap. Uniswap v2 documentation. <https://uniswap.org/docs/v2/protocol-overview/how-uniswap-works/>, 2020.
- [10] Synthetix Foundation. Synthetix documentation. <https://docs.synthetix.io/litepaper/#synthetixexchange>, 2020.
- [11] Seungki Min, Costis Maglaras, and Ciamac C Moallemi. Cross-sectional variation of intraday liquidity, cross-impact, and their effect on portfolio execution. *Columbia Business School Research Paper*, (19-4), 2018.
- [12] Uniswap: A good deal for liquidity providers? <https://medium.com/@pintail/uniswap-a-good-deal-for-liquidity-providers-104c0b6816f2>, January 2019.
- [13] Understanding uniswap returns. <https://medium.com/@pintail/understanding-uniswap-returns-cc593f3499ef>, February 2019.
- [14] Jan Hendrik Witte. Volatility harvesting: Extracting return from randomness. Preprint, 2015. Available at <https://arxiv.org/abs/1508.05241>.
- [15] T. Chitra, M. Quaintance, S. Haber, and W. Martino. Agent-based simulations of blockchain protocols illustrated via kadena’s chainweb. In *2019 IEEE European Symposium on Security and Privacy Workshops (EuroS PW)*, pages 386–395, June 2019.
- [16] Steve Yang, Mark Paddrik, Roy Hayes, Andrew Todd, Andrei Kirilenko, Peter Beling, and William Scherer. Behavior based learning in identifying high frequency trading strategies. In *2012 IEEE Conference on Computational Intelligence for Financial Engineering & Economics (CIFER)*, pages 1–8. IEEE, 2012.
- [17] Grzegorz Halaj. Agent-based model of system-wide implications of funding risk. 2018.
- [18] Anqi Liu, Mark Paddrik, Steve Y Yang, and Xingjia Zhang. Interbank contagion: An agent-based model approach to endogenously formed networks. *Journal of Banking & Finance*, 2017.
- [19] John Geanakoplos, Robert Axtell, J Doyne Farmer, Peter Howitt, Benjamin Conlee, Jonathan Goldstein, Matthew Hendrey, Nathan M Palmer, and Chun-Yi Yang. Getting at systemic risk via an agent-based model of the housing market. *American Economic Review*, 102(3):53–58, 2012.
- [20] Richard Bookstaber, Mark Paddrik, and Brian Tivnan. An agent-based model for financial vulnerability. *Journal of Economic Interaction and Coordination*, 13(2):433–466, 2018.
- [21] Adam Paszke, Sam Gross, Francisco Massa, Adam Lerer, James Bradbury, Gregory Chanan, Trevor Killeen, Zeming Lin, Natalia Gimelshein, Luca Antiga, et al. Pytorch: An imperative style, high-performance deep learning library. In *Advances in Neural Information Processing Systems*, pages 8024–8035, 2019.
- [22] Peter Nystrup, Stephen Boyd, Erik Lindström, and Henrik Madsen. Multi-period portfolio selection with drawdown control. *Annals of Operations Research*, 282(1-2):245–271, 2019.

F Authors

Victor Xu  is a Data Scientist at Gauntlet. Victor worked as a quantitative trader at Flow Traders. Prior to Flow Traders, Victor spent time at Citadel and Jane Street Capital. He studied Mathematics and Computer Science at Princeton University.

John Morrow   is Gauntlet's COO. Prior to Gauntlet, he worked on product and data engineering at Gigster, Microsoft, and Goldman Sachs. He also studied at Cornell where he received a B.S. in Applied Physics and Computer Science.

G Acknowledgements

The authors would like to thank John-Paul Thorbjornsen and Chad Barraford for helpful discussions. We would also like to thank the other members of Gauntlet team, in particular Rei Chiang and Hsien-Tang Kao for simulation design and best practices, Tarun Chitra for guidance and clarity, and Jonathan Reem for help with the simulation infrastructure.