MATH 253 – Discrete Mathematics
Final Exam – Study Guide

Schedule

The Final Exam will be given in class on **Thursday, December 17, 2020 @ 08:00 AM – 9:50 AM.**

Unlike the mid-term exams, the Final Exam will start exactly on time and end at the time specified.

Administrivia

This is an exam given online via Canvas. As such, there is no effective way to limit the resources you have available during the exam. I will, though, write the exam with the idea that you have a sheet of notes that you have prepared in advance. I will even make suggestions below about what that sheet of notes should have on it. But you can add whatever you think is useful to that sheet, and you are not required to turn it in. Hold onto that sheet, though, as the final exam will cover all material in the class and you can use it then, too!s

Keep in mind that the exam will be timed in a manner that does not allow extensive referrals to external materials. It will be geared towards testing your familiarity with the presented materials in class and in the online textbook. This is another good reason to condense your notes to a single "cheat sheet" you can use quickly and easily to keep pace with the problems in the time provided.

Format of exam

The majority of the questions presented will be multiple choice. There are significant limitations in giving an online exam, and one of them is the ability to write in a mathematical notation. Thus, the need for a question format like multiple choice is necessitated. These questions will not concentrate just on rote memorization, though – they are designed to allow students to show the skills of applying what has been learned in class to problems similar to, but not exactly the same as, those in the text. They are also designed to allow students to show analysis skills in order to solve problems presented to them.

You will want a pencil and lost of blank paper available to work through problems! Multiple choice doesn't mean "do it all in your head" – you'll need to write out a few things to get to many of the answers to the questions provided! There will be no need to submit the work you do on your scratch paper, though.

A multiple choice exam can be every bit as challenging as an "on paper" exam! But there will be no penalty in giving wrong answers, so answering <u>every</u> question will be to your benefit – don't leave any questions unanswered!

I <u>may</u> also ask you some questions that are not multiple choice. These questions, though, will be far fewer than the ones in multiple choice form.

Academic Honesty

While the exam may be open book, it is ABSOLUTELY required that the work you present on the exam is your own! No real-time collaboration with other students is permitted. Canvas can randomize the order of answers listed on multiple-choice tests, and even present questions in a different order for each student, so sharing of multiple-choice answers in A/B/C/D format between students is an utterly futile effort.

Material to be Covered

All material covered in the class, including material from the mid-term exams, may be on the final exam. Because of this, this Study Guide will be a supplement to the other two Study Guides, and all three Study Guides together constitute the material covered in the final exam.

You are responsible for all material covered in class and in the text from the following sections:

Chapter 8 "Introduction to Counting"
Chapter 9 "Advanced Counting"
Chapter 10 "Discrete Probability"
Chapter 11 "Graphs"
Chapter 12 "Trees"

Preparation Strategies

First and foremost, the best preparation strategy is to review the material in the text, and to drill through the provided exercises. Make that your go-to strategy! Understanding the answers there, and the methods presented to get to the answers, presents your best opportunity to do well on the exam.

Topics & Terminology

You should be familiar with and be able to answer questions about:

<u>Chapters 8 & 9 – "Introduction to Counting" and "Advanced Counting" (the Pigeonhole Principle)</u>

- The **Sum Rule** and the **Product Rule** when it comes to counting the elements in finite sets
- The ability to count **strings**, that are created as Cartesian products of elements from an **alphabet**
- Understanding that the concept of "**counting**" is actually the mapping (a bijection) with the set of positive integers (the "**counting numbers**")
- Using a **k-to-1 correspondence** when k elements of the domain map to 1 element of the target
- The **generalized product rule** is used when there are a specific number of choices for the first item to be selected from a set, and the second item, and so on, in a sequence of items.
- A **r-permutation** is a selection of r items from a set of size n, when the order of selection matters. The number of r-permutations in a set of n is calculated as n! \ (n – r)!
- A **r-combination** is a selection of r items from a set of size n, when the order of selection does NOT matter. The number of r-combinations in a set of n is calculated as n! / [r! (n – r)!]
- The two basic questions to ask when selecting r values from a set of n values are: "Does order matter?" and "Is repetition allowed?"
- The **pigeonhole principle** applies when the cardinality of a domain set is greater than the cardinality of a target set. This means the function cannot be one-to-one in those cases. The **generalized pigeonhole principle** applies when the domain is more than K times larger than the target, which means that there must be at least one value in the target that is mapped to by K+1 or more values in the domain.

Chapter 10 – "Discrete Probability"

- The definitions of a **experiment**, an **outcome**, an **event**, the **event space**, and the **sample space**
- The **probability** of a event *E*, written *P*(*E*), and the formula $P(E)=\frac{|E|}{|S|}$
- A set is **countably infinite** if there can be defined a bijection that maps the set of positive integers $\mathbf{Z}^+$ to the set. If such a bijection is impossible, then the set is **uncountably infinite**
- A **probability distribution** of an experiment with a countable sample space is a function from the sample space to the set of real numbers from 0 to 1, with the property that the sum of all the function values add up to 1. The probability of an outcome E in the sample space is the sum of the probabilities of the individual outcomes in the event space.
- A **uniform distribution** results when all outcomes have an equal probability of occurring.
- A **union** of **mutually exclusive** events is the sum of the probabilities of the events
- The **complement** of an event E that has a probability p(E) has a probability of 1 – p(E).
- The **conditional probability** p(E | F) is the probability of E given that a related event F has occurred. If the events are **mutually independent**, then the probability p(E | F) = p(E)
- Bayes' Theorem is important, but NO questions specifically about Bayes' Theorem will be on the exam!
- Counting a list of integers from *m* to *n*, including *m* and *n*, using the expression $n - m + 1$

Chapter 11 – "Graphs"

- A **graph** is defined by its set of **vertices** and a set of **edges**, each connecting two vertices
- Vertices connected by an edge are called **adjacent**
- A **simple graph** is one that contains no **loops** (edges whose ends connect to the same vertex) and no **parallel edges** (two or more edges that connect the same pair of vertices)
- An **undirected graph** has edges without direction (**two-way** edges), while a **directed graph** or **digraph** has edges that have a direction (**one-way** edges)
- A **complete graph** is one where every possible pair of vertices are adjacent
- A **bipartite graph** is one where there are two groups of vertices and all edges connect a vertex from one group to a vertex in another group (think of a function diagram with a domain and co-domain)
- The **degree of a vertex** is the number of connections it has to edges, that is, the number of edges **incident** upon that vertex (with a loop representing two connections)
- The **degree of a graph** is the sum total of the degrees of all its vertices
- The **Handshake Theorem** states that the total degree of a graph = twice the number of edges
- You should be able to determine whether a graph with specified degrees for its vertices can exist
- A **walk** is a series of adjacent vertices and edges with unlimited repetition of vertices and edges
- A **trail** is a walk where no edge is used more than once, but vertices may be visited repeatedly
- A **path** is a trail where no vertices may be visited more than once

- A **closed walk** is a walk where the starting point and ending point are the same vertex
- A **circuit** is a trail with at least one edge whose start point and end point are the same vertex
- A **simple circuit** is a path with at least one edge whose start point and end point are the same vertex
- A **connected graph** is one with a walk between any two vertices
- A **connected component** is a component (**subgraph**) with a walk between any two vertices in the component, but with no walk to any vertex outside the component
- An **Eulerian trail** is a trail that contains all edges of the graph
- An **Eulerian circuit** is a circuit that contains all edges of the graph
- You should be able to determine whether a connected graph has an Eulerian circuit or an Eulerian path
- A **Hamiltonian circuit** is a simple circuit that visits all vertices

Chapter 12 – "Trees"

- A **tree** is a graph that is connected and contains no circuits
- A **forest** is a graph that is not connected and contains no circuits (each connected component is a separate tree)
- A **trivial tree** is a single vertex with no incident edges
- A **leaf** is a vertex on a tree with degree 1
- An **internal vertex** is a vertex on a tree with degree greater than 1
- Properties of trees: every tree with n vertices has exactly $n - 1$ edges and total degree $2(n - 1)$
- Every connected graph has a subgraph that is a tree containing all vertices and a subset of edges
- A tree with a root and with every node having 2 or fewer children is called a **binary tree**.
- A tree can be traversed in three distinct ways – **preorder**, **inorder**, and **postorder**.