MATH 253 – Discrete Mathematics
Exam #2 – Study Guide

Schedule

Exam #1 will be given on **Thursday, November12, 2020. You may take the exam online in any 80-minute period between 9:00 AM and 5:00 PM. The instructor will be available for questions ONLY from 9:00 AM to 10:20 AM.** The exam is 80 minutes (+ accommodations).

Administrivia

This is an exam given online via Canvas. As such, there is no effective way to limit the resources you have available during the exam. I will, though, write the exam with the idea that you have a sheet of notes that you have prepared in advance. I will even make suggestions below about what that sheet of notes should have on it. But you can add whatever you think is useful to that sheet, and you are not required to turn it in. Hold onto that sheet, though, as the final exam will cover all material in the class and you can use it then, too!s

Keep in mind that the exam will be timed in a manner that does not allow extensive referrals to external materials. It will be geared towards testing your familiarity with the presented materials in class and in the online textbook. This is another good reason to condense your notes to a single "cheat sheet" you can use quickly and easily to keep pace with the problems in the time provided.

Format of exam

The majority of the questions presented will be multiple choice. There are significant limitations in giving an online exam, and one of them is the ability to write in a mathematical notation. Thus, the need for a question format like multiple choice is necessitated. These questions will not concentrate just on rote memorization, though – they are designed to allow students to show the skills of applying what has been learned in class to problems similar to, but not exactly the same as, those in the text. They are also designed to allow students to show analysis skills in order to solve problems presented to them.

You will want a pencil and lost of blank paper available to work through problems! Multiple choice doesn't mean "do it all in your head" – you'll need to write out a few things to get to many of the answers to the questions provided! There will be no need to submit the work you do on your scratch paper, though.

A multiple choice exam can be every bit as challenging as an "on paper" exam! But there will be no penalty in giving wrong answers, so answering <u>every</u> question will be to your benefit – don't leave any questions unanswered!

I <u>may</u> also ask you some questions that are not multiple choice. These questions, though, will be far fewer than the ones in multiple choice form.

Academic Honesty

While the exam may be open book, it is ABSOLUTELY required that the work you present on the exam is your own! No real-time collaboration with other students is permitted. Canvas can randomize the order of answers listed on multiple-choice tests, and even present questions in a different order for each student, so sharing of multiple-choice answers in A/B/C/D format between students is an utterly futile effort.

Material to be Covered

You are responsible for all material covered in class and in the text from the following sections:

Chapter 4 "Sets" – All sections

Chapter 5 "Functions" – All sections

Chapter 6 "Relations / Digraphs" – All sections

Chapter 7 "Integer Properties" – All sections, except there will be no questions about the Extended Euclidean Algorithm in Section 7.5 (there **will** be questions about the "regular" Euclidean Algorithm!)

Preparation Strategies

First and foremost, the best preparation strategy is to review the material in the text, and to drill through the provided exercises. Make that your go-to strategy! Understanding the answers there, and the methods presented to get to the answers, presents your best opportunity to do well on the exam.

Topics & Terminology

You should be familiar with and be able to answer questions about:

Chapter 4 - "Sets"

- The concept of a **set**, and the set notations – **roster notation** and **set builder notation**
- The **cardinality** of a set, and **finite sets** vs **infinite sets**
- The pre-defined sets of numbers – **N**, **Z**, **Q**, and **R**
- **Subset**: $A \subseteq B$ <==> if $x \in A$ then $x \in B$
- **Proper subset**: $A \subset B$ <==> if $x \in A$ then $x \in B$, and $A \neq B$
- **Set equality**: $A = B$ <==> $A \subseteq B$ and $B \subseteq A$
- **Venn diagrams** and their use in representing set relations graphically
- **Intersection** ($A \cap B$), **union** ($A \cup B$), **difference** ($A - B$) and **symmetric difference** ($A \oplus B$)
- The **complement** A^c of a set
- The **empty set** or **null set** $\varnothing$, and the **Universal Set** U
- **Disjoint sets** A and B : $A \cap B = \varnothing$
- **Partition** of a set A into pairwise disjoint sets $\{ A_1, A_2, \ldots, A_n \}$ where $A_1 \cup A_2 \cup \ldots \cup A_n = A$
- **Power set** of A: **P** (A) = the set of all subsets of A
- The **Cartesian product** of two sets: $A \times B = \{ (a, b) \mid a \in A \text{ and } b \in B \}$
- A set of **strings** composed of letters from an **alphabet** using **concatenation**
- The **Set Identities**

Chapter 5 – "Functions"

- The definition of a **function**, and the function notation ***f*: *X* → *Y***

- The **<u>domain</u>**, **<u>target</u>**, and **<u>range</u>** of a function
- The "**<u>circles and arrows</u>**" graphical representation of a function
- The test for showing whether two functions are **<u>equal</u>**
- The **floor** and **ceiling** functions with the domain of **R** and the target/range of **Z**
- The **<u>identity function</u>**
- The definition of a **<u>one-to-one</u>** function, or **<u>injection</u>**, and recognizing the diagram of an injection
- The definition of an **<u>onto</u>** function, or **<u>surjection</u>**, and recognizing the diagram of a surjection
- The definition of a function that's both one-to-one and onto, or a **<u>bijection</u>**
- Proving that a function is not one-to-one by counterexample
- Proving that a function is one-to-one by showing that if $f(x_1) = f(x_2)$, then x_1 must equal x_2
- The **<u>inverse function</u>** of a bijection, and the F^{-1} notation for an inverse function
- **<u>Function composition</u>** and the notation $(F \circ G)(x) = F(G(x))$
- Writing the composition of two functions in algebraic notation
- Composition with the identity function
- Composition of a function and its inverse
- **<u>Exponential</u>** functions and **<u>logarithmic</u>** functions, and how the two are inverses of each other

<u>Chapter 6 – "Relations / Digraphs"</u>

- A **<u>binary relation</u>** between two sets as a subset of the Cartesian product of the sets
- The differences in the properties of a **<u>binary relation</u>** and a **<u>function</u>**
- How to tell when a binary relation of a set A onto itself is **<u>reflexive</u>**, **<u>anti-reflexive</u>**, **<u>symmetric</u>**, **<u>anti-symmetric</u>**, or **<u>transitive</u>**
- How to represent a binary relation where the domain and the target are the same finite set by using a **<u>digraph</u>** or **<u>directed graph</u>**
- A **<u>digraph</u>** is defined by its set of **<u>vertices</u>** and a set of directed **<u>edges</u>**, each connecting two vertices
- A **<u>simple graph</u>** is one that contains no **<u>loops</u>** (edges whose ends connect to the same vertex) and no **<u>parallel edges</u>** (two or more edges that connect the same pair of vertices)
- The **<u>in-degree of a vertex</u>** is the number of connections coming into a vertex
- The **<u>out-degree of a vertex</u>** is the number of connections coming out of a vertex
- A **<u>walk</u>** is a series of adjacent vertices and edges with unlimited repetition of vertices and edges
- The difference between an **<u>open walk</u>** and a **<u>closed walk</u>**
- A **<u>trail</u>** is an open walk where no edge is traversed more than once, though vertices may be visited repeatedly
- A **<u>circuit</u>** is a trail whose start point and end point are the same vertex
- A **<u>path</u>** is a trail where no vertices may be visited more than once

- A **cycle** is a circuit where no vertices may be visited more than once (except for the first and last being the same vertex)
- The **composition** of relations (all on the same set) where an ordered pair (a,c) is in the composition S ° R if and only is there is a b where (a,b) is in R and (b,c) is in S.
- The **power** of a graph G, represented as $\mathbf{G}^k$, where (a,b) is in G^k if and only if there is a walk on length k from a to b in graph G.
- The **transitive closure** G^+ of a graph G, where (a,b) is in G^+ if and only if there is a walk of length 1 or greater from a to b in graph G.
- The **matrix** (or **adjacency matrix**) form of a graph, where a square matrix contains 1s and 0s to represent a graph
- Use of the **dot product** operation to create a **matrix product** that can denote a power of a graph G
- How the **sum** of the matrices of graphs G and H, both on the same set of vertices, can represent the **union** of the two graphs
- How the **transitive closure** in matrix form of a graph G is the sum of the matrices of the powers of G

Chapter 7 – "Integer Properties"

- The **div** function and the **mod** function
- How an integer y **divides** an integer x (y is a **multiple** of x) if y div x = 0
- The Division Algorithm, where, for any integer n and positive integer d, there are unique integers q and r where $0 \leq r \leq d - 1$ (the **quotient** and the **remainder**) where n = qr + d
- **Modular arithmetic**, where two integers can be added or multiplied modulo an integer m
- The permitted use of the mod m operation at any time during the computation of the sum or product of two integers mod m
- The property of **congruence mod m**, where a is congruent to b mod m if a mod m = b mod m
- **Prime** numbers and **composite** numbers
- The **prime factorization** of any positive integers, and the **Fundamental Theorem of Arithmetic** which states that every positive integer has a unique prime factorization
- The **Greatest Common Divisor** or **GCD** of two positive integers, and how two positive integers are **relatively prime** if their GCD is 1
- What it means to perform **primality testing** on a positive integer
- **Euclid's Algorithm** for efficiently determining the GCD of two positive integers (remember, the extended Euclidean Algorithm will NOT be on the exam!)
- Representation of numbers in different **bases**, and how to convert a value from one number base to another
- **Base 2** (**binary**), **base 10** (**decimal**), and **base 16** (**hexadecimal**)
- The **fast exponentiation algorithm** by the use of repeated squaring