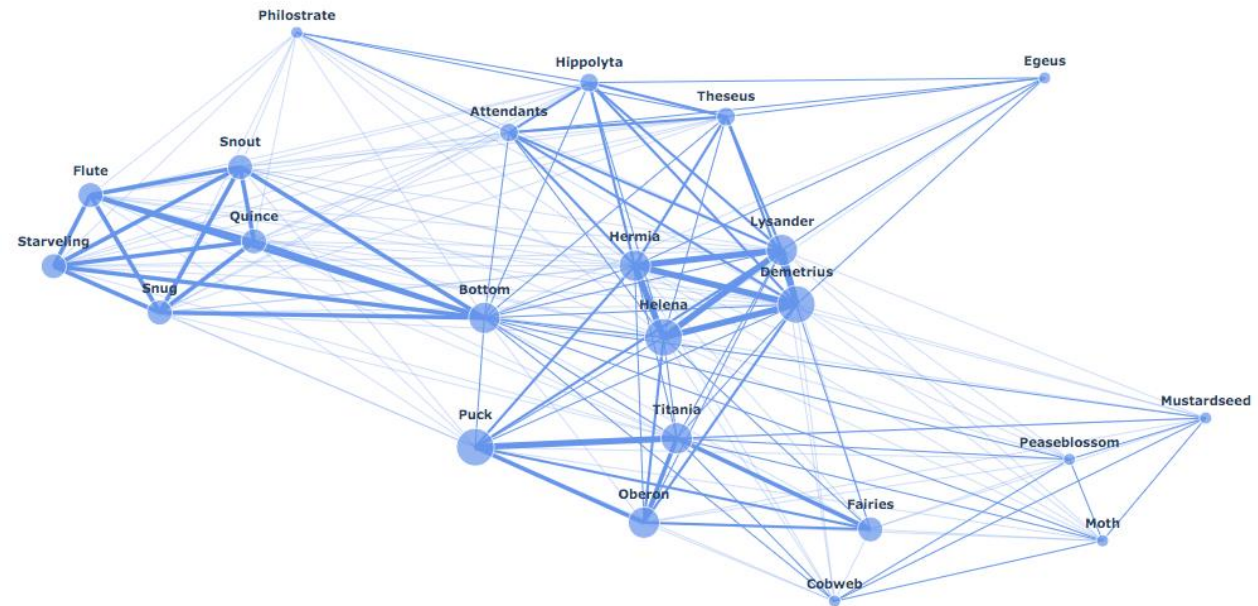


Graph visits

How to explore graphs

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Representing and visiting graphs



GRAPH VISITS

Visit Algorithms

- Visit =
 - Systematic exploration of a graph
 - Starting from a 'source' vertex
 - Reaching all reachable vertices
- Main strategies
 - Breadth-first visit (“in ampiezza”)
 - Depth-first visit (“in profondità”)

Breadth-First Visit

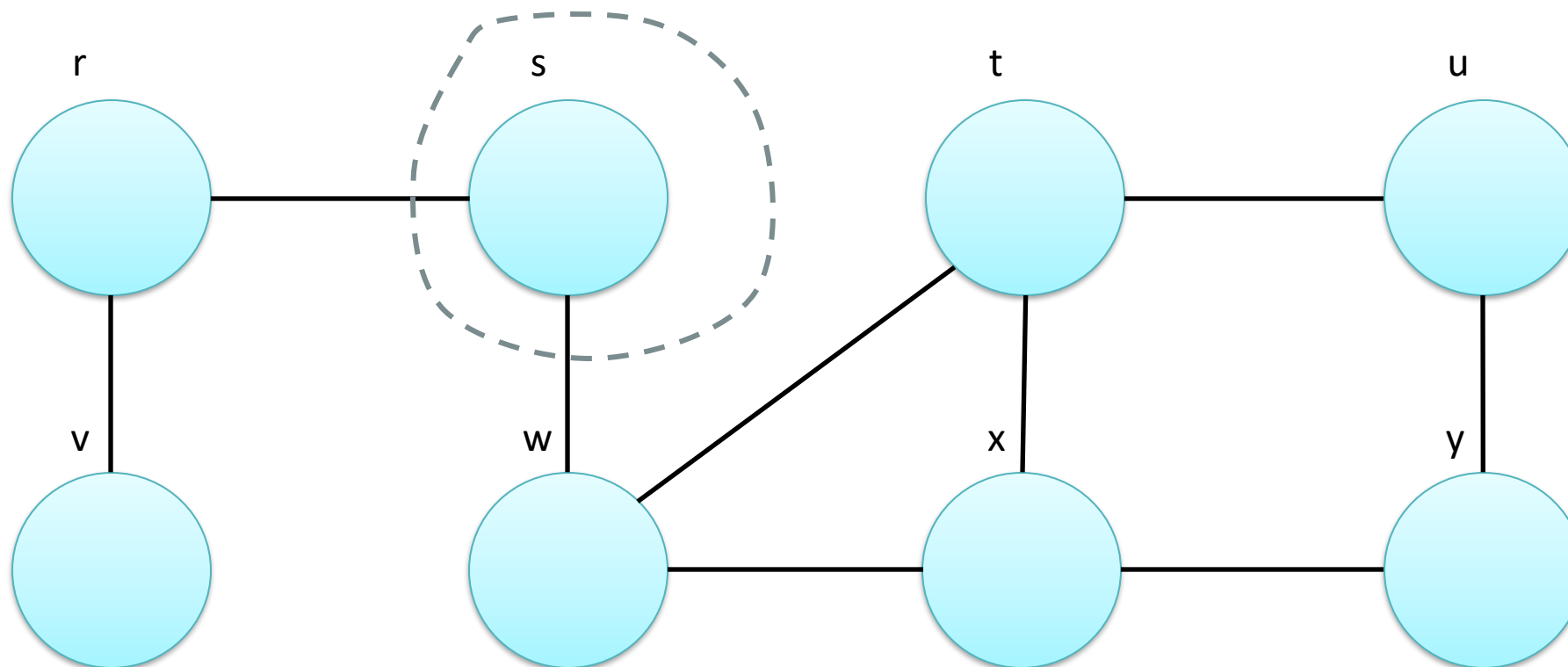
- Also called Breadth-first search (BFV or BFS)
- All reachable vertices are visited “by levels”
 - L – level of the visit
 - SL – set of vertices in level L
 - $L=0, S_0=\{v_{source}\}$
 - Repeat while SL is not empty:
 - $SL+1$ = set of all vertices:
 - not visited yet, and
 - adjacent to at least one vertex in SL
 - $L=L+1$

Example

Source = s

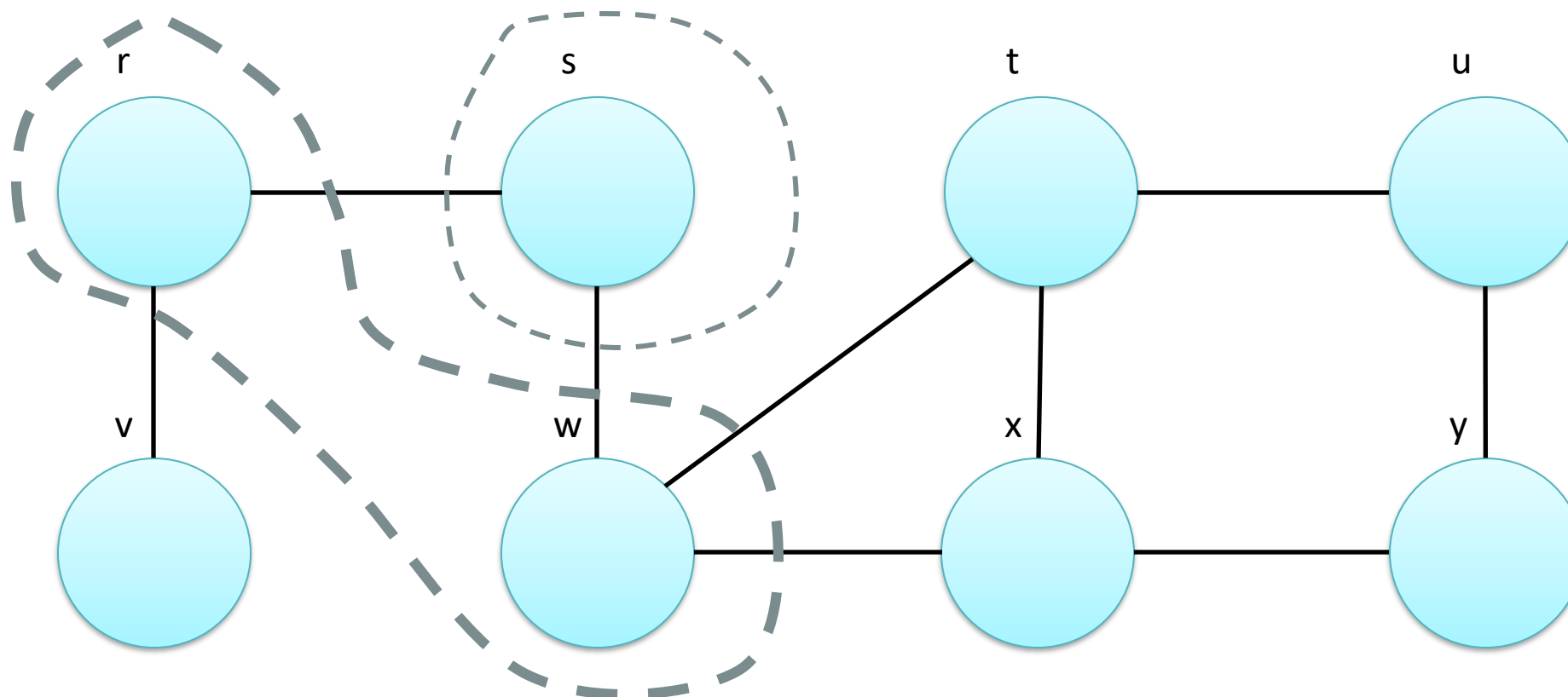
$L = 0$

$S_0 = \{s\}$



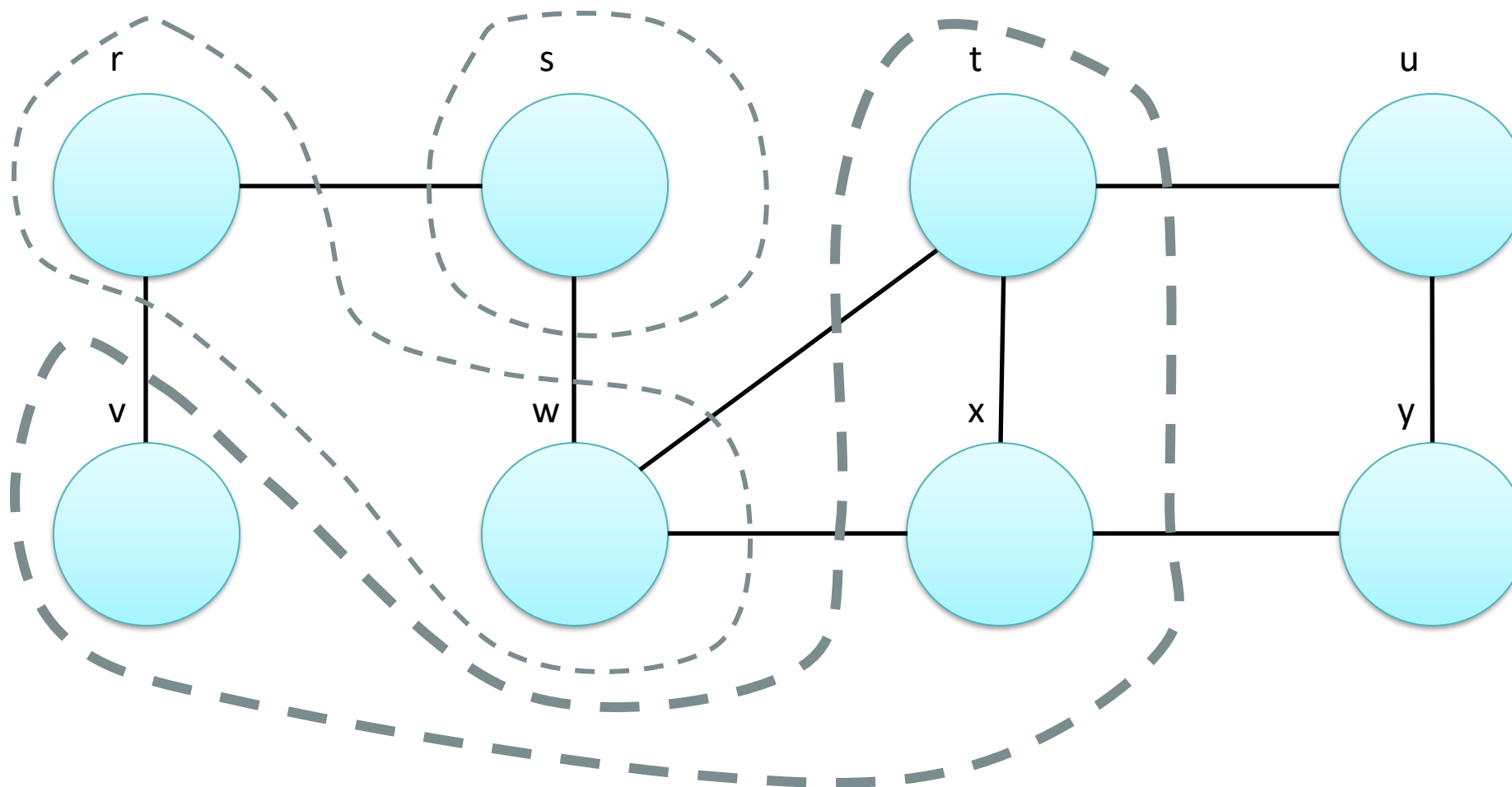
Example

$L = 1$
 $S_0 = \{s\}$
 $S_1 = \{r, w\}$



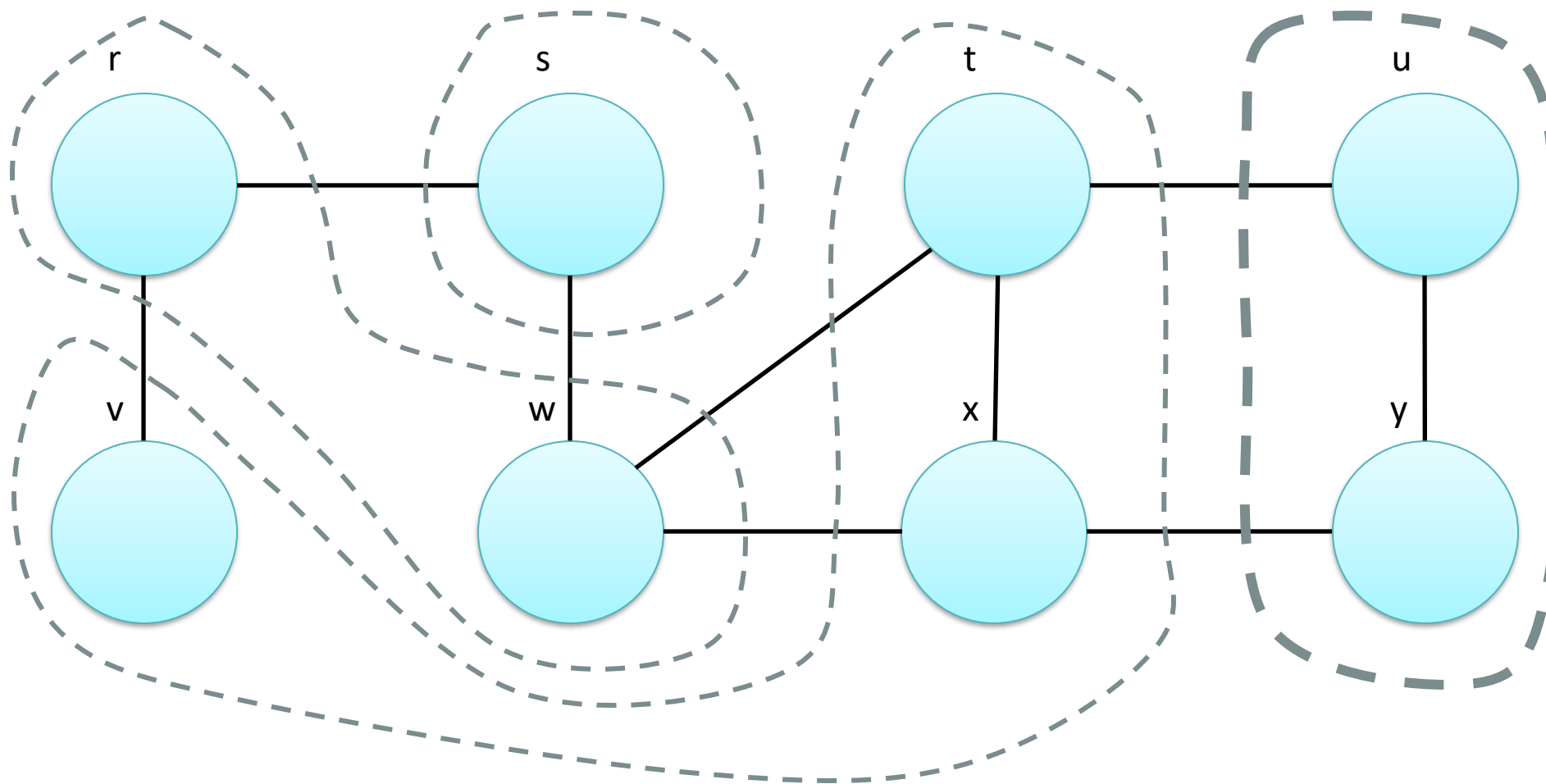
Example

$L = 2$
 $S_1 = \{r, w\}$
 $S_2 = \{v, t, x\}$



Example

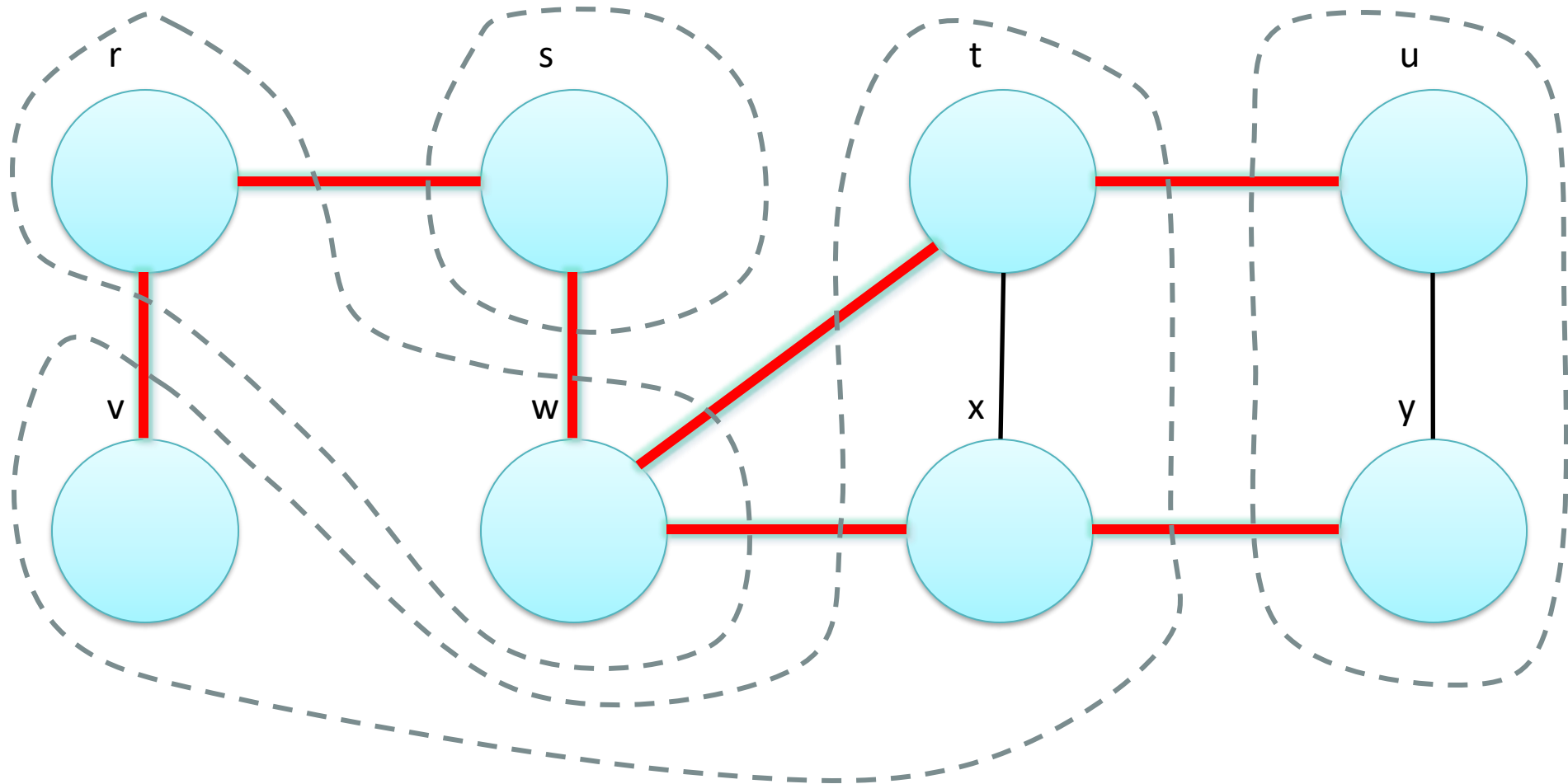
$L = 3$
 $S_2 = \{v, t, x\}$
 $S_3 = \{u, y\}$



BFS Tree

- The result of a BFV identifies a “visit tree” in the graph:
 - The tree root is the source vertex
 - Tree nodes are all graph vertices
 - (in the same connected component of the source)
 - Tree are a subset of graph edges
 - Those edges that have been used to “discover” new vertices.

BFS Tree



Minimum (shortest) paths

- Shortest path: the minimum number of edges on any path between two vertices
- The BFS procedure computes all minimum paths for all vertices, starting from the source vertex
- NB: unweighted graph : path length = number of edges

Depth First Visit

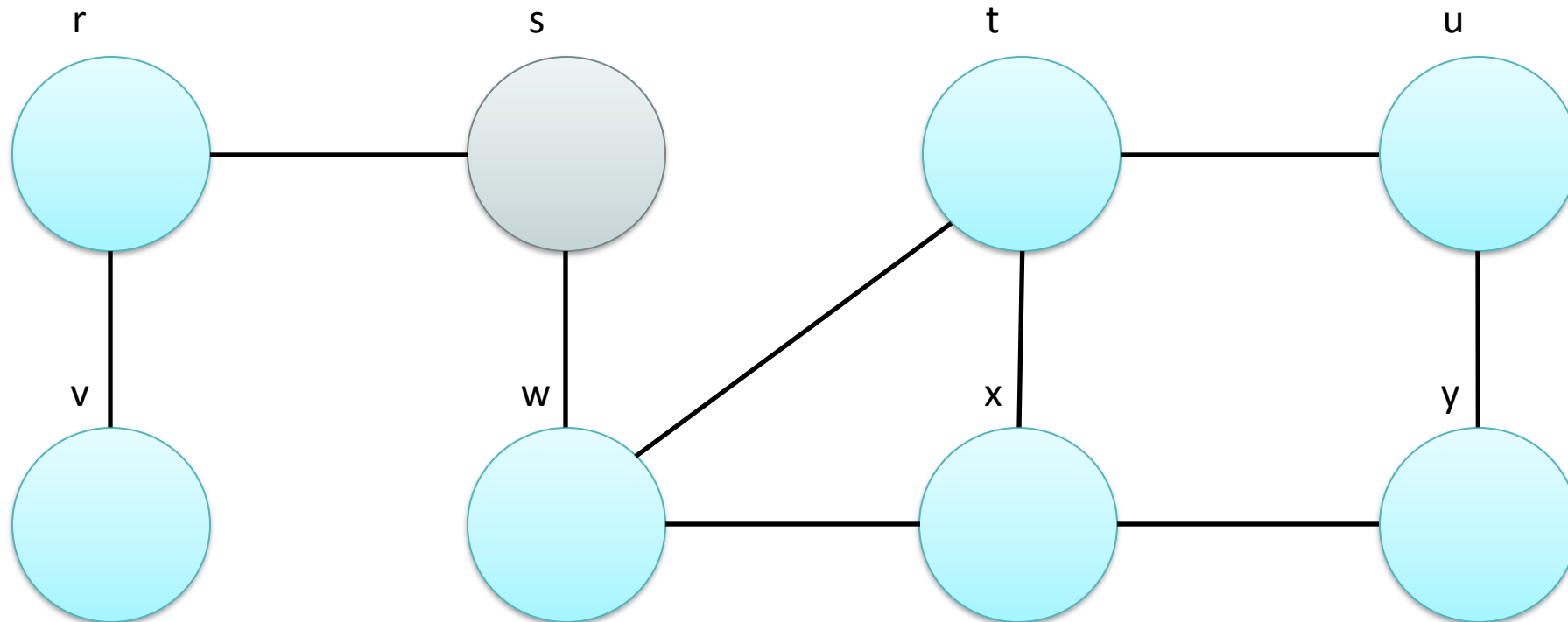
- Also called Depth-first search (DFV or DFS)
- Opposite approach to BFS
- At every step, visit one (yet unvisited) vertex, adjacent to the last visited one
- If no such vertex exist, go back one step to the previously visited vertex
- Lends itself to recursive implementation
 - Similar to tree visit procedures

DFS Algorithm

- DFS(Vertex v)
 - For all (w : adjacent_to(v))
 - If(not visited (w))
 - Visit (w)
 - DFS(w)
- Start with: DFS(source)

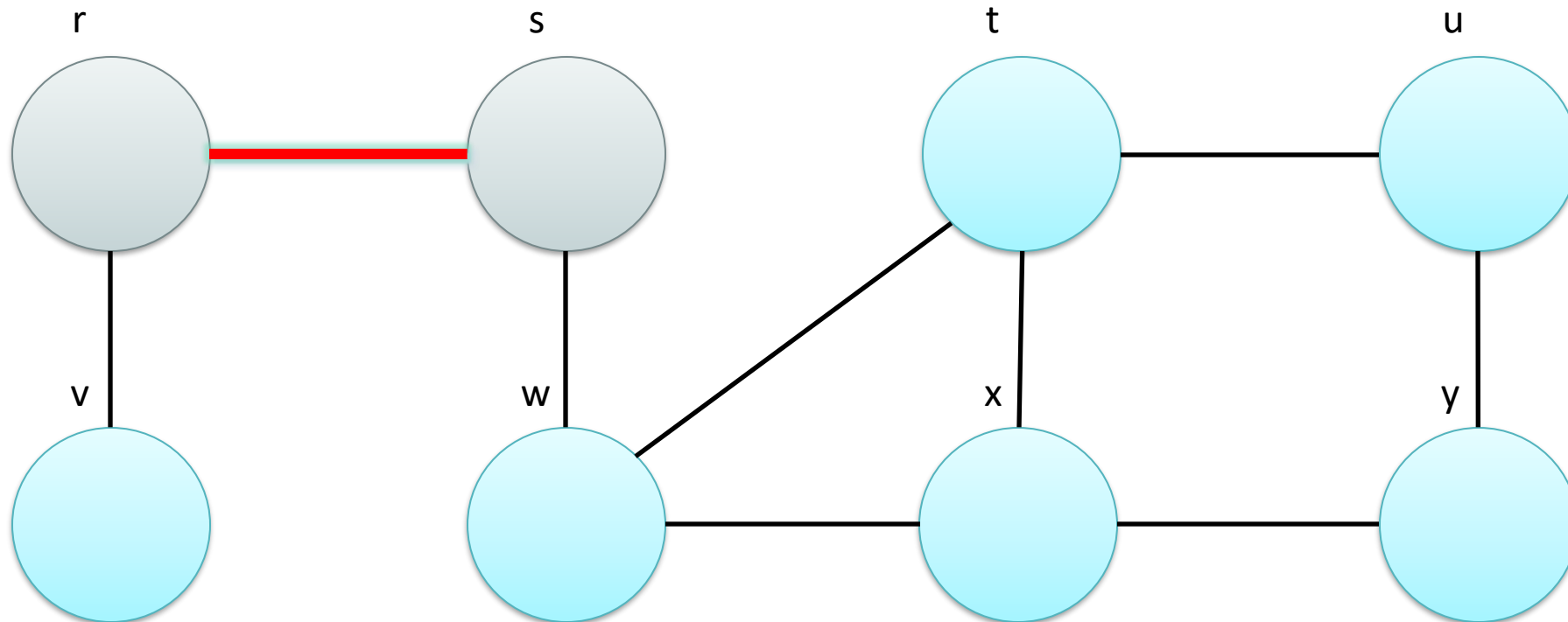
Example

Source = s



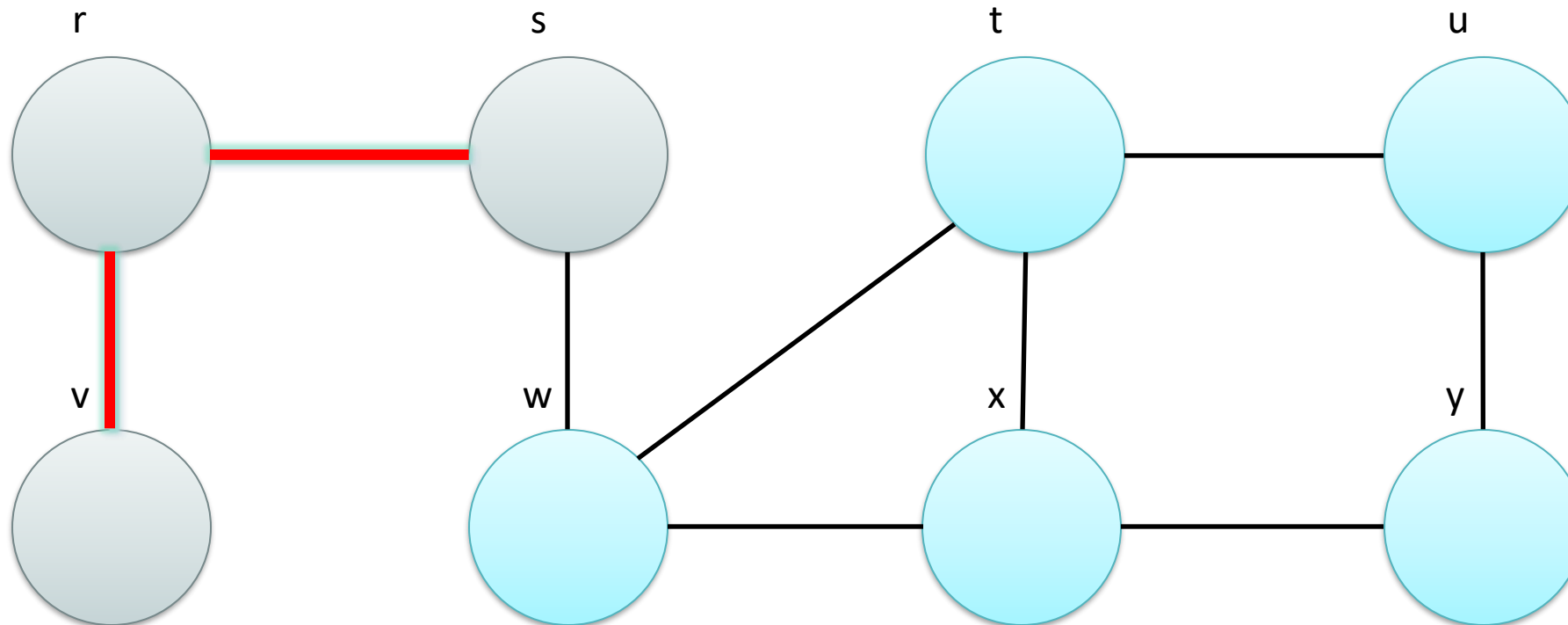
Example

Source = s
Visit r

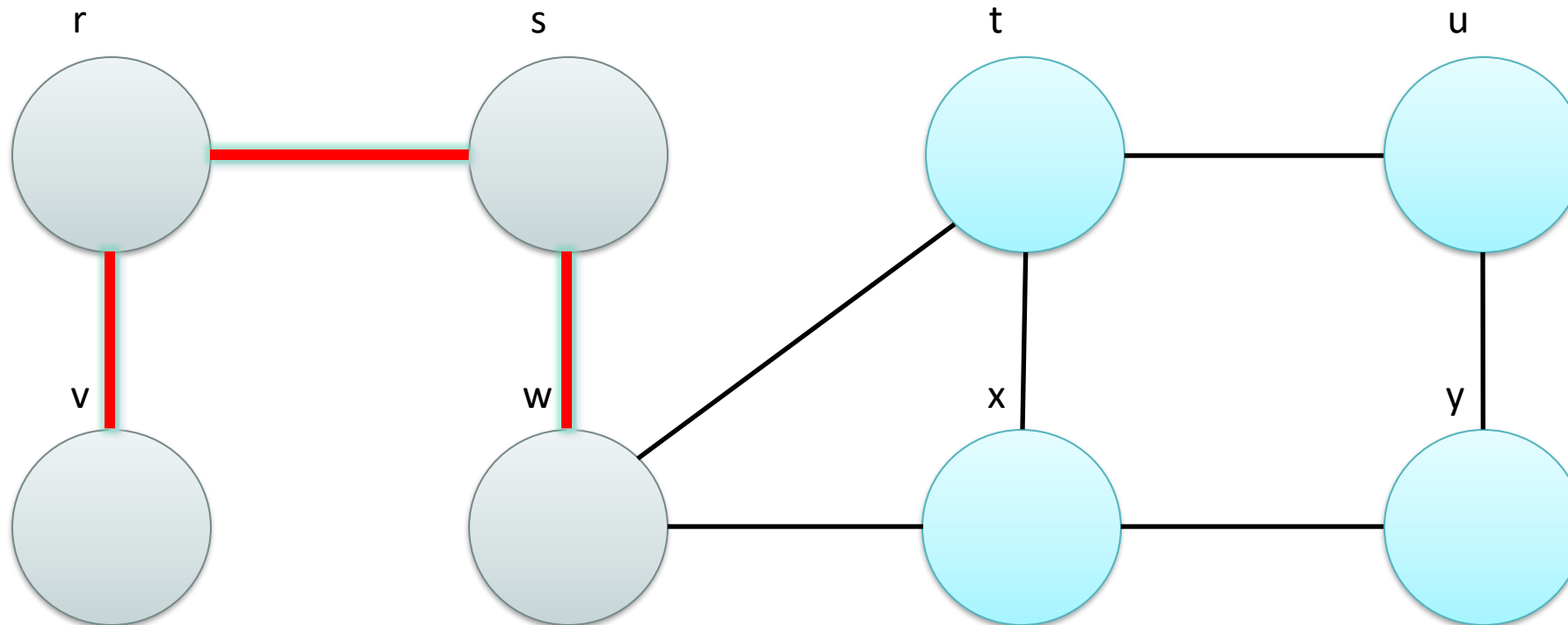


Example

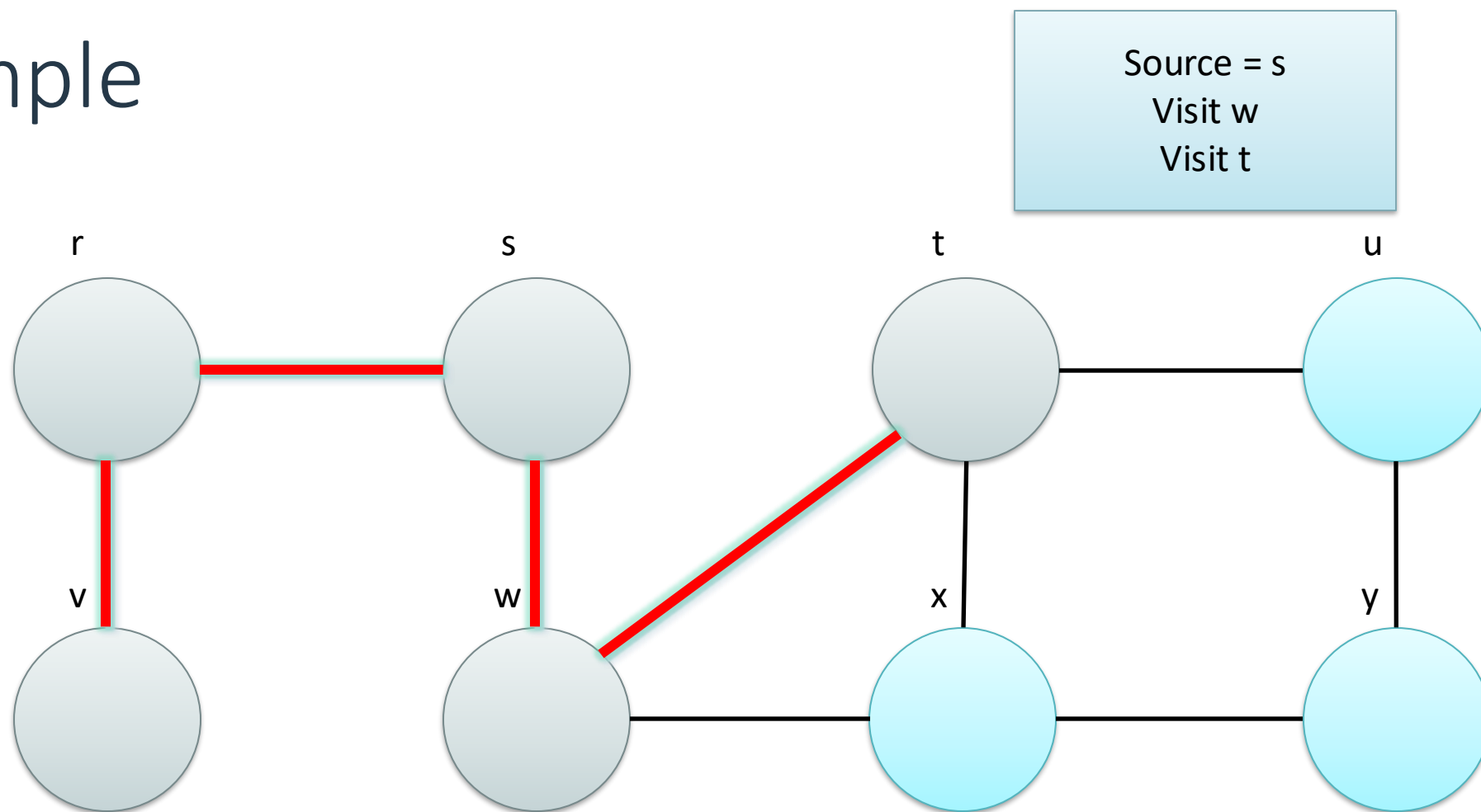
Source = s
Visit r
Visit v



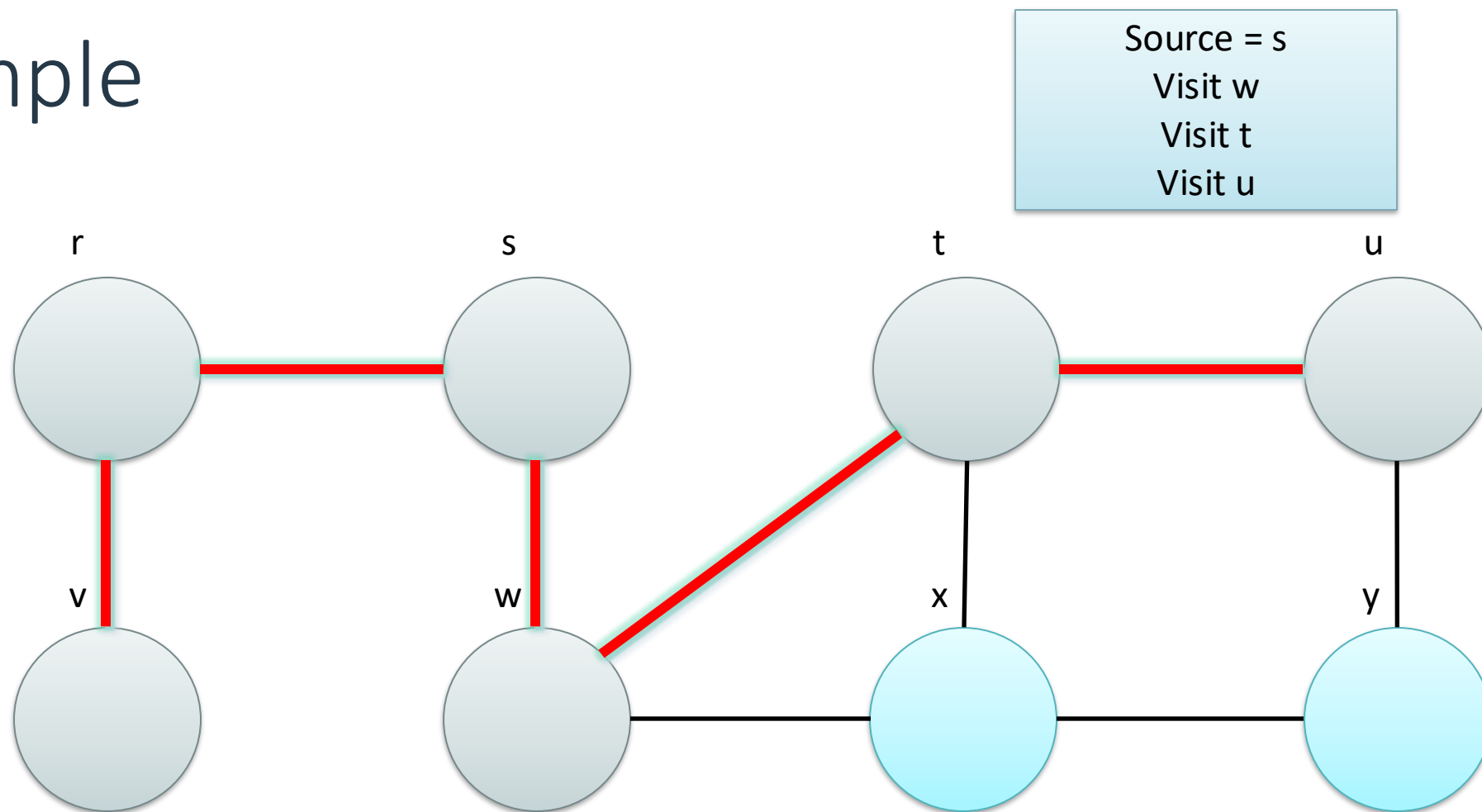
Example



Example

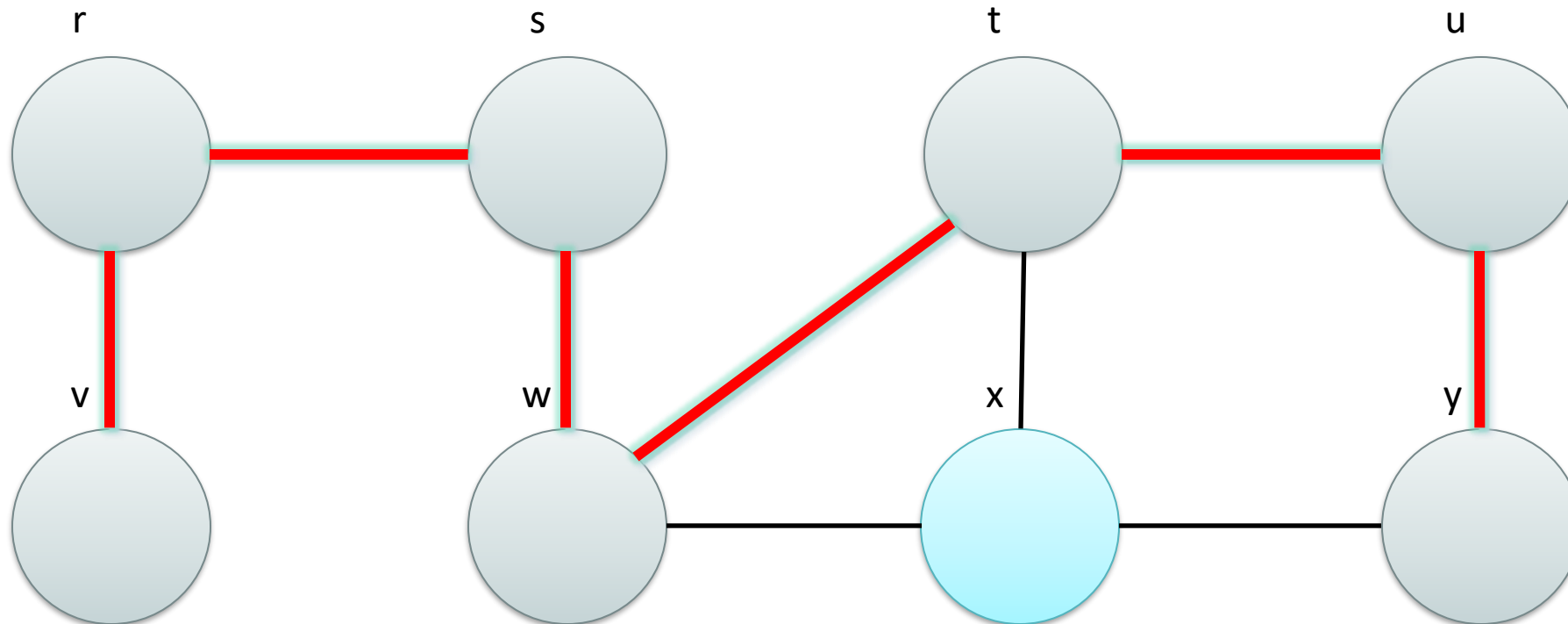


Example

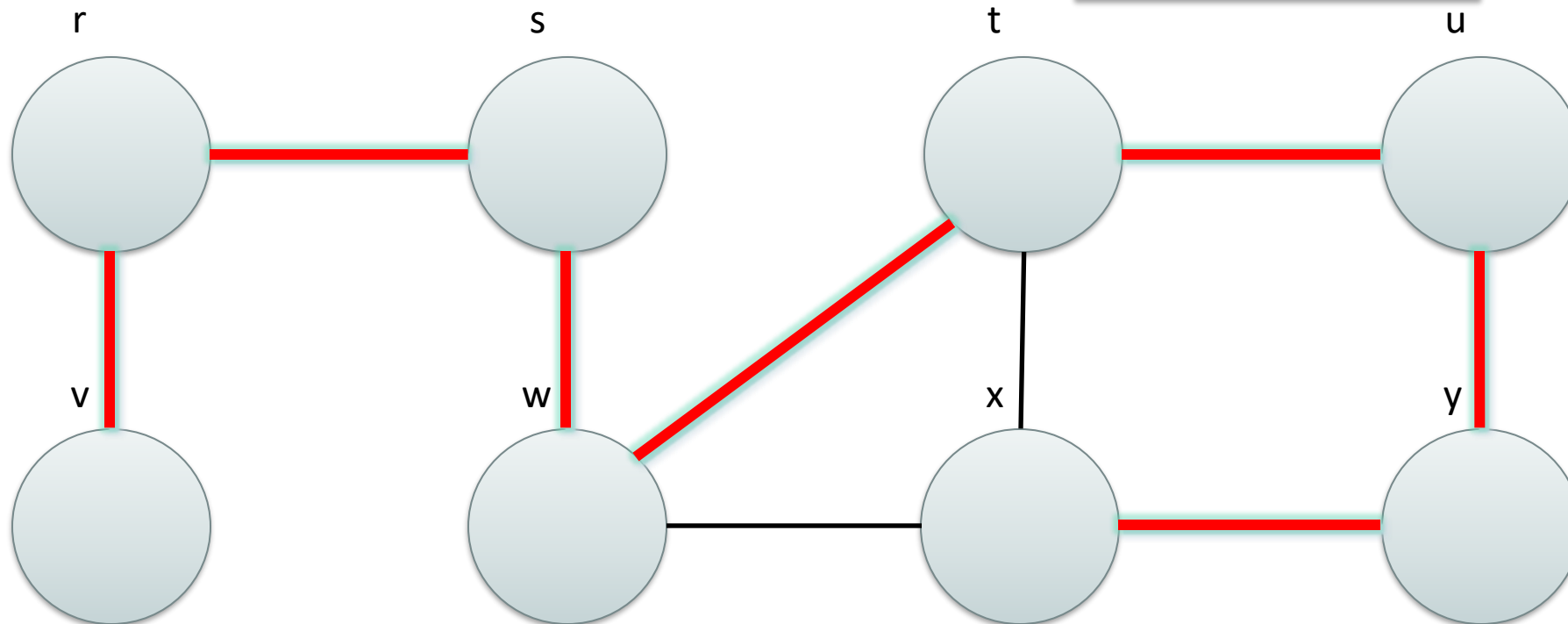


Example

Source = s
Visit w
Visit t
Visit u
Visit y

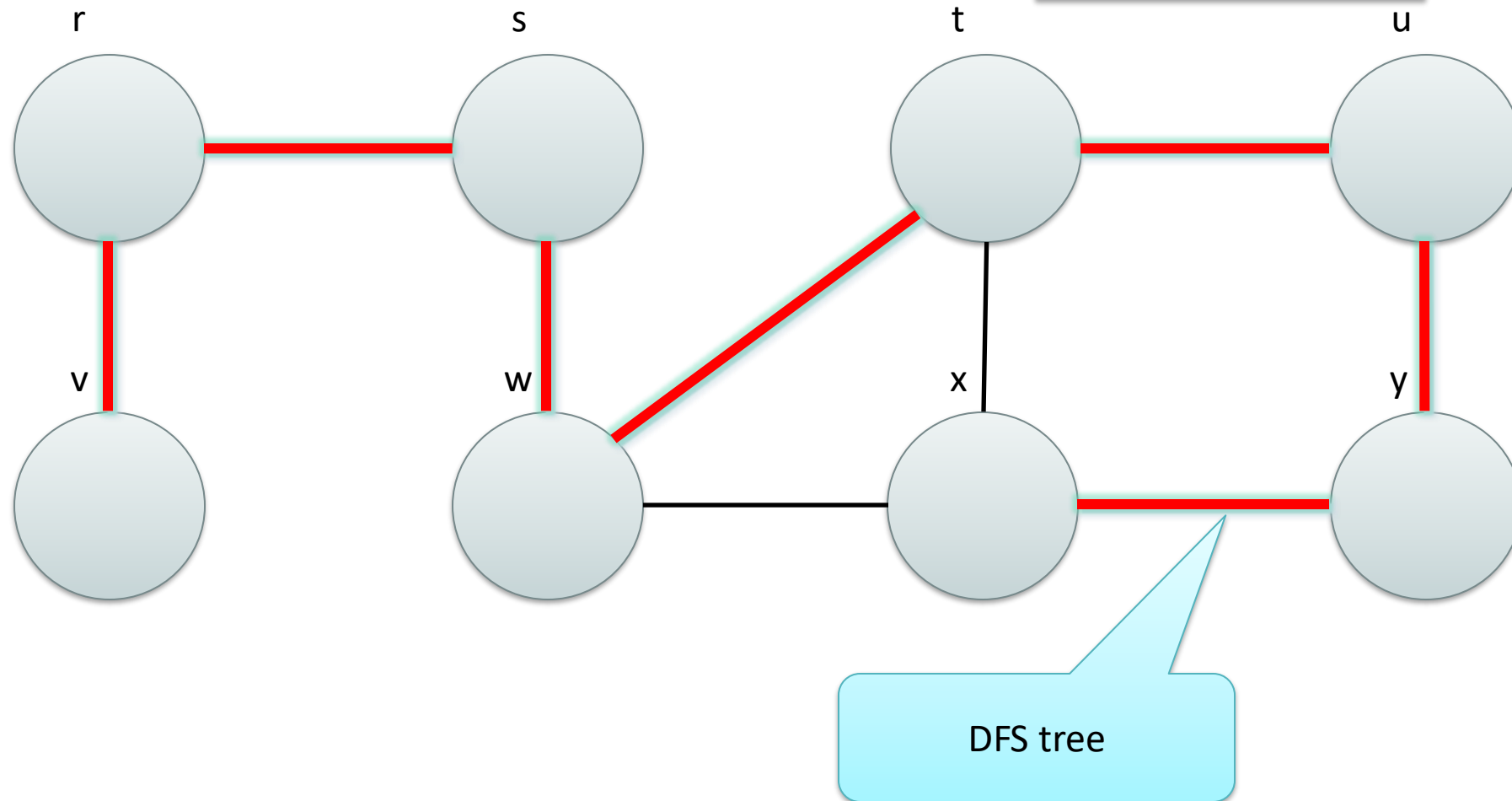


Example



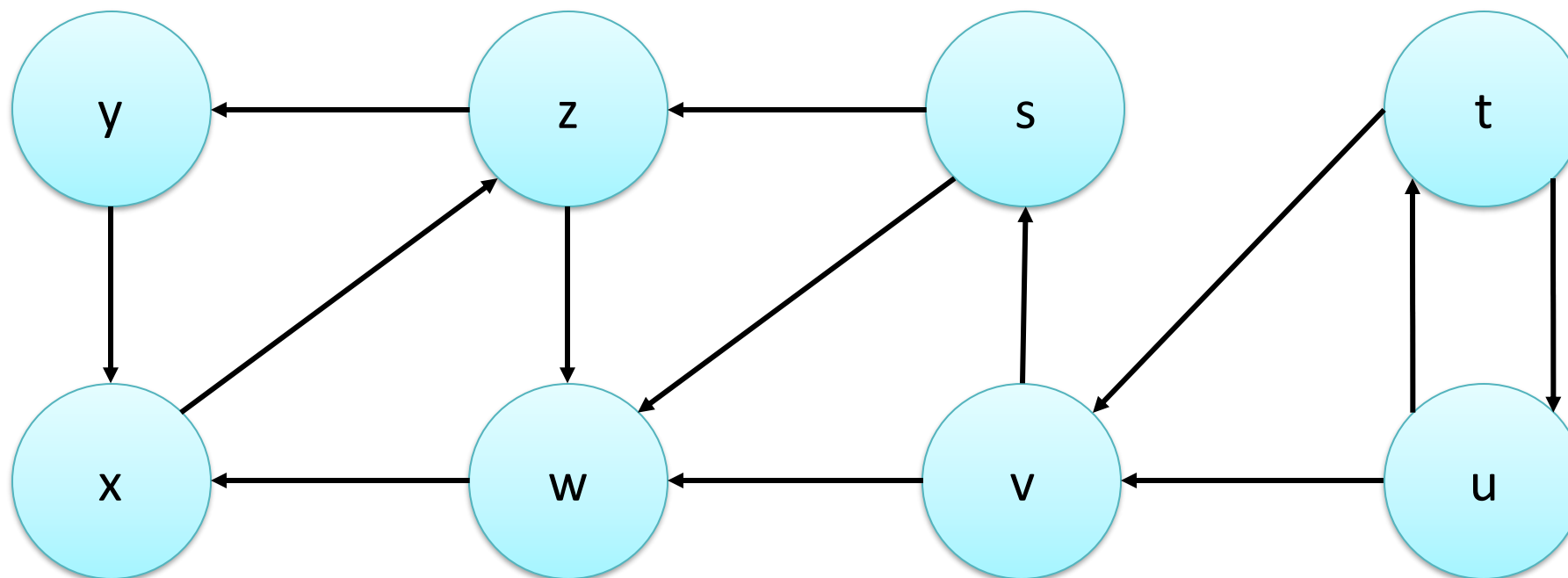
Source = s
Visit w
Visit t
Visit u
Visit y
Visit x

Example



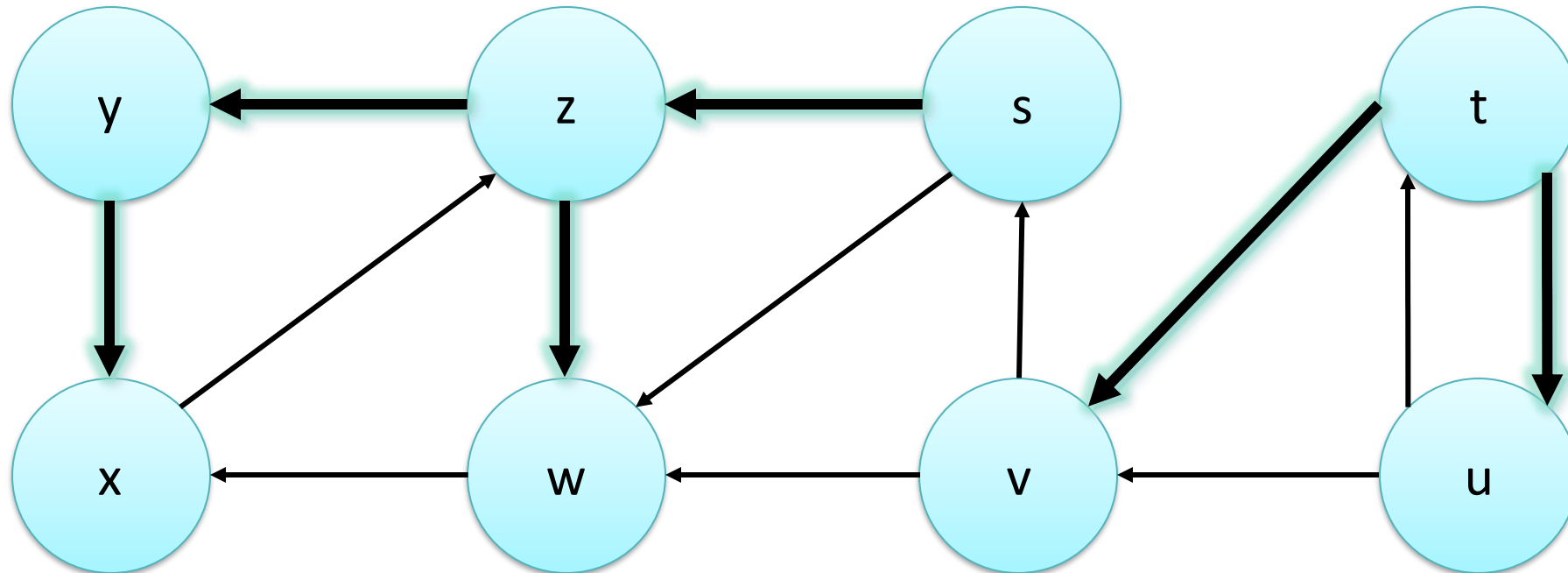
Example

Directed graph



Example

DFS visit
(sources: s, t)



Complexity

- Visits have linear complexity in the graph size
 - BFS : $O(V+E)$
 - DFS : $O(V+E)$
- N.B. for dense graphs, $E = O(V^2)$

Resources

- Maths Encyclopedia: <http://mathworld.wolfram.com/>
- Basic Graph Theory with Applications to Economics
<http://www.isid.ac.in/~dmishra/mpdoc/lecgraph.pdf>
- Application of Graph Theory in real world
- <http://prezi.com/tseh1wvpves-/application-of-graph-theory-in-real-world/>



Representing and visiting graphs



VISITS IN NETWORKX



Traversal

- Visits are called “traversals”
- NetworkX already provides implementations for BFV and DFV, together with other visits strategies

Graph traversal methods

Traversal

Depth First Search

Basic algorithms for depth-first searching the nodes of a graph.

<code>dfs_edges</code> (G[, source, depth_limit, ...])	Iterate over edges in a depth-first-search (DFS).
<code>dfs_tree</code> (G[, source, depth_limit, ...])	Returns oriented tree constructed from a depth-first-search from source.
<code>dfs_predecessors</code> (G[, source, depth_limit, ...])	Returns dictionary of predecessors in depth-first-search from source.
<code>dfs_successors</code> (G[, source, depth_limit, ...])	Returns dictionary of successors in depth-first-search from source.
<code>dfs_preorder_nodes</code> (G[, source, depth_limit, ...])	Generate nodes in a depth-first-search pre-ordering starting at source.
<code>dfs_postorder_nodes</code> (G[, source, ...])	Generate nodes in a depth-first-search post-ordering starting at source.
<code>dfs_labeled_edges</code> (G[, source, depth_limit, ...])	Iterate over edges in a depth-first-search (DFS) labeled by type.

Breadth First Search

Basic algorithms for breadth-first searching the nodes of a graph.

<code>bfs_edges</code> (G, source[, reverse, depth_limit, ...])	Iterate over edges in a breadth-first-search starting at source.
<code>bfs_layers</code> (G, sources)	Returns an iterator of all the layers in breadth-first search traversal.
<code>bfs_tree</code> (G, source[, reverse, depth_limit, ...])	Returns an oriented tree constructed from of a breadth-first-search starting at source.
<code>bfs_predecessors</code> (G, source[, depth_limit, ...])	Returns an iterator of predecessors in breadth-first-search from source.
<code>bfs_successors</code> (G, source[, depth_limit, ...])	Returns an iterator of successors in breadth-first-search from source.
<code>descendants_at_distance</code> (G, source, distance)	Returns all nodes at a fixed <code>distance</code> from <code>source</code> in <code>G</code> .
<code>generic_bfs_edges</code> (G, source[, neighbors, ...])	Iterate over edges in a breadth-first search.

<https://networkx.org/documentation/stable/reference/algorithms/traversal.html>



Example



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