UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism II (Phys. 402) — Prof. Leo C. Stein — Spring 2019

Problem Set 7

Due: Thursday, Apr. 11, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. **Resonant cavity modes**. Take what you've learned in the analysis of waveguides and apply it to a resonant cavity. Suppose we have a hollow rectangular box of side lengths $a \geq b \geq d$, and this box is made out of an excellent conductor. Solve Maxwell's equations subject to the appropriate boundary conditions at all the surfaces. Find the modes that are possible, which should now be labeled by three integers (l, m, n), and find the associated frequency ω_{lmn}. What is the general solution for $\boldsymbol{E}$ and $\boldsymbol{B}$ in one of these modes?

2. **Uniqueness of Lorenz gauge**. Suppose somebody hands you a $V, \boldsymbol{A}$ that solve Maxwell's equations in the potential formulation, and you check that they satisfy the Lorenz gauge condition $\nabla \cdot \boldsymbol{A} + \frac{1}{c^2}\frac{\partial V}{\partial t} = 0$. Can you perform a gauge transformation *generated* by a scalar function $\lambda(t, \boldsymbol{r})$ to a different gauge, but still satisfying the Lorenz gauge condition – what differential equation needs to be solved? How much freedom is there for λ that will take you between two different Lorenz gauges?

3. What are the electric and magnetic fields that correspond to
$$V = 0\,, \quad \boldsymbol{A} = \frac{-1}{4\pi\epsilon_0}\frac{qt}{r^2}\hat{\boldsymbol{r}}\,? \tag{1}$$
Find $V', \boldsymbol{A}'$ in another gauge via the gauge transformation function $\lambda = -(1/4\pi\epsilon_0)(qt/r)$. What is this new gauge?

4. **Practice with finding retarded time**. Suppose a particle follows the hyperbolic trajectory $\boldsymbol{\xi}(t) = \hat{\boldsymbol{z}}\sqrt{b^2 + c^2t^2}$.
   (a) Draw the space-time diagram with $z - t$ axes to show this particle's motion. Draw the light signals that would emanate from the particle.
   (b) Notice that there are some regions in spacetime that do not know about the existence of the particle! What points in the (t, z) plane (setting $x = y = 0$) haven't yet received a signal from the particle?
   (c) From the implicit equation for the definition of retarded time, show that $t_r(t, \boldsymbol{r})$ can be found by solving a quadratic equation.
   (d) Actually solve it in the simpler case where your position is along the z axis, $\hat{\boldsymbol{r}} = z\hat{\boldsymbol{z}}$, and in the appropriate region so that you've received a signal from the particle. You can be either above or below the particle; find the solution in both cases.

5. Start from the Liénard-Wiechart scalar potential for a charge in uniform linear motion,
$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{\sqrt{(ct - \boldsymbol{r}\cdot\boldsymbol{v}/c)^2 - (1 - v^2/c^2)(r^2 - c^2t^2)}}\,. \tag{2}$$
Show that this can be rewritten in terms of $\boldsymbol{R} \equiv \boldsymbol{r} - \boldsymbol{v}t$, the "instantaneous" separation, as
$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{R\sqrt{1 - (\sin^2\theta)v^2/c^2}}\,, \tag{3}$$
where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{R}$.

6. **Energy flux in the field of a uniformly moving charge**.

(a) For a charge in uniform linear motion, what is the Poynting vector? (I think in lecture I erroneously claimed it vanished!).

(b) Suppose the charge's velocity is purely in the $\hat{\boldsymbol{z}}$ direction, $\boldsymbol{v} = v_z \hat{\boldsymbol{z}}$. Integrate the energy flux through the entire $z = 0$ plane. What is the resulting dE/dt, as a function of the particle's position d along the $\boldsymbol{z}$ axis?