AE 3613

Orbits 1

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HW #02

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Problem 1

Looking at the Europa Clipper mission scheduled to launch around October 10th this year (2024) and arrive at Jupiter's moon, Europa, and land sometime in 2030, the projected path can be a bit intimidating and difficult to analyze. To simply basic calculations, it can be assumed that rather than the actual projected flight path (figure 1), the Clipper will take a much simpler path (figure 2). For ease of calculation, it can also be assumed that the Clipper can travel directly through celestial bodies, as well as that the only bodies exerting a significant gravitational force on the Clipper are the Sun, Earth, Jupiter, and Europa.

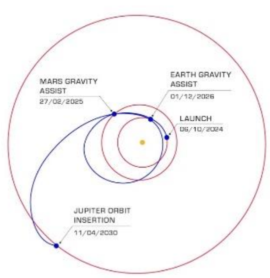

Figure 1

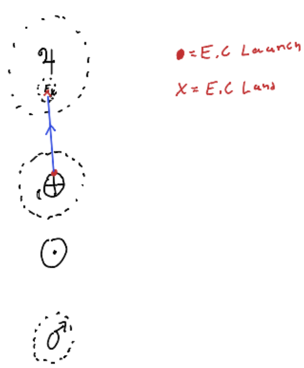

Figure 2

By defining sphere of influence (SOI) as when the distance at which one body is exerting a larger gravitational force than the other, the method of determining where the SOI ends is to use Newton's Universal Law of Gravitation, $F_G = -G\frac{m_1 m_2}{r^2}$, for both bodies being analyzed. Since the SOI of one body ends where the other begins, at the exact point where both SOI's end the gravitational forces will be equal. So, a hypothetical system would look like $F_{G1} = -G\frac{m_1 m_i}{r_{1i}{}^2}$, $F_{G2} = -G\frac{m_2 m_i}{r_{2i}{}^2}$, and $F_{G1} = F_{G2}$, so $-G$ and m_i divide out leaving only $\frac{m_1}{r_{1i}{}^2} = \frac{m_2}{r_{2i}{}^2}$, where r_{1i} and r_{2i} are the SOI of bodies 1 and 2 respectively. If one of the distances is put into terms of the other, say $r_{2i} = r_{1i} + r$, where r is a known value, the equation can be rearranged into a quadratic equation, $(m_1 - m_2)r_{1i}{}^2 + (2m_1 r)r_{1i} + (-m_1 r^2) = 0$ **(1)**. Using quadratic formula with $a = m_1 - m_2$, $b = 2m_1 r$, and $c = -m_1 r^2$, r_{1i} can be solved for.

For the SOI's being found here, I. Earth vs the Sun, II. the Sun vs Jupiter, and III. Jupiter vs Europa, the values for mass and distance can be plugged in and the SOI's can be solved for.

- First, define necessary information:

  NOTE Subscript E denotes Europa and subscript i denotes the Europa Clipper vessel)

  - *Masses (in kilograms)*
    - $m_\odot = 1988 \times 10^{24}, m_\oplus = 5.9722 \times 10^{24}$,
      $m_{♃} = 1898.13 \times 10^{24}, m_\mathrm{E} = 48.0 \times 10^{21}$
  - *Distances (in Astronomical Units)*
    - $r_{\odot\oplus} = 1, r_{\odot♃} = 5.2, r_{♃\mathrm{E}} = 0.00448535796$
  - *Relative distances (AU)*
    - $r_{\oplus♃} = r_{\odot♃} - r_{\odot\oplus}, r_{\odot i} = r_{\oplus\mathrm{i}} + r_{\odot\oplus}, r_{\odot\mathrm{i}} = r_{\odot♃} - r_{♃\mathrm{i}}$,
      $r_{♃\mathrm{i}} = r_{E\mathrm{i}} + r_{♃\mathrm{E}}$

I. Plugging in values for $m_1 = m_\oplus, m_2 = m_\odot, r_{1i} = r_{\oplus\mathrm{i}}$, and $r_{2i} =, r_{\odot i}$ into **(1)**, the equation becomes $(m_\oplus - m_\odot)r_{\oplus i}{}^2 + (2m_\oplus r_{\odot\oplus})r_{\oplus i} +$

$(m_{\oplus} r_{\odot\oplus}{}^2) = 0$, where $a = m_{\oplus} - m_{\odot}$, $b = 2m_{\oplus} r_{\odot\oplus}$, and $c = m_{\oplus} r_{\odot\oplus}{}^2$. Using MATLAB to solve the quadratic, it is found that $\boldsymbol{r_{\oplus i} = 1.736075502 \times 10^{-3}}$ AU, and $\boldsymbol{r_{\oplus i} = -1.730068443 \times 10^{-3}}$ AU, or that $\boldsymbol{r_{\oplus i} = 259713.19847}$ km and $\boldsymbol{r_{\oplus i} = -258814.5552}$ km.

II. Plugging in values for $m_1 = m_{\odot}$, $m_2 = m_{♃}$, $r_{1i} = r_{\odot \mathrm{i}}$, and $r_{2i} =, r_{♃i}$ into **(1)**, the equation becomes $(m_{\odot} - m_{♃}) r_{♃i}{}^2 + (2m_{♃} r_{\odot♃}) r_{♃i} + (-m_{♃} r_{\odot♃}{}^2) = 0$, where $a = m_{\odot} - m_{♃}$, $b = 2m_{♃} r_{\odot♃}$, and $c = -m_{♃} r_{\odot♃}{}^2$. Using MATLAB to solve the quadratic, it is found that $\boldsymbol{r_{♃i} = 0.0084876}$ AU and $\boldsymbol{r_{♃i} = -0.0085154}$ AU or that $\boldsymbol{r_{♃i} = 1269726.8874}$ km and $\boldsymbol{r_{♃i} = -1273885.7082}$ km.

III. Plugging in values for $m_1 = m_E$, $m_2 = m_{♃}$, $r_{1i} = r_{E\mathrm{i}}$, and $r_{2i} =, r_{♃i}$ into **(1)**, the equation becomes $(m_E - m_{♃}) r_{Ei}{}^2 + (2m_E r_{♃\mathrm{E}}) r_{Ei} + (m_E r_{♃\mathrm{E}}{}^2) = 0$, where $a = m_E - m_{♃}$, $b = 2m_E r_{♃\mathrm{E}}$, and $c = m_E r_{♃\mathrm{E}}{}^2$. Using MATLAB to solve the quadratic, it is found that $\boldsymbol{r_{Ei} = 0.0000227}$ AU and $\boldsymbol{r_{♃i} = -0.0000224}$ AU or that $\boldsymbol{r_{♃i} = 3395.8717}$ km and $\boldsymbol{r_{♃i} = -3350.9923}$ km.

Problem 2

The "Tilt-a-Tot" (figure 3) is a new and possibly dangerous toy being introduced on playgrounds near you. It's essentially a teeter-totter expanded to three dimensions, and the prototype consists of a 5-meter by 5-meter surface that has been placed on a fulcrum located at the geometric center of the surface.

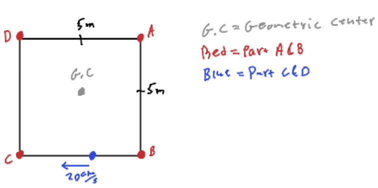

Figure 3

4 toddlers, toddler A, toddler B, toddler C, and toddler D are testing this prototype and weigh a respective 20 lbf, 25 lbf, 22 lbf, and 40 lbf. The toddlers have already been modeled on the "Tilt-a-Tot" in figure 3. To find the center of mass (C.M) of the loaded "Tilt-a-Tot", the equation $M\boldsymbol{R}_{CM} = \Sigma(m_i\boldsymbol{r}_i)$**,** where $\boldsymbol{R}_{CM}$ is the vector location of the C.M relative to the origin (which has been placed at the geometric center of the "Tilt-a-Tot"), M is the total mass of the system, and $\Sigma(m_i\boldsymbol{r}_i)$ is the summation of the individual masses multiplied by their position relative to the origin can be used. First it is rearranged to $\boldsymbol{R}_{CM} = \Sigma(m_i\boldsymbol{r}_i) \times M^{-1}$ **(2)** to solve for the vector location of the C.M.

- Define the position vectors of each toddler relative to the origin (in meters)
  - $\boldsymbol{r}_{OA} = 2.5\boldsymbol{i} + 2.5\mathbf{j}$, $\boldsymbol{r}_{OB} = 2.5\boldsymbol{i} - 2.5\mathbf{j}$**,** $\boldsymbol{r}_{OC} = -2.5\boldsymbol{i} - 2.5\mathbf{j}$**,** $\boldsymbol{r}_{OD} = -2.5\boldsymbol{i} + 2.5\mathbf{j}$
- Define the toddler's weights (in lbf)
  - $w_A = 20, w_B = 25, w_C = 22, w_D = 40,$ $W = w_A + w_B + w_C + w_D$
- **NOTE** it is not necessary to find the toddler's mass, as the gravitational will divide out when the values are plugged into **(2)**.
- Break **(2)** down into **x** and **y** (**i** and **j**) components
  - $\boldsymbol{R}_{CM_x} = \left(w_A\boldsymbol{r}_{OA_x} + w_B\boldsymbol{r}_{OB_x} + w_C\boldsymbol{r}_{OC_x} + w_D\boldsymbol{r}_{OD_x}\right) \times W^{-1}$
  - $\boldsymbol{R}_{CM_y} = \left(w_A\boldsymbol{r}_{OA_y} + w_B\boldsymbol{r}_{OB_y} + w_C\boldsymbol{r}_{OC_y} + w_D\boldsymbol{r}_{OD_y}\right) \times W^{-1}$
  - By plugging in the known values, the **x** component is found to be $\boldsymbol{R}_{CM_x} = -0.3983$ meters, and the **y** component is found to be $\boldsymbol{R}_{CM_y} = 0.3047$ meters, or $\boldsymbol{R}_{CM} = -0.3983\boldsymbol{i} + 0.3047\mathbf{j}$.

- Due to how the coordinate axes were defined, the distance from the geometric center is simply the magnitude of vector $\boldsymbol{R}_{CM}$.
    - $||R_{CM}|| = \sqrt{(}\boldsymbol{R}_{CM_x}{}^{\mathbf{2}} + \boldsymbol{R}_{CM_y}{}^{\mathbf{2}})$
  - By plugging the values for $\boldsymbol{R}_{CM_x}$ and $\boldsymbol{R}_{CM_y}$ into the equation for magnitude, it is found that $R_{CM} = 0.5000$ meters.

If toddler B is to start moving towards toddler C at a rate of 20 centimeters per second (0.20 meters per second), the center of mass can and its distance from the geometric center can now be plotted as a function of time. To do this, the position vector $\boldsymbol{r}_{OB}$ must be rewritten as a function of time using toddler B's velocity relative to toddler C.

- Rewrite vector $\boldsymbol{r}_{OB}$ as a function of time, $\boldsymbol{r}_{OB}(t)$
  - First find vector $\boldsymbol{r}_{BC}$
    - $\boldsymbol{r}_{BC_0} = \boldsymbol{r}_{OB_0} - \boldsymbol{r}_{OC}$
    - $\boldsymbol{r}_{BC_0} = (2.5\boldsymbol{i} - 2.5\mathbf{j}) - (-2.5\boldsymbol{i} - 2.5\mathbf{j})$, or $\boldsymbol{r}_{BC_0} = 5\boldsymbol{i} - 0\mathbf{j}$
  - Now find vector $\boldsymbol{r}_{BC}$ as a function of time, $\boldsymbol{r}_{BC}(t)$ using basic kinematics equations
    - $\boldsymbol{r}_{BC_{x_f}} = \boldsymbol{r}_{BC_{x_0}} + \boldsymbol{v}_{BC_x} \times t$
      - $\boldsymbol{r}_{BC_{x_f}} = 0, \boldsymbol{v}_{BC_x} = -0.2\ \frac{m}{s}$
      - Solving for t, it is found that it will take toddler B 25 seconds to reach toddler C.
    - $\boldsymbol{r}_{BC}(t) = (5 - 0.2t)\boldsymbol{i} + 0\boldsymbol{j}$
  - Using $\boldsymbol{r}_{BC}(t)$, find $\boldsymbol{r}_{OB}(t)$

- $\boldsymbol{r}_{BC}(t) = \boldsymbol{r}_{OB}(t) - \boldsymbol{r}_{OC}$, or $\boldsymbol{r}_{OB}(t) = \boldsymbol{r}_{BC}(t) + \boldsymbol{r}_{OC}$
- $\boldsymbol{r}_{OB}(t) = ((5 - 0.2t)\boldsymbol{i} + 0\boldsymbol{j}) + (-2.5\boldsymbol{i} - 2.5\mathbf{j})$
- $\boldsymbol{r}_{BC}(t) = (2.5 - 0.2)\boldsymbol{i} - 2.5\boldsymbol{j}$

- Plug the new $\boldsymbol{r}_{BC}(t)$ into the $\boldsymbol{R}_{CM_{\boldsymbol{x}}}$ and $\boldsymbol{R}_{CM_{\boldsymbol{y}}}$ and find the magnitude of the new C.M as a function of time
  - $\boldsymbol{R}_{CM_{\boldsymbol{x}}}(t) = \left(w_A \boldsymbol{r}_{OA_{\boldsymbol{x}}} + w_B \boldsymbol{r}_{OB_{\boldsymbol{x}}}(t) + w_C \boldsymbol{r}_{OC_{\boldsymbol{x}}} + w_D \boldsymbol{r}_{OD_{\boldsymbol{x}}}\right) \times W^{-1}$
  - $\boldsymbol{R}_{CM_{\boldsymbol{y}}}(t) = \left(w_A \boldsymbol{r}_{OA_{\boldsymbol{y}}} + w_B \boldsymbol{r}_{OB_{\boldsymbol{y}}}(t) + w_C \boldsymbol{r}_{OC_{\boldsymbol{y}}} + w_D \boldsymbol{r}_{OD_{\boldsymbol{y}}}\right) \times W^{-1}$
    - **NOTE** since the **y** component of $\boldsymbol{r}_{BC}(t)$ is zero, the **y** component of $\boldsymbol{R}_{CM}(t)$ does NOT change with time.
  - $\boldsymbol{R}_{CM}(t) = \boldsymbol{R}_{CM_{\boldsymbol{x}}}(t) + \boldsymbol{R}_{CM_{\boldsymbol{y}}}$
  - $||R_{CM}(t)|| = \sqrt{(}\boldsymbol{R}_{CM_{\boldsymbol{x}}}(t)^2 + \boldsymbol{R}_{CM_{\boldsymbol{y}}}(t)^2)$

- Finally plot the results using MATLAB
  - $\boldsymbol{R}_{CM}(t)$ ~ figure 4
  - $||R_{CM}(t)||$ ~ figure 5

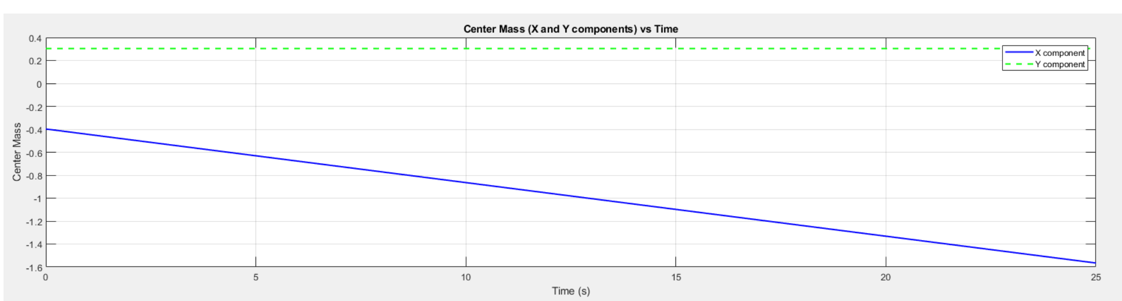

Figure 4

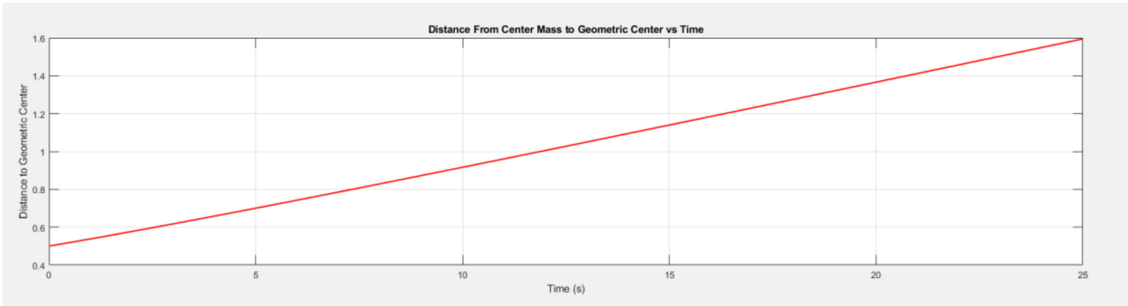

Figure 5

Toddler B probably didn't think this decision through, as by crawling over to the other side of the "Tilt-a-Tot", they will have made it much harder to do anything on the toy. This is due to the center of mass having shifted so far in the negative X direction, and this will make it so that Toddler A is stuck in the air and can't play.

Problem 3

TCM's, or Trajectory Correction Maneuver, are propulsive maneuvers spacecrafts due to make a small adjustment to its flight path. These maneuvers are typically done with cold gas thrusters, essentially just releasing compressed gases from a nozzle. These thrusters typically use gases like nitrogen, hydrogen, helium, or other lightweight gases, and the lack of adding heat or chemical reactions makes these thrusters less powerful, and much easier control; perfect for TCM's. When a TCM goes wrong different things can happen depending on what the TCM was being used for. If it were being used for correcting a small error in the middle of its path somewhere, it would need to be corrected using more of the gas from the cold gas thrusters. If it were being used to maintain orbit, the craft could spiral out of orbit and crash onto the body it was orbiting. If it were being used to dock at a space station, it could result in failure to dock as well as damaging the spacecraft. If the thrusters fail, it could also prevent the craft from making TCM's in the future, which could completely prevent the craft from following its desired path. While it's possible that a TCM fail is part of why the Starliner vessel is "stuck" at the ISS, it's unlikely that it's the root cause. Technically it would be because five of the cold gas thrusters used for these TCM's overheated and stopped functioning rather than a failed maneuver. Due to the overheating of these thrusters (which I find to be rather ironic due to the nature of cold gas thrusters and Gay-Lussac's Law), the seals failed, and the helium gas being used began to leak. This ultimately makes any travel with the craft very risky as it would need to function with less attitude adjustors than it was designed to function with.

Citations

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MATLAB

```
% Define Weights (Weights can be used rather than masses because

% gravitational constant cancels out in center mass equation


W_a = 20;

W_b = 25;

W_c = 22;

W_d = 40;


W = W_a + W_b + W_c + W_d;


% Define Vectors x and y components (relative to geometric center, o)
```

```
r_ax = 2.5;
r_ay = 2.5;

r_bx = 2.5;
r_by = -2.5;

r_cx = -2.5;
r_cy = -2.5;

r_dx = -2.5;
r_dy = 2.5;

% Time range from 0 to 25 seconds
t = linspace(0, 25, 1000); % 1000 points for smooth plots

% Replace these with the actual functions later
RCM_x = (((W_a * r_ax) + (W_c * r_cx) + (W_d * r_dx)) + (W_b * (2.5 - (0.2 *
t)))) ./ W;
RCM_y = (((W_a * r_ay) + (W_b * r_by) + (W_c * r_cy) + (W_d * r_dy))) ./ W
.* ones(size(t));
```

```
distance_to_center = sqrt(RCM_x.^2 + RCM_y.^2);

figure;
% Plot 1: X and Y Components of Center Mass vs Time
subplot(2, 1, 1); % Creates a 2x1 grid of plots, and this is the first plot
plot(t, RCM_x, 'b', 'LineWidth', 1.5); % Plot X component in blue
hold on; % Keep the current plot to overlay the Y component
plot(t, RCM_y, 'g--', 'LineWidth', 1.5); % Plot Y component in green dashed line
title('Center Mass (X and Y components) vs Time');
xlabel('Time (s)');
ylabel('Center Mass');
legend('X component', 'Y component');
grid on; % Add grid to the plot
hold off; % Release the plot hold

% Plot 2: Distance From Center Mass to Geometric Center vs Time
subplot(2, 1, 2); % This is the second plot in the 2x1 grid
plot(t, distance_to_center, 'r', 'LineWidth', 1.5); % Plot in red with line width of
1.5
```

```
title('Distance From Center Mass to Geometric Center vs Time');
xlabel('Time (s)');
ylabel('Distance to Geometric Center');
grid on; % Add grid to the plot
```