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解37：(1) 甲，乙种子发芽互相独立

设 A 为甲发芽，B 为乙发芽，则 $P(A)=0.8$，$P(B)=0.9$

$P(AB)=P(A)P(B)=0.72$，故两粒都发芽概率为0.72

(2) $P(A\cup B)=1-P(\bar{A}\bar{B})=1-P(\bar{A})P(\bar{B})=1-[1-P(A)][1-P(B)]$

$=1-0.2\times0.1=0.98$　故至少有1粒发芽概率为0.98

(3) $P(A\bar{B}\cup\bar{A}B)=P(A\bar{B})+P(\bar{A}B)=P(A)[1-P(B)]+P(B)[1-P(A)]$

$=0.8\times0.1+0.9\times0.2=0.26$　故恰有1粒发芽概率为0.26

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解43 (1)∵设A_i为第i个射手命中

$P(\bar{A_1}\bar{A_2}\bar{A_3}\cdots\bar{A_n}) = P(\bar{A_1})P(\bar{A_2})\cdots P(\bar{A_n}) = [1-P(A_1)][1-P(A_2)]$

$\cdots[1-P(A_n)] = (1-P_1)(1-P_2)(1-P_3)\cdots(1-P_n) = \prod_{i=1}^{n}(1-P_i)$

故都未命中的概率为 $\prod_{i=1}^{n}(1-P_i)$

(2) $P(A_1\cup A_2\cup A_3\cdots\cup A_n) = 1-P(\bar{A_1}\bar{A_2}\cdots\bar{A_n}) = 1-\prod_{i=1}^{n}(1-P_i)$

故至少有1个人命中的概率为 $1-\prod_{i=1}^{n}(1-P_i)$

解46：设5个系统正常的概率分别为$A_1, A_2\cdots A_5$

对系统1：$P[A_1\cap(A_2A_3\cup A_4A_5)]$

$= P(A_1A_2A_3\cup A_1A_4A_5) = P(A_1A_2A_3)+P(A_1A_4A_5)-P(A_1A_2A_3A_4A_5)$

$= P(A_1)P(A_2)P(A_3)+P(A_1)P(A_4)P(A_5)-P(A_1)P(A_2)P(A_3)P(A_4)P(A_5)$

$= P_1P_2P_3+P_1P_4P_5-P_1P_2P_3P_4P_5 = P_1(P_2P_3+P_4P_5-P_2P_3P_4P_5)$

对系统2：

$P[A_3[(A_1\cup A_4)\cap(A_2\cup A_5)]\cup\bar{A_3}(A_1A_2\cup A_4A_5)]$

$=P[A_3(A_1\cup A_4)(A_2\cup A_5)]+P(\bar{A_3}A_1A_2\cup\bar{A_3}A_4A_5)$

$=P(A_3)P(A_1\cup A_4)P(A_2\cup A_5)+P(\bar{A_3}A_1A_2)+P(\bar{A_3}A_4A_5)-P(A_1A_2\bar{A_3}A_4A_5)$

$=P(A_3)[P(A_1)+P(A_4)-P(A_1A_4)][P(A_2)+P(A_5)-P(A_2A_5)]$

$+P(\bar{A_3})P(A_1)P(A_2)+P(\bar{A_3})P(A_4)P(A_5)-P(A_1)P(A_2)P(\bar{A_3})P(A_4)P(A_5)$

$= P_3[P_1+P_4-P_1P_4)(P_2+P_5-P_2P_5)+(1-P_3)(P_1P_2+P_4P_5-P_1P_2P_4P_5)$

故系统2可靠性为 $P_3(P_1+P_4-P_1P_4)(P_2+P_5-P_2P_5)+(1-P_3)(P_1P_2+P_4P_5-P_1P_2P_4P_5)$

解47：

(1)设A_i表示一个零件由第i台车床生产

$P(A_1)=\frac{2}{3}$，$P(A_2)=\frac{1}{3}$ 设B为一件零件是合格品

$P(B)=P(A_1)P(B|A_1)+P(A_2)P(B|A_2)$

$=\frac{2}{3}\times0.97+\frac{1}{3}\times0.94=0.96$ 故一个零件合格品的概率为0.96

(2) $P(A_2|\bar{B}) = \dfrac{P(\bar{B}|A_2)P(A_2)}{P(\bar{B})} = P(\bar{B}|A_2)P(A_2) / [P(\bar{B}|A_1)P(A_1)+P(\bar{B}|A_2)P(A_2)]$

$= (\frac{1}{3}\times 0.06) / (\frac{2}{3}\times 0.03 + \frac{1}{3}\times 0.06) = 0.5$

故是由第二台车床生产的概率为0.5.

解49：设丢白球为A_1，丢黑球为A_2，摸2个球均为白球为B.

$P(A_1)=P(A_2)=\frac{1}{2}$　$P(B|A_1)=\dfrac{C_{m-1}^2}{C_{2m-1}^2}$　$P(B|A_2)=\dfrac{C_m^2}{C_{2m-1}^2}$

$P(A_1|B) = P(A_1)P(B|A_1) / [P(A_1)P(B|A_1)+P(A_2)P(B|A_2)] = \dfrac{P(B|A_1)}{P(B|A_1)+P(B|A_2)}$

$P(A_2|B) = P(A_2)P(B|A_2) / [P(A_1)P(B|A_1)+P(A_2)P(B|A_2)] = \dfrac{P(B|A_2)}{P(B|A_1)+P(B|A_2)}$

又$m\geq 3$，$P(B|A_1) = \dfrac{C_{m-1}^2}{C_{2m-1}^2} = \dfrac{m-2}{2(2m-1)} < P(B|A_2) = \dfrac{C_m^2}{C_{2m-1}^2} \dfrac{m}{2(2m-1)}$

$\Rightarrow P(A_1|B) < P(A_2|B)$

故丢黑球的概率更大

解53：设$A_1A_2A_3A_4$分别表示从甲中拿出的三个球为3白，2白1黑，1黑2白，3黑。B为从乙中摸出的是白球.

$P(A_1)=\dfrac{C_5^3}{C_{11}^3}=\dfrac{2}{33}$，$P(A_2)=\dfrac{C_5^2C_6^1}{C_{11}^3}=\dfrac{12}{33}$，$P(A_3)=\dfrac{C_5^1C_6^2}{C_{11}^3}=\dfrac{15}{33}=\dfrac{5}{11}$，$P(A_4)=\dfrac{C_6^3}{C_{11}^3}=\dfrac{4}{33}$

$P(B|A_1)=\dfrac{13}{25}$，$P(B|A_2)=\dfrac{12}{25}$　$P(B|A_3)=\dfrac{11}{25}$　$P(B|A_4)=\dfrac{10}{25}$

$P(B)=P(A_1)P(B|A_1)+P(A_2)P(B|A_2)+P(A_3)P(B|A_3)+P(A_4)P(B|A_4)$

$=\frac{2}{33}\times\frac{13}{25}+\frac{12}{33}\times\frac{12}{25}+\frac{15}{33}\times\frac{11}{25}+\frac{4}{33}\times\frac{10}{25}=\frac{5}{11}$

故摸出的球是白球的概率为$\frac{5}{11}$

解56：设A为该人带菌，B为该人2次阳性1次阴性.

$P(A)=0.1$，$P(\bar{A})=0.9$，$P(B|A)=C_3^2(0.95)^2\cdot 0.05$

$P(B|\bar{A})=C_3^2(0.01)^2\cdot 0.99$

$P(A|B)=\dfrac{P(A)\cdot P(B|A)}{P(A)P(B|A)+P(\bar{A})P(B|\bar{A})}$

$=\dfrac{0.1\cdot C_3^2\cdot(0.95)^2\cdot 0.05}{0.1\cdot C_3^2\cdot(0.95)^2\cdot 0.05+0.9\cdot C_3^2\cdot(0.01)^2\cdot 0.99}=0.9806.$

故该人为带菌者的概率为0.9806

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