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解:31(1)

(X,Y)	(0,0)	(0,1)	(0,2)	(1,0)	(1,1)	(1,2)
P(X+Y)	0.2	0.1	0.2	0.3	0.2	0
X+Y	0	1	2	1	2	3

⇒ X+Y的分布律为

X+Y	0	1	2
P	0.2	0.4	0.4

(2)

(X,Y)	(0,0)	(0,1)	(0,2)	(1,0)	(1,1)	(1,2)
XY	0	0	0	0	1	2
min(X,Y)	0	0	0	0	1	1
max(X,Y)	0	1	2	1	1	2
P	0.2	0.1	0.2	0.3	0.2	0

⇒ XY的分布律为

XY	0	1
	0.8	0.2

min(X,Y)与max(X,Y)的分布律为

min(X,Y)	0	1
	0.8	0.2

max(X,Y)	0	1	2
	0.2	0.6	0.2

解34:(1) X_1 X_2 X_3 的分布律为

X_i	0	1
	$1-P$	P

$$P(Y_1=1) = P(X_1=0, X_2=0) + P(X_1=1, X_2=1)$$
$$= P(X_1=0)P(X_2=0) + P(X_1=1)P(X_2=1) = (1-p)^2+p^2$$

$$P(Y_1=0) = P(X_1=0, X_2=1) + P(X_1=1, X_2=0)$$
$$= P(X_1=0)P(X_2=1) + P(X_1=1)P(X_2=0) = 2P(1-P)$$

同理 $P(Y_2=1) = (1-P)^2+P$, $P(Y_2=0) = (1-P)^2+P^2$

$$P(Y_1=1, Y_2=1) = P(X_2=0, X_1=0, X_3=0) + P(X_2=1, X_1=1, X_3=1)$$
$$= (1-P)^3+P^3 = 1-3P(1-P)$$

$$P(Y_1=1, Y_2=0) = P(X_2=0, X_1=0, X_3=1) + P(X_2=1, X_1=1, X_3=0)$$

$= p(1-p)^2 + p^2(1-p) = p - 2p^2 + p^3 + p^2 - p^3 = p(1-p)$

$P(Y_1=0, Y_2=1) = P(X_2=0, X_1=1, X_3=0) + P(X_2=1, X_1=0, X_3=1)$

$= (1-p)^2 p + p^2(1-p) = p(1-p)$

$P(Y_1=0, Y_2=0) = P(X_2=0, X_1=1, X_3=1) + P(X_2=1, X_1=0, X_3=0)$

$= p^2(1-p) + (1-p)^2 p = p(1-p)$

故 (Y_1, Y_2) 的联合分布律为

$Y_2 \backslash Y_1$	0	1
0	$p(1-p)$	$p(1-p)$
1	$p(1-p)$	$1-3p(1-p)$

(2)

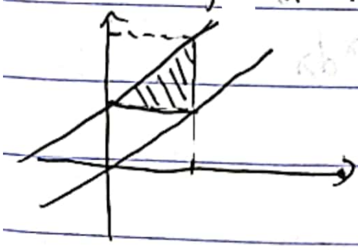
(Y_1, Y_2)	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$P(Y_1, Y_2)$	$p(1-p)$	$p(1-p)$	$p(1-p)$	$1-3p(1-p)$
$Y_1 Y_2$	0	0	0	1

$\Rightarrow$ Y_1Y_2 的分布律为

Y_1Y_2	0	1
P	$3p(1-p)$	$1-3p(1-p)$

解 35 $f_Z(z) = \int_{-\infty}^{+\infty} f(x, z-x)\,dx = \int_{-\infty}^{+\infty} 3x\,dx$

由 $1-y < x < 1$, $0 < y < 1 \Rightarrow x < z < x+1$ $x < 1$, $z > 1$



$\Rightarrow \int_{-\infty}^{+\infty} 3x\,dx = \int_{z-1}^{1} 3x\,dx = 3\left[\frac{1}{2} - \frac{(z-1)^2}{2}\right]$

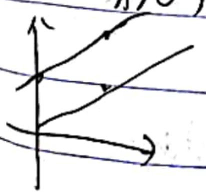
$= 3\left(\frac{1-z^2+2z-1}{2}\right) = 3z - \frac{3}{2}z^2$

$\Rightarrow f_Z(z) = \begin{cases} 3z - \frac{3}{2}z^2 & 1 < z < 2 \\ 0 & 其他 \end{cases}$

解 37：由 X、Y 相互独立，$\Rightarrow f(x,y) = f_X(x) f_Y(y) = 2y e^{-x}$

$f_Z(z) = \int_{-\infty}^{+\infty} f(x, z-x)\,dx = \int_{-\infty}^{+\infty} 2(z-x)e^{-x}\,dx$

由 $x > 0$, $0 < z-x < 1 \Rightarrow x > 0$ $x < z < x+1$, $z > 1$ 时,



$\Rightarrow 2\int_{-\infty}^{+\infty} (z-x)e^{-x}dx = 2\int_{z-1}^{z} (z-x)e^{-x}\,dx$

$= 2\left(-z\, e^{-x}\Big|_{z-1}^{z} + \int_{z-1}^{z} x\,d e^{-x}\right) = 2(-ze^{-z} + ze^{-z+1} +$

$xe^{-x}\big|_{z-1}^{z} - \int_{z-1}^{z} e^{-x}dx = 2(-ze^{-z}+ze^{-z+1}+ze^{-z}-(z-1)e^{-z+1}+e^{-z}-e^{-z+1})$

$= 2(e^{1-z}+e^{-z}-e^{-z+1}) = 2e^{-z}$

$0\le z\le 1$时 $f_Z(z)=2\int_0^z (z-x)e^{-x}dx = 2\int_0^z ze^{-x}dx - 2\int_0^z xe^{-x}dx$

$=2[-z(e^{-z}-1)+\int_0^z x\,de^{-x}] = 2[z(1-e^{-z}) + xe^{-x}\big|_0^z - \int_0^z e^{-x}dx$

$=2[z-ze^{-z}+ze^{-z}+(e^{-z}-1)] = 2(z+e^{-z}-1)$

故 $f_Z(z) = \begin{cases} 2e^{-z} & z>1 \\ 2(z+e^{-z}-1) & 0\le z\le 1 \\ 0 & \text{其他} \end{cases}$

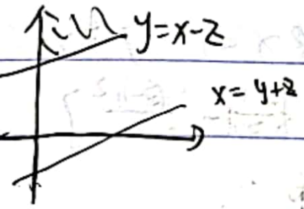
解39 (1)由正态分布函数的性质：X+Y 服从 $(0+1, 1+1)$ $=(1,2)$ 的正态分布，故X+Y密度函数为 $f_{X+Y}(x)=\frac{1}{2\sqrt{\pi}}e^{-\frac{(x-1)^2}{4}}$

(2) $-Y$服从 $N(-1,1)$

$\Rightarrow$ X-Y服从 $N(-1,1)$ $\Rightarrow$ 密度函数为 $f_{X-Y}(x)=\frac{1}{2\sqrt{\pi}}e^{-\frac{(x+1)^2}{4}}$

解41: $F(z)=P(Z\le z)=P(X-Y\le z)$

$= \iint_{x-y\le z} e^{-(x+y)}dxdy =$



当 $z<0$时, $F(z)=\int_0^{+\infty}\int_{x-z}^{+\infty} e^{-(x+y)}dy\,dx$

$=\int_0^{+\infty} e^{-x}\cdot e^{z-x}dx = \int_0^{+\infty} e^{z-2x}dx$

$=e^{z}\cdot\frac{1}{2}e^{-2x}\big|_{+\infty}^{0} = \frac{1}{2}e^{z}$

当 $z\ge 0$时, $F(z)=\int_0^{+\infty}\int_0^{y+z} e^{-(x+y)}dx\,dy$

$=\int_0^{+\infty} e^{-y}\int_0^{y+z} e^{-x}dx\,dy = \int_0^{+\infty} e^{-y}(1-e^{-(y+z)})dy$

$=\int_0^{+\infty} e^{-y}-e^{-2y-z}dy = 1-\frac{1}{2}e^{-z}\cdot e^{-2y}\big|_{\infty}^{0} = 1-\frac{1}{2}e^{-z}$

故 $F(z)=\begin{cases}\frac{1}{2}e^{z} & z<0 \\ 1-\frac{1}{2}e^{-z} & z\ge 0\end{cases} \Rightarrow f(z)=\begin{cases}\frac{1}{2}e^{z} & z<0 \\ \frac{1}{2}e^{-z} & z\ge 0\end{cases}$

即 $f(z)=\frac{1}{2}e^{-|z|}$

解45: (1) $f_X(x)=\int_{-\infty}^{+\infty} f(x,y)dy = \int_{-\infty}^{+\infty}(x+y)dy = \int_0^1 (x+y)dy$

$= \int_0^1 x\,dy + \int_0^1 y\,dy = x + \frac{1}{2}$，同理 $f_Y(y) = y + \frac{1}{2}$

由 $f(x,y) \neq f_X(x) f_Y(y) \Rightarrow$ X与Y不相互独立.

(2) $M = \max(X,Y)$　$F(m) = P(M \le m) = P(\max(X,Y) \le m)$

$= P(X \le m, Y \le m) = \iint_{x \le m, y \le m} (x+y)\,dx\,dy$

当 $m \geq 1$ 时，$F(m) = 1$　当 $m < 0$ 时，$F(m) = 0$，$f(m) = 0$.

当 $0 < m < 1$ 时，$F(m) = \int_0^m \int_0^m (x+y)\,dx\,dy = \int_0^m \left(\frac{m^2}{2} + ym\right) dy$

$= \frac{m^3}{2} + \frac{m^3}{2} = m^3$　$f(m) = F'(m) = 3m^2$

$\Rightarrow$ M的密度函数为 $f(m) = \begin{cases} 3m^2 & 0 < m < 1 \\ 0 & 其他 \end{cases}$

解46：$F(z) = P(Z \le z) = P\left(\frac{X}{2} + \frac{Y}{2} \le z\right)$

当 $z < 0$ 时，$F(z) = 0$，$z \geq 1$ 时，$F(z) = 1$，$f(z) = 0$，

当 $0 < z < 1$ 时，$F(z) = P\left(\frac{X}{2} + \frac{Y}{2} \le z\right) = P(X + Y \le 2z)$.

$= P(X+Y \le 2z, Y=0) + P(X+Y \le 2z, Y=1)$

$= P(X+Y \le 2z)P(Y=0) + P(X+Y \le 2z)P(Y=1)$

$= P(X \le 2z) \cdot \frac{1}{2} + \frac{1}{2} P(X \le 2z-1)$

若 $0 < z \le \frac{1}{2}$　$F(z) = \frac{1}{2}\left(\frac{2z}{1} + 0\right) = z$

若 $\frac{1}{2} < z < 1 \Rightarrow F(z) = \frac{1}{2}\left(1 + \frac{2z-1}{1}\right) = z$

$\Rightarrow f(z) = F'(z) = 1 \ (0 < z < 1)$ 故 $f(z) = \begin{cases} 1 & 0 < z < 1 \\ 0 & 其他 \end{cases}$

解2：设3张中5元券的个数为Y张，Y可取0,1,2，

$X = 5Y + 2(3-Y) = 3Y + 6$.

Y服从超几何分布　$E(Y) = 3 \cdot \frac{2}{10} = \frac{3}{5}$

$E(X) = 3E(Y) + 6 = \frac{39}{5} = 7.8$

故X的数学期望 $E(X) = 7.8$

解 8：(1) $\int_0^2 (Ax+\frac{1}{3})dx = 2A+\frac{2}{3} = 1 \Rightarrow A=\frac{1}{6}$

(2) $E(X) = \int_0^2 x(\frac{x}{6}+\frac{1}{3})dx = \int_0^2 (\frac{x^2}{6}+\frac{x}{3})dx$

$= \frac{1}{6}\cdot\frac{1}{3}x^3\Big|_0^2 + \frac{1}{3}\cdot\frac{1}{2}\cdot x^2\Big|_0^2 = \frac{1}{18}\cdot 8 + \frac{1}{6}\cdot 4 = \frac{10}{9}$

故 $E(X) = \frac{10}{9}$

解 12：$P(X>3) = \int_3^{+\infty} 2^{-x}\ln 2\,dx = 2^{-x}\Big|_{+\infty}^{3} = 2^{-3} = \frac{1}{8}$

$P(Y=k) = C_{k-1}^{1}\frac{1}{8}\cdot(1-\frac{1}{8})^{k-2}\cdot\frac{1}{8} = (k-1)\frac{1}{64}(\frac{7}{8})^{k-2} \quad (k\geq 2)$

$\Rightarrow E(Y) = \sum_{k=2}^{\infty} k(k-1)\frac{1}{64}\cdot(\frac{7}{8})^{k-2}$.

由 $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \Rightarrow \sum_{k=2}^{\infty} x^{k-2}\cdot k\cdot(k-1) = (\frac{1}{1-x})'' = \frac{2}{(1-x)^3}$

令 $x=\frac{7}{8} \Rightarrow \sum_{k=2}^{\infty}\frac{1}{64}k(k-1)\cdot(\frac{7}{8})^{k-2} = \frac{1}{64}\cdot\frac{2}{(1-\frac{7}{8})^3} = 16$ 故 $E(Y)=16$

解 18

$E(Z) = \int_{-\infty}^{+\infty}\frac{1}{x^2}\frac{4}{81}x^3\,dx = \int_0^3 \frac{4x}{81}dx = \frac{2}{9}$

故 $E(Z) = \frac{2}{9}$

解 21：由 X Y 相互独立：$E(XY^2-2X^2Y+1) = E(X)E(Y^2) - 2E(X^2)E(Y) + 1$

$E(X) = 10\times 0.4 = 4 \quad E(X^2) = 100\cdot 0.4 = 40$

$E(Y) = \int_{-\infty}^{+\infty}\frac{1}{2}e^{-|y|}\cdot y\,dy$

由 $\frac{1}{2}ye^{-|y|}dy$ 是奇函数 $\Rightarrow E(Y)=0$

$E(Y^2) = \int_{-\infty}^{+\infty} y^2\frac{1}{2}e^{-|y|}dy = \int_0^{+\infty} y^2 e^{-y}dy = \int_0^{+\infty} -y^2\,de^{-y}$

$= y^2e^{-y}\Big|_{+\infty}^{0} + \int_0^{+\infty} e^{-y}\,2y\,dy = -2\int_0^{+\infty} y\,de^{-y}$

$= -2(ye^{-y}\Big|_0^{+\infty} - \int_0^{+\infty} e^{-y}dy) = -2(0 - e^{-y}\Big|_{+\infty}^{0})$

$= 2\,e^{-y}\Big|_{+\infty}^{0} = 2$

$\Rightarrow E(XY^2-2X^2Y+1) = 4\times 2 - 2\times 40\times 0 + 1 = 9$

解 26：$E(X) = 1(0.2+0.1+0.1) + 2(0.1+0.1) + 3(0.3+0.1) = 2$

$E(Y) = -1(0.2+0.1) + 0\times(0.1+0.3) + 1(0.1+0.1+0.1) = 0$

$E(Z) = E[(X-Y)^2] = (X-Y)^2\cdot P[(X-Y)^2] = 4\times 0.2 + 0.1 + 0 +$

$9\times0.1+4\times0+0.1+9\times0.3+4\times0.1=0.8+0.1+0.9+0.1+0.2.7+0.4$

$=5 \Rightarrow E(Z)=5$

解 30

(1) 由 $Y\sim N(0,\frac{1}{2}) \Rightarrow -Y\sim(0,\frac{1}{2})$

$\Rightarrow X-Y\sim N(0,1)$ 令 $Z=X-Y \Rightarrow Z\sim N(0,1)$

即 $f(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$

$\Rightarrow E(|X-Y|)=E(|Z|)=\int_{-\infty}^{+\infty}|z|e^{-\frac{z^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}dz$

$=\frac{2}{\sqrt{2\pi}}\int_0^{+\infty}ze^{-\frac{z^2}{2}}dz=-\frac{2}{\sqrt{2\pi}}\int_0^{+\infty}e^{-\frac{z^2}{2}}d(-\frac{z^2}{2})=\sqrt{\frac{2}{\pi}}$

(2) $D(|X-Y|)=D(Z)=E[(|Z|-E(|Z|))^2]=\int_{-\infty}^{+\infty}(|z|-\sqrt{\frac{2}{\pi}})^2e^{-\frac{z^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}dz$

$=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}z^2e^{-\frac{z^2}{2}}dz-2\sqrt{\frac{2}{\pi}}\int_{-\infty}^{+\infty}|z|e^{-\frac{z^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}dz+\frac{2}{\pi}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz$

$=2\int_0^{+\infty}-\frac{1}{\sqrt{2\pi}}z\,de^{-\frac{z^2}{2}}-2\sqrt{\frac{2}{\pi}}\cdot\sqrt{\frac{2}{\pi}}+\frac{2}{\pi}$

$=\frac{-2}{\sqrt{2\pi}}(ze^{-\frac{z^2}{2}}\Big|_0^{+\infty}-\int_0^{+\infty}e^{-\frac{z^2}{2}}dz)-\frac{2}{\pi}$

$=\frac{2}{\sqrt{2\pi}}\int_0^{+\infty}e^{-\frac{z^2}{2}}dz-\frac{2}{\pi}=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz-\frac{2}{\pi}=1-\frac{2}{\pi}$

(3) $U=\frac{1}{2}(X+Y+|X-Y|)\Rightarrow E(U)=\frac{1}{2}(E(X)+E(Y)-E(|X-Y|))$

$=\frac{1}{2}(0+0-\sqrt{\frac{2}{\pi}})=\sqrt{\frac{1}{2\pi}}$

(4) $V=\frac{1}{2}(X+Y-|X-Y|)\Rightarrow E(V)=\frac{1}{2}(E(X)+E(Y)-E(X+Y))=-\frac{1}{\sqrt{2\pi}}$

(5) $E(U+V)=E(U)+E(V)=0$

(6) $E(U-V)=E(U)-E(V)=\sqrt{\frac{2}{\pi}}$

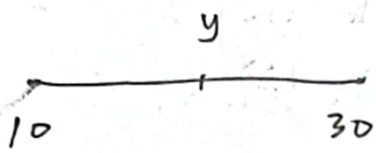
(7) $E(UV)=\frac{1}{4}E([(X+Y)+|X-Y|][(X+Y)-|X-Y|])$

$=\frac{1}{4}E(X^2+2XY+Y^2-X^2-Y^2+2XY)=E(XY)$

$=E(X)\cdot E(Y)=0$

故 (1) $\sqrt{\frac{2}{\pi}}$ (2) $1-\frac{2}{\pi}$ (3) $\sqrt{\frac{1}{2\pi}}$ (4) $-\frac{1}{\sqrt{2\pi}}$ (5) 0. (6) $\sqrt{\frac{2}{\pi}}$ (7) 0

解34：设进货量为 y



$$E=\int_{10}^{y}\left[500x-100(y-x)\cdot\frac{1}{20}\right]dx+\int_{y}^{30}\left[500y+300(x-y)\cdot\frac{1}{20}\right]dx$$

$$=5\left(\int_{10}^{y}5x-(y-x)\,dx+\int_{y}^{30}5y+3x-3y\right)dx$$

$$=5\left(\int_{10}^{y}6x-y\,dx+\int_{y}^{30}2y+3x\,dx\right)$$

$$=5\left[\left(3x^2-yx\right)\Big|_{10}^{y}+\left(2yx+\frac{3}{2}x^2\right)\Big|_{y}^{30}\right]$$

$$=5\left[(3y^2-y^2)-(300-10y)+(60y+1350)-\left(2y^2+\frac{3}{2}y^2\right)\right]$$

$$=5\left(2y^2-300+10y+60y+1350-2y^2-\frac{3}{2}y^2\right)$$

$$=5\left(-\frac{3}{2}y^2+70y+1050\right)$$

若 $E\geq 9280$

即 $5\left(-\frac{3}{2}y^2+70y+1050\right)\geq 9280$

$$-\frac{3}{2}y^2+70y+1050\geq 1856$$

$$-\frac{3}{2}y^2+70y-806\geq 0$$

$$3y^2-140y+1612\leq 0$$

$$(y-26)(3y-62)\leq 0$$

$$20.66=\frac{62}{3}\leq y\leq 26$$

故若平均值不少于9280元，进货量最小值为21