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解10：$f(x)=F'(x)=\begin{cases}\frac{1}{2}e^{x} & x<0\\ 0 & 0\le x<1\\ \frac{1}{4}e^{-(x-1)/2} & x\ge 1\end{cases}$

$\Rightarrow E(X)=\int_{-\infty}^{+\infty} f(x)x\,dx=\int_{-\infty}^{0}\frac{1}{2}xe^{x}dx+\int_{1}^{+\infty}\frac{1}{4}xe^{-\frac{(x-1)}{2}}dx$

$=\frac{1}{2}\int_{-\infty}^{0}x\,de^{x}+\frac{1}{4}(-2)\int_{1}^{+\infty}x\,de^{-\frac{(x-1)}{2}}$

$=\frac{1}{2}\left(xe^{x}\Big|_{-\infty}^{0}-\int_{-\infty}^{0}e^{x}dx\right)-\frac{1}{2}\left(xe^{-\frac{(x-1)}{2}}\Big|_{1}^{+\infty}-\int_{1}^{+\infty}e^{-\frac{x-1}{2}}dx\right)$

$=\frac{1}{2}\left(-e^{x}\Big|_{-\infty}^{0}\right)-\frac{1}{2}\left(-1-(-2)e^{-\frac{x-1}{2}}\Big|_{1}^{+\infty}\right)$

$=-\frac{1}{2}-\frac{1}{2}[-1-(-2)\cdot(0-1)]$

$=-\frac{1}{2}-\frac{1}{2}(-1-2)=-\frac{1}{2}+\frac{3}{2}=1$

$E(X^2)=\int_{-\infty}^{+\infty}x^2f(x)\,dx=\int_{-\infty}^{0}\frac{1}{2}x^2e^{x}dx+\int_{1}^{+\infty}\frac{1}{4}x^2e^{-\frac{(x-1)}{2}}dx$

$=\frac{1}{2}\int_{-\infty}^{0}x^2de^{x}+\frac{1}{4}\int_{1}^{+\infty}x^2(-2)\,de^{-\frac{x-1}{2}}$

$=\frac{1}{2}\left(x^2e^{x}\Big|_{-\infty}^{0}-2\int_{-\infty}^{0}x\,de^{x}\right)+\frac{(-2)}{4}\left(x^2e^{-\frac{x-1}{2}}\Big|_{1}^{+\infty}-\int_{1}^{+\infty}2xe^{-\frac{x-1}{2}}dx\right)$

$=\frac{1}{2}\left[-2\left(xe^{x}\Big|_{-\infty}^{0}-e^{x}\Big|_{-\infty}^{0}\right)\right]+\left(-\frac{1}{2}\right)\left(-1+4\int_{1}^{+\infty}x\,de^{-\frac{x-1}{2}}\right)$

$=\frac{1}{2}[-2(0-1)]-\frac{1}{2}\left[-1+4\left(xe^{-\frac{x-1}{2}}\Big|_{1}^{+\infty}-\int_{1}^{+\infty}e^{-\frac{x-1}{2}}dx\right)\right]$

$=1-\frac{1}{2}\left[-1+4\left[1-(-2)e^{-\frac{x-1}{2}}\Big|_{1}^{+\infty}\right]\right]$

$=1-\frac{1}{2}[-1+4[-1-(-2)(-1)]]$

$=1-\frac{1}{2}[-1+4(-3)]=1-\frac{1}{2}\times(-13)=\frac{15}{2}$

$\Rightarrow D(X)=E(X^2)-E^2(X)=\frac{15}{2}-1=\frac{13}{2}$

解13：$P(Y=1)=P(X>0)=\frac{2}{3}$

$P(Y=0)=P(X=0)=0$

$P(Y=-1)=P(X<0)=\frac{1}{3}$

$\Rightarrow E(Y)=1\times\frac{2}{3}+(-1)\times\frac{1}{3}=\frac{1}{3}$

$E(Y^2)=1\times\frac{2}{3}+1\times\frac{1}{3}=1$

$D(Y)=E(Y^2)-E^2(Y)=1-\frac{1}{9}=\frac{8}{9}$

解14: $E(X) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{+\infty} x e^{-x^2} dx = -\frac{1}{\sqrt{\pi}}\frac{1}{2}\int_{-\infty}^{+\infty} e^{-x^2} d(-x^2)$

$= -\frac{2}{\sqrt{\pi}} e^{-x^2}\Big|_{-\infty}^{+\infty} = 0$

$f(x) = \frac{1}{\sqrt{\pi}} e^{-x^2} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \Rightarrow \sigma = \frac{\sqrt{2}}{2}$

即方差 $D(X) = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

解17: $f(x) = F'(x) = 0.3\varphi(x) + 0.35\varphi\left(\frac{x-1}{2}\right)$

$E(X) = \int_{-\infty}^{+\infty} x f(x) dx$

$= 0.3\int_{-\infty}^{+\infty} x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx + 0.35\times2\int_{-\infty}^{+\infty} x \frac{1}{\sqrt{2\pi}\times2} e^{-\frac{(x-1)^2}{8}} dx$

$= 0.3\times0 + 0.7\times1 = 0.7$

$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x) dx$

$= 0.3\int_{-\infty}^{+\infty} x^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx + 0.35\times2\int_{-\infty}^{+\infty} x^2 \frac{1}{\sqrt{2\pi}\times2} e^{-\frac{(x-1)^2}{8}} dx$

$= 0.3(1^2+0^2) + 0.7(2^2+1^2)$

$= 0.3 + 3.5 = 3.8$

$\Rightarrow D(X) = E(X^2) - E^2(X) = \quad = 3.8 - 0.49 = 3.31$

故 $E(X) = 0.7 \quad D(X) = 3.31$

解20: $E(Y) = \int_{-\infty}^{+\infty} \min(|x|, 1) f(x) dx$

$= \int_{-\infty}^{-1} \frac{1}{\pi(1+x^2)} dx + \int_{1}^{+\infty} \frac{dx}{\pi(1+x^2)} + \int_{-1}^{1} |x| \frac{1}{\pi(1+x^2)} dx$

$= 1 - \int_{-1}^{1} \frac{1}{\pi(1+x^2)} dx + \int_{-1}^{1} \frac{|x|}{\pi(1+x^2)} dx$

$= 1 - \frac{2}{\pi}\int_0^1 \frac{1}{1+x^2} dx + \frac{2}{\pi}\int_0^1 \frac{x}{1+x^2} dx$

$= 1 - \frac{2}{\pi}\arctan x\Big|_0^1 + \frac{1}{\pi}\ln(1+x^2)\Big|_0^1$

$= 1 - \frac{2}{\pi}\cdot\frac{\pi}{4} + \frac{1}{\pi}\ln 2 = \frac{1}{\pi}\ln 2 + \frac{1}{2}$

$E(Y^2) = \int_{-\infty}^{+\infty} \min(x^2, 1) f(x) dx$

$= \int_{-\infty}^{-1} \frac{1}{\pi(1+x^2)} dx + \int_{1}^{+\infty} \frac{1}{\pi(1+x^2)} dx + \int_{-1}^{1} x^2 \frac{1}{\pi(1+x^2)} dx$

$= 1 - \int_{-1}^{1} \frac{1}{\pi(1+x^2)} dx + \frac{2}{\pi}\int_0^1 \frac{x^2}{1+x^2} dx$

$= 1 - \frac{2}{\pi}\cdot\arctan\Big|_0^1 + \frac{2}{\pi}\int_0^1 \left(\frac{x^2+1}{1+x^2} - \frac{1}{x^2+1}\right) dx$

$= \frac{1}{2} + \frac{2}{\pi}\int_0^1 1-\frac{1}{1+x^2}\,dx = \frac{1}{2}+\frac{2}{\pi} - \frac{2}{\pi}\int_0^1 \frac{1}{1+x^2}dx$

$= \frac{1}{2} + \frac{2}{\pi} - \frac{2}{\pi}\cdot\frac{\pi}{4} = \frac{2}{\pi}$

$\Rightarrow D(Y) = E(Y^2) - E^2(Y) = \frac{2}{\pi} - (\frac{1}{\pi^2}\ln^2 2 + \frac{1}{\pi}\ln 2 + \frac{1}{4})$

$= \frac{1}{\pi}(2-\ln 2) - \frac{1}{\pi^2}\ln^2 2 - \frac{1}{4}$

解 24:

$$F_i(x) = \begin{cases} x & 0<x<1 \\ 0 & \text{其他} \end{cases}$$

$\Rightarrow U = \max\{X_1, X_2 \cdots X_n\}$

$F_U(u) = P(U\le u) = P(X_1\le u, X_2\le u, \cdots X_n\le u)$

$= P(X_1\le u)\ P(X_2\le u) \cdots P(X_n\le u)$

$$\Rightarrow F_U(u) = \begin{cases} u^n & 0\le u\le 1 \\ 1 & u>1 \\ 0 & u<0 \end{cases} \Rightarrow f_U(u) = \begin{cases} nu^{n-1} & 0<u<1 \\ 0 & \text{其他} \end{cases}$$

$\Rightarrow E(U) = \int_0^1 nu^{n-1}\cdot u\,du = \int_0^1 nu^n du = \frac{n}{n+1}u^{n+1}\Big|_0^1 = \frac{n}{n+1}$

$E(U^2) = \int_0^1 nu^{n-1}\cdot u^2 du = \int_0^1 nu^{n+1}du = \frac{n}{n+2}u^{n+2}\Big|_0^1 = \frac{n}{n+2}$

$\Rightarrow D(U) = E(U^2) - E^2(U) = \frac{n}{n+2} - (\frac{n}{n+1})^2 = \frac{n}{n+2} - \frac{n^2}{(n+1)^2} = \frac{n}{(n+2)(n+1)^2}$

$V = \min\{X_1, X_2 \cdots X_n\}$

$F_V(v) = P(V\le v) = 1 - P(V\ge v)$

$= 1 - P(X_1\ge v, X_2\ge v \cdots X_n\ge v)$

$= 1 - P(X_1\ge v)\ P(X_2\ge v) \cdots P(X_n\ge v)$

$= 1 - [1-P(X_1\le v)][1-P(X_2\le v)] \cdots [1-P(X_n\le v)]$

$$\Rightarrow F_V(v) = \begin{cases} 1-(1-v)^n & 0<v\le 1 \\ 1 & v>1 \\ 0 & v<0 \end{cases} \Rightarrow f_V(v) = \begin{cases} n(1-v)^{n-1} & 0<v\le 1 \\ 0 & \text{其他} \end{cases}$$

$\Rightarrow E(V) = \int_0^1 v\,n(1-v)^{n-1}dv$. 令 $t=1-v$, $\Rightarrow v = 1-t$

$\Rightarrow E(V)=\int_0^1 v\,n(1-v)^{n-1}dv = \int_1^0 n(1-t)\,t^{n-1}d(1-t) = n\int_0^1 (1-t)\,t^{n-1}dt$

$=n\int_0^1 t^{n-1}dt - n\int_0^1 t^n dt = n\left(\frac{1}{n}-\frac{1}{n+1}\right) = 1-\frac{n}{n+1} = \frac{1}{n+1}$

$E(V^2)=\int_0^1 v^2 n(1-v)^{n-1}dv = n\int_0^1 (1-t)^2 t^{n-1}dt$

$=n\int_0^1 (t^{n+1}-2t^n+t^{n-1})dt$

$=n\left(\frac{1}{n+2}-\frac{2}{n+1}+\frac{1}{n}\right)$

$\Rightarrow D(V)=E(V^2)-E^2(V)$

$=\frac{n}{n+2}-\frac{2n}{n+1}+1-\frac{1}{(n+1)^2}=\frac{2n+2}{n+2}-\frac{2n^2+2n+1}{(n+1)^2}=\frac{n}{(n+2)(n+1)^2}$

故 $E(U)=\frac{n}{n+1}$ $\quad E(V)=\frac{1}{n+1}$

$D(U)=\frac{n}{(n+2)(n+1)^2}$ $\quad D(V)=\frac{n}{(n+2)(n+1)^2}$

解35： 设恰有一个白球 为事件A.

$\Rightarrow p(A)=\sum_{m=0}^{N} P(A\mid X=m)\,P(X=m)$

$=\sum_{m=0}^{N}\frac{1}{C_N^2}C_m^1 C_{N-m}^1\cdot P(X=m)$

$=\frac{1}{C_N^2}\sum_{m=0}^{N} m(N-m)\,p(X=m)$

$=\frac{1}{C_N^2}E[m(N-m)]$

$=\frac{1}{C_N^2}[E(Nm)-E(m^2)]=\frac{1}{C_N^2}[NE(m)-E(m^2)]$

$=\frac{1}{C_N^2}[N\cdot n-(6^2+n^2)]$

$=\frac{2(nN-6^2-n^2)}{N(N-1)}$

故恰有一个白球的概率为 $P=\frac{2(nN-6^2-n^2)}{N(N-1)}$

解36： $E(XY)=1\alpha+0\cdot\frac{1}{8}-1\cdot\frac{1}{4}-1\cdot\frac{1}{8}+0+1\cdot\beta$

$=\alpha+\beta-\frac{3}{8}$

而 $\alpha+\beta=1-\frac{1}{8}-\frac{1}{4}-\frac{1}{8}-\frac{1}{8}=\frac{3}{8}$ $\quad\Rightarrow E(XY)=0$

$P(X=-1)=\alpha+\frac{1}{8}$ $\quad P(Y=-1)=\alpha+\frac{1}{4}+\frac{1}{8}=\alpha+\frac{3}{8}$

$P(X=-1,Y=-1)=\alpha$ $\quad$ 由 X、Y 独立 $\Rightarrow \alpha=(\alpha+\frac{1}{8})(\alpha+\frac{3}{8})$

$\Rightarrow \alpha = \frac{1}{8}$

$p(X=1) = \frac{1}{4}+\beta \quad p(Y=1)=\beta+\frac{1}{4},\ p(X=1,Y=1)=\beta$

由 X,Y 相互独立 $\Rightarrow \beta = (\beta+\frac{1}{4})(\frac{1}{4}+\beta) \Rightarrow \beta = \frac{1}{4}$

由 X和Y不相关 $\Rightarrow \mathrm{cov}(X,Y)=0$

$\mathrm{cov}(X,Y) = E(XY) - E(X)E(Y) = -E(X)E(Y) = 0.$

$E(X) = -1\cdot(\alpha+\frac{1}{8}) + 1(\frac{1}{4}+\beta) = \frac{1}{8}+\beta-\alpha$

$E(Y) = -1(\frac{3}{8}+\alpha) + \frac{1}{4}+\beta = \beta-\alpha - \frac{1}{8}$

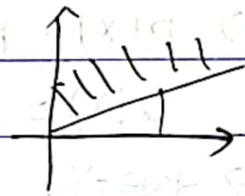
由 $E(X)E(Y)=0 \Rightarrow \beta-\alpha = -\frac{1}{8}$ 或 $\beta-\alpha=\frac{1}{8}$

而 X,Y 独立时要求 $\beta=\frac{1}{4},\ \alpha=\frac{1}{8}$，即 $\beta-\alpha=\frac{1}{8}$

不满足 $\beta-\alpha=-\frac{1}{8}$，则 X,Y 不一定独立.

故 $E(XY)=0$，$\alpha=\frac{1}{8}, \beta=\frac{1}{4}$，不一定独立

解 40: $E(X) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x\,2e^{-2y}\,dx\,dy$



$= \int_0^{+\infty}\int_{\frac{x}{2}}^{+\infty} x\,2e^{-2y}\,dy\,dx$

$= -\int_0^{+\infty} x\cdot (e^{-2y}\Big|_{\frac{x}{2}}^{+\infty}$

$= -\int_0^{+\infty} x(0-e^{-x})\,dx$

$= \int_0^{+\infty} xe^{-x}\,dx$

$= -\left(xe^{-x}\Big|_0^{+\infty} - \int_0^{+\infty} e^{-x}\,dx\right)$

$= -(0\ e^{-x}\Big|_0^{+\infty}) = 1$

$E(y) = \int_0^{+\infty}\int_{\frac{x}{2}}^{+\infty} y\,2e^{-2y}\,dy\,dx = \int_0^{+\infty}\int_x^{+\infty} \frac{1}{2}te^{-t}\,dt\,dx$

$= \int_0^{+\infty}\frac{1}{2}\int_x^{+\infty} te^{-t}\,dt\,dx$

$\int_x^{+\infty} te^{-t}\,dt = -\int_x^{+\infty} t\,de^{-t} = -(te^{-t}\Big|_x^{+\infty} + e^{-t}\Big|_x^{+\infty}) = -(-xe^{-x} - e^{-x})$

$= e^{-x}(1+x)$

$E(y) = \frac{1}{2}\int_0^{+\infty} e^{-x}(1+x)\,dx = \frac{1}{2}(\int_0^{+\infty} xe^{-x}\,dx + \int_0^{+\infty} e^{-x}\,dx)$

$= \frac{1}{2}[1 - e^{-x}\Big|_0^{+\infty}] = \frac{1}{2}[1-(-1)] = 2\times\frac{1}{2} = 1$

$E(XY) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy\,2e^{-2y}\,dy\,dx = \int_0^{+\infty} x \int_{\frac{x}{2}}^{+\infty} y\,2e^{-2y}\,dy\,dx$

$= \int_0^{+\infty}\frac{1}{2}xe^{-x}(x+1)\,dx = \frac{1}{2}\int_0^{+\infty}x^2e^{-x}\,dx + \frac{1}{2}\int_0^{+\infty}xe^{-x}\,dx$

$= -\frac{1}{2}\int_0^{+\infty}x^2\,de^{-x} + \frac{1}{2}\int_0^{+\infty}xe^{-x}\,dx = -\frac{1}{2}\left(x^2e^{-x}\Big|_0^{+\infty} - \int_0^{+\infty}2xe^{-x}\,dx\right) + \int_0^{+\infty}xe^{-x}\,dx$

$= -\frac{1}{2}\left(-\int_0^{+\infty}2xe^{-x}\,dx\right) + \frac{1}{2}\int_0^{+\infty}xe^{-x}\,dx = \frac{1}{2}3\int_0^{+\infty}xe^{-x}\,dx = 3\cdot\frac{1}{2} = \frac{3}{2}$

$E(X^2) = \int_0^{+\infty}x^2\int_{\frac{x}{2}}^{+\infty}2e^{-2y}\,dy\,dx = \int_0^{+\infty}x^2e^{-x}\,dx = 2\int_0^{+\infty}xe^{-x}\,dx = 2$

$E(Y^2) = \int_0^{+\infty}\int_{\frac{x}{2}}^{+\infty}2y^2e^{-2y}\,dy\,dx$

$\int_{\frac{x}{2}}^{+\infty}2y\cdot y\,e^{-2y}\,dy = \int_x^{+\infty}t\cdot\frac{t}{2}e^{-t}\,d\frac{t}{2} = \frac{1}{4}\int_x^{+\infty}t^2e^{-t}\,dt$

$= -\frac{1}{4}(x^2+2x+2)e^{-x}\Big|_x^{+\infty} = \frac{1}{4}(x^2+2x+2)e^{-x}$

$E(Y^2) = \frac{1}{4}\int_0^{+\infty}(x^2+2x+2)e^{-x}\,dx = \frac{1}{4}\left(\int_0^{+\infty}x^2e^{-x}\,dx + 2\int_0^{+\infty}xe^{-x}\,dx + 2\int_0^{+\infty}e^{-x}\,dx\right)$

$= \frac{1}{4}\left(4\int_0^{+\infty}xe^{-x}\,dx + (-2)\int_0^{+\infty}de^{-x}\right)$

$= 1 + \frac{1}{4}(-2)\,e^{-x}\Big|_0^{+\infty} = 1 + \frac{1}{4}(-2)(0-1) = \frac{3}{2}$

$\Rightarrow D(X) = E(X^2) - E^2(X) = 2-1 = 1 \Rightarrow \sqrt{D(X)} = 1$

$D(Y) = E(Y^2) - E^2(Y) = \frac{3}{2} - 1 = \frac{1}{2} \Rightarrow \sqrt{D(Y)} = \sqrt{\frac{1}{2}}$

$\Rightarrow cov(X,Y) = E(XY) - E(X)E(Y) = \frac{3}{2} - 1 = \frac{1}{2}$

$\rho_{XY} = \frac{cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = \frac{1}{2}/1\cdot\sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

故 $\rho_{XY} = \frac{\sqrt{2}}{2}$

解44: $E(X) = 3$ $\quad D(X) = \frac{1}{12}(6-0)^2 = 3$

$E(Y) = 0,$ $\quad D(Y) = 3$

$E(Z) = E(2X+3Y) = 2E(X) + 3E(Y) = 6$

$\rho_{XY} = \frac{cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = \frac{1}{3}cov(X,Y) = \frac{1}{3} \Rightarrow cov(X,Y) = 1$

$\Rightarrow E(XY) - E(X)E(Y) = 1 \Rightarrow E(XY) = 1$

$E(X^2) = E^2(X) + D(X) = 12 \quad E(Y^2) = E^2(Y) + D(Y) = 3$

$E(Z^2) = E(4X^2 + 9Y^2 + 12XY) = 4E(X^2) + 9E(Y^2) + 12E(XY)$

$= 4\times12 + 9\times3 + 12\times1 = 87$

$D(Z) = E(Z^2) - E^2(Z) = 51$

$E(ZX) = E(2X^2+3XY) = 2E(X^2) + 3E(XY)$

$= 2\times 12 + 3\times 1 = 27$

$cov(X,Z) = E(XZ) - E(X)E(Z) = 27 - 3\times 6 = 9$

$\Rightarrow \rho_{XZ} = cov(XZ) / \sqrt{D(X)}\cdot\sqrt{D(Z)} = 9/\sqrt{3}\cdot\sqrt{51} = \frac{3\sqrt{3}}{\sqrt{51}} = \frac{3}{\sqrt{17}}$

故 $\rho_{XZ} = \frac{3}{\sqrt{17}}$

解45：$E(X) = 20$，$D(X) = 4$ $E(X^2) = E^2(X) + D(X) = 404$

$E(Y) = \int_0^{+\infty} 2ye^{-2y}dy = \int_0^{+\infty} te^{-t}d\frac{t}{2} = \frac{1}{2}\int_0^{+\infty} te^{-t}dt = \frac{1}{2}$

$E(Y^2) = \int_0^{+\infty} 2y^2e^{-2y}dy = \int_0^{+\infty} t\frac{t}{2}e^{-t}d\frac{t}{2} = \frac{1}{4}\int_0^{+\infty} t^2e^{-t}dt = \frac{1}{2}$

$D(Y) = E(Y^2) - E^2(Y) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$\rho_{XY} = cov(XY) / \sqrt{D(X)}\sqrt{D(Y)} = cov(X,Y)/2\cdot\frac{1}{2} = cov(X,Y) = \frac{1}{8}$

$cov(X,Y) = E(XY) - E(X)E(Y) \Rightarrow E(XY) = \frac{81}{8}$

$E(X-Y) = E(X) - E(Y) = \frac{39}{2}$

$E[(X-Y)^2] = E(X^2 - 2XY + Y^2) = E(X^2) - 2E(XY) + E(Y^2)$

$= 404 - \frac{81}{4} + \frac{1}{2} = \frac{1537}{4}$

$D(X-Y) = E[(X-Y)^2] - E(X-Y) = \frac{1537}{4} - \frac{1521}{4} = \frac{16}{4} = 4.$

故 $D(X-Y) = 4$

解42：$E(X) = 0.5$ $D(X) = \frac{1}{12}$ $E(X^2) = D(X) + E^2(X) = \frac{1}{3} = E(Y)$

$E(Y^2) = E(X^4) = \int_0^1 x^4 dx = \frac{1}{5}$

$D(Y) = E(Y^2) - E^2(Y) = \frac{1}{5} - \frac{1}{9} = \frac{4}{45}$

$E(XY) = E(X^3) = \int_0^1 x^3 dx = \frac{1}{4}$

$cov(X,Y) = E(XY) - E(X)E(Y) = \frac{1}{4} - \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{12}$

$\rho_{XY} = cov(X,Y) / \sqrt{D(X)}\cdot\sqrt{D(Y)} = \frac{1}{12} / \sqrt{\frac{1}{12}}\cdot\sqrt{\frac{4}{45}} = \frac{3\sqrt{5}}{2\sqrt{12}} = \frac{\sqrt{15}}{4}$

故 $\rho_{XY} = \frac{\sqrt{15}}{4}$