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解6：设A为恰好选出1位女生和2位男生.

$P(A)= C_{10}^{1}C_{20}^{2}/C_{30}^{3} = \frac{95}{203} = 0.468$

故恰好选出1位女生和2位男生的概率为0.468

解7：(1)设 A_i 表示第 i 次取得白球.

$P(A_1A_2 \cup \bar{A_1}\bar{A_2}) = P(A_1A_2)+P(\bar{A_1}\bar{A_2})$

$= P(A_1)P(A_2|A_1)+P(\bar{A_1})P(\bar{A_2}|\bar{A_1}) = \frac{5}{8}\cdot\frac{4}{7}+\frac{3}{8}\cdot\frac{2}{7} = \frac{13}{28}$

(2) $P(A_1 \cup A_2) = 1-P(\overline{A_1 \cup A_2}) = 1-P(\bar{A_1}\bar{A_2})$

$=1-P(\bar{A_1})P(\bar{A_2}|\bar{A_1}) = 1-\frac{3}{8}\cdot\frac{2}{7} = \frac{25}{28}$

故(1)两球同色概率为 $\frac{13}{28}$ (2)至少有1个白球的概率为 $\frac{25}{28}$

解9：$P(A) = C_{12}^{10}\cdot 10!/12^{10}$　　$P(B) = C_{12}^{2}\cdot(2^{10}-2)/12^{10}$

故 $P(A) = \frac{C_{12}^{10}\,10!}{12^{10}}$　　$P(B) = \frac{C_{12}^{2}(2^{10}-2)}{12^{10}}$

解10 (1)：设 A_i 表示第 i 次抽到1号球，由独立性.

$P(\bar{A_1}\bar{A_2}\cdots\bar{A_r}) = P(\bar{A_1})P(\bar{A_2})\cdots P(\bar{A_r}) = (\frac{n-1}{n})^r$

(2) 设取到1号球为事件A，取到2号球为事件B.

B_i 为第 i 次取到2号球.

$P(AB) = 1-P(\overline{AB}) = 1-P(\bar{A}\cup\bar{B}) = 1-[P(\bar{A})+P(\bar{B})-P(\bar{A}\bar{B})]$

$=1-P(\bar{A_1}\bar{A_2}\cdots\bar{A_r})-P(\bar{B_1}\bar{B_2}\cdots\bar{A_r})+P(\bar{A_1}\bar{B_1}\,\bar{A_2}\bar{B_2}\cdots\bar{A_r}\bar{B_r})$

$=1-(\frac{n-1}{n})^r-(\frac{n-1}{n})^r+(\frac{n-2}{n})^r = 1-2(\frac{n-1}{n})^r+(\frac{n-2}{n})^r$

故 (1) 1号球取不到的概率为 $(\frac{n-1}{n})^r$.

(2) 1号球和2号球均被取到的概率为 $1-2(\frac{n-1}{n})^r+(\frac{n-2}{n})^r$

解11：由题知，样本空间共有36个样本点.

设A为方程有实根. 即 $b^2>4c \Rightarrow A=\{(5,1),(6,1),(4,1)$

$(6,2)\ (2,1)\ (4,4)$ $(6,5),(5,6)$

$(3,1)\ (3,2)\ (4,2)\ (5,2)\ (4,3)\ (5,3)\ (6,3)\ (5,4)\ (6,4)\ (5,5),(6,6)$

设B为有方程有重根，即 $b^2=4c \Rightarrow B=\{(2,1),(4,4)\}$

$(\cancel{4,4}) \Rightarrow P(A)=\frac{19}{36}$ $P(B)=\frac{1}{18}$

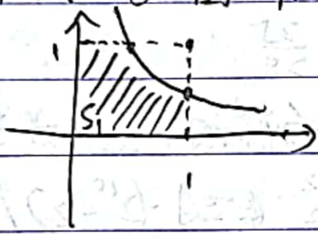
故有实根的概率为 $\frac{19}{36}$，有重根的概率为 $\frac{1}{18}$

解13：设A为至少有一个女生

$$P(A)=1-P(\bar{A})=1-\frac{C_{20}^5}{C_{25}^5}=0.95$$

故至少有1个女生的概率为0.95

解14：如图所示：$xy<0.5$ 即 $y<\frac{0.5}{x}$，区域如图所示



$$S_1=\int_{0.5}^{1}\frac{0.5}{x}dx+\frac{1}{2}\cdot 0.5\cdot 1$$
$$=\frac{1}{2}\ln x\Big|_{\frac{1}{2}}^{1}+\frac{1}{4}=\frac{\ln 2}{2}+\frac{1}{4}$$
$$S=1 \Rightarrow P=\frac{S_1}{S}=\frac{\ln 2}{2}+\frac{1}{4}$$

故两个数乘积小于0.5的概率为 $\frac{\ln 2}{2}+\frac{1}{4}$

解17：上周作业已写。 解23：上周作业已写

解25：$P(AB|\bar{C})=\frac{P(AB\bar{C})}{P(\bar{C})}$ 又A与C互不相容 $\Rightarrow$ AB与C互不相容

$\Rightarrow$ AB发生则 $\bar{C}$ 一定发生 $\Rightarrow P(AB\bar{C})=P(AB) \Rightarrow P(AB|\bar{C})=\frac{P(AB)}{P(\bar{C})}$

$=\frac{P(AB)}{1-P(C)}=\frac{\frac{1}{2}}{\frac{2}{3}}=\frac{3}{4}=0.75$ 故 $P(AB|\bar{C})=0.75$

解26：(1) 设A为一个学生高数不及格，B为一个学生线代不及格.

$$P(B|A)=\frac{P(AB)}{P(A)}=\frac{0.05}{0.15}=\frac{1}{3}$$

故高数不及格，则线代也不及格的概率为 $\frac{1}{3}$

(2) $P(A|B)=\frac{P(AB)}{P(B)}=\frac{0.05}{0.1}=\frac{1}{2}$

故线代不及格，则高数也不及格的概率为 $\frac{1}{2}$

解29 (1) 设A为甲系统有效，B为乙系统有效.

$$P(A\cup B)=1-P(\overline{A\cup B})=1-P(\bar{A}\bar{B})=1-P(\bar{A})P(\bar{B}|\bar{A})$$
$$=1-P(\bar{A})[1-P(B|\bar{A})]=1-[1-P(A)][1-P(B|\bar{A})]$$

$= 1-(1-0.92)(1-0.85) = 0.988$

故至少有1个有效的概率为 0.988

(2) $P(A|\bar{B}) = P(A\bar{B})/P(\bar{B})$

$P(A\bar{B}) = 1-P(\overline{A\bar{B}}) = 1-P(\bar{A}\cup B) = 1-[P(\bar{A})+P(B)-P(\bar{A}B)]$

$= 1-[P(\bar{A})+P(B) - P(\bar{A})\cdot P(B|\bar{A})]$

$= 1-[[1-P(A)]+P(B) - [1-P(A)]P(B|\bar{A})]$

$= 1-[(1-0.92)+0.93 - (1-0.92)\times 0.85] = 0.058$

$\Rightarrow P(A|\bar{B}) = P(A\bar{B})/P(\bar{B}) = P(A\bar{B})/[1-P(B)] = \frac{0.058}{0.07} = 0.829$

故乙失灵条件下甲有效的概率为 0.829.

解30：设A为母亲得病，B为孩子得病，C为父亲得病

$P(B)=0.6$, $P(A|B)=0.5$, $P(C|AB)=0.4$

$P(AB\bar{C}) = P(B)P(A|B)P(\bar{C}|AB)$

$= P(B)P(A|B)[1-P(C|AB)]$

$= 0.6\times 0.5\times(1-0.4) = 0.18$

故母亲及孩子得病但父亲未得病的概率为 0.18