Homework 1

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Problem 1: The convergence of Katz centrality is equivalent to the solvability of the following equation:

$$(\mathbf{I} - \alpha \mathbf{A})\mathbf{c}_{Katz} = \beta \mathbf{I} \ , \tag{1}$$

which is also equivalent to $\det(\mathbf{I} - \alpha \mathbf{A}) \neq 0$. Thus, we have:

$$\frac{1}{\alpha}\mathbf{I} \neq \mathbf{A}\mathbf{I} \ , \tag{2}$$

which means $\frac{1}{\alpha}$ **cannot be** an eigenvalue of the adjacency matrix $\mathbf{A}$.

Problem 2: By definition, the total number of common neighbors $|\mathcal{N}(v_i) \cap \mathcal{N}(v_j)|$ should be exactly equal to the number of walk of length 2, between node v_i and v_j. As a result, we have:

$$|\mathcal{N}(v_i) \cap \mathcal{N}(v_j)| = [\mathbf{A}^2]_{ij} \ , \tag{3}$$

where $\mathbf{A}$ is the adjacency matrix of the current graph.

Problem 3: Via Problem 2 above, the Jaccard's local overlap similarity measure can be rewritten in terms of the adjacency matrix $\mathbf{A}$ as:

$$S_{ij}^{\text{Jaccard}} = \frac{|\mathcal{N}(v_i) \cap \mathcal{N}(v_j)|}{|\mathcal{N}(v_i) \cup \mathcal{N}(v_j)|} \tag{4}$$

$$= \frac{[\mathbf{A}^2]_{ij}}{d_i + d_j - [\mathbf{A}^2]_{ij}} \tag{5}$$

$$= \frac{[\mathbf{A}^2]_{ij}}{\sum_j \mathbf{A}_{ij} + \sum_i \mathbf{A}_{ji} - [\mathbf{A}^2]_{ij}} \tag{6}$$

The result as:

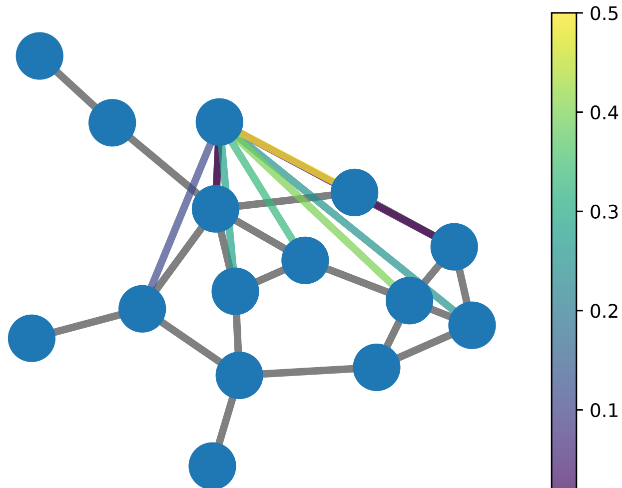

Figure 1: Jaccard similarity score