Comparison of several finite difference schemes applied to the 1-D Euler's equation

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1. Introduction

This project aims to realize the results of a highly-cited paper by Dr. Sod[1], in which the numerical performances of a few famous finite difference schemes utilized in a 1-D shock tube are evaluated. Exact solutions of that 1-D problem are also given so as to comparing with the results computed by different space and time discretizations. At the end of this paper, the Glimm's method is favored in the sense of its high resolution near the discontinuity region generated by shocks.

2. Plan

Part1: The realization based on the results of the paper[1]. Differencing schemes to be used include Godunov's method, two-step Lax-Wendroff method, Macormack's method, Rusanov's method, the upwind scheme, and Glimm's method.

Part2: Comparison of the numerical behaviors of the above methods with the exact solutions in terms of accuracy, stability consistency and efficiency.

Part3: A short literature review of the nonlinear finite difference discretization methods in recent years.

Part4: Based on all above, trying to come up with a new differencing method and evaluating it with the exact solutions.

Part5(If time permits): Extending the best finite difference scheme to a 2-D shock tube problem with the help of the finite volume method and the Riemann solver[2].

3. Reference

[1]: Gary. A. Sod, A survey of several finite difference methods for systems of nonlinear hyperbolic conservation laws, *Journal of Computational physics,* Volume 27, Issue 1, page 1-31, 1978.

[2]: Eleuterio F. Toro, Riemann Solvers and Numerical Methods for Fluid Dynamics-A Practical Introduction, *Springer,* 2009.