Stochastic Methods for Finance: Report 4

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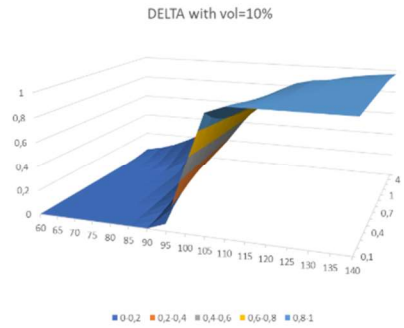
After having provided a VBA script for the Black Scholes model and its Greeks, I've set the following parameters: strike price K=100, risk free rate r=1%.

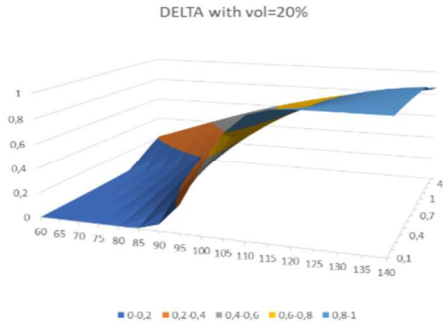
1. Greeks changing time and price

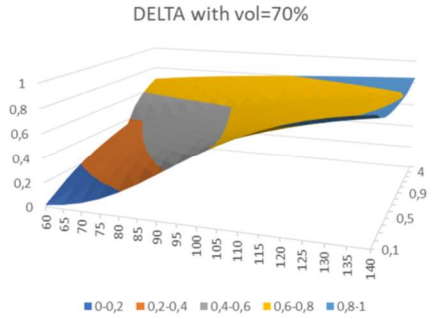
I've established the stock price fluctuating between 60 and 140 in periods of 5, the time of maturity be equal to 0.1,0.2,…,1, 2, 3, 4, 5 years and absence of dividends. The Greeks are the numerical measures for the different dimensions of risk related to own options. In particular: the **delta** is defined as $\frac{\partial V}{\partial S}$ so it's the first derivative of the value V of the option with respect to the underlying instrument's price S; **gamma,** $\frac{\partial^2 V}{\partial S^2}$, determines the rate of changing in relation to changes in the underlying price , this is the second derivative of the value function with regard to the underlying price; **vega** is the derivative of the option value with respect to the volatility of the underlying asset, $\frac{\partial V}{\partial \sigma}$, so it's the sensitivity to volatility; **theta,** $-\frac{\partial V}{\partial \tau}$, is the measure for the sensitivity of the option's value to the passage of time; finally, **rho,** is the derivative of the option value with

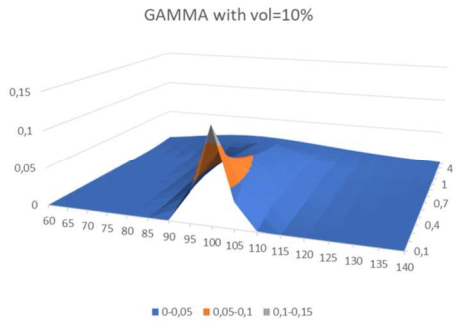
respect to the risk-free interest rate, so it's the sensitivity to the interest rate ($\frac{\partial V}{\partial r}$).

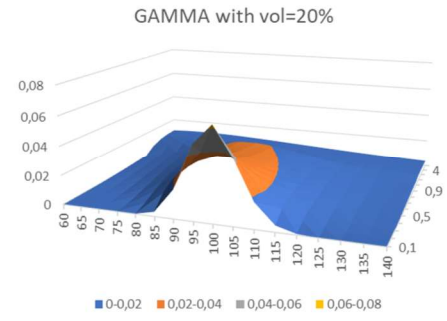
I've so implemented the VBA and computed the Greeks, for volatility equal to 20%, 10% and 70% and these are the graphical results:

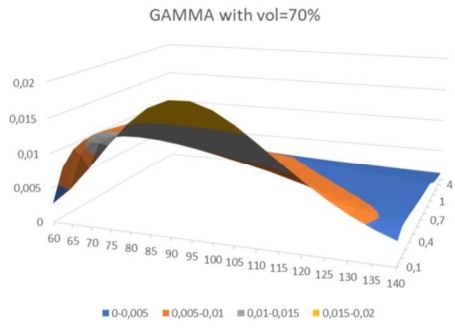


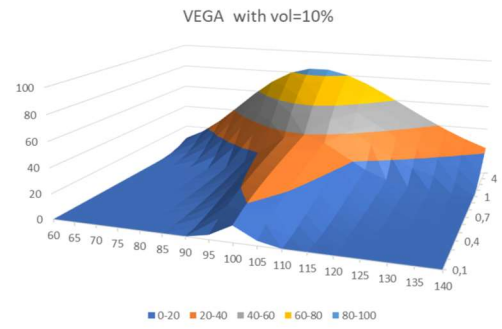


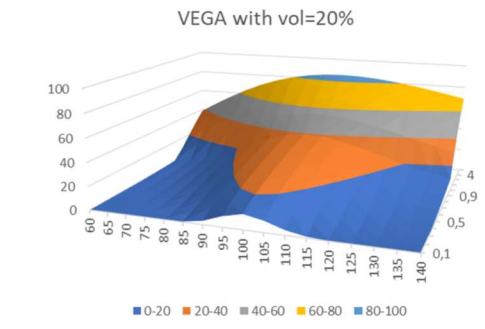


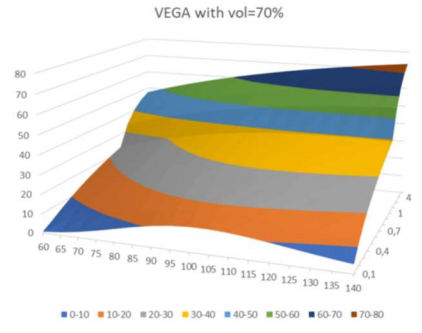


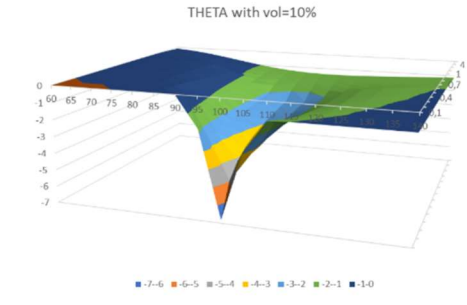


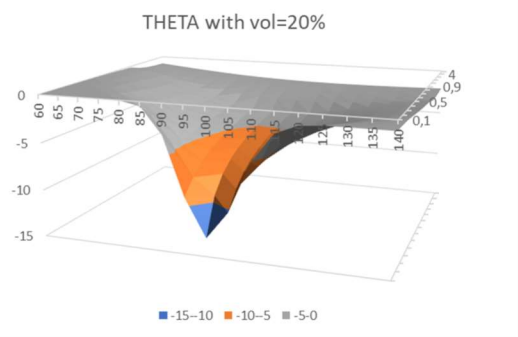


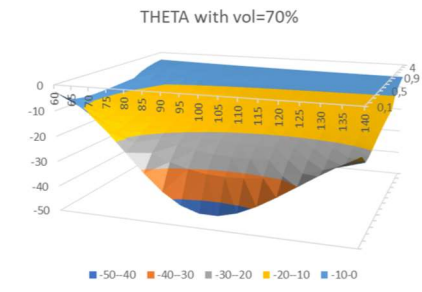


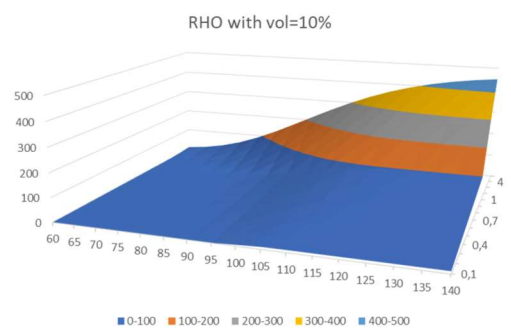


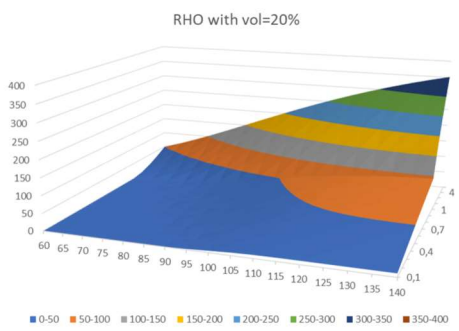


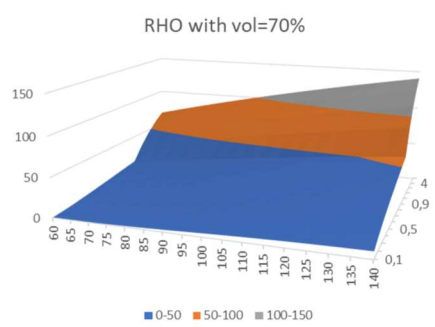












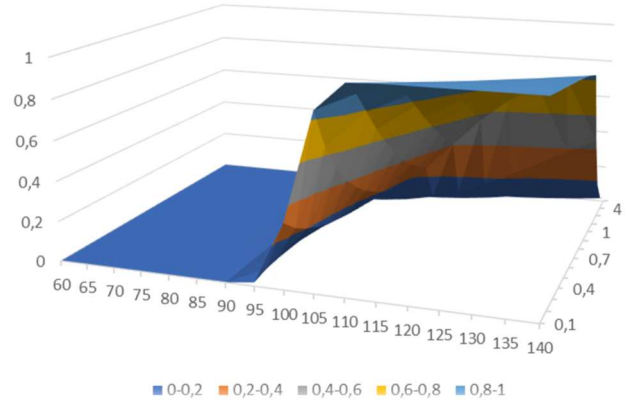
It's clear to see how the delta increases proportionally to the volatility because increases the probability of a large price movement. On the contrary, when the volatility increases the values of Gamma are reduced, because of their inverse proportionality. Theta increases when the volatility becomes higher. This is because higher volatility leads to larger potential price movements. Finally, I've noticed that all the graphs tend to become flatter when the time to maturity increases, except the rho's one that becomes higher, as expected.

2. Greeks changing dividend rate (q)

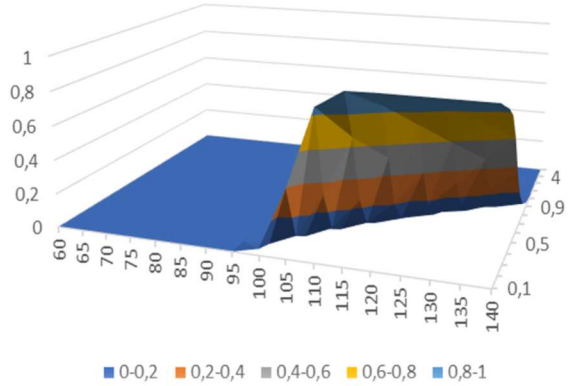
Until this point, I've considered the absence of dividends. Now I'll try, using the same parameters as before, to change the dividend rate also for every change in volatility.

Vol=10%

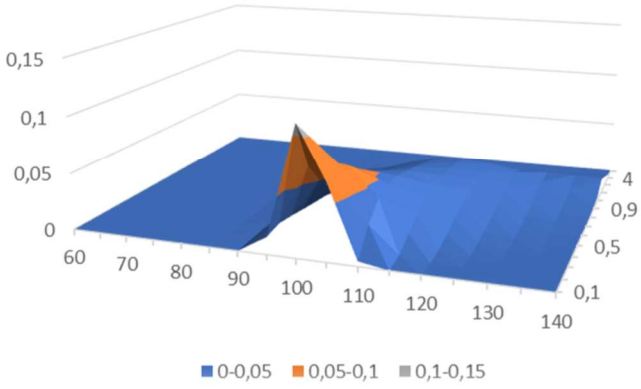
DELTA with q=0,16



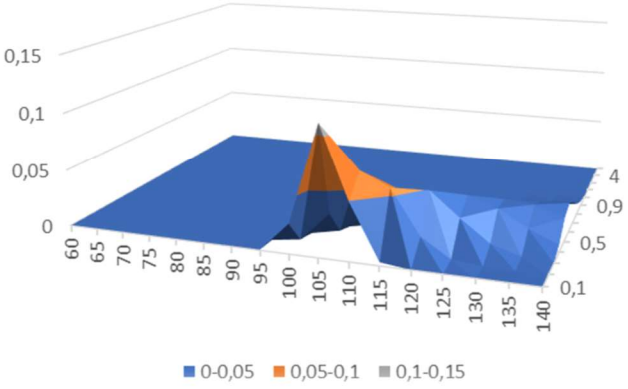
DELTA q=0,57



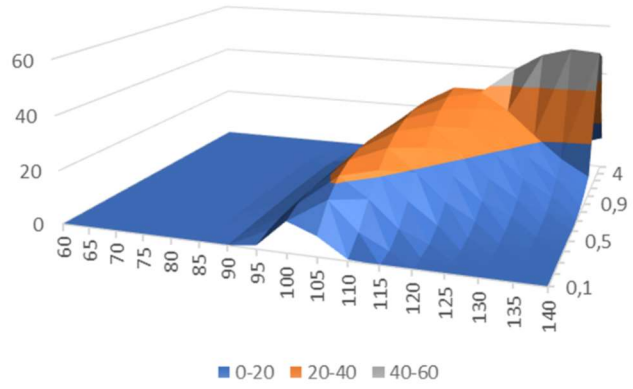
GAMMA with q=0,16



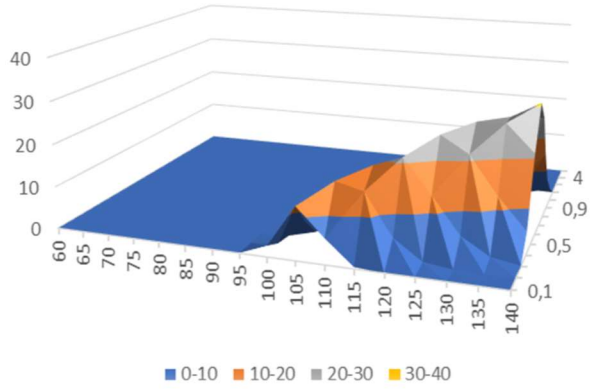
GAMMA with q=0,57

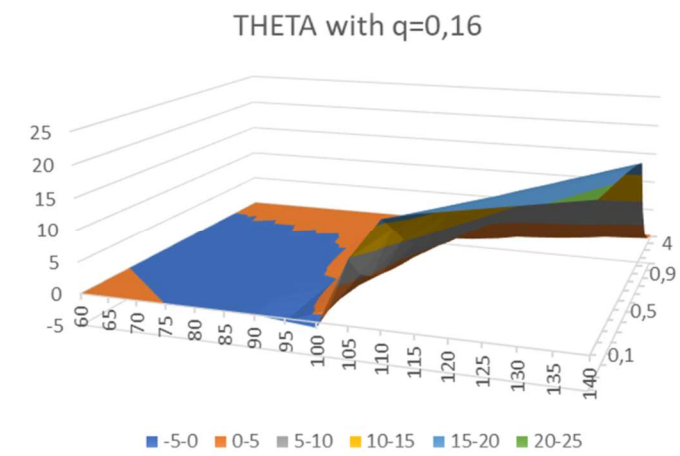


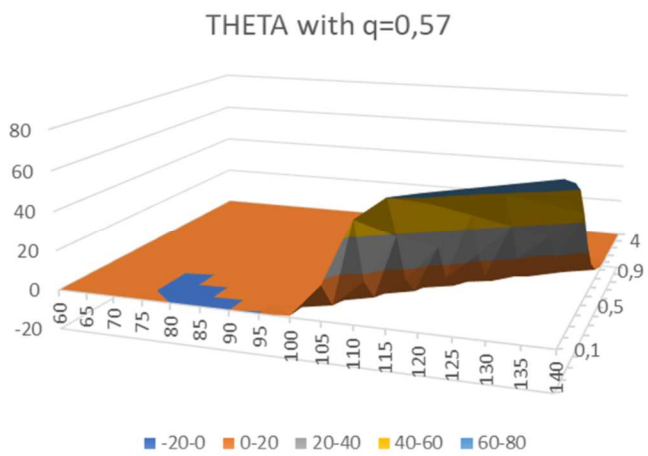
VEGA with q=0,16

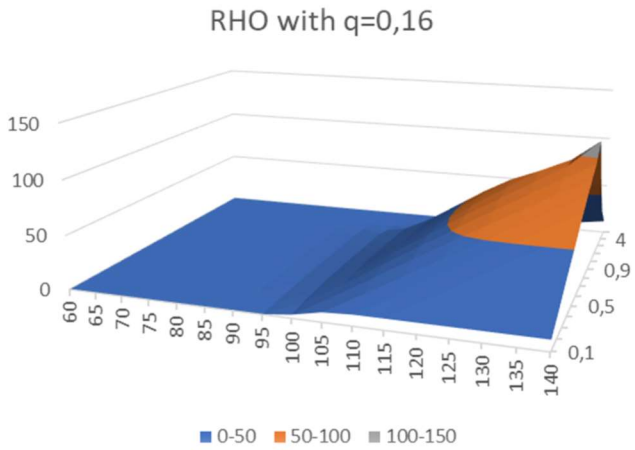


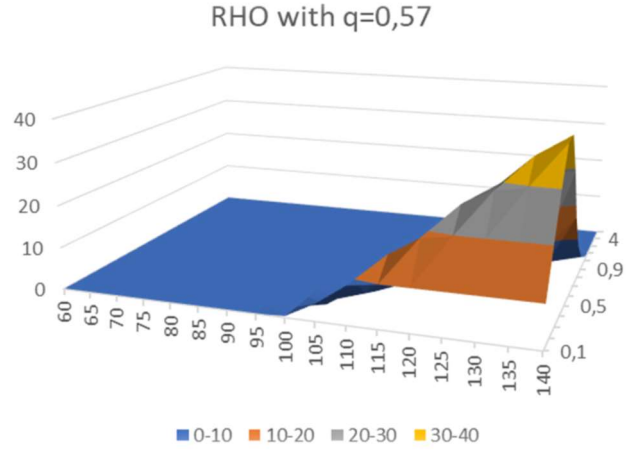
VEGA with q=0,57



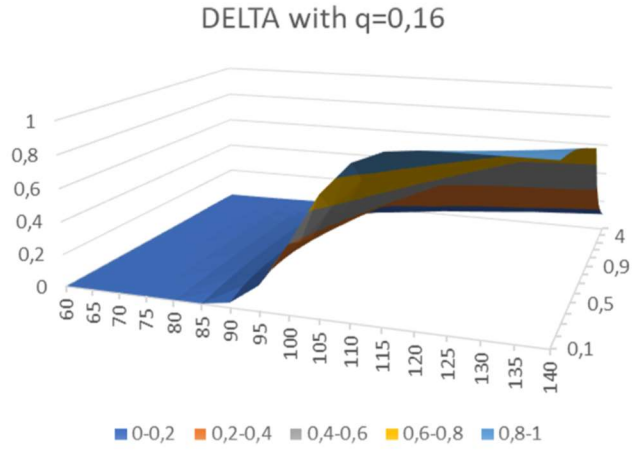

THETA with q=0,16
25
20
15
10
5
0
-5
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
-5-0
0-5
5-10
10-15
15-20
20-25

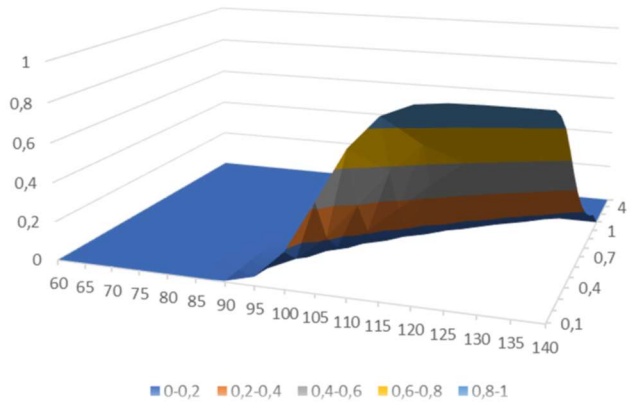

THETA with q=0,57
80
60
40
20
0
-20
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
-20-0
0-20
20-40
40-60
60-80


RHO with q=0,16
150
100
50
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
0-50
50-100
100-150

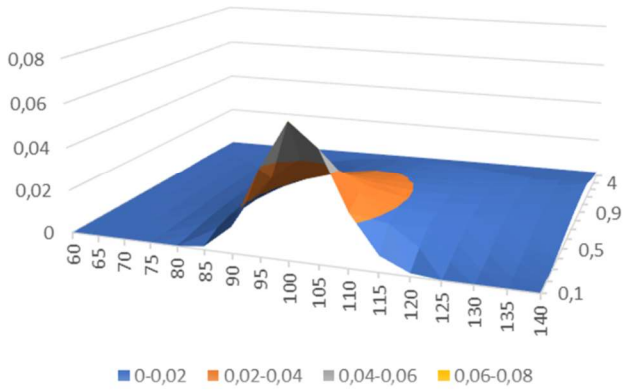

RHO with q=0,57
40
30
20
10
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
0-10
10-20
20-30
30-40

Vol=20%

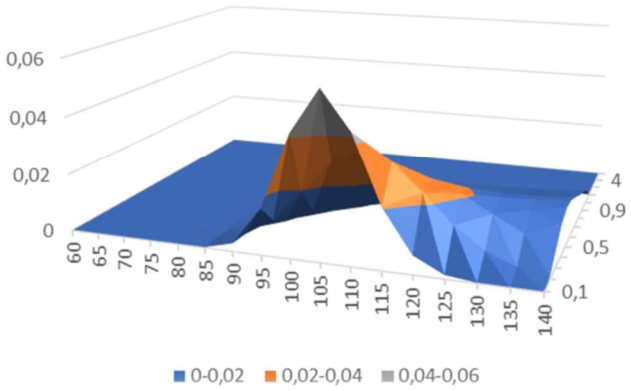

DELTA with q=0,16
1
0,8
0,6
0,4
0,2
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
0,9
0,5
0,1
0-0,2
0,2-0,4
0,4-0,6
0,6-0,8
0,8-1


DELTA with q=0,57
1
0,8
0,6
0,4
0,2
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4
1
0,7
0,4
0,1
0-0,2
0,2-0,4
0,4-0,6
0,6-0,8
0,8-1

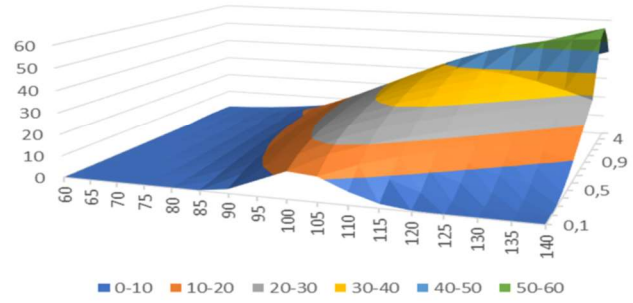
GAMMA with q=0.16



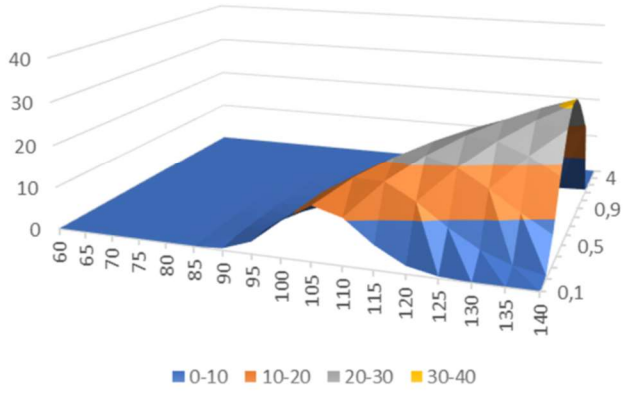
GAMMA with q=0,57



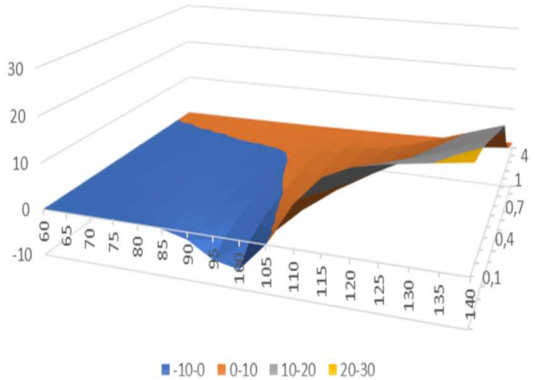
VEGA with q=0,16



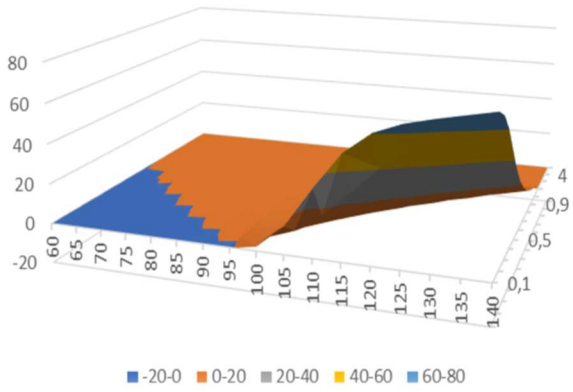
VEGA with q=0,57

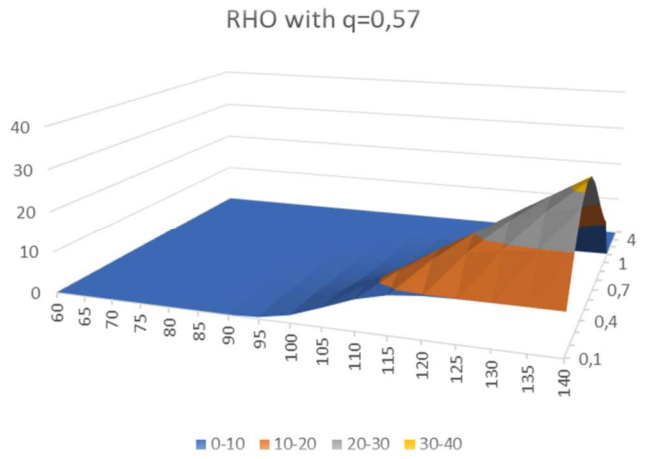


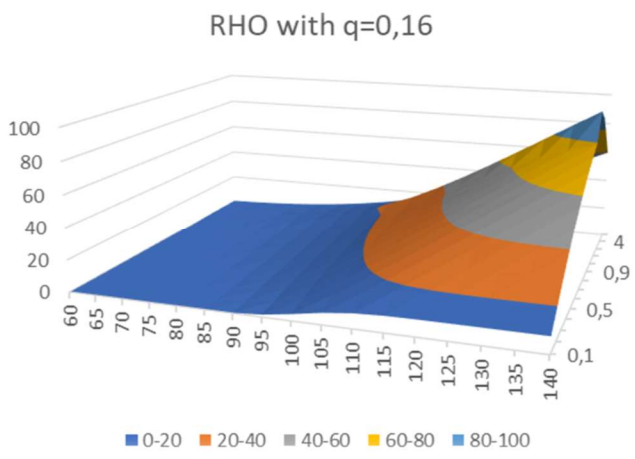
THETA with q=0.16



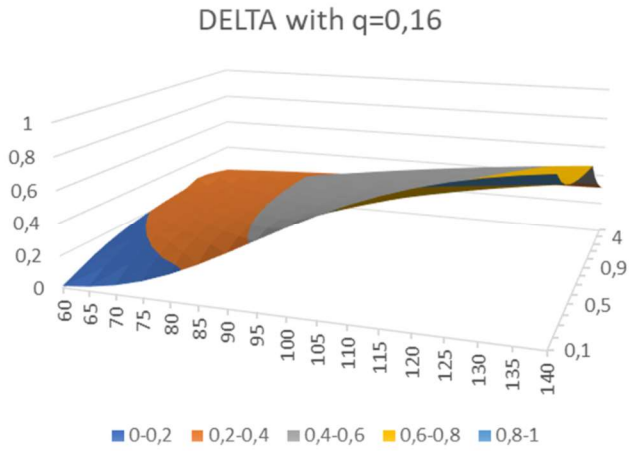
THETA with q=0,57

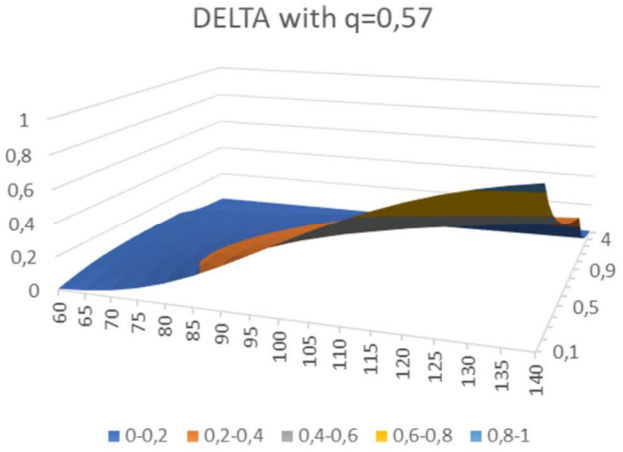


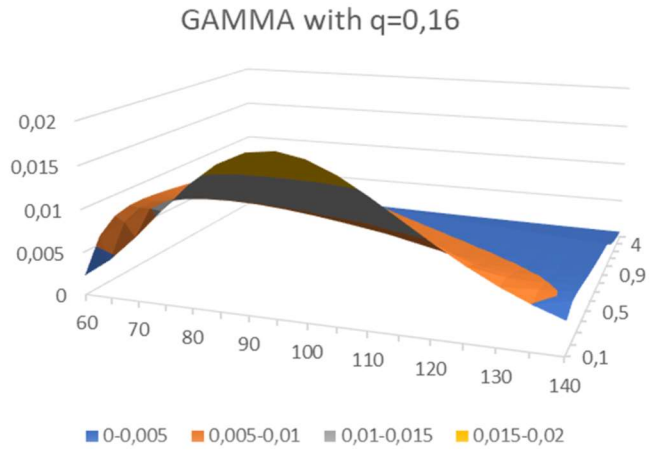

RHO with q=0,57
40
30
20
10
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4 1 0,7 0,4 0,1
0-10 10-20 20-30 30-40

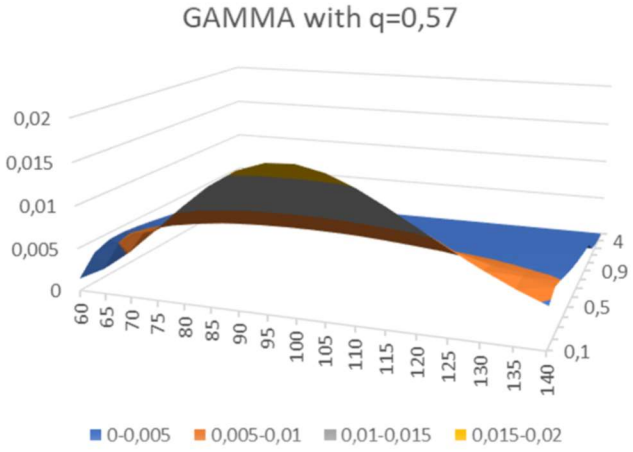

RHO with q=0,16
100
80
60
40
20
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4 0,9 0,5 0,1
0-20 20-40 40-60 60-80 80-100

Vol=70%

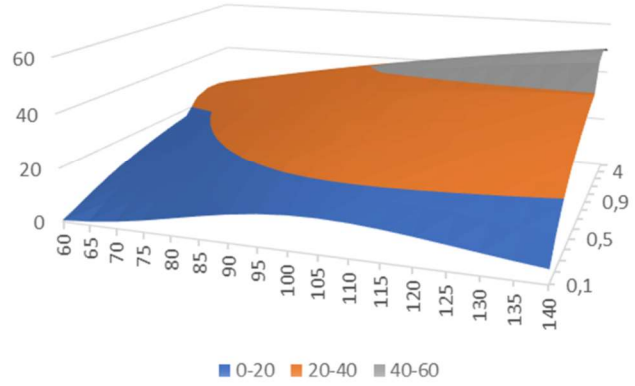

DELTA with q=0,16
1
0,8
0,6
0,4
0,2
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4 0,9 0,5 0,1
0-0,2 0,2-0,4 0,4-0,6 0,6-0,8 0,8-1


DELTA with q=0,57
1
0,8
0,6
0,4
0,2
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4 0,9 0,5 0,1
0-0,2 0,2-0,4 0,4-0,6 0,6-0,8 0,8-1

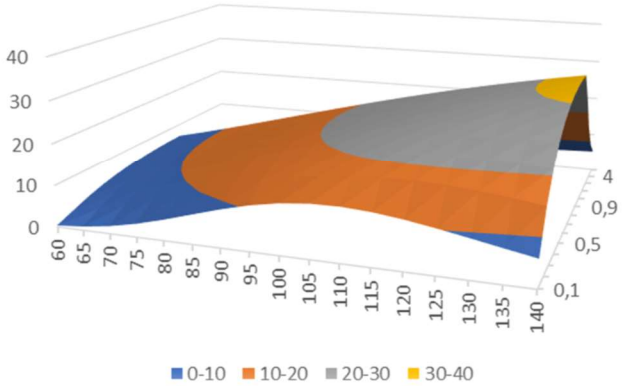

GAMMA with q=0,16
0,02
0,015
0,01
0,005
0
60 70 80 90 100 110 120 130 140
4 0,9 0,5 0,1
0-0,005 0,005-0,01 0,01-0,015 0,015-0,02


GAMMA with q=0,57
0,02
0,015
0,01
0,005
0
60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140
4 0,9 0,5 0,1
0-0,005 0,005-0,01 0,01-0,015 0,015-0,02

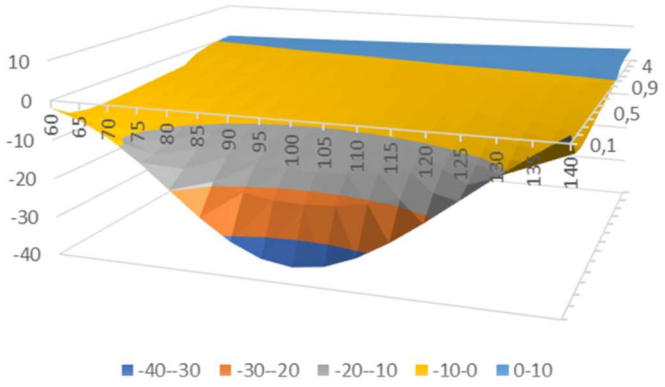
VEGA with q=0,16



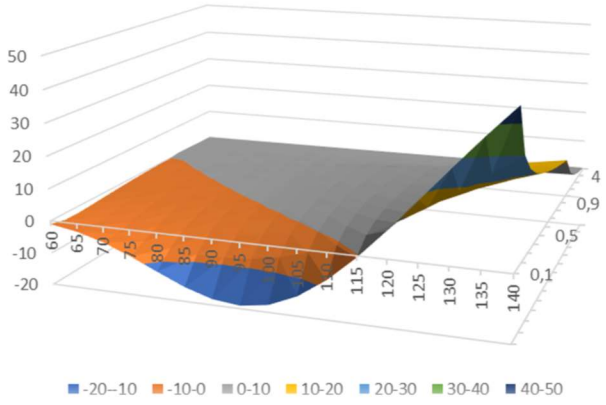
VEGA with q=0,57



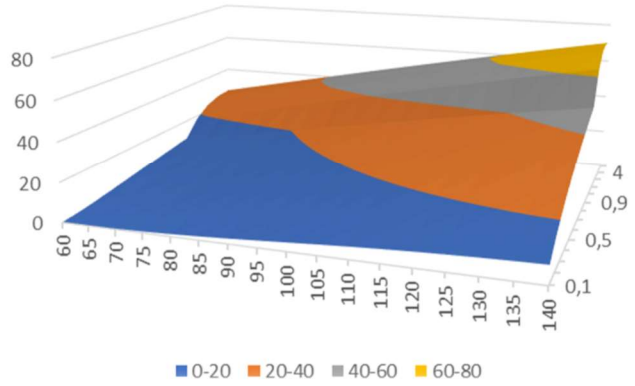
THETA with q=0,16



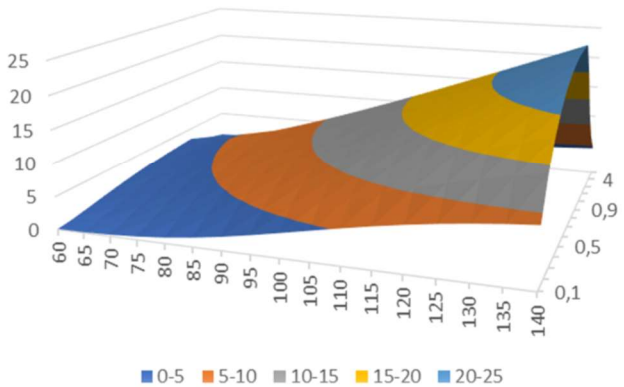
THETA with q=0,57



RHO with q=0,16



RHO with q=0,57



So, in all the three cases, when the dividend rate increases: the Delta of a call option decreases, this means the call option becomes less sensitive to changes in the stock price. As the delta decreases, the Gamma also decreases. Finally, the effect of dividend changes on Rho is usually: higher dividend rate reduces the value of a call option and the change in Rho will be negative, and vice versa. In summary, when the dividend rate increases, the Greeks of the options alter. Traders must be aware of these changes and modify their trading tactics to consider the altered risk and reward.