Stochastic Methods for Finance: Report 6

Graziana Capurso

12 May 2023

I've fixed some parameters of Strike price, Stock price, volatility, risk-free rate, time to maturity and time step for the discretization for the Black Scholes market model. In particular: **K**=100; **S**=120; volatility **σ**=0,3(30%); **r**= 0.05 (5%); **T**= 1 year; **dt**=30 days.

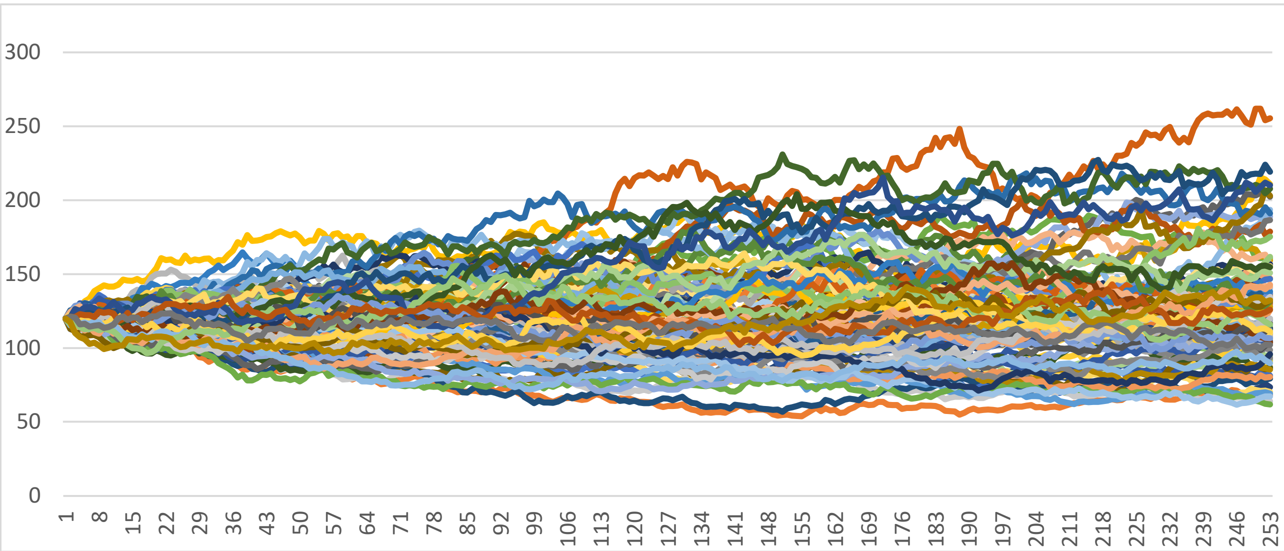
1. Geometric Brownian Motion

The Geometric Brownian Motion (GBM) is described by the stochastic differential equation:

$$dS = \mu S\,dt + \sigma S\,dW$$

Where S is the stock price, μ is the expected return or drift, σ is the volatility of the stock price, and dW is the increment of a standard Brownian motion or Wiener process. So, the process experiences random fluctuations in its value over time that are proportional to its current value.

For N=100 trajectories we have:



2. Vanilla Options prices at Monte Carlo

A Vanilla option grants the holder the right, but not the obligation, to buy the underlying asset a on or before the expiration date at a predefined price, known as the strike price.

Using the Monte Carlo method, as VBA, I've computed the price for call and put options, having **price(call)** = 28,47 and **price(put)** = 4,03. I've compared these with the one by the Black Scholes formula, resulting **price(call)** = 28,88 and **price(put)** = 4. They're very similar.

3. Vanilla Options prices at Monte Carlo with Euler method

The Euler method is a numerical approach for approximating differential equation solutions. The process entails approximating the solution at a series of discrete points and calculating the value at each subsequent location using the derivative function and values from the preceding ones. If the step size is too big or the underlying function is nonlinear, it may result in severe inaccuracies. So, in this case, I've found the stock price for 520 simulations, and I've computed the call price and the put one, using first the payoff formulas:

$$\boldsymbol{payoffT(call)} = (S - K)^+$$

$$\boldsymbol{payoffT(put)} = (K - S)^+$$

And then, I've computed the average value of these, respectively for the call and the put (payoff(call) = 30,99 and payoff(put) = 4,58).

Finally, to find the prices, I've computed the **discount factor** as : e^{-r*T} , that was 0,95. Multiplying this with the two previous average terms, I've obtained the Call and Put prices, given by the multistep Euler Scheme, also in this case, they're so similar to the ones given by the Black Scholes formula.

4. Asian option price using Monte Carlo approach

Asian options are based on the underlying asset's average price during a predetermined time frame, usually a few days to a few weeks. Using the same computation as the one for the Euler Method, I've found the **call price** = 23,54 and the **put price** = 0,85. Comparing these with the real values, Monte Carlo pricing, respectively equal to 23 and 0,98, has been clear to see how, also in this case, the prices are so close.

5. Lookback option price using Monte Carlo approach

A lookback option allows the investor to exercise the option by considering the historical price movements of the underlying asset, enabling them to secure profits or reduce losses. Investors find this option attractive as it allows them to reap the benefits of the most advantageous asset price. Nevertheless, lookback options are more intricate than conventional options. Using again the same calculations, I've found the **call price** = 25,35 and the **put price** = 24,09. Finally, I've compared these with the real values, Monte Carlo pricing, respectively equal to 24,56 and 22,45. In this case they're similar, but not so close to the previous one.

6. Conclusions

For a few reasons, the call and put option prices determined by the Black-Scholes formula and the Monte Carlo method are frequently very similar.

Similar presumptions are made by both techniques regarding how underlying asset prices will behave. They assume that the risk-free rate and volatility will remain constant throughout the life of the option and that asset prices will follow a log-normal distribution.

Both the Euler method and the Black-Scholes formula use the same underlying assumptions about the behavior of the underlying asset, namely that it follows a log-normal distribution and that the options are European-style (can only be exercised on the expiration date), so they arrive at similar conclusions about the prices of call and put options. The same factors, including the underlying asset price, strike price, time until expiration, volatility, and risk-free interest rate, are also considered by both techniques.