Stochastic Methods for Finance: Report 3

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I've set the parameters S=133, K=133, r=0.03, vol=20% and T=1. This with the aim of creating a formula for the call option by the Binomial model and compare it with the Black Scholes one and see the convergence between the two.

1) BINOMIAL MODEL

$$C_{\boldsymbol{bin}} = \frac{1}{(1+r)^{n}} \sum_{j=0}^{n} \binom{j}{n} q^{j} (1-q)^{n-j} \left(S_0 u^{j} d^{n-j} - K\right)^{+}$$

This is the formula pricing the call option in a binomial model. Where **S** denotes the starting stock price, **K** denotes the strike price, **r** denotes the risk-free rate, **q** denotes the market probability measure, **u** and **d** denote the movements of the asset, and **n** is the number of iterations. Starting from that, I've implemented a VBA code also using the measures of maturity (**T**) and volatility ($\boldsymbol{\sigma}$).

2) BLACK SCHOLES FORMULA

In this case the formula is

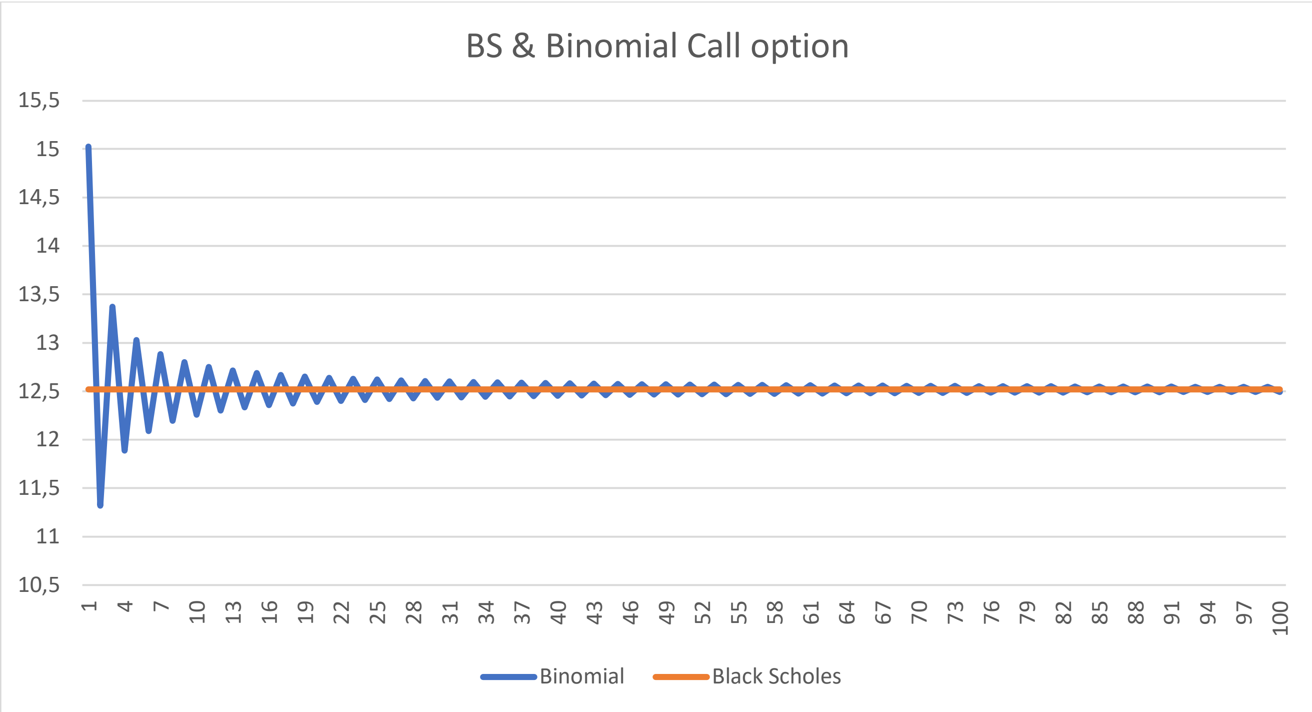
$$d_1 = \frac{\ln\left[\frac{S_t}{K}\right] + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

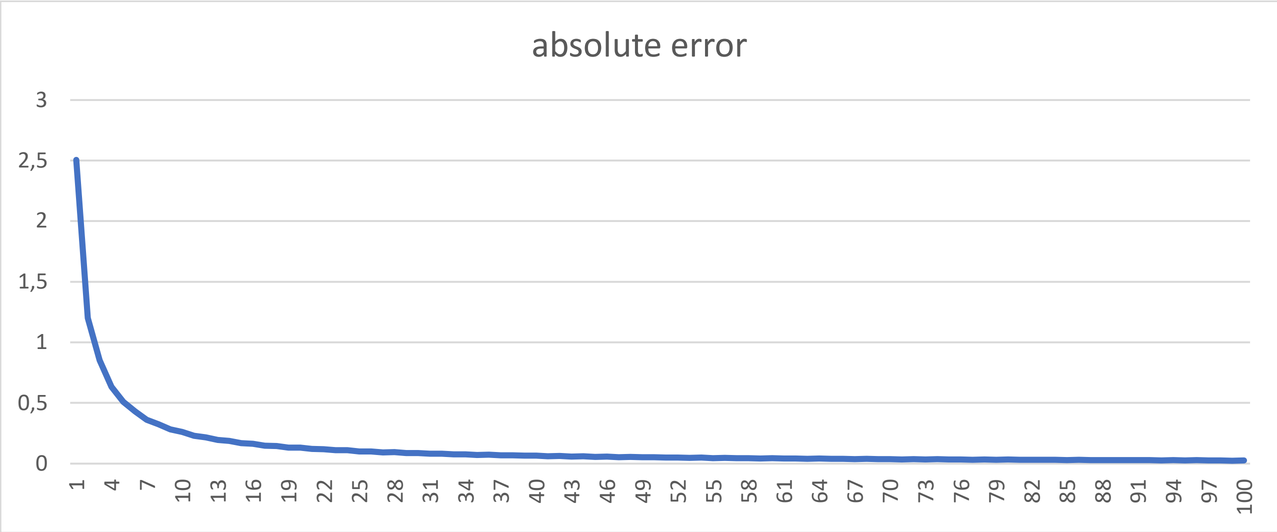
$$C_{BS} = SN(d_1) - Ke^{-rT}N(d_2)$$

Where the cumulative normal distribution is denoted by N. Using the same parameters, C_{BS} is equal to 12,52.

3) COMPARISON

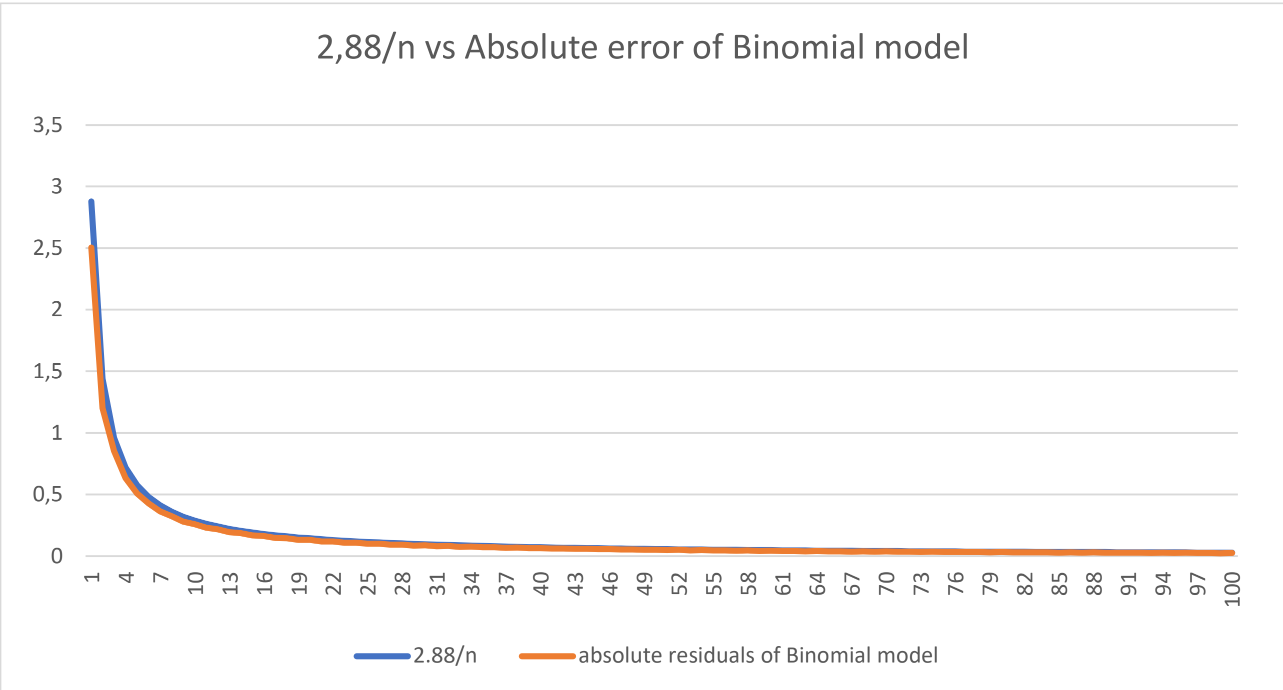


In this graph It's possible see the convergence of the two on 100 iterations. The graph of the absolute error, between the two formulations, shows how the Binomial model's residuals have a linear rate of convergence.



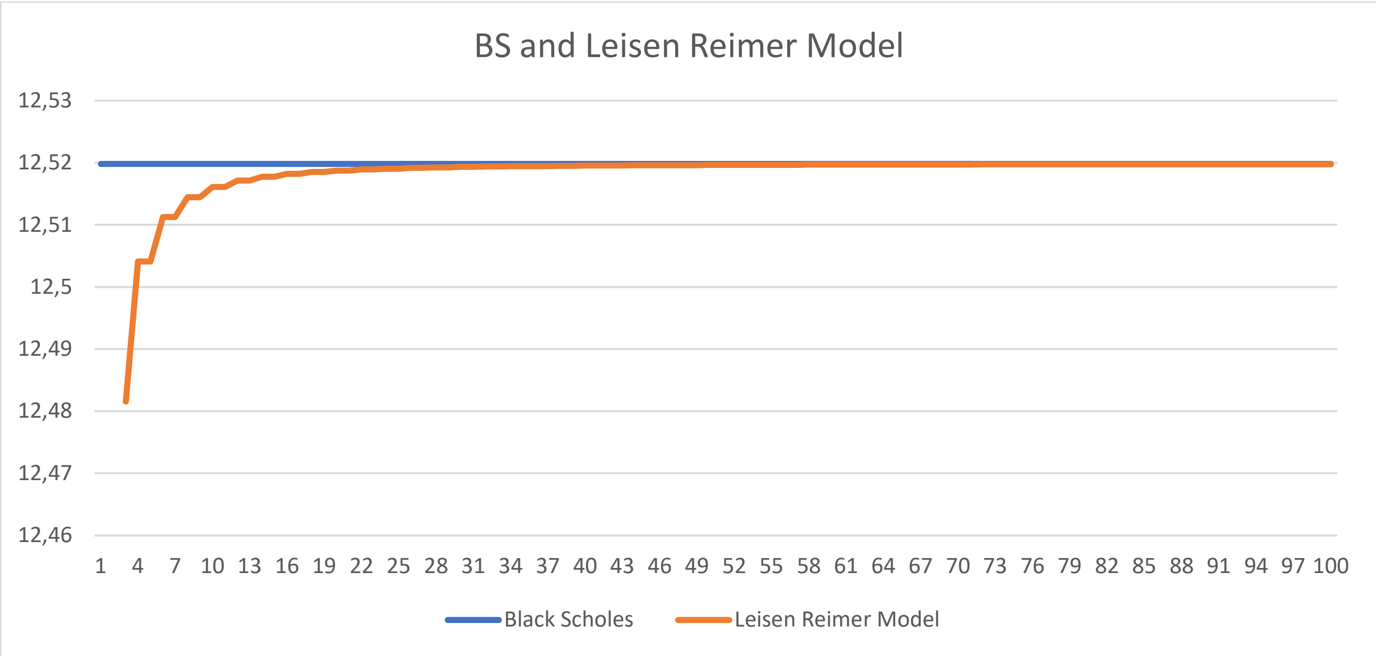
It tends to decrease as the iteration number increases.

I've compared the graph above with the course of 2.88/n, the result is a high correspondence.



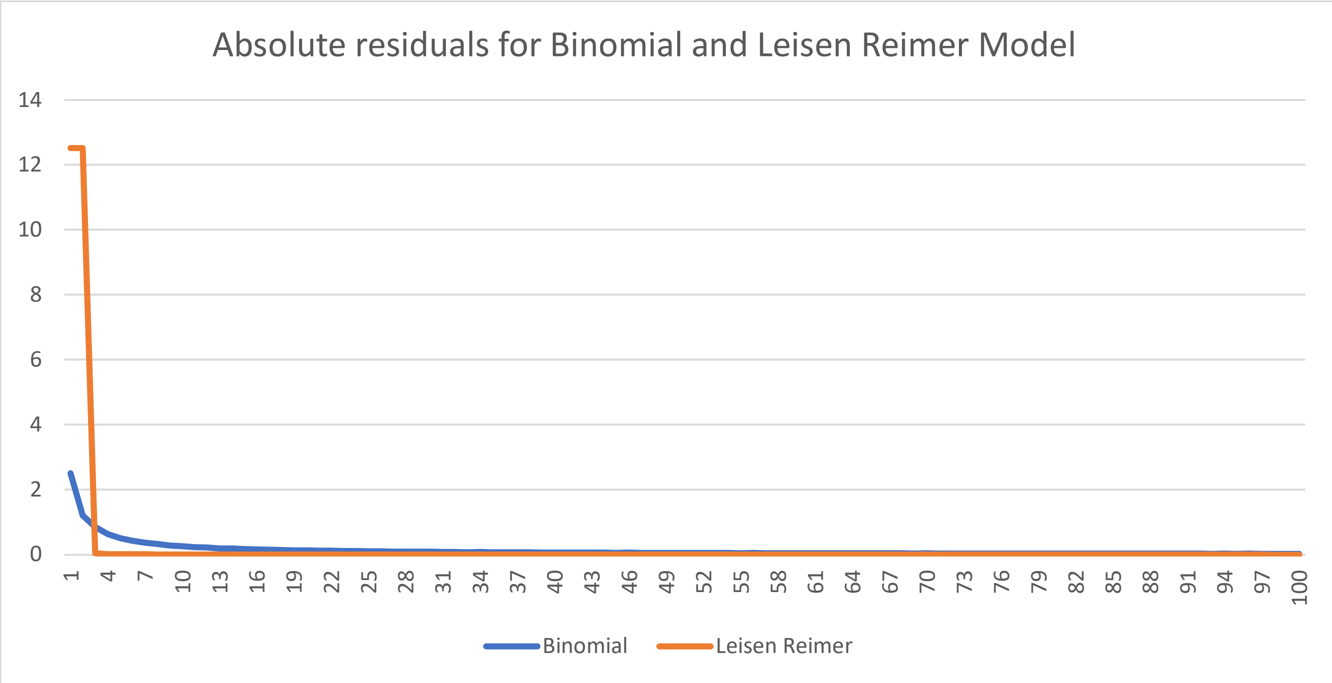
4) LEISEN REIMER MODEL

With the Leisen-Reimer approach is possible to improve the velocity of the convergence, in fact Leisen-Reimer allows the binomial tree's parameters u, d, and q to be changed to optimize the convergence behavior. Leisen-Reimer recommends using the inversion formula instead of the conventional approach, and they employ normal approximations to find the binomial distribution B(n,p). They specifically recommend the Camp-Paulson-Inversion Formula and the Peizer-Pratt Inversion Formulas 1 and 2. I'd cut the zero region of the Leisen-Reimer tree using a VBA implementation that was optimized.



It is evident how the convergence has changed.

5)CONCLUSION



The graph above explains the difference between the two models in terms of error from the Black Scholes formula.