Discrete Distributions

Uniform Distribution

$$f(k) = \frac{1}{b-a+1}.$$

$$M(t) = \frac{1}{b-a+1}\frac{e^{at}(1-e^{b-a+1})}{1-e^t}.$$

$$\mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a+1)^2-1}{12}.$$

Geometric Distribtuion

$$f(k) = (1-p)^{k-1}p.$$

$$M(t) = \frac{pe^t}{1-(1-p)e^t}.$$

$$\mu = \frac{1}{p}, \quad \sigma^2 = \frac{1-p}{p^2}.$$

Hypergeometric Distribution

$$f(k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}.$$

where N is the population size, K is the number of success states, n is the number of draws, k is the number of observed successes.

$$\mu = n\frac{K}{N}, \quad \sigma^2 = n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1}.$$

Binomial Distribution

$$f(k) = \binom{n}{k}p^k(1-p)^{n-k}.$$

$$\mu = np, \quad \sigma^2 = np(1-p).$$

Negative Binomial Distribution

$$f(k) = \binom{k-1}{r-1}p^n(1-p)^{k-r}.$$

When $n=1$, it is a geometric distribution.

$$M(t) = \left(\frac{pe^t}{1-(1-p)e^t}\right)^r.$$

$$M'(t) = r(pe^t)^r[1-(1-p)e^t]^{-r-1}.$$

$$\mu = \frac{r}{p}, \quad \sigma^2 = \frac{r(1-p)}{p^2}.$$

Poisson Distribution

$$f(k) = \frac{\lambda^x e^{-\lambda}}{x!}.$$

$$M(t) = e^{\lambda(e^t-1)}.$$

$$M'(t) = \lambda e^t e^{\lambda(e^t-1)}.$$

$$M''(t) = (\lambda e^t)^2 e^{\lambda(e^t-1)} + \lambda e^t e^{\lambda(e^t-1)}.$$

$$\mu = \lambda, \quad \sigma^2 = \lambda.$$

Continuous Distributions

Uniform Distribution

$$f(x) = \frac{1}{b-a}, \quad F(x) = \frac{x-a}{b-a}.$$

$$M(t) = \frac{e^{tb}-e^{ta}}{t(b-a)}.$$

$$M'(t) = \frac{(bt-1)e^{bt}-(at-1)e^{at}}{t^2(b-a)}.$$

$$M''(t) = \frac{[(bt-1)^2-1]e^{bt}-[(at-1)^2-1]e^{at}}{t^3(b-a)}.$$

$$\mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{(b-a)^2}{12}.$$

Expotential Distribution

$$\theta = \frac{1}{\lambda}.$$

where λ is the mean number of occurrences in the unit time interval so θ is the meaning waiting time for the first occurrence.

$$f(x) = \frac{1}{\theta}e^{-x/\theta}, \quad F(x) = 1-e^{-x/\theta}.$$

$$M(t) = \frac{1}{1-\theta t}.$$

$$M'(t) = \frac{\theta}{(1-\theta t)^2}, \quad M''(t) = \frac{2\theta^2}{(1-\theta t)^3}.$$

$$\mu = \theta, \quad \sigma^2 = \theta^2.$$

The forgetfulness property

$$P(X>b|X>a) = P(X>b-a).$$

Gamma Distribution

α is the number of the total occurrences.

$$\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}\,\mathrm{d}x, \quad \Gamma(n) = (n-1)!.$$

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha}x^{\alpha-1}e^{-x/\theta}.$$

$$M(t) = \frac{1}{(1-\theta t)^\alpha}.$$

$$\mu = \alpha\theta, \quad \sigma^2 = \alpha\theta^2.$$

Chi-Square Distribution

A special case of the gamma distribution that

$$\theta = 2, \quad \alpha = \frac{r}{2}.$$

r is the number of degrees of freedom.

$$\mu = r, \quad \sigma^2 = 2r.$$

NormalDistribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp[-\frac{(x-\mu)^2}{2\sigma^2}].$$

$$M(t) = \exp\left(\mu t + \frac{\sigma^2t^2}{2}\right).$$

$$M'(t) = (\mu+\sigma^2 t)\exp\left(\mu t + \frac{\sigma^2t^2}{2}\right).$$

$$M''(t) = [(\mu+\sigma^2t)^2+\sigma^2]\exp\left(\mu t + \frac{\sigma^2t^2}{2}\right).$$

If X is $N(\mu,\sigma^2)$, then $Z=(X-\mu)/\sigma$ is $N(0,1)$. If X is $N(\mu,\sigma^2)$, then $V=(X-\mu)^2/\sigma^2 = Z^2$ is $\chi^2(1)$.