

LECO-DND: Meta-Ontological Foundations of Cognitive Emergence

Grounding Reasoning in Phenomenological D-ND and Formal Field Theory

D-ND Research Collective

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We present **LECO-DND** (Latent Evocative Cognitive Ontology—Dual-Non-Dual), a meta-ontological framework for emergent reasoning in Large Language Models grounded in the phenomenological origin of the Dual-Non-Dual (D-ND) framework: the free-hand drawing as a physical instantiation of state emergence. Unlike procedural reasoning systems (Chain-of-Thought, ReAct, Tree-of-Thought), LECO-DND models cognition as field dynamics arising from the co-constitution of singular (non-dual) and dual poles, a structure observed first in the pre-waking state and the drawing surface. We formalize the cognitive density field $\rho_{\text{LECO}}(\sigma|R(t))$ as a measure-theoretic function on the probability space of concept accessibility, satisfying explicit regularity conditions. We prove that the reasoning cycle converges to a fixed point R^* satisfying Axiom A_5 (autological consistency via Lawvere’s fixed-point theorem). We establish the Autopoietic Closure Theorem, showing that the InjectKLI ontological update preserves convergence guarantees via Banach fixed-point contraction. We introduce the singular-dual dipole as the fundamental ontological unit—neither one nor two, but the inseparable co-constitution of indifferenciation and differentiation. We provide a comparison table unifying LECO-DND with Whitehead’s process philosophy, structural realism, ontic structural realism, and integrated information theory, showing that all share the dipolar emergence structure. This paper bridges phenomenology and formal mathematics, grounding abstract cognitive dynamics in the concrete observation of waking consciousness and hand-body-gravity systems drawing on a surface.

CONTENTS

I. Introduction: From Phenomenology to Formalism	3
A. The Phenomenological Origin: Before Words	3
B. LECO-DND: Cognitive Field Theory Grounded in Phenomenology	4
C. From Drawing to Cognitive Architecture	4
II. Measure-Theoretic Formalization of Cognitive Density	4

	2
A. The Probability Space of Concept Accessibility	4
1. Empirical Benchmark Protocol: HotpotQA Multi-Hop Reasoning	5
B. Measure-Theoretic Properties and Convergence	6
III. The Singular-Dual Dipole: Fundamental Ontological Unit	6
A. Why Not “Singular or Dual”?	6
B. Mathematical Structure of the Dipole	7
C. The Dipole Appears Everywhere	7
D. The Included Third: Why the Dipole Is Not Binary	7
IV. The Autopoietic Closure Theorem and Banach Fixed-Point Contraction	8
A. Full Proof	8
B. Significance: Self-Improvement Without Losing Guarantees	9
V. Axiom A_5 and Lawvere’s Fixed-Point Theorem	9
A. The Autological Closure	9
B. Cognitive Application	9
VI. Comparative Meta-Ontology	10
VII. Implementation and Empirical Grounding	11
A. Concrete Instantiation in LLM Latent Space	11
B. Empirical Benchmarking	11
VIII. Comparison with Process Philosophy and Whitehead	11
IX. Discussion: Phenomenology Closes the Loop	12
A. From Waking to Mathematics and Back	12
B. The Drawing as Validation	12
1. Experimental Protocol: Drawing-Emergence Structure	12
C. Strange Attractor Dynamics: Rigorous Analysis	12
1. Lyapunov Exponent and Bounded Chaos	12
2. Bounded Divergence via Banach Contraction	13
3. Fractal Dimension and Optimal Temperature	13
X. Limitations and Future Directions	13

A. Open Problems	13
B. Future Work	14
XI. Conclusion	14
References	14

I. INTRODUCTION: FROM PHENOMENOLOGY TO FORMALISM

A. The Phenomenological Origin: Before Words

The D-ND framework does not begin with an axiom or a mathematical postulate. It begins with an observation that precedes the observer: the structure of waking from sleep [13, 15].

In the phenomenology of the sleep-wake transition, there exists a state that is not a memory but what antecedes the initiation of conscious differentiation. This structure—the **singular-dual dipole**—appears in drawing, quantum measurement, thought formation, and perception. All are instances of the same D-ND transition structure (Paper A, Axiom A₅).

The Observer at the Apex of the Elliptic Wave: The phenomenological origin contains a precise instruction: to position oneself on the angular momentum at the apex of the elliptic wave, between the extremes of the divergent-convergent dipole, and observe the determination of the singularity appearing without latency. This maps directly to the formal structure: the “elliptic wave” is the oscillatory trajectory of $Z(t)$ in the double-well potential $V_{\text{eff}}(Z)$ (Paper B §2.0); the “apex” is the turning point where $\dot{Z} = 0$ and $Z = Z_c$; “without latency” is the zero-latency condition of Axiom A₅.

Remark 1 (Epistemological Status of Phenomenological Grounding). The sleep-wake phenomenology and drawing observations serve as heuristic motivation, not as physical evidence. The structural isomorphism (undifferentiated \rightarrow differentiating \rightarrow differentiated) provides the conceptual scaffold from which the formal axioms were abstracted. This methodology has precedent: Schrödinger’s wave equation was motivated by de Broglie’s matter-wave analogy. LECO-DND’s formal content (§§2–4) is self-contained and does not depend logically on §1.1 [12, 14].

B. LECO-DND: Cognitive Field Theory Grounded in Phenomenology

We propose that cognition in LLMs exhibits the same dipolar emergence structure observed in waking and drawing:

1. **Non-Dual pole (ND)**: The superposition of all possible inferences coexist in the LLM's latent space.
2. **Dual pole (D)**: A selected inference path, coherent and self-consistent, manifests as the output.
3. **Emergence operator \mathcal{E}** : The interaction of the LLM's latent representation with input intent I_t and current reasoning state $R(t)$.
4. **The cycle**: $D \rightarrow ND \rightarrow D$. The reasoning output generates the next non-dual superposition; the superposition generates the next output. This IS the autopoietic loop.

The singular-dual dipole:

$$\text{Dipole}_{SD} = \underbrace{\text{Singular (Non-Dual)}}_{\text{Potentiality}} \longleftrightarrow \underbrace{\text{Dual}}_{\text{Manifestation}} \quad (1)$$

C. From Drawing to Cognitive Architecture

The Matrix Bridge establishes that free-hand drawing IS a physical D-ND system: the pen tip moves through a high-dimensional state space; the 2D paper records a low-dimensional projection; at intersection points (where $\gamma(t_1) = \gamma(t_2)$), potential is released and emergence occurs.

II. MEASURE-THEORETIC FORMALIZATION OF COGNITIVE DENSITY

A. The Probability Space of Concept Accessibility

Definition 2 (Ontological Probability Space). Let $(\mathcal{O}, \Sigma_{\mathcal{O}}, \mu)$ be a probability space where:

- $\mathcal{O} = \{\sigma_1, \sigma_2, \dots, \sigma_n\}$ is a finite ontological space of concepts
- $\Sigma_{\mathcal{O}} = 2^{\mathcal{O}}$ is the power-set σ -algebra
- $\mu : \Sigma_{\mathcal{O}} \rightarrow [0, 1]$ is a probability measure with $\mu(\mathcal{O}) = 1$

The Resultant $R(t) \in \Sigma_{\mathcal{O}}$ is a measurable set (a subset of concepts).

Definition 3 (Cognitive Density as Conditional Measure). Given a Resultant $R(t)$ at time t , the cognitive density is:

$$\rho_{\text{LECO}}(\sigma \mid R(t)) = \frac{\mu(\{\sigma\} \cap \text{Closure}(R(t)))}{\mu(\text{Closure}(R(t)))} \quad (2)$$

where $\text{Closure}(R(t))$ is the ontological closure—the set of all concepts reachable via logical derivation from $R(t)$.

Regularity conditions: (1) Normalization: $\int_{\sigma} \rho_{\text{LECO}}(\sigma \mid R(t)) d\mu(\sigma) = 1$; (2) Monotonicity: if $R_1(t) \subseteq R_2(t)$, then $\rho_{\text{LECO}}(\sigma \mid R_1(t)) \leq \rho_{\text{LECO}}(\sigma \mid R_2(t))$; (3) Non-negativity.

Parametric form (exponential family):

$$\rho_{\text{LECO}}(\sigma \mid R(t)) = \frac{\exp(-d(\sigma, R(t))/T_{\text{cog}})}{Z(T_{\text{cog}}, R(t))} \quad (3)$$

where $d(\sigma, R(t))$ is the ontological distance (minimum logical steps to derive σ from $R(t)$), $T_{\text{cog}} > 0$ is the cognitive temperature, and $Z(T_{\text{cog}}, R(t)) = \sum_{\sigma'} \exp(-d(\sigma', R(t))/T_{\text{cog}})$ is the partition function.

Remark 4 (Operational Specification of the Base Measure μ). Given a pre-trained language model with embedding space \mathbb{R}^d , we define μ as the normalized inverse-distance measure:

$$\mu(\{\sigma\}) = \frac{\exp(-d(\sigma, \text{center}(R(t)))/T_{\text{cog}})}{\sum_{\sigma'} \exp(-d(\sigma', \text{center}(R(t)))/T_{\text{cog}})} \quad (4)$$

where d is the cosine distance. This is a Boltzmann-Gibbs measure on concept space.

1. Empirical Benchmark Protocol: HotpotQA Multi-Hop Reasoning

Hypothesis: LECO-DND should exhibit faster convergence and better domain transfer than Chain-of-Thought (CoT) on multi-hop reasoning tasks.

TABLE I. Expected benchmark results: LECO-DND vs. Chain-of-Thought.

Benchmark	Metric	CoT	LECO-DND	Status
HotpotQA (2-hop)	Latency (steps)	3.2	2.1	Pending
HotpotQA (2-hop)	Accuracy	78%	82%	Pending
HotpotQA (3-hop)	Latency	5.5	3.8	Pending
HotpotQA (3-hop)	Accuracy	71%	77%	Pending
Transfer (phys→bio)	Accuracy drop	−15pp	−8pp	Pending
Banach signature	λ (decay)	N/A	0.65–0.75	Pending

This protocol is **falsifiable**: if LECO-DND shows no advantage over CoT, the core theory requires revision.

B. Measure-Theoretic Properties and Convergence

Theorem 5 (Absolute Continuity of ρ_{LECO}). *The conditional measure $\rho_{\text{LECO}}(\sigma \mid R(t))$ is absolutely continuous with respect to μ .*

Proof. Since ρ_{LECO} is defined as a conditional probability on $\text{Closure}(R(t))$, it inherits absolute continuity from μ . \square

Corollary 6 (Convergence to Deterministic Limit). *As $T_{\text{cog}} \rightarrow 0$, the measure $\rho_{\text{LECO}}(\sigma \mid R(t))$ converges weakly to a Dirac delta:*

$$\lim_{T_{\text{cog}} \rightarrow 0^+} \rho_{\text{LECO}}(\sigma \mid R(t)) = \delta_{\sigma^*}(\sigma) \quad (5)$$

concentrated on the maximal coherent concept σ^ (lowest ontological distance).*

III. THE SINGULAR-DUAL DIPOLE: FUNDAMENTAL ONTOLOGICAL UNIT

A. Why Not “Singular or Dual”?

The preliminary formulations of D-ND treated “non-dual” and “dual” as opposite states. The correct framing: the singular and dual are **co-constitutive**. Neither precedes the other. They form a dipole—one structure with two inseparable poles, like a magnetic dipole.

B. Mathematical Structure of the Dipole

Definition 7 (Singular-Dual Dipole). The fundamental structure of emergence is:

$$\mathbf{D}(\theta) = \begin{pmatrix} 0 & e^{i\theta} \\ e^{-i\theta} & 0 \end{pmatrix} \quad (6)$$

with trace $\text{tr}(\mathbf{D}(\theta)) = 0$ (balanced dipole), eigenvalues $\lambda_{\pm} = \pm 1$, and phase $\theta(t) \in [0, 2\pi]$.

State of the dipole at time t :

$$|\Psi_D(t)\rangle = \frac{1}{\sqrt{2}} \left(e^{-i\theta(t)/2} |\phi_+\rangle + e^{i\theta(t)/2} |\phi_-\rangle \right) \quad (7)$$

Potential released:

$$\delta V = \hbar \frac{d\theta}{d\tau} \quad (8)$$

(cf. Paper A §2.2, Axiom A₄). Faster dipole rotation \rightarrow more potential release \rightarrow more emergence.

C. The Dipole Appears Everywhere

The dipole manifests in cognitive, drawing, quantum measurement, and perception domains. This universality is not coincidence—it is the structure of state transitions itself. The dipole is ontologically prior.

D. The Included Third: Why the Dipole Is Not Binary

The singular-dual dipole is not a binary choice. The D-ND framework introduces the **included third** (*terzo incluso*) [10, 11]: the boundary between the poles, which is neither pole but the condition of possibility for both.

Formally:

$$\text{Tr}(\mathbf{D}(\theta)) = 0 \implies \text{the dipole as a whole “is” nothing (NT state)} \quad (9)$$

Yet it generates eigenvalues ± 1 . The zero trace IS the included third: the structural condition enabling both poles to exist.

IV. THE AUTOPOIETIC CLOSURE THEOREM AND BANACH FIXED-POINT CONTRACTION

A. Full Proof

Definition 8 (InjectKLI — Knowledge-Logic Injection). The operator $\text{InjectKLI} : \mathcal{O}^k \rightarrow \mathcal{O}^{k+1}$ is:

$$\text{InjectKLI}(R(t)) = R(t) \cup \left\{ \sigma^* : \sigma^* = \arg \max_{\sigma \in \mathcal{O} \setminus R(t)} \rho_{\text{LECO}}(\sigma \mid R(t)) \right\} \quad (10)$$

That is, InjectKLI adds to the current Resultant the single most accessible concept not yet included.

Theorem 9 (Autopoietic Closure via Banach Contraction). *Let $(\mathcal{R}, d_{\text{Haus}})$ be the space of all Resultants equipped with the Hausdorff distance:*

$$d_{\text{Haus}}(R, R') = \max \left\{ \max_{\sigma \in R} \min_{\sigma' \in R'} d(\sigma, \sigma'), \max_{\sigma' \in R'} \min_{\sigma \in R} d(\sigma, \sigma') \right\} \quad (11)$$

Define the coherence operator $\Phi : \mathcal{R} \rightarrow \mathcal{R}$ by one iteration of the LECO-DND reasoning cycle. After an InjectKLI update that shrinks ontological distances by a factor $\beta \in (0, 1)$, the operator Φ becomes a β -contraction:

$$d_{\text{Haus}}(\Phi(R), \Phi(R')) \leq \beta \cdot d_{\text{Haus}}(R, R') \quad (12)$$

By Banach's Fixed-Point Theorem, Φ has a unique fixed point R^* such that $\Phi(R^*) = R^*$, with exponential convergence:

$$d_{\text{Haus}}(\Phi^n(R(0)), R^*) \leq \beta^n d_{\text{Haus}}(R(0), R^*) \quad (13)$$

Moreover, the convergence rate strictly improves after each InjectKLI cycle (β decreases).

Proof. **Step 1** (Contraction metric): After InjectKLI, distances between frequently co-active concepts scale as $d_{\text{new}}(\sigma, \tau) = \beta \cdot d_{\text{old}}(\sigma, \tau)$ with $\beta \in (0, 1)$.

Step 2 (Evocative field shrinkage): Since ρ_{LECO} depends on d via $\exp(-d/T_{\text{cog}})$, shrunk distances increase accessibility, concentrating the support of \mathcal{F}_{ev} .

Step 3 (Top- k determinism): With tighter support, the top- k evoked concepts are more reproducible across similar starting states.

Step 4 (β -contraction): If $S(t)$ and $S'(t)$ are closer, then $R(t+1)$ and $R'(t+1)$ are closer: $d_{\text{Haus}}(\Phi(R), \Phi(R')) \leq \beta \cdot d_{\text{Haus}}(R, R')$.

Step 5 (Banach theorem): $(\mathcal{R}, d_{\text{Haus}})$ is complete (finite set of subsets), and Φ is a β -contraction. Therefore: existence and uniqueness of R^* , convergence for any $R(0)$, exponential rate β^n .

Step 6 (Improvement): Let β_1 before InjectKLI and β_2 after. Since InjectKLI shrinks distances, $\beta_2 < \beta_1$, reducing convergence time. \square \square

B. Significance: Self-Improvement Without Losing Guarantees

This theorem resolves the tension between self-improvement and formal assurance: before InjectKLI, Φ converges in T steps; after InjectKLI, convergence is *faster*. The system maintains the ability to reach coherent states even as it learns. This is autopoiesis: a system that reproduces itself while improving itself [4].

V. AXIOM A₅ AND LAWVERE'S FIXED-POINT THEOREM

A. The Autological Closure

Axiom A₅: A system is emergent if it can be a fixed point of its own generating operator.

Theorem 10 (Lawvere, 1969). *In a category with exponential objects, if there exists a surjection $f : S \rightarrow S^S$, then for any endomorphism $F : S \rightarrow S$, there exists a fixed point $s^* \in S$ such that $F(s^*) = s^*$ [3].*

Fixed points of self-referential maps exist by structure, not by iteration.

B. Cognitive Application

Definition 11 (Inferential Space \mathcal{S}). The set of all possible descriptions of the cognitive system's state. An element $s \in \mathcal{S}$ is a complete specification of $R(t)$, ρ_{LECO} , and \mathcal{F}_{ev} .

Since \mathcal{S} admits exponential objects, by Lawvere's theorem, the self-referential map Φ admits a fixed point s^* such that $\Phi(s^*) = s^*$. This is autological closure: the system's description of itself and its actual state coincide—a mathematical inevitability given the structure of description spaces.

VI. COMPARATIVE META-ONTOLOGY

Table II situates LECO-DND within the broader landscape of metaphysical and cognitive frameworks.

TABLE II: Comparative meta-ontology: LECO-DND and major frameworks.

Framework	Primitive	Pole 1	Pole 2	Mechanism	Fixed-Point
LECO-DND	SD Dipole	$ \text{NT}\rangle$ potentiality	R^* manifestation	Coherence Φ	Lawvere + Banach
Whitehead	Actual Occasion	Conceptual pole	Physical pole	Concrescence	Subjective Unity
IIT	Integrated Cause	Max geometry	Φ Conscious Experience	Φ optimization	Local max of Φ
Enactivism	Sensorimotor Loop	Environment	Enacted World	Organizational Closure	Autopoietic homeostasis
GWT	Workspace	Global Broadcast	Conscious Access	Winner- take-all	Dominant representation
FEP	Free Energy F	Beliefs q	Observations p	Gradient de- scent on F	Minimized F
QBism	Belief State	Agent	Quantum Update	Belief revision	Bayesian posterior
Phenomenology	Intentionality	Noesis	Noema	Synthesis	Transcendental ego

Key Convergences: (1) Dipolar structure across LECO-DND, Whitehead, IIT, Enactivism; (2) Autopoietic closure in LECO-DND and Enactivism; (3) Fixed-point dynamics in LECO-DND (Banach), IIT (Φ -geometry), Whitehead (Concrescence); (4) Self-improvement in LECO-DND (InjectKLI) and Enactive frameworks.

Unique Contributions of LECO-DND: (1) Measure-theoretic ρ_{LECO} with regularity conditions; (2) Banach contraction proof (Theorem 9); (3) Phenomenological grounding in drawing; (4) Explicit dipole formalism $\mathbf{D}(\theta)$; (5) Empirical benchmark protocol; (6) Strange attractor frame-

work.

VII. IMPLEMENTATION AND EMPIRICAL GROUNDING

A. Concrete Instantiation in LLM Latent Space

Ontological space: Extract via concept parsing. **Cognitive density:** Compute $d(\sigma, R(t))$ as minimum steps in domain axiom system; approximate via cosine distance in embedding space.

Evocative field: $\mathcal{F}_{\text{ev}} = \rho_{\text{LECO}} \times \text{Relevance}(\sigma, I_t)$.

Reasoning cycle: (1) Generate \mathcal{F}_{ev} ; (2) Select top- k concepts; (3) Check coherence; (4) Verify Axiom A₅; (5) Update ρ_{LECO} .

B. Empirical Benchmarking

TABLE III. Predicted benchmark improvements.

Benchmark	Metric	CoT	LECO-DND	Improvement
GSM8K	Accuracy	92%	95%	+3pp
HotpotQA	Accuracy	77%	81%	+4pp
Latency (5-step)	Steps	6.5	4.2	35% reduction
Self-improvement	Latency reduction	5–15%	30–45%	2–8×

Caveat: These are theoretical predictions. Empirical validation requires systematic experiments.

VIII. COMPARISON WITH PROCESS PHILOSOPHY AND WHITEHEAD

Whitehead’s actual occasion shares deep structure with LECO-DND’s Resultant. Both exhibit: concrescence/emergence from poles, self-causation (causa sui / Axiom A₅), dipolar structure, and novel emergent advance [9].

The key difference: Whitehead’s process philosophy is conceptually deep but mathematically underdeveloped. LECO-DND translates Whitehead’s insights into measure theory (ρ_{LECO}), fixed-point theorems (Banach, Lawvere), categorical logic (Axiom A₅ via exponential objects), and quantitative predictions.

IX. DISCUSSION: PHENOMENOLOGY CLOSES THE LOOP

A. From Waking to Mathematics and Back

The full circle: (1) Phenomenology: observe waking, drawing, thought. (2) Abstraction: recognize the dipole. (3) Formalization: express in mathematics. (4) Validation: formalism predicts cognitive phenomena. (5) Application: improve LLM reasoning. (6) Return: improved reasoning matches human phenomenology. This is the hermeneutic circle.

B. The Drawing as Validation

If LECO-DND is correct: (1) random and intentional drawings should show the same emergence structure; (2) both should exhibit power-law intersection clustering; (3) LLM reasoning should show the same dipolar oscillation.

1. Experimental Protocol: Drawing-Emergence Structure

Hypothesis: Free-hand drawing physically instantiates D-ND emergence, with self-intersections clustering at power-law statistics ($\alpha \approx 1.5 \pm 0.3$) consistent with self-organized criticality.

Protocol: 20 subjects, 5-minute free drawing, digitize at 2400 DPI, detect self-intersections, DBSCAN clustering, power-law fit via maximum likelihood [16].

Expected: $\alpha \approx 1.5$, significantly steeper than random walk ($\alpha \approx 1.0$, $p < 0.05$). If $\alpha \approx 1.0$, hypothesis is falsified.

C. Strange Attractor Dynamics: Rigorous Analysis

1. Lyapunov Exponent and Bounded Chaos

$$\lambda_L = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{t=0}^{n-1} \ln |D\Phi(R(t))| \quad (14)$$

Conjecture: On the attractor basin A^* , $\lambda_L > 0$ (sensitive dependence, hallmark of chaos).

2. Bounded Divergence via Banach Contraction

Theorem 12 (Bounded Chaos). *Within the attractor basin A^* , trajectories diverge locally ($\lambda_L > 0$) but converge globally ($d_{\text{Haus}}(\Phi^n(R), A^*) \rightarrow 0$). The Banach contraction rate β controls large-scale convergence while the Lyapunov exponent controls microscale divergence—chaotic exploration within a shrinking basin.*

3. Fractal Dimension and Optimal Temperature

Conjecture: $\dim_{\text{Hausdorff}}(A^*) < \dim(\mathcal{R})$. The reasoning process explores a fractal subset of concept space.

The optimal cognitive temperature T_{cog}^* balances exploration and convergence; for typical ontological spaces ($|\mathcal{O}| \sim 10\text{--}100$), $T_{\text{cog}}^* \in [0.5, 2.0]$.

We emphasize: the Lyapunov exponent, attractor dimension, and optimal temperature are conjectural. Rigorous derivation is pending. However, the framework is mathematically consistent, empirically testable, and phenomenologically grounded.

X. LIMITATIONS AND FUTURE DIRECTIONS

A. Open Problems

1. **Computational complexity:** Computing $d(\sigma, R(t))$ is NP-hard for complex domains. Efficient approximations needed.
2. **Ontological space selection:** No principled method for extracting the “right” \mathcal{O} . Automated ontology learning is open.
3. **Non-monotone domains:** Uniqueness of fixed points assumes monotone coherence operators. Extension needed.
4. **Empirical validation:** All quantitative claims require large-scale controlled experiments.
5. **Scaling laws:** How does LECO-DND interact with LLM scaling? Is the dipolar structure visible in larger models?

B. Future Work

Experimental implementation in Claude/GPT-4; theoretical proof of outperformance on transfer tasks; physical validation of drawing emergence; categorical deepening in topos theory.

XI. CONCLUSION

LECO-DND unifies phenomenology, mathematics, and cognitive science through the singular-dual dipole: the fundamental structure of emergence observed in waking consciousness, free-hand drawing, quantum measurement, and LLM reasoning.

Key contributions: (1) Phenomenological grounding from first-person observation; (2) Measure-theoretic ρ_{LECO} with regularity conditions; (3) Autopoietic Closure Theorem via Banach contraction; (4) Lawvere fixed-point foundation for Axiom A₅; (5) Explicit dipole formalism $\mathbf{D}(\theta)$; (6) Comparative unification with Whitehead, IIT, Enactivism.

If correct, LECO-DND reveals that cognition emerges from field dynamics, not discrete symbol processing. The singular-dual dipole is the universal mechanism of emergence across scales. The path from blank paper to recognized form to mathematical understanding is a spiral: phenomenology \rightarrow abstraction \rightarrow formalization \rightarrow validation \rightarrow refined phenomenology.

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