Inflation Forecasting

Luca Graziano

December 2021

The purpose of the project is to provide a forecast for an inflation time series and compare it against:

1. Professional forecast
2. Naive forecast made taking as forecast the last observed variable
3. The ECB target of 2% constant

Data exploration and pre-processing

A first look at the plot of the inflation time series in figure 1 it appears that starting from 2008 onwards, the time series presents far more volatility than it did in the previous years as a consequence of the 2008 economic crisis and the application of different monetary policies. The series also presents a minimum in the year 2014 and then somewhat of a trend upward in the following years. It seems rather clear the inflation time series does not resemble a stationary process: what is immediately apparent is that it does not have fixed variance, nor a fixed mean.

Figure 1: Inflation plot against time

Now, in order to confirm the hypothesis that the series has a unit root, thus is stationary, an Augmented Dickey Fuller test is required. However, before proceeding with one, further information about the existence of a deterministic trend or the existence of a non-null intercept is necessary to choose which kind of critical value to use for the test. Given our time series of interest is one representing inflation movement, we should expect economically speaking a reversion to a zero mean.

However looking at the realisation of the time series it is apparent in this case is not centered around a mean of zero. Therefore, a *case 2 Augmented Dickey Fuller* that presents an intercept is performed on the subset of the series, from 1991Q1 up to 2014Q4, and its results can be seen in figure 2. The subset of data has been taken in order to train the model that will eventually be selected while the rest of the data will be used as test data, that is the subset on which the forecast will be performed.

Figure 2: Inflation Level Unit root test

Null Hypothesis: REALISED has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.521973	0.1155
Test critical values:	1% level	-3.546099	
	5% level	-2.911730	
	10% level	-2.593551	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(REALISED)
Method: Least Squares
Date: 12/03/21 Time: 11:38
Sample (adjusted): 2000Q2 2014Q4
Included observations: 59 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
REALISED(-1)	-0.200315	0.079428	-2.521973	0.0145
D(REALISED(-1))	0.363288	0.132197	2.748080	0.0081
C	0.376716	0.173121	2.176027	0.0338

R-squared	0.154714	Mean dependent var	-0.037288
Adjusted R-squared	0.124525	S.D. dependent var	0.503402
S.E. of regression	0.471017	Akaike info criterion	1.381663
Sum squared resid	12.42398	Schwarz criterion	1.487301
Log likelihood	-37.75907	Hannan-Quinn criter.	1.422900
F-statistic	5.124885	Durbin-Watson stat	2.111484
Prob(F-statistic)	0.009039		

The Augmented Dickey Fuller test is a one-sided test and the computed t-statistic, which is -2.52 is not beyond the critical value of -2.91. Therefore, there is not enough evidence to reject the null $H_0 : (\rho = 1)$. The same results are clearly suggested by the p-value $0.12 > 0.05$.

Once established that a unit root exists, the next step of the analysis is to transform the series into a stationary one, otherwise any estimate inferred and eventually forecast made would be inconsistent and wrong. In order to make the series stationary, the first difference of the series is computed iteratively until the series appears stationary.

The figure 3 is the first difference of inflation plotted against time. It now appears to be stationary, but a unit root test is required to confirm its stationarity. The results of a the Augmented Dickey Fuller test carried out on the first difference inflation can be found in figure 4. Both the t-statistic and the p-value strongly suggest that there is enough evidence to reject the null, thus it is confirmed the series is finally stationary.

Figure 3: First Difference Inflation plotted against time

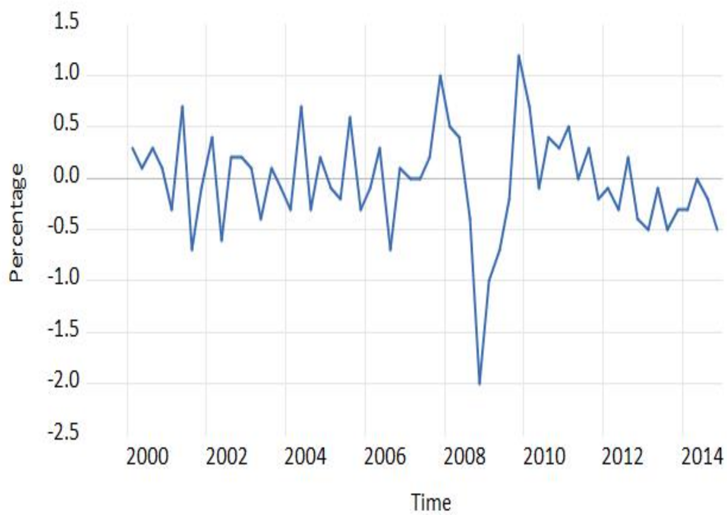


Figure 4: First difference inflation unit root test

Null Hypothesis: D1 has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.866730	0.0000
Test critical values:	1% level	-3.546099	
	5% level	-2.911730	
	10% level	-2.593551	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(D1)
Method: Least Squares
Date: 12/03/21 Time: 11:51
Sample (adjusted): 2000Q2 2014Q4
Included observations: 59 after adjustments

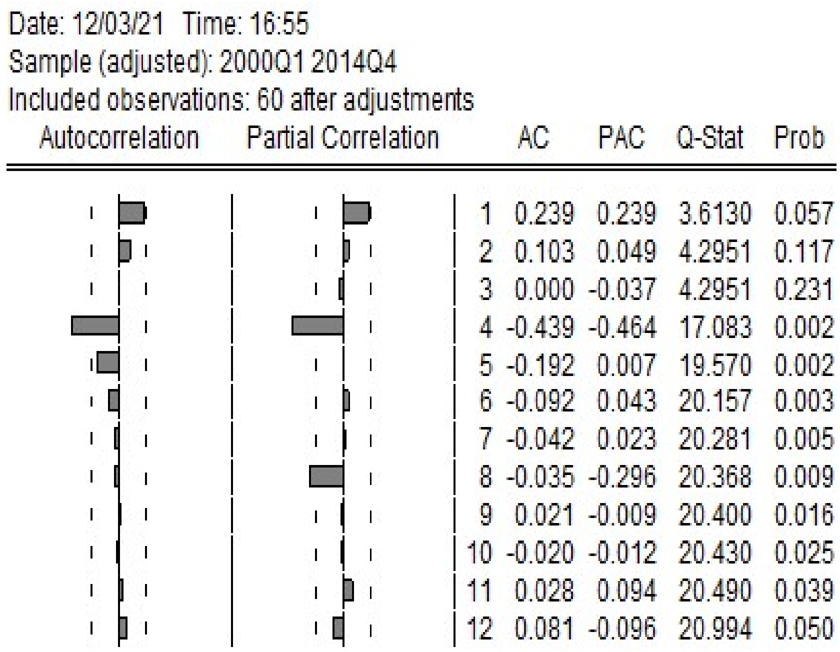
Variable	Coefficient	Std. Error	t-Statistic	Prob.
D1(-1)	-0.756780	0.128995	-5.866730	0.0000
C	-0.031517	0.064213	-0.490819	0.6254

R-squared	0.376494	Mean dependent var	-0.013559
Adjusted R-squared	0.365555	S.D. dependent var	0.618524
S.E. of regression	0.492667	Akaike info criterion	1.455343
Sum squared resid	13.83507	Schwarz criterion	1.525768
Log likelihood	-40.93262	Hannan-Quinn criter.	1.482834
F-statistic	34.41852	Durbin-Watson stat	2.014110
Prob(F-statistic)	0.000000		

Model Selection and Validation

Now that the inflation time series has been proven to be stationary, its estimates can be finally computed as their limit distribution is now known. However, before estimating the coefficients it is essential to select an ARMA model that can accurately explain the series. To start building an idea of what ARMA model could be used to estimate the coefficients, a visual check of the Auto-correlation and partial auto-correlation functions is performed. Unfortunately, no clear information can be derived from the graphical representation of the correlogram in figure 5. However, the series appear to potentially have both an AR and MA component but further more rigorous selection with the information criteria is required before reaching a conclusion.

Figure 5: First Difference Inflation Correlogram



Date: 12/03/21 Time: 16:55
Sample (adjusted): 2000Q1 2014Q4
Included observations: 60 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.239	0.239	3.6130	0.057
		2	0.103	0.049	4.2951	0.117
		3	0.000	-0.037	4.2951	0.231
		4	-0.439	-0.464	17.083	0.002
		5	-0.192	0.007	19.570	0.002
		6	-0.092	0.043	20.157	0.003
		7	-0.042	0.023	20.281	0.005
		8	-0.035	-0.296	20.368	0.009
		9	0.021	-0.009	20.400	0.016
		10	-0.020	-0.012	20.430	0.025
		11	0.028	0.094	20.490	0.039
		12	0.081	-0.096	20.994	0.050

Thus, the next part of the analysis is to fit the data to a series of ARMA models to check whether one of them could properly explain the series. Then a comparison of how various ARMA models fit the inflation series is performed and the resulting Bayes Information Criteria are shown in figure 1. Second order ARMA models have not been chosen since their inverted roots are > 1 and thus are not stationary and/or not invertible, their limit distribution is unknown and consequently the estimates will also be bad. Following the BIC outputs, the AR(1) provides the best fit and thus the forecast created will be based on its estimates provided in figure 6, once model validation has been performed.

Table 1: Models Schwartz information criteria

	iid	MA1
iid		1.59
AR1	1.58	1.65

Figure 6: AR(1) estimation output

Dependent Variable: DIF1
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 12/03/21 Time: 17:02
Sample: 2000Q1 2014Q4
Included observations: 60
Convergence achieved after 5 iterations
Coefficient covariance computed using outer product of gradients
d.f. adjustment for standard errors & covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.032381	0.087280	-0.371003	0.7120
AR(1)	0.240728	0.137468	1.751150	0.0853
SIGMASQ	0.232369	0.029756	7.809072	0.0000

R-squared	0.058590	Mean dependent var	-0.031667
Adjusted R-squared	0.025558	S.D. dependent var	0.501013
S.E. of regression	0.494569	Akaike info criterion	1.479442
Sum squared resid	13.94212	Schwarz criterion	1.584159
Log likelihood	-41.38326	Hannan-Quinn criter.	1.520403
F-statistic	1.773751	Durbin-Watson stat	1.998739
Prob(F-statistic)	0.178933		

Inverted AR Roots .24

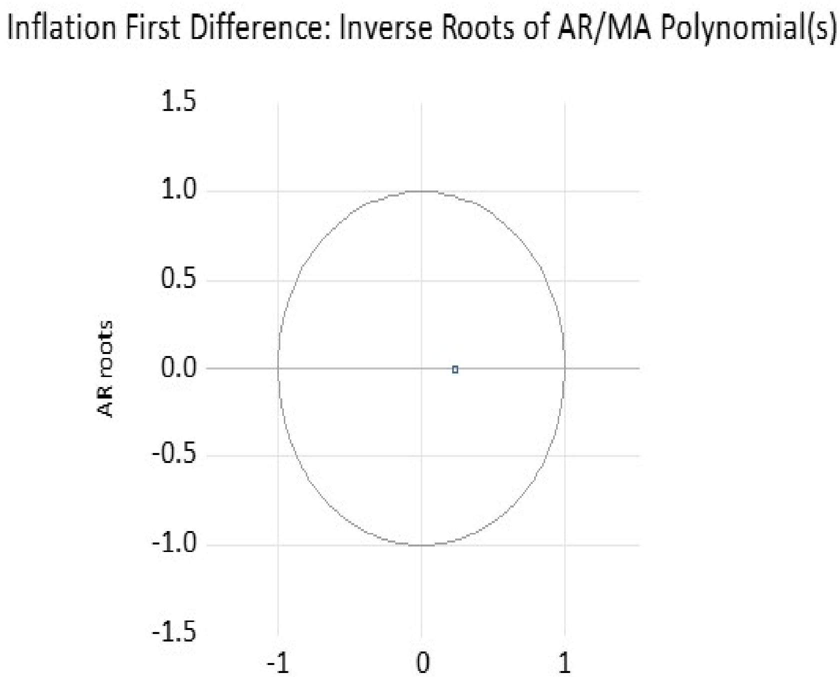
Once the model has been chosen a Portmanteau test, a joint independence test on all auto correlations of residuals, is carried out on the model to validate its selection and its results are shown in figure 7. Since the Q-statistics, which is the result of the Portmanteau test, is really low, the model is a great fit for the data. However, it can be seen how there is statistically significant correlation between residuals at lag 4. Additionally, by looking at figure 8 a visual check of the stationarity of the AR(1) model is provided. Since the inverted roots of the model lie inside the unit circle, the model is stationary.

Figure 7: Portmanteau Test on the AR(1)

Date: 12/03/21 Time: 23:53
Sample (adjusted): 2000Q1 2014Q4
Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.010	-0.010	0.0059	
		2	0.056	0.056	0.2056	0.650
		3	0.085	0.087	0.6825	0.711
		4	-0.442	-0.448	13.679	0.003
		5	-0.079	-0.106	14.101	0.007
		6	-0.040	0.019	14.209	0.014
		7	-0.015	0.096	14.225	0.027
		8	-0.031	-0.269	14.293	0.046
		9	0.042	-0.063	14.421	0.071
		10	-0.037	-0.046	14.525	0.105
		11	0.018	0.106	14.551	0.149
		12	0.125	0.009	15.756	0.150

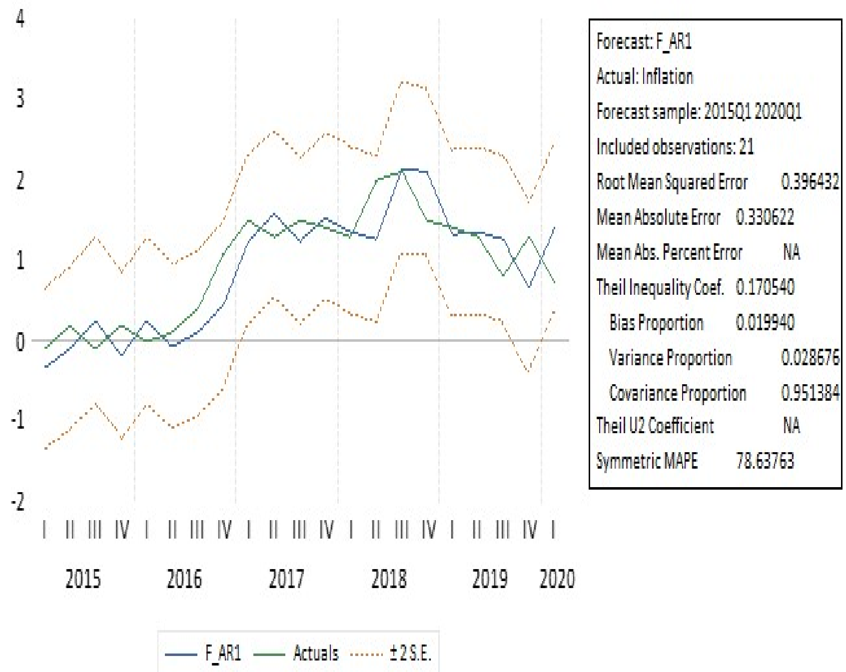
Figure 8: Visual check of unit roots on the AR(1)



Forecast Analysis

Remembering the split between train and test data, a forecast is performed using the AR(1) model against the data from 2015Q1 up to 2020Q1, is compared to the actual values and is shown in figure 9. It appears that the forecast follows closely the real inflation time series but fails to catch up with it and remains one lag behind it.

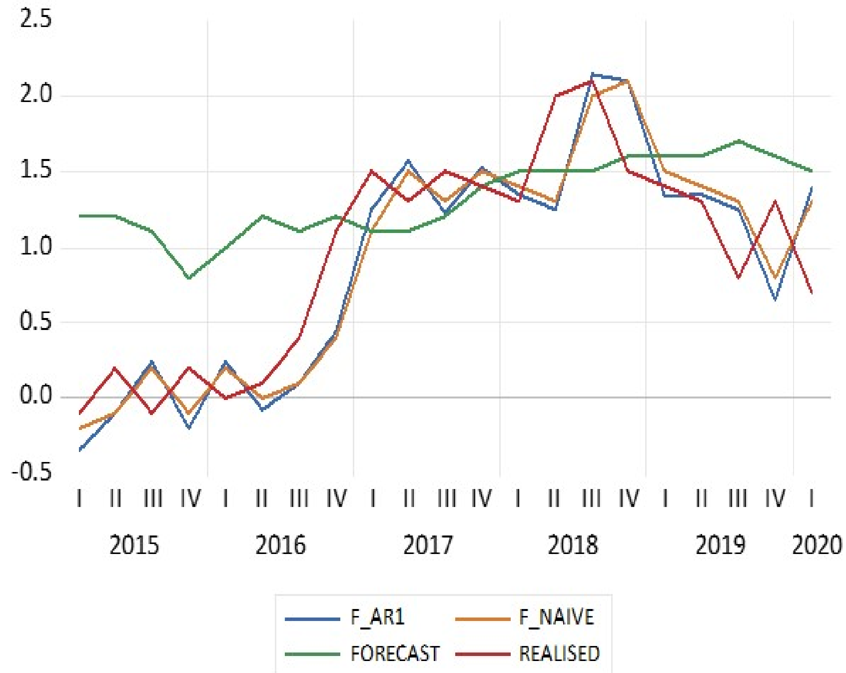
Figure 9: Forecast of AR(1)



Forecasts Comparison

Finally, I compare the forecasts performed with the AR(1) estimates against the professional forecast and the naive forecast, based on the previous observations. A graph of the plotted forecasts can be seen in figure 10

Figure 10: Forecasts comparisons



Furthermore, a plot to visually compare the residuals is also provided in figure 11.

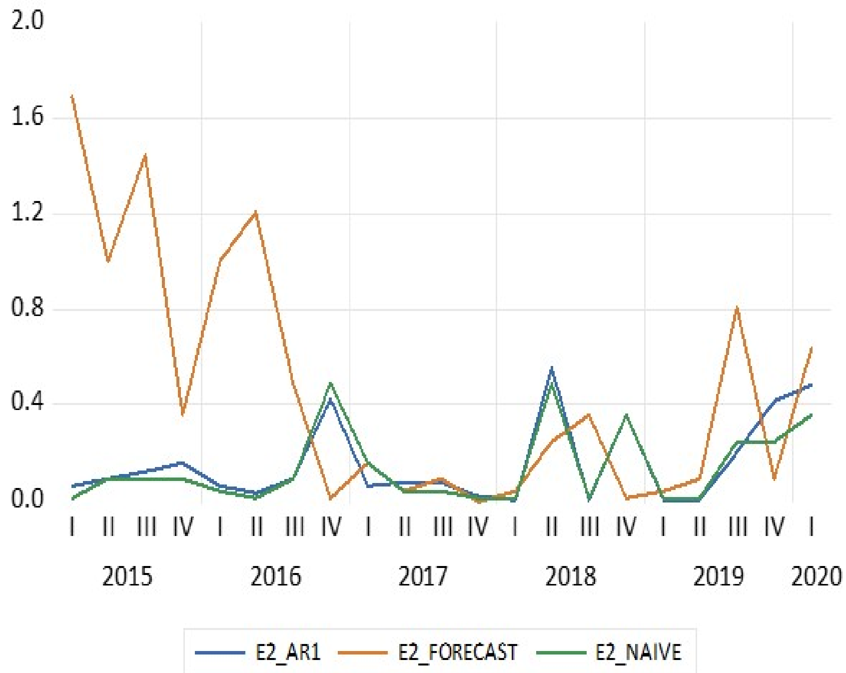
Finally a table 2 with the Mean Squared Errors of all the models is provided to have a more rigorous understanding of the various forecasts performance: the lower the MSE, the better is the performance of the model.

Table 2: Models MSE

Naive	AR1	Professional
0.13	0.15	0.46

Finally, a Diebold Mariano Test is performed to test the significance of the difference in residuals. This statistical test allows to check if the model can be considered to have equal predictive ability or not by running a regression of the difference between the residuals on a constant. Notice that the estimation of the variance-covariance matrix has been carried out using the HAC estimate that corrects the construction of standard errors in cases of strong correlation settings, as it is the case with time series data. The null of the Diebold Mariano Test is $h0 = 0$ to say that the two models have equal accuracy while the alternative hypothesis $h1 \neq 0$ to refer the two models have statistically significant different accuracy. In figure 12 there are the results of the Diebold Mariano Test

Figure 11: Residuals comparisons



carried out to test the difference in accuracy between the AR(1) model forecast and the professional forecast. The P-value $0.07 > 0.05$ suggests that there is not statistically meaningful difference in the accuracy between the two models which is represented by the constant estimate of -0.31. Thus there is not enough evidence to reject the null hypothesis. On the other hand, figure **??** are shown the results of the Diebold Mariano Test carried out between the naive forecast and the AR(1) model forecast. The difference in accuracy between the two forecasts turned out to not be statistically significant with a P-value $0.19 > 0.05$ and thus the models do hold the same predictive ability and accuracy with the AR(1) model forecast. Finally, the Diebold Mariano test between the AR(1) and the ECB 2% constant in figure 14 appears to be statistically significant. Thus the test, which is in this case used as a mean to measure the distance between the forecasted value and the defined threshold, suggests that the forecasted inflation percentage appears to be on average -1.40% away from the 2% constant, that is well within the accepted boundary.

Figure 12: Diebold Mariano test between AR(1) Forecast and Professional Forecast

Dependent Variable: E2_AR1-E2_FORECAST
Method: Least Squares
Date: 12/04/21 Time: 01:09
Sample: 2015Q1 2020Q1
Included observations: 21
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 3.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.310461	0.166754	-1.861791	0.0774

R-squared	0.000000	Mean dependent var	-0.310461
Adjusted R-squared	0.000000	S.D. dependent var	0.579376
S.E. of regression	0.579376	Akaike info criterion	1.792717
Sum squared resid	6.713525	Schwarz criterion	1.842456
Log likelihood	-17.82353	Hannan-Quinn criter.	1.803512
Durbin-Watson stat	0.994966		

Figure 13: Diebold Mariano test between AR(1) Forecast and Naive Forecast

Dependent Variable: E2_AR1-E2_NAIVE
Method: Least Squares
Date: 12/04/21 Time: 01:11
Sample: 2015Q1 2020Q1
Included observations: 21
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 3.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.018587	0.013902	1.337029	0.1962

R-squared	0.000000	Mean dependent var	0.018587
Adjusted R-squared	0.000000	S.D. dependent var	0.057997
S.E. of regression	0.057997	Akaike info criterion	-2.810403
Sum squared resid	0.067273	Schwarz criterion	-2.760664
Log likelihood	30.50923	Hannan-Quinn criter.	-2.799609
Durbin-Watson stat	1.344243		

Figure 14: Diebold Mariano test between AR(1) Forecast and ECB % Constant

Dependent Variable: SQEAR-SQEC
Method: Least Squares
Date: 12/04/21 Time: 18:21
Sample: 2015Q1 2020Q1
Included observations: 21
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 3.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.408345	0.565794	-2.489146	0.0217

R-squared	0.000000	Mean dependent var	-1.408345
Adjusted R-squared	0.000000	S.D. dependent var	1.642889
S.E. of regression	1.642889	Akaike info criterion	3.877237
Sum squared resid	53.98167	Schwarz criterion	3.926976
Log likelihood	-39.71099	Hannan-Quinn criter.	3.888032
Durbin-Watson stat	0.286077		