Conditional Probability

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The Conditional Probability $\Pr(A|B)$ denotes the probability of the event A occurring given that the event B has occurred,

$$\Pr(A|B) = \frac{\Pr(A \bigcap B)}{\Pr(B)}.$$

Example: The rain in Ireland

A normal probability would be what is the probability it is going to rain $\Pr(\text{rain})$. A conditional probability would, be what is the probability it is going to rain **given** that you are in Ireland, $\Pr(\text{rain}|\text{Ireland})$,

$$\Pr(\text{rain}|\text{Ireland}) = \frac{\Pr(\text{rain} \bigcap \text{Ireland})}{\Pr(\text{Ireland})},$$

where the probability of rain is $\Pr(\text{rain}) = 0.3$, the probability of being in Ireland is $\Pr(\text{Ireland}) = 0.4)$ and the probability of being in Ireland and it raining is $\Pr(\text{rain} \bigcap \text{Ireland}) = 0.2$,

$$\Pr(\text{rain}|\text{Ireland}) = \frac{0.2}{0.4} = 0.5,$$

You could be interested in the probability that you are in Ireland **given** that it is raining,

$$\Pr(\text{Ireland}|\text{rain}) = \frac{\Pr(\text{rain} \bigcap \text{Ireland})}{\Pr(\text{rain})} = \frac{0.2}{0.4} = 0.75.$$