Mathematical Probability

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Definitions

Define some event A that can be the outcome of an experiment. $\Pr(A)$ is the probability of a given event A will happen. Rules:

1. $\Pr(A)$ is between 0 and 1, $0 \leq Pr(A) \leq 1$;

2. $\Pr(A) = 1$, means it will definitely happen;

3. $\Pr(A) = 0$, means it will definitely **not** happen;

4. $\Pr(A) = 0.05$, is arbitrarily considered unlikely.

Sample Space and Events

The **Sample Space**, S, of an experiment is the universal set of all possible outcomes for that experiment, defined so, no two outcomes can occur simultaneously. For example:

Axioms of Probabilities

For an event A subset S associated a number $Pr(A)$, the probability of A, which must have the following properties

1. If $Pr(A \bigcap B) = 0$, then $\Pr(A \bigcup B) = \Pr(A) + \Pr(B)$;

2. Probability of the Null Event $\Pr(\emptyset) = 0$;

3. The probability of the complement of A, $\Pr(\bar{A}) = 1 - \Pr(A)$;

4. $\Pr(A \bigcup B) = \Pr(A) + \Pr(B) - \Pr(A \bigcap B)$.

Dice

For a die the Sample Space is all the faces of the die

$$S = \{1, 2, 3, 4, 5, 6\}.$$

The probability of each event is

$$\frac{1}{6}.$$

```r
Events<-c("1","2","3","4","5","6") # List of 7 numbers
## Cacluate the mean in the long form
Pr<-c(1/6,1/6,1/6,1/6,1/6,1/6)
## Cacluate the mean using the function mean
Die<-data.frame(Events,Pr)
Die
```

```
##   Events        Pr
## 1      1 0.1666667
## 2      2 0.1666667
## 3      3 0.1666667
## 4      4 0.1666667
```

```
## 5        5 0.1666667
## 6        6 0.1666667
```

An event, A, is a subset, $\subset$, of the Sample Space, such as $A = \{2, 3\} = 2\bigcup 3$. Given $Pr(2\bigcap 3) = 0$, then

$$\Pr(2\bigcup 3) = \Pr(2) + \Pr(3) = \frac{1}{6} + \frac{1}{6}.$$