

Measures of Location: First Moment

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Introduction

A different aspects of a distribution of data can be summarised by the measures of location:

1. The First Moment: Middle.
2. The Second Moment: Spread.
3. The Third Moment: Symmetry.

All that being said, I would always recommend plotting the data first before anything else.

A picture (histogram) is worth a thousand words.

First Moment: Middle

Mean (average)

Definition:

The mean \bar{x} , μ is the point around which the sum of the deviations is 0. The formula is given by

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

where x_i is each element and n is the number of elements. In words, add the numbers together and divided by how many numbers are in the list.

Example

Given the list of 7 ages at a concert {19,18,20,18,18,18,20}, the mean is:

$$\bar{x} = \frac{20 + 18 + 19 + 18 + 18 + 18 + 20}{7} = 18.71429.$$

In R code:

```
Age=c(20, 18,19, 18,18,18,20) # List of 7 numbers
## Cacluate the mean in the long form
(20+18+19+18+18+18+20)/7
```

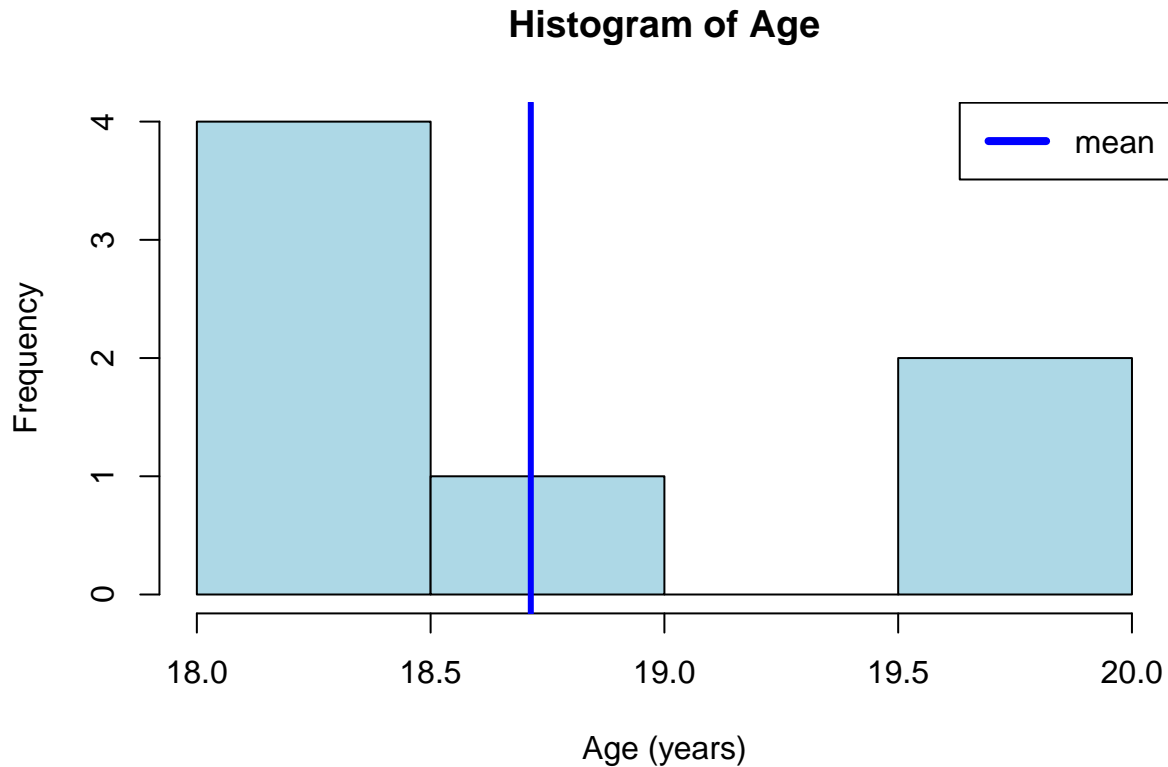
```
## [1] 21.42857
```

```
## Cacluate the mean using the function mean
mean(Age) # The mean of the list
```

```
## [1] 18.71429
```

Graphical representation of the mean:

```
hist(Age,col="lightblue",xlab="Age (years)") # Histogram of the 7 numbers
abline(v=mean(Age),col="blue",lwd=3) # line indicating the mean
legend("topright",c("mean"),lwd=4,col=c("blue"))
```



Mean Pros and Cons

Pros of the mean:

- The mean is easy to calculate.
- The mean is well known and understood.

Cons of the mean: * The mean is sensitive to extreme values.

Given the list of the ages of 8 people at a concert {20,18,19, 18,56, 18,18,20}, the mean is:

$$\bar{x} = \frac{20 + 18 + 19 + 18 + 18 + 56 + 18 + 20}{8} = 23.375.$$

In R code:

```
Age=c(20,18,19, 18,56, 18,18,20) # List of 8 ages
## Calculate the mean using the function mean
mean(Age) # The mean of the list
```

```
## [1] 23.375
```

Median

Definition:

The median is the middle values of an ordered list.

Example

Given the list of 7 ages at a concert {19,18,20,18,18,18,20}.

1. First the list has to be ordered {18,18,18,18,19,20,20}.
2. Then the middle of the list found {18,18,18,18,19,20,20}.

The median is 18.

In R code:

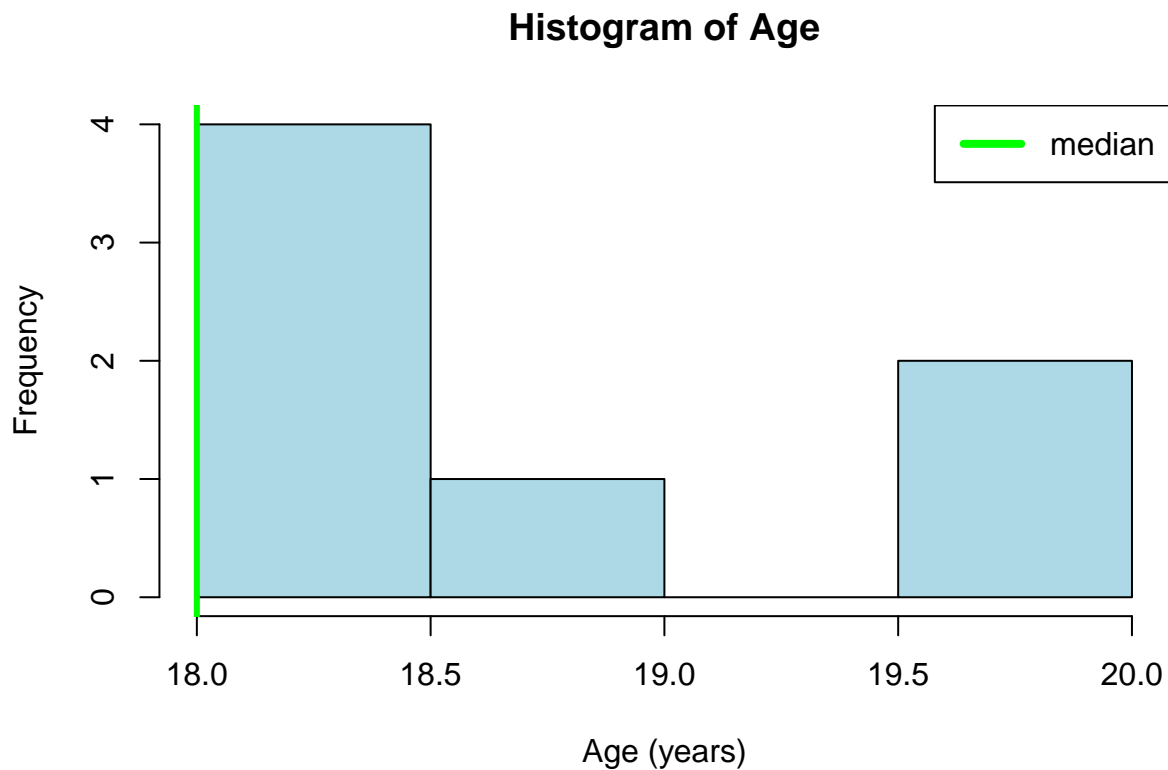
```
Age=c(20,18,19,18,18,18,20) # List of 7 numbers
```

```
median(Age)
```

```
## [1] 18
```

Graphical representation of the median:

```
hist(Age,col="lightblue",xlab="Age (years)") # Histogram of the 7 numbers
abline(v=median(Age),col="green",lwd=3)# line indicating the median
legend("topright",c("median"),lwd=4,col=c("green"))
```



Median Pros and Cons Pros of the median:

- The median is the centre of the data - very intuitive.
- Not as sensitive as the mean to extreme values.

Cons of median: * The calculation of the median does not use all data. * Sorting a long list can be computationally expensive.

Mode

Definition:

The mode is the most commonly occurring value in the distribution.

Example

Given the list of 7 ages at a concert {19,18,20,18,18,18,20}. The most frequent values is 18, so 18 is the mode.

In R code:

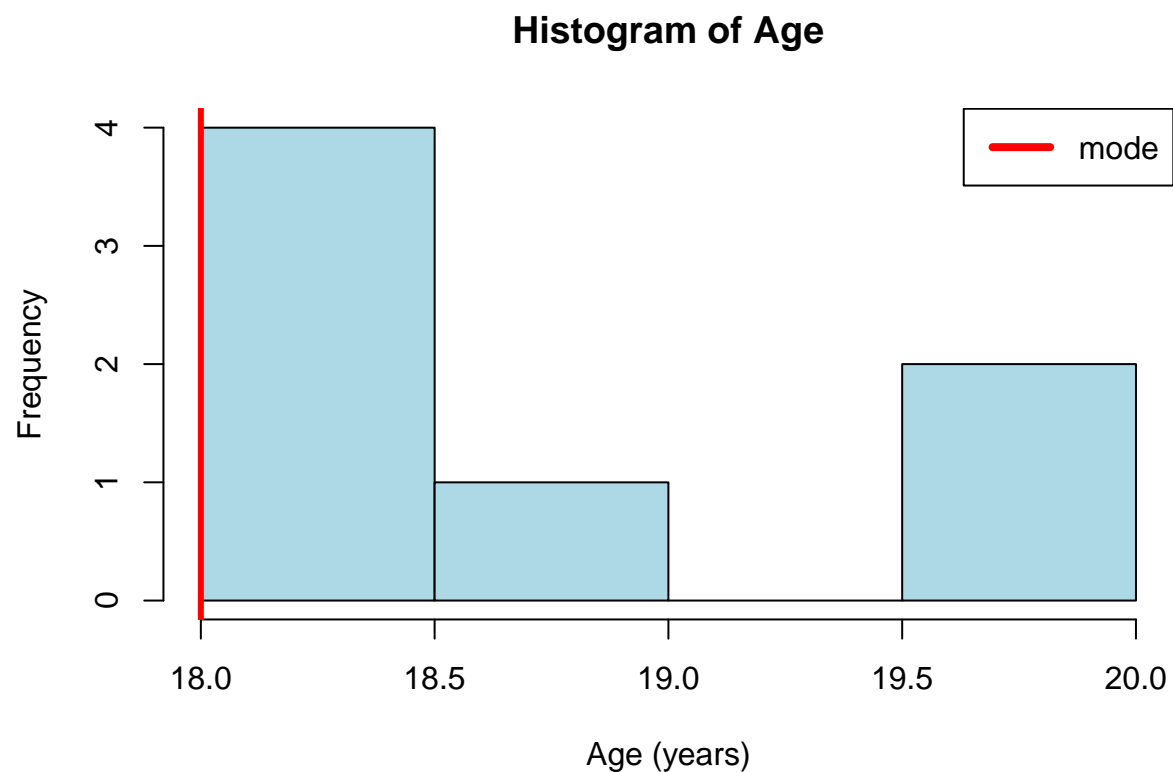
```
Age=c(20,18,19,18,18,18,20) # List of 7 numbers
# Function to calculate mode
Mode <- function(x) {
  ux <- unique(x)
  ux[which.max(tabulate(match(x, ux)))]
}
```

```
}
Mode(Age)
```

```
## [1] 18
```

Graphical representation of the mode:

```
hist(Age,col="lightblue",xlab="Age (years)") # Histogram of the 7 numbers
abline(v=Mode(Age),col="red",lwd=3)# line indicating the mode
legend("topright",c("mode"),lwd=4,col=c("red"))
```



Median Pros and Cons

Pros of the mode:

- The mode shows where the data is concentrated.

Cons of the mode:

- More than one Mode in a dataset.
- The calculation of the mode does not use all the data.