# Discrete Distributions

*John S Butler (TU Dublin)*

## Introduction

## Binomial Distribution

A Binomial Distribution is used to describe the probability distribution of $n$ experiments that have two possible outcomes "success" or "failure". The probability of "success" is $p$, the probability of failure is $q = 1-p$. This gives the general deifintion of the distribution as:

$$\Pr(k) = \left( \begin{array}{c} n \\ k \end{array} \right) p^k q^{n-k}, \quad k = 0, 1, 2, ...n,$$

where $k$ is the number of "successes". If the data can be described using a Binomial Distribution then the expected outcome, $E[k]$ of the distribution is given by:

$$E[k] = np,$$

and the variance of the distribution $Var[k]$, is

$$Var[k] = npq.$$

#### Example

The example we shall use to illustrate the discrete distibutions is the New Zealand vs Ireland Wolrd Cup Rugby Quarter Final. Let's assume the probability of "success", Ireland beating New Zealand is

$$p = 0.15,$$

the probability of failure is

$$q = 1 - p = 1 - 0.15 = 0.85.$$

Now imagine that Ireland have to play New Zealand 10 times, this gives the Binomial Distribution

$$\Pr(k) = \left( \begin{array}{c} 10 \\ k \end{array} \right) (0.15)^k (0.85)^{10-k}, \quad k = 0, 1, 2, ...10,$$

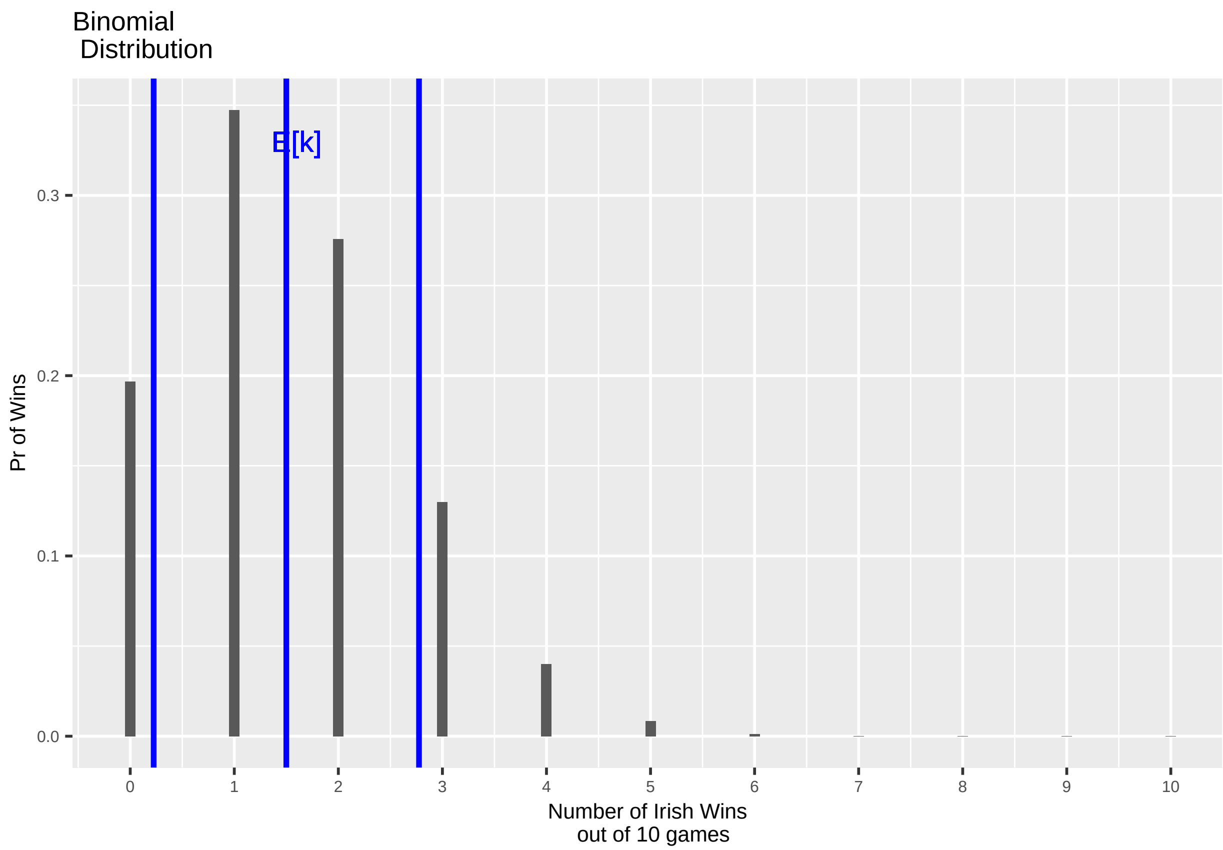
where k is the number of times Ireland beats New Zealand. As it is a binomial Distribution we can state that the expected number of times Ireland will win is

$$E[k] = 10 \times 0.15 = 1.5,$$

ie Ireland is only expected to win once, the variance of the distribution is

$$Var[k] = 10 \times 0.15 \times 0.85 = 1.35.$$

```
## Warning: Ignoring unknown parameters: line
## Warning: Ignoring unknown parameters: lines
## Warning: Duplicated aesthetics after name standardisation: vjust
## Warning: Ignoring unknown parameters: text
```



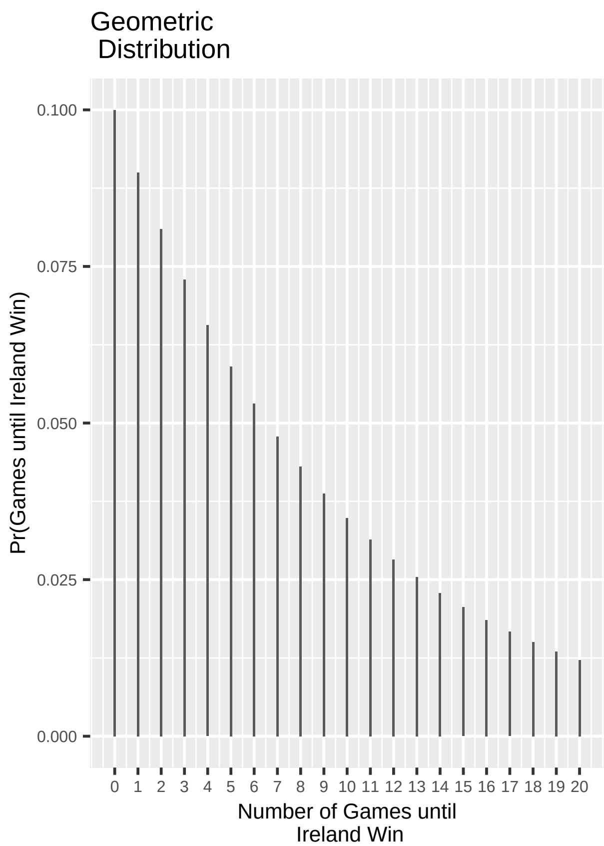

## Geometric Distribution

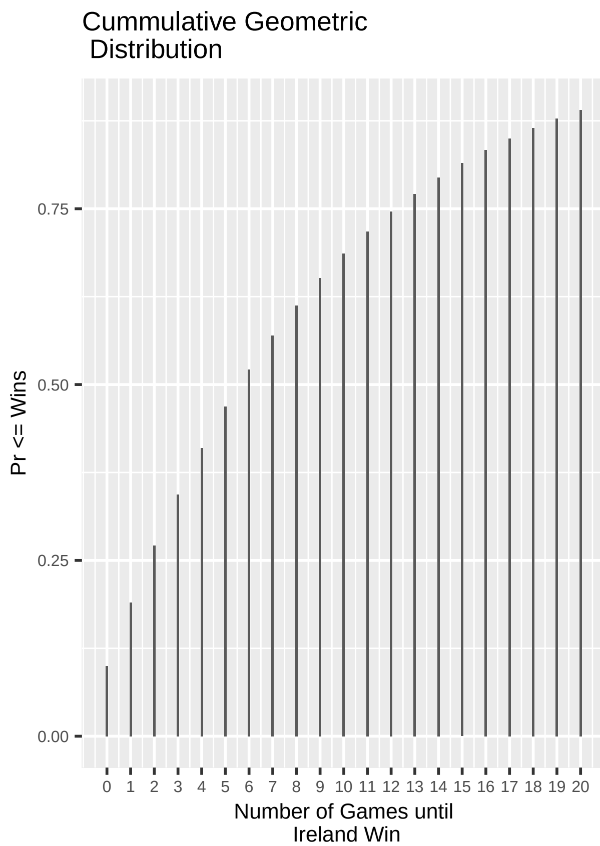
### Definition:

$$\Pr(k) = q^{(k-1)}p, \quad k = 1, 2, ...$$

$$E[k] = \frac{1}{p}, \quad Var[k] = \frac{q}{p^2}.$$

In R code:



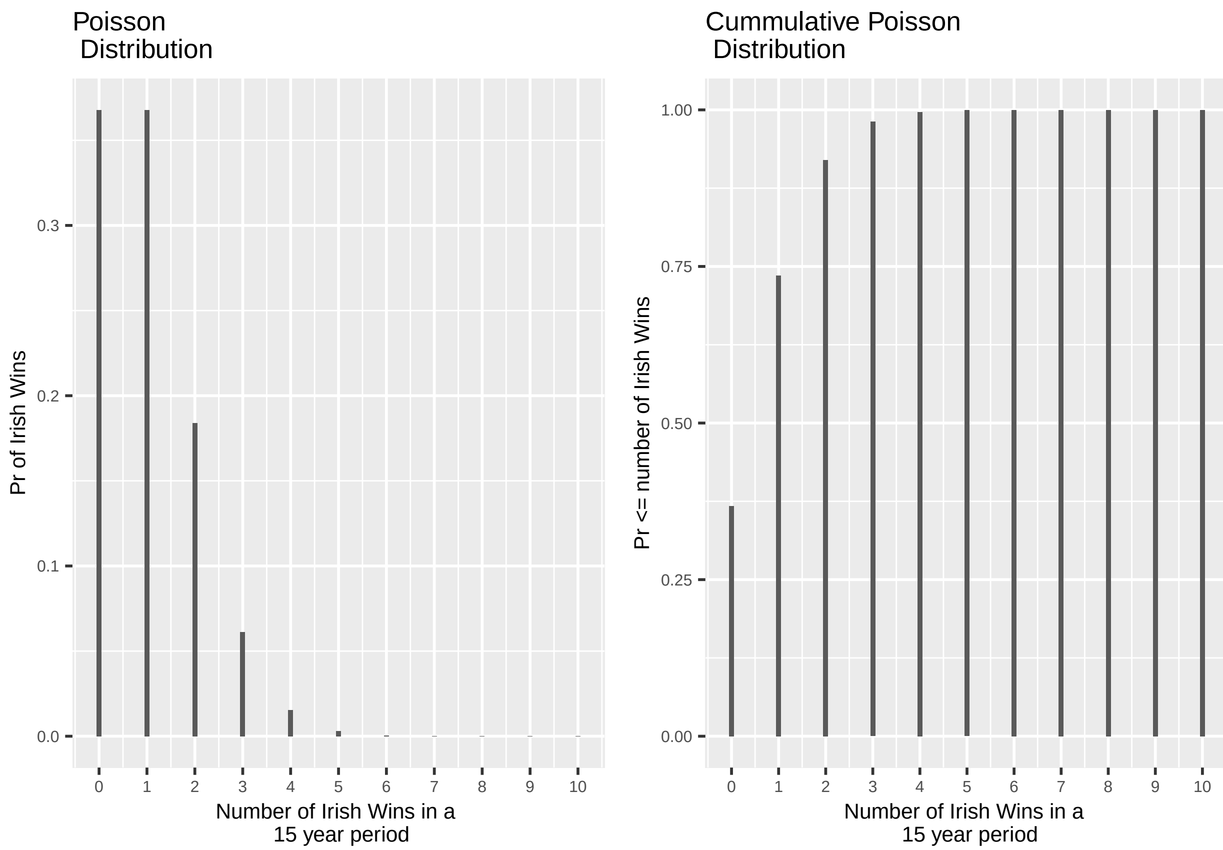

# Poisson Distribution

## Definition:

$$\Pr(k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, ...$$

$$E[k] = \lambda, \quad Var[k] = \lambda.$$

In R code:




# Normal Distribution

## Definition:

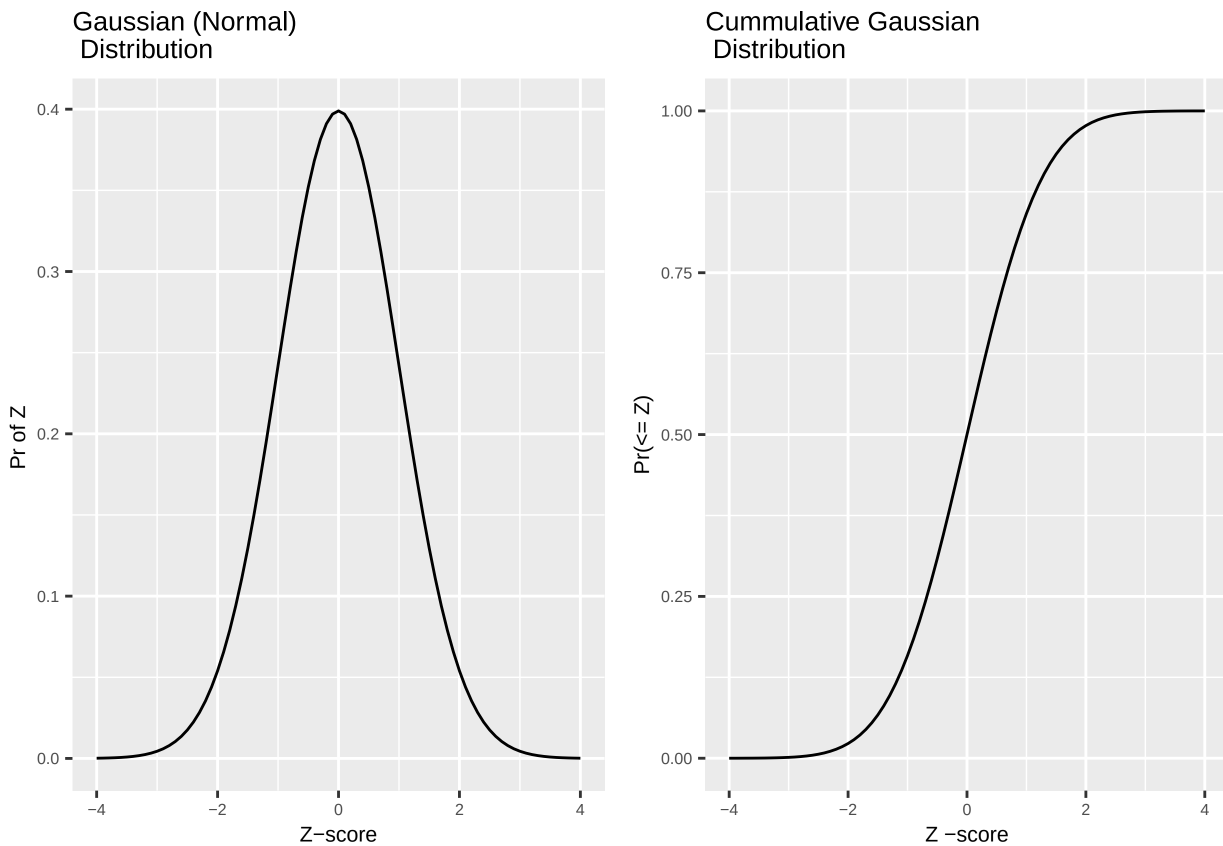
$$\Pr(k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad k = 0, 1, 2, ...$$

$$E[k] = \lambda, \quad Var[k] = \lambda.$$

In R code:

```
## Warning: Ignoring unknown parameters: width
```

```
## Warning: Ignoring unknown parameters: width
```


Gaussian (Normal)
Distribution
Pr of Z
Z−score
Cummulative Gaussian
Distribution
Pr(<= Z)
Z −score

## Confidence Intervals