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Automatic Parameter Setting Method for an Accurate Kalman Filter Tracker Using an Analytical Steady-State Performance Index

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ABSTRACT We present an automatic parameter setting method to achieve an accurate second-order Kalman filter tracker based on a steady-state performance index. First, we propose an efficient steady-state performance index that corresponds to the root-mean-square (rms) prediction error in tracking. We then derive an analytical relationship between the proposed performance index and the generalized error covariance matrix of the process noise, for which the automatic determination using the derived relationship is presented. The model calculated by the proposed method achieves better accuracy than the conventional empirical model of process noise. Numerical analysis and simulations demonstrate the effectiveness of the proposed method for targets with accelerating motion. The rms prediction error of the tracker designed by the proposed method is 63.8% of that with the conventional empirically selected model for a target accelerating at 10 m/s^2.

INDEX TERMS Tracking filter, Kalman filter, parameter setting, steady-state performance, process noise.

I. INTRODUCTION

Various remote monitoring systems for machines such as cars and robots require accurate tracking of moving targets. Representative tracking methods include the Kalman filter [1] and variants such as the extended Kalman and particle filters [2]–[9]. These can accurately track movement based on adaptive filtering, which minimizes the error in the predicted state vector based on dynamical and measurement models. Thus, tracking filters have many applications across a range of technical areas including radar [3], [4], visual tracking with video [5], [6], underwater vehicle navigation [7], sonar [8], and global positioning systems [9].

For practical uses of a Kalman filter tracker, we must determine a dynamical model of the target to achieve sufficient tracking performance. Effective models have been proposed for various target motions such as those based on constant turn rate [10]. For the tracker to predict both position and velocity (second-order tracker), the most commonly used dynamical system is a constant velocity (CV) model [1], [10], [11]. This model assumes that the target velocity is constant during a sampling interval in which the interval values are a random sequence known as process noise. A large number of applications have used this model because of its effectiveness and simplicity. However, in conventional tracking systems, the selection of process noise and its parameters is conducted empirically [4]. This is because conventional studies assume that process noise takes one of a limited number of forms, known as appropriate selections. Thus, despite the large number of investigations into Kalman filter trackers over the past few decades, the optimality of the selection of process noise model has not been discussed.

To ensure an appropriate setting of process noise, a reasonable evaluation of the filter performance is required. However, no valid index of tracking performance has been established. The most general and practical filter performance index is the root-mean-square (RMS) tracking error [12]. However, this is calculated from Monte Carlo simulations with a trial-and-error process, and there are many cases where it is difficult to employ appropriate simulation settings. As an analytical performance evaluation method, the use of an α-β filter (a steady-state Kalman filter) has been considered [11]–[17]. Various steady-state performance indices have been proposed, such as the error covariance matrix [13] and the closed-form tracking error for basic motions [11], [14]–[17]. However, these conventional studies again only assumed limited types of process noise. In addition, it is difficult to determine appropriate or optimal model parameters with some representative performance indices, because the relationships between the preset parameters and most indices are monotonic. Hence, empirical filter design is unavoidable. Note that a detailed review of these parameter setting and performance evaluation aspects of the Kalman filter tracker is provided in Section III.

The general problems of model selection and filter performance evaluation for Kalman filter trackers were reviewed and discussed by Ekstrand in 2012 [1]. In the years since that paper, further research on these points has been conducted. However, no sufficient solution to the above problems has been presented. Crouse [18] described a general solution for optimal trackers with steady-state Kalman gains, such as α-β filters. However, this method also requires appropriate selection of the dynamical models to calculate optimal gains. For various applications including global navigation satellite systems [19] and video tracking systems [20], a detailed analysis of the Kalman filter tracker is given. These studies investigated the relationship between performance and model parameters. However, only a limited system was considered, and no concrete parameter setting procedure for the tracker was provided. In our previous work [21], [22], the relationship between the analytical performance index and the filter gains are investigated for a position/velocity measured tracker. However, the motion model selection criterion is not presented. Consequently, although various studies and simulations of the Kalman filter tracker have been reported, these suffer from the problems described above. Thus, the establishment of an efficient and analytical filter performance index to achieve accurate Kalman filter trackers is important.

As a solution to these problems, this paper proposes an efficient steady-state performance index, and describes its application in an appropriate parameter setting method for process noise of the dynamical model of a second-order Kalman filter tracker. The proposed index corresponds to the steady-state RMS error for a target moving under constant acceleration, and is analytically calculated with a single, simple parameter. The analytical relationship between the proposed index and the generalized error covariance matrix for process noise is derived. Based on this result, the proposed process noise setting method is presented. Numerical analyses show that we can automatically determine process noise that provides better tracking accuracy than conventional models. The validity and effectiveness of the proposed method for a moving target is verified in these numerical simulations.

The remainder of this paper is organized as follows. Section II introduces the basics of Kalman filter trackers, and Section III reviews aspects of their process noise setting and performance evaluation. The typical covariance matrix of process noise and conventional filter performance indices are summarized, and their problems are clarified. Section IV presents the proposed steady-state performance index and parameter setting method. The validity and effectiveness of the proposed method is investigated in Section V via theoretical analyses and numerical simulations. Section VI presents our conclusions and plans for future work.

II. KALMAN FILTER TRACKER

This section introduces the Kalman filter for target tracking, and defines the model assumed in this paper.

A. BASIC MODEL AND ALGORITHM

The Kalman filter estimates a state vector composed of the target's parameters such as position and velocity based on a dynamic/measurement model. The typical dynamical model is

$$\boldsymbol{x}_{\mathbf{t}k} = \boldsymbol{\Phi}\boldsymbol{x}_{\mathbf{t}k-1} + \boldsymbol{w}_k, \tag{1}$$

where $\boldsymbol{x}_{\mathbf{t}k}$ denotes the true target state vector at time kT, T the sampling interval, $\boldsymbol{\Phi}$ the transition matrix from kT to $(k+1)T$, and $\boldsymbol{w}_k$ the process noise with covariance matrix $\boldsymbol{Q}$. The measurements of the target are modeled as

$$z_k = \boldsymbol{H}\boldsymbol{x}_{\mathbf{t}k} + \boldsymbol{v}_k, \tag{2}$$

where z_k denotes the measurement vector, $\boldsymbol{H}$ the measurement matrix, and $\boldsymbol{v}_k$ the measurement noise with covariance matrix $\boldsymbol{R}$.

The Kalman filter tracker for the above model sequentially estimates state vectors via the following equations:

$$\tilde{\boldsymbol{x}}_k = \boldsymbol{\Phi}\hat{\boldsymbol{x}}_{k-1}, \tag{3}$$

$$\hat{\boldsymbol{x}}_k = \tilde{\boldsymbol{x}}_k + \boldsymbol{K}_k(z_k - \boldsymbol{H}\tilde{\boldsymbol{x}}_k), \tag{4}$$

where forecasts and estimates are denoted by $\tilde{}$ and $\hat{}$, respectively, and $\boldsymbol{K}_k$ denotes the Kalman gain that minimizes the errors in the estimated state vectors. $\boldsymbol{K}_k$ is calculated from

$$\boldsymbol{K}_k = \tilde{\boldsymbol{P}}_k\boldsymbol{H}^{\mathrm{T}}(\boldsymbol{H}\tilde{\boldsymbol{P}}_k\boldsymbol{H}^{\mathrm{T}} + \boldsymbol{R})^{-1}, \tag{5}$$

where $^{\mathrm{T}}$ denotes the transpose, and $\boldsymbol{P}_k$ the covariance matrix of errors determined from

$$\tilde{\boldsymbol{P}}_k = \boldsymbol{\Phi}\hat{\boldsymbol{P}}_{k-1}\boldsymbol{\Phi}^{\mathrm{T}} + \boldsymbol{Q}, \tag{6}$$

$$\hat{\boldsymbol{P}}_k = \tilde{\boldsymbol{P}}_k - \boldsymbol{K}_k\boldsymbol{H}\tilde{\boldsymbol{P}}_k. \tag{7}$$

Equations (3)–(7) are known as the Kalman filter equations.

B. MODEL FOR TRACKER

Although models have been proposed for various trackers [10], this paper deals with the most commonly used second-order model, which has the state vector

$$\boldsymbol{x}_k = (x \quad v)^{\mathrm{T}}, \tag{8}$$

where x and v denote position and velocity, respectively. In this setting, one-dimensional (1D) tracking is only considered. However, applications to two-dimensional (2D) tracking and more than 2D tracking are easily realized by implementing the tracker for each axis. Although this paper mainly considers a 1D tracking problem, an application to 2D tracking with a numerical simulation is also presented in the final section. The common model assumes that the velocity is constant during the sampling interval. The measurement variable is the position, and its error is white Gaussian noise. Thus, $\boldsymbol{\Phi}$, $\boldsymbol{H}$, and $\boldsymbol{R}$ are expressed as

$$\boldsymbol{\Phi} = \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix}, \tag{9}$$

$$\boldsymbol{H} = (\,1\ 0\,), \tag{10}$$

$$\boldsymbol{R} = (\sigma_{\mathrm{x}}^2), \tag{11}$$

where σ_{x}^2 is the variance of the measurement errors. The dynamical model using (9) is called a CV model.

As indicated in (3)–(7), we must set the covariance matrix of the process noise $\boldsymbol{Q}$ to realize tracking errors that are as small as possible. A detailed discussion on the conventional setting method of $\boldsymbol{Q}$ and its problems is given in the next section.

III. ASPECTS OF PROCESS NOISE SETTING FOR KALMAN FILTER TRACKER

The main questions in the parameter setting of the Kalman filter with the CV model are:

- How can we select the most appropriate $\boldsymbol{Q}$?
- What is "appropriate"? That is, how can we evaluate the filter performance?

Ekstrand [1] pointed out that, although the study of Kalman filter trackers has a long history, answers to these questions have not been sufficiently debated. This is because most studies on tracking filters deal with estimation problems (i.e., present new tracking algorithms) rather than problems in its parameter setting. That is, the selection of $\boldsymbol{Q}$ is a problem that has not been given much attention in previous studies. Consequently, the following subsections provide a detailed review on the selection of $\boldsymbol{Q}$, performance evaluations for filter parameter setting, and the problems encountered in conventional studies.

A. COVARIANCE MATRIX OF PROCESS NOISE

In this section, typical covariance matrices of the process noise for the CV model are explained. The most commonly used process noise is random acceleration, for which the covariance matrix is expressed as [4], [10]

$$\boldsymbol{Q}_{\mathrm{DNCV}} = \begin{pmatrix} T^4/4 & T^3/2 \\ T^3/2 & T^2 \end{pmatrix} \sigma_{\mathrm{aw}}^2. \tag{12}$$

In this model, $\boldsymbol{w}_k = (T^2/2 \quad T)^{\mathrm{T}} w_{\mathrm{a}}$, where w_{a} is white Gaussian acceleration with variance σ_{aw}^2. This model is defined directly in discrete time. This paper refers to the CV model with $\boldsymbol{Q}_{\mathrm{DNCV}}$ as a discrete-time nearly constant velocity (DNCV) model. The same model as in (12) can also be defined in continuous time, and the covariance matrix for this model is [10]

$$\boldsymbol{Q}_{\mathrm{CNCV}} = \begin{pmatrix} T^4/3 & T^3/2 \\ T^3/2 & T^2 \end{pmatrix} \sigma_{\mathrm{aw}}^2. \tag{13}$$

The CV model with $\boldsymbol{Q}_{\mathrm{CNCV}}$ is referred to as a continuous-time nearly constant velocity (CNCV) model. Random velocity process noise, introduced by Benedict and Bordner [15], is also an option in which $\boldsymbol{Q}$ is expressed as [11], [15], [21]

$$\boldsymbol{Q}_{\mathrm{BB}} = \begin{pmatrix} T^2 & T \\ T & 1 \end{pmatrix} \sigma_{\mathrm{vw}}^2. \tag{14}$$

In this model, $\boldsymbol{w}_k = (T \quad 1)^{\mathrm{T}} w_{\mathrm{v}}$ where w_{v} is white Gaussian velocity of variance σ_{vw}^2. We refer to this CV model using $\boldsymbol{Q}_{\mathrm{BB}}$ as the Benedict–Bordner (BB) model.

The appropriate selection of these models and their parameters (σ_{aw} or σ_{vw}) is important, because it determines the performance of the tracking filter with the CV model. However, in conventional studies, parameter values are empirically selected, and the validity of the selection is discussed only casually [4], [20]. Many conventional tracking systems select the DNCV model, with variance σ_{aw} set based on the assumed target motion. For example, if the target has a relatively complicated motion (i.e., the target has a large range of acceleration), a relatively large value of σ_{aw} is selected [10]. However, no concrete method of determining σ_{aw} has been established. Hence, there is no guarantee that the setting of σ_{aw} in conventional systems is appropriate. Moreover, the validity in using the DNCV model is also questionable. Various other forms of $\boldsymbol{Q}$ are known and have been used for different target motions [10]. However, for the reasons discussed above, the difference in performance between the various models is not known. Consequently, Kalman filter trackers require a clear and objective method for setting $\boldsymbol{Q}$.

B. STEADY-STATE PERFORMANCE INDEX

To set $\boldsymbol{Q}$, an objective performance index is required. The most important performance index of the tracker is the steady-state prediction error. Several researchers have investigated various σ_{aw} or gains for a steady-state Kalman filter (known as an α-β filter [14], [21]) based on some steady-state performance index [11]–[17]. In the following, representative performance indices and their problems are explained.

1) $\tilde{P}$ OF THE KALMAN FILTER EQUATION

This is typically used to evaluate the Kalman filter tracker [11], [13]. However, $\tilde{\boldsymbol{P}}$ overrates the variance in the errors that is caused by measurement noise, as verified by Ekstrand (see [1, Sec. 9.8]).

2) RMS PREDICTION ERROR

The RMS prediction error calculated with Monte Carlo simulations is an effective metric for the filter performance, because it represents a natural and practical index. Therefore, most studies conduct Monte Carlo simulations to demonstrate improvements in the RMS prediction error using their proposed algorithms. However, for the filter parameter setting, an iterative trial-and-error process with varying filter parameters is required. Moreover, there are some cases in which appropriate simulation settings are difficult to implement. Some researchers have used an analytically calculated RMS prediction error [12], but their effectiveness and the selection of dynamical models using these errors have not been sufficiently discussed.

3) MEASUREMENT ERROR FOR CONSTANT VELOCITY TARGET (SMOOTHING PERFORMANCE INDEX)

An important function of the tracking filter is the reduction of random errors caused by measurement noise. One index of this performance is the steady-state error of a target under constant velocity considering sensor noise. We assume that the steady-state Kalman gain is [21]

$$\boldsymbol{K}_\infty = \begin{pmatrix} \alpha \\ \beta/T \end{pmatrix}. \tag{15}$$

The measured position contains noise with variance σ_x^2, and the target moves with constant velocity. The variance of the predicted target position in the steady state is calculated as a function of σ_x, α, and β as [16], [17]:

$$\sigma_p^2 = E[(x_{tvk} - x_{pk})^2] = \frac{2\alpha^2 + 2\beta + \alpha\beta}{\alpha(4 - 2\alpha - \beta)}\sigma_x^2, \tag{16}$$

where $E[\ ]$ denotes the mean with respect to k, and x_{pk}, x_{tvk} are the predicted and true positions, respectively, of the constant velocity target. Note that the mean error $E[x_{pk} - x_{tk}]$ is zero, because the assumed target motion is the same as the motion model. σ_p^2 is called the smoothing performance index [22]. This index is more effective in evaluating the steady-state tracking accuracy than the error covariance matrix in the Kalman filter equation, because no overestimation occurs [1].

4) BIAS ERROR FOR CONSTANT ACCELERATION TARGET (TRACKING PERFORMANCE INDEX)

The tracking filter is required to track motion that may include acceleration. If the Kalman filter tracker involves the CV model, some steady-state bias error occurs when tracking a target moving with constant acceleration. This error is an index of the tracking performance. If $x_{tak} = a_c(kT)^2/2$ (where a_c is the constant acceleration) and the measurement errors are not considered, the steady-state predicted error is expressed as [16], [17]:

$$e_{fin} = \lim_{k\to\infty} (x_{tak} - x_{pk}) = a_c T^2/\beta. \tag{17}$$

Here e_{fin} is called the tracking performance index [22], and effectively evaluates the performance for an accelerating target tracking.

C. DISCUSSION AND SUMMARY

As described in the previous section, $\tilde{\boldsymbol{P}}$ is misleading as an evaluator of tracker performance, and the calculation of the RMS prediction error requires the iterative Monte Carlo simulations. However, e_{fin} and σ_p^2 might be effective for parameter setting of the Kalman filter based on steady-state performance. The smaller these indices are, the better the tracking filter. However, there is a trade-off between e_{fin} and σ_p^2. Figure 1 shows example calculations of e_{fin}, σ_p^2 for the DNCV model, where

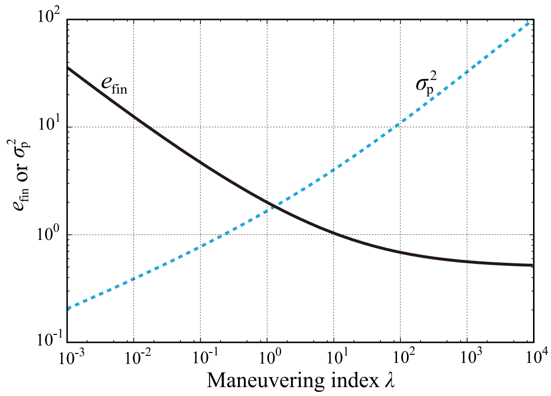


FIGURE 1. Example calculation of e_{fin} and σ_p^2 for DNCV model ($\sigma_x = T = a_c = 1$).

$$\lambda = \sigma_{aw} T^2/\sigma_x \tag{18}$$

is the so-called target maneuvering index [14], with $\sigma_x = T = a_c = 1$. The following known relations [1] between λ and the filter gains were used to calculate the curves in Figure 1:

$$\alpha = -(\lambda^2 + 8\lambda - (\lambda + 4)\sqrt{\lambda^2 + 8\lambda})/8, \tag{19}$$
$$\beta = (\lambda^2 + 4\lambda - \lambda\sqrt{\lambda^2 + 8\lambda})/4. \tag{20}$$

The trade-off between e_{fin} and σ_p^2 is clearly illustrated in Figure 1. Moreover, e_{fin} and σ_p^2 are monotonically decreasing and increasing functions of λ (σ_{aw}). This means that no optimal selection with respect to these indices exists, and we cannot avoid the empirical setting of trackers. Tenne and Singh [17] used circular trajectories based on these indices to set optimal gains for the α-β filter. However, the parameter setting of a Kalman filter was not discussed in their study.

In addition, conventional analyses of these indices assume only the limited models described in Section III-A. Therefore, the relationship between some arbitrary $\boldsymbol{Q}$ and the performance indices has not been clarified. To summarize, the problems with the parameter selection of a CV Kalman filter tracker in most conventional systems are:

- A few limited models such as $\boldsymbol{Q}_{DNCV}$ are empirically used without adequate consideration.
- No appropriate filter performance index has been established.

IV. PROPOSED METHOD FOR SETTING Q

To solve the problems discussed in previous sections, an automatic setting method for $\boldsymbol{Q}$ of the Kalman filter tracker is now presented. We first propose a steady-state performance index based on the tracking/smoothing performance indices of (16) and (17). The relationship between an arbitrary $\boldsymbol{Q}$ and the proposed performance index is then clarified analytically. Finally, the determination of the optimal $\boldsymbol{Q}$ with respect to the proposed performance index is presented as the proposed parameter setting method.

A. PROPOSED PERFORMANCE INDEX

1) DEFINITION

As described in the previous section, the RMS prediction error is the practical performance index. The proposed performance index corresponds to the RMS prediction error for a target moving with constant acceleration in the steady state. Based on the derivation of the RMS prediction error presented in Appendix A, a proposed performance index is defined as the function of the steady-state gains of (15) as

$$\mu \equiv \sqrt{\frac{2\alpha^2+2\beta+\alpha\beta}{\alpha(4-2\alpha-\beta)}+\frac{a_{\mathrm{D}}^2}{\beta^2}}, \tag{21}$$

where

$$a_{\mathrm{D}} \equiv a_{\mathrm{c}}T^2/\sigma_{\mathrm{x}}. \tag{22}$$

Clearly, the smaller the value of μ, the better the tracking filter. As shown in (16), (17), (21), and (22), the proposed index is $(\sqrt{\sigma_{\mathrm{p}}^2+e_{\mathrm{fin}}^2})/\sigma_{\mathrm{x}}^2$. Consequently, the optimal point is generated by adding the smoothing and tracking performance indices as indicated in Figure 1 and the parameter a_{D} is considered to control the balance of these indices in the performance evaluation using the proposed index μ.

We now discuss the difference between the proposed index and the conventional preset parameter and cost function presented in related works. The proposed index μ is similar to the cost function defined in [17] for the α-β filters. However, μ and the parameter a_{D} have a clear physical meaning for Kalman filter tracking compared with this cost function and its parameter (μ and a_{D} correspond to the RMS error and target acceleration). Additionally, a_{D} is similar to the maneuvering index λ of (18). However, a_{D} is directly given by the target acceleration, unlike λ, which is expressed as a standard deviation [14]. Therefore, the proposed index can be easy to use compared with these conventional preset parameters and functions.

2) EVALUATION EXAMPLES FOR COMMON MODELS

First, examples of performance evaluation using the proposed index are presented for the conventional DNCV, CNCV, and BB models. The steady-state gains α and β are calculated using their relationship with $\boldsymbol{Q}$, which is derived in the next section. Figure 2 shows some example calculations of μ for $a_{\mathrm{D}} = 0.1$ and 10. Here, the maneuvering index for the BB model is defined as $\lambda = \sigma_{\mathrm{vw}}T/\sigma_{\mathrm{x}}$. As shown in Figure 2(a), we can select an optimal λ for each a_{D}, and the optimal performance of each model is very similar. This means that there is no performance difference between the conventional models for relatively small a_{D} when we set an (almost) optimal value of λ. In contrast, Figure 2(b) indicates that the DNCV model realizes better performance than the other models for large values of a_{D}. Overall, the DNCV model realizes the best performance of the conventional models for various a_{D}. This indicates that the empirical selection of DNCV in conventional tracking systems was appropriate. Moreover, we can determine the optimal value

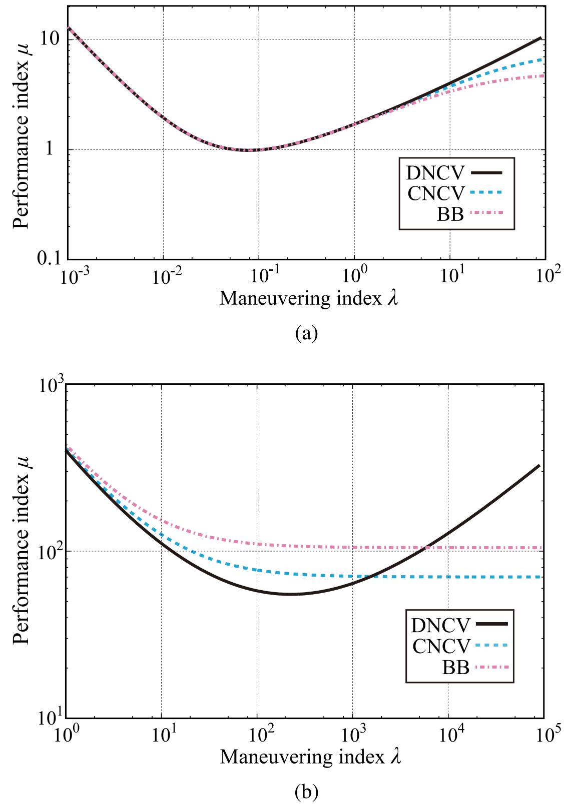


FIGURE 2. Calculation examples of μ in conventional models. (a) $a_{\mathrm{D}} = 0.1$. (b) $a_{\mathrm{D}} = 10$.

of λ in the DNCV model with respect to μ by setting a_{D} using the assumed target acceleration.

3) COMPARISON WITH MONTE CARLO SIMULATION

Next, we investigate the validity of the evaluation using μ by comparing the results with those from Monte Carlo simulations. In the simulations, $T = 1$ s and $\sigma_{\mathrm{x}} = 1$ m. The target is assumed to be moving with nearly-constant acceleration, for which the true acceleration is $a_{\mathrm{c}tk} = a_{\mathrm{c}}+w_{\mathrm{ac}k}$, where $w_{\mathrm{ac}k}$ is a zero-mean white Gaussian random acceleration with standard deviation σ_{ac}. This target motion is intended to demonstrate the effectiveness of the proposed method for motion other than constant acceleration, including random variations in acceleration and jerk. The DNCV model is assumed, and λ is determined as the argument that minimizes μ for each a_{D}. The minimum calculated value of μ is defined as μ_{dncv}. We compare μ_{dncv} with the mean RMS prediction error given by the simulations. The RMS prediction error is evaluated over 1000 Monte Carlo simulations, and is defined as

$$\mathrm{RMS}_k = \sqrt{\frac{1}{1000}\sum_{m=1}^{1000}(x_{\mathrm{t}k}-x_{\mathrm{p}mk})^2}, \tag{23}$$

where $x_{\mathrm{p}mk}$ is the predicted position in the m-th Monte Carlo simulation.

Table 1 presents the results of analyses and simulations for $a_{\mathrm{c}} = 0.1$, 1, and 10 m/s^2 (these values are equal to a_{D},

TABLE 1. Comparison of analytical error μ_{dncv} with the RMS prediction error in simulations.

a_{D}	μ_{dncv}	$E[\mathrm{RMS}_k]$ ($\sigma_{\mathrm{ac}}=0$)	$E[\mathrm{RMS}_k]$($\sigma_{\mathrm{ac}}=a_{\mathrm{c}}/20$)	$E[\mathrm{RMS}_k]$($\sigma_{\mathrm{ac}}=a_{\mathrm{c}}/10$)	$E[\mathrm{RMS}_k]$($\sigma_{\mathrm{ac}}=a_{\mathrm{c}}/5$)
0.1	0.986	0.986 m	0.986 m	0.985 m	0.984 m
1	4.49	4.49 m	4.49 m	4.47 m	4.56 m
10	55.2	55.1 m	55.7 m	56.8 m	60.7 m

TABLE 2. Summary of the conventional and proposed setting method for $\boldsymbol{Q}$.

	Empirical model selection	Preset parameter	Analytical performance index	Tracking accuracy
Conventional	Required	λ	Not so used	Low
Proposed	Not required (Automatically determined)	a_{D}	Used	High

as shown in (22)) and $\sigma_{\mathrm{ac}} = 0$, $a_{\mathrm{c}}/20$, $a_{\mathrm{c}}/10$, and $a_{\mathrm{c}}/5$. From this table, we see that the mean RMS prediction error $E[\mathrm{RMS}_k]$ almost matches the theoretical values of μ_{dncv}, even for cases in which the target acceleration includes random variations. However, if $a_{\mathrm{D}} = 10$ and $\sigma_{\mathrm{ac}} = a_{\mathrm{c}}/5$, there is a relatively large difference between the analysis and simulation values. This suggests that the proposed index is ineffective for targets that have both comparatively large acceleration and jerk. However, with the exception of such cases, the proposed index accurately indicates the RMS prediction error. In selecting the CV model, targets with large jerk values are generally not assumed. Thus, we have verified that the proposed index μ is an effective means of evaluating the tracking accuracy.

However, we can only determine the optimal σ_{aw} or σ_{vw} in the conventional models from the above results. To determine a more effective $\boldsymbol{Q}$, the relationship between an arbitrary $\boldsymbol{Q}$ and μ must be derived.

B. ARBITRARY PROCESS NOISE AND PROPOSED PERFORMANCE INDEX

This section considers the arbitrary $\boldsymbol{Q}$ defined as

$$\boldsymbol{Q}_{\mathrm{gen}} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}, \tag{24}$$

where $a > 0$, $b > 0$, $c > 0$, and the dimensions of a, b, and c are [m^2], [m^2/s], and [m^2/s^2], respectively. We derive the relationship between (a, b, c) and the steady-state gains (α, β) to obtain $\mu(a, b, c, a_{\mathrm{D}})$ from the Kalman filter equations of (3)–(7). Using this relationship, we can calculate the optimal (a, b, c) for various a_{D}, which is defined as $(a_{\mathrm{des}}, b_{\mathrm{des}}, c_{\mathrm{des}})$.

The derivation of $\mu(a, b, c, a_{\mathrm{D}})$ is given in Appendix B and its result is

$$\mu(a, b, c, a_{\mathrm{D}}) = \left[\frac{D^4 - 2CD^3 - 32CD^2 + 96C^2D + 256C^2}{(D^3 + 16C - 2CD)(16C - D^2)} + \frac{16a_{\mathrm{D}}^2}{D^2}\right]^{1/2}, \tag{25}$$

where,

$$C \equiv cT^2/\sigma_{\mathrm{x}}^2. \tag{26}$$

$$D \equiv C + D_1 - \sqrt{2(D_1^3 + D_2)}, \tag{27}$$

$$D_1 \equiv \sqrt{C(16 + 4A - 4B + C)}, \tag{28}$$

$$D_2 \equiv C(2A - 2B + C), \tag{29}$$

$$A \equiv a/\sigma_{\mathrm{x}}^2, \tag{30}$$

$$B \equiv bT/\sigma_{\mathrm{x}}^2. \tag{31}$$

By seeking the minimum of $\mu(a, b, c, a_{\mathrm{D}})$ for the given a_{D}, we have a Kalman filter tracker that realizes optimal steady-state accuracy corresponding to the RMS prediction error.

C. PROCEDURE AND FEATURE

The proposed setting method of $\boldsymbol{Q}$ is composed of the following two steps:

1) Preset a_{D} based on the target's assumed acceleration.
2) Calculate $(a_{\mathrm{des}}, b_{\mathrm{des}}, c_{\mathrm{des}})$ by

$$\begin{aligned} &\arg\min_{a,b,c} \ \mu(a, b, c, a_{\mathrm{D}}) \\ &\text{sub. to } a > 0,\ b > 0,\ c > 0. \end{aligned} \tag{32}$$

Table 2 summarizes the features of the conventional and proposed methods. As shown in this table, although the number of preset parameter is the same (one) for both methods, the proposed method does not require the empirical selection of the process noise model. The proposed method automatically determines the optimal $\boldsymbol{Q}$ by solving (32) from the preset parameter a_{D}, which is determined from the target acceleration, to be discussed in the next section. In contrast, the process noise model (such as DNCV, CNCV, and BB) is empirically selected without objective performance index in the conventional method. Moreover, the proposed method can achieve better tracking accuracy than the conventional method and this is verified in the next section.

V. APPLICATION EXAMPLES AND DISCUSSION

In this section, numerical analyses and simulations are presented to demonstrate the parameter setting of a Kalman filter tracker using the proposed method and its application to a maneuvering target. These examples show that the proposed method determines $\boldsymbol{Q}$ so as to achieve better tracking accuracy than the conventional DNCV model.

A. PERFORMANCE ANALYSIS OF PROPOSED METHOD AND COMPARISON WITH CONVENTIONAL MODELS

We present examples of the Kalman filter tracker with the proposed method for various a_{D}. We assume that T and σ_{x}

TABLE 3. Design results given by the proposed method and comparison with DNCV model.

a_D	a_{des}	b_{des}	c_{des}	μ_{opt}	μ_{dncv}
0.01	5.13×10^{-4}	1.03×10^{-3}	2.05×10^{-3}	0.315	0.315
0.1	0.135	0.464	0.0633	0.909	0.986
1	0.470	2.48	1.39	3.82	4.49
10	7.01	13.0	9.20	35.2	55.2
100	10.2	5.01×10^3	2.00×10^4	2.79×10^3	2.79×10^3

are normalized to 1. Table 3 presents the calculation results for $(a_{des}, b_{des}, c_{des})$ and the corresponding minimum performance index value μ_{min}. This table also gives the results for the DNCV model μ_{dncv}. For $a_D = 0.1, 1, 10$, the optimal $\boldsymbol{Q}$ determined by the proposed method achieves better performance than the DNCV model. As indicated in the previous section, DNCV is the best of the conventional models. Hence, the proposed method determines a better setting for $\boldsymbol{Q}$ than conventional empirical models. The conventional models of (12)–(14) do not obtain optimal settings $(a_{des}, b_{des}, c_{des})$ for $a_D = 0.1$, 1, and 10. Figure 3 shows the relationship between the ratio μ_{min}/μ_{dncv} and a_D. If $10^{-2} < a_D < 100$, the proposed method achieves better accuracy. In particular, for $a_D = 10$, we can confirm a significant performance improvement using the proposed method: μ_{min} is 63.8% of μ_{dncv}. The above results indicate that the proposed method automatically calculates the optimal $\boldsymbol{Q}$ from the single preset parameter a_D, and the performance with this $\boldsymbol{Q}$ is better than that given by conventional models. Note that $(a_{des}, b_{des}, c_{des})$ are easily calculated for various a_D using the simple gradient descent method [23]. The required time to solve (32) is less than 10 s for all cases (using an Intel CORE i7-4600U CPU@2.70 GHz). This time is acceptable, because the calculation of $\boldsymbol{Q}$ is conducted during filter parameter selection, before its application in a tracking system.

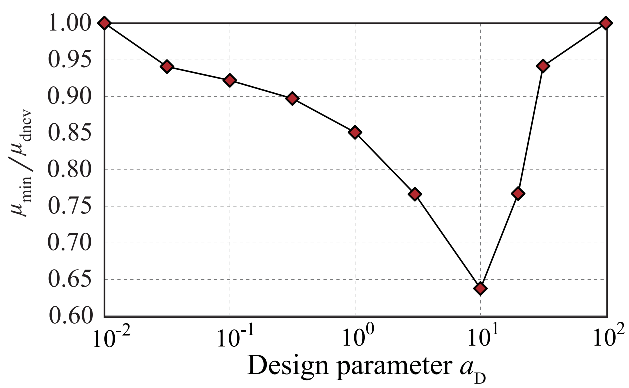


FIGURE 3. Relationship between the ratio μ_{min}/μ_{dncv} and a_D.

We now consider why $\boldsymbol{Q}$ is more accurate in the proposed method than in conventional models. The DNCV, CNCV, and BB models assume white Gaussian process noise. In contrast, the proposed index μ is defined by the tracking/smoothing performance indices e_{fin} and σ_p^2 based on two motion models: CV and constant acceleration. We believe that μ effectively harmonizes the performance of these antithetical models and this harmonization is considered by assuming arbitrary $\boldsymbol{Q}$. Moreover, the assumption of white process noise is not used in the derivation of e_{fin} or σ_p^2 [16]. Hence, it seems that conventional models cannot obtain optimal $\boldsymbol{Q}$ with respect to μ. In contrast, if a_D is quite small or large, there is no difference between μ given by the conventional and proposed models, as shown in Figure 3. In these cases, one of the performance indices (e_{fin} or σ_p^2) is dominant in calculating μ, and the considered motion is therefore similar to a CV or constant acceleration model. Consequently, the harmonics of the performance indices are not considered, and there is no performance difference between the conventional and proposed models. However, to clarify the precise reason behind the results in Table 3, further analysis considering dynamical models other than the CV model is required.

B. SIMULATION ASSUMING MANEUVERING TARGET

This section presents the application example of the numerical simulation of a maneuvering target. First, the appropriate setting of a_D for a non-constant accelerating target is discussed with 1D tracking simulation. Then, the performance for a realistic 2D system is evaluated.

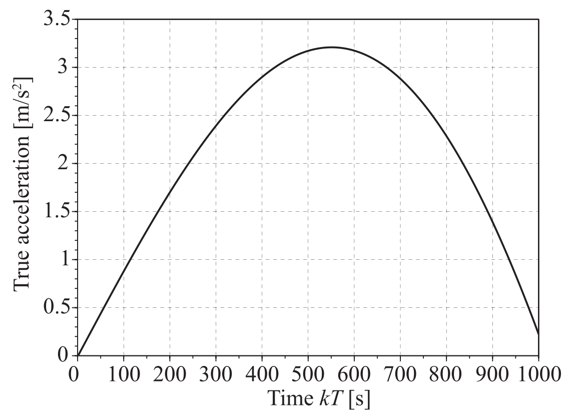


FIGURE 4. True acceleration of maneuvering target in 1D simulation.

1) 1D TRACKING EXAMPLE AND DISCUSSION FOR SETTING a_D

Figure 4 shows the true acceleration of the assumed target. The motion of this target is $x_{tk} = (k)^2 \sin(2\pi k/(4200))$. We set $T = 1$ s and $\sigma_x = 1$ m. The initial values of the state vector and error covariance matrix are all zero. The RMS prediction errors from the DNCV model with optimal λ are compared with those given by the optimal $\boldsymbol{Q}$ determined by the proposed method. The RMS prediction error is calculated by 1000 Monte Carlo simulations, as in (23).

We evaluate two cases: $a_D = 1$ and 3. These values can be considered as rough approximate values of the target acceleration in Figure 4. Figure 5 shows the results of the DNCV model and the optimal tracker determined by the proposed method. In both cases, the proposed method achieves better performance than the DNCV model with optimal λ.

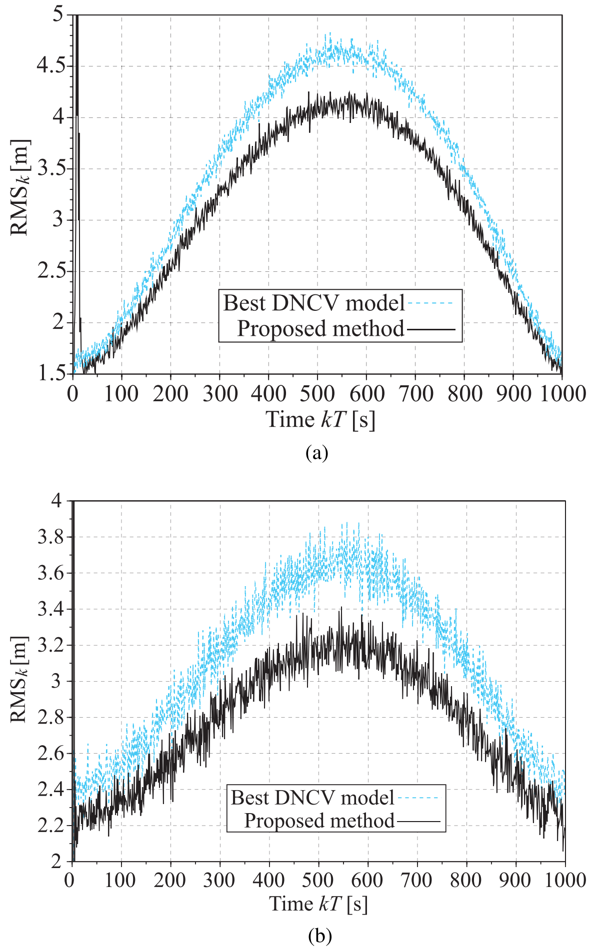


FIGURE 5. Simulation results for 1D maneuvering target. (a) $a_D = 1$. (b) $a_D = 3$.

This implies that the proposed method is effective for a maneuvering target, and achieves optimal RMS prediction errors with the automatically calculated $\boldsymbol{Q}$ from the appropriate a_{D}.

Based on these results, the appropriate setting for a_{D} in practical use is discussed. The matrix $\boldsymbol{Q}$ determined by the proposed method is only optimal if a_{D} is matched to the target acceleration and the target is moving with constant acceleration corresponding to a_{D}. This is because μ corresponds to the RMS prediction error for a target moving under constant acceleration. However, using the proposed method, the tracking accuracy is always better than when using conventional models, as shown in Figure 5 for both $a_{\mathrm{D}} = 1$ and 3. Consequently, the proposed method achieves appropriate accuracy, even if a_{D} is not matched to the true target acceleration. This means that the relatively small difference between the true and set acceleration is acceptable. Thus, in practical use, we estimate an approximate or typical value for the acceleration (eg., mean and maximum) in advance based on the assumed motion of the target, and then set a_{D} using this estimated value. Note that we can realize more accurate tracking of a maneuvering target using an interacting multiple model approach [24]. Applying the proposed parameter setting method to such an approach will be considered in future work.

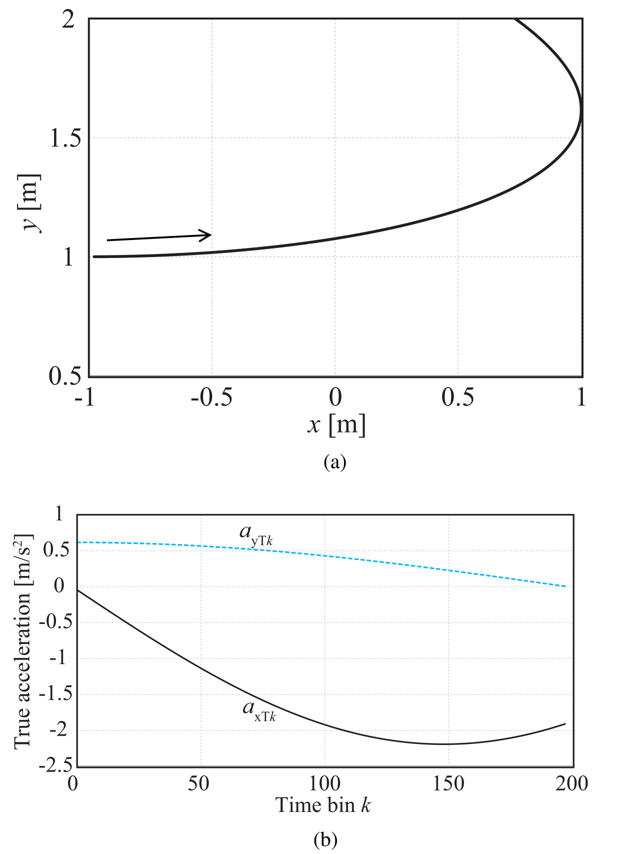


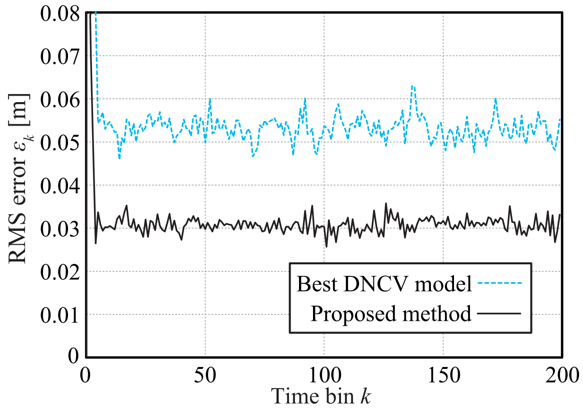
FIGURE 6. True orbit and acceleration of target for 2D simulation. (a) True orbit. (b) True acceleration.

2) 2D TRACKING EXAMPLE

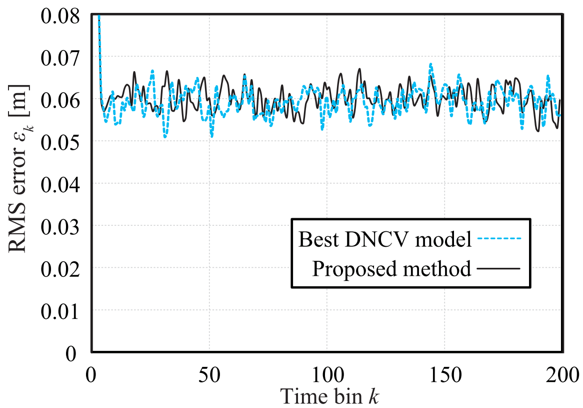
Finally, the 2D tracking simulation results are presented to show the effectiveness of the proposed method. We assume 2D tracking in xy plane. Figure 6(a) and (b) show the true orbit and acceleration of the assumed target. The orbit of this target is ($x_{\mathrm{t}k} = -1 + 2\sin(2\pi kT/6)$, $y_{\mathrm{t}k} = 2 - \cos(2\pi kT/8)$. We set $T = 0.01$ s, $\sigma_{\mathrm{x}} = 0.01$ m, and a standard deviation of measurement noise in y-axis σ_{y}^2 is 0.01 m. Measurement position is generated by adding white Gaussian noise to true position. To show the effectiveness of the 1D tracking-based approach in this paper, the Kalman filter tracker is implemented for each axis and we use same $\boldsymbol{Q}$ for both axes. The initial values of the state vector and error covariance matrix are all zero. The RMS prediction error is calculated by 1000 Monte Carlo simulations, which is defined as

$$\epsilon_k = \sqrt{\frac{1}{1000}\sum_{m=1}^{1000}(x_{\mathrm{t}k} - x_{\mathrm{p}mk})^2 + (y_{\mathrm{t}k} - y_{\mathrm{p}mk})^2}, \tag{33}$$

where $y_{\mathrm{p}mk}$ is the predicted position in y-axis in the m-th Monte Carlo simulation.



(a)



(b)

FIGURE 7. Simulation results for 2D maneuvering target. (a) $a_D = 3$. (b) $a_D = 30$.

First, we evaluate the $a_D = 3$ ($a_c = 3$ m/s^2) case. This value can be considered as a rough approximate value of the maximum acceleration of target and we can predict that this setting realizes appropriate tracking accuracy based on the discussions in the previous section. Figure 7(a) shows the results of the tracker for which $\boldsymbol{Q}$ is determined by the conventional and the proposed methods. This figure verifies that the proposed method achieves better accuracy than the conventional method. Therefore, the proposed method can determine an appropriate $\boldsymbol{Q}$ for a realistic case assuming non-constant accelerating target in 2D problem. However, if the preset value of a_D has large error, the tracking accuracy also deteriorates. Figure 7(b) shows the simulation results for $a_D = 30$ ($a_c = 30$ m/s^2). Note that the accuracy of the proposed method has deteriorated because a_D has been miss-set. Moreover, the results of the proposed method are almost the same as that of the best conventional DNCV model. This means that the proposed method cannot improve the tracking accuracy if the preset value of a_D includes a large error. Consequently, although the proposed method achieves accurate tracking even for 2D problems, the appropriate presetting of a_D is important in practical use. However, to achieve more accurate tracking, consideration of a general 2D tracking model is an important task for the future.

VI. CONCLUSION

This paper has proposed an automatic parameter setting method for a Kalman filter tracker with the CV model assuming an arbitrary covariance matrix of process noise $\boldsymbol{Q}$. The efficient steady-state performance index μ, corresponding to the RMS prediction error of a target moving with constant acceleration, was defined and its validity was demonstrated via numerical simulations. The analytical relationship between μ and an arbitrary $\boldsymbol{Q}$ was then derived, and a method of determining the optimal $\boldsymbol{Q}$ that minimizes μ proposed. The proposed method has only one parameter, a_D, corresponding to the target acceleration. Numerical analyses showed that the $\boldsymbol{Q}$ given by the proposed method can achieve more accurate tracking than conventional models with optimal settings. For $a_D = 10$, the optimal μ given by the proposed method is 63.8% of that given by the DNCV model. Moreover, numerical simulations verified the effectiveness of the proposed method for a maneuvering target, as well as the appropriate setting of a_D.

For simplicity, the interacting multiple model approach was not considered. The relaxation of this assumption will be important in future work enabling the realization of more accurate tracking filters. Moreover, although the effectiveness of the proposed method is shown in a 2D tracking simulation, the proposed method is developed based on a 1D tracking model. Thus, the consideration of more practical 2D and 3D tracking models is also an important task for the future. Further, as described in Section V-A, clarification of the strict mechanism of model determination with μ using other dynamical models represents another important area of research.

APPENDIX A
DERIVATION OF THE PROPOSED INDEX OF (21)

The proposed index corresponds to the RMS prediction error for a constant acceleration target. This RMS prediction error is expressed as

$$\epsilon_p = \sqrt{\lim_{k\to\infty} E[(x_{tcak} - x_{pk})^2]}, \tag{34}$$

where x_{tcak} is the true position of the constant acceleration target, which is expressed as

$$x_{tcak} = x_{tcak-1} + v_{tcak-1} + a_c T^2/2, \tag{35}$$

where v_{tcak} and a_c are the true velocity and acceleration. We now define the predicted and estimated states as:

$$\tilde{\boldsymbol{x}}_k = (x_{pk} \quad v_{pk})^T, \tag{36}$$
$$\hat{\boldsymbol{x}}_k = (x_{sk} \quad v_{sk})^T. \tag{37}$$

Substituting (9), (36), and (37) into (3) gives:

$$x_{pk} = x_{sk-1} + Tv_{sk-1}. \tag{38}$$
$$v_{pk} = v_{sk-1}. \tag{39}$$

With (35) and (38), the prediction error is obtained as

$$x_{tcak} - x_{pk} = \Delta x_{sk-1} + T\Delta v_{sk-1} + a_c T^2/2, \tag{40}$$

where

$$\Delta x_{\mathrm{x}sk} \equiv x_{\mathrm{tca}k} - x_{\mathrm{s}k}, \tag{41}$$
$$\Delta v_{\mathrm{v}sk} \equiv v_{\mathrm{tca}k} - v_{\mathrm{s}k}. \tag{42}$$

Consequently, the mean-square prediction error is calculated as

$$\begin{aligned} E[(x_{\mathrm{tca}k} - x_{\mathrm{p}k})^2] = E[\Delta x_{\mathrm{s}k-1}^2] + T^2 E[\Delta v_{\mathrm{s}k-1}^2] \\ + a_\mathrm{c}^2 T^4/4 + 2TE[\Delta x_{\mathrm{s}k-1}\Delta v_{\mathrm{s}k-1}] \\ + T^2 a_\mathrm{c} E[\Delta x_{\mathrm{s}k-1}] + T^3 a_\mathrm{c} E[\Delta v_{\mathrm{s}k-1}], \end{aligned} \tag{43}$$

As shown in this equation, calculations of mean errors and mean square errors in the estimated state ($x_{\mathrm{s}k}$ and $v_{\mathrm{s}k}$) are required to derive the RMS prediction error.

First, mean errors of $x_{\mathrm{s}k}$ and $v_{\mathrm{s}k}$ are calculated. Substituting (10), (15), and (36) into (4), we have:

$$x_{\mathrm{s}k} = (1-\alpha)x_{\mathrm{p}k} + \alpha x_{\mathrm{o}k} \tag{44}$$
$$v_{\mathrm{s}k} = v_{\mathrm{p}k} + (\beta/T)(x_{\mathrm{o}k} - x_{\mathrm{p}k}). \tag{45}$$

Substituting (38) into (44) gives

$$x_{\mathrm{s}k} = (1-\alpha)(x_{\mathrm{s}k-1} + Tv_{\mathrm{s}k-1}) + \alpha x_{\mathrm{o}k}, \tag{46}$$

and substituting (38) and (39) into (45) and its simplification gives

$$v_{\mathrm{s}k} = (1-\beta)v_{\mathrm{s}k-1} + (\beta/T)(x_{\mathrm{o}k} - x_{\mathrm{s}k-1}). \tag{47}$$

Here, the true velocity can be expressed as

$$v_{\mathrm{tca}k} = v_{\mathrm{tca}k-1} + Ta_\mathrm{c}, \tag{48}$$

and this can be rewritten as

$$v_{\mathrm{tca}k} = (x_{\mathrm{tca}k} - x_{\mathrm{tca}k-1})/T + Ta_\mathrm{c}/2. \tag{49}$$

With (35), (46), (47), (48), and (49), the errors in estimated position and velocity defined as (41) and (42) are calculated as:

$$\Delta x_{\mathrm{s}k} = (1-\alpha)(\Delta x_{\mathrm{s}k-1} + T\Delta v_{\mathrm{s}k-1} + a_\mathrm{c}T^2/2) + \alpha\Delta x_{\mathrm{o}k}, \tag{50}$$

$$\begin{aligned} \Delta v_{\mathrm{s}k} &= (1-\beta)(\Delta v_{\mathrm{s}k-1} + Ta_\mathrm{c}) \\ &\quad + \beta\left(\frac{\Delta x_{\mathrm{o}k} - \Delta x_{\mathrm{s}k-1}}{T} + \frac{Ta_\mathrm{c}}{2}\right), \end{aligned} \tag{51}$$

where $\Delta x_{\mathrm{o}k} \equiv x_{\mathrm{tca}k} - x_{\mathrm{o}k}$. Means of these errors are:

$$\begin{aligned} E[\Delta x_{\mathrm{s}k}] = (1-\alpha)(E[\Delta x_{\mathrm{s}k-1}] + TE[\Delta v_{\mathrm{s}k-1}] + a_\mathrm{c}T^2/2) \\ + \alpha E[\Delta x_{\mathrm{o}k}], \end{aligned} \tag{52}$$

$$\begin{aligned} E[\Delta v_{\mathrm{s}k}] &= (1-\beta)(E[\Delta v_{\mathrm{s}k-1}] + Ta_\mathrm{c}) \\ &\quad + \beta\left(\frac{E[\Delta x_{\mathrm{o}k}] - E[\Delta x_{\mathrm{s}k-1}]}{T} + \frac{Ta_\mathrm{c}}{2}\right). \end{aligned} \tag{53}$$

Because the steady-state assumption ($k \to \infty$), the following relations are satisfied:

$$E[\Delta x_{\mathrm{s}k}] = E[\Delta x_{\mathrm{s}k-1}], \tag{54}$$
$$E[\Delta v_{\mathrm{s}k}] = E[\Delta v_{\mathrm{s}k-1}], \tag{55}$$

and because white Gaussian measurement noise is assumed, we find

$$E[\Delta x_{\mathrm{o}k}] = 0. \tag{56}$$

The simplification of (52) and (53) using (54)–(56), we have the following linear system:

$$(1-\alpha)(E[\Delta v_{\mathrm{s}k}] + T^2 a_\mathrm{c}/2) - \alpha E[\Delta x_{\mathrm{s}k}] = 0, \tag{57}$$

$$(1-\beta)Ta_\mathrm{c} - \beta E[\Delta v_{\mathrm{s}k}] + \beta(Ta_\mathrm{c}/2 - E[\Delta x_{\mathrm{s}k}]/T) = 0. \tag{58}$$

The solution of this system is:

$$E[\Delta x_{\mathrm{s}k}] = \frac{1-\alpha}{\beta} a_\mathrm{c} T^2, \tag{59}$$
$$E[\Delta v_{\mathrm{s}k}] = \frac{2\alpha - \beta}{2\beta} a_\mathrm{c} T. \tag{60}$$

Then, the mean-square errors and mean correlated error are calculated. Similar to (50) and (51), the square errors of $x_{\mathrm{s}k}$ and $v_{\mathrm{s}k}$ and their correlated error are calculated as:

$$\begin{aligned} \Delta x_{\mathrm{s}k}^2 &= (1-\alpha)^2(\Delta x_{\mathrm{s}k-1}^2 + T^2\Delta v_{\mathrm{s}k-1}^2 \\ &\quad + a_\mathrm{c}^2 T^4/4 + 2T\Delta x_{\mathrm{s}k-1}\Delta v_{\mathrm{s}k-1} \\ &\quad + T^2 a_\mathrm{c}(\Delta x_{\mathrm{s}k-1} + \Delta v_{\mathrm{s}k-1})) + \alpha^2\Delta x_{\mathrm{o}k}^2 \\ &\quad + 2\alpha(1-\alpha)\Delta x_{\mathrm{o}k}(\Delta x_{\mathrm{s}k-1} + T\Delta v_{\mathrm{s}k-1} \\ &\quad + a_\mathrm{c}T^2/2), \end{aligned} \tag{61}$$

$$\begin{aligned} \Delta v_{\mathrm{s}k}^2 &= (1-\beta)^2(\Delta v_{\mathrm{s}k-1}^2 + 2Ta_\mathrm{c}\Delta v_{\mathrm{s}k-1} + T^2 a_\mathrm{c}^2) \\ &\quad + \beta^2((\Delta x_{\mathrm{o}k}^2 + \Delta x_{\mathrm{s}k-1}^2 + 2\Delta x_{\mathrm{o}k}\Delta x_{\mathrm{s}k-1})/T^2 \\ &\quad + T^2 a_\mathrm{c}^2/4 + a_\mathrm{c}(\Delta x_{\mathrm{o}k} - \Delta x_{\mathrm{s}k-1})) \\ &\quad + 2\beta(1-\beta)/(2T)(a_\mathrm{c}^2 T^3 + a_\mathrm{c}T^2\Delta v_{\mathrm{s}k-1} \\ &\quad + 2Ta_\mathrm{c}(\Delta x_{\mathrm{o}k} - \Delta x_{\mathrm{s}k-1}) \\ &\quad - 2\Delta x_{\mathrm{s}k-1}\Delta v_{\mathrm{s}k-1} + 2\Delta x_{\mathrm{o}k}\Delta v_{\mathrm{s}k-1}), \end{aligned} \tag{62}$$

$$\begin{aligned} \Delta x_{\mathrm{s}k}\Delta v_{\mathrm{s}k} &= (1-\alpha)(1-\beta)(T^3 a_\mathrm{c}^2/2 + 3T^2 a_\mathrm{c}\Delta v_{\mathrm{s}k-1}/2 \\ &\quad + T(a_\mathrm{c}\Delta x_{\mathrm{s}k-1} + \Delta v_{\mathrm{s}k-1}^2) + \Delta x_{\mathrm{s}k-1}\Delta v_{\mathrm{s}k-1} \\ &\quad + \alpha(1-\beta)\Delta x_{\mathrm{o}k}(Ta_\mathrm{c} + \Delta v_{\mathrm{s}k-1}) \\ &\quad + \beta(1-\alpha)(T^3 a_\mathrm{c}^2/4 + T^2 a_\mathrm{c}\Delta v_{\mathrm{s}k-1}/2 \\ &\quad + Ta_\mathrm{c}\Delta x_{\mathrm{o}k}/2 + \Delta v_{\mathrm{s}k-1}(\Delta x_{\mathrm{o}k} - \Delta x_{\mathrm{s}k-1}) \\ &\quad - \Delta x_{\mathrm{s}k-1}^2/T + \Delta x_{\mathrm{o}k}\Delta x_{\mathrm{s}k-1}) \\ &\quad + \alpha\beta(\Delta x_{\mathrm{o}k}^2/T - \Delta x_{\mathrm{o}k}\Delta x_{\mathrm{s}k-1}/T + a_\mathrm{c}T) \end{aligned} \tag{63}$$

Because of the steady-state assumption and because the smoothed parameters are a linear combination of the measured parameters, the following relations hold:

$$E[\Delta x_{\mathrm{s}k}^2] = E[\Delta x_{\mathrm{s}k-1}^2], \tag{64}$$
$$E[\Delta v_{\mathrm{s}k}^2] = E[\Delta v_{\mathrm{s}k-1}^2], \tag{65}$$
$$E[\Delta x_{\mathrm{s}k}\Delta v_{\mathrm{s}k}] = E[\Delta x_{\mathrm{s}k-1}\Delta v_{\mathrm{s}k-1}], \tag{66}$$
$$E[\Delta x_{\mathrm{o}k}\Delta x_{\mathrm{s}k-1}] = 0, \tag{67}$$
$$E[\Delta x_{\mathrm{o}k}\Delta v_{\mathrm{s}k-1}] = 0, \tag{68}$$

with the variance of measurement noise given by

$$\sigma_\mathrm{x}^2 = E[\Delta x_{\mathrm{o}k}^2]. \tag{69}$$

With calculations of the means for (61)–(63) using (54)–(56) and (64)–(69) and their simplification, we have a linear system composed of the following three equations:

$$\alpha(\alpha-2)E[\Delta x_{sk}^2]+(1-\alpha)^2(+T^2E[\Delta v_{sk}^2] + T^4a_c^2/4+2TE[\Delta x_{sk}\Delta v_{sk}]+T^2a_c(E[\Delta x_{sk}] + E[\Delta v_{sk}]))+\alpha^2\sigma_x^2=0, \tag{70}$$

$$\beta(\beta-2)E[\Delta v_{sk}^2]+\beta^2E[\Delta x_{sk}^2]/T^2 + 2\beta(\beta-1)E[\Delta x_{sk}\Delta v_{sk}]/T+a_c\beta(3\beta-2)E[\Delta x_{sk}] + Ta_c(\beta-1)(3\beta-2)E[\Delta v_{sk}]+T^2a_c^2(3\beta-2)^2/4 + \beta^2\sigma_x^2/T^2=0, \tag{71}$$

$$(2\alpha\beta-\alpha-2\beta)E[\Delta x_{sk}\Delta v_{sk}]+\beta(\alpha-1)E[\Delta x_{sk}^2]/T + T(\alpha-1)(\beta-1)E[\Delta v_{sk}^2]+Ta_c(\alpha-1)(2\beta-1)E[\Delta x_{sk}] + T^2a_c(\alpha-1)(4\beta-3)E[\Delta v_{sk}]/2 + T^3a_c^2(\alpha-1)(3\beta-2)/4+\alpha\beta\sigma_x^2/T. \tag{72}$$

By solving this linear system after substituting (59) and (60), we have:

$$E[\Delta x_{sk}^2]=\frac{2\alpha^2-3\alpha\beta+2\beta}{\alpha(4-2\alpha-\beta)}\sigma_x^2 + \frac{(1-\alpha)^2}{\beta^2}a_c^2T^4, \tag{73}$$

$$E[\Delta v_{sk}^2]=\frac{2\beta^2}{\alpha(4-2\alpha-\beta)}\frac{\sigma_x^2}{T^2} + \frac{(2\alpha-\beta)^2}{4\beta^2}a_c^2T^2, \tag{74}$$

$$E[\Delta x_{sk}\Delta v_{sk}]=\frac{\beta(2\alpha-\beta)}{\alpha(4-2\alpha-\beta)}\frac{\sigma_x^2}{T} + \frac{(1-\alpha)^2(2\alpha-\beta)^2}{2\beta^2}a_c^2T^3. \tag{75}$$

Substituting (59), (60), and (73)–(75) into (43) and using (54), (55), and (64)–(66), the RMS prediction error of (34) is calculated as

$$\epsilon_p=\sqrt{\frac{2\alpha^2+2\beta+\alpha\beta}{\alpha(4-2\alpha-\beta)}\sigma_x^2+\frac{a_c^2T^4}{\beta^2}}. \tag{76}$$

With ϵ_p/σ_x and (22), we obtain the proposed steady-state performance index of (21).

APPENDIX B
DERIVATION OF (25)

The steady-state assumption means that $\boldsymbol{P}$ and $\boldsymbol{K}$ converge as $k\rightarrow\infty$, and so time k in (3)–(7) is omitted from the following. The i-th row and j-th column of a matrix $\boldsymbol{P}$ is denoted as $P^{i,j}$. With (6) and (24), $\tilde{\boldsymbol{P}}$ can be calculated as

$$\tilde{\boldsymbol{P}}=\begin{pmatrix}\hat{P}^{1,1}+2T\hat{P}^{1,2}+T^2\hat{P}^{2,2}+a & \hat{P}^{1,2}+T\hat{P}^{2,2}+b\\ \hat{P}^{1,2}+T\hat{P}^{2,2}+b & \hat{P}^{2,2}+c\end{pmatrix}. \tag{77}$$

Equation (5) can also be written as [25]

$$\boldsymbol{K}_k=\hat{\boldsymbol{P}}_k\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}. \tag{78}$$

From (7), (10), (11), (15), and (78), we have

$$\boldsymbol{K}=\begin{pmatrix}\alpha\\ \beta/T\end{pmatrix}=\begin{pmatrix}\hat{P}^{1,1}/\sigma_x^2\\ \hat{P}^{1,2}/\sigma_x^2\end{pmatrix}, \tag{79}$$

$$\hat{\boldsymbol{P}}=\begin{pmatrix}(1-\alpha)\tilde{P}^{1,1} & (1-\alpha)\tilde{P}^{1,2}\\ (1-\alpha)\tilde{P}^{1,2} & \tilde{P}^{2,2}-(\beta/T)\tilde{P}^{1,2}\end{pmatrix}. \tag{80}$$

Equation (79) indicates that

$$\hat{P}^{1,1}=\alpha\sigma_x^2, \tag{81}$$

$$\hat{P}^{1,2}=\beta\sigma_x^2/T. \tag{82}$$

Substituting (77), (81), and (82) into (80), we obtain

$$\hat{\boldsymbol{P}}=\begin{pmatrix}\alpha\sigma_x^2 & \beta\sigma_x^2/T\\ \beta\sigma_x^2/T & \hat{P}^{2,2}\end{pmatrix}=\begin{pmatrix}\hat{P}^{1,1'} & \hat{P}^{1,2'}\\ \hat{P}^{1,2'} & \hat{P}^{2,2'}\end{pmatrix}, \tag{83}$$

where

$$\hat{P}^{1,1'}=(1-\alpha)(\alpha\sigma_x^2+2\beta\sigma_x^2+T^2\hat{P}^{2,2}+a), \tag{84}$$

$$\hat{P}^{1,2'}=(1-\alpha)(\beta\sigma_x^2/T+T\hat{P}^{2,2}+b), \tag{85}$$

$$\hat{P}^{2,2'}=\hat{P}^{2,2}+c-(\beta/T)(\beta\sigma_x^2T+T\hat{P}^{2,2}+b). \tag{86}$$

Examining each element of (83), the following equations are obtained:

$$2\beta\sigma_x^2+T^2\hat{P}^{2,2}+a-\alpha(\alpha\sigma_x^2+2\beta\sigma_x^2+T^2\hat{P}^{2,2}+a)=0, \tag{87}$$

$$T\hat{P}^{2,2}+b-\alpha(\beta\sigma_x^2/T+T\hat{P}^{2,2}+b)=0, \tag{88}$$

$$\hat{P}^{2,2}=(c-b\beta)/\beta-\beta\sigma_x^2/T^2. \tag{89}$$

Substituting (89) into (87) and (88) and simplifying, we obtain:

$$(1-\alpha)(A\beta-B\beta+C)-\beta(\alpha^2+\alpha\beta-\beta)=0, \tag{90}$$

$$\alpha=1-\beta^2/C, \tag{91}$$

where A, B, and C are defined as (30), (31), and (26), respectively. Substituting (91) into (90), we obtain a quartic equation with respect to β. Its real solution satisfying the filter stability conditions [16] is

$$\beta=\frac{C+\sqrt{C(16+4A-4B+C)}}{4} -\sqrt{\frac{C^2(16+4A-4B+C)}{8\sqrt{C(16+4A-4B+C)}}+\frac{C(2A-2B+C)}{8}}. \tag{92}$$

With (27), (28), (29), and (92), we have

$$\beta=D/4. \tag{93}$$

Substituting (93) into (91), we have

$$\alpha=\frac{16C-D^2}{16C}. \tag{94}$$

Note that substituting (12) ($a=T^4\sigma_{aw}^2/4$, $b=T^3\sigma_{aw}^2/2$, and $c=T^2\sigma_{aw}^2$) into (93) and (94) gives (20) and (19). Substituting (93) and (94) into (21), we arrive at (25).

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