Medical Image Analysis KU, WS24
Assignment 1
CT Reconstruction

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1 Task Description:

The main task of this assignment is to reconstruct the provided CT thorax slice image from a sinogram via different methods. This was clearly structured into the following points:

- Perform a reconstruction via backprojection summation image method.
- Perform a reconstruction via filtered backprojection method.
- Compare the output of the filtered backprojection method with the provided image via computing the root mean squared error (RMSE) between both.
- Study and describe the influences of the influence of the number of discrete angles and different interpolation schemes.

Figure 1: CT thorax slice

2 Backprojection summation image method

The main objective of this method is to extract every projection out of the sinogram, "smear" it into a 2D image, called a single backprojection at a given angle. This step is repeated for every projection of the sinogram and to then "overlay" all of this single 2D images to get the backprojection summation image as shown in Figure 2

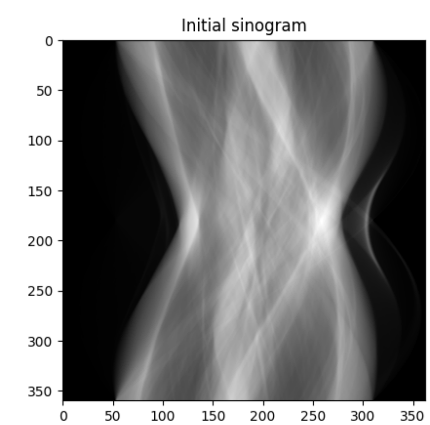
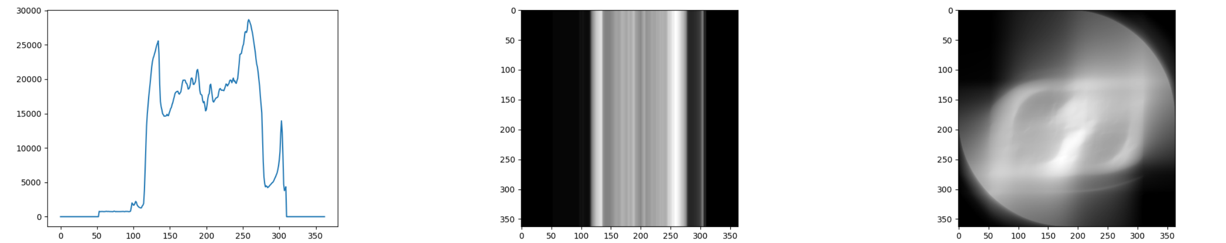


Figure 2: Sinogram of the CT thorax slice image

Starting from the provided imageof the CT thorax slice, we can compute the sinogram (code already provided) where the output *Numpy* array has two dimensions. The fist dimension represents the number of angles a projection was captured and the second one all the summed up values the CT detector has seen at this specific angle hence the projection. To perform this task properly the loaded image is padded ($\sqrt{2} *$ number of pixels) before and after the image in every dimension. This ensures that there is no unintentional data loss when computing the sinogram.



(a) Sample projection at 180 degree (b) Sample smear at 180 degree (c) Summed proj. at 180 degree

Figure 3: Back projection figures

To compute the backprojection we start with extracting every single projection from the sinogram, meaning that we extract a single row of the array as shown in Figure 3a.
As mentioned before, this projection then gets "smeared" back into the third dimension. Every value is kept the same and generates a line in this third dimension as you can see in Figure 3b.
Once we have all projections smeared back we then can sum up all of them. This is done by rotating every "smeared" projection with its according angle. An example of this can be seen in Once we have all projections smeared back we then can sum up all of them. This is done by rotating every "smeared" projection with its according angle. An example of this can be seen in Figure 3c.

The final output of the backprojection summation image method can be seen in Figure 7a. It is clearly visible that by summing up all "smeared" back projections build up a glowing effect that lets the image look blurry.

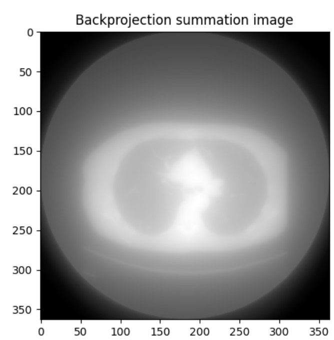


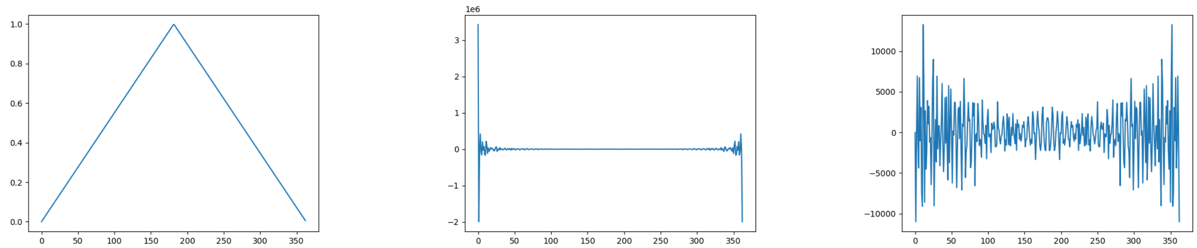
Figure 4: Backprojection summation image

3 Filtered backprojection method

To further improve on the image quality and visibility another method is introduced, the filtered backprojection.
For this the input is the same sinogram that we generated for the backprojection summation method from the CT thorax scan. (Figure 2)

The main difference between the two methods is that filtered backprojection adds a filtering step before the image is backprojected.

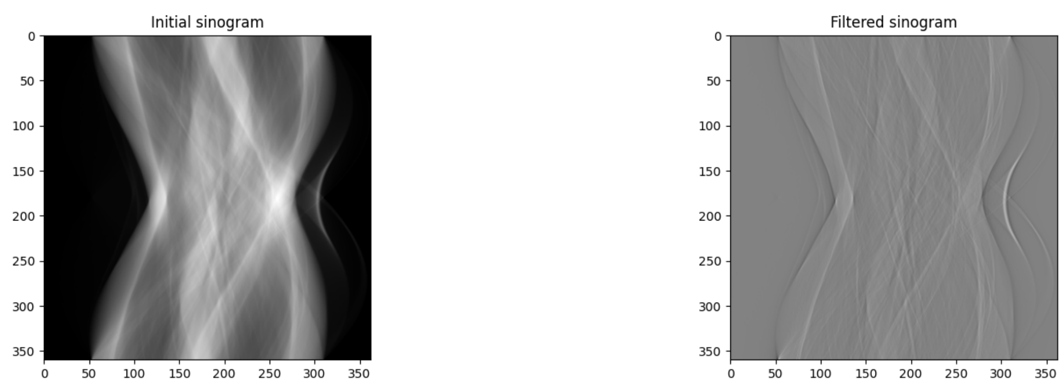
To perform the filtering step we take every projection from the sinogram (in our case every row) and compute a 1-D Fourier transform of it, which puts the projections into the frequency domain. This can be seen in Figure 5b, that represents the Fourier transformed projection of the sinogram at angle 180 degree. As the main intention of this step is to amplify high-frequencies and attenuate low-frequencies to enhance



(a) Ramp filter (b) FT projection at 180 degree (c) Filtered proj. at 180 degree

Figure 5: Filtering step

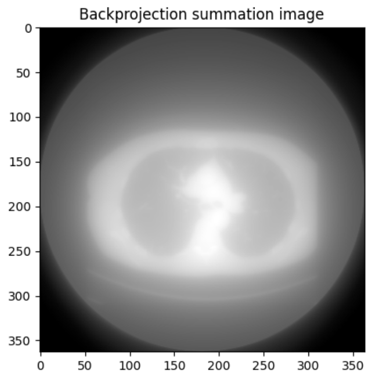
details and edges in the image. To do this we have to know that in the frequency domain *low frequency components are found at the central regions while high frequency components are in peripherals.* [1]
Whit is in mind we can multiply every Fourier transformed projection of the sinogram with our chosen filter. The type of filter influences the outcome of the filtration and in the end the final reconstructed image. For this step a ramp filter was used which can be seen in Figure 5a. Once we have all the filtered projections (Figure 5c shows exemplarily the filtered projection at 180 degree) we can inverse Fourier transform all of them and get a filtered sinogram. A comparison of the initial sinogram and the filtered sinogram can be seen in Figure 6.



(a) Initial sinogram (b) Filtered sinogram

Figure 6: Comparison of sinograms

As soon as we have the filtered sinogram we again can us the backprojection summation method to compute a reconstructed image of the CT thorax scan. This image (Figure 7a) is less blurry due to the filtration when compared to the backprojection mathod.



(a) Backprojection reconstruction

(b) Filtered backprojection reconstruction

Figure 7: Comparison of reconstructed images

4 Comparison of the output

The final step to conclude this reconstruction step is to evaluate how close the reconstruction is to the initial provided picture. For this we use the root mean square error (RMSE). But before we can do this we have to ensure that we compare two images that actually represent the same thing. This means that we firstly have to get rid of the padding that we initially put onto the image and further more normalize the reconstructed image to the same value range (in the case of the provided image from 0 to 255).
After these two steps we can subtract the images from each other to visualize the differences, as shown in Figure 8. (Darker - lower difference; Lighter - higher difference)

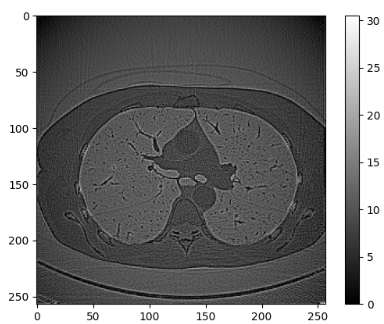
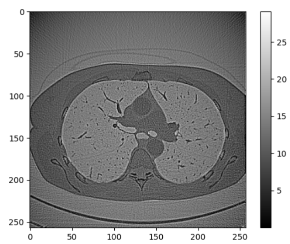


Figure 8: Differences of reconstructed to initial image

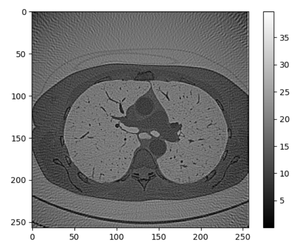
When computing the root mean square error (RMSE) of the initial image and the reconstructed image (without interpolation and a ramp filter) a value of $RMSE = \mathbf{9.1699}$ is achieved (lower is better).

5 Influences of the influence of the number of discrete angles and different interpolation schemes

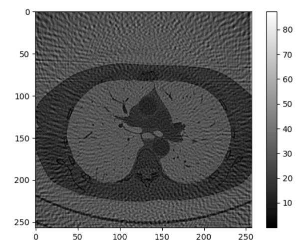
This section investigates the impact of the amount of discrete angle steps that determine how the sinogram and all following steps are computed as well as how different interpolation schemes in the rotate function impacts the final reconstructed image.



(a) Difference with 270 discrete angles(RMSE = 12.31)



(b) Difference with 180 discrete angles(RMSE = 16.06)



(c) Difference with 90 discrete angles (RMSE = 26.48)

Figure 9: Differences of final to initial image with different angle steps

In Figure 9 the difference in discrete angle steps is visualized by computing a subtraction of the reconstructed image and the original image. The basis was 360 angle steps, as discussed in section 4. It is clearly noticeable that with fewer steps the reconstructed image gets worse in quality indicated by the RMSE.

Regarding the interpolation scheme two parameters can me modified within the rotate function:

- Order
- Mode

Order represents the degree of spline interpolation used while resampling the image during rotation. It can range from 0 to 5, zero representing "Nearest-neighbor interpolation", one - "Linear interpolation", two - "Quadratic interpolation", three - " Cubic interpolation" and four & five - "Higher-order interpolation". In 1 the different computed values for different interpolation schemes are shown as well as the according computation time. It can be noticed that the higher the order the better the value hence the quality of the reconstructed image but this comes at a higher computational cost.

Order	RMSE	Time [sec]
0	11.762	1.39
1	10.653	2.05
2	9.154	3.70
3	9.167	5.35
4	9.059	7.97
5	9.026	9.55

Table 1: RMSE values of different order values

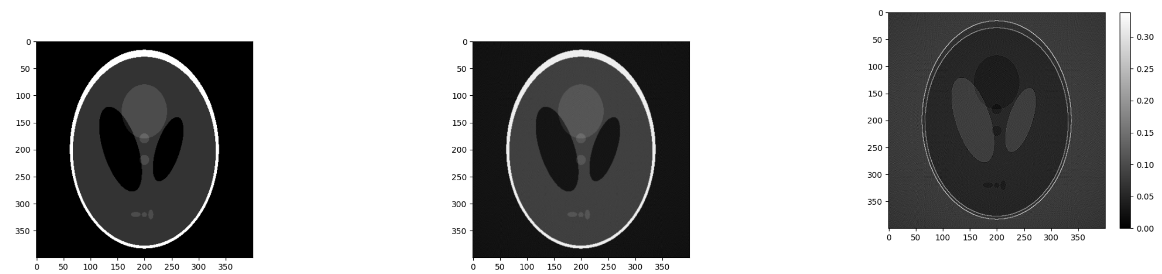
Mode represents how the function handles the interpolation of out of the boundary pixels, meaning how to calculate values for missing pixels that fall outside the original image boundaries during rotation. In 2 the corresponding RMSE values for the different modes are shown.

Order	Mode	RMSE
5	reflect	8.920
5	grid-mirror	8.920
5	grid-constant	9.012
5	nearest	8.951
5	mirror	9.026
5	grid-wrap	9.067
5	wrap	9.026

Table 2: RMSE values of different mode values

6 Test on additional images

As a final step the codes was also tested on the Shepp Logan phantom image. In Figure 10 you can see the inital image, the reconstructed one (order=3, mode=constant) and the difference of these two. Interestingly the on this picture the code could get a $RMSE = \mathbf{0.075}$ with this settings.



(a) Shepp Logan phantom initial

(b) Reconstructed image

(c) Difference between initial and reconstructed image

Figure 10: Shepp Logan phantom reconstruction

7 Conclusion

This report has explored how backprojection and filtered backprojection to reconstruct a CT thorax slice can work.

Within the analysis of these two methods, it could be observed that different factors have an influence on the result of the result of the filtered backprojection.
First and foremost the filtering itself, where it has to be made sure that the Fourier transformation is performed on the projections and not the angles. Then the position of the frequencies has to be attuned with the filter to ensure that when the filter applied, high-frequencies are amplify and low-frequencies are attenuate. Other factors like the interpolation schemes and number of discrete angle steps also play a roll in the performance of the code and how the reconstructed image looks like. Here a trade-off between output quality and time has to be made.

Further steps may include the investigation of different filters that can be applied in the filtering step and also how they could perform on other images.

References

[1] Assistant. Prof. Muhammed Rushdi/ Cairo University. *Week 3: Images in Frequency Domain*. Spring 2018/ Spring 2018. URL: `https://sbme-tutorials.github.io/2018/cv/notes/3_week3.html`.