This file shows (though it does not prove) that using only even order derivatives on the edges to constrain the terms of an interpolating polynomial results in a series in h, the sample spacing, that is purely even.

```
Maxima 5.33.0 http://maxima.sourceforge.net
using Lisp SBCL 1.1.18
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
```

*** My very own personal maxima-init.mac has been loaded. ***

```
STYLE-WARNING: redefining MAXIMA::MAIN-PROMPT in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-STRIPDOLLAR in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-MEXPT in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-CHOOSE in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-INT in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-SUM in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-LSUM in DEFUN
```

```
(%i1) polyorder :2;
```

(%o1) 2

```
(%i2) pint(x) := sum( a[i] * x^(i) / (i)!, i, 0, (polyorder + 1)-1 +
      2*ceiling((polyorder+1)/2) );
```

(%o2) $\mathrm{pint}(x) := \mathrm{sum}\left(\frac{a_i\,x^i}{i!}, i, 0, \mathrm{polyorder}+1-1+2\left\lceil\frac{\mathrm{polyorder}+1}{2}\right\rceil\right)$

```
(%i3) pm(x) := sum( b[i] * x^(i) / (i)!, i, 0, polyorder );
```

(%o3) $\mathrm{pm}(x) := \mathrm{sum}\left(\frac{b_i\,x^i}{i!}, i, 0, \mathrm{polyorder}\right)$

```
(%i4) p0(x) := sum( c[i] * x^i / i!, i, 0, polyorder );
```

(%o4) $\mathrm{p0}(x) := \mathrm{sum}\left(\frac{c_i\,x^i}{i!}, i, 0, \mathrm{polyorder}\right)$

```
(%i5) pp(x) := sum(d[i] * x^(i) / (i)!, i, 0, polyorder );
```

(%o5) $\mathrm{pp}(x) := \mathrm{sum}\left(\frac{d_i\,x^i}{i!}, i, 0, \mathrm{polyorder}\right)$

```
(%i6) ders(f) := makelist( diff(f(x), x, i), i, 0, polyorder );
```

(%o6) $\mathrm{ders}(f) := \mathrm{makelist}(\mathrm{diff}(f(x), x, i), i, 0, \mathrm{polyorder})$

```
(%i7) derseven(f) := makelist( diff(f(x), x, 2*i), i, 0, ceiling((polyorder-
      1)/2));
```

(%o7) $\mathrm{derseven}(f) := \mathrm{makelist}\left(\mathrm{diff}(f(x), x, 2\,i), i, 0, \left\lceil\frac{\mathrm{polyorder}-1}{2}\right\rceil\right)$

```
(%i8) dpint : ders(pint);
```

(%o8) $\left[\frac{a_6\,x^6}{720}+\frac{a_5\,x^5}{120}+\frac{a_4\,x^4}{24}+\frac{a_3\,x^3}{6}+\frac{a_2\,x^2}{2}+a_1\,x+a_0, \frac{a_6\,x^5}{120}+\frac{a_5\,x^4}{24}+\frac{a_4\,x^3}{6}+\frac{a_3\,x^2}{2}+a_2\,x+a_1, \frac{a_6\,x^4}{24}+\frac{a_5\,x^3}{6}+\frac{a_4\,x^2}{2}+a_3\,x+a_2\right]$

```
(%i9) dpinteven : derseven(pint);
```

(%o9) $\left[\frac{a_6\,x^6}{720}+\frac{a_5\,x^5}{120}+\frac{a_4\,x^4}{24}+\frac{a_3\,x^3}{6}+\frac{a_2\,x^2}{2}+a_1\,x+a_0,\frac{a_6\,x^4}{24}+\frac{a_5\,x^3}{6}+\frac{a_4\,x^2}{2}+a_3\,x+a_2\right]$

(%i10)
```
dpm : derseven(pm);
```

(%o10) $\left[\frac{b_2\,x^2}{2}+b_1\,x+b_0,b_2\right]$

(%i11)
```
dp0 : ders(p0);
```

(%o11) $\left[\frac{c_2\,x^2}{2}+c_1\,x+c_0,c_2\,x+c_1,c_2\right]$

(%i12)
```
dpp : derseven(pp);
```

(%o12) $\left[\frac{d_2\,x^2}{2}+d_1\,x+d_0,d_2\right]$

(%i13)
```
eqns : makelist( ev(dpinteven[i], x=-h) = ev(dpm[i], x=0), i, 1,
length(dpm));
```

(%o13) $\left[\frac{a_6\,h^6}{720}-\frac{a_5\,h^5}{120}+\frac{a_4\,h^4}{24}-\frac{a_3\,h^3}{6}+\frac{a_2\,h^2}{2}-a_1\,h+a_0=b_0,\frac{a_6\,h^4}{24}-\frac{a_5\,h^3}{6}+\frac{a_4\,h^2}{2}-a_3\,h+a_2=b_2\right]$

(%i14)
```
eqns : append( eqns, makelist( ev(dpint[i], x=0) = ev(dp0[i], x=0), i, 1,
length(dp0)));
```

(%o14) $\left[\frac{a_6\,h^6}{720}-\frac{a_5\,h^5}{120}+\frac{a_4\,h^4}{24}-\frac{a_3\,h^3}{6}+\frac{a_2\,h^2}{2}-a_1\,h+a_0=b_0,\frac{a_6\,h^4}{24}-\frac{a_5\,h^3}{6}+\frac{a_4\,h^2}{2}-a_3\,h+a_2=b_2,a_0=c_0,a_1=c_1,a_2=c_2\right]$

(%i15)
```
eqns : append( eqns, makelist( ev(dpinteven[i], x=h) = ev(dpp[i], x=0), i,
1, length(dpp)));
```

(%o15) $\left[\frac{a_6\,h^6}{720}-\frac{a_5\,h^5}{120}+\frac{a_4\,h^4}{24}-\frac{a_3\,h^3}{6}+\frac{a_2\,h^2}{2}-a_1\,h+a_0=b_0,\frac{a_6\,h^4}{24}-\frac{a_5\,h^3}{6}+\frac{a_4\,h^2}{2}-a_3\,h+a_2=b_2,a_0=c_0,a_1=c_1,a_2=c_2,\frac{a_6\,h^6}{720}+\frac{a_5\,h^5}{120}+\frac{a_4\,h^4}{24}+\frac{a_3\,h^3}{6}+\frac{a_2\,h^2}{2}+a_1\,h+a_0=d_0,\frac{a_6\,h^4}{24}+\frac{a_5\,h^3}{6}+\frac{a_4\,h^2}{2}+a_3\,h+a_2=d_2\right]$

(%i16)
```
vars : makelist( a[i], i, 0, (polyorder+1)-1 +
2*ceiling((polyorder+1)/2));
```

(%o16) $[a_0,a_1,a_2,a_3,a_4,a_5,a_6]$

(%i17)
```
soln : solve(eqns, vars);
```

(%o17) $\left[\left[a_0=c_0,\ a_1=c_1,\ a_2=c_2,\ a_3=\frac{(3\,b_2-3\,d_2)\,h^2-120\,c_1\,h+60\,d_0-60\,b_0}{14\,h^3},\ a_4=-\frac{(2\,d_2+56\,c_2+2\,b_2)\,h^2-60\,d_0+120\,c_0-60\,b_0}{3\,h^4},\ a_5=-\frac{(30\,b_2-30\,d_2)\,h^2-360\,c_1\,h+180\,d_0-180\,b_0}{7\,h^5},\ a_6=\frac{(20\,d_2+200\,c_2+20\,b_2)\,h^2-240\,d_0+480\,c_0-240\,b_0}{h^6}\right]\right]$

(%i18)
```
avec : transpose(matrix( makelist( rhs(soln[1][i]), i, 1, (polyorder+1) +
2*ceiling((polyorder+1)/2))));
```

(%o18) $$\begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \frac{(3\,b_2-3\,d_2)\,h^2-120\,c_1\,h+60\,d_0-60\,b_0}{14\,h^3} \\ -\frac{(2\,d_2+56\,c_2+2\,b_2)\,h^2-60\,d_0+120\,c_0-60\,b_0}{3\,h^4} \\ -\frac{(30\,b_2-30\,d_2)\,h^2-360\,c_1\,h+180\,d_0-180\,b_0}{7\,h^5} \\ \frac{(20\,d_2+200\,c_2+20\,b_2)\,h^2-240\,d_0+480\,c_0-240\,b_0}{h^6} \end{pmatrix}$$

```
(%i19) xvec : transpose(matrix(makelist( x^n/n!, n, 0, (polyorder+1)-1 +
       2*ceiling((polyorder+1)/2))));
```

(%o19) $$\begin{pmatrix} 1 \\ x \\ \frac{x^2}{2} \\ \frac{x^3}{6} \\ \frac{x^4}{24} \\ \frac{x^5}{120} \\ \frac{x^6}{720} \end{pmatrix}$$

```
(%i20) intpoly : transpose(xvec).avec;
```

(%o20) $$\frac{((20\,d_2+200\,c_2+20\,b_2)\,h^2-240\,d_0+480\,c_0-240\,b_0)\,x^6}{720\,h^6}-\frac{((30\,b_2-30\,d_2)\,h^2-360\,c_1\,h+180\,d_0-180\,b_0)\,x^5}{840\,h^5}-\frac{((2\,d_2+56\,c_2+2\,b_2)\,h^2-60\,d_0+120\,c_0-60\,b_0)\,x^4}{72\,h^4}+\frac{((3\,b_2-3\,d_2)\,h^2-120\,c_1\,h+60\,d_0-60\,b_0)\,x^3}{84\,h^3}+\frac{c_2\,x^2}{2}+c_1\,x+c_0$$

```
(%i21) meanPoly : integrate(intpoly, x, -h, h)/(2*h);
```

(%o21) $$\frac{\frac{(7\,d_2+256\,c_2-23\,b_2)\,h^3-1080\,c_1\,h^2+(-120\,d_0+3840\,c_0+1320\,b_0)\,h}{5040}-\frac{(23\,d_2-256\,c_2-7\,b_2)\,h^3-1080\,c_1\,h^2+(-1320\,d_0-3840\,c_0+120\,b_0)\,h}{5040}}{2\,h}$$

```
(%i22) meanPoly : expand(meanPoly);
```

(%o22) $$-\frac{d_2\,h^2}{630}+\frac{16\,c_2\,h^2}{315}-\frac{b_2\,h^2}{630}+\frac{5\,d_0}{42}+\frac{16\,c_0}{21}+\frac{5\,b_0}{42}$$

```
(%i23) meanPoly : fullratsimp(meanPoly);
```

(%o23) $$-\frac{(d_2-32\,c_2+b_2)\,h^2-75\,d_0-480\,c_0-75\,b_0}{630}$$

```
(%i59) meanCoeffs : makelist(diff(meanPoly, h, i)/i!, i, 0, polyorder);
```

(%o59) $$\left[-\frac{(d_2-32\,c_2+b_2)\,h^2-75\,d_0-480\,c_0-75\,b_0}{630},-\frac{(d_2-32\,c_2+b_2)\,h}{315},-\frac{d_2-32\,c_2+b_2}{630}\right]$$

```
(%i60) meanCoeffs : ev(meanCoeffs, h = 0);
```

(%o60) $$\left[-\frac{-75\,d_0-480\,c_0-75\,b_0}{630},0,-\frac{d_2-32\,c_2+b_2}{630}\right]$$

```
(%i61) meanCoeffs : factor(meanCoeffs);
```

(%o61) $\left[\frac{5\,d_0+32\,c_0+5\,b_0}{42}, 0, -\frac{d_2-32\,c_2+b_2}{630}\right]$

(%i62) `meanCoeffs, expand, numer;`

(%o62) $[0.119047619047619\,d_0+0.7619047619047619\,c_0+0.119047619047619\,b_0, 0, -0.001587301\backslash$
$587301587\,d_2+0.05079365079365079\,c_2-0.001587301587301587\,b_2]$

(%i28) `kill(all);`

(%o0) **done**

(%i1)