What follows is a derivation of the interpolation basis functions for interpolating a function known at 3 equally spaced points, separated by unit distances. The function and the first 3 derivatives of it must be known at these points to be useful. The c-basis functions of x multiply the appropriate derivatives of the function near the center point, the the d-basis functions multiply the derivatives to the right of the center, and the b-basis is identical to the d-basis evaluated at -x (by symmetry).

```
Maxima 5.33.0 http://maxima.sourceforge.net
using Lisp SBCL 1.1.18
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
```
*** My very own personal maxima-init.mac has been loaded. ***
```
STYLE-WARNING: redefining MAXIMA::MAIN-PROMPT in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-STRIPDOLLAR in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-MEXPT in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-CHOOSE in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-INT in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-SUM in DEFUN
STYLE-WARNING: redefining MAXIMA::TEX-LSUM in DEFUN
```

```
(%i1) polyorder : 3;
```

(%o1) 3

```
(%i2) pint(x) := sum( a[i] * x^i / i!, i, 0, 3*(polyorder + 1)-1 );
```

$$\text{(\%o2)}\quad \text{pint}(x) := \text{sum}\left(\frac{a_i\,x^i}{i!}, i, 0, 3\,(\text{polyorder}+1)-1\right)$$

```
(%i3) ders(f) := makelist( diff(f(x), x, i), i, 0, polyorder );
```

$$\text{(\%o3)}\quad \text{ders}(f) := \text{makelist}(\text{diff}(f(x), x, i), i, 0, \text{polyorder})$$

```
(%i4) dpint : ders(pint);
```

$$\text{(\%o4)}\quad \left[\frac{a_{11}\,x^{11}}{39916800}+\frac{a_{10}\,x^{10}}{3628800}+\frac{a_9\,x^9}{362880}+\frac{a_8\,x^8}{40320}+\frac{a_7\,x^7}{5040}+\frac{a_6\,x^6}{720}+\frac{a_5\,x^5}{120}+\frac{a_4\,x^4}{24}+\frac{a_3\,x^3}{6}+\frac{a_2\,x^2}{2}+a_1\,x+a_0,\frac{a_{11}\,x^{10}}{3628800}+\frac{a_{10}\,x^9}{362880}+\frac{a_9\,x^8}{40320}+\frac{a_8\,x^7}{5040}+\frac{a_7\,x^6}{720}+\frac{a_6\,x^5}{120}+\frac{a_5\,x^4}{24}+\frac{a_4\,x^3}{6}+\frac{a_3\,x^2}{2}+a_2\,x+a_1,\frac{a_{11}\,x^9}{362880}+\frac{a_{10}\,x^8}{40320}+\frac{a_9\,x^7}{5040}+\frac{a_8\,x^6}{720}+\frac{a_7\,x^5}{120}+\frac{a_6\,x^4}{24}+\frac{a_5\,x^3}{6}+\frac{a_4\,x^2}{2}+a_3\,x+a_2,\frac{a_{11}\,x^8}{40320}+\frac{a_{10}\,x^7}{5040}+\frac{a_9\,x^6}{720}+\frac{a_8\,x^5}{120}+\frac{a_7\,x^4}{24}+\frac{a_6\,x^3}{6}+\frac{a_5\,x^2}{2}+a_4\,x+a_3\right]$$

```
(%i5) eqns : makelist( ev(dpint[i], x=-h) = b[i-1], i, 1, polyorder+1);
```

$$\text{(\%o5)}\quad \left[-\frac{a_{11}\,h^{11}}{39916800}+\frac{a_{10}\,h^{10}}{3628800}-\frac{a_9\,h^9}{362880}+\frac{a_8\,h^8}{40320}-\frac{a_7\,h^7}{5040}+\frac{a_6\,h^6}{720}-\frac{a_5\,h^5}{120}+\frac{a_4\,h^4}{24}-\frac{a_3\,h^3}{6}+\frac{a_2\,h^2}{2}-a_1\,h+a_0=b_0,\frac{a_{11}\,h^{10}}{3628800}-\frac{a_{10}\,h^9}{362880}+\frac{a_9\,h^8}{40320}-\frac{a_8\,h^7}{5040}+\frac{a_7\,h^6}{720}-\frac{a_6\,h^5}{120}+\frac{a_5\,h^4}{24}-\frac{a_4\,h^3}{6}+\frac{a_3\,h^2}{2}-a_2\,h+a_1=b_1,-\frac{a_{11}\,h^9}{362880}+\frac{a_{10}\,h^8}{40320}-\frac{a_9\,h^7}{5040}+\frac{a_8\,h^6}{720}-\frac{a_7\,h^5}{120}+\frac{a_6\,h^4}{24}-\frac{a_5\,h^3}{6}+\frac{a_4\,h^2}{2}-a_3\,h+a_2=b_2,\frac{a_{11}\,h^8}{40320}-\frac{a_{10}\,h^7}{5040}+\frac{a_9\,h^6}{720}-\frac{a_8\,h^5}{120}+\frac{a_7\,h^4}{24}-\frac{a_6\,h^3}{6}+\frac{a_5\,h^2}{2}-a_4\,h+a_3=b_3\right]$$

```
(%i6) eqns : append( eqns, makelist( ev(dpint[i], x=0) = c[i-1], i, 1,
      polyorder+1));
```

$$\text{(\%o6)}\quad \left[-\frac{a_{11}\,h^{11}}{39916800}+\frac{a_{10}\,h^{10}}{3628800}-\frac{a_9\,h^9}{362880}+\frac{a_8\,h^8}{40320}-\frac{a_7\,h^7}{5040}+\frac{a_6\,h^6}{720}-\frac{a_5\,h^5}{120}+\frac{a_4\,h^4}{24}-\frac{a_3\,h^3}{6}+\frac{a_2\,h^2}{2}-a_1\,h+a_0=b_0,\frac{a_{11}\,h^{10}}{3628800}-\frac{a_{10}\,h^9}{362880}+\frac{a_9\,h^8}{40320}-\frac{a_8\,h^7}{5040}+\frac{a_7\,h^6}{720}-\frac{a_6\,h^5}{120}+\frac{a_5\,h^4}{24}-\frac{a_4\,h^3}{6}+\frac{a_3\,h^2}{2}-\right.$$

$$a_2\,h+a_1=b_1,-\frac{a_{11}\,h^9}{362880}+\frac{a_{10}\,h^8}{40320}-\frac{a_9\,h^7}{5040}+\frac{a_8\,h^6}{720}-\frac{a_7\,h^5}{120}+\frac{a_6\,h^4}{24}-\frac{a_5\,h^3}{6}+\frac{a_4\,h^2}{2}-a_3\,h+a_2=b_2,$$
$$\frac{a_{11}\,h^8}{40320}-\frac{a_{10}\,h^7}{5040}+\frac{a_9\,h^6}{720}-\frac{a_8\,h^5}{120}+\frac{a_7\,h^4}{24}-\frac{a_6\,h^3}{6}+\frac{a_5\,h^2}{2}-a_4\,h+a_3=b_3,a_0=c_0,a_1=c_1,a_2=c_2,a_3=$$
$$c_3\Bigg]$$

```
(%i7) eqns : append( eqns, makelist( ev(dpint[i], x=h) = d[i-1], i, 1,
      polyorder+1));
```

(%o7)
$$\Bigg[-\frac{a_{11}\,h^{11}}{39916800}+\frac{a_{10}\,h^{10}}{3628800}-\frac{a_9\,h^9}{362880}+\frac{a_8\,h^8}{40320}-\frac{a_7\,h^7}{5040}+\frac{a_6\,h^6}{720}-\frac{a_5\,h^5}{120}+\frac{a_4\,h^4}{24}-\frac{a_3\,h^3}{6}+$$
$$\frac{a_2\,h^2}{2}-a_1\,h+a_0=b_0,\frac{a_{11}\,h^{10}}{3628800}-\frac{a_{10}\,h^9}{362880}+\frac{a_9\,h^8}{40320}-\frac{a_8\,h^7}{5040}+\frac{a_7\,h^6}{720}-\frac{a_6\,h^5}{120}+\frac{a_5\,h^4}{24}-\frac{a_4\,h^3}{6}+\frac{a_3\,h^2}{2}-$$
$$a_2\,h+a_1=b_1,-\frac{a_{11}\,h^9}{362880}+\frac{a_{10}\,h^8}{40320}-\frac{a_9\,h^7}{5040}+\frac{a_8\,h^6}{720}-\frac{a_7\,h^5}{120}+\frac{a_6\,h^4}{24}-\frac{a_5\,h^3}{6}+\frac{a_4\,h^2}{2}-a_3\,h+a_2=b_2,$$
$$\frac{a_{11}\,h^8}{40320}-\frac{a_{10}\,h^7}{5040}+\frac{a_9\,h^6}{720}-\frac{a_8\,h^5}{120}+\frac{a_7\,h^4}{24}-\frac{a_6\,h^3}{6}+\frac{a_5\,h^2}{2}-a_4\,h+a_3=b_3,a_0=c_0,a_1=c_1,a_2=c_2,a_3=$$
$$c_3,\frac{a_{11}\,h^{11}}{39916800}+\frac{a_{10}\,h^{10}}{3628800}+\frac{a_9\,h^9}{362880}+\frac{a_8\,h^8}{40320}+\frac{a_7\,h^7}{5040}+\frac{a_6\,h^6}{720}+\frac{a_5\,h^5}{120}+\frac{a_4\,h^4}{24}+\frac{a_3\,h^3}{6}+\frac{a_2\,h^2}{2}+a_1\,h+$$
$$a_0=d_0,\frac{a_{11}\,h^{10}}{3628800}+\frac{a_{10}\,h^9}{362880}+\frac{a_9\,h^8}{40320}+\frac{a_8\,h^7}{5040}+\frac{a_7\,h^6}{720}+\frac{a_6\,h^5}{120}+\frac{a_5\,h^4}{24}+\frac{a_4\,h^3}{6}+\frac{a_3\,h^2}{2}+a_2\,h+a_1=$$
$$d_1,\frac{a_{11}\,h^9}{362880}+\frac{a_{10}\,h^8}{40320}+\frac{a_9\,h^7}{5040}+\frac{a_8\,h^6}{720}+\frac{a_7\,h^5}{120}+\frac{a_6\,h^4}{24}+\frac{a_5\,h^3}{6}+\frac{a_4\,h^2}{2}+a_3\,h+a_2=d_2,\frac{a_{11}\,h^8}{40320}+$$
$$\frac{a_{10}\,h^7}{5040}+\frac{a_9\,h^6}{720}+\frac{a_8\,h^5}{120}+\frac{a_7\,h^4}{24}+\frac{a_6\,h^3}{6}+\frac{a_5\,h^2}{2}+a_4\,h+a_3=d_3\Bigg]$$

```
(%i8) vars : makelist( a[i], i, 0, 3*(polyorder+1)-1 );
```

(%o8) $[a_0,a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11}]$

```
(%i9) soln : solve(eqns, vars);
```

(%o9)
$$\Bigg[\Bigg[a_0=c_0,a_1=c_1,a_2=c_2,a_3=c_3,a_4=$$
$$\frac{(b_3-d_3)\,h^3+(21\,d_2-192\,c_2+21\,b_2)\,h^2+(165\,b_1-165\,d_1)\,h+480\,d_0-960\,c_0+480\,b_0}{4\,h^4},a_5=-$$
$$\frac{(5\,d_3+320\,c_3+5\,b_3)\,h^3+(120\,b_2-120\,d_2)\,h^2+(1065\,d_1+4800\,c_1+1065\,b_1)\,h-3465\,d_0+3465\,b_0}{4\,h^5},$$
$$a_6=-$$
$$\frac{(45\,b_3-45\,d_3)\,h^3+(855\,d_2-4320\,c_2+855\,b_2)\,h^2+(5895\,b_1-5895\,d_1)\,h+14400\,d_0-28800\,c_0+14400\,b_0}{2\,h^6},$$
$$a_7=$$
$$\frac{(315\,d_3+10080\,c_3+315\,b_3)\,h^3+(6930\,b_2-6930\,d_2)\,h^2+(55125\,d_1+201600\,c_1+55125\,b_1)\,h-155925\,d_0+155925\,b_0}{2\,h^7},$$
$$a_8=$$
$$\frac{(1260\,b_3-1260\,d_3)\,h^3+(21420\,d_2-80640\,c_2+21420\,b_2)\,h^2+(132300\,b_1-132300\,d_1)\,h+302400\,d_0-604800\,c_0+302400}{h^8}$$
$$a_9=-$$
$$\frac{(11340\,d_3+241920\,c_3+11340\,b_3)\,h^3+(226800\,b_2-226800\,d_2)\,h^2+(1644300\,d_1+5443200\,c_1+1644300\,b_1)\,h-4365900\,c}{h^9}$$
$$a_{10}=-$$
$$\frac{(37800\,b_3-37800\,d_3)\,h^3+(567000\,d_2-1814400\,c_2+567000\,b_2)\,h^2+(3288600\,b_1-3288600\,d_1)\,h+7257600\,d_0-145152}{h^{10}}$$
$$a_{11}=$$
$$\frac{(415800\,d_3+6652800\,c_3+415800\,b_3)\,h^3+(7484400\,b_2-7484400\,d_2)\,h^2+(51143400\,d_1+159667200\,c_1+51143400\,b_1)\,h-}{h^{11}}$$

```
(%i10) avec : transpose(matrix( makelist( rhs(soln[1][i]), i, 1,
       3*(polyorder+1))));
```

(%o10)

$$\begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ \frac{(b_3-d_3)\,h^3+(21\,d_2-192\,c_2+21\,b_2)\,h^2+(165\,b_1-165\,d_1)\,h+480\,d_0-960\,c_0+480\,b_0}{4\,h^4} \\ -\frac{(5\,d_3+320\,c_3+5\,b_3)\,h^3+(120\,b_2-120\,d_2)\,h^2+(1065\,d_1+4800\,c_1+1065\,b_1)\,h-3465\,d_0+3465\,b_0}{4\,h^5} \\ -\frac{(45\,b_3-45\,d_3)\,h^3+(855\,d_2-4320\,c_2+855\,b_2)\,h^2+(5895\,b_1-5895\,d_1)\,h+14400\,d_0-28800\,c_0+14400\,b_0}{2\,h^6} \\ \frac{(315\,d_3+10080\,c_3+315\,b_3)\,h^3+(6930\,b_2-6930\,d_2)\,h^2+(55125\,d_1+201600\,c_1+55125\,b_1)\,h-155925\,d_0+155925\,b_0}{2\,h^7} \\ \frac{(1260\,b_3-1260\,d_3)\,h^3+(21420\,d_2-80640\,c_2+21420\,b_2)\,h^2+(132300\,b_1-132300\,d_1)\,h+302400\,d_0-604800\,c_0+302400\,b_0}{h^8} \\ -\frac{(11340\,d_3+241920\,c_3+11340\,b_3)\,h^3+(226800\,b_2-226800\,d_2)\,h^2+(1644300\,d_1+5443200\,c_1+1644300\,b_1)\,h-4365900\,d_0+4365900\,b_0}{h^9} \\ -\frac{(37800\,b_3-37800\,d_3)\,h^3+(567000\,d_2-1814400\,c_2+567000\,b_2)\,h^2+(3288600\,b_1-3288600\,d_1)\,h+7257600\,d_0-14515200\,c_0+7257600\,b_0}{h^{10}} \\ \frac{(415800\,d_3+6652800\,c_3+415800\,b_3)\,h^3+(7484400\,b_2-7484400\,d_2)\,h^2+(51143400\,d_1+159667200\,c_1+51143400\,b_1)\,h-130977000\,d_0+1309770}{h^{11}} \end{pmatrix}$$

```
(%i11) xvec : transpose(matrix(makelist( x^n/n!, n, 0, 3*(polyorder+1)-1)));
```

(%o11)

$$\begin{pmatrix} 1 \\ x \\ \frac{x^2}{2} \\ \frac{x^3}{6} \\ \frac{x^4}{24} \\ \frac{x^5}{120} \\ \frac{x^6}{720} \\ \frac{x^7}{5040} \\ \frac{x^8}{40320} \\ \frac{x^9}{362880} \\ \frac{x^{10}}{3628800} \\ \frac{x^{11}}{39916800} \end{pmatrix}$$

```
(%i12) intpoly : transpose(xvec).avec;
```

(%o12)

$$\frac{((415800\,d_3+6652800\,c_3+415800\,b_3)\,h^3+(7484400\,b_2-7484400\,d_2)\,h^2+(51143400\,d_1+159667200\,c_1+51143400\,b_1)\,h}{39916800\,h^{11}}$$
$$\frac{((37800\,b_3-37800\,d_3)\,h^3+(567000\,d_2-1814400\,c_2+567000\,b_2)\,h^2+(3288600\,b_1-3288600\,d_1)\,h+7257600\,d_0-145152}{3628800\,h^{10}}$$
$$\frac{((11340\,d_3+241920\,c_3+11340\,b_3)\,h^3+(226800\,b_2-226800\,d_2)\,h^2+(1644300\,d_1+5443200\,c_1+1644300\,b_1)\,h-4365900}{362880\,h^9}$$
$$\frac{((1260\,b_3-1260\,d_3)\,h^3+(21420\,d_2-80640\,c_2+21420\,b_2)\,h^2+(132300\,b_1-132300\,d_1)\,h+302400\,d_0-604800\,c_0+30240}{40320\,h^8}$$
$$\frac{((315\,d_3+10080\,c_3+315\,b_3)\,h^3+(6930\,b_2-6930\,d_2)\,h^2+(55125\,d_1+201600\,c_1+55125\,b_1)\,h-155925\,d_0+155925\,b_0)\,x^7}{10080\,h^7}$$
$$\frac{((45\,b_3-45\,d_3)\,h^3+(855\,d_2-4320\,c_2+855\,b_2)\,h^2+(5895\,b_1-5895\,d_1)\,h+14400\,d_0-28800\,c_0+14400\,b_0)\,x^6}{1440\,h^6}-$$
$$\frac{((5\,d_3+320\,c_3+5\,b_3)\,h^3+(120\,b_2-120\,d_2)\,h^2+(1065\,d_1+4800\,c_1+1065\,b_1)\,h-3465\,d_0+3465\,b_0)\,x^5}{480\,h^5}+$$
$$\frac{((b_3-d_3)\,h^3+(21\,d_2-192\,c_2+21\,b_2)\,h^2+(165\,b_1-165\,d_1)\,h+480\,d_0-960\,c_0+480\,b_0)\,x^4}{96\,h^4}+$$
$$\frac{c_3\,x^3}{6}+\frac{c_2\,x^2}{2}+c_1\,x+c_0$$

```
(%i13) bbases : makelist( diff(intpoly, b[i]), i, 0, polyorder );
```

(%o13) $\left[\frac{105\,x^{11}}{32\,h^{11}}-\frac{2\,x^{10}}{h^{10}}-\frac{385\,x^9}{32\,h^9}+\frac{15\,x^8}{2\,h^8}+\frac{495\,x^7}{32\,h^7}-\frac{10\,x^6}{h^6}-\frac{231\,x^5}{32\,h^5}+\frac{5\,x^4}{h^4},\frac{41\,x^{11}}{32\,h^{10}}-\frac{29\,x^{10}}{32\,h^9}-\frac{145\,x^9}{32\,h^8}+\frac{105\,x^8}{32\,h^7}+\frac{175\,x^7}{32\,h^6}-\frac{131\,x^6}{32\,h^5}-\frac{71\,x^5}{32\,h^4}+\frac{55\,x^4}{32\,h^3},\frac{3\,x^{11}}{16\,h^9}-\frac{5\,x^{10}}{32\,h^8}-\frac{5\,x^9}{8\,h^7}+\frac{17\,x^8}{32\,h^6}+\frac{11\,x^7}{16\,h^5}-\frac{19\,x^6}{32\,h^4}-\frac{x^5}{4\,h^3}+\frac{7\,x^4}{32\,h^2},\frac{x^{11}}{96\,h^8}-\frac{x^{10}}{96\,h^7}-\frac{x^9}{32\,h^6}+\frac{x^8}{32\,h^5}+\frac{x^7}{32\,h^4}-\frac{x^6}{32\,h^3}-\frac{x^5}{96\,h^2}+\frac{x^4}{96\,h}\right]$

(%i14) cbases : makelist(diff(intpoly, c[i]), i, 0, polyorder);

(%o14) $\left[\frac{4\,x^{10}}{h^{10}}-\frac{15\,x^8}{h^8}+\frac{20\,x^6}{h^6}-\frac{10\,x^4}{h^4}+1,\frac{4\,x^{11}}{h^{10}}-\frac{15\,x^9}{h^8}+\frac{20\,x^7}{h^6}-\frac{10\,x^5}{h^4}+x,\frac{x^{10}}{2\,h^8}-\frac{2\,x^8}{h^6}+\frac{3\,x^6}{h^4}-\frac{2\,x^4}{h^2}+\frac{x^2}{2},\frac{x^{11}}{6\,h^8}-\frac{2\,x^9}{3\,h^6}+\frac{x^7}{h^4}-\frac{2\,x^5}{3\,h^2}+\frac{x^3}{6}\right]$

(%i15) dbases : makelist(diff(intpoly, d[i]), i, 0, polyorder);

(%o15) $\left[-\frac{105\,x^{11}}{32\,h^{11}}-\frac{2\,x^{10}}{h^{10}}+\frac{385\,x^9}{32\,h^9}+\frac{15\,x^8}{2\,h^8}-\frac{495\,x^7}{32\,h^7}-\frac{10\,x^6}{h^6}+\frac{231\,x^5}{32\,h^5}+\frac{5\,x^4}{h^4},\frac{41\,x^{11}}{32\,h^{10}}+\frac{29\,x^{10}}{32\,h^9}-\frac{145\,x^9}{32\,h^8}-\frac{105\,x^8}{32\,h^7}+\frac{175\,x^7}{32\,h^6}+\frac{131\,x^6}{32\,h^5}-\frac{71\,x^5}{32\,h^4}-\frac{55\,x^4}{32\,h^3},-\frac{3\,x^{11}}{16\,h^9}-\frac{5\,x^{10}}{32\,h^8}+\frac{5\,x^9}{8\,h^7}+\frac{17\,x^8}{32\,h^6}-\frac{11\,x^7}{16\,h^5}-\frac{19\,x^6}{32\,h^4}+\frac{x^5}{4\,h^3}+\frac{7\,x^4}{32\,h^2},\frac{x^{11}}{96\,h^8}+\frac{x^{10}}{96\,h^7}-\frac{x^9}{32\,h^6}-\frac{x^8}{32\,h^5}+\frac{x^7}{32\,h^4}+\frac{x^6}{32\,h^3}-\frac{x^5}{96\,h^2}-\frac{x^4}{96\,h}\right]$

(%i21) bbases - ev(dbases, x = -x)*makelist((-1)^i, i, 0, length(dbases)-1);

(%o21) $[0,0,0,0]$

(%i20) makelist((-1)^i, i, 1, length(dbases)-1);

(%o20) $[-1,1,-1]$

(%i18) %, factor;

rat: replaced 0.3102264404296875 by 20331/65536 = 0.3102264404296875

rat: replaced -0.0902252197265625 by -5913/65536 = -0.0902252197265625

rat: replaced 0.0098876953125 by 81/8192 = 0.0098876953125

rat: replaced -   4.119873046875e-4 by -27/65536 = -   4.119873046875e-4

(%o18) $\left[\frac{3^4\,251}{2^{16}},-\frac{3^4\,73}{2^{16}},\frac{3^4}{2^{13}},-\frac{3^3}{2^{16}}\right]$

(%i19) meanPoly : integrate(intpoly, x, -h, h)/(2*h);

(%o19) $-\frac{(205\,d_3-2464\,c_3-51\,b_3)\,h^4+(-5638\,d_2-16384\,c_2-1018\,b_2)\,h^3+(65355\,d_1-147840\,c_1-7605\,b_1)\,h^2+(-374475\,d_0-491520\,c_0-21045\,b_0)\,h}{887040}-$

$2\,h$

(%i20) meanPoly: expand(meanPoly);

(%o20) $-\frac{d_3\,h^3}{6930}+\frac{b_3\,h^3}{6930}+\frac{13\,d_2\,h^2}{3465}+\frac{64\,c_2\,h^2}{3465}+\frac{13\,b_2\,h^2}{3465}-\frac{19\,d_1\,h}{462}+\frac{19\,b_1\,h}{462}+\frac{103\,d_0}{462}+\frac{128\,c_0}{231}+\frac{103\,b_0}{462}$

(%i21) meanPoly;

(%o21) $-\frac{d_3\,h^3}{6930}+\frac{b_3\,h^3}{6930}+\frac{13\,d_2\,h^2}{3465}+\frac{64\,c_2\,h^2}{3465}+\frac{13\,b_2\,h^2}{3465}-\frac{19\,d_1\,h}{462}+\frac{19\,b_1\,h}{462}+\frac{103\,d_0}{462}+\frac{128\,c_0}{231}+\frac{103\,b_0}{462}$

(%i22) meanPoly : fullratsimp(meanPoly);

(%o22) $$-\frac{\left(d_3-b_3\right)h^3+\left(-26\,d_2-128\,c_2-26\,b_2\right)h^2+\left(285\,d_1-285\,b_1\right)h-1545\,d_0-3840\,c_0-1545\,b_0}{6930}$$

```
(%i23) meanCoeffs : makelist(diff(meanPoly, h, i)/i!, i, 0, polyorder);
```

(%o23) $$\left[-\frac{\left(d_3-b_3\right)h^3+\left(-26\,d_2-128\,c_2-26\,b_2\right)h^2+\left(285\,d_1-285\,b_1\right)h-1545\,d_0-3840\,c_0-1545\,b_0}{6930},-\frac{3\left(d_3-b_3\right)h^2+2\left(-26\,d_2-128\,c_2-26\,b_2\right)h+285\,d_1-285\,b_1}{6930},-\frac{6\left(d_3-b_3\right)h+2\left(-26\,d_2-128\,c_2-26\,b_2\right)}{13860},-\frac{d_3-b_3}{6930}\right]$$

```
(%i24) meanCoeffs : ev(meanCoeffs, h = 0);
```

(%o24) $$\left[-\frac{-1545\,d_0-3840\,c_0-1545\,b_0}{6930},-\frac{285\,d_1-285\,b_1}{6930},-\frac{-26\,d_2-128\,c_2-26\,b_2}{6930},-\frac{d_3-b_3}{6930}\right]$$

```
(%i25) meanCoeffs : factor(meanCoeffs);
```

(%o25) $$\left[\frac{103\,d_0+256\,c_0+103\,b_0}{462},-\frac{19\left(d_1-b_1\right)}{462},\frac{13\,d_2+64\,c_2+13\,b_2}{3465},-\frac{d_3-b_3}{6930}\right]$$

```
(%i26) meanCoeffs, expand, numer;
```

(%o26) $[0.2229437229437229\,d_0+0.5541125541125541\,c_0+0.2229437229437229\,b_0, 0.0411255411\backslash$
$2554113\,b_1-0.04112554112554113\,d_1, 0.003751803751803752\,d_2+0.01847041847041847\,c_2+0.00\backslash$
$3751803751803752\,b_2, 1.4430014430014432\times10^{-4}\,b_3-1.4430014430014432\times10^{-4}\,d_3]$

```
(%i27) kill(all);
```

(%o0) **done**

(%i1)