

Solving Differential Equations representing Simple Harmonic Motion

Amritpal Singh

July 31, 2015

Contents

1	Simple Harmonic Motion	1
1.1	Derivation	1
1.2	Numericals	2

Chapter 1

Simple Harmonic Motion

1.1 Derivation

Consider m be the mass of object, k be spring constant, x be a displacement from equilibrium state of a spring and t is time.

Therefore, the given differential equation of Simple Harmonic Motion is

$$\frac{d^2x}{dt^2} + \frac{kx}{m} = 0.$$

Solving the above differential equation, we get,

$$x = K_2 \cos\left(\frac{\sqrt{k}t}{\sqrt{m}}\right) + K_1 \sin\left(\frac{\sqrt{k}t}{\sqrt{m}}\right)$$

When $m = 1$, $k = 1$, then the graph is

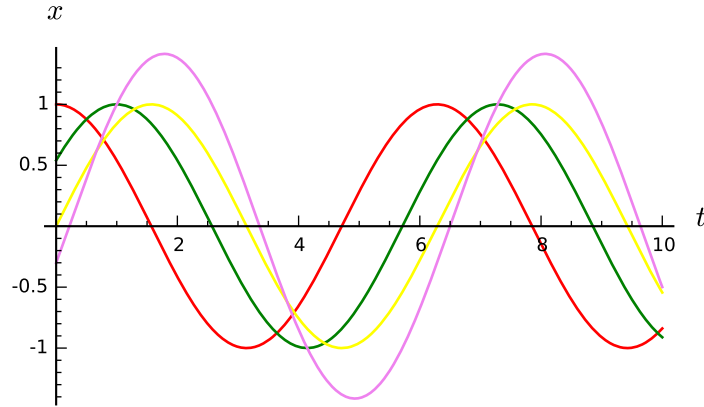


Figure 1.1: Graph

here red, green, yellow and violet curves are drawn when the initial or boundary conditions are $[0,1,0]$, $[1,1,0]$, $[0,0,1]$ and $[1,1,1]$ respectively.

The initial or boundary conditions means for a second-order differential equation, specify the initial x , y , and $\frac{dx}{dt}$, i.e. write $[t, x(t), \frac{dx}{dt}]$

1.2 Numericals

Example:

A spring at rest is suspended from the ceiling without mass. A 2 kg weight is then attached to this spring, stretching it 9.8 cm. From a position $2/3$ m above equilibrium the weight is given a downward velocity of 5 m/s.

(a) Find the equation of motion.

(b) What is the amplitude?

(c) At what times does the mass first reach equilibrium?

Sol: Note $m = 2\text{ kg}$, $x = 9.8\text{ cm}$, and $k = mg/x = 200.0$. Therefore, the general solution $x = K_2 \cos(10t) + K_1 \sin(10t)$. Then by computing the above equation from the initial conditions $x(0) = -2/3$ (down is positive, up is negative), $x'(0) = 5$ we get,

$$x = -\frac{2}{3} \cos(10t) + \frac{1}{2} \sin(10t)$$

Now we write this in the more compact and useful form

$$x = A \sin(\omega t + \phi) = K_2 \cos(\omega t) + K_1 \sin(\omega t)$$

where $A = \sqrt{K_1^2 + K_2^2}$ denotes the *amplitude*

$$A = \frac{5}{6}$$

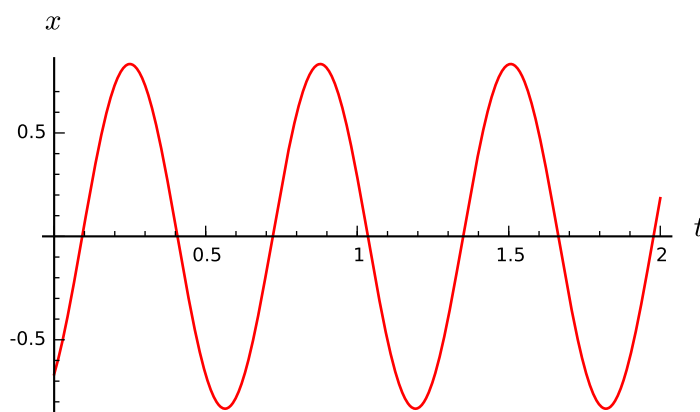


Figure 1.2: Graph