Solving Differential Equations representing Simple Harmonic Motion

Amritpal Singh

July 31, 2015

Contents

Chapter 1

Simple Harmonic Motion

1.1 Derivation

Consider m be the mass of object, k be spring constant, x be a displacement from equilibrium state of a spring and t is time.
Therefore, the given differential equation of Simple Harmonic Motion is

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \frac{kx(t)}{m} = 0.$$

Solving the above differential equation, we get,

$$x = K_2 \cos\left(\frac{\sqrt{k}t}{\sqrt{m}}\right) + K_1 \sin\left(\frac{\sqrt{k}t}{\sqrt{m}}\right)$$

When $m = 1$, $k = 1$, then the graph is

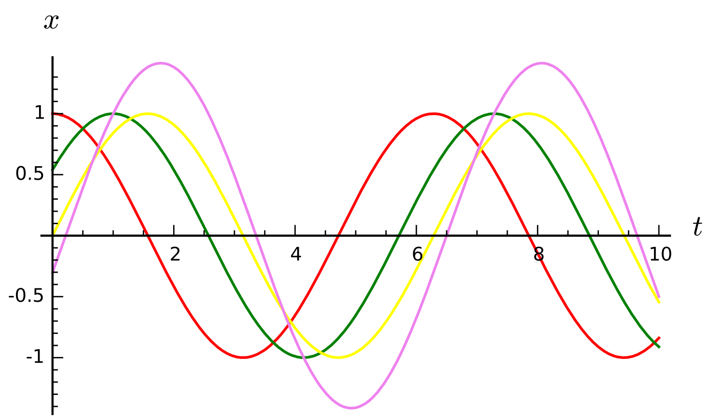

Figure 1.1: Graph

here red, green, yellow and violet curves are drawn when the initial or boundary conditions are [0,1,0], [1,1,0], [0,0,1] and [1,1,1] respectively.
The initial or boundary conditions means for a second-order differential equation, specify the initial x, y, and $\frac{\mathrm{d}x}{\mathrm{d}t}$, i.e. write $[t,\, x(t),\, \frac{\mathrm{d}x}{\mathrm{d}t}]$

1.2 Numericals

Example:
A spring at rest is suspened from the ceiling wthout mass. A 2 kg weight is then attached to this spring, stretching it 9.8 cm. From a position 2/3 m above equilibrium the weigth is give a downward velocity of 5 m/s.
(a) Find the equation of motion.
(b) What is the amplitude?
(c) At what times the mass first equilibrium?

Sol: Note $m = 2kg$, $x = 9.8cm$, and $k = mg/x = 200.0$. Therefore, the general solution $x = K_2 \cos(10\,t) + K_1 \sin(10\,t)$. Then by computing the above equation from the initial conditions $x(0) = -2/3$ (down is positive, up is negative), $x'(0) = 5$ we get,

$$x = -\frac{2}{3}\cos(10\,t) + \frac{1}{2}\sin(10\,t)$$

Now we write this in the more compact and useful form

$$x = A\sin(\omega t + \phi) = K_2\cos(\omega t) + K_1\sin(\omega t)$$

where $A = \sqrt{K_1^2 + K_2^2}$ denotes the *amplitude*

$$A = \frac{5}{6}$$

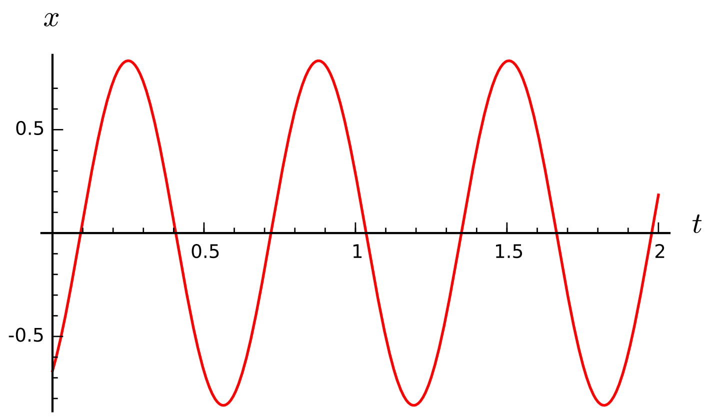

Figure 1.2: Graph