Solving Differential Equations representing Simple Harmonic Motion

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Consider m be the mass of object, k be spring constant, x be a displacement from equilibrium state of a spring and t is time.

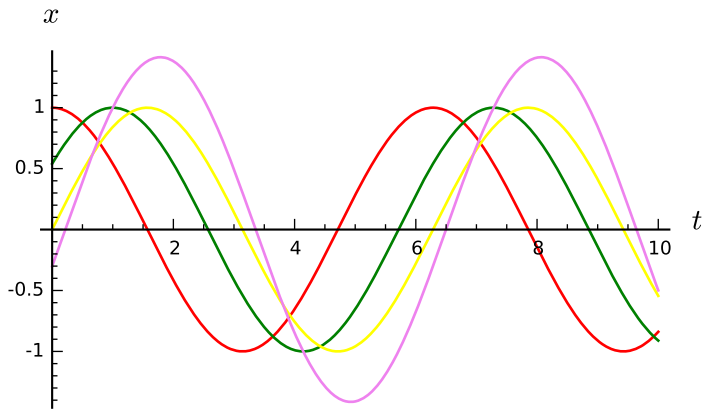
Therefore, the given differential equation of Simple Harmonic Motion is

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{kx\,(t)}{m} = 0.$$

Solving the above differential equation, we get,

$$x = K_2 \cos\left(\frac{\sqrt{k}t}{\sqrt{m}}\right) + K_1 \sin\left(\frac{\sqrt{k}t}{\sqrt{m}}\right)$$

When $m = 1$, $k = 1$, then the graph is



where red, green, yellow and violet curves are drawn when the initial or boundary conditions are [0,1,0], [1,1,0], [0,0,1] and [1,1,1] respectively.

The initial or boundary conditions means for a second-order equation, specify the initial x, y, and $\frac{\mathrm{d}x}{\mathrm{d}t}$, i.e. write [$t$, $x(t)$, $\frac{\mathrm{d}x}{\mathrm{d}t}$]