Solving Differential Equations representing Simple Harmonic Motion

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Chapter 1

Simple Harmonic Motion

1.1 Derivation

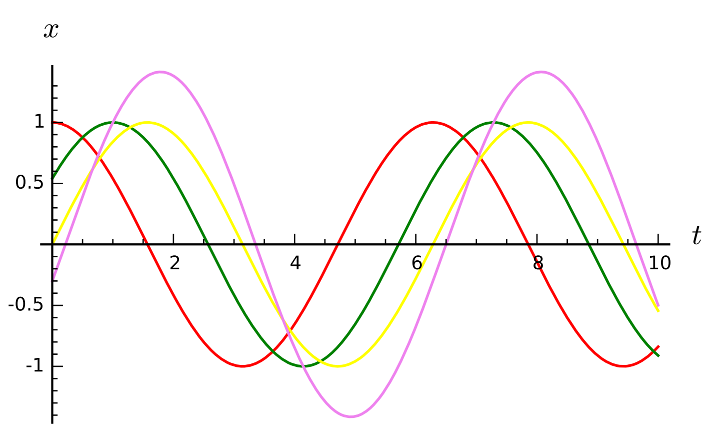
Consider m be the mass of object, k be spring constant, x be a displacement from equilibrium state of a spring and t is time.
Therefore, the given differential equation of Simple Harmonic Motion is

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{kx(t)}{m} = 0.$$

Solving the above differential equation, we get,

$$x = K_2 \cos\left(\frac{\sqrt{k}t}{\sqrt{m}}\right) + K_1 \sin\left(\frac{\sqrt{k}t}{\sqrt{m}}\right)$$

When $m = 1$, $k = 1$, then the graph is



where red, green, yellow and violet curves are drawn when the initial or boundary conditions are [0,1,0], [1,1,0], [0,0,1] and [1,1,1] respectively.

The initial or boundary conditions means for a second-order equation, specify the initial x, y, and $\frac{\mathrm{d}x}{\mathrm{d}t}$, i.e. write $[t,\ x(t),\ \frac{\mathrm{d}x}{\mathrm{d}t}]$

1.2 Numericals

Example:
A spring at rest is suspened from the ceiling wthout mass. A 2 kg weight is then attached to this spring, stretching it 9.8 cm. From a position 2/3 m above equilibrium the weigth is give a downward velocity of 5 m/s.
(a) Find the equation of motion.
(b) What is the amplitude and period of motion?
(c) At what times the mass first equilibrium?
(d) At what time is the mass first exactly 1/2 m below equilibrium?

Sol: Note $m = 2kg$, $x = 9.8cm$, and $k = mg/x = 200.0$. Therefore, the general solution $x = K_2 \cos(10\,t) + K_1 \sin(10\,t)$. Then by computing the above equation from the initial conditions $x(0) = -2/3$ (down is positive, up is negative), $x'(0) = 5$ we get,

$$-\frac{2}{3}\cos(10\,t) + \frac{1}{2}\sin(10\,t)$$

Now we write this in the more compact and useful form

$$x = A\sin(\omega t + \phi) = K_2\cos(\omega t) + K_1\sin(\omega t)$$

where $A = \sqrt{K_1^2 + K_2^2}$ denotes the *amplitude*

$$A = \frac{5}{6}$$