Lab Nr. 4, Probability and Statistics

Random number generators; RND;
Computer simulations of discrete random variables
Numerical characteristics of random variables

1. Function **rnd** in Statistics Toolbox; special functions **rand** and **randn**.

2. Using a $\mathcal{U}(0,1)$ (standard Uniform) random number generator, generate the common discrete probability distributions:

a. Bernoulli Distribution $Bern(p)$, with parameter $p \in (0,1)$: $X \begin{pmatrix} 0 & 1 \\ 1-p & p \end{pmatrix}$;

b. Binomial Distribution $Bino(p)$, with parameters $n \in \mathbb{N}, p \in (0,1)$: $X \begin{pmatrix} k \\ C_n^k p^k q^{n-k} \end{pmatrix}_{k=\overline{0,n}}$;

Hint: A Binomial $Bino(n,p)$ variable is the sum of n independent $Bern(p)$ variables;

c. Geometric Distribution $Geo(p)$, with parameter $p \in (0,1)$: $X \begin{pmatrix} k \\ pq^k \end{pmatrix}_{k\in\mathbb{N}}$;

Hint: A Geometric $Geo(p)$ variable represents the number of failures (i.e. the number of Bernoulli trials that ended up being failures) needed to get the first success;

d. Pascal Distribution $NB(n,p)$ with parameters $n \in \mathbb{N}, p \in (0,1)$: $X \begin{pmatrix} k \\ C_{n+k-1}^k p^n q^k \end{pmatrix}_{k\in\mathbb{N}}$;

Hint: A Pascal $NB(n,p)$ variable is the sum of n independent $Geo(p)$ variables;

3. Numerical characteristics of random Variables: in *Statistics Toolbox* **stat**

The means and variances of the following distributions (fill in the table):

Distribution	**Notation**	**Mean** $E(X)$	**Variance** $V(X)$
Discrete Uniform	$U(m)$	(m-1)/2	(m^2 - 1) / 2
Binomial	$B(n,p)$	n*p	n * p * (1 - p)
Hypergeometric	$H(N,n_1,n)$	n1 * n / N	n1*n*(N-n)*(N-n1)/(N^2*(N-1)
Poisson	$P(\lambda)$	λ	λ
Pascal (Neg. Bin.)	$NB(n,p)$	n*(1-p)/p^2	n*(1-p)/p^2
Geometric	$G(p)$	1/p	(1-p)/p^2
Uniform	$U(a,b)$	(a+b)/2	(b-a)^2/12
Normal	$N(\mu,\sigma)$	μ	σ
Gamma	$Ga(a,b)$	a*b	a*b^2
Exponential	$Exp(\lambda)$	1/λ	1/λ^2
Beta	$\beta(a,b)$	a/(a+b)	ab/((a+b)^2(a+b+1))
Student	$T(n)$	0	n/(n-2)
Chi squared	$\chi^2(n)$	n	2n
Fisher	$F(m,n)$	n/(n-2)	2n^2(m+n-2)/(m*(n-2)^2(n - 4)